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Analysis of a two-echelon inventory system with two supply modes

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Abstract

In this paper, we consider a serial two-echelon periodic review inventory system with two supply modes at the most upstream stock point. As control policy for this system, we propose a natural extension of the dual-index policy, which has three base-stock levels. We consider the minimization of long run average inventory holding, backlogging, and both per unit and fixed emergency ordering costs. We provide nested newsboy characterizations for two of the three base-stock levels involved and show a separability result for the difference with the remaining base-stock level. We use results for the single-echelon system to efficiently approximate the distributions of random variables involved in the newsboy equations and find an asymptotically correct approximation for both the per unit and fixed emergency ordering costs. Based on these results, we provide an algorithm for setting base-stock levels in a computationally efficient manner. In a numerical study, we investigate the value of dual-sourcing in supply chains and show that it is useful to decrease upstream stock levels. In cases with high demand uncertainty, high backlogging cost or long lead times, we conclude that dual-sourcing can lead to significant savings.

Keywords: inventory, dual-sourcing, dual-index policy, Markov chain, lead times, multi-echelon

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1. Introduction

Modern supply networks are complex and often consist of many manufacturing facilities and inventory locations spread over the continents. In recent decades many European companies have switched production to Asia and have built new production plants there, adding to the globalization of supply chains. However, in order to stay competitive and be flexible, they maintain the possibility of manufacturing in Europe as well, albeit at a higher price. As a consequence, inventory managers have two different options for inventory replenishments, differing mainly in costs and lead times. With the growing complexity of supply chains, situations with one buyer and several supply options have become increasingly common. Nevertheless, quantitative modeling approaches to analyze these supply networks are limited. Although there is a large body of literature on inventory management in supply chains, most authors consider single vendor/single buyer relationships or single vendor/multiple buyer relationships. Furthermore, most research on multiple supplier inventory systems is restricted to a single inventory location.

In this article, we extend the existing literature and present a serial two-echelon model, where the most upstream stockpoint has two supply options: an expensive one with a short lead time as well as a cheaper one with a longer lead time. The first stockpoint is supplied by the second via a single mode. The aim is to determine the replenishment quantities based on the inventory status to minimize the operational costs of the system. We assume that the inventory is reviewed periodically, and both stockpoints apply the same review period. Since no restrictive assumptions for the lead times are made, such as a one period lead time difference for the most upstream stockpoint, we cannot expect the optimal control policy to have a simple structure. Not even for the single-echelon case can the optimal policy structure be obtained or computed for general lead time differences (Fukuda, 1964; Whittmore and Saunders, 1977; Feng et al., 2006a,b). Therefore, we restrict ourselves to a class of policies which is optimal in case of single supply modes and has been shown to work reasonably well in case of dual sourcing. Specifically, we consider base-stock policies for both stockpoints combined with a dual-index policy at the most upstream stockpoint, and we show how to compute near optimal base-stock levels in an efficient manner.

Our research is related to two streams of literature. In the first stream of literature, dual-sourcing for single-echelon models is studied. Since the excellent review of Minner (2003), much research has been done to generate new knowledge and results in this area. Because the optimal replenishment policy is complex in case of two or more suppliers, most of the papers present heuristic policies and the computation of good or optimal policy parameters.

The so-called constant order policy, where each period a constant amount is ordered at the regular supplier, is studied in Rosenshine and Obee (1976), Chiang (2007), Janssen and de Kok (1999), and Allon and Van Mieghem (2010). Although the constant order policy performs well when the regular lead time is long (Klosterhalfen et al., 2010), the dual-index policy (DIP) performs well in general (Veeraraghavan and Scheller-Wolf, 2008). The DIP tracks two inventory positions: a regular inventory position (on-hand stock + all outstanding orders-backlog) and an emergency inventory position (on-hand stock + outstanding orders that will arrive within the emergency lead time - backlog). In each period, ordering decisions are made to raise both inventory positions to their order-up-to levels. A fast algorithm to compute near optimal policy parameters for the dual-index policy can be found in Arts et al. (2010). Extensions of the dual-index policy are investigated in Sheopuri et al. (2010). In addition to periodic review policies, models in continuous time with two or more suppliers are studied in Song and Zipkin (2009) and Plambeck and Ward (2007).

The second stream of literature related to our work is devoted to serial multi-echelon systems where there is only one way of replenishing each stockpoint. Since the seminal work of Clark and Scarf (1960), many contributions have been added to this stream of literature. While some researchers have derived bounds (e.g. Chen and Zheng (1994), Shang and Song (2003), and Chao and Zhou (2007)), others concentrate on computational efficiency (e.g. Gallego and Özer (2006)). For an extensive discussion of the existing literature and important results in this field, we also refer to Axsäter (2003) and van Houtum (2006).

Both research streams are merged in the new field of serial multi-echelon inventory systems with multiple supply modes. To the best of our knowledge, there are only a few contributions in this field. The first extension of the classical Clark and Scarf model is presented in Lawson and Porteus (2000). They allow for two different transportation modes between the stockpoints where the emergency delivery mode has lead time zero and the regular mode a lead time of one period. For such a system they are able to characterize the optimal policy under linear holding and backorder costs. Muharrenoglu and Tsitsiklis (2003) also allow for supermodular shipping costs and derive the optimal policy. The optimal policy under physical storage constraints is derived in Xu (2009). In a more recent paper by Zhou and Chao (2010), the case of arbitrary regular lead times and a one period shorter emergency lead time is studied. They also provide bounds and heuristics based on newsvendor equations. Although their model is a clear extension of the model of Lawson and Porteus, an environment where a product can either be shipped in three weeks over sea or in one day by plane is not included in the modeling approach. Therefore, there is a clear need for models with more general lead time assumptions.

The main contribution of this paper to the literature is as follows. We provide a two-echelon inventory system with two supply options for the most upstream stockpoint and, in contrast to the papers discussed above, we allow for general lead time difference between regular and emergency supply. We show how to compute near optimal policy parameters in an efficient manner, and we quantify the added value of the emergency supply mode. In a numerical study we investigate when an emergency supply source is most beneficial.

The remainder of the paper is organized as follows. In Section 2 we present the model, which is analyzed in Section 3. In Section 4 numerical results are presented, and the benefit of the second supply source is analyzed. The paper concludes with a summary and directions for future research in Section 5.

2. Model

We consider a two-echelon serial supply chain that faces stochastic demand for a single stock keeping unit (SKU). The most upstream stockpoint (stockpoint 2) has a regular and an emergency supply mode. The system is periodic and within a period the order of events is as follows for each stockpoint: (1) holding and backlogging costs are incurred, (2) orders are placed and received, and (3) demand is realized and filled or backlogged. The price and lead time for a SKU, ordered via the regular (emergency) supply mode, are c_r (c_e) [\$/SKU] and $l^{2,r}$ ($l^{2,e}$) [periods], $c_e > c_r$, $l^{2,r} > l^{2,e}$. For convenience we also define $\ell := l^{2,r} - l^{2,e} \geq 1$. Additionally, there is a fixed set-up cost k [\$/emergency order] for each order placed through the emergency supply mode. The lead time from stockpoint 2 to stockpoint 1 is l^1. Demand per review period is a sequence of non-negative i.i.d. discrete random variables $\{D_t\}_{t=0}^{\infty}$, where t is a period index. We will need the regularity condition $\mathbb{P}(D > 0) > 0$, where D is the generic single period demand random variable. Also for notational convenience, we let $D_{t_1,t_2} = \sum_{t=t_1}^{t_2} D_t$. The regular (emergency) order placed by stockpoint 2 in period t are denoted $Q_t^{r,2}$ ($Q_t^{e,2}$), and the order placed by stockpoint 1 is Q_t^1. We let I_t^1 denote the physical stock minus backorders at stockpoint 1 at the beginning of period t while I_t^2 denotes the physical stock at stockpoint 2 at the beginning of period t. We also define echelon 1 and 2 inventory levels at the beginning of a period as

$$IL_t^1 = I_t^1 \tag{1}$$

$$IL_t^2 = IL_t^1 + \sum_{n=t-l^1}^{t-1} Q_n^1 + I_t^2 \tag{2}$$

Each period a holding cost of h_2 [\$/unit] is charged to all items in stockpoint 2 and downstream therefrom, $h_2(IL_t^2 + (IL_t^1)^-)$. For units in stockpoint 1, an additional charge of h_1 [\$/unit] is applied, $h_1(IL_t^1)^+$. If backorders exist in stockpoint 1 at the beginning of a period, a penalty cost p [\$/unit] is charged for each unit in backorder which amounts to $p(IL_t^1)^-$. Here we use the standard notations $x^+ = \max(0, x)$ and $x^- = \max(0, -x)$. By using the fact that $x = x^+ - x^-$, we can write the total incurred holding and penalty costs in a period t as

$$h_2(IL_t^2 + (IL_t^1)^-) + h_1(IL_t^1)^+ + p(IL_t^1)^- = h_2 IL_t^2 + h_1 IL_t^1 + (p + h_1 + h_2)(IL_t^1)^-. \quad (3)$$

For the emergency ordering costs, we observe that any reasonable policy (including the policy we will consider) will order $\mathbb{E}(D)$ per period on average to stockpoint 2. Thus, we have that the average purchasing cost per period equals $c_r\mathbb{E}(D) + (c_e - c_r)\mathbb{E}(Q^{2,e})$, where we dropped the time index to indicate steady state. Since $c_r\mathbb{E}(D)$ is a fixed cost term regardless of policy operation, we omit it from the cost function. If we define $c = c_e - c_r$, then the relevant variable ordering costs in a period t are $cQ_t^{2,e}$. Also, for notational convenience we allow $Q_t^{2,e} = 0$ but only account for the fixed emergency ordering cost k whenever $Q_t^{2,e} > 0$. Thus, in a period t the total relevant ordering costs are given by

$$k\mathbb{I}(Q_t^{2,e} > 0) + cQ_t^{2,e} \quad (4)$$

where $\mathbb{I}(\xi)$ is the indicator function of the event ξ.

The sequence of events in a period t can be summarized as follows:

1. Inventory holding and backlogging costs are incurred according to equation (3).
2. Stockpoint 2 places orders $Q_t^{2,e}$ and $Q_t^{2,r}$ and incurs ordering costs according to equation (4).
3. Stockpoint 2 receives orders $Q_{t-l^{2,e}}^{2,e}$ and $Q_{t-l^{2,r}}^{2,r}$.
4. Stockpoint 1 places order Q_t^1, which is constrained by the on hand inventory in stock point 2: $I_t^2 + Q_{t-l^{2,e}}^{2,e} + Q_{t-l^{2,r}}^{2,r}$.
5. Stockpoint 1 receives order $Q_{t-l^1}^1$.
6. Demand at stockpoint 1 occurs and is satisfied except for possible backorders.

Now we describe the operation of the two-echelon dual-index policy. This policy uses three inventory positions in its operation, which are defined at the beginning of a period right before the corresponding orders are placed. The first inventory position is simply the echelon 1 inventory position IP^1:

$$IP_t^1 = I_t^1 + \sum_{n=t-l^1}^{t-1} Q_n^1 \tag{5}$$

For echelon 2, we distinguish the emergency ($IP^{2,e}$) and the regular ($IP^{2,r}$) echelon inventory positions:

$$IP_t^{2,e} = IP_t^1 + I_t^2 + \sum_{n=t-l^{2,e}}^{t-1} Q_n^{2,e} + \sum_{n=t-l^{2,r}}^{t-\ell} Q_n^{2,r} \tag{6}$$

$$IP^{2,r} = IP_t^1 + I_t^2 + \sum_{n=t-l^{2,e}}^{t} Q_n^{2,e} + \sum_{n=t-l^{2,r}}^{t-1} Q_n^{2,r} = IP_t^{2,e} + \sum_{n=t-\ell+1}^{t-1} Q_n^{2,r} + Q_t^{2,e} \tag{7}$$

Notice that the last term in $IP_t^{2,e}$ includes only regular orders that will arrive to stockpoint 2 within the emergency lead time, whereas $IP_t^{2,r}$ includes all outstanding regular orders. Moreover, $IP_t^{2,r}$ *includes* the emergency order placed in period t, $Q_t^{2,e}$. A graphical representation of these inventory positions in period t and the rest of the model and its notations is given in Figure 1. Orders that are placed in period t have a dashed line indicating the place where they will be in the pipeline upon placement.

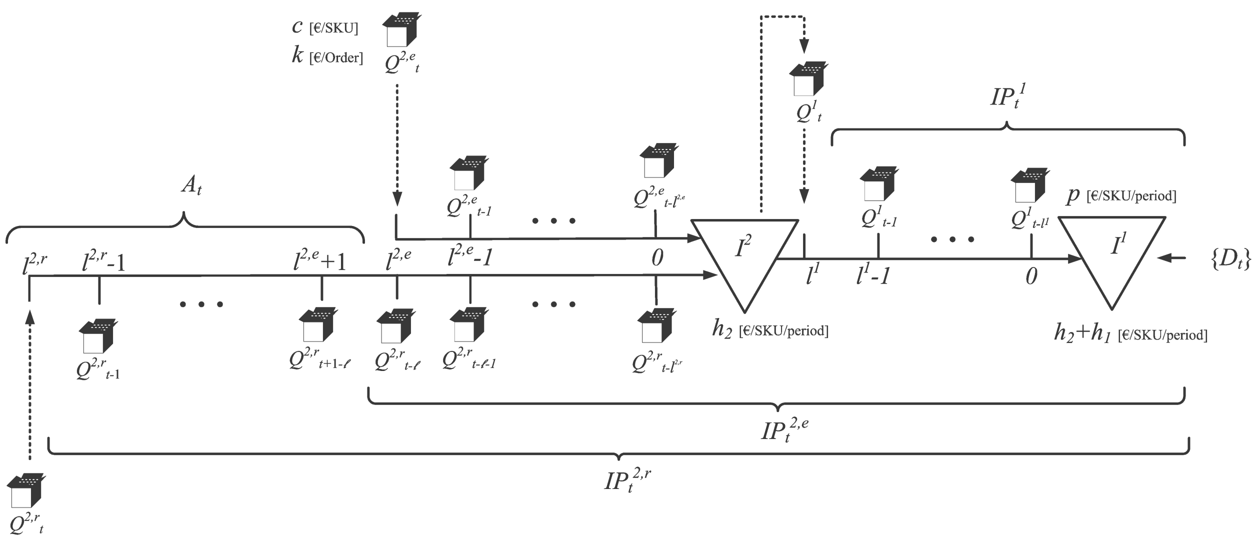

Figure 1: *Graphical representation of the model at the beginning of period t*

The ordering decisions in period t are given by comparing the inventory positions with their order-up-to-levels $S^{2,r}$, $S^{2,e}$ and S^1. For convenience we shall assume the regularity condition that $IP_t^1 \leq S^1$ and $IP_t^{2,r} \leq S^{2,r}$ for all $t \geq 0$. This can be assumed without loss of generality because if this assumption is violated, the number of periods where this assumption is violated is finite with probability 1 if we do not order until these assumptions are met. Consequently we can always renumber periods such that this assumption holds.

$$
\begin{aligned}
Q_t^1 &= \min(S^1 - IP_t^1, I_t^2 + Q_{t-l^{2,e}}^{2,e} + Q_{t-l^{2,r}}^{2,r}) &(8)\\
Q_t^{2,e} &= (S^{2,e} - IP_t^{2,e})^+ &(9)\\
Q_t^{2,r} &= S^{2,r} - IP^{2,r} = S^{2,r} - (IP_t^{2,e} + Q_t^{2,e} + \textstyle\sum_{n=t+1-\ell}^{t_1} Q_n^{2,r}). &(10)
\end{aligned}
$$

Notice that $Q_t^{2,e}$ is not simply the difference between the inventory position and its order-up-to-level $S^{2,e}$. The reason is that $IP_t^{2,e}$ is usually not below its order-up-to-level $S^{2,e}$. In fact, it is usually larger and the excess is called the overshoot O_t defined as

$$O_t = (IP_t^{2,e} - S^{2,e})^+ = IP_t^{2,e} + Q_t^{2,e} - S^{2,e}. \tag{11}$$

As with the single stage DIP, the evaluation of the stationary distribution of O_t plays a crucial role in the evaluation and optimization of the echelon DIP.

Now we can state the average cost function $C(S^1, S^{2,e}, S^{2,r})$ as

$$
\begin{aligned}
C(S^1, S^{2,e}, S^{2,r}) &= c\mathbb{E}\left(Q^{2,e}\right) + k\mathbb{P}\left(Q^{2,e} > 0\right) + \\
&\quad h_2\mathbb{E}\left(IL^2\right) + h_1\mathbb{E}\left(IL^1\right) + (p + h_1 + h_2)\mathbb{E}\left[\left(IL^1\right)^-\right]. &(12)
\end{aligned}
$$

3. Analysis

The analysis will proceed along the following lines. In section 3.1 we show that when $\Delta := S^{2,r} - S^{2,e}$ is fixed, the stationary distributions of O, $Q^{2,r}$ and $Q^{2,e}$ are fixed. This result allows us to decompose the problem and provide newsvendor characterizations for optimal S^1 and $S^{2,e}$ for fixed Δ. In section 3.2 we show how the distribution of O and $\mathbb{P}(Q^{2,e} > 0)$ can be accurately approximated.

3.1 Optimization

Let us define $\Delta = S^{2,r} - S^{2,e}$. With this definition, a dual-index policy is fully defined by Δ, $S^{2,e}$ and S^1 as $(S^1, S^{2,e}, S^{2,r}) = (S^1, S^{2,e}, S^{2,e} + \Delta)$. Furthermore, let us consider the outstanding regular orders that will not arrive to stockpoint 2 within the emergency lead time in period t just *after* all orders have been placed, A_t:

$$A_t = \textstyle\sum_{n=t+1-\ell}^{t} Q_n^{2,r} \tag{13}$$

With these preliminaries we have the following result.

Lemma 3.1. (Separability result) *The following statements hold:*

(i) Suppose $t \geq 0$. Then the following relations hold:

$$\Delta = O_t + A_t \tag{14}$$

$$O_{t+1} = (O_t - D_t + Q^{2,r}_{t+1-\ell})^+ \tag{15}$$

$$Q^{2,e}_{t+1} = (D_t - O_t - Q^{2,r}_{t+1-\ell})^+ \tag{16}$$

$$Q^{2,r}_{t+1} = D_t - Q^{2,e}_{t+1}. \tag{17}$$

(ii) The stationary distribution of O, $Q^{2,e}$ and $Q^{2,r}$ depend on $S^{2,e}$ and $S^{2,r}$ only through their difference $\Delta = S^{2,r} - S^{2,e}$.

Proof. For part (i), recall the regular echelon 2 inventory position as given in equation (7). Adding $Q^{2,r}_t$ to both sides of this equation and substituting equation (11) yields:

$$IP^{2,r}_t + Q^{2,r}_t = S^{2,e} + O_t + \textstyle\sum_{n=t+1-\ell}^{t} Q^{2,r}_n.$$

Now by supposition $t \geq 0$ and so $IP^{2,r} \leq S^{2,r}$. Consequently $Q^{2,r}_t = S^{2,r} - IP^{2,r}_t$ and with the definition of A_t in (13) we obtain:

$$S^{2,r} = S^{2,e} + O_t + A_t.$$

Rearrangement and substitution of $\Delta = S^{2,r} - S^{2,e}$ proves (14).

For the proof of equations (15)-(17), we rewrite the emergency echelon 2 inventory position:

$$\begin{aligned} IP^{2,e}_{t+1} &= IP^{2,e}_t + Q^{2,e}_t - D_t + Q^{2,r}_{t+1-\ell} \\ &= S^{2,e} + O_t - D_t + Q^{2,r}_{t+1-\ell}. \end{aligned} \tag{18}$$

The second equality follows from the definition of the overshoot (11). Now by rewriting we have:

$$\begin{aligned} O_{t+1} &= (IP^{2,e}_{t+1} - S^{2,e})^+ \\ &= (S^{2,e} + O_t - D_t + Q^{2,r}_{t+1-\ell} - S^{2,e})^+ \\ &= (O_t - D_t + Q^{2,r}_{t+1-\ell})^+. \end{aligned} \tag{19}$$

For the echelon 2 emergency order quantity, we can write similarly:

$$\begin{aligned} Q^{2,e}_{t+1} &= (S^{2,e} - IP^{2,e}_{t+1})^+ \\ &= (D_t - O_t - Q^{2,r}_{t+1-\ell})^+. \end{aligned} \tag{20}$$

Lastly, $Q^{2,r}_{t+1} = D_t - Q^{2,e}_{t+1}$ follows immediately from the fact that the dual-index policy ensures that every period the total amount ordered to stock point 2 equals demand from the previous period.

To prove part (ii), we substitute equation (14) into equations (15) to (17) to find

$$O_{t+1} = (\Delta - D_t - \textstyle\sum_{n=t-\ell+2}^{t} Q^{2,r}_n)^+ \tag{21}$$

$$Q^{2,e}_{t+1} = (D_t + \textstyle\sum_{n=t-\ell+2}^{t} Q^{2,r}_n - \Delta)^+ \tag{22}$$

$$Q^{2,r}_{t+1} = D_t - Q^{2,e}_{t+1}. \tag{23}$$

From these equations, we see that the stochastic processes $\{O_t\}$, $\{Q^{2,e}_t\}$, and $\{Q^{2,r}_t\}$ can be described completely using $S^{2,r}$ and $S^{2,e}$ only through their difference Δ. Consequently, their stationary distributions can depend only on Δ. □

From this Lemma, we immediately have that the cost terms $c\mathbb{E}(Q^{2,e}) + k\mathbb{P}(Q^{2,e} > 0)$ can be fixed, by fixing Δ. Thus, for fixed Δ, we only need to optimize the holding and penalty costs $h_2\mathbb{E}\left(IL^2\right) + h_1\mathbb{E}\left(IL^1\right) + (p + h_1 + h_2)\mathbb{E}\left[(IL^1)^-\right]$. We will now investigate how to evaluate these costs for fixed Δ. We will do this by analyzing replenishment cycles.

Let $C_{t,2} = h_2 IL^2_t$ denote the holding costs associated with echelon 2 that are incurred in period t. Suppose we start in some period $t_0 \geq 0$ and order according to the echelon dual-index policy, we obtain:

$$\begin{aligned} \mathbb{E}\left[C_{t_0+l^{2,e}+1,2} | IP^{2,e}_{t_0} + Q^{2,e}_{t_0} = S^{2,e} + O\right] &= \mathbb{E}\left[h_2(S^{2,e} + O - D_{t_0,t_0+l^{2,e}})\right] \\ &= h_2(S^{2,e} + \mathbb{E}(O) - \mathbb{E}\left[D_{t_0,t_0+l^{2,e}}\right]). \end{aligned} \tag{24}$$

Next, we let $C_{t,1} = h_1 IL^1_t + (p + h_1 + h_2)(IL^1_t)^-$ denote the holding and penalty costs associated with echelon 1 in period t. Then we have:

$$\begin{aligned} &\mathbb{E}\left[C_{t_0+l^{2,e}+l^1+1,1} | IP^1_{t_0+l^{2,e}} + Q^1_{t_0+l^{2,e}} = \min(S^1, IP^{2,e}_{t_0} + Q^{2,e}_{t_0} - D_{t_0,t_0+l^{2,e}-1})\right] \\ = \;& \mathbb{E}\left[C_{t_0+l^{2,e}+l^1,1} | IP^1_{t_0+l^{2,e}} + Q^1_{t_0+l^{2,e}} = S^1 - (D_{t_0,t_0+l^{2,e}-1} - O - (S^{2,e} - S^1))^+\right] \\ = \;& h_1\left(S^1 - \mathbb{E}[(D_{t_0,t_0+l^{2,e}-1} - O - (S^{2,e} - S^1))^+] - \mathbb{E}\left[D_{t_0+l^{2,e},t_0+l^{2,e}+l^1}\right]\right) + \\ & (p + h_1 + h_2)\mathbb{E}\left[\left(\left[D_{t_0,t_0+l^{2,e}-1} - O - (S^{2,e} - S^1)\right]^+ + D_{t_0+l^{2,e},t_0+l^{2,e}+l^1} - S^1\right)^+\right] \end{aligned} \tag{25}$$

With the conventions

$$
\begin{aligned}
B_1 &= (D_{t_0,t_0+l^{2,e}-1} - O - (S^{2,e} - S^1))^+ && (26)\\
B_0 &= (B_1 + D_{t_0+l^{2,e},t_0+l^{2,e}+l^1} - S^1)^+, && (27)
\end{aligned}
$$

the average holding and penalty cost function for fixed Δ, $C_{hp}(S^1, S^{2,e}|\Delta)$, can be written succinctly as

$$
\begin{aligned}
C_{hp}(S^1, S^{2,e}|\Delta) &= h_2\left(S^{2,e} + \mathbb{E}(O) - \mathbb{E}(D_{t_0,t_0+l^{2,e}-1})\right) + \\
&\quad h_1\left(S^1 - \mathbb{E}(B_1) - \mathbb{E}(D_{t_0+l^{2,e},t_0+l^{2,e}+l^1})\right) + (p + h_1 + h_2)\mathbb{E}(B_0). \quad (28)
\end{aligned}
$$

The following result follows immediately from Lemma 3.1, equation (28), and results for serial supply chains with discrete demand (Doĝru et al., 2004; van Houtum, 2006).

Theorem 3.2. (Newsvendor inequalities for $S^{2,e}$ and S^1) *Suppose $\Delta = S^{2,r} - S^{2,e}$ has been fixed. Then the optimal choice for S^1 is the smallest integer that satisfies the following newsvendor inequality:*

$$
\mathbb{P}\left(B_0^{(1)} = 0\right) \geq \frac{p + h_2}{p + h_2 + h_1}. \quad (29)
$$

where

$$
B_0^{(1)} = (D_{t_0+l^{2,e},t_0+l^{2,e}+l^1} - S^1)^+.
$$

Now let S^{1*} denote the optimal S^1 and let $\varepsilon(S^{1*}) = \mathbb{P}\left(D_{t_0+l^{2,e},t_0+l^{2,e}+l^1} \leq S^{1*}\right) - \frac{p+h_2}{p+h_2+h_1}$. Then the optimal $S^{2,e}$ is the smallest integer that satisfies the following newsvendor inequality:

$$
\mathbb{P}\left(B_0^* = 0\right) \geq \frac{p}{p + h_1 + h_2} + \mathbb{P}(B_1^* = 0)\varepsilon(S_1^*) \quad (30)
$$

where

$$
B_1^* = (D_{t_0,t_0+l^{2,e}-1} - O - (S^{2,e} - S^{1*}))^+
$$

and

$$
B_0^* = (B_1^* + D_{t_0+l^{2,e},t_0+l^{2,e}+l^1} - S^{1*})^+.
$$

Remark Note that while Theorem 3.2 pertains to two-echelon systems, analogous results can easily be obtained for N-echelon systems as long as dual sourcing only occurs at the most upstream stock point.

3.2 Overshoot and emergency ordering probability

In Lemma 3.1, we have established that the stationary distributions of O, $Q^{2,e}$, and $Q^{2,r}$ are uniquely defined for fixed Δ. However, to evaluate the performance of a given DIP, we need to evaluate these distributions to find the cost terms $k\mathbb{P}(Q^{2,e} > 0)$, $c\mathbb{E}(Q^{2,e})$ and apply Theorem 3.2. In this section we provide an accurate approximation for the needed results.

The overshoot distribution behaves in a manner entirely identical to the overshoot distribution in a single stage system as an inspection of Lemma 3.1 will reveal. Consequently, we can adopt the approximation proposed by Arts et al. (2010) to determine the overshoot distribution. To be self-contained, we briefly outline how to do this.

Observe that equation (14) in Lemma 3.1 implies that that the stationary distribution of O can be obtained from the stationary distribution of A as $\mathbb{P}(O = x) = \mathbb{P}(A = \Delta - x)$. For A_t the following recursion holds (Arts et al., 2010, provide a detailed derivation):

$$A_{t+1} = \min(\Delta, A_t - Q^{2,r}_{t+1-l} + D_t). \tag{31}$$

From this equation, it is readily verified that a one-dimensional Markov chain for A_t has transition probabilities $p_{ij} = \mathbb{P}(A_{t+1} = j | A_t = i)$:

$$p_{ij} = \begin{cases} \sum_{k=0}^{j} \mathbb{P}(Q^{2,r}_{t+1-\ell} = i + k - j | A_t = i)\mathbb{P}(D = k), & \text{if } j < \Delta; \\ \sum_{k=0}^{i} \mathbb{P}(Q^{2,r}_{t+1-\ell} = k | A_t = i)\mathbb{P}(D \geq \Delta + k - i), & \text{if } j = \Delta. \end{cases} \tag{32}$$

Note that this is an aggregated Markov chain for A_t. A full Markov chain for A_t would require storing regular orders $Q^{2,r}_{t-\ell+1}$ to $Q^{2,r}_t$ in the state space. Thus, the state-space grows exponentially in ℓ. In making this aggregation, we require the probability $\mathbb{P}(Q^r_{t+1-\ell} = k | A_t = i)$, which is in fact unknown. However, this probability can be approximated using the following limiting result which is proven in Arts et al. (2010).

Proposition 3.3. *The following statements hold:*

(i) *As* $\Delta \to \infty$, $\mathbb{P}(Q^{2,r}_t = x) \to \mathbb{P}(D_{t-1} = x)$

(ii) *As* $\Delta \to \infty$, $\mathbb{P}\left(Q^{2,r}_{t+1-\ell} = x | A_t = y\right) \to \mathbb{P}\left(D_{t+1-\ell} = x | \sum_{n=t+1-\ell}^{t} D_n = y\right)$.

(iii) *For* $\Delta = 1$, $\mathbb{P}\left(Q^{2,r}_{t+1-\ell} = x | A_t = y\right) = \mathbb{P}\left(D_{t+1-\ell} = x | \sum_{n=t+1-\ell}^{t} D_n = y\right)$.

Using the limiting results in Proposition 3.3 by approximating $\mathbb{P}\left(Q^{2,r}_{t+1-\ell} = x | A_t = y\right)$ with $\mathbb{P}\left(D_{t+1-\ell} = x | \sum_{n=t+1-\ell}^{t} D_n = y\right)$, we can compute an approximation to $\pi_x = \mathbb{P}(A = x)$ by solving the linear balance equations together with the normalization equation:

$$\pi_x = \sum_{j=0}^{\Delta} \pi_j p_{xj} \quad x = 0, 1, ..., \Delta - 1 \qquad \sum_{j=0}^{\Delta} \pi_j = 1 \tag{33}$$

An approximation for the stationary distribution of O is now $\mathbb{P}(O = x) = \pi_{\Delta - x}$. This result can be used in the newsvendor characterizations in Theorem 3.2.

Next, to evaluate the term $c\mathbb{E}(Q^{2,e})$ we note that $\mathbb{E}(A) = \ell\mathbb{E}(Q^{2,r})$ and $\mathbb{E}(D) = \mathbb{E}(Q^{2,r}) + \mathbb{E}(Q^{2,e})$. Combining these relations, we have $c\mathbb{E}(Q^{2,e}) = c(\mathbb{E}(D) - \mathbb{E}(A)/\ell)$.

Evaluating $k\mathbb{P}(Q^{2,e} > 0)$ can be done by conditioning as follows:

$$\begin{aligned} k\mathbb{P}(Q^{2,e} > 0) &= k\mathbb{P}((D - O - Q^{2,r}_{t+1-\ell})^+ > 0 | A_t = \Delta - O) \\ &= k\mathbb{P}(D - O - Q^{2,r}_{t+1-\ell} > 0 | A_t = \Delta - O) \\ &= k\textstyle\sum_{y=0}^{\Delta} \mathbb{P}(D - Q^{2,r}_{t+1-\ell} > y | A_t = \Delta - y)\mathbb{P}(O = y) \\ &= k\textstyle\sum_{y=0}^{\Delta}\sum_{z=0}^{\Delta-y} \mathbb{P}(D > y + z)\mathbb{P}(Q^{2,r}_{t+1-\ell} = z | A_t = \Delta - y)\mathbb{P}(O = y). \quad (34) \end{aligned}$$

From equation (34), one can readily compute an approximation for $k\mathbb{P}(Q^{2,e} > 0)$ using Proposition 3.3 again in the same manner. The costs associated with emergency ordering $C_e(\Delta)$ are:

$$C_e(\Delta) = c(\mathbb{E}(D) - \mathbb{E}(A)/\ell) + k\textstyle\sum_{y=0}^{\Delta}\sum_{z=0}^{\Delta-y} \mathbb{P}(D > y + z)\mathbb{P}(Q^{2,r}_{t+1-\ell} = z | A_t = \Delta - y)\mathbb{P}(O = y). \quad (35)$$

In Arts et al. (2010), it is shown that the approximations suggested here are extremely accurate for a single stage system in that the solutions obtained are statistically not distinguishable from simulation estimates. An efficient algorithm to optimize the parameters of the dual-index policy for the system under study is now straightforward, and an outline for an algorithm to do this is given in Figure 2.

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1. Initialize Δ
2. Determine the approximate stationary distribution of the overshoot using equations (32), (33) and Proposition 3.3.
3. Find the optimal S^1 and $S^{2,e}$ for this Δ using Theorem 3.2.
4. Compute the (approximate) average cost of this policy using equations (28) and (35).
5. Stop or update Δ according to some search procedure and proceed to step 2

Figure 2: Outline of an algorithm to optimize the two-echelon dual-index policy

4. Numerical results

In this section, we investigate the effect of different problem parameters on optimal costs, order-up-to-levels, and savings compared to the equivalent single sourcing system that uses only the best supply source, which can be solved to optimality. For the computation of the optimal policy parameters in a serial system with single delivery modes, we use the approach described in van Houtum (2006). For our numerical study, we define a base instance and then unilaterally vary different parameters. The base case has the following parameters: $\mathbb{E}(D) = 10$, $cv_D = 1$, $l^1 = 2$, $l^{2,e} = 1$, $\ell = l^{2,r} - l^{2,e} = 10$, $h_1 = 0.6$, $h_2 = 0.4$, $p = 19$, $c = 10$ and $k = 0$. Here cv_D is the coefficient of variance of demand $\sigma(D)/\mathbb{E}(D)$. The demand distributions we use are either mixtures of negative binomial or geometric distributions as fitted on the first two moments by the procedure of Adan et al. (1996).

First, we investigate the effect of demand variability. We let cv_D range from 0.3 to 2 and plot cost, base-stock levels, and percentage saving compared to the optimal single sourcing policy (Fig 3). The plot of cost is further divided to see the share of different cost terms. Figure 3 shows that costs increase rapidly with demand variability, but that savings compared to single sourcing do too. Furthermore, it is noteworthy that the emergency base-stock level $S^{2,e}$ is mostly below the echelon 1 base-stock level S^1. Further results show that this is usual behavior.

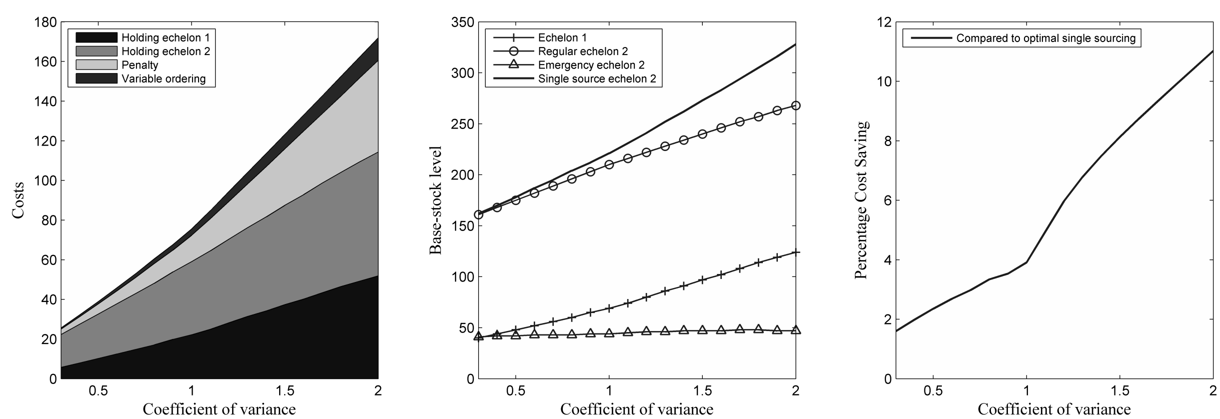

Figure 3: *Effect of demand variability*

Second, we investigate the effect of the lead time difference $\ell = l^{2,r} - l^{2,e}$ on the system. Results are shown in Figure 4. Here we see that costs do not increase rapidly with ℓ, but savings compared to single sourcing do. This saving is possible because more units are ordered via the emergency channel as evidenced from the increase in variable ordering costs.

Thus, dual-sourcing is especially efficient when regular lead times are long compared to emergency lead times.

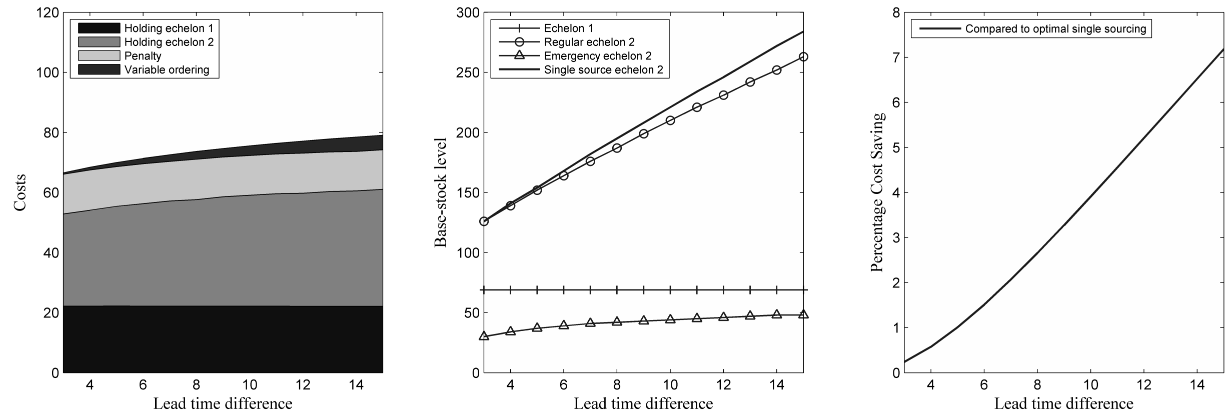

Figure 4: *Effect of lead time difference*

Third, the service level defined as $SL = \frac{p}{p+h_1+h_2}$ is varied by varying p. In Figure 5, we see that dual-sourcing is more beneficial when customers require high service, while the added value when service is relatively unimportant is negligible.

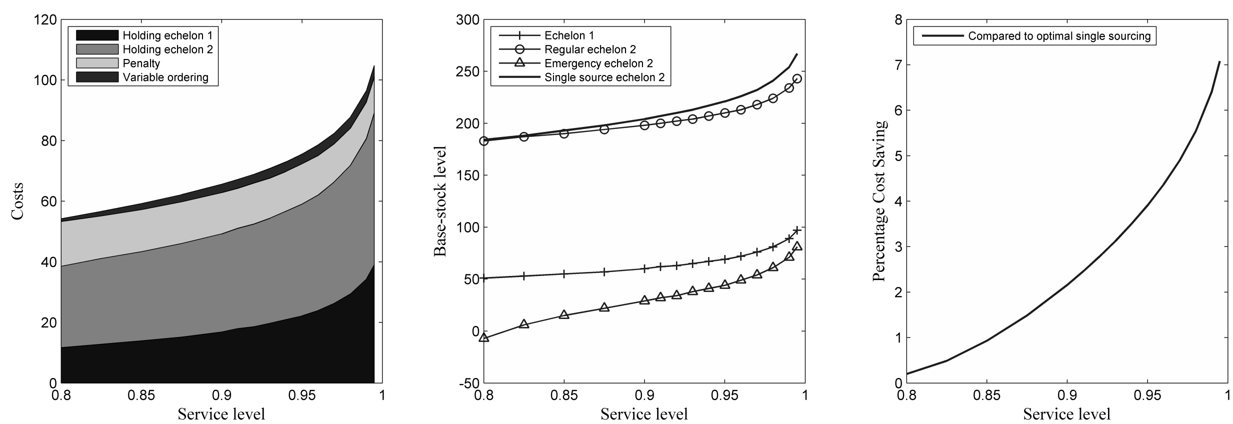

Figure 5: *Effect of required service level*

Fourth, the unit emergency costs c are varied. The results, shown in Figure 6, indicate that variable ordering and echelon 2 holding costs act as substitutes. This highlights previous findings (Veeraraghavan and Scheller-Wolf, 2008) that the dual-index policy saves money by reducing holding, not penalty costs. Also, as expected, we see that the dual-index policy approaches a single-sourcing base-stock policy as c increases.

In the next experiment, we keep $h_1 + h_2 = 1$ while varying h_1. This represents the incremental value added from stage 2 to stage 1 of the supply chain. In Figure 7, we see that,

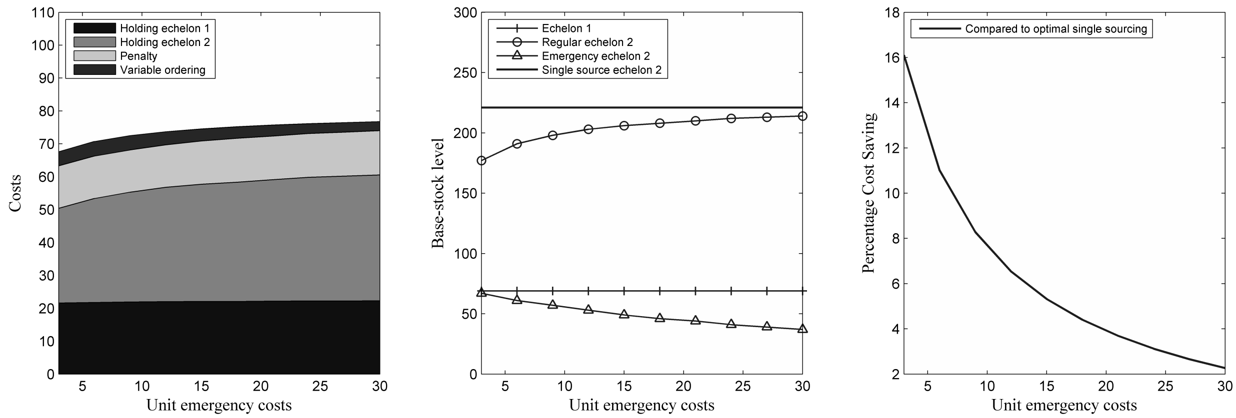

Figure 6: *Effect of unit emergency costs*

as the incremental value of stage 1 inventory increases, the value of dual sourcing decreases (although total costs decrease). The explanation for this is that, as observed in Figure 6, the benefit from the dual-index policy is the reduction of echelon 2 stock. As the importance of this cost decreases, so does the benefit of the dual-index policy over single-sourcing.

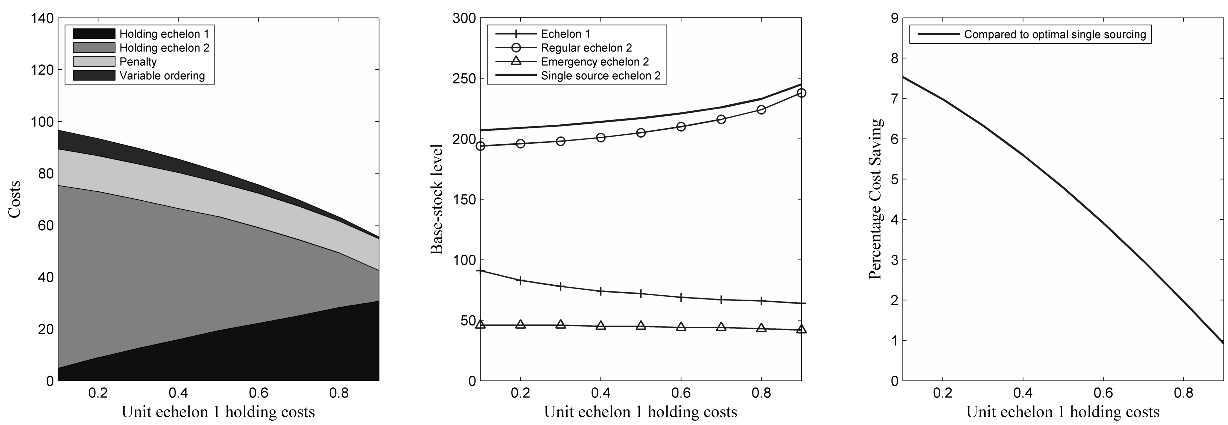

Figure 7: *Effect of holding cost division*

Finally, we investigate the effect of having a fixed emergency ordering cost k. A fixed cost can represent such things as shipping over air instead of over sea. In many cases, shipping tariffs are based mostly on the modality and less so on the order quantity as long as the order fits in a standard container. To model this, we reduce the variable ordering costs c in the base instance to 1 and let the fixed ordering costs k range from 5 to 95. The results are shown in Figure 8. We see a large saving potential compared to single sourcing. This is striking because the dual-index policy is not especially fitted for fixed ordering costs. For example, it is quite possible to place an emergency order of size one under this policy. This effect is shown by the division of fixed and variable ordering costs for large k. A more apt policy may avoid fixed costs associated with placing very small emergency orders. Despite this, the dual-index policy makes large cost savings possible. Note also that when both c and k are small, it is possible for $S^{2,e}$ to exceed S^1.

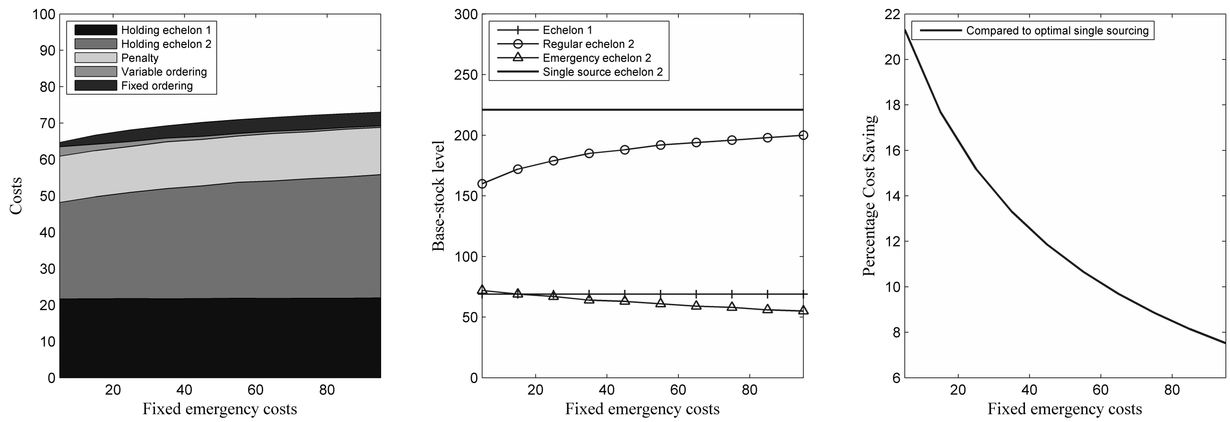

Figure 8: *Value of dual-sourcing under fixed ordering costs*

The computations were implemented in MATLAB and run on a 2.4 GHz dual core processor. The computation times for finding optimal dual-index policy parameters for one instance were 0.5 seconds on average and always within 1 second. The short computation times, intuitive structure, and cost saving potential of the dual-index policy make it especially fit for use in practice.

5. Summary and directions for future research

In this paper, we have studied a two-echelon serial inventory system where the most upstream stockpoint has two suppliers. Replenishment orders are placed following a dual-index and a base-stock policy. For the case of a discrete demand distribution, we have derived newsvendor inequalities for two base-stock levels. For the overshoot distribution, we rely on an approximation based on a Markov chain. These results enable us to compute near-optimal policies. We have further illustrated that the second supply option can lead to considerable cost savings in case of high demand variability, large lead time difference, and small cost difference. However, the second supply option is less worthwhile for systems where the larger part of the value is added at stockpoint one, since the second supply options mostly results in inventory reduction at the most upstream stockpoint.

Our model is the first serial multi-echelon model where two supply options with arbitrary lead times can be used. However, lead time flexibility for a higher price is only considered at the most upstream stockpoint. In a next step we plan to investigate two delivery options at intermediate stockpoints.

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References

I.J.B.F. Adan, M.J.A. van Eenige, and J.A.C. Resing. Fitting discrete distributions on the first two moments. *Probability in the Engineering and Informational Sciences*, 9:623–632, 1996.

G. Allon and J.A. Van Mieghem. Global dual sourcing: tailored base surge allocation to near and offshore production. *Management Science*, 56(1):110–124, 2010.

J.J. Arts, M. Van Vuuren, and G.P. Kiesmüller. Efficient optimization of the dual-index policy using markov chains. *IIE Transactions*, accepted for publication, 2010.

S. Axsäter. Supply chain operations: Serial and distribution inventory systems. In A.G. de Kok and S.C. Graves, editors, *Supply Chain Management: Design, Coordination and Operation*, pages 525–559. Elsevier, 2003.

X. Chao and S.X. Zhou. Probabilistic solution and bounds for serial systems with discounted and average costs. *Naval Research Logistics*, 54:623–631, 2007.

F. Chen and Y-S. Zheng. Lower bounds of multi-echelon stochastic inventory systems. *Management Science*, 40(11):1426–1443, 1994.

C. Chiang. Optimal control policy for a standing order inventory system. *European Journal of Operational Research*, 182:695–703, 2007.

A.J. Clark and H. Scarf. Optimal policies for a multiechelon inventory problem. *Management Science*, 6(4):475–490, 1960.

M.K. Doĝru, A.G. de Kok, and G.J. van Houtum. A note on newsboy inequalities in an n-echelon serial inventory/production system with discrete demand. *BETA Working Paper, Eindhoven University of Technology*, 2004.

Q. Feng, S.P. Sethi, H. Yan, and H. Zhang. Optimality and nonoptimality of the base-stock policy in inventory problems with multiple delivery modes. *Journal of Industrial and Management Optimization*, 2(1):19–42, 2006a.

Q. Feng, S.P. Sethi, H. Yan, and H. Zhang. Are base-stock policies optimal in inventory problems with multiple delivery modes? *Operations Research*, 54(4):801–807, 2006b.

Y. Fukuda. Optimal policies for the inventory problem with negotiable leadtime. *Management Science*, 10(4):690–708, 1964.

G. Gallego and O. Özer. A new algorithm and a new heursitic for serial supply systems. *Operations Research Letters*, 33:349–362, 2006.

F. Janssen and T. de Kok. A two-supplier inventory model. *International Journal of Production Economics*, 59:395–403, 1999.

S. Klosterhalfen, G.P. Kiesmüller, and S. Minner. A comparison of the constant-order and dual-index policy for dual sourcing. *International Journal of Production Economics*, in press, 2010.

D.G. Lawson and E.L. Porteus. Multi-stage inventory management with expediting. *Operations Research*, 48(6):878–893, 2000.

S. Minner. Multiple supplier inventory models in supply chain management: A review. *International Journal of Production Economics*, 81-82:265–279, 2003.

A. Muharrenoglu and J.N. Tsitsiklis. Dynamic leadtime management in supply chains. *Working Paper Columbia University*, 2003.

E.M. Plambeck and A.R. Ward. Note: a separation principle for a class of assemble-to-order systems with expediting. *Operations Research*, 55(3):603–609, 2007.

M. Rosenshine and D. Obee. Analysis of a standing order system with emergency orders. *Operations Research*, 24(6):1143–1155, 1976.

K.H. Shang and J.S. Song. Newsvendor bounds and heurstics for optimal policies in serial supply chains. *Management Science*, 49(5):618–638, 2003.

A. Sheopuri, G. Janakiraman, and S. Seshadri. New policies for the stochastic inventory control problem with two supply sources. *Operations Research*, 58(3):734–745, 2010.

J.S. Song and P. Zipkin. Inventories with multiple supply sources and networks of queues with overflow bypasses. *Management Science*, 55(3):362–372, 2009.

G.J. van Houtum. Multiechelon production/inventory systems: optimal policies, heuristics and algorithms. *Tutorials in Operations Research INFORMS*, pages 163–199, 2006.

S. Veeraraghavan and A. Scheller-Wolf. Now or later: a simple policy for effective dual sourcing in capacitated systems. *Operations Research*, 56(4):850–864, 2008.

A.S. Whittmore and S.C. Saunders. Optimal inventory under stochastic demand with two supply options. *SIAM journal of applied mathematics*, 32(2):293–305, 1977.

N. Xu. Optimal policy for a two-facility inventory problem with storage constraints and two freight modes. *European Journal of Operational Research*, 194:78–84, 2009.

S.X. Zhou and X. Chao. Newsvendor bounds and heuristics for serial supply chains with regular and expedited shipping. *Naval Research Logistics*, 57:71–87, 2010.

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