

Pricing and Market Segmentation Using Opaque Selling Mechanisms

Chris K. Anderson and Xiaoqing Xie

School of Hotel Administration, Cornell University, Ithaca, NY 14853, USA
cka9@cornell.edu

In opaque pricing certain characteristics of the product or service are hidden from the consumer until after purchase, transforming a differentiated good into somewhat of a commodity. Opaque pricing has become popular in service pricing as it allows firms to sell their differentiated products at higher prices to regular brand loyal customers while simultaneously selling to non loyal customers at discounted prices. We develop a stylized model of consumer choice that illustrates the role of opaque pricing in market segmentation. We model a monopolist selling a product via three selling channels: a regular full information channel, an opaque posted price channel and an opaque bidding channel where consumers specify the price they are willing to pay. We illustrate the segmentation created by opaque pricing as well as compare optimal revenues and prices for sellers using regular full information channels with those using opaque selling mechanisms in conjunction with regular channels. We also study the segmentation and policy changes induced by capacity constraints.

Keywords: Revenue Management, Marketing:Pricing, Segmentation, Auctions, Buyer behavior

1. Online Travel Sales

The pricing of services (rooms, rental cars, airline seats, etc...) online has dramatically changed how service firms reach customers, with online travel sales now exceeding offline (or traditional sales channels). Initial thoughts about pricing online were very positive as firms had new channels to reach customers enabling increased opportunities for segmentation. Over time service providers have increased efforts to move customers back to company direct distribution channels (company websites and call centers) in an effort to control sales costs and commissions while maintaining direct contact with the customer to facilitate loyalty programs and other marketing efforts.

Hotwire and Priceline, unlike other online travel sales channels such as Expedia, Travelocity and Orbitz, offer customers opaque products with aspects of the service provider concealed

until the transaction has been completed. Figure 1 shows a typical service provider listing (here hotels) on a full information channel like Expedia. Figures 2 and 3 display information available to someone using Hotwire’s opaque mechanism. For instance a customer purchasing a hotel room through Hotwire can not specify the hotel they wish to stay at, but rather only its star rating and general location within the destination city. Customers do not know the identity or exact location of their non-refundable choice property until after purchase. Opaque travel sites offer service providers a convenient channel to segment customers and distribute discounted products without cannibalizing or diluting full priced products. The opaque channels naturally segment customers as regular full price paying customers desiring to stay at the hotel of their choice with full cancellation flexibility are unique from those willing to purchase the discounted, non-refundable opaque product at the unknown service provider. Similar to the opaque posted price model of Hotwire, Priceline offers opaque services but without posted prices. Priceline’s name-your-own-price model is similar to Hotwire where consumers, as shown in Figure 4, only know the star level and region for a hotel. On Priceline, consumers post bids for the opaque service as shown in Figure 5, having to then wait for the service provider to accept to reject their offer. For a more detailed description of Priceline’s name-your-own-price model see Anderson (2009). While the illustrations provided in Figures 1-5 use hotels as examples, opaque services are also offered for other travel services. With air travel, the consumer is unaware of the itinerary (connections and layover durations) or airline and with rental cars, the consumer does not know the type of car or rental firm until after paying for the service. Lastminute.com, another online travel agent, also offers opaque posted price services similar to those of Hotwire.

The level of opacity varies across the different opaque channels as some choose to offer cancellation opportunities as in the case of Lastminute.com, provide user generated feedback as in the case of Hotwire.com, or list some of the amenities offered by the service provider. Similarly the degree of opacity may also be impacted by the market, as markets with fewer similar competitors offer decreased opacity over markets with a larger number of service providers.

Opaque selling has recently started to receive interest in the the academic literature, most of the early research has focused on models similar to Priceline’s name-your-own-price (NYOP) bidding mechanism where customers post bids for opaque services. Anderson (2009) provides a detailed background on the nature of Priceline’s NYOP model as well as a dynamic programming based model for the setting of prices by firms on Priceline. Fay (2004) develops

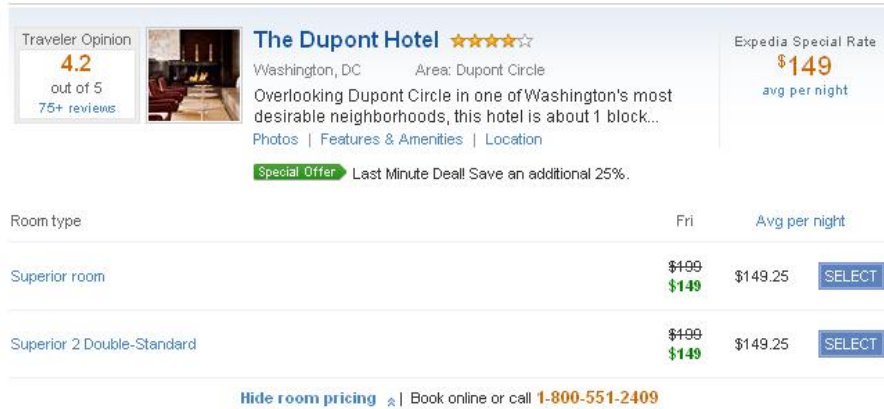


Figure 1: Typical Full Information Hotel Listing

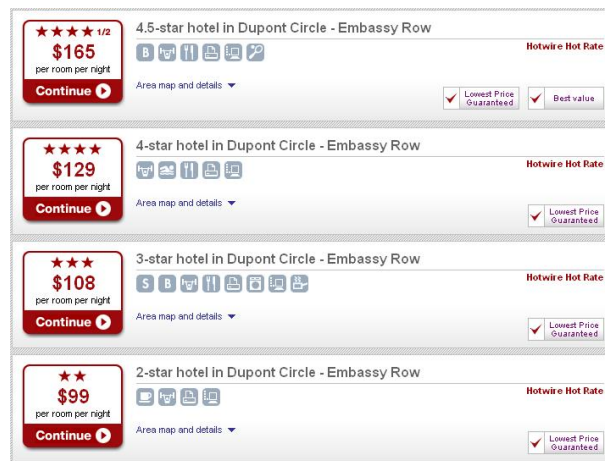


Figure 2: Posted Opaque Hotel Listing

a stylized model of a monopolist firm using a NYOP channel and investigates whether repeat bidding should be allowed. Strictly speaking, Priceline does not allow repeat bidding within a 24 hour period but there are numerous methods to circumvent this limitation, see BiddingforTravel.com for examples. Fay indicates that partial repeat bidding, i.e. repeat bidding by knowledgeable customers may be less profitable than complete repeat bidding. Fay (2008) extends the monopolist model to a duopoly model with firms pricing into two consumer segments. One segment is loyal to a particular service provider, the second has preferences distributed between the two firms along a line as in the traditional Hotelling model (Hotelling, 1929). Fay (2008) is the first paper to investigate how product opacity affects the market. Fay studies two competing service providers selling products to two types



Figure 3: Posted Opaque City Areas

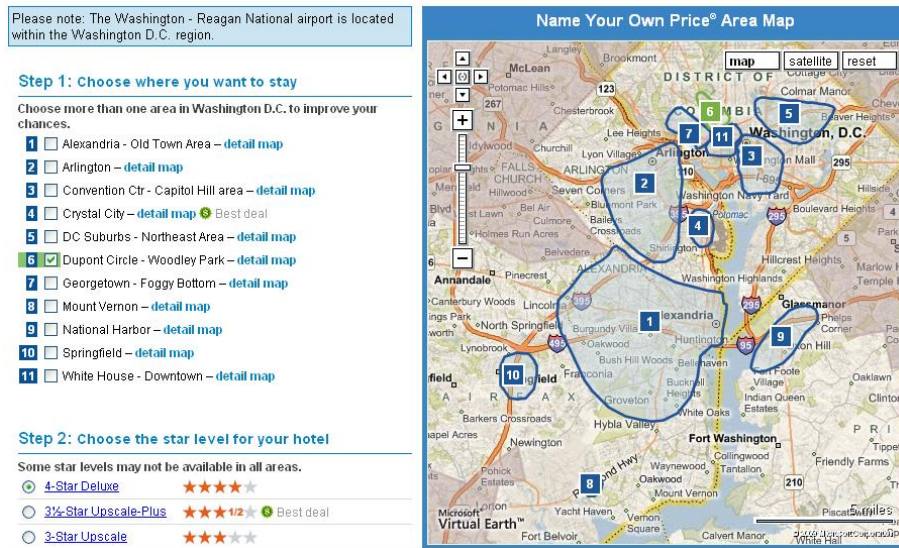


Figure 4: Opaque Bidding Hotel Listing

Step 3: Name Your Own Price® (per room night)

Total charges, including taxes and service fees, are shown on the next page.

<p>Name Your Own Price®</p> <p>Per Room, Per Night (\$USD)</p> <p>\$ <input type="text"/> .00</p>	<p>Median retail price for a 4 star hotel in the area selected is \$199. Name a lower price, or click here to shop and compare prices.</p>
--	--

Figure 5: Submitting Opaque Bid

of customers (business and leisure) on both an opaque posted price channel and a traditional distribution channel. Fay shows that opaque selling benefits the monopoly service provider when customers have heterogeneous values for products. Shapiro and Shi (2008) extend the model of Fay (2008) to N firms with the number of firms indicating the degree of opacity - uncertainty in knowledge of service provider increases with number of firms. Shapiro and Shi focus on providing a rationale for opaque selling. They explain why service providers are willing to distribute products through opaque travel sites such as Priceline and Hotwire and lose the advantage of product differentiation.

Hann and Terwiesch (2003) use data from a European NYOP retailer to investigate consumer transactions costs (the cost of resubmitting bids) of using a repeat bidding NYOP channel. In a related paper Spann et al. (2004) investigate consumers' frictional or transactions costs as well as their willingness to pay using data from a German NYOP seller of flights from Germany to Spain.

Wang et al. (2009) develop a game theoretic model of a supplier using both regular posted price full information channels as well as a NYOP channel to reach heterogeneous customers. They develop a two-stage game where suppliers set posted prices in period 1 and after observing demand in period 1, set minimally acceptable prices at the NYOP channel in period 2. Posted prices are rigid in period 2. Consumers observe posted prices in the first period then decide to buy or bid in period 2. The rigidity of posted prices combined with demand uncertainty results in the NYOP channel generating improved revenues for the service provider. Wilson and Zhang (2008) look at a retailer setting prices on a NYOP channel. They develop ϵ optimal policies for the retailer that encourage the customer to bid their maximum reservation price.

Related research looks more generally at opaque selling where prices are posted but some aspect of the service or service provider is hidden i.e. the selling mechanism similar to that provided by Hotwire.com. Jiang (2007) develops a Hotelling type model to illustrate how a firm should price on regular full information channels versus opaque channels. Jiang indicates that opaque selling can be Pareto improving for both customers and suppliers when customers are differentiated in their willingness to pay. Jiang compares opaque selling and regular selling (selling full-information products), providing insight when to implement opaque selling. Jerath, Netessine and Veeraraghavan (2007) compare opaque selling with last-minute direct selling and obtain the conditions under which opaque selling is preferred. In their model two firms of equal capacity offer a differentiated service via three channels:

regular posted price, posted last-minute sales, and last-minute sales through an opaque intermediary. Their goal is to investigate under what market conditions a firm should directly offer last-minute discounts versus offer those discounts through an intermediary. Jerath et al. relax the posted price rigidity of Wang et al. (2005) through introduction of the direct last-minute discounts. They conclude that direct last-minute selling is preferred over the opaque intermediary when consumer valuations are high or if the service offerings are relatively homogeneous.

While there is an extensive body of research on the use of auctions, very little of this research looks at the simultaneous use of auctions and posted prices. Firms can use auctions to reach customers whom may not otherwise purchase, as posted prices may be too high. Conversely auctions potentially dilute revenues as customers willing to pay posted (full prices) may purchase (at lower prices) via the auction. The opaque nature of Priceline's NYOP model helps to avoid this dilution. Etizon, Pinker and Seidmann (2006) is one of the few auction related papers that looks at the simultaneous use of auctions and posted prices. Similar to our development they look at a firm with excess supply facing consumers who strategically choose to purchase at posted prices or bid (resorting to posted prices if their bid fails). Different from our model, consumers do not face any product opacity with the auction but do incur a waiting cost associated with bidding. Van Ryzin and Vulcano (2004) look at firm using posted prices as well as an auction mechanism, unlike our model of endogenous channel choice (strategic customers similar to Etizon et al.) they assume separate streams of customers to each channel with the seller deciding on inventory allocation across the channels.

We develop a stylized model of consumers looking to acquire travel services through either full information or opaque channels (both posted price and bidding). Consumers choose their channel or sequence of channels (in the case of bidding first followed by posted prices) that maximizes their surplus. Our paper is unique from the literature in that it is the only paper that investigates a firm using two opaque (posted and bidding) channels simultaneously with regular full information posted price channels. Second, prior research assumes two or more exogenous customer segments (i.e. business and leisure) with the opaque channels targeted at the leisure or price sensitive segment; whereas we develop endogenous consumer segments where consumers choose the channel of their choice by maximizing their surplus. Our goal is to illustrate how opaque channels naturally segment consumers as well as how firms should use and price into these channels as a function of the degree of their opacity. We also discuss

the segmentation and policy changes induced by capacity constraints.

2. Model Development

We develop a model of a firm selling to strategic consumers - consumers are strategic as they choose the channel or sequence of channels which maximizes their surplus. The seller can potentially offer its products across three selling mechanisms: a posted full information market, posted opaque market with certain aspects of the product hidden and a name your price opaque auction mechanism. Unlike previous research which assumes exogenous consumer behavior we model endogenous consumer behavior where all consumers act strategically as they optimally choose the channel (or sequence of channels) that maximizes their surplus. For ease of exposition we will refer to the full information channel as the regular (REG), the opaque posted price channel as opaque (OPQ) and the opaque channel with bidding as BID. For comparison purposes, think of our regular channel as a firm's website (Marriott.com, Hilton.com or USAirways.com) or a typical online travel agent similar to Expedia, Orbitz or Travelocity, the posted opaque channel analogous to Hotwire.com, and our bidding model similar to Priceline's name-your-own-price model. We do not model competition in the full information market as the firm is selling a differentiated/branded product desired by consumers.

Each customer i looking to acquire service has an independent reference price or valuation v_i for the service provider. Similar to Wang et al. (2009) we assume v_i uniformly distributed between 0 and 1, i.e. its density function $f(v_i)$ is 1 for $0 \leq v_i \leq 1$ and 0 otherwise. The service provider posts a price P_1 on the regular channel and fully discloses all service provider characteristics. The service provider posts price P_2 on the opaque posted price channel and reveals the full information until after the purchase. The service provider also sets a threshold price R on the opaque bidding channel. The customer, if they choose to bid, bids B_i on the bidding channel. Similar to Hann and Terwiesch (2003), Spann et al. (2004) and Ding et al. (2005), with limited knowledge of the value of the threshold R , customers expect R to be distributed uniformly over $[0, 1]$. As a result customers believe their bid of B will be accepted by the service provider with a probability of B .

When a consumer pays P_1 at the regular full information channel they are purchasing the product from the service provider of choice, here assuming the consumer has an affinity for this branded service provider. When the consumer pays P_2 at a posted opaque channel they

know they are receiving a similar product but they don't know from which service provider - e.g. could be any of 10 3-Star hotels in Times Square NYC. Typically posted price opaque channels like Hotwire.com display online the service provider whom has provided them the lowest price - e.g. if all 10 of the aforementioned 3-Star Times Square hotels offered inventory to Hotwire only the one with the cheapest price would be posted with the opportunity for a sale. Which property is displayed would change over time as transactions occur and inventory is sold. Priceline's opaque bidding channel behaves in a similar fashion except the consumer submits an offer, B_i , for a 3-Star Times Square hotel, Priceline then randomly selects from the firms that have provided it with inventory to see if they have a price that is less than the consumers offer price. Priceline randomly rotates through all the qualifying hotels (3-Star Times Square) until either a hotel with a price low enough is found or no service provider meets the consumer's bid. Online boards such as BiddingForTravel.com provide resources and historic bid results to help consumers in determining how to bid on Priceline. For a more exhaustive discussion of Priceline see Anderson (2009).

The service provider looks to augment its full information channel with the opaque channels in an effort to sell surplus inventory. The service provider looks to use the opaque channels even though they yield considerably lower revenues (typical discounts at Hotwire and Priceline range from 25-50%). Figure 6 shows a set of sample reservations buildup for a 3.5 star hotel in Dupont Circle Washington DC. The figure shows the average percentage of reservations over the last week prior to arrival for 6 weeks of arrival dates in the fall of 2008. The figure displays total reservations as well as those through each of Hotwire and Priceline. As can be seen from the figure Hotwire and Priceline are typically only used very close to the arrival day. Virtually no reservations are accepted on opaque channels prior to 7 days before arrival whereas approximately half of total reservations have been received prior to the last week. The service provider is using the deeply discounted opaque channels to sell distressed inventory, inventory that would otherwise not be sold, over these final few days prior to arrival. During these last few days prior to the service becoming worthless (hotel bed not occupied or airline seat flying empty) the firm is in essence pricing without capacity considerations (able to meet all demand). Whereas earlier on in the selling process (several weeks or months prior to arrival at the hotel or departure of the aircraft) the firm may not use opaque channels as it prices in consideration of capacity constraints - hoping to sell all inventory at higher prices to the brand loyal customers on the full information channels. As we will also see in later sections, the firm also tends not to use the opaque channels if they

are not very opaque. The opaque channels become increasingly less opaque earlier on in the selling process as fewer firms may tend to use them - with opacity as in Shapiro and Shi (2008) directly related to the number of service providers using the opaque channels.

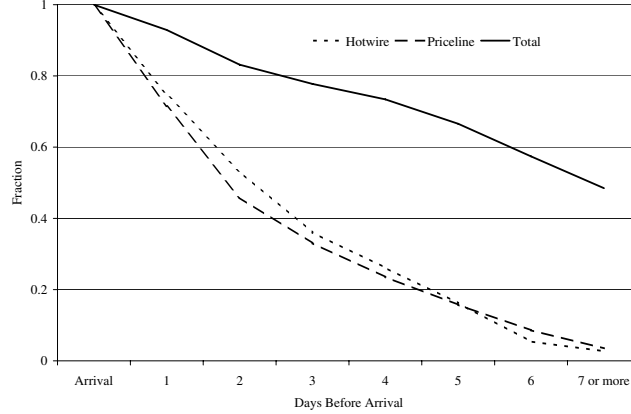


Figure 6: Reservations buildup at Hotwire, Priceline and all channels for 3.5 star DC hotel

In the following sections we outline optimal prices and the resulting market segmentation for a service provider who has the opportunity to release their products on the regular full information channel, an opaque posted price channel and an opaque channel with bidding. We illustrate our modeling approach when the service provider chooses to list only on the full information channel, optimal prices and the resulting revenue provide a basis to later compare multi-channel strategies. Initially we focus on a firm with no capacity constraint, later extending the formulation to a firm where demand exceeds capacity. For ease of presentation, and without loss of generality, all revenues are normalized to a market of one.

2.1 Customer Segmentation

The service provider chooses to release products only on the REG and set its price as P_1 . Consumer i has surplus $CS_i = v_i - P_1$, so only consumers with valuation higher than the price P_1 will purchase on this channel (Table 1 summarizes all model notation).

Therefore, the expected revenue for the service provider π is given by

$$\pi = \int_{P_1}^1 P_1 f(v_i) dv_i = P_1(1 - P_1) = P_1 - P_1^2 \quad (1)$$

Table 1: Notations in the model

v_i	consumer i 's valuation of the product
P_1	price set by the service provider on the REG channel
P_2	price set by the service provider on the OPQ channel
R	bidding threshold set by the service provider on the BID channel
d_1	the discount factor for purchasing on OPQ
d_2	the discount factor for bidding on BID
$B_{i1}(v_i)$	the bid of consumer i in the segment of BID then REG market segment
$B_{i2}(v_i)$	the bid of consumer i in the segment of BID then OPQ market segment
$B_{i3}(v_i)$	the bid of consumer i in the segment of BID only market segment
V_1, V_2, V_4	the critical value points in the market segmentation
V_3, V_5, V_6	the critical value points in the revenue segmentation
C	the capacity constraint

Taking the derivative of π with respect to P_1 and setting it to be zero, we can solve for the optimal price should be posted on REG: $P_1^* = 1/2$.

Since $\frac{d^2\pi}{dP_1^2} = -2 < 0$, we substitute P_1^* back into (1) and get the maximum revenue $\pi^* = \frac{1}{4}$. Moreover, from (1), it is straightforward to see that the maximum revenue is concave in the prices. Figure 7 summarizes the segmentation created by only pricing on the REG.

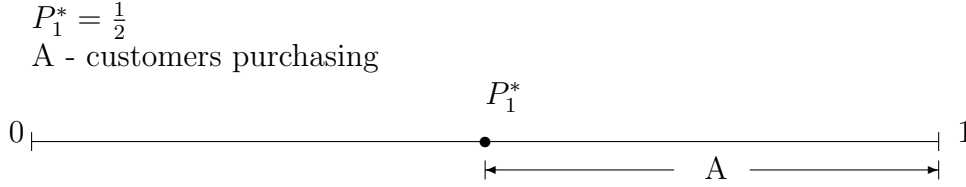


Figure 7: Segmentation resulting from full information posted prices

The service provider now release products on the REG, OPQ and BID simultaneously. They set price P_1 on REG, P_2 on OPQ, a bidding threshold i.e. the minimum acceptable bid R on BID, which is unknown to the consumers. However, as mentioned previously that consumers expect R to follow a uniform distribution over $[0, 1]$.

The consumer surplus from purchasing on the REG is $CS_i = v_i - P_1$. To allow comparison of consumer surplus across channels we adopt a utility framework, where the utility, $U(CS)$, resulting from a surplus CS is assumed to be linear, i.e. $U(CS) = d_j CS + b_j$ for $j = 0, 1, 2$ with j being a channel specific index (0 = REG, 1 = OPQ, 2 = BID). For simplicity, but without loss of generality, we assume $b_j = 0$ and $d_0 = 1$ for consumers acquiring service from the full

information channel. The utility for a consumer purchasing on REG is simply $U(CS_i) = v_i - P_1$. As the consumer is not fully aware of all the service provider's characteristics when purchasing through OPQ we discount the consumer surplus from purchasing on OPQ. Let d_1 denote the discount factor for purchasing on OPQ resulting in utility $U(CS_i) = d_1(v_i - P_2)$ from purchasing on OPQ, where $0 \leq d_1 < 1$. Here $1 - d_1$ represents the opacity of the opaque channel, implying as d_1 approaches 1 the channel becomes less opaque as the consumer discounts the surplus less. Similarly, we denote the degree of opacity of the products on the BID channel by $1 - d_2$. As indicated in Shapiro and Shi (2008) that the degree of opacity is related to the numbers of competitors using the opaque channel. More specifically, for example, if there are N service providers listing their products on the opaque channel, i.e. not disclosing their identity, then in general the consumer's chance of purchasing from one of them is $\frac{1}{N}$. And so, the degree of opacity can be interpreted as a function of $\frac{1}{N}$.

If consumer i 's valuation v_i satisfying $v_i - P_1 \geq d_1(v_i - P_2)$ and $v_i \geq P_1$, then the consumer will prefer to purchase on the REG versus OPQ. If $v_i - P_1 < d_1(v_i - P_2)$ and $v_i \geq P_2$, then they will choose OPQ to make the purchase. The customer will be indifferent to purchasing on REG and OPQ when $v_i = \frac{P_1 - d_1 P_2}{1 - d_1} := V_1$.

Some consumers may bid first and switch to the REG channel if their bid gets rejected and their valuations are higher than P_1 . Suppose B_i is the bid that consumer i submits to BID, and he expects it to be accepted with a probability of B_i . If the bid is rejected (with probability of $1 - B_i$ in consumers' belief), the consumer will go to the REG and purchase the product at P_1 . Given $v_i \geq P_1$, the utility for consumer i is then the sum of the utilities from a possible opaque bidding purchase and in the case their bid is rejected the utility from purchasing at regular prices,

$$U(CS_i) = d_2(v_i - B_i)B_i + (1 - B_i)(v_i - P_1). \quad (2)$$

As $U(CS_i)$ is a concave quadratic function of B_i , it is straightforward to show $U(CS_i)$ is maximized when $B_i = B_{i1}^*(v_i)$, where

$$B_{i1}^*(v_i) = \frac{P_1 - (1 - d_2)v_i}{2d_2}. \quad (3)$$

As bids must be positive, i.e. $B_i > 0$, this results in

$$B_{i1}^*(v_i) > 0 \implies v_i < \frac{P_1}{1 - d_2} := V_2, \quad (4)$$

It is easy to show that the optimal bid is less than P_1 and is decreasing in the opacity degree on the BID channel as the products on the BID channel become less valuable for the customers while the BID channel becomes more opaque.

For the consumer i who chooses to bid $B_{i1}^*(v_i)$, by substituting the bid back into (2) we obtain their maximum expected surplus $U(CS_{i1}^*(v_i)) = d_2(B_{i1}^*(v_i))^2 + (v_i - P_1)$, which exceeds the utility, $(v_i - P_1)$, from buying directly from the posted full information channel as we expected. Therefore, consumers with valuation $P_1 \leq v_i < V_2$ will choose to bid $B_{i1}^*(v_i) = \frac{P_1 - (1-d_2)v_i}{2d_2}$ first and then go to the REG channel if their bids fails.

However, from the service provider's perspective, the bid $B_{i1}^*(v_i)$ will be accepted only if $B_{i1}^*(v_i) > R$ i.e. $v_i < \frac{P_1 - 2d_2R}{1-d_2} := V_3$. This means that customers with valuations $v_i \in [V_3, V_2)$ will lose the bid (note that they do not know it before they bid) and go back to purchase on REG. Customers with valuation $v_i \in [P_1, V_3)$ will win the bid.

A subset of consumers may choose to bid first and switch to purchase at the OPQ channel if their bid is rejected and $v_i \geq P_2$. Assume B_i is the bid that consumer i submits to BID and he believes the accepting probability is B_i . If the bid is rejected, the consumer will go to the OPQ and purchase at P_2 . Given $v_i \geq P_2$, the surplus for consumer i is

$$U(CS_i) = d_2(v_i - B_i)B_i + (1 - B_i)(v_i - P_2) \quad (5)$$

It is straightforward to show $U(CS_i)$ is maximized when $B_i = B_{i2}^*(v_i)$, where

$$B_{i2}^*(v_i) = \frac{d_1P_2 - (d_1 - d_2)v_i}{2d_2}. \quad (6)$$

As consumers bids must be positive, i.e. $B_i > 0$, this results in

$$B_{i2}^*(v_i) > 0 \implies v_i < \frac{d_1P_2}{d_1 - d_2} := V_4, \quad (7)$$

One can easily show that $B_{i2}^*(v_i)$ is less than P_2 and we now take the first derivative of $B_{i2}^*(v_i)$ with respect to d_1 and d_2 respectively and get

$$\frac{dB_{i2}^*(v_i)}{d d_1} = -\frac{v_i - P_2}{2d_2} \leq 0 \text{ since } v_i \geq P_2, \quad (8)$$

$$\frac{dB_{i2}^*(v_i)}{d d_2} = \frac{d_1(v_i - P_2)}{2d_2^2} \geq 0 \text{ since } v_i \geq P_2. \quad (9)$$

The optimal bid for customers who choose bid first and go purchase at the OPQ channel if the bid fails is decreasing in the opacity degree on BID, but increasing in the opacity degree

on OPQ. This is because the products on the BID channel becomes less valuable for the customers while the BID channel becomes more opaque, but becomes more valuable when the OPQ channel becomes more opaque.

We substitute $B_{i2}^*(v_i)$ back and obtain the maximum expected utility for consumer i is $U(CS_{i1}^*(v_i)) = d_2(B_{i2}^*(v_i))^2 + d_1(v_i - P_2)$, which exceeds the surplus, $d_1(v_i - P_2)$, from buying directly from the OPQ channel as desired.

However, similar to the segment of BID then purchase at REG after the bid fails, the bid $B_{i2}^*(v_i)$ will be accepted only if $B_{i2}^*(v_i) > R$ i.e. $v_i < \frac{d_1 P_2 - 2d_2 R}{d_1 - d_2} := V_5$. Thus customers with valuations $v_i \in [V_5, V_4]$ will lose the bid (again, they do not know it before they bid) and switch to purchase at OPQ. Customers with valuations $v_i \in [P_2, V_5]$ will win their bid.

For consumers with valuations lower than P_2 , their only choice is to bid. Their surplus is $U(CS_i) = d_2(v_i - B_i)B_i$, which is maximized with $B_i = B_{i3}^*(v_i) = v_i/2$. Service provider will only accept the bid when $B_{i3}^*(v_i) > R$ i.e. $v_i > 2R := V_6$. This means that customers with valuations $v_i \in [0, V_6]$ will lose the bid and leave empty handed, while customers with valuations $v_i \in [V_6, P_2]$ will win their bid and get the product.

We now summarize the consumer self-selected market segmentation when the service provider can list products on all three channels : REG, OPQ, and BID and illustrate it by using critical points V_1, P_1, V_2, V_4, P_2 . Based on the relationship between the discount factors d_1, d_2 and prices P_1, P_2 posted on channels REG and OPQ as well as the previous analysis, there are two cases of consumer market segmentation as follows:

Case I. $(d_1 - d_2)P_1 - d_1 P_2(1 - d_2) \leq 0 \Leftrightarrow V_4 \geq V_2 \geq V_1$.

The three channels REG, OPQ and BID partition consumers into four potential segments as shown in Table 2.

Table 2: Market Segmentation - Case I

Valuation (v_i)	Segment	Critical Points		
		$1 > V_2$	$V_2 \geq 1 > V_1$	$V_1 \geq 1$
$[V_2, 1]$	REG	Present	Absent	Absent
$[V_1, \min(V_2, 1))$	BID then REG	Present	Present	Absent
$[P_2, \min(V_1, 1))$	BID then OPQ	Present	Present	Present
$[0, P_2)$	BID	Present	Present	Present

Where, REG denotes buying on REG only; BID then REG denotes bidding then purchasing at REG if bid fails; BID then OPQ denotes bidding then purchasing at OPQ if bid fails;

BID denotes bidding only.

Case II. $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0 \Leftrightarrow V_4 < V_2 < V_1$ and $V_1 \geq P_1 \geq P_2$.

Note that this case only exists when $d_1 > d_2$, since when $d_1 \leq d_2$, $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq 0$.

In this case, the three channels REG, OPQ and BID partition consumers into four potential segments as displayed in Table 3.

Table 3: Market Segmentation - Case II

Valuation (v_i)	Segment	Critical Points		
		$1 > V_1$	$V_1 \geq 1 > V_3$	$V_3 \geq 1$
$[V_1, 1]$	REG	Present	Absent	Absent
$[V_3, \min(V_1, 1))$	OPQ	Present	Present	Absent
$[P_2, \min(V_3, 1))$	BID then OPQ	Present	Present	Present
$[0, P_2)$	BID	Present	Present	Present

Where, REG denotes buying on REG only; OPQ denotes buying on OPQ only; BID then OPQ denotes bidding then purchasing at OPQ if bid fails; BID denotes bidding only.

Figure 8 displays the market segmentation of these two cases. Since the market segmentation is formed by consumer self-selection, it will not change when the capacity constraint C added in the model.

2.2 Optimal Service Provider Policies

In this section, we solve for the optimal prices and threshold set on the channels REG, OPQ and BID respectively and the resulting maximum expected revenue for a service provider under both segmentation cases discussed previously. As mentioned before we assume all revenues are normalized to a market of one, as such expected revenue values are per customer, and they do not face a capacity constraint.

As discussed earlier that $B_{i1}^*(v_i)$, $B_{i2}^*(v_i)$, and $B_{i3}^*(v_i)$ are the optimal bids for the consumers in the segments of BID and purchase at REG if the bid fails, BID and purchase at OPQ if the bid fails and BID only respectively. However, from the perspective of the service provider, those bids can be accepted only when they are more than the threshold R , i.e. $B_{i1}^*(v_i) > R$, $B_{i2}^*(v_i) > R$, and $B_{i3}^*(v_i) > R$. This implies consumers in those three segments will win the bidding if their valuations $v_i < V_3$, $v_i < V_5$, and $v_i > V_6$ respectively. Hence, V_3, V_5, V_6 are critical points for determining which channels the revenue is actually coming

Case I

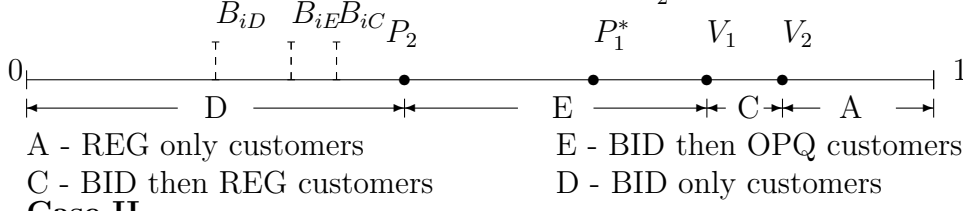
$$V_1 = \frac{P_1 - d_1 P_2}{1 - d_1}$$

$$V_2 = \frac{P_1}{1 - d_2}$$

$$B_{iC} = \frac{P_1 - (1 - d_2)v_i}{2d_2}$$

$$B_{iE} = \frac{d_1 P_2 - (d_1 - d_2)v_i}{2d_2}$$

$$B_{iD} = \frac{v_i}{2}$$



Case II

$$V_1 = \frac{P_1 - d_1 P_2}{1 - d_1}$$

$$V_4 = \frac{d_1 P_2}{d_1 - d_2}$$

$$B_{iE} = \frac{d_1 P_2 - (d_1 - d_2)v_i}{2d_2}$$

$$B_{iD} = \frac{v_i}{2}$$

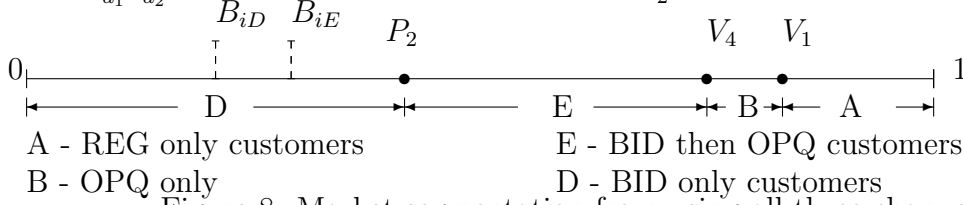


Figure 8: Market segmentation from using all three channels

from. Recall that V_1, P_1, V_2, V_4, P_2 are the critical points for consumer market segmentation and based on the relations among d_1, d_2, P_1 , and P_2 there are the two segmentation cases. From the perspective of the service provider, We now have several sub-scenarios in each segmentation case according to the relations among d_1, d_2, P_1, P_2 and R , and display the scenarios using all the critical points $V_1, V_2, V_3, V_4, V_5, V_6, P_1$, and P_2 as discussed in the following.

Case I. $(d_1 - d_2)P_1 - d_1 P_2(1 - d_2) \leq 0 \Leftrightarrow V_4 \geq V_2 \geq V_1$.

Recall that the consumer segmentation given in Table 2.

There are three revenue segmentation scenarios in this market segmentation case.

Case I - Scenario I. $(d_1 - d_2)P_1 - d_1 P_2(1 - d_2) \leq -2d_2 R(1 - d_1)$

Consumers in the first segment $[V_2, 1]$ (if $V_2 < 1$) buy on REG directly. It is straightforward to check that $(d_1 - d_2)P_1 - d_1 P_2(1 - d_2) \leq -2d_2 R(1 - d_1)$ is equivalent to

$$\frac{P_1 - 2d_2 R}{1 - d_2} \geq \frac{P_1 - d_1 P_2}{1 - d_1}, \text{ i.e. } V_3 \geq V_1.$$

Thus the segment BID then REG ($[V_1, \min(V_2, 1))$) is divided into two groups of customers with the first group $v_i \in [\min(V_3, 1), \min(V_2, 1))$ purchases on REG and second group $v_i \in [V_1, \min(V_3, 1))$ wins the bid.

One can also easily show that $V_3 \geq V_1 \Rightarrow V_5 \geq V_1$, then in the segment of BID then OPQ ($v_i \in [P_2, \min(V_1, 1))$), all consumers will win their bids since their optimal bids are above the threshold R as long as their valuation $v_i \leq V_5$.

$V_5 \geq V_1 \geq P_1 \geq P_2$ (since if $P_1 < P_2$, no one would buy on REG, i.e. there is no REG only segment existing) $\Rightarrow P_2 \geq V_6$, then in the segment of bidding only, consumers with valuations $v_i \in [V_6, P_2)$ win their bidding and consumers with valuations $v_i \in [0, V_6)$ lose.

Table 4 summarizes this revenue segmentation.

Table 4: Revenue segmentation, Case I - Scenario I				
Valuation (v_i)	Transaction channel	Critical Points		
		$1 > V_3$	$V_3 \geq 1 > V_1$	$V_1 \geq 1$
$[V_3, 1]$	REG (P_1)	Present	Absent	Absent
$[V_1, \min(V_3, 1))$	BID ($B_{i1}^*(v_i)$)	Present	Present	Absent
$[P_2, \min(V_1, 1))$	BID ($B_{i2}^*(v_i)$)	Present	Present	Present
$[V_6, P_2)$	BID ($B_{i3}^*(v_i)$)	Present	Present	Present

Therefore, if $1 \geq V_3 \geq V_1$ i.e. $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq -2d_2R(1 - d_1)$ and $P_1 - 2d_2R \leq 1 - d_2$, then the expected revenue π for the service provider in this scenario is:

$$\begin{aligned}
\pi &= \int_{V_3}^1 P_1 f(v_i) dv_i + \int_{V_1}^{V_3} B_{i1}^*(v_i) f(v_i) dv_i + \int_{P_2}^{V_1} B_{i2}^*(v_i) f(v_i) dv_i + \int_{V_6}^{P_2} B_{i3}^*(v_i) f(v_i) dv_i \\
&= \frac{[-d_1(P_1 - P_2)^2 - 4(-1 + d_1)d_2^2P_1(-1 + 2R) + d_2((-3 + 4d_1)P_1^2 + d_1P_2^2 \\
&\quad - 2P_1(-2 + d_1(2 + P_2)) - 4R^2 + 4d_1R^2)]}{4(-1 + d_1)(-1 + d_2)d_2}
\end{aligned} \tag{10}$$

where, recall that

$$\begin{aligned}
B_{i1}^*(v_i) &= \frac{P_1 - (1 - d_2)v_i}{2d_2}, B_{i3}^*(v_i) = \frac{v_i}{2}, B_{i2}^*(v_i) = \frac{d_1P_2 - (d_1 - d_2)v_i}{2d_2}, \\
V_1 &= \frac{P_1 - d_1P_2}{1 - d_1}, V_3 = \frac{P_1 - 2d_2R}{1 - d_2}, V_6 = 2R.
\end{aligned}$$

We take the derivatives of π in (10) with respect to P_1, P_2, R and set equal to zero, and solve for the optimal solutions as the follows:

$$P_1^* = P_2^* = \frac{2(1 - d_2)}{3 - 4d_2^2}, R^* = \frac{2d_2(1 - d_2)}{3 - 4d_2^2} = d_2P_1^* \tag{11}$$

Furthermore, one can show that the Hessian matrix is negative-definite. Substituting optimal prices P_1^*, P_2^* and R^* into (10), we have the maximum expected revenue π^* ,

$$\pi^* = \frac{1 - d_2}{3 - 4d_2^2} \tag{12}$$

We can see that although there are three channels and four customer segments in this scenario, the service provider only has two sources of revenue: REG and BID as the OPQ channel is not generating sales. This is because the price on OPQ is set the same as that on REG and the threshold on the BID channel is set relatively low so that all the consumers in the segment of BID then OPQ will win their bids and will not switch to OPQ.

Parameters d_1, d_2 need to satisfy constraints obtained by substituting optimal prices P_1^*, P_2^* and R^* as shown in (11) into the conditions in this scenario: $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq -2d_2R(1 - d_1)$, $P_1 - 2d_2R \leq 1 - d_2$, and $P_1 \in [0, 1], P_2 \in [0, 1]$. Here, the constraint is just simply $0 \leq d_2 \leq 1/2$, which has nothing to do with d_1 since the results are only functions of d_2 . Under this constraint, one can show that both the optimal full information price and the maximum expected revenue in this scenario are more than those values in the situation where there is only the REG channel, which are $\frac{1}{2}$ and $\frac{1}{4}$ respectively. Please see the Appendix for the detailed derivation of the results discussed above.

Case I - Scenario II. $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ and $P_2 \geq 2R$.

As the same as in previous scenario, the consumers in the first segment $[V_2, 1]$ (if $V_2 < 1$) buy on REG directly. It is easy to see that $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ is equivalent to

$$\frac{P_1 - 2d_2R}{1 - d_2} < \frac{P_1 - d_1P_2}{1 - d_1}, \quad \text{i.e. } V_3 < V_1.$$

Thus, all the consumers in the segment of BID then REG ($[V_1, \min(V_2, 1))$) lose their bids and go to purchase at REG as their bids are below the threshold R if their valuation is more than V_3 .

One can easily show that $V_3 < V_1$ implies $V_5 < V_1$ and if $P_2 \geq 2R$, then $P_2 \leq V_5$, so the segment of BID then OPQ ($v_i \in [P_2, \min(V_1, 1))$) consists of two groups of consumers. The first group of consumers with valuations $v_i \in [V_5, \min(V_1, 1))$ purchase on OPQ and second group $v_i \in [P_2, \min(V_5, 1))$ wins the bid.

$P_2 \geq 2R$ indicates $P_2 \geq V_6$, then in the segment of bidding only, consumers with valuations $v_i \in [V_6, P_2)$ win their bidding and consumers with valuations $v_i \in [0, V_6)$ lose. The revenue segmentation for the service provider in this scenario is summarized in Table 5.

Thus, if $1 \geq V_1 > V_3$ and $1 \geq V_5 \geq P_2$ i.e. $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$, $P_1 - d_1P_2 \leq 1 - d_1$ and $P_2 \geq 2R$, then the expected revenue π for the service provider

Table 5: Revenue segmentation, Case I - Scenario II

Valuation (v_i)	Transaction channel	Critical Points		
		$1 > V_1$	$V_1 \geq 1 > V_5$	$V_5 \geq 1$
$[V_1, 1]$	REG (P_1)	Present	Absent	Absent
$[V_5, \min(V_1, 1))$	OPQ (P_2)	Present	Present	Absent
$[P_2, \min(V_5, 1))$	BID ($B_{i2}^*(v_i)$)	Present	Present	Present
$[V_6, P_2)$	BID ($B_{i3}^*(v_i)$)	Present	Present	Present

in this scenario is:

$$\begin{aligned}
\pi &= \int_{V_1}^1 P_1 f(v_i) dv_i + \int_{V_5}^{V_1} P_2 f(v_i) dv_i + \int_{P_2}^{V_5} B_{i2}^*(v_i) f(v_i) dv_i + \int_{V_6}^{P_2} B_{i3}^*(v_i) f(v_i) dv_i \\
&= \frac{[4d_2(P_1 - P_1^2 + P_1 P_2 - 2P_2 R) + d_1^2(-4P_1(-1 + P_2) + P_2^2 - 4R^2)]}{4(-1 + d_1)(d_1 - d_2)} \\
&\quad + \frac{d_1(4P_1^2 + 4P_1(-1 + d_2(-1 + P_2) - P_2) + (3 - 4d_2)P_2^2 + 8d_2 P_2 R + 4R^2)]}{4(-1 + d_1)(d_1 - d_2)} \quad (13)
\end{aligned}$$

where, recall that

$$\begin{aligned}
B_{i3}^*(v_i) &= \frac{d_2 v_i}{2}, B_{i2}^*(v_i) = \frac{P_2 - (d_1 - d_2)v_i}{2}, \\
V_1 &= \frac{P_1 - d_1 P_2}{1 - d_1}, V_5 = \frac{d_1 P_2 - 2d_2 R}{d_1 - d_2}, V_6 = 2R.
\end{aligned}$$

As earlier, taking the derivatives of π in (13) with respect to P_1, P_2, R and setting to zero, we solve for the optimal solutions.

$$\begin{aligned}
P_1^* &= \frac{d_1^3 + d_1^2(3 - 4d_2) - 4d_2^2 + 4d_1 d_2^2}{2(d_1^3 - d_1^2(-2 + d_2) + d_1 d_2 - 4d_2^2)} \\
P_2^* &= \frac{d_1(1 + d_1)(d_1 - d_2)}{d_1^3 - d_1^2(-2 + d_2) + d_1 d_2 - 4d_2^2}, \\
R^* &= \frac{d_2(1 + d_1)(d_1 - d_2)}{d_1^3 - d_1^2(-2 + d_2) + d_1 d_2 - 4d_2^2} \quad (14)
\end{aligned}$$

One can show that the Hessian matrix is negative-definite and substituting optimal prices P_1^*, P_2^* and R^* in (13), one can get the maximum expected revenue π^* ,

$$\pi^* = \frac{d_1^3 + d_1^2(3 - 4d_2) - 4d_2^2 + 4d_1 d_2^2}{4(d_1^3 - d_1^2(-2 + d_2) + d_1 d_2 - 4d_2^2)} \quad (15)$$

As before parameters d_1, d_2 need to satisfy a set of constraints; which are obtained by substituting optimal prices P_1^*, P_2^* and R^* as shown in (14) into: $0 \geq (d_1 - d_2)P_1 - d_1 P_2(1 - d_2) > -2d_2 R(1 - d_1), P_1 - d_1 P_2 \leq 1 - d_1, P_2 \geq 2R$, and $P_1 \in [0, 1], P_2 \in [0, 1]$.

Hence, the constraints that d_1, d_2 need to satisfy are shown in (16), (17), (18), and (19). Similar to the previous scenario, under these constraints, one can show that both the optimal

full information price and the maximum expected revenue in this scenario are larger than $\frac{1}{2}$ and $\frac{1}{4}$ respectively. The appendix provides the detailed derivation.

$$d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2 \geq 0 \quad (16)$$

$$d_2 \leq \frac{d_1}{2} \quad (17)$$

$$4d_2^3 + d_1^2(-1 + 2d_2) + d_1(d_2 - 6d_2^2) < 0 \quad (18)$$

$$4d_1d_2^2 - 4d_2^3 + d_1^3(-1 + 2d_2) + d_1^2(d_2 - 2d_2^2) \geq 0 \quad (19)$$

Case I - Scenario III. $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ and $P_2 < 2R$.

As in previous scenarios, consumers in the first segment $[V_2, 1]$ (if $V_2 < 1$) buy on REG directly. And $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$ implies

$$\frac{P_1 - 2d_2R}{1 - d_2} < \frac{P_1 - d_1P_2}{1 - d_1}, \quad \text{i.e. } V_3 < V_1.$$

Hence, all the consumers in the segment BID then REG ($[V_1, \min(V_2, 1))$) lose their bids and purchase at the REG channel as their bids are below the threshold R if their valuation is more than V_3 .

If $P_2 < 2R$, then $V_5 < P_2$, and so all the consumers in the segment of BID then OPQ lose their bids and so switch back to the OPQ channel to buy as their bids are less than the threshold R if the valuation $v_i > V_5$.

$P_2 < 2R$ also indicates $P_2 < V_6$, then in the segment of bidding only, all consumers will lose their bid as $B_i < R$ if their valuation is less than V_6 . In summary, the revenue segmentation in this scenario is given in Table 6.

Table 6: Revenue segmentation, Case I - Scenario III

Valuation (v_i)	Transaction channel	Critical Points	
		$1 > V_1$	$V_1 \geq 1 > P_2$
$[V_1, 1]$	REG (P_1)	Present	Absent
$[P_2, \min(V_1, 1))$	OPQ (P_2)	Present	Present

Therefore, if $1 \geq V_1$ and $P_2 \geq V_5$ i.e. $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$, $P_1 - d_1P_2 \leq 1 - d_1$ and $P_2 < 2R$, then every possible transaction interval in Table 6 is

present and produces π :

$$\begin{aligned}\pi &= \int_{V_1}^1 P_1 f(v_i) dv_i + \int_{P_2}^{V_1} P_2 f(v_i) dv_i \\ &= \frac{(1 - d_1 - P_1 + d_1 P_2)P_1 + P_2(P_1 - P_2)}{1 - d_1}\end{aligned}\quad (20)$$

Taking the first partial derivatives of π with respect to P_1 and P_2 respectively and setting them to zero,

$$P_1^* = \frac{2}{3 + d_1}, P_2^* = \frac{1 + d_1}{3 + d_1} \quad (21)$$

It is easy to show that the Hessian matrix is again negative-definite, substituting P_1^* and P_2^* in (21) into (20), and we have the maximum expected revenue π^* is:

$$\pi^* = \frac{1}{3 + d_1} \quad (22)$$

As shown above REG and OPQ are the only two channels with sales. This happens when the threshold on the BID channel is set so high that the consumers in both BID then OPQ segment and the BID only segment lose their bid. In fact, from the conditions $0 \geq (d_1 - d_2)P_1 - d_1 P_2(1 - d_2) > -2d_2 R(1 - d_1)$, $P_1 - d_1 P_2 \leq 1 - d_1$ and $P_2 < 2R$ we can see that

$$R^* > \max \left\{ \frac{P_2^*}{2}, \frac{-(d_1 - d_2)P_1^* + d_1 P_2^*(1 - d_2)}{2d_2(1 - d_1)} \right\}.$$

By substituting P_1^* and P_2^* shown in (21) back into the inequality above, we have

$$R^* > \max \left\{ \frac{1 + d_1}{2(3 + d_1)}, \frac{2d_2 - d_1(1 - d_2)}{2d_2(3 + d_1)} \right\}.$$

Since

$$\frac{2d_2 - d_1(1 - d_2)}{2d_2(3 + d_1)} - \frac{1 + d_1}{2(3 + d_1)} = \frac{d_2 - d_1}{6d_2 + 2d_1 d_2},$$

so the lower bound of the optimal threshold

$$R_L^* = \begin{cases} \frac{2d_2 - d_1(1 - d_2)}{2d_2(3 + d_1)} & \text{if } d_1 < d_2 \\ \frac{1 + d_1}{2(3 + d_1)} & \text{otherwise} \end{cases} \quad (23)$$

Therefore, the optimal threshold $R^* \in [R_L^*, P_2^*]$. Substituting the optimal solutions P_1^*, P_2^* , and $R^* > P_2^*/2$ back in the conditions $0 \geq (d_1 - d_2)P_1 - d_1 P_2(1 - d_2) > -2d_2 R(1 - d_1)$, $P_1 - d_1 P_2 \leq 1 - d_1$ and $P_2 < 2R$ to get the constraint that d_1 and d_2 need to satisfy is

$d_1 \leq 2d_2/(1 - d_2)$. It is easy to see that $\leq 2d_2/(1 - d_2) > d_2$, thus, the lower bound of the optimal threshold becomes

$$R_L^* = \begin{cases} \frac{2d_2 - d_1(1-d_2)}{2d_2(3+d_1)} & \text{if } d_1 < d_2 \\ \frac{1+d_1}{2(3+d_1)} & \text{if } d_2 \leq d_1 \leq \frac{2d_2}{1-d_2} \end{cases} \quad (24)$$

One can check that under this constraint the optimal REG price and the maximum expected revenue in this scenario are greater than or equal to those values in the situation where there is only REG channel i.e. $\frac{1}{2}$ and $\frac{1}{4}$ respectively. The details are provided in the Appendix.

Case II. $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0 \Leftrightarrow V_4 < V_2 < V_1$.

The market segmentation in Case II was previously summarized in Table 3. Note that this case only exists when $d_1 > d_2$, since when $d_1 \leq d_2$, $d_1P_2(1 - d_2) - (d_1 - d_2)P_1 > 0$.

$V_4 = \frac{d_1P_2}{d_1-d_2}$, $V_5 = \frac{P_2-2R}{d_1-d_2}$ implies $V_4 \geq P_2$, and $V_4 \geq V_5$. And recall that the consumers with valuations $v_i < V_5$ and $v_i > V_6$ will win the bidding in the segments of BID then OPQ and BID only respectively. Hence, we only need to compare the critical points V_5 , P_2 and V_6 to decide for the revenue segments in this subcase. There are two revenue segmentation scenarios based on the relations among d_1, d_2, P_1, P_2 , and R as discussed in the following .

Case II - Scenario I. $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$ and $2R \leq P_2$.

Recall that the segment $[V_1, 1]$ (if $V_1 < 1$) and $[V_4, \min(V_1, 1))$ are the segment of REG only and OPQ only segments respectively. If $2R \leq P_2$, then $V_5 \geq P_2 \geq V_6$, then the segment of BID then OPQ ($v_i \in [P_2, \min(V_4, 1))$) consists of two groups of consumers. The first group of consumers with valuation $v_i \in [V_5, \min(V_4, 1))$ purchases on OPQ and second group $v_i \in [P_2, \min(V_5, 1))$ wins the bid. And in the segment of bidding only, consumers with valuation $v_i \in [V_6, P_2)$ win their bid and consumers with valuation $v_i \in [0, V_6)$ lose. In summary, the revenue segmentation for the service provider in this scenario is given in Table 7.

Table 7: Revenue segmentation, Case II - Scenario I

Valuation (v_i)	Transaction channel	Critical Points		
		$1 > V_1$	$V_1 \geq 1 > V_5$	$V_5 \geq 1$
$[V_1, 1]$	REG (P_1)	Present	Absent	Absent
$[V_5, \min(V_1, 1))$	OPQ (P_2)	Present	Present	Absent
$[P_2, \min(V_5, 1))$	BID ($B_{i2}^*(v_i)$)	Present	Present	Present
$[V_6, P_2)$	BID ($B_{i3}^*(v_i)$)	Present	Present	Present

Thus, if $V_1 \leq 1$ and $V_5 \geq P_2 \geq V_6$ i.e. $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$ and $P_1 - d_1P_2 \leq 1 - d_1$ and $2R \leq P_2$, then every possible transaction interval in Table 7 is present, with π for the service provider in this scenario:

$$\pi = \int_{V_1}^1 P_1 f(v_i) dv_i + \int_{V_5}^{V_1} P_2 f(v_i) dv_i + \int_{P_2}^{V_5} B_{i2}^*(v_i) f(v_i) dv_i + \int_{V_6}^{P_2} B_{i3}^*(v_i) f(v_i) dv_i \quad (25)$$

where, recall that

$$B_{i3}^*(v_i) = \frac{d_2 v_i}{2}, B_{i2}^*(v_i) = \frac{P_2 - (d_1 - d_2)v_i}{2},$$

$$V_1 = \frac{P_1 - d_1 P_2}{1 - d_1}, V_5 = \frac{d_1 P_2 - 2d_2 R}{d_1 - d_2}, V_6 = 2R.$$

As in Case I, Scenario II above that the service provider uses all three channels by setting the appropriate threshold and prices on the channels. And it has the same revenue expression (25) but with slightly different parameter constraints. Therefore, the optimal prices and expected revenue in this scenario are also given by (14), (15) above respectively. One can derive the parameter constraints that need to be satisfied in this scenario by substituting P_1^*, P_2^* and R^* in (14) into conditions $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$, $P_1 - d_1P_2 \leq 1 - d_1$, $2R \leq P_2$, $P_1 \in [0, 1]$, $P_2 \in [0, 1]$, and $P_2 \leq P_1$.

Specifically, the constraints are (16), (17), (18), and

$$4d_1 d_2^2 - 4d_2^3 + d_1^3(-1 + 2d_2) + d_1^2(d_2 - 2d_2^2) < 0, \quad (26)$$

whose inequality sign is just in the opposite direction from the fourth condition (19) in Case I, Scenario II.

Case II - Scenario II. $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$ and $P_1 - d_1P_2 \leq 1 - d_1$ and $2R > P_2$.

Consumers in the segment of $[V_1, 1]$ (if $V_1 < 1$) and $[V_4, \min(V_1, 1))$ buy at REG only and OPQ only respectively. If $2R > P_2$, then $V_5 < P_2 < V_6$, then for both the segments of BID then OPQ ($v_i \in [P_2, \min(V_4, 1))$) and BID only ($v_i \in [0, P_2)$), there are no consumers will win the bids.

Table 8 shows the revenue segmentation in this scenario.

Table 8: Revenue segmentation, Case II - Scenario II			
Valuation (v_i)	Transaction channel	Critical Points	
		$1 > V_1$	$V_1 \geq 1 > P_2$
$[V_1, 1]$	REG (P_1)	Present	Absent
$[P_2, \min(V_1, 1))$	OPQ (P_2)	Present	Present

Therefore, if $1 \geq V_1$ and $P_2 \geq V_5$ i.e. $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$ and $P_1 - d_1P_2 \leq 1 - d_1$ and $2R > P_2$, then every possible transaction interval in Table 8 is present producing expected revenue π :

$$\pi = \int_{V_1}^1 P_1 f(v_i) dv_i + \int_{P_2}^{V_1} P_2 f(v_i) dv_i \quad (27)$$

Substituting optimal prices P_1^* and P_2^* , as shown in 21 and $R^* > P_2^*/2$ back in the conditions $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > 0$ and $P_1 - d_1P_2 \leq 1 - d_1$ and $2R > P_2$ and $P_2 < 2R$ we get $d_1 > 2d_2/(1 - d_2)$ as the parameter constraints in scenario. Note that this is just with an opposite sign from the parameter condition in Case I - Scenario III. As $R^* > P_2^*/2$, we have the lower bound of the optimal threshold

$$R_L^* = P_2^*/2 = \frac{1 + d_1}{2(3 + d_1)},$$

and the optimal threshold will still be $R^* \in [R_L^*, P_2^*]$.

Overall, in the situation where the service provider releases their products on all three channels: REG, OPQ and BID, we have five scenarios of revenue segmentation as summarized in Table 9.

Table 9: Results in revenue segmentation sub-scenarios

Scenario	Optimal price on REG (P_1^*)	Optimal price on OPQ (P_2^*)	Optimal threshold on BID (R^*)	Maximum expected revenue (π^*)	Parameter conditions (d_1, d_2)
Case I-I	$\frac{2(1-d_2)}{3-4d_2^2}$	$\frac{2(1-d_2)}{3-4d_2^2}$	$\frac{2d_2(1-d_2)}{3-4d_2^2}$	$\frac{1-d_2}{3-4d_2^2}$	$0 \leq d_2 \leq \frac{1}{2}$
Case I-II	$\frac{d_1^3 + d_1^2(3-4d_2) - 4d_2^2 + 4d_1d_2^2}{2(d_1^3 - d_1^2(-2+d_2) + d_1d_2 - 4d_2^2)}$	$\frac{d_1(1+d_1)(d_1-d_2)}{d_1^3 - d_1^2(-2+d_2) + d_1d_2 - 4d_2^2}$	$\frac{d_2(1+d_1)(d_1-d_2)}{d_1^3 - d_1^2(-2+d_2) + d_1d_2 - 4d_2^2}$	$\frac{d_1^3 + d_1^2(3-4d_2) - 4d_2^2 + 4d_1d_2^2}{4(d_1^3 - d_1^2(-2+d_2) + d_1d_2 - 4d_2^2)}$	(16), (17), (18), (19)
Case I-III	$\frac{2}{3+d_1}$	$\frac{1+d_1}{3+d_1}$	$\left[\frac{2d_2 - d_1(1-d_2)}{2d_2(3+d_1)}, \frac{1+d_1}{3+d_1} \right]$	$\frac{1}{3+d_1}$	$d_1 < d_2$
	$\frac{2}{3+d_1}$	$\frac{1+d_1}{3+d_1}$	$\left[\frac{1+d_1}{2(3+d_1)}, \frac{1+d_1}{3+d_1} \right]$	$\frac{1}{3+d_1}$	$d_2 \leq d_1 \leq \frac{2d_2}{1-d_2}$
Case II-I	Same as Case I-II				(16), (17), (18), (26)
Case II-II	$\frac{2}{3+d_1}$	$\frac{1+d_1}{3+d_1}$	$\left[\frac{1+d_1}{2(3+d_1)}, \frac{1+d_1}{3+d_1} \right]$	$\frac{1}{3+d_1}$	$d_1 > \frac{2d_2}{1-d_2}$

2.2.1 Illustration of the Optimal Policies

In this section we illustrate optimal policies and the resulting segmentation substituting different values of d_1 and d_2 into the closed-form solutions discussed previously and plot them to illustrate the impact of channel opacity on the revenue, prices, and threshold. As mentioned before that given the values of d_1 and d_2 the optimal policy that the service provider will implement is to set the prices and threshold at the values of the solutions in the revenue segmentation scenario that achieves the maximum expected revenue. Of course, this

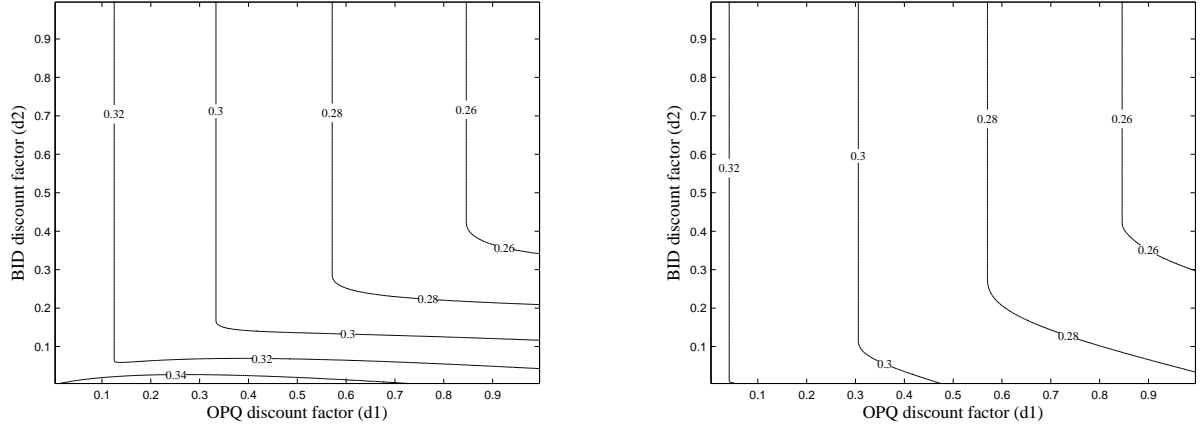


Figure 9: Optimal expected revenue - left unconstrained, right limited capacity

revenue segmentation scenario is based on the corresponding market segmentation formed by consumers self-selection according to the values of d_1 , d_2 , and the prices P_1 and P_2 . Therefore, the service provider sets the prices and threshold on REG, OPQ and BID such that they can induce the market segmentation and produce the desired revenue segmentation given the values of d_1 and d_2 .

We plot the maximum expected revenue that the service provider can obtain while releasing products on REG, OPQ and BID channels and the corresponding optimal prices and threshold set on those channels as given in (9). It is analytically illustrated in the Appendix that the maximum expected revenue and the posted price on REG in all five scenarios are more than those in the base case: REG only, so the maximum expected revenue and corresponding posted price on REG in this three channel case are also greater than those values in the REG only case, which are $\frac{1}{4}$ and $\frac{1}{2}$ respectively. One can also see this property in the plots.

The optimal revenues the firm receives from posting optimal prices at the full information channel (REG) and the opaque channel (OPQ) and the threshold set on the bidding channel (BID) are displayed in Figures 9, 10, 11, 12 respectively.

As shown in Figures 10, 11 prices on REG decrease as BID and OPQ become less opaque, conversely OPQ prices increase (and converge to REG prices) as opacity on OPQ decreases, but decreases as opacity on BID decreases. Figure 12 shows that BID thresholds increase as opacity on BID decreases, but decreases as opacity on OPQ decreases. The impacts of channel opacity on optimal prices and threshold indicate that when the products on the opaque channels (OPQ and BID) become more valuable (less opaque), the price on the full

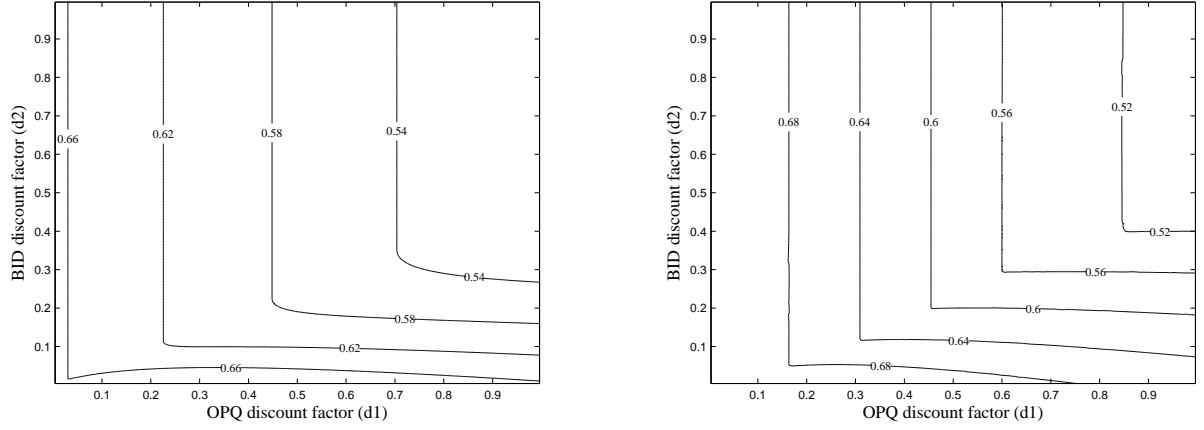


Figure 10: Optimal regular prices - left unconstrained, right limited capacity

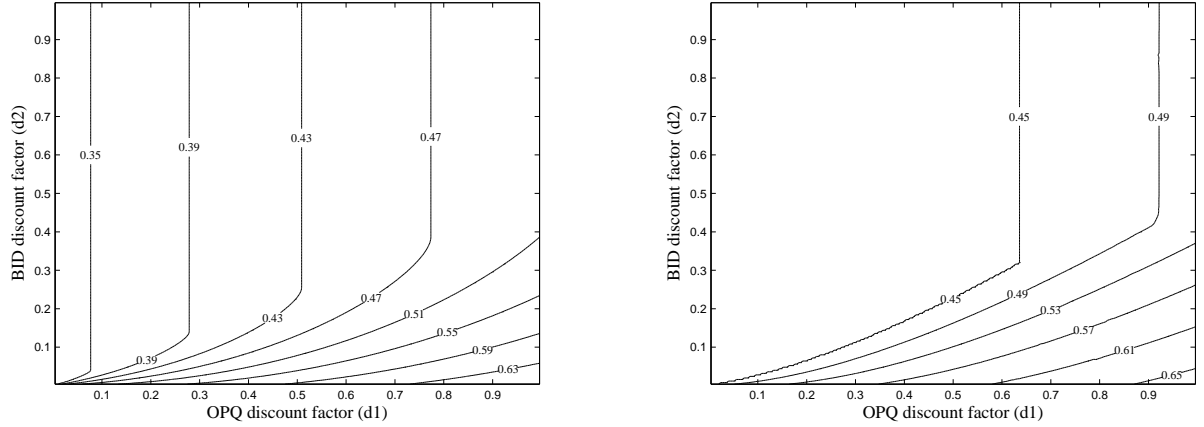


Figure 11: Optimal opaque prices - left unconstrained, right limited capacity

information channel (REG) can not be set too high to lose consumers. Similarly, if the BID channel becomes less opaque, some consumers on OPQ may switch to bid on the BID channel due to a potential chance of getting a better value with less price, vice versa.

As displayed in Figure 9 that the maximum expected revenue decreases when either OPQ or BID channel's opacity degree decreases (d 's increase). This implies that the more opaque those opaque channels are, the more segments in the market and so the service provider can capture more consumers because of their heterogenous valuations of the product.

2.3 Optimal Service Provider Policies with Limited Capacity

In this section, we consider the setting where capacity is limited and as such the firm would logically limit sales at lower prices. We assume that the service provider has a limited

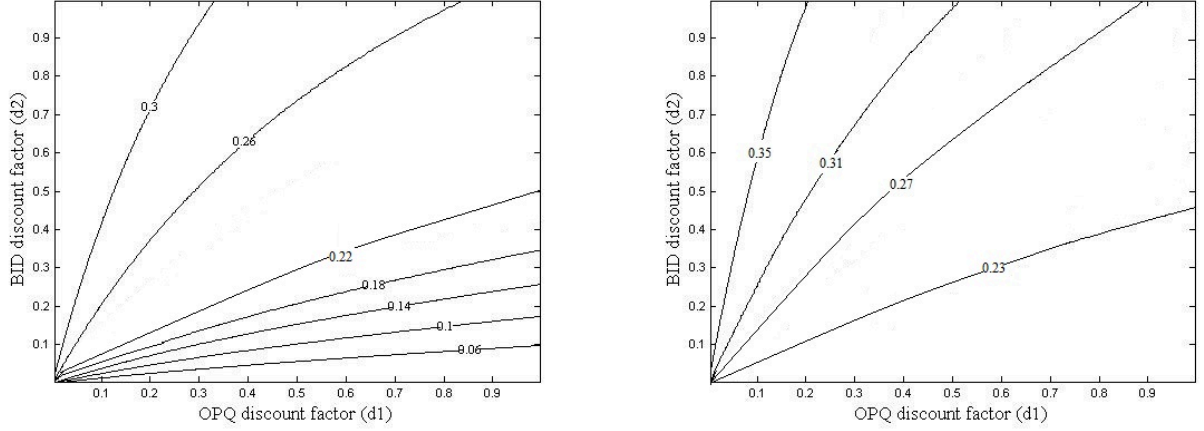


Figure 12: Optimal bidding thresholds - left unconstrained, right limited capacity

inventory of capacity $C < 1$. Although we have the capacity constraint in this case, the consumer segmentation and the revenue segmentation will still be the same as the case with abundant capacity discussed in earlier sections. However, the optimal pricing policy and the maximum expected revenue that the service provider can achieve will depend on capacity.

Here, we assume the customers arrive in a random order, i.e. first come first serve. Thus, the capacity C will be allocated to each segment of the market proportional to its size relative to the total demand in the market, otherwise referred to as random or proportionate splitting.

As an illustration on how the limited capacity influences the service provider's pricing policy and maximum revenue that can be achieved, we assume that the service provider sells the products only on the REG channel and set its price as P_1 .

Similar to the situation with no capacity constraint, consumers with valuation higher than the price P_1 will purchase through this channel. However, demand can be met only up to C . In other words, when the total demand $1 - P_1 \leq C$, the situation is exactly the same as the case with no capacity constraint discussed previously, and so the revenue is $\pi_C = (1 - P_1)P_1$, which reaches the maximum value $\pi_C^* = \frac{1}{4}$ at $P_{1C}^* = \frac{1}{2}$. But when $1 - P_1 > C$, the revenue is $\pi_C = CP_1$. Thus, the price P_1 increases until it reaches the upper bound $1 - C$ and the maximum revenue $\pi_C^* = C(1 - C)$ is achieved.

We summarize the results as the following:

If $1 - C \leq \frac{1}{2}$ i.e. $C \geq \frac{1}{2}$, then $\pi_C^* = \frac{1}{4}$ and $P_{1C}^* = \frac{1}{2}$;

If $1 - C > \frac{1}{2}$ i.e. $C < \frac{1}{2}$, then $\pi_C^* = C(1 - C)$ and $P_{1C}^* = 1 - C$.

One can easily show that when $C < \frac{1}{2}$, $\pi_C^* = C(1 - C)$ is an increasing function in the capacity C , and P_{1C}^* is decreasing in C . These are quite intuitive as when we can not meet all demand we receive less revenue but through higher prices.

The service provider now lists the products on the REG, OPQ and BID simultaneously. They set price P_1 on REG, P_2 on OPQ, a bidding threshold R on BID, and consumers expect that it follows a uniform distribution over $[0, 1]$. In this constrained capacity case, we have three cases as follows, recalling that in the case of no capacity constraint, we have two cases of market segmentation and up to three scenarios in each case based on relations between d_1, d_2 .

Constrained Case I.

Capacity C is allocated proportionally and from (10) we know that the total demand that is supposed to be met if we have enough capacity is $1 - 2R$. Thus, if the conditions $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq -2d_2R(1 - d_1)$, $P_1 - 2d_2R \leq 1 - d_2$, are satisfied and $C \geq 1 - 2R$, then the expected revenue π_C is the same as the revenue π in the case of no capacity constraint. If $C < 1 - 2R$, then the expected revenue π_C is :

$$\begin{aligned}
\pi_C &= \frac{\int_{V_3}^1 f(v_i)dv_i}{1 - 2R} \cdot C \cdot P_1 + \frac{\int_{V_1}^{V_3} f(v_i)dv_i}{1 - 2R} \cdot C \cdot \frac{\int_{V_1}^{V_3} B_{i1}^*(v_i)f(v_i)dv_i}{\int_{V_1}^{V_3} f(v_i)dv_i} \\
&\quad + \frac{\int_{P_2}^{V_1} B_{i2}^*(v_i)f(v_i)dv_i}{1 - 2R} \cdot C \cdot \frac{\int_{P_2}^{V_1} B_{i2}^*(v_i)f(v_i)dv_i}{\int_{P_2}^{V_1} f(v_i)dv_i} \\
&\quad + \frac{\int_{V_6}^{P_2} B_{i3}^*(v_i)f(v_i)dv_i}{1 - 2R} \cdot C \cdot \frac{\int_{V_6}^{P_2} B_{i3}^*(v_i)f(v_i)dv_i}{\int_{V_6}^{P_2} f(v_i)dv_i} \\
&= \frac{C}{1 - 2R} \left[\int_{V_3}^1 P_1 f(v_i)dv_i + \int_{V_1}^{V_3} B_{i1}^*(v_i)f(v_i)dv_i + \int_{P_2}^{V_1} B_{i2}^*(v_i)f(v_i)dv_i \right. \\
&\quad \left. + \int_{V_6}^{P_2} B_{i3}^*(v_i)f(v_i)dv_i \right] \\
&= \frac{C}{1 - 2R} \cdot \pi
\end{aligned} \tag{28}$$

where,

$$\begin{aligned}
B_{i1}^*(v_i) &= \frac{P_1 - (1 - d_2)v_i}{2d_2}, B_{i3}^*(v_i) = \frac{v_i}{2}, B_{i2}^*(v_i) = \frac{d_1P_2 - (d_1 - d_2)v_i}{2d_2}, \\
V_1 &= \frac{P_1 - d_1P_2}{1 - d_1}, V_3 = \frac{P_1 - 2d_2R}{1 - d_2}, V_6 = 2R,
\end{aligned}$$

π = the revenue of the Case I-Scenario I without a capacity constraint.

Constrained Case II

Case I - Scenario II and Case II - Scenario I have the same revenue functions in terms of P_1, P_2, R, d_1 and d_2 as shown in (13), but with different parameter constraints. Thus, in the case with capacity constraint C we combine these two scenarios together, and similar to the Constrained Case I above, the total demand that we need to meet if we have abundant capacity is $1 - 2R$.

If the conditions of the scenarios $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$, $P_1 - d_1P_2 \leq 1 - d_1$, $P_2 \geq 2R$, are satisfied and $C \geq 1 - 2R$, the expected revenue π_C is the same as the revenue π in the case of no capacity constraint. If $C < 1 - 2R$, the expected revenue π_C with capacity C is given as below:

$$\begin{aligned}
\pi_C &= \frac{\int_{V_1}^1 f(v_i)dv_i}{1 - 2R} \cdot C \cdot P_1 + \frac{\int_{V_5}^{V_1} f(v_i)dv_i}{1 - 2R} \cdot C \cdot P_2 + \frac{\int_{P_2}^{V_5} f(v_i)dv_i}{1 - 2R} \cdot C \cdot \frac{\int_{P_2}^{V_5} B_{i2}^*(v_i)f(v_i)dv_i}{\int_{P_2}^{V_5} f(v_i)dv_i} \\
&\quad + \frac{\int_{V_6}^{P_2} f(v_i)dv_i}{1 - 2R} \cdot C \cdot \frac{\int_{V_6}^{P_2} B_{i3}^*(v_i)f(v_i)dv_i}{\int_{V_6}^{P_2} f(v_i)dv_i} \\
&= \frac{C}{1 - 2R} \left[\int_{V_1}^1 P_1 f(v_i)dv_i + \int_{V_5}^{V_1} P_2 f(v_i)dv_i + \int_{P_2}^{V_5} B_{i2}^*(v_i)f(v_i)dv_i \right. \\
&\quad \left. + \int_{V_6}^{P_2} B_{i3}^*(v_i)f(v_i)dv_i \right] \\
&= \frac{C}{1 - 2R} \cdot \pi
\end{aligned} \tag{29}$$

where,

$$\begin{aligned}
B_{i3}^*(v_i) &= \frac{d_2 v_i}{2}, B_{i2}^*(v_i) = \frac{P_2 - (d_1 - d_2)v_i}{2}, \\
V_1 &= \frac{P_1 - d_1 P_2}{1 - d_1}, V_5 = \frac{d_1 P_2 - 2d_2 R}{d_1 - d_2}, V_6 = 2R,
\end{aligned}$$

π = the revenue of the Case II-Scenario I or Case II-Scenario I in the case with no capacity constraint.

Constrained Case III.

Recall that Case I-Scenario III and Case II-Scenario II have the same revenue functions of P_1, P_2, R, d_1 and d_2 as given in (20) but with different parameter constraints. Hence, we combine these two scenarios together in the setting with constrained capacity and note the total demand that we need to meet if we have abundant capacity is now $1 - P_2$.

Thus, if the conditions $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1)$, $P_1 - d_1P_2 \leq 1 - d_1$, $P_2 < 2R$ in the scenarios are satisfied and $C \geq 1 - P_2$, then the expected revenue π_C with capacity

C is the same as the expected revenue π with no constrained capacity. If $C < 1 - P_2$, then the expected revenue π_C given as the follows.

$$\begin{aligned}
\pi_C &= \frac{\int_{V_1}^1 f(v_i) dv_i}{1 - P_2} \cdot C \cdot P_1 + \frac{\int_{P_2}^{V_1} f(v_i) dv_i}{1 - P_2} \cdot C \cdot P_2 \\
&= \frac{C}{1 - P_2} \left[\int_{V_1}^1 P_1 f(v_i) dv_i + \int_{P_2}^{V_1} P_2 f(v_i) dv_i \right] \\
&= \frac{C}{1 - P_2} \cdot \pi
\end{aligned} \tag{30}$$

where,

$$V_1 = \frac{P_1 - d_1 P_2}{1 - d_1},$$

π = the revenue of the Case I-Scenario II or Case II Scenario II sub-scenario I-ii with no capacity constraint.

Similar to the abundant capacity case, we will choose the one out of these three cases that achieves the highest maximum expected revenue when it is feasible and so set the corresponding optimal prices and threshold which are functions of capacity C in this situation. We solve for the optimal prices, threshold and so the resulting maximum expected revenue numerically. The resulting optimal revenues, prices and thresholds for a capacity of 0.55 are shown in concert with the earlier unconstrained results in Figures 9, 10, 11 and 12 respectively.

3. Discussion

Figure 13 displays the channels across which the service provider conducts transactions provided they set optimal prices and thresholds as displayed in Figures 10, 11 and 12. These transactions are a function of consumer self-selected segmentation as illustrated in Figure 14. Figure 15 displays optimal expected revenues for increasing capacity for a selection of d_1 and d_2 values.

Together these figures illustrate the impacts of opaque selling and under what conditions it appears fruitful to consumers and service providers. Firms should always adopt at least two channels, selling via opaque posted prices in addition to regular full information prices. The opaque posted prices simply approach regular full information prices as the opaque channel becomes less opaque - this is consistent across unlimited and constrained capacity settings. Similarly firms should employ opaque bidding but only when opacity of this channel

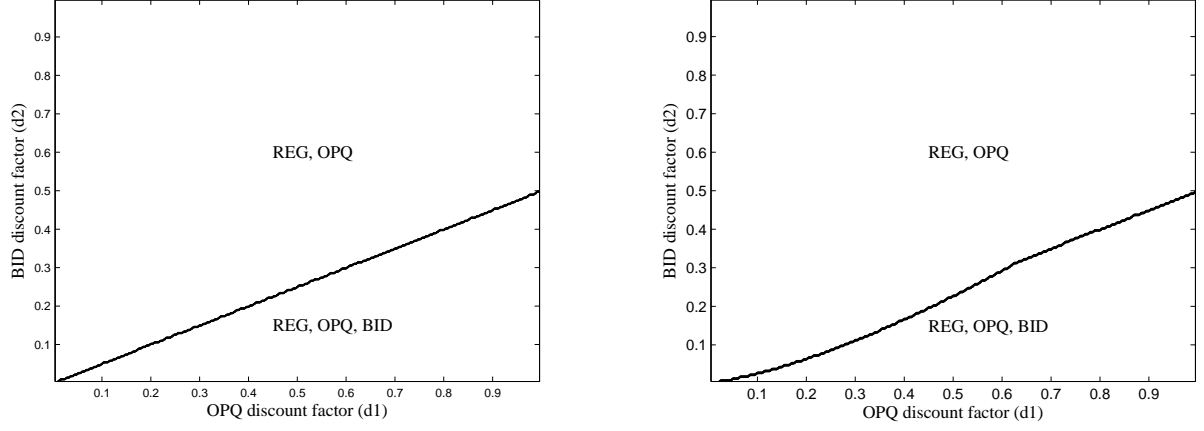


Figure 13: Optimal channel strategies - left unconstrained, right limited capacity

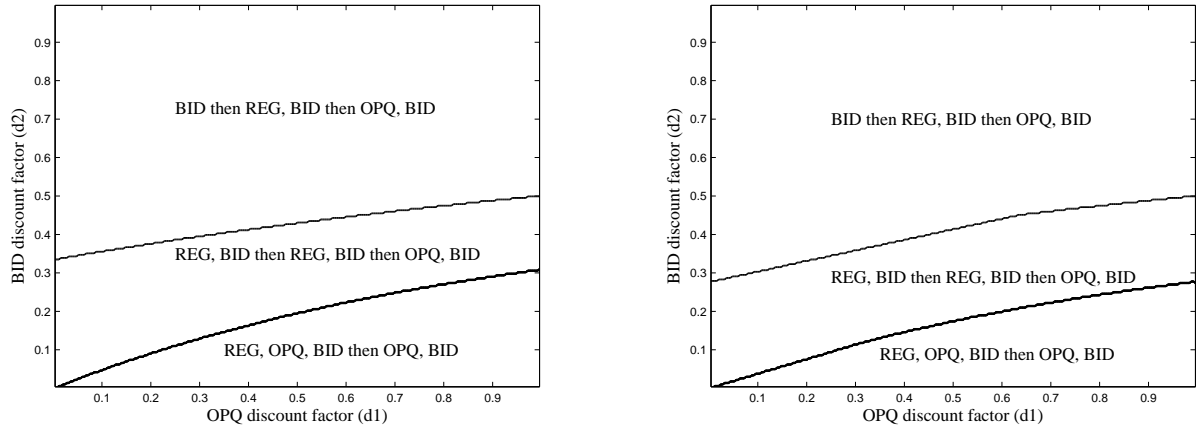


Figure 14: Market segmentation - left unconstrained, right limited capacity

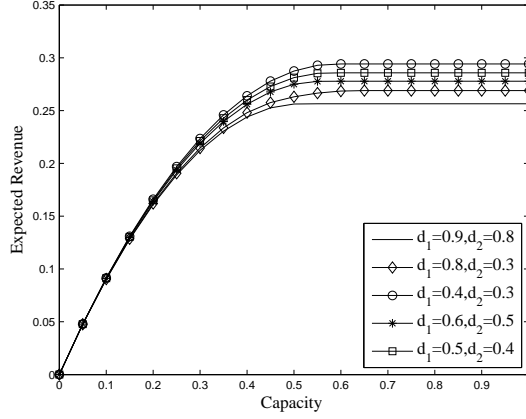


Figure 15: Optimal expected revenue as a function of capacity

is significant - here $d_2 < \frac{1}{2}$. It is important to realize that the firm should always be using all three channels, with posted opaque prices/thresholds set too high such that no transactions occur under conditions of decreased capacity. As capacity becomes tighter, the required degree of opacity increases (for continued use of opaque channels) as do prices and thresholds.

As indicated earlier, and as displayed in Figure 9 that the maximum expected revenue decreases as opacity decreases. This implies that the more opaque those opaque channels are, the more segments in the market and so the service provider can capture more consumers because of their heterogenous valuations of the product. This is consistent with what we see in practice as opaque channels tend to separate themselves along degrees of opacity, for example Hotwire.com provides information of hotel amenities as well as feedback from recent guests whereas Priceline.com provides neither on its NYOP bidding channel indicating Hotwire is probably less opaque than Priceline.

In summary we have developed a stylized model of when and how to deploy an opaque selling strategy in concert with regular full information pricing. Unlike previous research which usually assumes an exogenous consumer separation into regular consumers and opaque consumers we endogenously model this channel selection process as a function of prices and channel characteristics (opacity). We have shown that even in the face of capacity constraints firms should be using opaque channels whereas historically focus has been on using opaque channels to sell distressed or otherwise unsellable inventory (surplus capacity). The simultaneous use of opaque selling with regular full information selling, effectively segments consumers - allowing firms to sell at higher prices to higher valuation/brand loyal consumers

and at lower prices to lower valuation/brand agnostic shoppers via opaque channels.

4. References

- Anderson, C.K. 2009. Setting Prices on Priceline. *Interfaces*. 39(4) pp. 307-215.
- Anderson, C.K. and X. Xie. 2011. A Choice Based Dynamic Programming Approach for Setting Opaque Prices. Forthcoming *Production and Operations Management*.
- Ding, M., Eliashberg, J., Huber, J., Saini, R. 2005. Emotional bidders: An analytical and experimental examination of consumers behavior in a priceline-like reverse auction. *Management Science* 51(3): 352-364.
- Etzion, H., Pinker, E. and A. Seidmann. 2006. Analyzing the Simultaneous Use of auctions and Posted Prices for Online Selling. *Manufacturing and Service Operations Management* 8(1) pp.68-91.
- Fay, S. 2004. Partial-Repeat-Bidding in the Name-Your-Own-Price Channel. *Marketing Science* 23(3) pp. 207-418.
- Fay, S. 2008. Selling an Opaque Product through an Intermediary: The Case of Disguising One's Product. *Journal of Retailing* 84(1) pp. 59-75.
- Hann, I., and C. Terwiesch. 2003. Measuring the Frictional costs of Online Transactions: The Case of a Name-Your-Own-Price Channel. *Management Science* 49(11) pp. 1563-1579.
- Hotelling, H. 1929. Stability in competition. *Economic Journal* 29 pp. 41-57.
- Jerath, K., S. Netessine, and S.K. Veeraraghavan. 2007. Revenue Management with Strategic Customers: Last-Minute Selling and Opaque Selling. Working Paper The Wharton School, University of Pennsylvania.
- Jiang, Y. 2007. Price Discrimination with opaque products. *Journal of Revenue and Pricing Management* 6(2) pp. 118-134.
- Shapiro, D. and X. Shi. 2008. Market Segmentation: The role of opaque travel agencies. Working paper, Belk College of Business, University of North Carolina.
- Spann, M., B. Skiera, and B. Schaefers. 2004. Measuring Individual Frictional Costs and Willingness to Pay via Name-Your-Own-Price Mechanisms. *Journal of Interactive Marketing* 18(4) pp.22-36.

- van Ryzin, G. and G. Vulcano. 2004. Optimal auctioning and ordering in an infinite horizon inventory-pricing system. *Operations Research* **52(3)** pp. 346-367.
- Wang, T., E. Gal-Or, and R. Chatterjee. 2005. Why a Name-Your-Own-Price Channel Makes Sense For Service Providers (Or: Who Needs Priceline, Anyway?). Working Paper, Katz Graduate School of Business, University of Pittsburgh.
- Wilson, J. G., G. Zhang. 2008. Optimal design of a name-your-own price channel. *J. Revenue Pricing Management* **7(3)** 281–290.

Appendix

This section includes the detailed discussion and derivation for some of the results in Case I - Scenarios I, II and III in section 2.2.

Case I - Scenario I.

Here, we derive the constraints that parameters d_1, d_2 need to satisfy in this sub-scenario by substituting optimal prices P_1^*, P_2^* and R^* as shown in (11) into the conditions in this scenario: $(d_1 - d_2)P_1 - d_1P_2(1 - d_2) \leq -2d_2R(1 - d_1), P_1 - 2d_2R \leq 1 - d_2, P_1 \in [0, 1], P_2 \in [0, 1],$.

$$\begin{aligned}
0 \leq P_1 \leq 1 &\Rightarrow 0 \leq d_2 \leq \frac{\sqrt{5} + 1}{4}; \\
(d_1 - d_2)P_1 - d_1P_2(1 - d_2) &\leq -2d_2R(1 - d_1) \\
\Rightarrow (d_1 - d_2) - d_1(1 - d_2) &\leq -2d_2^2(1 - d_1) \Rightarrow 0 \leq d_2 \leq \frac{1}{2}; \\
P_1 - 2d_2R &\leq 1 - d_2 \\
\Rightarrow \frac{2(1 - 2d_2^2)}{3 - 4d_2^2} &\leq 1 \text{ which is true given } 0 \leq d_2 \leq \frac{1}{2} \text{ above.} \tag{31}
\end{aligned}$$

Therefore, we obtain the constraint that the parameter d_2 needs to satisfy, which is $0 \leq d_2 \leq 1/2$. Under this constraint, one can show that both the optimal full information price and the maximum expected revenue in this sub-scenario are more than those values in case I where there is only the REG channel, which are $1/2$ and $1/4$ respectively, since we have the follows:

$$\begin{aligned}
P_1^* - \frac{1}{2} &= \frac{2(1 - d_2)}{3 - 4d_2^2} - \frac{1}{2} = \frac{(1 - 2d_2)^2}{3 - 4d_2^2} \geq 0 \text{ if } d_2 \leq \frac{1}{2} \\
\pi^* - \frac{1}{4} &= \frac{1 - d_2}{3 - 4d_2^2} - \frac{1}{4} = \frac{(1 - 2d_2)^2}{4(3 - 4d_2^2)} \geq 0 \text{ if } d_2 \leq \frac{1}{2}
\end{aligned}$$

Case I - Scenario II.

Similar to the above scenario, we derive the constraints that parameters d_1, d_2 need to satisfy by substituting optimal prices P_1^*, P_2^* and R^* as shown in (14) into the conditions in this sub-scenario: $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1), P_1 - d_1P_2 \leq 1 - d_1, P_2 \geq 2R,$ and $P_1 \in [0, 1], P_2 \in [0, 1].$

$$P_2 \geq 0 \Rightarrow d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2 \geq 0 \tag{32}$$

$$P_2 \geq 2R \Rightarrow d_2 \leq \frac{d_1}{2} \tag{33}$$

$$\begin{aligned}
& (d_1 - d_2)P_1 - d_1P_2(1 - d_2) > -2d_2R(1 - d_1) \\
\Rightarrow & \frac{(-1 + d_1)d_1(4d_2^3 + d_1^2(-1 + 2d_2) + d_1(d_2 - 6d_2^2))}{2(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} > 0 \\
\Rightarrow & 4d_2^3 + d_1^2(-1 + 2d_2) + d_1(d_2 - 6d_2^2) < 0 \tag{34} \\
& \text{since constraint (32) above and } d_1 \leq 1
\end{aligned}$$

$$\begin{aligned}
& 0 \geq (d_1 - d_2)P_1 - d_1P_2(1 - d_2) \\
\Rightarrow & \frac{(-1 + d_1)(4d_1d_2^2 - 4d_2^3 + d_1^3(-1 + 2d_2) + d_1^2(d_2 - 2d_2^2))}{2(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} \leq 0 \\
\Rightarrow & 4d_1d_2^2 - 4d_2^3 + d_1^3(-1 + 2d_2) + d_1^2(d_2 - 2d_2^2) \geq 0 \tag{35} \\
& \text{since constraint (32) above and } d_1 \leq 1
\end{aligned}$$

On the other hand, constraint (33) $d_2 \leq d_1/2 \Rightarrow d_1^2 + 2d_1d_2 - 4d_2^2 \geq 0$, so combining this with constraint (32) gives us the following:

$$\begin{aligned}
P_1 - 1 &= -\frac{(1 + d_1)(d_1^2 + 2d_1d_2 - 4d_2^2)}{2(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} \leq 0 \\
\frac{P_1 - d_1P_2}{1 - d_1} - 1 &= -\frac{d_1^2 + 2d_1d_2 - 4d_2^2}{2(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} \leq 0
\end{aligned}$$

Therefore, (32), (33), (34), and (35) are the constraints that parameters d_1, d_2 need to satisfy in this scenario in order that the optimal solutions (14) and optimal expected revenue (15) are feasible.

Under the constraints (32), (33), (34), and (35), one can show that both the optimal full information price and the maximum expected revenue in this scenario are also more than $1/2$ and $1/4$ respectively. In fact, we have

$$\begin{aligned}
\pi^* - \frac{1}{3 + d_1} &= \frac{(1 + d_1)^2(d_1 - 2d_2)^2}{4(3 + d_1)(d_1^3 - d_1^2(-2 + d_2) + d_1d_2 - 4d_2^2)} \\
&\geq 0 \quad \text{given condition (32)} \tag{36}
\end{aligned}$$

On the other hand,

$$\frac{1}{3 + d_1} \geq \frac{1}{4} \text{ for } 0 \leq d_1 < 1.$$

Therefore, $\pi^* \geq 1/4$. Since $P_1^* = \pi^*/2$, so $P_i^* \geq 1/2$.

Case I - Scenario III. Similarly, we obtain the constraints that d_1, d_2 need to satisfy by plugging the optimal solutions P_1^*, P_2^* , and R^* into the conditions $0 \geq (d_1 - d_2)P_1 - d_1P_2(1 -$

$d_2) > -2d_2R(1 - d_1)$, $P_1 - d_1P_2 \leq 1 - d_1$ and $P_2 < 2R$. Note that $P_1^* = 2P_2^*/(1 + d_1)$, $R^* > P_2^*/2$, $P_1 \in [0, 1]$, and $P_2 \in [0, 1]$.

In fact,

$$\begin{aligned}
& (d_1 - d_2)P_1^* - d_1P_2^*(1 - d_2) + 2d_2R(1 - d_1) \\
> & P_2^* \left[\frac{2}{1 + d_1} (d_1 - d_2) - d_1(1 - d_2) + d_2(1 - d_1) \right] \\
= & P_2^* \left[\frac{2}{1 + d_1} (d_1 - d_2) - (d_1 - d_2) \right] \geq P_2^* [d_1 - d_2 - (d_1 - d_2)] \geq 0;
\end{aligned} \tag{37}$$

$$\frac{P_1^* - d_1P_2^*}{1 - d_1} - 1 = \frac{-1}{3 + d_1} < 0;$$

$$(d_1 - d_2)P_1^* - d_1P_2^*(1 - d_2) = \frac{(1 - d_1)(d_1 - d_1d_2 - 2d_2)}{3 + d_1} \leq 0 \Rightarrow d_1 \leq \frac{2d_2}{1 - d_2} \tag{38}$$

Thus, $d_1 \leq 2d_2/(1 - d_2)$ is the parameter constraint in this scenario.

It is easy to see that the optimal full information price $\frac{2}{3+d_1}$ and the maximum expected revenue $\frac{1}{3+d_1}$ in this scenario are greater than or equal to those values in the situation where there is only the REG channel i.e. $1/2$ and $1/4$ respectively, since we have the following:

$$P_1^* = \frac{2}{3 + d_1} > \frac{2}{3 + 1} = \frac{1}{2}, \text{ and } \pi^* = \frac{1}{3 + d_1} > \frac{1}{3 + 1} = \frac{1}{4}$$