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DISCUSSION PAPER

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Revenue management for operations with urgent orders

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Abstract

This article is motivated by the case of a company manufacturing industrial equipment that faces two types of demand: on the one hand there are the so-called regular orders for installations or refurbishing of existing facilities, these orders have a relatively long lead time; on the other hand there are urgent orders mostly related to spare parts when a facility has a breakdown, the delay in such case is much shorter but higher margins can be obtained. We study the order acceptance problem for a firm that serves two classes of demand over an infinite horizon. The firm has to decide whether to accept a regular order (or equivalently how much capacity to set aside for urgent orders) in order to maximize its profit. We formulate this problem as a multi-dimensional Markovian Decision Process (MDP). We propose a family of approximate formulations to reduce the dimension of the state space via aggregation. We show how our approach can be used to compute bounds on the profit associated with the optimal order acceptance policy. Finally, we show that the value of revenue management is commensurate with the operational flexibility of the firm.

Keywords: order acceptance, revenue management, Markov decision process, heuristics, flexibility.

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1 Introduction

This article is motivated by the case of a cast iron manufacturer. This company is specialized in the production of cast iron pieces for industrial equipments. Most of its orders are either for the preventive maintenance of installations or for the building of new facilities. Such projects are scheduled with long lead times, but it is very important that the pieces be delivered on time because the plant where the pieces have to be installed will have to be (at least partially) stopped for the maintenance or installation activities to take place and obviously the duration of such stoppage should be minimized. A different type of orders received by the company corresponds to corrective maintenance when a breakdown occurs in a plant, in those cases a new cast iron piece is needed to restart the facility. Given that production at the customer is stopped because of the breakdown, a much faster service is required but the company can charge higher prices for such "emergency" orders. Moreover, the bargaining power of the customer is much weaker in such circumstances. Given its finite production capacity, the company cannot always accept all orders and should sometimes forego a regular order in order to keep some possibility to accept an urgent order later on. This dilemma is faced by many suppliers confronted with urgent requests that are potentially very profitable but could be very disruptive if not taken into account in their planning. Typical examples from the service sector include heating ventilation air-conditioning (HVAC) companies. The installation of new systems is typically a large project with a relatively long lead time. In contrast, when a system fails it could block the operations of a customer that is then willing to pay a higher fee for speedy action. In a very different context, suppliers of the fashion industry are known to combine orders from large vendors and more profitable orders for high fashion clothes which often require fast delivery (see e.g. Harris and Pinder, 1995; Barut and Sridharan, 2005). The question facing the supplier is how much capacity should be set aside for the urgent high margin demand, given the inherent unpredictability of this type of orders.

To address this question, we build a model with a supplier that handles two demand classes, that we will refer to as regular and urgent respectively. The regular orders are typically characterized by longer processing times, longer lead times but lower margins, while the urgent orders have shorter processing times, shorter lead-times and higher margins. If the supplier accepts orders without foresight it is likely that at some point, when an urgent order arrives, the supplier will be unable to accept this order as her short term capacity is already entirely committed for regular orders (that were booked earlier with a longer lead time). Given the difference in margin between the two classes, this situation causes some loss of revenue. On the other hand, rejecting a regular order in anticipation for potential urgent orders that do not materialize, also causes some revenue loss.

This tradeoff has clear similarities with other revenue management problems. The distinct feature is that when an order is accepted the supplier keeps some flexibility. For example, if a regular order necessitates 10 days of work and the lead time is 20 days, in most cases the customer does not care about the days during which the order is effectively produced as long as it is finished on time. In revenue management terms, if we consider that the capacity available during each period is a distinct *product*, an order requiring more than one period of work is in fact reserving several *products*. But the supplier has some flexibility in assigning the *products* to the order and does not need to make a commitment at the time of reservation. One can draw a parallelism with the network revenue management problem but where the supplier can accept a reservation without committing to specific legs in the network, the only commitment is on the origin and destination points in the network.

Our first contribution is to derive a Markov Decision Process (MDP) formulation of the problem. Likewise for the network revenue management problem, the size of this formulation quickly excludes the possibility of solving it exactly for larger size instances. We develop a family of approximate formulations parametrizable to range from a coarse approximation to the original full formulation. This makes it possible to choose between speed of solution and precision of result. What is of particular interest is that for each formulation, we can compute an upper and lower bound on the exact result. This last feature is rather uncommon for revenue management problems and is particularly interesting to make sure the adequate level of approximation is chosen in the proposed family of formulations. Through a numerical study we show that the proposed heuristics allow to obtain near-optimal solutions in a tractable time.

A second contribution is to show how the potential benefit of revenue management is commensurate with operational flexibility. In our setting operational flexibility consists in the slack between the promised lead time for an order class and the processing time needed for such order. To the best of our knowledge, this link between flexibility and revenue management has not been studied so far.

1.1 Related Literature

Our work belongs to the growing literature of Perishable Asset Revenue Management (PARM) which deals with the problem of allocation of scarce resources to different demand classes. Talluri and van Ryzin (2004) give a comprehensive overview of this topic. The first applications were for the airline industry by Littlewood (1972), and extended by Belobaba (1987), Wollmer (1992), and Brumelle and McGill (1993). In addition to airlines, typical service applications are in hotel management and car rental (Kimes, 1989; Bertsimas and Popescu, 2003; Talluri and van Ryzin, 2004; Bitran and Mondschein, 1995; Geraghty and Johnson, 1997). Gradually, new applications appeared for very different environments such as: MTO manufacturing (Balakrishnan et al., 1996; Barut and Sridharan, 2005; Spengler et al., 2007), project management (Herbots et al., 2007, 2010) and health care (Gupta and Wang, 2008; Dobson et al., 2011). A noteworthy example is Kapuscinski and Tayur (2007) that propose a dynamic programming approach to address the problem of lead-time quotation for multiple demand classes when customers are not equally sensitive to waiting. Once the lead time is quoted, enough capacity must be reserved to ensure on-time delivery. In contrast, our work considers the lead-time decisions as exogenously given and it focuses on order acceptance decisions.

Gupta and Wang (2007) consider an acceptance decision problem in which the lead time of urgent orders is a soft operational constraint. The authors formulate the problem by assuming that a tardiness cost is incurred if the orders are not filled at the end of each period. The authors propose a multi-dimensional MDP whose optimal solution turns out to be a threshold based policy. This solution property is a consequence of the well-structured value function that describes the problem. In contrast to their work, we consider the order lead time of both demand streams as hard operational constraints, in other words, tardiness is not allowed.

The following references focus on acceptance decision problems where the lead times must be strictly respected. Germs and Foreest (2011) study an order acceptance problem with multiple customer classes with a common deadline, setup times and scheduling constraints. The problem is modelled as a Markov chain controlled by a threshold policy. The authors provide a numerical study for small instances which are computationally tractable. In contrast to their work, we provide efficient alternative methods to treat the state space explosion. Barut and Sridharan (2005) study an order acceptance problem involving multiple demand classes that differ in terms of price, lead time and demand pattern. The authors propose a nested rationing policy which fulfils incoming orders as much as possible while preserving a certain level of capacity for more profitable future orders. The proposed policy is computed using a myopic heuristic method, that does not take the evolution of the capacity into account. Consequently, the efficiency of the heuristic is hurt by the simplified estimation of the future available capacity. Our formulation keeps track more accurately of the capacity evolution for this type of problem, leading to highly efficient policies.

Our work is closely related to the approximate dynamic programming literature, where the aim is to reduce the computation time required to solve large instances of dynamic problems. For a comprehensive review of the different existing techniques refer to Powell (2011). Specifically, several studies introduce state aggregation techniques in order to deal with the "curse of dimensionality" for problems involving decisions over an infinite horizon (see e.g. Bean et al., 1987; Alden and Smith, 1992). The solution procedures provided in these studies suppose a finite rolling horizon whose length is a parameter which controls a trade-off between the optimality and the computation time. Thus, the optimal solution for the finite horizon is computed, implemented, and then the process is repeated for the next period. In this paper, we apply aggregation techniques for a problem of order acceptance when customer classes differ in their lead times. We aggregate some information of the states of the system within the lead time window of the incoming orders. Thereby, the infinite horizon problem is solved, but the state space of the problem is reduced.

There are some similarities with the network revenue management problem where the dimension of the MDP models quickly make it impossible to solve exactly even small size problems. Consequently, research in this area has concentrated on different approximation techniques, notable examples of this stream of investigation include Bitran and Mondschein (1995); Talluri and van Ryzin (1998); Bertsimas and Popescu (2003); Adelman (2007); Kunnumkal and Topaloglu (2008); Zhang and Adelman (2009); Zhang (2011). The flexibility aspect present in the problem studied here calls for different types of approximations.

The remainder of the paper is structured as follows. In Section 2 we provide a detailed description of our problem. We introduce an MDP formulation and discuss its resolution to obtain the optimal admission policy in Section 3. In Section 4 we propose two heuristic formulations of the problem based on different levels of state aggregation and report on a numerical study of our proposed formulations in Section 5. Section 6 investigates the impact of operational flexibility on the benefit of revenue management. Finally, Section 7 summarizes the main conclusions and identifies future research directions.

2 Model Description

We consider the order acceptance problem for a firm serving two customer classes with different profit margins and lead times. The time horizon is infinite and consists of discrete periods. In each period, the firm is subject to a limited processing capacity, which is normalized to 1.

We consider that all uncertainty about the processing time is known when the order is placed. This assumption is motivated by two reasons: on the one hand, in practice most uncertainty is resolved during the ordering process; on the other hand, if the remaining uncertainty is too large it is not possible to promise due dates without either large safety lead times or a low utilization. In the cases that motivated our work, we observed that the remaining processing time uncertainty after the order is placed is dealt with using some type of recourse action such as overtime or renegotiation of the due-date. These recourse actions play only a secondary role in the management of capacity and are beyond the scope of our investigation.

The two classes of demand will be denominated, urgent and regular orders (indexed by $k = \{1, 2\}$, respectively). The demand (in terms of processing time) for class k at time t will be denoted D_{kt}. We suppose the random variables take integer values and are iid between time periods and independent between classes. If $D_{kt} = 0$ there is no demand for class k in period t. The profit margin per unit capacity of a class k customer is r_k and its lead time is L_k. Urgent orders are more lucrative but come with a short lead time, while regular orders are not as profitable, yet have a looser lead time. Accordingly, we assume $r_1 > r_2 > 0$ and $L_1 < L_2$. Figure 1 shows the structure of the problem. We suppose that only a single order of each class can arrive during any period t, this means that the demand D_{kt} cannot be partially accepted, the firm either accepts the order and hence commits to deliver the order before its due date or declines the order and gets no revenue.

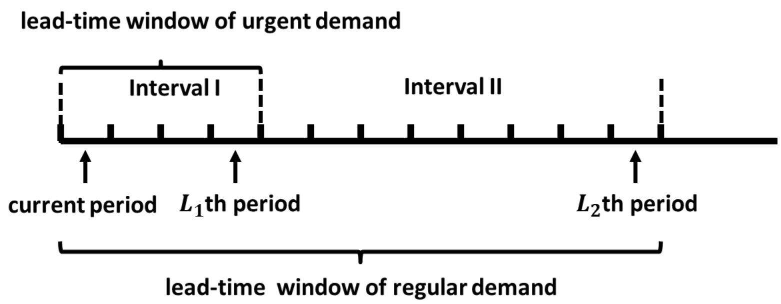


Figure 1: Partition of Lead Time Window of Regular Demand.

Consequently, the main decision faced by the firm is how many regular orders to accept in order to maximize its long-run net profit. In each period, the sequence of events is as follows. A regular order, if any, arrives first, and the firm decides whether to accept it or not. Then, an urgent order, if any, arrives, it will be accepted as long as there is enough available capacity. Finally, the firm uses the capacity in the current period for processing the order with the earliest due date. In fact, the sequence of arrivals does not matter; one can assume that the urgent order arrives before the regular order, or both arrive simultaneously, without complicating the model.

In the remainder of the paper we will assume a somewhat simpler structure for the demand. We assume

$$D_{kt} = \begin{cases} 0 & \text{with probability } 1 - p_k \\ B_k & \text{with probability } p_k \end{cases}$$

This simplifies the notations in the following sections and our numerical tests indicate that the distribution of the demand has no qualitative impact on our results.

3 MDP Formulation for the Optimal Steady-State Policy

The crux of developing the MDP formulation of this problem is to find an efficient representation of the usage of future capacity. Our representation builds on the idea that when an order is accepted, some future capacity will be "reserved". By reservation we mean the provisional allocation of capacity to meet the requirements of an accepted order on time. The reservation consists of allocating provisionally the available capacity of the latest periods to process an order without incurring tardiness. This gives the maximum flexibility to accept the future orders. Note that, the allocation is provisional because the firm may start processing earlier the order if there is no order to deliver before this one. When an order is (partially) processed before its provisionally allocated time slot, some capacity is freed to process future orders. For example, when accepting an order with a lead time of 5 and an order size of 2, the capacity of 4th and 5th periods from the current period shall be reserved (provided that they are available for reservation), but this capacity reservation could change in the next 4 periods if there is no order with an earlier due date.

To reserve capacity for coming orders, the firm needs to calculate the total available capacity for reservation within the lead time windows of regular and urgent orders. Since the lead time window of urgent orders is contained in that of the regular orders, it suffices to calculate the total available capacity within L_1 periods, and that between (L_1+1)th and L_2th periods, i.e. Interval I and Interval II of Figure 1, respectively. However, this aggregate information cannot fully characterize the evolution of the system. Note that the (L_1+1)th period, (i.e., the first period of Interval II), will be shifted by one period and thus become the L_1th period in the next period, (i.e., the last period of Interval I). Without the information regarding how reserved capacity is distributed in Interval II, it is impossible to know whether the shifted capacity is reserved or not in order to update the available capacity in Interval I and II in the next period. Therefore, it is necessary to keep a track of the distributional information in Interval II, but only of aggregate information in Interval I.

We now introduce the notation to be used in our formulation.

- $\mathbf{x}$: is the reservation vector, it keeps track of capacity that has been reserved for processing; $\mathbf{x}[0] \in \{0, 1, \ldots, L_1\}$ denotes the total reserved capacity till L_1th period, (i.e., in Interval I). For $j = 1, 2, \ldots, L_2 - L_1$, $\mathbf{x}[j] = 1$ if the capacity of (L_1+j)th period is reserved, and $\mathbf{x}[j] = 0$ otherwise. Note that in a reservation vector we do not distinguish whether the capacity is reserved for urgent orders or regular orders.

- $\mathbf{y}$: is the cumulative (available capacity) vector[1] of $\mathbf{x}$; for $j = 0, 1, \ldots, L_2 - L_1$, $\mathbf{y}[j]$ denotes the *total* available capacity till the (L_1+j)th period, i.e., $\mathbf{y}[j] = L_1 + j - \sum_{i=0}^{j} \mathbf{x}[i]$. For a given cumulative vector $\mathbf{y}$, its corresponding reservation vector $\mathbf{x}$ can be calculated as follows: $\mathbf{x}[0] = L_1 - \mathbf{y}[0]$, and for $j = 1, 2, \ldots, L_2 - L_1$, $\mathbf{x}[j] = \mathbf{y}[j-1] - \mathbf{y}[j] + 1$.

- a: represents the admission decision for regular orders; $a = 1$ if the firm "admits" the regular order, and $a = 0$ otherwise.

- $\mathbf{D} \equiv (D_1, D_2)$: is the demand vector in each period.

- $R(\mathbf{x}, \mathbf{D}, a)$: denotes the profit generated from $\mathbf{D}$ for a given admission decision a, if the reservation vector at the beginning of the current period is $\mathbf{x}$.

We define the system state as the reservation vector $\mathbf{x}$ at the beginning of a period, before the arrival of regular and urgent orders. It is easy to check that $\mathbf{x}[L_2 - L_1] = 0$ for any system state, because the last element of the system state cannot be reserved by orders that arrived in earlier periods. Thus, the system state space essentially involves $L_2 - L_1$ variables and its size is $(L_1+1) \cdot 2^{L_2 - L_1 - 1}$. Let $\widetilde{\mathbf{x}}$ be the reservation vector updated from $\mathbf{x}$ after accepting/rejecting D_2. Additionally, let $\widehat{\mathbf{x}}$ be the system state in the next period, this is the reservation vector updated from $\widetilde{\mathbf{x}}$ after accepting/rejecting D_1 and processing, if any, is carried out in the current period. Further, let $\widetilde{\mathbf{y}}$ and $\widehat{\mathbf{y}}$ be the corresponding cumulative vectors of $\widetilde{\mathbf{x}}$ and $\widehat{\mathbf{x}}$, respectively. Thus, $\widehat{\mathbf{x}}$ and $\widehat{\mathbf{y}}$ are functions of $\mathbf{x}$, $\mathbf{D}$ and a: $\widehat{\mathbf{x}} = \widehat{\mathbf{x}}(\mathbf{x}, \mathbf{D}, a)$, $\widehat{\mathbf{y}} = \widehat{\mathbf{y}}(\mathbf{x}, \mathbf{D}, a)$. According to the average reward criteria (Ross 1995, Chapter 5), we provide the dynamic programming formulation as follows,

$$V(\mathbf{x}) + g = \mathbb{E}_{\mathbf{D}} \left\{ \max_a \left\{ R(\mathbf{x}, \mathbf{D}, a) + V\left(\widehat{\mathbf{x}}(\mathbf{x}, \mathbf{D}, a)\right) \right\} \right\}, \forall \mathbf{x}, \tag{1}$$

[1] $\mathbf{y}$ is introduced to facilitate the notations related to transitions between system states, which are composed of reservation vectors. There is a one-to-one correspondence between a reservation vector and its cumulative vector.

together with the dynamics described by equations (2)-(7),

$$\widetilde{\mathbf{y}}[j] = \begin{cases} \mathbf{y}[j] - (D_2 - (\mathbf{y}[L_2 - L_1] - \mathbf{y}[j]))^+, & \text{if } a = 1 \text{ and } \mathbf{y}[L_2 - L_1] \geq D_2, \\ \mathbf{y}[j], & \text{otherwise,} \end{cases} \tag{2}$$

$$\text{for } j = 0, 1, , \ldots, L_2 - L_1,$$

$$\widehat{\mathbf{y}}[j] = \begin{cases} \min\{\widetilde{\mathbf{y}}[j+1] - D_1, L_1 + j\}, & \text{if } \widetilde{\mathbf{y}}[0] \geq D_1, \\ \min\{\widetilde{\mathbf{y}}[j+1], L_1 + j\}, & \text{otherwise,} \end{cases} \tag{3}$$

$$\text{for } j = 1, 2, \ldots, L_2 - L_1 - 1,$$

$$\widehat{\mathbf{y}}[L_2 - L_1] = \widehat{\mathbf{y}}[L_2 - L_1 - 1] + 1, \tag{4}$$

$$R(\mathbf{x}, \mathbf{D}, a) = R_1(\widetilde{\mathbf{x}}, D_1) + R_2(\mathbf{x}, D_2, a), \tag{5}$$

$$R_2(\mathbf{x}, D_2, a) = \begin{cases} r_2 \cdot D_2, & \text{if } a = 1 \text{ and } \mathbf{y}[L_2 - L_1] \geq D_2, \\ 0, & \text{otherwise,} \end{cases} \tag{6}$$

$$R_1(\widetilde{\mathbf{x}}, D_1) = \begin{cases} r_1 \cdot D_1, & \text{if } \widetilde{\mathbf{y}}[0] \geq D_1, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

In the solution of Equation (1), g represents the steady state expected profit per period. Equations (2)-(4) characterize the transition between system states $\mathbf{x}$ and $\widehat{\mathbf{x}}$, with the help of their corresponding cumulative vectors $\mathbf{y}$ and $\widehat{\mathbf{y}}$. Specifically, Equation (2) describes how $\mathbf{y}$ is updated to $\widetilde{\mathbf{y}}$, and Equations (3)-(4) further describe how $\widetilde{\mathbf{y}}$ is updated to $\widehat{\mathbf{y}}$. When updating the system state, all accepted orders are scheduled in the reservation vector as late as possible within their lead time windows to allow for maximal flexibility to process the orders. Equations (5)-(7) calculate the profit generated during transitions. Specifically, $R_1(\widetilde{\mathbf{x}}, D_1)$ represents the profit from urgent orders and $R_2(\mathbf{x}, D_2, a)$ represents the profit from regular orders.

Example 1. *Consider an instance with the following parameters:* $L_1 = 4$*,* $L_2 = 8$*,* $B_1 = 2$*,* $B_2 = 3$*. Suppose the reservation vector at the beginning of current period is* $\mathbf{x} = [2, 0, 1, 1, 0]$*, meaning there are 2 units of capacity reserved in Interval I and 2 units reserved in Interval II, totaling 4 units of available capacity for fulfilling regular orders. If a regular order arrives (*$D_2 = B_2 = 3$*), and the decision is to accept it, 1 unit of available capacity in Interval I and 2 units in Interval II will be reserved for processing the order, and the reservation vector will be updated to* $\widetilde{\mathbf{x}} = [3, 1, 1, 1, 1]$ *or* $\widetilde{\mathbf{y}} = [1, 1, 1, 1, 1]$ *(equation* (2)*). However, this leaves with only 1 unit of available capacity within the lead time window of urgent orders, (i.e.,*$\widetilde{\mathbf{y}}[0] = 1$*) and therefore, there is not enough room to accommodate any urgent order (since* $B_1 = 2$*). Finally, the capacity of current period is used to process one unit of order in Interval I, and one unit of reserved capacity is shifted from Interval II into Interval I. Therefore, the reservation vector observed at the beginning of next period becomes* $\widehat{\mathbf{x}} = [3, 1, 1, 1, 0]$ *or* $\widehat{\mathbf{y}} = [1, 1, 1, 1, 2]$ *(equations* (3) *and* (4)*).*

Linear programming (LP) is a common approach for solving MDP problems (Ross 1995, Chapter 5). To see how it tailors to our problem, optimality condition (1) can be easily transformed into a pair of linear inequalities. Thus, we have the following LP model that is equivalent to (1).

$$\max g \tag{8}$$

$$s.t. \quad V(\mathbf{x}) + g \leq \mathbb{E}_{\mathbf{D}}\{R(\mathbf{x}, \mathbf{D}, 1) + V(\widehat{\mathbf{x}}(\mathbf{x}, \mathbf{D}, 1))\}, \forall \mathbf{x}, \tag{9}$$

$$V(\mathbf{x}) + g \leq \mathbb{E}_{\mathbf{D}}\{R(\mathbf{x}, \mathbf{D}, 0) + V(\widehat{\mathbf{x}}(\mathbf{x}, \mathbf{D}, 0))\}, \forall \mathbf{x}. \tag{10}$$

The variables in the LP formulation are $\{V(\mathbf{x})\}$ and g, and thus the formulation contains $(L_1 + 1) \cdot 2^{L_2 - L_1 - 1} + 1$ variables and $(L_1 + 1) \cdot 2^{L_2 - L_1}$ constraints.

The optimal admission policy is derived from the dual problem of (8)-(10). Let $\lambda^*(\mathbf{x}, a)$ be the optimal dual variables associated with constraints (9)-(10). Define $I^* = \{\mathbf{x} | \lambda^*(\mathbf{x}, 1) + \lambda^*(\mathbf{x}, 0) \geq 0\}$ and $\overline{I}^* = \{\mathbf{x} | \mathbf{x} \notin I^*\}$. An optimal stationary admission policy $a^*(\mathbf{x})$ is given by: for $\mathbf{x} \in I^*$,

$$a^*(\mathbf{x}) = \begin{cases} 1, & \text{if } \lambda^*(\mathbf{x}, 1) > \lambda^*(\mathbf{x}, 0), \\ 0, & \text{otherwise;} \end{cases} \tag{11}$$

for $\mathbf{x} \in \overline{I}^*$, the optimal decisions are arbitrary, meaning the firm can either accept or reject regular orders in these states.

4 State Reduction Heuristics

The formulation described in Section 3 leads to an optimal steady-state policy for accepting/rejecting regular orders. However, solving the LP formulation becomes prohibitively hard when $L_2 - L_1$ is large, because the number of variables and constraints increases exponentially in $L_2 - L_1$. For example, if $L_1 = 5$ and $L_2 = 25$, the resulting formulation has 3,145,729 variables and 6,291,456 constraints. Despite the striking efficiency of state-of-art LP solvers, problems with such level of complexity cannot be solved in a reasonable amount of time. Thus, we seek to develop more efficient heuristics.

The complexity of the formulation is largely related to keeping track of the distributional information in Interval II. To reduce the complexity, one plausible idea is to somehow aggregate the distributional information in Interval II, so that the MDP formulation can be reduced to involve fewer variables. The reduced formulation can then be used to generate heuristic policies.

We start with the Full Aggregation Heuristic (FAH) that completely ignores the distributional information in Interval II. Though being extremely fast, the FAH does not always achieve near-optimal solutions. Consequently, we propose the Partial Aggregation Heuristic (PAH) which keeps the most "important" distributional information intact while aggregating the rest in Interval II. A major advantage of this approach is that one can easily control the tradeoff between computation efforts and optimality.

4.1 Full Aggregation Heuristic

We propose a new MDP formulation based on *aggregate reservation vectors* as opposed to reservation vectors in the original formulation. For a reservation vector $\mathbf{x}$, we define its corresponding aggregate reservation vector as $\mathbf{x}_f = (\mathbf{x}_f[0], \mathbf{x}_f[1])$, in which $\mathbf{x}_f[0] = \mathbf{x}[0]$ and $\mathbf{x}_f[1] = \sum_{j=1}^{L_2-L_1} \mathbf{x}[j]$, i.e., $\mathbf{x}_f[0]$ corresponds to the total reserved capacity in Interval I, and $\mathbf{x}_f[1]$ corresponds to that in Interval II. Note that there can be multiple reservation vectors mapping to the same aggregate reservation vector. Further, we define $\mathbf{y}_f = (\mathbf{y}_f[0], \mathbf{y}_f[1])$ as the *aggregate cumulative vector*, in which $\mathbf{y}_f[0] = L_1 - \mathbf{x}_f[0]$ and $\mathbf{y}_f[1] = L_2 - \mathbf{x}_f[0] - \mathbf{x}_f[1]$, i.e., $\mathbf{y}_f[0]$ (respectively $\mathbf{y}_f[1]$) corresponds to the total available capacity in the lead time window of urgent (respectively regular) demands. The one-to-one mapping between an aggregate reservation vector and its aggregate cumulative vector still holds.

The new system state is defined as the aggregate reservation vector in the beginning of a period, and therefore only involves two dimensions. One way to think of the new system state is that each one groups multiple system states in the original formulation into a "super state" (Figure 2), resulting in a significantly shrunk state space. In other words, the size of the aggregated state space is reduced from $(L_1 + 1) \cdot 2^{(L_2 - L_1 - 1)}$ to $(L_1 + 1) \cdot (L_2 - L_1)$.

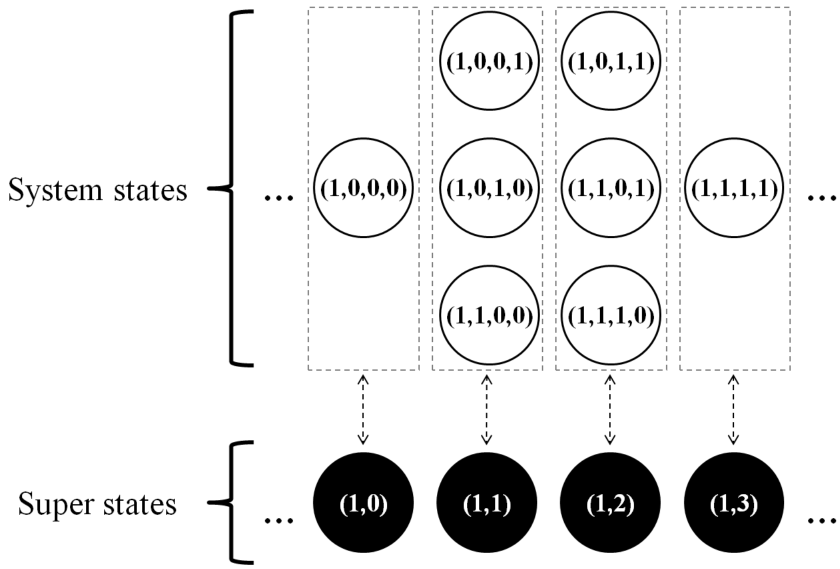


Figure 2: The New System States Defined by "Super States". This illustrative example shows the states whose first element is 1 for an instance in which $L_2 - L1 = 4$.

Then we characterize how one new system state transits to another for a given admission policy a and demand pattern D under the new state space. Note that transitions triggered by accepting/rejecting demands and processing can be easily characterized, but there is one step of transition that cannot be properly defined due to the aggregation, that is, how the aggregated reservation vector at the end of one period evolves to the one at the beginning of the next period. As discussed previously, without distributional information in Interval II, it is impossible to decide whether the shifted unit of capacity is reserved or not. To address this issue, we heuristically account for the transfer of capacity based on assumptions regarding how the reserved capacity is distributed in Interval II. We consider three scenarios as follows.

- **Optimistic scenario.** Assuming that all reserved capacity in Interval II (excluding the last unit) are distributed *as late as possible*, the shifted unit of capacity is reserved only if Interval II (excluding the last unit) is "full", i.e., all the capacity in this area has been reserved. "Optimistic" refers to the fact that the assumption results in an overestimation of the available capacity in Interval I.

- **Pessimistic scenario.** Assuming that all reserved capacity in Interval II (excluding the last unit) are distributed *as early as possible*, the shifted unit of capacity is available only if Interval II (excluding the last unit) is "empty", i.e., none of the capacity in this area has been reserved. "Pessimistic" refers to the fact that the assumption results in an underestimation of the available capacity in Interval I.

- **Realistic scenario.** Assuming that all reserved capacity in Interval II (excluding the last unit) are distributed *uniformly*, the shifted unit of capacity is reserved with a probability, defined as the proportion of reserved capacity in Interval II (excluding the last unit). "Realistic" refers to the fact that the assumption accounts for distribution in this area in a probabilistic way, resulting in a less extreme estimation than the other two scenarios.

These assumptions do not deal with the last unit of capacity in Interval II, because whether it is reserved or not can be explicitly characterized without introducing additional dimensions: it is always available at the beginning of one period, and it will become reserved whenever some regular demand is accepted in the current period.

Let $\widetilde{\mathbf{x}}_f$, $\widehat{\mathbf{x}}_f$, $\widetilde{\mathbf{y}}_f$ and $\widehat{\mathbf{y}}_f$ be the aggregate version of vectors $\widetilde{\mathbf{x}}$, $\widehat{\mathbf{x}}$, $\widetilde{\mathbf{y}}$ and $\widehat{\mathbf{y}}$, respectively. In addition, we define $\overline{\mathbf{y}}_f$ as the aggregate cumulative vector after accepting/rejecting urgent orders but before processing is carried out, i.e. $\overline{\mathbf{y}}_f$ serves as an intermediary between $\widetilde{\mathbf{y}}_f$ and $\widehat{\mathbf{y}}_f$. We characterize the transition between $\mathbf{x}_f$ and $\widehat{\mathbf{x}}_f$ as follows.
Accepting/rejecting a regular order:

$$(\widetilde{\mathbf{y}}_f[0], \widetilde{\mathbf{y}}_f[1]) = \begin{cases} (\mathbf{y}_f[0] - (D_2 - (\mathbf{y}_f[1] - \mathbf{y}_f[0]))^+, \mathbf{y}_f[1] - D_2), & \text{if } a = 1 \text{ and } \mathbf{y}_f[1] \geq D_2, \\ (\mathbf{y}_f[0], \mathbf{y}_f[1]), & \text{otherwise.} \end{cases} \tag{12}$$

Accepting/rejecting an urgent order:

$$(\overline{\mathbf{y}}_f[0], \overline{\mathbf{y}}_f[1]) = \begin{cases} (\widetilde{\mathbf{y}}_f[0] - D_1, \widetilde{\mathbf{y}}_f[1] - D_1), & \text{if } \widetilde{\mathbf{y}}_f[0] \geq D_1, \\ (\widetilde{\mathbf{y}}_f[0], \widetilde{\mathbf{y}}_f[1]), & \text{otherwise.} \end{cases} \tag{13}$$

Processing and updating to the next period:

$$(\widehat{\mathbf{y}}_f[0], \widehat{\mathbf{y}}_f[1]) = \begin{cases} (\min\{\overline{\mathbf{y}}_f[0], L_1 - 1\}, \min\{\overline{\mathbf{y}}_f[1] + 1, L_2\}), & \text{with probability } \pi \\ (\min\{\overline{\mathbf{y}}_f[0], L_1 - 1\} + 1, \min\{\overline{\mathbf{y}}_f[1] + 1, L_2\}), & \text{with probability } 1 - \pi. \end{cases} \tag{14}$$

in which the value of π is contingent on whether the last unit of capacity in Interval II is reserved or not. Some additional notation follows: let ψ be the total available capacity in Interval II after processing, i.e., $\psi = \overline{\mathbf{y}}_f[1] - \min\{\overline{\mathbf{y}}_f[0], L_1 - 1\}$. Next, we discuss the value of π in the different cases.
Case 1: $\widetilde{\mathbf{y}}_f = \mathbf{y}_f$ (the last unit of capacity in Interval II is available)

- In the optimistic scenario, if $\psi = 1$, then $\pi = 1$; otherwise, $\pi = 0$.
- In the pessimistic scenario, if $\psi \neq L_2 - L_1$, then $\pi = 1$; otherwise, $\pi = 0$.
- In the realistic scenario, $\pi = 1 - \frac{\psi - 1}{L_2 - L_1 - 1}$.

Case 2: $\widetilde{\mathbf{y}}_f \neq \mathbf{y}_f$ (the last unit of capacity in Interval II is reserved)

- In the optimistic scenario, if $\psi = 0$, then $\pi = 1$; otherwise, $\pi = 0$.
- In the pessimistic scenario, if $\psi \neq L_2 - L_1 - 1$, then $\pi = 1$; otherwise, $\pi = 0$.
- In the realistic scenario, $\pi = 1 - \frac{\psi}{L_2 - L_1 - 1}$.

Figure 3 shows an example of how an aggregated reservation vector evolves in different scenarios. In this example, $L_1 = 2$ and $L_2 = 7$, the aggregated reservation vector at the end of one period, (i.e., the one after accepting/rejecting demands and processing) is $(1, 3)$. Assuming some regular demand is accepted in this period, the last unit in Interval II is reserved. In the optimistic scenario, the shifted unit of capacity is available because the Interval II (excluding the last unit) is not full, and thus the aggregated reservation vector at the start of next period is $\widehat{\mathbf{y}}_f = (1, 3)$; in the pessimistic scenario, the shifted unit is reserved, leading to $\widehat{\mathbf{y}}_f = (2, 2)$; in realistic scenario, with probability $\pi = 2/4 = 0.5$, the shifted unit is reserved, leading to $\widehat{\mathbf{y}}_f = (2, 2)$, and with probability $1 - \pi = 0.5$, the shifted unit is available, leading to $\widehat{\mathbf{y}}_f = (1, 3)$.

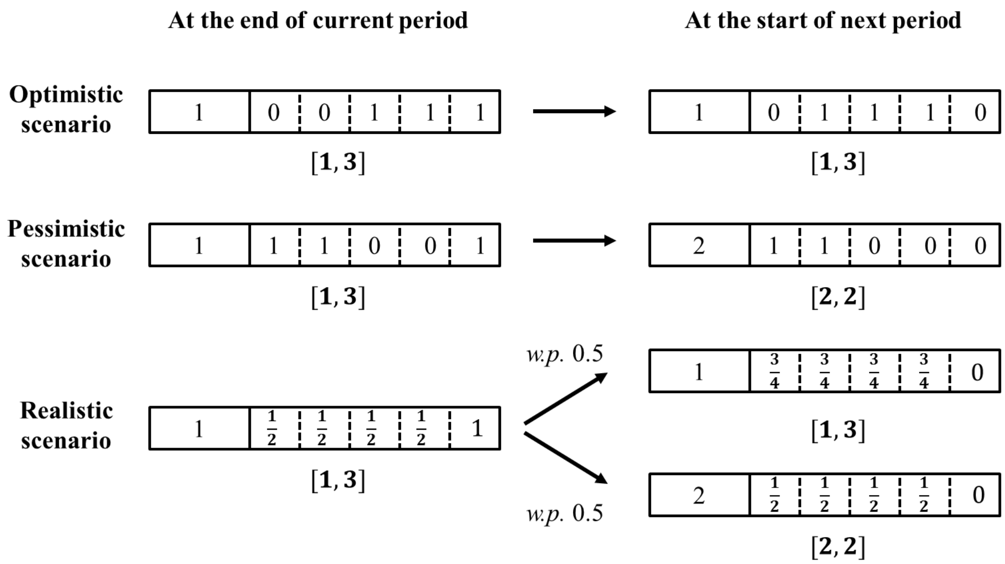


Figure 3: Transition of Aggregated Reservation Vectors Between Two Periods.

The profit generated in each aggregated state can be defined in a similar fashion as in (5)-(7), and a new MDP formulation can be further obtained from optimality equation (1) by replacing the system states with their aggregated versions. For simplicity, we omit their representations. We can still use LP to solve the new MDP formulation, which is much smaller than the original formulation, as we already showed.

Finally, once an optimal admission policy for the new MDP is obtained, a heuristic policy can be constructed in the following way: for each aggregate state, apply its admission decision to all the corresponding states in the original formulation.

4.2 Partial Aggregation Heuristic

The lack of accuracy in characterizing the transitions between aggregate states sometimes leads to significant profit gaps in comparison with the optimal profit, as we will show in the numerical experiments. We wonder whether a more precise characterizing of the distributional information in Interval II could improve the performance of the heuristic. We propose a generalized heuristic, the PAH, that partially aggregates the distributional information in Interval II. Specifically, we split Interval II into two parts: (i) Sub-Interval II-A, where the distributional information is precisely tracked, and (ii) Sub-Interval II-B, where the distributional information is fully aggregated. Figure 4 shows an example of reservation vectors for different levels of aggregation.

However, we face the same issue as in the full aggregation case, that is, how to account for the transfer of capacity between Sub-Interval II-A and Sub-Interval II-B. Again, we address this issue by heuristic approaches in which the reserved capacity in Sub-Interval II-B is distributed according to the pessimistic, realistic and optimistic scenarios as described in Section 4.1. The MDP formulation is derived in the same way; for simplicity, we omit their presentation.

Let $z \in \{0, \ldots, L_2 - L_1 - 1\}$ be the number of periods in Sub-Interval II-A, hereafter referred to as the disaggregation level. This parameter controls the tracking accuracy of the distributional information in Sub-Interval II-A: if $z = 0$, the resulting formulation is actually identical to the full aggregation; if $z = L_2 - L_1 - 1$, the resulting formulation coincides with the one without aggregation that provides the optimal steady-state policy. For a given disaggregation level z, the size of the

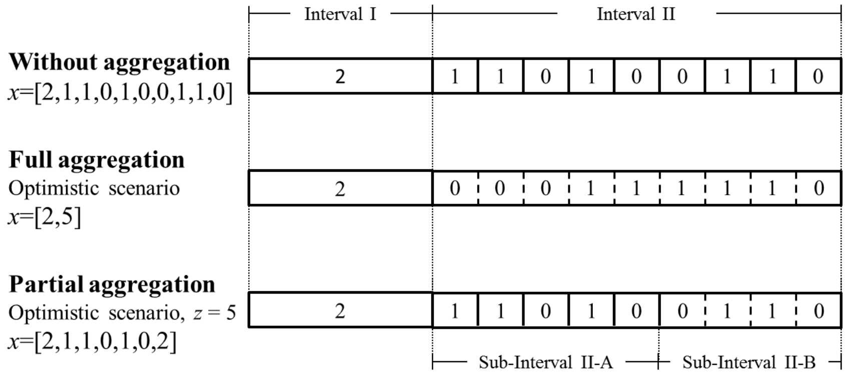


Figure 4: An Example of Reservation Vectors for Different Levels of Aggregation.

resulting state space is $(L_1 + 1) \cdot (L_2 - L_1 - z) \cdot 2^z$, obviously more disaggregation leads to greater computational efforts. Moreover, we have the following results comparing the optimal profits for different partially aggregated MDP formulations.

Proposition 1. *Let $g_o^*(z)$, $g_p^*(z)$ and $g_r^*(z)$ be the optimal expected profit obtained by solving the partial aggregation model with a level of disaggregation z for the optimistic, pessimistic and realistic scenarios, respectively. We have*

$$g_p^*(z) \leq \{g_r^*(z), g^*\} \leq g_o^*(z), \forall z,$$

and $g_o^*(z)$ and $g_p^*(z)$ are non-increasing and non-decreasing with z, respectively.

Proof. The proof is based on two facts: (i) it is always feasible to shorten the lead times of the already accepted orders given that there is available capacity in earlier periods, and (ii) for any aggregate state, the pessimistic scenario implies shorter lead times for already accepted orders than the realistic scenario, which further implies shorter lead times than the optimistic scenario. Therefore, any transition between two aggregate states of scenarios implying shorter lead times can also be achieved with scenarios implying more relaxed lead times. Consequently, for any sample path for the formulation of scenarios implying shorter lead times, we can obtain an identical sample path that provides the same profit with more relaxed scenarios. Thus, the profit for the formulation of the optimistic scenario is at least as large as the profit for the formulation of the realistic scenario, which is further at least as large as the profit for the formulation of the pessimistic scenario. The same logic can be applied for proving the monotonicity property of $g_o^*(z)$ and $g_p^*(z)$, i.e., more aggregation implies shorter (longer) lead times for the pessimistic (optimistic) scenario. □

Proposition 1 indicates that $g_p^*(z)$ and $g_o^*(z)$ are the lower and upper bounds for $g_r^*(z)$ and g^*, but $g_r^*(z)$ and g^* are not directly comparable. It also shows that the proposed bounds get tighter as the level of disaggregation z increases.

5 Numerical Results

In this section we investigate the actual performance of the policies presented above, more specifically we analyze:

- the computation time required to obtain an acceptance policy with the different formulations.

- the relative profits obtained with the FAH, the PAH, the optimal steady-state policy and two benchmark policies often cited in the literature: the First-Come-First-Served (FCFS) policy and the protection level based (PLB) policy (see Section 5.3 for a detailed description of this policy).
- the tightness of the bounds on the profit derived for the FAH and the PAH.

In order to answer those questions, we first compute the heuristic acceptance policies and the optimal policy for the instances small enough to do so. We then simulate the different heuristic policies in order to determine their performance in terms of average gain per period. Indeed, although the gains $g_o^*(z)$ and $g_p^*(z)$ computed for the optimistic and pessimistic policies constitute upper and lower bounds on the optimal gain, the actual gains achieved by those policies – as well as the other policies – cannot be determined from the Markov Decision Process.

We simulate the long-run net profit for an acceptance policy by generating demand realizations. The simulation consists of an initial warm-up interval of 100,000 periods. Afterwards, the simulation incorporates an additional 100,000 periods for which the accumulated net profit is recorded. We repeat the process until the simulated net profit converges with a precision of 0.001%. We denote $\hat{g}_f$ and $\hat{g}_m$ the simulated long-run net profit achieved by the FCFS and the PLB policies, and $\hat{g}_o(z)$, $\hat{g}_p(z)$ and $\hat{g}_r(z)$ the simulated long-run net profit achieved by the PAH with a disaggregation level z assuming the optimistic, pessimistic and realistic scenarios, respectively. Note that the gain of the optimal steady-state policy is given directly by the solution of the Markov Decision Process, its value is equal to g^*.

The Markov Decision Process is solved by formulating it as linear program that is solved using the Gurobi 4.6.1 software. The Acceptance policy simulation routine was implemented in the Java language. The computer used was a 6-Core Intel Xeon 2×2.66 GHz with 48 GB of RAM.

In the following subsections we introduce certain characteristics of the demand classes that we used to generate instances of the problem (Section 5.1). Afterwards, we describe the results of three studies: (1) the computation time for the construction of an acceptance policy for different instances (Section 5.2); (2) the profits achieved by the optimal steady-state policy, the FCFS and the PLB policies, and the aggregation heuristics (see Section 5.3 and Section 5.4 for the FAH and the PAH, respectively); (3) the profits of the FAH and the PAH for large instances where the optimal steady-state policy cannot be computed (Section 5.5).

5.1 Experiment Settings

The instances we generated are characterized by the following attributes:

Profit structure (ρ). It represents the ratio between the net profits of both demand classes, i.e. $\rho = r_2/r_1$. Without loss of generality, the value of r_1 is normalized to 1, so the value of r_2 is obtained directly from ρ. In our experiment we test the following values for $\rho \in \{0.25, 0.50, 0.75\}$ in order to explore high, moderate and low differences in the profit structure, respectively.

Lead-time structure L_1 *and* L_2. We investigate two aspects of our model that depend on the values of L_1 and L_2. On the one hand, it is clear from the formulation of our model that the computational complexity is closely related to the difference between L_1 and L_2. On the other hand we will see that L_1 influences the performance of the different policies.

Order size structure B_1 *and* B_2. The sizes of the orders determine the operational flexibility when an order is accepted. We also did some tests with stochastic order sizes, we do not report on this here as the results do not really differ from the deterministic case. The only difference is larger computation times, as a result the computation of the optimal policy is restricted to even smaller instances.

Demand structure (β). It is defined as the ratio between the expected demand rates of both classes, i.e. $\beta = (p_1 \cdot B_1)/(p_2 \cdot B_2)$. We study how this ratio impacts the performance of the proposed formulations. We chose the following values of $\beta \in \{1/2, 2/3, 1, 3/2, 2/1\}$ (i.e. the expected demand of one class is 100% or 50% greater than the other, or the expected demand rates of both classes are equal).

Global demand rate (τ). It corresponds to the total expected demand for both classes per period i.e. $\tau = p_1 \cdot B_1 + p_2 \cdot B_2$. We focus in scenarios in which $\tau > 1$, inasmuch as in these scenarios the acceptance decision is most meaningful as some demand will have to be refused. If $\tau < 1$, the decision to not accept a demand is not very relevant. We chose the following values for $\tau \in \{1.2, 1.6, 2.0\}$

Note that, the values of p_1 and p_2 are functions of β, τ, B_1 and B_2. Their values will be derived from these parameters.

5.2 Computation Times

In this section, we compare the average CPU times needed to calculate the optimal steady-state policy and the heuristic policies. We analyze the FAH and the PAH for the so-called realistic scenario, which is the most demanding in terms of computation time among the three scenarios considered. Note that, the PAH is tested for different values for the disaggregation level ($z \in \{2, 4, 6\}$). In this experiment, we fix the order sizes to 1 because this leads to the longest computation times. Similar insights can be obtained for any other combination of values of B_1 and B_2. As already explained, the difference between lead times and their combinations have a direct impact on the size of the state space of the system and, thus, the computation time. To explore this dependency we test a wide range of values: $L_1 \in \{1, 3, 5, 7\}$ and $L_2 \in \{13, 15, 17, 19\}$. For the other parameters, we perform a full factorial experiment based on the different values presented in Section 5.1 (though discarding the combinations that result in p_1, $p_2 > 1$). In total this experiment consisted of 342 instances.

Table 1 shows the average CPU times and the size of the system state space for each combination of L_1 and L_2. We observe that the time needed to find the optimal steady-state policy increases very quickly when the difference between L_2 and L_1 increases. In fact, for $L_2 - L_1 \geq 15$ we could not determine the optimal order acceptance policy.

5.3 Efficiency of the FAH

We compare the efficiency of the FAH with the optimal steady-state policy, the FCFS and the PLB policies by measuring their relative profits. We also compare the quality of the FAH under each of the three scenarios. In order to compare the efficiencies we compute the optimality gap of an acceptance policy constructed by the heuristic i as follows $Gap_i = (g^* - \hat{g}_i)/g^* \times 100\%$. Note that the FAH is represented by $i = \{o(0), p(0), r(0)\}$ and the FCFS and PLB policy by $i = \{f, m\}$, respectively.

The protection level based (PLB) policy applied in this paper is an adaptation of the well-known revenue management approaches that divide the available capacity into two portions: protected (reserved for the high net-profit class) and unprotected (used for both classes). These approaches are commonly applied for the finite-horizon problems. If they are directly implemented over an infinite horizon, the construction of the policy is computationally as intensive as the construction of the optimal steady-state policy. Therefore, in order to reduce the complexity of the existing approaches while at the same time capturing their essence we implement a myopic method which determines the amount of protected capacity q that maximizes the expected net profit within L_2. This corresponds

Table 1: Average CPU Time and Dimension of the State Space of the System for the Optimal Steady-State Policy, the FAH and the PAH Under the Realistic Scenario.

		Average CPU (seconds)					Size	
				PAH				PAH
L_2	L_1	Optimal	FAH	$z=2$	$z=4$	$z=6$	Optimal	$z=6$
13	1	2.55	0.00	0.01	0.02	0.17	$4,096$	768
13	3	2.99	0.00	0.01	0.06	0.38	$2,048$	$1,024$
13	5	0.26	0.00	0.01	0.06	0.20	768	768
13	7	0.03	0.00	0.01	0.03	$-$	256	$-$
15	1	194	0, 00	0.01	0.03	0.30	$16,384$	$1,024$
15	3	22	0.00	0.01	0.10	1.02	$8,192$	$1,536$
15	5	10	0.00	0.02	0.14	1.20	$3,072$	$1,536$
15	7	0.47	0.01	0.02	0.12	0.39	$1,024$	$1,024$
17	1	$*$	0.00	0.01	0.05	0.51	$65,536$	$1,280$
17	3	$*$	0.00	0.02	0.16	2.11	$32,768$	$2,048$
17	5	105	0.01	0.03	0.28	1.72	$12,288$	$2,304$
17	7	8	0.01	0.03	0.29	3.03	$4,096$	$2,048$
19	1	$*$	0.00	0.01	0.06	0.73	$262,144$	$1,536$
19	3	$*$	0.00	0.02	0.24	2.45	$131,072$	$2,560$
19	5	$*$	0.01	0.04	0.44	3.24	$49,152$	$3,072$
19	7	$1,015$	0.01	0.05	0.54	3.52	$16,384$	$3,072$

"$*$" symbolizes instances for which the optimal steady-state policy cannot be obtained in 1 hour and "$-$" represents the cases where the PAH is equivalent to the optimal policy because $z \geq L_2 - L_1 - 1$.

to the approximations used in the literature see e.g. Barut and Sridharan (2005). The expected net profit within L_2 is computed with the following expression: $r_1 \cdot E[\min(\bar{D}_1, \max(q, y[L_2] - \bar{D}_2))] + r_2 \cdot E[\min(\bar{D}_2, y[L_2] - q)]$, where the random variable $\bar{D}_i$ represents the amount of demand of class i during L_2 periods. The PLB policy consists in protecting the capacity q that maximizes the previous expression.

In order to evaluate the efficiency of the proposed policies with respect to the optimal steady-state policy, we fix $L_2 = 15$ and $L_1 \in \{3, 7\}$. Under this setting the optimal steady-state policy can be obtained within a tractable time. In order to capture the full essence of different degrees of heterogeneity in demand classes, we consider the following values of order sizes: $B_1 \in \{1, 3, 5, 7\}$ and $B_2 \in \{3, 5, 7, 9\}$ (note that for $L_1 = 3$ $B_1 \in \{1, 3\}$). For the other parameters we do a full factorial experiment based on the values presented in Section 5.1. In this experiment, our analysis is based on 984 instances (the instances with $p_1, p_2 \geq 1$ are discarded).

Table 2 reports on the average optimality gaps for the two values of L_1. The average optimality gap of the FAH is lower than that of the FCFS and the PLB policies. There is also a significant difference between the different implementations of the FAH (realistic, pessimistic and optimistic).

The lowest average optimality gap of the FAH is achieved with the realistic scenario. The strength of the realistic scenario lies in the balance between the excess of protection of capacity for high profitability orders (pessimistic scenario) and the assumption of maximum flexibility for processing incoming orders (optimistic scenario).

The results also reveal that the PLB and FAH tend to perform a bit better when L_1 is larger.

Table 2: Average Optimality Gap of the FCFS, the PLB Policy and the FAH Under the Three Scenarios for the Two Values of L_1.

	FCFS	PLB	FAH		
			Optimistic	Pessimistic	Realistic
$L_1 = 3$	12.92%	7.27%	5.00%	1.90%	0.45%
$L_1 = 7$	13.52%	5.13%	4.98%	0.57%	0.36%
Overall	13.33%	5.81%	4.99%	0.99%	0.39%

We also study the reliability of the different policies when the values of the parameters vary. Figure 5 shows the dispersion of the optimality gap of the analyzed methods. The limits of each box represent the first and third quartiles of the measured gaps. The central line corresponds to the mean result. Finally, the bottom and top whiskers represent the fifth and the ninety-fifth percentiles of the optimality gaps. In addition to the advantages achieved in terms of the average optimality gap, the implementation of the FAH with the realistic scenario provides the most reliable performance.

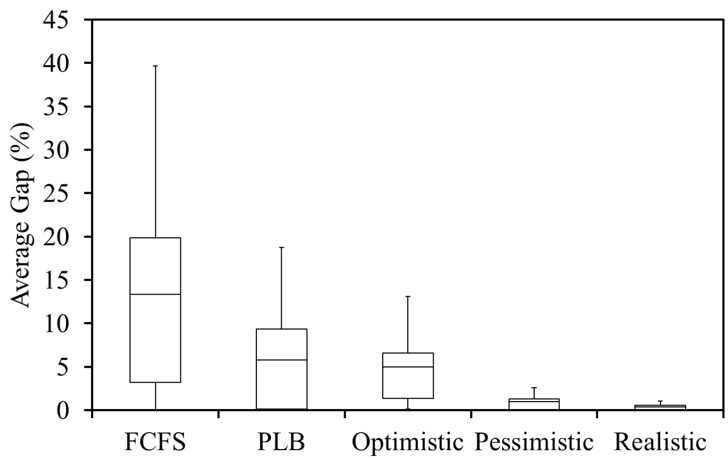


Figure 5: Dispersion of the Optimality Gap of Different Heuristic Methods and Distributional Scenarios for the FAH.

5.4 Efficiency of the PAH

Despite the excellent overall performance of the FAH with the realistic scenario, the optimality gap remains significant for 9.76% of the instances generated ($Gap_r(0) \geq 1\%$). For these instances, we study how the systematic disaggregation of information related to already accepted orders can improve the quality of the acceptance policies found. For this, we calculate $Gap_r(z)$ for $z \in \{1, 6\}$. The average values of $Gap_r(z)$ are displayed in Figure 6. The optimality gap of the PAH decreases rapidly as the value of z grows. In particular, we note from Figure 6 that when z increases from 0 to 4, the average gap decreases from 1.54% to 0.73% when $L_1 = 3$ and from 2.05% to 0.17% when $L_1 = 7$. Thus, the optimality gap of the FAH is greatly reduced while the computation time remains small (see Table 1). Note that for the remaining 90.24% of the instances studied, the average optimality gap is also improved when the value of z increases.

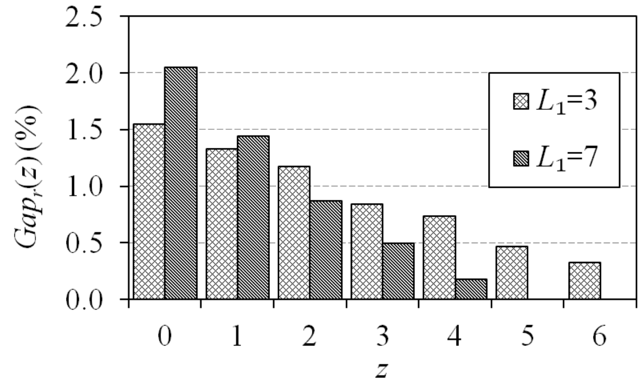


Figure 6: Average Optimality Gap of the Acceptance Policies Under the Realistic Scenario for the Instances with $Gap_r(0) \geq 1\%$ for Different Values of z and L_1.

5.5 Efficiency of the FAH for Large Lead Times

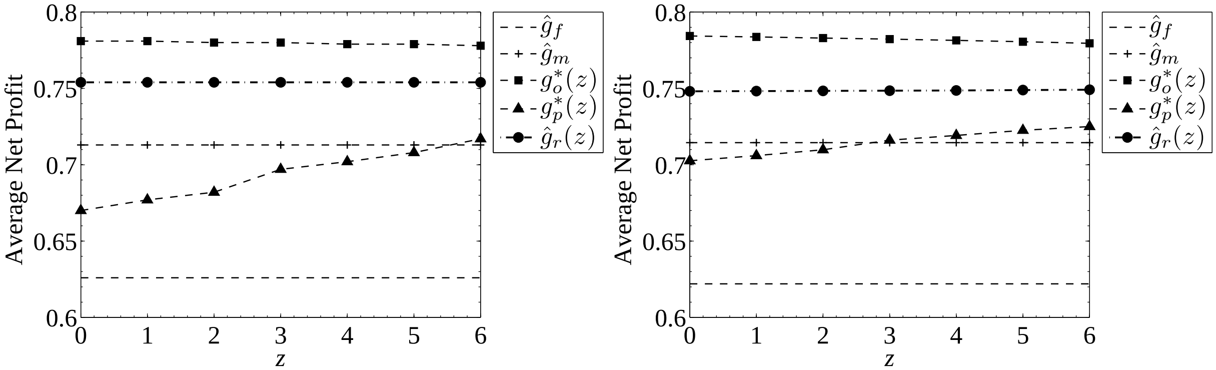
As shown in Table 1 an advantage of the FAH and the PAH is to construct the policies in tractable time even for instances for which the optimal steady-state policies cannot be obtained. In order to evaluate the efficiency of the studied heuristics for these instances, we fix the value of L_2 to be large i.e. $L_2 = 29$ while the other parameters take the same values as in Section 5.3. As a result our experiments includes again 984 instances. We also compute the upper and lower bounds obtained from Proposition 1.

The average net profits of the different policies are plotted as a function of z in Figure 7a and Figure 7b for $L_1 = 3$ and $L_1 = 7$, respectively. We observe the following: first, the FAH and the PAH with the realistic scenario significantly outperform the PLB policy. Second, the effect of increasing z seems much stronger on the quality of the bounds than on the performance of the realistic policy. For a vast majority of instances the disaggregation does not bring any significant improvement. However, in a very similar fashion to Section 5.4, we observe that for the 10% of instances with the worst performance for the FAH, the profit increased by at least 0.5% between the PAH heuristic with $z = 0$ and $z = 6$. The average improvement for these instances is 0.781%. This seems to indicate that the main usefulness of the PAH is to give the possibility of controlling the quality of the proposed solution.

6 Operational Flexibility

In the previous section, we illustrated the efficiency of the proposed algorithms for a large set of instances. Here, we try to gain some further insight into the circumstances where revenue management would have the most significant impact. We will focus on instances following the pattern of the cases that motivated our work (namely, a class of urgent orders with relatively low demand and a class of regular orders with longer lead times and lower revenues).

Unsurprisingly, we observe in Figure 8 that the benefit of revenue management is strongly correlated with the intensity of demand. The more interesting observation is that the potential benefit of revenue management compared to the FCFS policy increases with the difference between L_2 and B_2. This means that the advantage of revenue management is larger when there is more flexibility in processing orders of class 2. Note that, the effect of the flexibility for orders of class 1 is more limited.



(a) $L_1 = 3$.

(b) $L_1 = 7$.

Figure 7: Effect of Disaggregation of the Information on the Average Long-Run Net Profits Achieved with Different Policies for the Large Lead Time of Class 2.

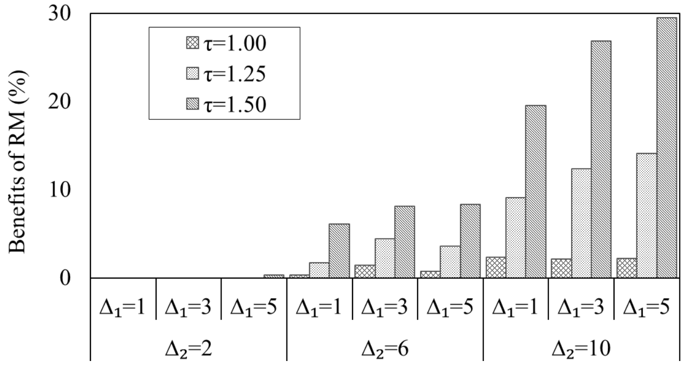


Figure 8: Benefits of the Proposed Revenue Management Approach (RM) Compared to the FCFS Policy for Different Characteristics of the Demand Streams. Results for $L_1 = 6$, $L_2 = 12$, $\rho = 0.50$ and $\beta = 0.50$. $\Delta_i = L_i - B_i$ where $i = 1, 2$. The Benefits of RM are Computed by $(g^* - \hat{g}_f)/\hat{g}_f \times 100\%$.

In order to gain more insights into the impact of revenue management on the operations we compare the acceptance rates of both classes with and without revenue management. Table 3 shows the proportion of accepted orders for each demand class. The results illustrate how the revenue management technique is giving gradually higher priority to the urgent orders when ρ (the relative margin of class 2 orders) is decreasing. We also observe again that the operational flexibility once an order is accepted plays an important role, in this experiment when $B_2 = 10$ the impact of revenue management is minimal, and more generally the smaller B_2 (for a fixed value of L_2) the larger the impact. Of particular interest is the case where $B_1 = 5$ and $B_2 = 10$, that is there is hardly any flexibility for both classes. In that case we observe that for $\rho = 0.75$ or 0.5 the optimal policy is essentially FCFS, while for $\rho = 0.25$ the policy is to accept only the high revenue class. In other words, given the small amount of operational flexibility there is an abrupt switch in the acceptance policy from accept all orders (whenever feasible) to accept only high margin urgent orders when the difference in margin is high enough.

Table 3: Acceptance Rate Under the FCFS and the Optimal Policy and the Benefits of RM for Different Order Sizes and Revenues.

		FCFS		Optimal, $\rho = 0.75$			Optimal, $\rho = 0.50$			Optimal, $\rho = 0.25$		
B_1	B_2	μ_1	μ_2	μ_1	μ_2	b	μ_1	μ_2	b	μ_1	μ_2	b
1	2	55.6%	91.8%	96.6%	70.8%	4.9%	99.3%	68.9%	14.1%	99.9%	67.7%	31.8%
1	6	77.1%	70.7%	85.1%	65.8%	0.3%	93.4%	59.8%	3.6%	97.6%	55.2%	11.4%
1	10	85.0%	52.4%	85.1%	52.3%	0.0%	85.1%	52.3%	0.0%	90.2%	45.6%	1.6%
3	2	44.8%	95.4%	80.4%	75.2%	2.8%	87.3%	70.3%	12.5%	92.4%	62.3%	33.6%
3	6	57.7%	74.2%	57.8%	74.2%	0.0%	80.5%	57.4%	4.4%	88.5%	45.6%	17.5%
3	10	60.4%	56.5%	60.4%	56.5%	0.0%	60.4%	56.5%	0.1%	97.1%	0.0%	9.6%
5	2	36.1%	97.1%	66.0%	79.3%	1.8%	66.0%	79.3%	9.1%	71.9%	70.1%	26.4%
5	6	40.4%	77.1%	40.4%	77.1%	0.0%	53.4%	66.1%	1.8%	71.0%	48.4%	20.6%
5	10	44.4%	56.8%	44.4%	56.8%	0.0%	44.4%	56.8%	0.0%	79.6%	0.0%	9.3%

Results for $L_1 = 6$, $L_2 = 12$, $\tau = 1.25$ and $\beta = 0.50$. Note that, μ_i is the service level of demand stream i and b is the benefit of RM compared to the FCFS.

7 Conclusions and Future Research

In this paper we studied the order acceptance problem for a firm serving two classes of demand that differ in net profit and lead time over an infinite horizon. We obtained the optimal order acceptance policy by formulating the problem as a multi-dimensional Markov Decision Process. However the construction of this policy can involve high computational requirements. To overcome this difficulty, we proposed an efficient heuristic consisting in a parametric aggregation of the state space. The parameter makes it possible to find the best trade-off between the computation time and the quality of the solution. We propose several variants based on different assumptions about the dynamics of the aggregate state. The variant which assumes that all reserved capacity is distributed uniformly in the aggregation interval gives significantly better solutions than the other approaches studied in the extant literature. The other variants give lower and upper bounds that make it possible to obtain a guarantee about the quality of the solution and the possible gap with respect to the optimal solution. Finally, the computation time remains very short even for instances with large order lead times.

We showed that the amount of operational flexibility (in our case this means how much slack there is between the promised lead time and the effective processing time needed) has a large impact on the performance of revenue management. The more flexibility there is the larger the potential benefit of implementing a revenue management based order acceptance policy. The study of the potential benefits of revenue management associated with flexibility in other contexts is an interesting avenue for further research. In the airline industry for example, the hub and spoke organization is widespread. The major airlines have several hubs and for a journey that is not starting or ending in a hub, it might be interesting to keep some flexibility in terms of the legs traveled by a passenger (i.e. through which hub) as long as a time-slot is respected for the departure and arrival times at the origin and destination respectively. It would be interesting to investigate how the state aggregation policy presented in this article could be extended for the more general network structure of airline operations.

Another direction for future work is the exploration of some properties of the value functions in the MDP formulation. Based on the concept of $L^\natural$-convexity, some recent works (see e.g. **??**) partially characterize the optimal policies for inventory problems with lead time issues. We conjecture that similar properties exist in the context of our problem. If so, it might be possible to develop more efficient computational methods.

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