



Decision Support

An optimization framework for the development of efficient one-way car-sharing systems

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ABSTRACT

Electric vehicle-sharing systems have been introduced to a number of cities as a means of increasing mobility, reducing congestion, and pollution. Electric vehicle-sharing systems can offer one or two-way services. One-way systems provide more flexibility to users since they can be dropped-off at any station. However, their modeling involves a number of complexities arising from the need to relocate vehicles accumulated at certain stations. The planning of one-way electric vehicle-sharing systems involves a host of strongly interacting decisions regarding the number, size and location of stations, as well as the fleet size.

In this paper we develop and solve a multi-objective MILP model for planning one-way vehicle-sharing systems taking into account vehicle relocation and electric vehicle charging requirements. For real world problems the size of the problem becomes intractable due to the extremely large number of relocation variables. In order to cope with this problem we introduce an aggregate model using the concept of the virtual hub. This transformation allows the solution of the problem with a branch-and-bound approach.

The proposed approach generates the efficient frontier and allows decision makers to examine the trade-off between operator's and users' benefits. The capabilities of the proposed approach are demonstrated on a large scale real world problem with available data from Nice, France. Extensive sensitivity analysis was performed by varying demand, station accessibility distance and subsidy levels. The results provide useful insights regarding the efficient planning of one-way electric vehicle-sharing systems and allow decision makers to quantify the trade-off between operator's and users' benefits.

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1. Introduction

According to Federal Highway Administration (FHWA) studies a private vehicle travels on average 40 kilometers per day, which is approximately 90 minutes time. For the rest of the time this vehicle is idle and occupies a parking spot (FHWA, 2010). An alternative is car-sharing (also known as shared-use vehicle) systems, which have attracted considerable attention with multiple implementations worldwide due to their potential to improve mobility and sustainability (Shaheen & Cohen, 2013). These systems provide benefits both to their users and the society as a whole. Reduced personal transportation cost and mobility enhancement have been cited as the two most notable benefits to individual users. Recent

studies show that, car-sharing also decreases average vehicle kilometers traveled and, it is likely to decrease congestion (Crane, Ecola, Hassell, & Natarah, 2012) and emissions (Shaheen & Cohen, 2013). Provision of affordable mobility to economically disadvantaged groups with on-demand and public transportation systems is another societal benefit (Duncan, 2011).

The attractiveness of car-sharing systems is determined by the level of service offered and the cost associated with the use of the system. The level of service is influenced by the accessibility of vehicle stations by the potential users, i.e. (i) the distance between user's origin and destination from pick-up and drop-off vehicle stations respectively, and (ii) the availability of vehicles at stations. On the other hand, station number and size, as well as fleet size and availability of vehicles, at the "right time" at the "right station", influence the cost of establishing and operating a car-sharing system.

The car-sharing systems can be classified into flexible "one-way" and the more restricted "two-way" types, according to

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whether the users should return the rented vehicle at a different or at the location they picked it up. The “one-way” systems are also classified as “free-floating” and “non-floating” according to parking spot restrictions. The former refers to a system without restricted parking spots. Users can pick-up or drop-off vehicles in any parking spot restricted within the borders of an area. The latter is used for defining systems in which pick-up and drop-off locations of the vehicles should be designated parking spots. In “free-floating” models, reservation is not possible whereas “non-floating” models provide users both the ability to make reservation and the flexibility of one-way trips. While two-way systems allow users for reservations, the state of practice in one-way systems is renting-based on real-time availability or with short term reservations (e.g. 30 minutes in advance). A recent study showed with an agent based simulation that parking reservations can improve the quality of service for one-way car sharing systems (Kaspi, Raviv, & Tzur, 2014).

The problem of ensuring vehicle availability and fulfilling reservation becomes more prominent when the vehicles can be rented and used on a one-way basis in non-floating systems. The one-way operation of the vehicles coupled with the imbalance of demand for vehicles, both at the origin of the trip (pick-up station) and at the destination (drop-off station), may result to a situation where vehicles are accumulated to stations where they are not needed, while at the same time there is vehicle shortage at the stations where more vehicles are needed (Barth, Todd, & Xue, 2004).

Vehicle relocation, i.e. transfer of vehicles from stations with high vehicle accumulation to stations where shortage is experienced, is a technique that has been proposed to improve the performance of one-way car-sharing systems (e.g. Cucu, Ion-Boussier, Ducq, & Boussier, 2009; Jorge, Correia, & Barnhart, 2014; Kek, Cheu, & Chor, 2006). The lack of efficient vehicle relocation coupled with the need to guarantee a given level of vehicle availability may lead to an unnecessary increase of the fleet size and vehicle underutilization. The efficient and cost-effective strategic planning, and the operation of one-way car-sharing systems require models that will optimally determine the number and location of the service stations, the fleet size, and the dynamic allocation of vehicles to stations. These models should assist decision makers to strike an optimum balance between the level of service offered and the total cost (including vehicle relocation costs) for implementing and operating the car-sharing system.

However, the literature currently lacks a model that can consider simultaneously decisions related to the determination of station location, size and number, and fleet size, while taking into account the dynamics of vehicle relocation and balancing for a system with reservations. Existing models (Correia & Antunes, 2012; Lin & Yang, 2011) either look at station locations without due consideration to vehicle relocation decisions (Lin & Yang, 2011), or consider station locations assuming that only the demand in the catchment area of opened stations needs to be served (Correia & Antunes, 2012). In the case where vehicle relocation is modeled (Correia & Antunes, 2012), the relocation of the vehicles and the associated costs are considered only at the end of the operating period (usually a day), and therefore they are influencing the fleet size.

The objective of this paper is twofold: (i) to develop and solve a mathematical model for determining the optimum fleet size, and the number and location of the required stations of one-way non-floating reservation-based, for both pick-up and drop-off, car-sharing systems by taking into account the dynamic repositioning (relocation) of vehicles, and (ii) to apply the proposed model for planning and operating a one-way electrical car-sharing system in the city of Nice, France.

The remainder of this paper is organized as follows. Section 2 provides an overview of previous related work and further

elaborates on the arguments justifying the need for the proposed model, Section 3 presents the formulation and the solution approach of the proposed model, Section 4 describes the application of the proposed model for planning and operating a one-way electrical car-sharing system in Nice, France, Section 5 highlights practical considerations while Section 6 discusses the research conclusions and provides recommendations for future research.

2. Previous related research

Models related to the planning and operation of car-sharing systems can be classified into the following two broad categories: (i) models addressing strategic planning decisions and (ii) models supporting operational decisions.

2.1. Models for strategic planning decisions

Strategic planning decisions seek to determine the number, size and location of stations, and the number of the vehicles that should be assigned to each station, in order to optimize a measure or a combination of measures for system performance. Station location models have been developed to locate bicycle stations (Lin & Yang, 2011) and car stations (Correia & Antunes, 2012). Although the focus of our work is on electrical car-sharing systems, we also review models that address the station location of shared-use bicycles, given some similarities of the two systems.

The problem of locating stations for shared-use bicycles has been studied recently (Lin & Yang, 2011). This paper presents a model for determining the number and location of bicycle stations and the structure of the network of bicycle paths that should be developed to connect the bicycle stations. The problem is formulated as a non-linear integer model. The objective function used expresses the total yearly cost encountered by the operator and the users. A small scale example was used to illustrate the model and a branch and bound algorithm was used to solve it. This model does not consider the daily variation of demand and the problems arising from the dynamic accumulation/shortage of bicycles due to the variation of demand in time and space.

The optimization of vehicle depot locations and the definition of the number of parking spaces (size) for each depot has been also addressed (Correia & Antunes, 2012). The number of parking spaces at each depot is determined by the maximum number of vehicles that are allocated to each station throughout an operating day. Vehicle relocation (and the associated relocation cost) is considered only at the end of the entire operating period (i.e. day). Thus, this model does not treat explicitly the dynamic imbalance created by the one-way operation and therefore it does not rebalance the vehicles at the end of each operating sub-interval (e.g. hour). This model assumes that the vehicle imbalance problem is by-passed through the optimum depot location and size. The objective function of the model seeks to maximize the profit of the operating agency and takes into account the depreciation, maintenance and relocation (at the end of the operating period) costs of the vehicles, the maintenance cost of the depots, and the revenues generated by the system operations. This model makes the assumption that only trips associated with open stations need to be served. Thus, the demand (trips) that falls outside the catchment area of open stations associated with the stations that are not open is ignored. As a consequence, this model does not consider the access and egress cost of the potential users to/from the candidate station locations. A direct implication of this assumption is that, the proposed model cannot be used to study the trade-off between station accessibility cost and system benefits. Finally, this model does not consider the dynamic relocation

of vehicles throughout the operating period. The proposed model was used to analyze a case study in Lisbon and an optimizer based on branch-and-cut algorithms was used to solve the problem.

A recent work also models one-way car-sharing problem with an MILP considering relocation throughout the day (Jorge et al., 2014). Similar to Correia and Antunes (2012), the model exogenously associates trips to stations. Different than the previous work, the model allows relocations at any time of the day. The objective function maximizes the profit of the operating agency. The model is tested on three different scenarios and the results are supported with a simulation. In simulation, cost of relocation is minimized with a minimum cost flow algorithm. Results on different scenarios show that, with dynamic relocation, car-sharing system modeled on the demand in Lisbon, Portugal starts profiting.

The problem of determining the fleet size and the distribution of vehicles among the stations of a car-sharing system was studied in relation to the Personal Intelligent City Accessible Vehicles (PICA Vs). This system uses a homogeneous fleet of eco-friendly vehicles and allows one-way trips (Cepolina & Farina, 2012). The stations are parking lots that offer vehicle recharging services and are located at inter-modal transfer points and near major attraction sites within a pedestrian area. The number, location and capacity of stations are not determined by the model, hence constitute inputs to the simulated annealing process. To cope with the imbalance of vehicle accumulation of the one-way system, this model introduces the concept of supervisor. The task of the supervisor is to direct users that are flexible in returning the vehicle to alternative stations, as to achieve a balanced operation and fulfill a maximum waiting time constraint. The objective function of this model includes the minimization of the daily system and user costs subject to a maximum waiting time constraint. The value of the objective function of the model was estimated through micro-simulation. A simulated annealing approach was used for determining the fleet size and for allocating vehicles among system stations.

Models for evaluating the performance of a network of car-sharing stations has been introduced in the literature (Fassi, Awasthi, & Viviani, 2012; George & Cathy, 2011). This problem arises when the demand for car-sharing services changes (increases) and as a consequence the network of stations should be adapted to serve better the emerging demand profile. In response to this need, a decision support tool was developed, which allows decision makers to simulate alternative strategies leading to different network configurations. Such strategies include opening and/or closing stations, and increasing the capacity of stations. This tool is based on discrete event simulation and seeks to maximize the satisfaction level of the users and to minimize the number of vehicles used (Fassi et al., 2012). This model does not address vehicle relocation as it is based on a system that does not allow one-way use of vehicles. Performance analysis for shared-use vehicles systems has been proposed in the literature using a closed queuing network model (George & Cathy, 2011). In this approach, both exact and approximate solution methods are proposed to evaluate the bike sharing system Vélib operating in Paris, France with over 20,000 bicycles and 1500 locations.

Recently, the impact of user flexibility in choosing their pick-up and drop-off station, and the impact of using real time information regarding vehicle and parking space availability at stations was studied (Correia, Jorge, & Antunes, 2014). This research extends the MILP optimization model proposed in Correia and Antunes (2012), by considering scenarios regarding user flexibility and availability of information, i.e. inflexible, flexible, and flexible users with vehicle stock availability

information. The model was applied to the city of Lisbon, Portugal. The results emerging from this case study suggest that if users are willing to choose one of the three closest stations to their origins and destinations to pick-up and drop-off vehicles respectively instead of insisting on using the closest stations, the satisfied demand ratio will increase from 33% to 65%. In addition, making the vehicle stock information available to flexible customers will increase the satisfied demand ratio to 83% (Correia et al., 2014).

2.2. Operational decisions

A major decision associated with the operation of one-way car-sharing systems is how to relocate vehicles. The vehicle relocation problem arises from the imbalanced accumulation of vehicles at stations when the car-sharing system allows their one-way use. Different strategies and models have been proposed in the literature to cope with the vehicle relocation problem.

The relocation of shared vehicles can be realized by using operating staff (Kek et al., 2006) or it can be user-based (Barth et al., 2004). Shortest time, and inventory balancing strategies have been used (Kek et al., 2006) for staff-based vehicle relocation. The shortest time strategy relocates vehicles from other stations to minimize the travel time needed for a staff member from his/her current location to the station where the vehicle is available plus the travel time needed from the station that the vehicle is available to the station where the vehicle is needed. The inventory balancing strategy relocates vehicles from stations with over-accumulated vehicles to stations that experience vehicle shortages. Both strategies were tested through a simulation model which was validated using data from an operational car-sharing system (Kek et al., 2006). An optimization-trend-simulation decision support system (Kek, Cheu, Meng, & Fung, 2009) is proposed which uses the same simulation model. In this three-phase decision support system, the effectiveness of different relocation policies are evaluated according to zero-vehicle time (duration of the vehicle shortage), full-port time (shortage of empty parking space when needed) and number of relocations.

The dynamic allocation of vehicles among stations of a car-sharing system to maximize profit has been modeled in Fan, Randy, and Lownes (2008). The fleet size, the location of stations, and the demand for trips for a given planning horizon are known in advance. Penalties associated with unserved trip requests are not considered. A multistage stochastic linear model with recourse has been proposed to address this problem. A stochastic optimization method based on Monte Carlo simulation was used to solve the proposed model Fan et al. (2008). This model considers only the vehicle relocation decisions. Furthermore, vehicle relocation is performed at the end of the day.

Chance constraint modeling has been used to study fleet redistribution (Nair & Miller, 2011). This model assumes that system configuration, current inventory of each station, costs and demand at each station are known in advance. The model aims to find the minimum cost fleet redistribution plan for the demand expected in the near future. The chance constrained model with reliability p (CCM- p) is constructed and solved by utilizing a special technique involving p -efficient points (PEPs) (Prékopa, 2003). The model is applied on the Intelligent Community Vehicle System in Singapore, a one-way system with 14 stations, 202 parking spaces and 94 vehicles.

Two user-based relocation strategies namely, trip-joining and trip-splitting have been proposed for a system operating at a university (Barth et al., 2004). The trip-joining strategy is used when two users have common pick-up and drop-off stations and there is a shortage of vehicles at the pick-up station. In this case,

the users are asked to share the ride. The trip-splitting strategy is used when there is a surplus of vehicles at the pick-up station(s) and there are users that are traveling as a group. Under this condition, the users are asked to use separate vehicles when there is a shortage of vehicles at their destination (Barth et al., 2004). However these strategies would be difficult to be implemented in open access systems with numerous origins and destinations, where people hardly know each other.

Relocation operations in bike-sharing systems are also investigated in the OR literature. Asymmetric demand creates problem of imbalance for bike-sharing systems. This results in increase in the number of users (i.e. who try to rent bikes from empty stations or to leave bikes to full stations) who cannot utilize the system properly. As a result redistribution of bikes becomes inevitable. The literature contains solutions for both static and dynamic balancing problems. Static balancing problem disregards customer demand and assumes the system does not operate during redistribution (e.g. during the night). Whereas in dynamic balancing problem, demand varies with time and redistribution operations are performed accordingly. The static balancing problem has been modeled as a single vehicle one-commodity capacitated pickup and delivery problem (SVOCPPD) and was solved with an exact algorithm based on column-generation (Chemla, Meunier, & Wolfier Calvo, 2012). Additional formulations of the static balancing problem have been proposed in Raviv, Tzur, and Forma (2013). Dantzig and Wolfe (1960) and Benders (1962) decompositions have been also used to solve the dynamic balancing problem (Contardo, Morency, & Rousseau, 2012).

In the literature, there are also other types of problems that share common structures with the one-way car-sharing problem. The multiple depot vehicle scheduling problem with time windows (MDVSPTW) is one of the examples (Desaulniers, Lavigne, & Soumis, 1998). In the MDVSPTW, each customer has a request of tight time windows with a precise start and end time of operations, and a fleet of vehicles serves these customers one at a time. Each vehicle in the fleet belongs to a depot and the vehicles have to return to their depot at the end of the service. The objective of the problem is to minimize the number of vehicles and empty trips.

The literature review revealed that existing modeling efforts make a sharp separation between strategic and tactical decisions. This means that strategic decision-making models do not integrate in their structure aspects of tactical and operational decisions (e.g. vehicle relocation, fleet size) which, as we demonstrate in this paper have a significant bearing on the cost and performance of the car-sharing system. On the other hand, operational models are focused on the detailed modeling of different types of relocation strategies, assuming that the location, number, and station and fleet size are exogenously defined.

In reality, strategic, tactical, and operational decisions are interweaved and therefore there is a strong interaction between the three decision-making levels. Strategic decisions are primarily related to the definition of the location, number, and size of stations and interact with the tactical decision of fleet size determination. In turn, the fleet size is affected by vehicle relocation which is an operational decision. Here it is important to stress the fact that both fleet size and vehicle relocation influence the strategic level decisions. The above discussion suggests that there is a need for a model that will be able to address the strategic and tactical decisions by taking into account (at a macroscopic level) the impact of vehicle relocation. Fig. 1 illustrates these interactions. The above discussion suggests that there is a need for a model that will be able to address the strategic and tactical decisions by taking

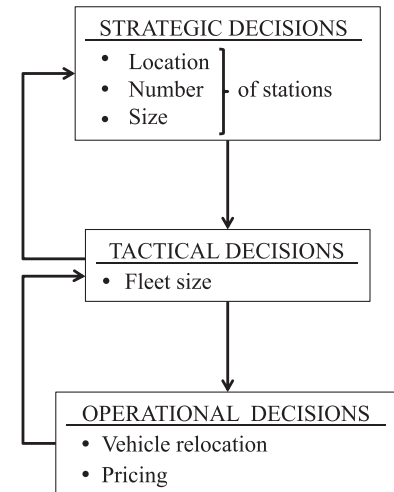


Fig. 1. Relationship between strategic, tactical and operational decisions.

into account the impact of vehicle relocation. In what follows we are presenting such a model.

3. Model description

The proposed model is motivated from the planning of electrical one-way non-floating reservation-based, for both pick-up and drop-off, car-sharing system. Shared-use electric vehicles are used to serve trips within a given geographical area. In what follows, we provide a description of the system in terms of its demand and supply characteristics before introducing the problem formulation.

3.1. System characteristics

i. Vehicles: A homogeneous fleet of electric vehicles is used to provide the services. Any type of trip request can be accommodated by any available vehicle.

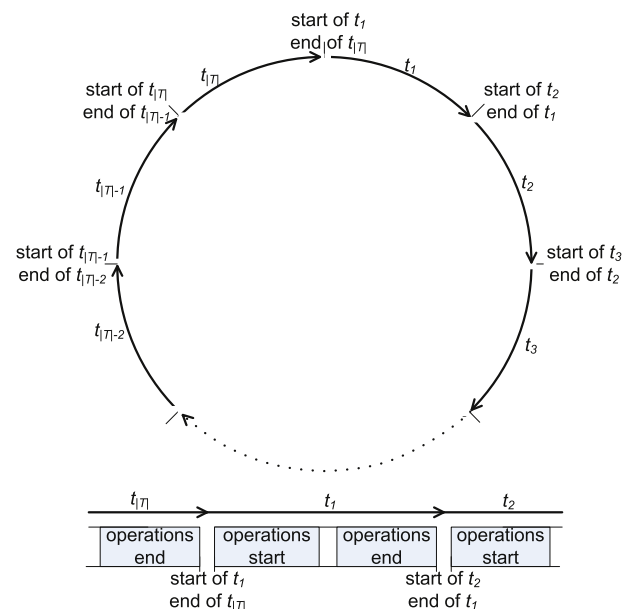


Fig. 2. The relationship between time intervals and operations where $T = \{t_1, t_2, \dots, t_{l-1}\}$ is the set of time intervals.

ii. Stations: Vehicles are picked-up and dropped-off at designated stations. Stations have the necessary infrastructure for parking and recharging the vehicles. Each station provides a specific number of parking places which defines the station size. Station size varies among stations and the size of each station determines its capacity.

iii. Time intervals: An operating day is divided into time intervals (not necessarily equally long) and each operation (i.e. rental, relocation, charging) starts at the beginning and finishes at the end of a time interval. The first time interval of a given day starts after the last time interval of the previous day (Fig. 2).

iv. Operations: The system involves three types of operations: rental, relocation and charging.

a. Rental: The system operates on the basis of reservations and allows one-way rental of vehicles. Reservations are made in advance of the pick-up time. Origin and destination locations, and pick-up and drop-off times are also known. Vehicles are picked-up/dropped-off from/at a station that is accessible to the initial origin/destination location of the respective user at pre-specified (when reservation is made) periods. It is assumed that each rental starts at the beginning of a time interval and ends at the end of the same or a subsequent time interval (Fig. 2).

b. Relocation: The system allows one way rental of vehicles. As a result, there might be accumulation and/or shortage of vehicles at stations. Relocation is used to rebalance the system resources, i.e. vehicles. Relocations can last more than one time interval (Fig. 2). During relocation, the vehicle is not available with the exception of extremely closely located stations (i.e. less than 2 kilometers per second), in which case rental and relocation can take place at the same time interval. The total time spend for relocation operations during a time interval cannot exceed the total available time of the staff assigned to a working shift.

c. Charging: The system modeled in this paper uses electric vehicles. In order to model the electric vehicles charging period, it is assumed that after a vehicle is returned from a rental operation, it has to stay in the station for a fixed period of time, which represents the charging period of the vehicle.

v. Working shift: A set of consecutive time intervals defines a working shift. The personnel needed for relocation operations is assigned to working shifts.

vi. Demand centers: In the model, demand centers (in the rest of the paper referred as centers) represent demand points that can be served by the same set of (candidate) stations. To illustrate how the centers are defined, we are using the example shown in Fig. 3. Fig. 3a depicts the origin and destination of demand and the station locations. Fig. 3b shows the stations that are accessible by different origin and destination locations. Please note that more than one station may be accessible from a given origin/destination point. The origin/destination points that can access the same set of stations are clustered together and constitute a center. Fig. 3c illustrates two centers (shaded areas) and trips (demand) associated with these centers. The grouping of demand into centers decreases the number of variables since the trips with the same origin and destination centers are grouped together. This grouping allows the solution of larger instances of problems. The distance between a center and a station is the average of all distances defined by the demand points of a given center and the associated station.

vii. Demand: Demand has a temporal and a spatial dimensions. Demand represents an aggregation of trip reservations

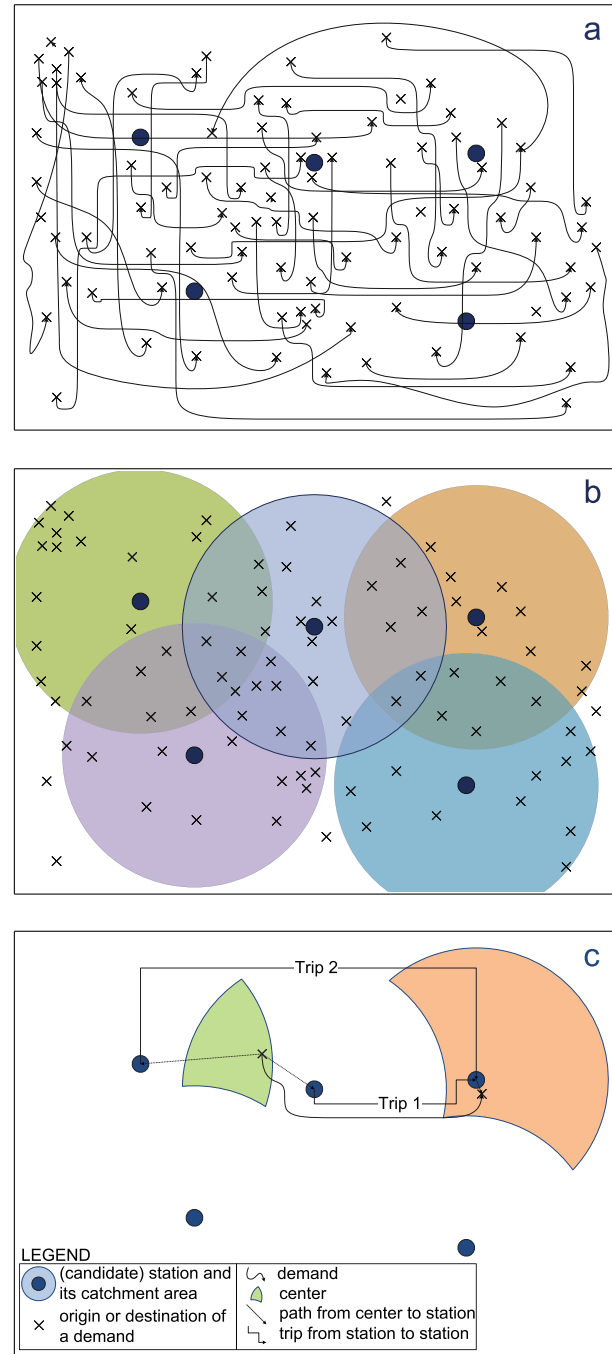


Fig. 3. (a) Location of stations and historical trips generated between origins and destinations; (b) origins and destinations are grouped according to the set of accessible (candidate) stations; and (c) based on this aggregation, a specific demand can be served in two different ways (trip 1 and 2).

(orders) of rentals that are associated with the same set of origin and destination centers and have common departure and arrival time intervals. In order to satisfy an “order” (i) a vehicle from a station that is accessible from the origin location (or equivalently center) at the beginning of the departure time interval, and (ii) a parking space at a station that is accessible from the destination location (or equivalently center) at the end of the arrival time interval have to be available. Note that “orders” do not have to be assigned to the closest station, but to accessible ones.

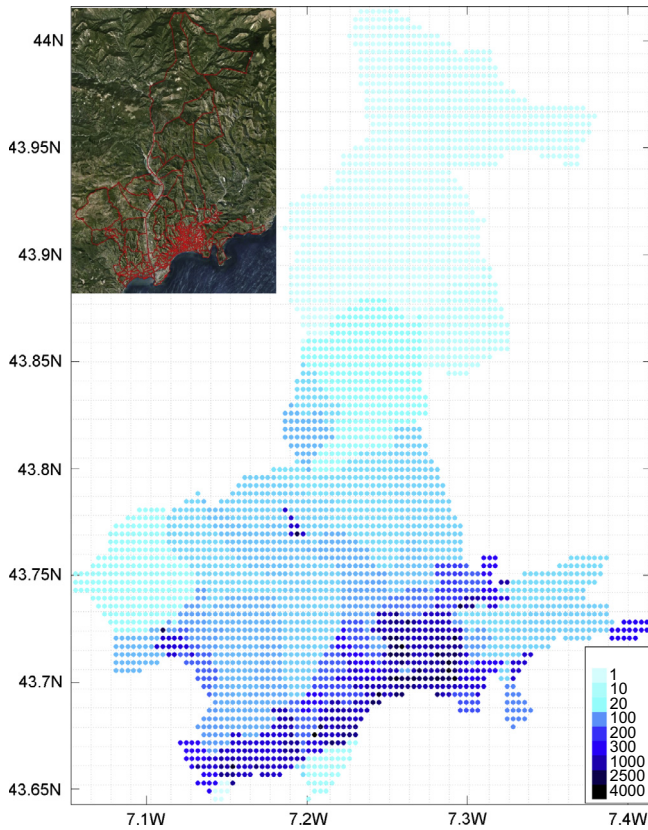


Fig. 4. Atoms used in population coverage.

viii. Atoms: An atom represents a small geographical area with known population. The atoms are used to model the population coverage of the car-sharing system. In our model, we assume that there is a maximum distance that determines if an atom is covered. Thus, if there is an open station closer than the predefined maximum value (coverage distance), the atom is covered. Atoms used in population coverage of the example problem can be seen in Fig. 4. These atoms represent the cells of a grid with an area of 0.1 square kilometer.

ix. Costs and revenues: The model includes two objective functions expressing the objectives of the users and the operator. The operator's benefits include vehicle rental revenues and subsidies, while costs include maintenance, operation and relocation of vehicles, and station opening costs. Users' net benefit is calculated as the difference between the utility gain in terms of monetary value, and the sum of vehicle rental and accessibility costs. In what follows (see items a–h below) we define all these terms.

a. Vehicle rental cost: The amount paid by the users to the operator to rent a vehicle expressed in euro per hours

b. Subsidy: It represents money paid directly to the operator, by public agencies, to cover revenue deficits per rental in euro per unit time.

c. Fixed vehicle cost: The cost encountered by the operator expressed in euro per day (e.g. depreciation, insurance)

d. Variable vehicle cost: The cost of the operator per km vehicle rented (e.g. cost of energy, maintenance cost due to wear-and-tear).

e. Vehicle relocation cost: The cost related to the relocation operations of the vehicles. It has two components: the relocation personnel cost (per shift) and the cost for driving vehicles between stations.

f. Station operating cost: The cost of operating a station. It is a function of the number of operating parking spaces.

g. User utility: The monetary value of the utility gained by the users by each satisfied trip expressed in euro per unit time.

h. Accessibility cost: The monetary value of time of the users required to reach a station from their origin and from stations to their destination expressed in euro per distance.

x. Scenarios: We use scenarios, to cope with the stochasticity and the seasonality of the demand. Alternative scenarios are defined by varying the input parameters of the model (e.g. weekdays, weekends).

xi. Scenario groups: The set of scenarios which addresses the same strategic decisions and parameters (e.g. number of vehicles, relocation personnel cost) belongs to the same scenario group. In order to account for daily variation within the same season (e.g. summer, autumn, winter), each season is set as a scenario group and more than one scenario are generated according to day of the week (e.g. weekdays, weekends).

3.2. Mathematical model

In this part, we represent the mathematical structure of the proposed model. We first define the sets and indices used to describe the model as well as the functions, variables and parameters in Section 3.2.1. In Section 3.2.2, the detailed multi-objective mathematical model is given and its objective functions and constraints are described in detail. The aggregate model and the rational for to have an aggregate model are presented in Section 3.2.3.

3.2.1. Inputs

Sets and indices:

i and $k \in I$: center indices

j and $l \in J$: (candidate) station indices

t, u and $w \in T$: time interval indices

$f \in F$: working shift index

$a \in A$: atom index

$s \in S$: scenario index

$g \in G$: scenario group index

Functions:

cover(a): set of stations that are accessible from atom a

btwn(t, u): set of time intervals from t to u

close(j): set of stations that relocation with station j is possible during the same time interval

Parameters:

SOC_j: cost for establishing station j

PSC_j: cost per parking space at station j

VFC^g: fixed vehicle cost per vehicle-day in scenario group g

VOC^{stu}_{jl}: operating cost of a vehicle rented at time interval t from station j to reach station l at time interval u in scenario s

VRC^{gt}_{jl}: relocation cost of moving a vehicle from station j to l starting at time interval t in scenario group g

AC^{gt}_{ij}/AC^{gt}_{ij}: accessing/egressing cost from/to center i to/from station j at time interval t in scenario group g

RPC^g: cost of relocation personnel for working shift f in scenario group g

RC^{gtu}_{jl}/SA^{gtu}_{jl}: rental charge/subsidy when a vehicle is rented at time interval t from station j to reach station l at time interval u in scenario group g

UC_{jl}^{stu} : user utility when a vehicle is rented at time interval t from station j to reach station l at time interval u in scenario s
CAP_j: maximum number of available for opening parking spaces at station j

COV: minimum percentage of population need to be covered by open stations

PR_a: percent of population inhabiting in atom a

OD_{ik}^{stu}: number of orders starting at the beginning of time interval t from center j ending at the end of time interval u at center k for scenario s

RI_{jl}^{gt}: time intervals needed to relocate a vehicle from station j to l starting at the beginning of time interval t in scenario group g

LR_{jl}^{gt}: last time interval of relocation if a vehicle is relocated from station j to l starting at the beginning time interval t in scenario group g

SI_f^g: time intervals included in working shift f in scenario group g

RT_{jl}^{gt}: time spent to relocate a vehicle from station j to l at the beginning of time interval t in scenario group g

WH^{gt}: total available working hours for a shift operating during time interval t in scenario group g

SW^s: weight of the net benefit of scenario s in the objective function

CT_{jl}^{stu}: charging periods of vehicles rented at time interval t from station j to reach station l at time interval u in scenario s

N: maximum number of open stations

$S(g)$: scenarios belonging to scenario group g

$G(s)$: scenario group of scenario s

Decision variables:

x_j : binary variable indicating if (candidate) station j is open or not

c_j : number of parking spaces at station j

v^g : number of vehicles used in scenario group g

d_a : binary variable indicating if atom a is covered by a station or not

h_f^g : number of relocation personnel needed during shift f in scenario group g

Auxiliary variables:

n_j^{st} : number of available vehicles in station j at the beginning of time interval t in scenario s

y_{ikjl}^{stu} : number of trip orders satisfied from center i renting vehicle from station j to make a trip at the beginning of time interval t to reach center k through station l at the end of time interval u in scenario s

z_{jl}^{stu} : number of vehicles rented from station j at the beginning of time interval t to reach station l at the end of time interval u in scenario s

m_{ik}^{stu} : number of unserved orders of OD_{ik}^{stu}

$p_{ij}^{st}/\bar{p}_{ij}^{st}$: number of vehicles rented/left from/to station j at the beginning/end of time interval t to/from center i in scenario s
 q_j^{st}/\bar{q}_j^{st} : number of vehicles rented/left from/to station j at the beginning/end of time interval t in scenario s

b_t^s : number of vehicles rented before time interval t which are still rented during time interval t in scenario s

e_t^s : number of vehicles being relocated during time interval t for which their relocation started before t in scenario s

r_{jl}^{st} : number of vehicles relocated from station j to l starting from the beginning of time interval t in scenario s

3.2.2. Detailed model

$$\max \sum_{(s,j,l,t)} SW^s \left[\underbrace{\sum_u (RC_{jl}^{stu} + SA_{jl}^{stu} - VOC_{jl}^{stu}) z_{jl}^{stu}}_{\text{rental charge+subsidy - vh. operating costs}} - \underbrace{VRC_{jl}^{G(s)t} r_{jl}^{st}}_{\text{vh. relocation cost}} \right. \\ \left. - \sum_g \sum_{s \in S(g)} SW^s \left(\underbrace{\sum_j RPC_j^g h_f^g}_{\text{personnel cost}} + \underbrace{\sum_j VFC_j^g v^g}_{\text{vehicle maintenance cost}} \right) - \sum_j \underbrace{(SOC_j x_j + PSC_j n_j^s)}_{\text{st. operating and parking costs}} \right] \quad (1)$$

$$\max \sum_s SW^s \left[\underbrace{\sum_{(j,l,t,u)} (UC_{jl}^{stu} - RC_{jl}^{stu}) z_{jl}^{stu}}_{\text{utility - rental charge}} - \underbrace{\sum_{(i,j,t)} (AC_{ij}^{G(s)t} p_{ij}^{st} + \bar{AC}_{ij}^{G(s)t} \bar{p}_{ij}^{st})}_{\text{accessibility cost}} \right] \quad (2)$$

$$\text{s.t. } c_j \leq CAP_j x_j \quad (\mathbf{a}) \quad n_j^{st} \leq c_j \quad (\mathbf{b}) \quad \sum_j x_j \leq N \quad (\mathbf{c}) \quad \forall j \text{ and } \forall s, j, t \quad (3)$$

$$c_j \geq x_j \quad (\mathbf{a}) \quad \sum (s, t) n_j^{st} \geq x_j \quad (\mathbf{b}) \quad \forall j \quad (4)$$

$$d_a \leq \sum_{j \in \text{cover}(a)} x_j \quad (\mathbf{a}) \quad \sum_a PR_a d_a \geq COV \quad (\mathbf{b}) \quad \forall a \quad (5)$$

$$\sum_{(j,l)} y_{ikjl}^{stu} + m_{ik}^{stu} = OD_{ik}^{stu} \quad \forall s, i, k, t, u \quad (6)$$

$$\sum_{(i,k)} y_{ikjl}^{stu} = z_{jl}^{stu} \quad \forall s, j, l, t, u \quad (7)$$

$$\sum_{(k,l,u)} y_{ikjl}^{stu} = p_{ij}^{st} \quad (\mathbf{a}) \quad \sum_{(k,l,u)} y_{ikjl}^{stu} = \bar{p}_{ij}^{st} \quad (\mathbf{b}) \quad \forall s, i, j, t \quad (8)$$

$$\sum_{(i,k,l,u)} y_{ikjl}^{stu} = q_j^{st} \quad (\mathbf{a}) \quad \sum_{(i,k,l,u)} y_{ikjl}^{stu} = \bar{q}_j^{st} \quad (\mathbf{b}) \quad \forall s, j, t \quad (9)$$

$$q_j^{st} \leq n_j^{st} - \sum_t r_{jl}^{st} + \sum_{l \in \text{close}(j)} r_{lj}^{st} \quad \forall s, j, t \quad (10)$$

$$n_j^{st} - q_j^{st} + \bar{q}_j^{st} - \sum_t r_{jl}^{st} + \sum_{\substack{(l,u): \\ LR_{jl}^{G(s)t} = t}} r_{jl}^{su} = n_j^{\text{next}(t,1)} \quad \forall s, j, t \quad (11)$$

$$\sum_{\substack{(j,l,u,w): \\ t \in \text{between}(u,w) \cap u}} z_{jl}^{suw} = b^{st} \quad (\mathbf{a}) \quad \sum_{\substack{(j,l,w): \\ t \in \text{between}\left(u, LR_{jl}^{G(s)t}\right) \cap u}} r_{jl}^{su} = e^{st} \quad (\mathbf{b}) \quad \forall s, t \quad (12)$$

$$\sum_j n_j^{st} + b^{st} + e^{st} = v^{G(s)} \quad \forall s, t \quad (13)$$

$$\sum_{\substack{(j,l,w): \\ t \in LR_{jl}^{G(s)t} \cap u}} RT_{jl}^{G(s)t} r_{jl}^{su} \leq WH^{G(s)t} h_f^{G(s)t} \quad \forall s, f, t \in SI_f^{G(s)} \quad (14)$$

$$r_{jl}^{st} \leq c_j \quad (\mathbf{a}) \quad r_{ij}^{st} \leq c_j \quad (\mathbf{b}) \quad \forall s, j, l, t \quad (15)$$

$$n_j^{st} \geq \sum_{(s,j,l,u,w): t \in CT_{ij}^{suw}} z_{ij}^{suw} \quad \forall s, j, t \quad (16)$$

$$x_j, d_a \in \{0, 1\} \quad \forall j, a \quad (17)$$

$$n_j^{st}, c_j, p_{ij}^{st}, \bar{p}_{ij}^{st}, r_{jl}^{st}, m_{ik}^{stu}, z_{jl}^{stu}, y_{ikjl}^{stu}, q_j^{st}, \bar{q}_j^{st}, b^{st}, e^{st}, v^s, h_f^g \in \mathcal{N} \quad \forall s, i, k, j, l, t, u, g, f \quad (18)$$

The problem formulation is described in Eqs. (1)–(18). The first objective function (Eq. (1)) expresses the maximization of the net revenue for the operator. Net revenue is calculated as the difference between the sum of total rental revenue and subsidy minus station, vehicle and relocation costs. Note that all of the values in both objective functions except station opening cost are weighted analogously to the number of days (e.g. five for weekdays, two for weekends) of each scenario (SW^s). This is due to the fact that the location of the stations and the number of parking spaces are regarded as strategic decisions and therefore have to be the same in all scenarios. However the rest of the parameters are scenario specific (e.g. the number of vehicles). The net revenue for the trip starting from station j to station l from the beginning of time interval t to time interval u in scenario s of given type, equals the rental charge per trip (RC_{jl}^{stu}) plus subsidy (SA_{jl}^{stu}) minus operating cost (VOC_{jl}^{stu}) times the number of trips of the same type served (z_{jl}^{stu}).

The relocation cost has two components: (i) The vehicle cost related to the total km driven to relocate and (ii) the labor cost associated with the cost of the personnel used to relocate the vehicles. The total vehicle relocation cost is equal to the expenses of all the relocation operations. The vehicle relocation cost for the relocation starting from station j at time interval t to station l in scenario s is equal to the sum per relocation ($VRC_{jl}^{G(s)t}$) times the number of relocations (r_{jl}^{st}). Similarly, the relocation personnel cost equals the sum of all personnel costs. The total personnel cost for shift f in scenario group g equals the unit personnel cost (RPC_f^g) times the number of staff hired for this shift (h_f^g).

The fixed vehicle cost depends on the total number of vehicles operating in the system. For scenario s , this cost is equal to the product of the unit fixed vehicle cost (VFC^g) and the number of vehicles in the system (v^g) in scenario group g . Note that, for scenarios belonging to the same (scenario) group, the number of vehicles is the same, since we regard the number of vehicles as a tactical decision.

The station operating and parking space costs are the costs dedicated to station operations. There is a fixed cost for operating a station (SOC_j) and a variable cost (PSC_j) for each parking space (n_j^p) operating at a given station j .

The second objective (Eq. (2)) expresses the maximization of the users' net benefit. UG_{jl}^{stu} can be defined as the monetary value (i.e. euro) of the utility gain for each realized trip starting from station j to station l from the beginning of time interval t to time interval u in scenario s of the same type. Similarly, the rental fee is the money paid to the operator for the rental of vehicles by the users (REV_{jl}^{stu}) and total rental charge equals the sum of them. The accessibility cost is the cost associated with the access or egress of a station from a center.

Constraints (3a) and (3b) restrict the number of parking spaces (station capacity constraint), and the number of available vehicles for each time interval and station. For each open station there is an upper bound (CAP_j) for its capacity. Constraint (3c) limits the total number of operating stations. Constraints (4a) and (4b) assign at least one parking space and an operation (i.e. rental, relocation) to each open station. These constraints are essential in order to guarantee the coverage of the demand by an open station. Constraints (5a) and (5b) are the atom coverage constraints, i.e. if an atom is covered or not, and population coverage constraints, i.e. the car-sharing system is accessible by a given percentage of the population, respectively. Constraints (6) ensure that the total number of orders is equal to the sum of the satisfied demand (served orders) and unserved (lost) orders.

A trip order consists of three segments (see Fig. 5). A segment connecting any origin center with an origin station is called *access segment*. A segment connecting any origin station with a destination station is called *rental segment*. A segment connecting any destination station with a destination center is called *egress segment*. The total number of trip orders using a segment is called *segment flow*. Constraints (7) ensure that for a given pair of origin–destination stations, the rental segment flow should be equal to the number of trip orders from the given origin to destination station. Constraints (8a) require, for a given center–station pairs, that the access segment flow should be equal to the number of trip orders originated from the given center and served by the given origin station. Similarly, constraints (8b) require, for a given station–center pair, that the egress segment

flow should be equal to the number of trip orders served by the given destination station and destined to the given center. Constraints (9a) ensure that the sum of the flows of the access segment to a station should be equal to the sum of the vehicles leaving the station. The latter coincides with the sum of the flows of the rental segments that originate from the given station. Similarly, constraints (9b) require that the sum of the flows of the egress segments from a station should be equal to the number of vehicles entering the station. The latter coincides with the sum of the flows of the rental segments destined to the given station. Note that, these constraints hold for all time intervals and scenarios.

Constraints (10) require that the number of vehicles leaving a station (due to rental and relocation) at the beginning of interval t cannot exceed the number of vehicles available at that stations at the same time interval. Constraints (11) are the “vehicle conservation” constraints for each station.

Constraints (12a) and (12b) ensure that, if a vehicle is under rental or relocation for more than one time interval, it will still be accounted for by variables b_t^s and e_t^s respectively. We do not need to specially keep track of vehicles under relocation or rental for one period only since they are already counted when they were parked before they picked up for either operation. Constraints (13) are used to ensure the conservation of the number of vehicles. Stated otherwise, it requires that, for a given time interval, the sum of vehicles at each station and the vehicles under rental or relocation should be equal to the number of vehicles available for the scenario.

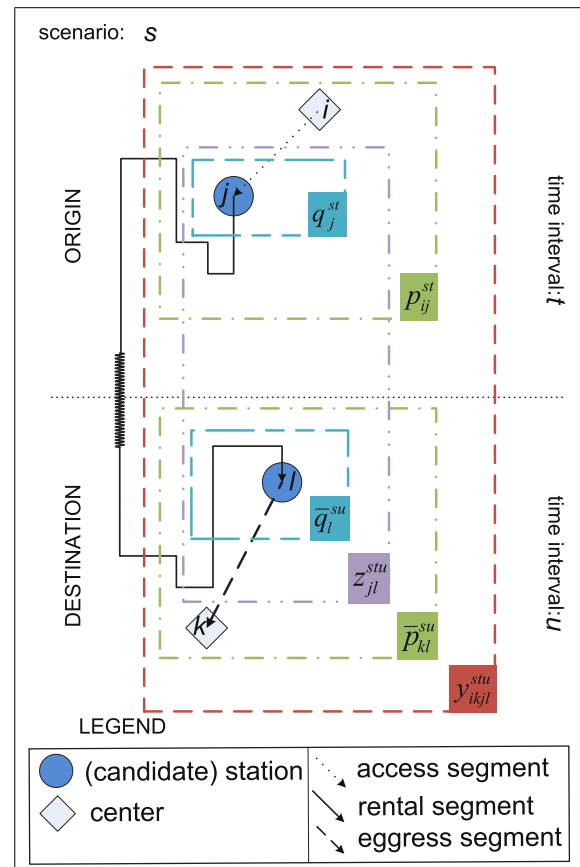


Fig. 5. Access, rental and egress segments for a single trip order with related variables shown in frames' right bottom corners.

Constraints (15a) and (15b) set an upper bound to relocation from and to every station respectively. This upper bound equals to the number of operating parking spaces in all open stations.

Constraints (16) are restrictions specific to electric-car-sharing systems. These constraints require the vehicles to stay and be charged, after each rental operation, at the station they arrived. These constraints require that the number of vehicles in the station should be greater than or equal to the number of vehicles requiring charging.

3.2.3. Aggregate model

Eqs. (2)–(9), (12a), (16)–(18) (19)

$$\max \sum_{(s,j,t)} SW^s \left[\sum_u \left(RC_{jl}^{stu} + SA_{jl}^{stu} - VOC_{jl}^{stu} \right) z_{jl}^{stu} \right] - \overbrace{\sum_{(s,j,t)} SW^s VRC_j^{G(s)t} (r_j^{st} + \bar{r}_j^{st})}^{\text{new vh. relocation cost}} - \sum_g \sum_{s \in S(g)} SW^s \left(\sum_f RPC_f^g h_f^g + VFC^g v^g \right) - \sum_j \left(SOC_j x_j + PSC_j n_j^s \right) \quad (20)$$

$$q_j^{st} \leq n_j^{st} - r_j^{st} + \sum_{l \in \text{close}(j)} r_l^{st} \quad \forall s, j, t \quad (21)$$

$$q_j^{st} + \sum_{l \in \text{close}(j)} q_l^{st} \leq n_j^{st} + \sum_{l \in \text{close}(j)} n_l^{st} \quad \forall s, j, t \quad (22)$$

$$n_j^{st} - q_j^{st} + \bar{q}_j^{st} - r_j^{st} + \sum_{(j,u): t = LR_j^{G(s)u}} \bar{r}_j^{su} = n_j^{\text{next}(t,1)} \quad \forall s, j, t \quad (23)$$

$$\sum_{(j,t): t = LR_j^{G(s)u}} r_j^{su} = \sum_j \bar{r}_j^{st} \quad \forall s, j, t \quad (24)$$

$$r_j^{st} \leq c_j \quad (a) \quad \bar{r}_j^{st} \leq c_j \quad (b) \quad \forall s, j, t \quad (25)$$

$$\sum_{(j,u): t = \text{btwn}(u, LR_j^{G(s)t}) \setminus u} r_j^{su} + \bar{r}_j^{su} = e^{st} \quad \forall s, t \quad (26)$$

$$\sum_{(j,l,u): t = LR_j^{G(s)u}} RT_j^{G(s)t} (r_j^{su} + \bar{r}_j^{su}) \leq WH^{G(s)t} h_f^{G(s)} \quad \forall s, f, t \in S_f^{G(s)} \quad (27)$$

$$r_j^{st} \geq 0 \quad \bar{r}_j^{st} \geq 0 \quad \forall s, j, t \quad (28)$$

In real life instances, the model described by Eqs. (1)–(18). may result in problem sizes that are not possible to be efficiently solved. Although for most of the variables, we only generate those that have positive values and construct the corresponding constraints accordingly, we do not have this opportunity for the relocation variables r_{jl}^{st} . As the relocations can happen between any station pairs, we need to generate $|J|^2|S||T|$ number of variables which renders problem instances found in real world cases difficult to solve. For example, for the case of Nice, France an instance of 142 candidate stations, 12 scenarios and 15 time intervals needs more than 3.6 millions of relocation variables, r_{jl}^{st} only. In order to cope with this issue, we assume that the relocated vehicles are firstly accumulated at an imaginary hub and then distributed from that hub to the stations. For this issue, two new variables, r_j^{st} and \bar{r}_j^{st} are defined expressing the number of vehicles relocated from/to station j starting from the beginning/finishing at the end of time interval t in scenario s . Using this transformation, the number of relocation variables decreases to $2|J||S||T|$ which translates to 51,120 variables instead of over 3.6 millions. As we demonstrate in the subsequent section of the paper (Section 4), in the case of Nice, this transformation results to an error of less than 2% of the operator's net revenue.

In addition, we substitute the constraints (10), (11), (12b) and (14) and (15), with the following constraints (21)–(28). Moreover, the vehicle relocation cost part of the operator's objective function (Eq. (1)) is replaced with Eq. (20). Note that, parameters LR_j^{st} , VRC_j^{st} and RT_j^{st} shows the last time interval, the vehicle relocation cost and time spent when a vehicle is relocated from/to station j to/from hub starting in time interval t in scenario group g respectively. The aggregate model is expressed by Eqs. (19)–(28).

Constraints (21) and (22) replace constraints (10). Constraints (21) postulate that the total number of trips starting from station j going to station l during time interval t to u in scenario s cannot be more than the number of available vehicles at the beginning of the time interval t ; minus the number of relocations from station j ; plus the number of relocations from the stations that are close enough to station j to have relocations at the same time interval. Constraints (22) set an upper bound for each station group close enough to have relocations to the same station. For each set of stations, the total number of trips started from the corresponding set of stations cannot be more than the total number of available vehicles at these stations.

Constraints (23) replace constraints (11) of the first model. Constraints (24) require that the total number of relocations from stations to the imaginary hub ending in time interval t should be equal to the number of relocations to the stations from the imaginary hub starting in time interval t . This is applicable for each time interval and scenario.

Constraints (25a) and (25b) replace constraints (15a) and (15b). They set the number of relocations to the number of operating parking spaces. Constraints (26) and (27) work the same as constraints (12b) and (14) respectively. The former constraints calculate the number of vehicles under relocation whereas the latter constraints decide on the manpower needs for each time interval in each scenario.

4. Model application

The model presented in Section 3.2.3 was applied to plan a one-way electric-car-sharing system in Nice, France. The study area is 294.19 square kilometer and has a population 327,188 inhabitants between ages 15–64 with a density 1112 persons/square kilometer. The area under consideration consists of 210 regions. The population of each region was obtained from 2009 census data (INSEE, 2010). We assume that the population is uniformly distributed inside regions and calculate the population of each atom accordingly. The atoms and their population can be seen in Fig. 4.

The whole model is implemented in C#.NET environment. IBM ILOG Cplex Version 12.5 with Concert Technology is used for solving MILPs. To cope with the enormous number of relocation variables, the aggregate model (Section 3.2.3) is used. The exact and aggregated relocation costs will be compared later. For each station, half of the average distance of closest n stations is calculated and regarded as the distance of the same station to the imaginary hub. This approach generates values that closely approximate real relocation distances. To further investigate the performance of the approximation, a simulation environment that compares average real and hub relocation distance for 1000 cases was generated with different n values. In Fig. 6, the error for different values of the number of relocations (n) are compared. We use $n = 20$ which results to an average minimum error. In other words, when distance for relocation is calculated, the distance from a station to the hub is assumed half of the average distance of 20 closest (candidate) stations. Note that in the aggregate model a relocation is composed of two legs in aggregate model: relocating vehicle from its old station to the imaginary hub and to its new destination from the hub. A similar approach is used for the second leg. The number of relocations per personnel has values between 7 and 15 which results in an error not more than 0.7 kilometer per relocation. Since distance per relocation observed is around 4 kilometers and the total cost of relocation is not more than 20% of the objective function value of each case (see in Figs. 10 and 11), this relaxation might not create an error more than 3.5%. Also post-analysis showed that the difference between the cost of relocation operations calculated by the aggregate model and the exact model

is less than 2% of the operator's revenue on average. In order to deal with the extremely large size of the problem, we take advantage of the sparsity of the matrices of the variables and we do not generate the variables that have zero value. This decreases the number of variables of aggregate model in order of magnitude from 10 to 5.

To guarantee generation of feasible solutions in reasonable time, extra cuts are generated with CPLEX. The runs are taken on a computer with 3.00 gigahertz Intel Core 2 Quad CPU and 8 gigabytes of RAM. All runs are realized as single-threaded programs and every run is terminated when either they reach 2% optimality gap or 9 hours run time. Most of the runs that are represented here were terminated in less than three hours and all of the runs had an optimality gap less than 8%.

The summary of the methodology for the entire approach can be seen in Fig. 7 where w_{operator} and w_{users} express the weights of operator and users benefit respectively. The terms *superior* and *inferior* are related to station coverage. If a candidate station covers an additional origin or destination location as compared to another candidate station's covered locations, the former candidate station is superior to the latter.

4.1. Car-sharing system in nice

The current system operating in Nice is a two-way car-sharing system (no need for relocation operations). However, the proposed model deals with the case of one-way car-sharing, which makes the implementation more demanding. Therefore, there was a need to convert the existing two-way car-sharing data into one-way. This conversion was achieved by looking at the current database and creating one-way data by splitting the trips into one-way legs when the idle time of the rented vehicle at a given location was exceeding one hour, and the location was accessible from a station (i.e. the distance between the location and the stations is less than 500 meters). The problem formulation and solution procedure of Section 3 are not affected by this conversion and other methods could be utilized to generate the one-way demand (e.g. population surveys) Efthymiou, Antoniou, and Waddell (2013).

We use the origin and destination locations of the real demand in two steps. First, we solve a *maximal set covering problem* (Church & ReVelle, 1974) to identify the candidate station locations for the aggregate model. For each origin and destination, the (existing or candidate) stations that are accessible (the distance between two points is less than the maximum accessibility distance) are estimated. In addition to existing 42 stations, the model was forced

to choose 100 new candidate locations for the stations. Second, the locations are grouped into centers. This grouping was done according to the (existing or candidate) stations that are accessible to them. The locations with the same accessible stations were assigned to the same centers. The accessibility distance between a center and a station is calculated by taking the average of the distance between the elements of the center and the station (Fig. 3). The graph showing the locations of the origin and destination of the trips (crosses), the operating (blue) and candidate (red, gray and black) stations' locations (dots) and their catchment areas (circles with the same colors) can be seen in Fig. 8 in which x-axis shows the longitude and y-axis shows the latitude values. Note that, the covered origin and destination locations by already operating and/or selected candidate stations have dark gray color, and each grid is a square with sides of 1 kilometer.

After solving set covering problems, the set of candidate locations for the aggregate model (defined in Section 3.2.3) is produced. The aggregate model is solved with different weights (of users' and operator's benefit) in order to generate an efficient frontier for the given case. A total of 8 different scenarios of four seasons (spring, summer, autumn, winter) for two different day groups (weekdays, weekends) were selected. A working shift is assigned for each time interval. It was also assumed that the number of operating vehicles and relocation personnel for the same season is the same. This is because the fleet and crew size decisions are considered tactical and do not change within the same season. Each scenario was constructed by using two days of the real demand of the same day group in the same season. The capacity of each station was set to five vehicles and the model was asked to choose 28 more stations (from a set of 100 candidates) in addition to 42 stations that are already operating. Each day was divided into 15 time intervals. The time intervals are generated in such a way that the total duration of rental time (vehicle-hours) in each time interval in the historical demand are almost equal.

Given that: (i) each vehicle has a maximum range of 120 kilometers, (ii) the average trip length is 30 kilometers and (iii) it takes 8 hours to fully charge an empty battery, it follows that each vehicle should be charged at least for 2 hours before it becomes operationally available. An average value for the charging duration is utilized for all trips as the operator is not aware of the distance that will be traveled by the driver at the beginning of the trip. A more detailed model can be solved in the operational problem, where uncertainty in the duration of charging can be considered. The values for some of the other parameters applied in the model are presented in Table 1. The fuel cost is low because the system is operating with electric vehicles. Note that, the stated values have been properly modified to ensure data confidentiality.

Using the parameters presented in Table 1, we solved the model and generated the efficient frontier provided in Fig. 9 by using weighted sum method Cohon (2004). The selected candidate stations can also be seen in Fig. 8. The candidate stations shown with red color are the candidates that are not selected, the ones with gray and black are the stations selected at least once. The intensity of the color given to the selected candidate stations increases as the frequency of their appearance in the efficient frontier increases. For instance, black means the candidate station appears in all the efficient solutions whereas the lightest gray suggests that it appeared in only one of them. The circles around each station shows the stations' accessibility area which is a circle with 500 meters of radius.

As it can be seen in Fig. 8, although the part of the data used to create the efficient frontier composed of 16 of the 421 days, selected candidate stations manage to cover locations with high demand. For instance, there is an accumulation of demand around

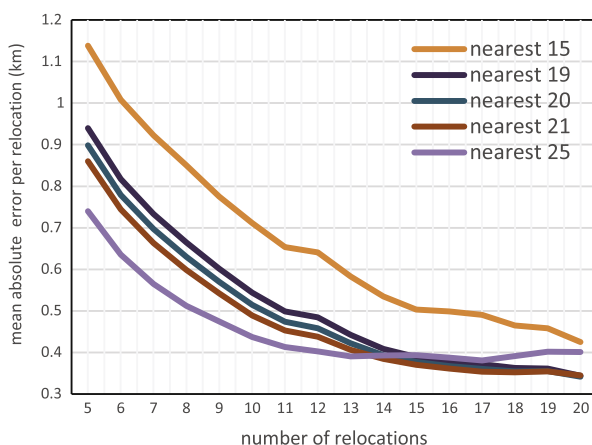


Fig. 6. Average absolute error of imaginary hub usage in relocation for different number of relocations. Different n values are compared in order to find the most suitable value for our case.

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|---|--|
| <ol style="list-style-type: none"> 1. Data reading and parameter creation <ol style="list-style-type: none"> (a) Read population data and create atoms (b) Read historical demand data (c) Conversion from two-way demand data to one-way <ol style="list-style-type: none"> i. For each historical demand datum <ol style="list-style-type: none"> A. If waiting time is greater than predefined value <ul style="list-style-type: none"> • Split the historical demand and create two new demands (d) Time Interval Selection <ol style="list-style-type: none"> i. Set working shifts ii. Find time intervals consistent with working shifts that minimizes the variation of demand count in each time interval 2. Finding candidate locations <ol style="list-style-type: none"> (a) Finding all candidate locations for set covering model <ol style="list-style-type: none"> i. Iterate over all demand origin and destination locations <ol style="list-style-type: none"> A. If current location is <i>superior</i> to any previously added location <ul style="list-style-type: none"> • Remove previously added <i>inferior</i> solution B. If current location is not inferior to any previously added location <ul style="list-style-type: none"> • Add current location to locations for candidate locations for set covering model (b) Finding candidate locations for aggregate model <ol style="list-style-type: none"> i. Set $i = N$ ii. While $J < \text{maximum number of stations}$ <ul style="list-style-type: none"> • Run <i>maximal set covering problem</i> [27] with number of sets = i | <ul style="list-style-type: none"> • Add candidate locations found in the solution to J • Set $i \leftarrow i + 1$ <ol style="list-style-type: none"> 3. Mathematical model <ol style="list-style-type: none"> (a) Select predefined number of days from historical demand and create scenarios (b) Set values of $w_{\text{operator}} > 0$ and $w_{\text{users}} > 0$ (c) Variable creation <ol style="list-style-type: none"> i. Create variables d_a, x_j, n_j, c_j and b_i^s ii. For each demand <ul style="list-style-type: none"> • Create (or increment upper bound if already created) variables $y_{ik}^{stu}, m_{ik}^{stu}$ and z_{jl}^{stu} • Create (or increment upper bound if already created) variables $p_{ij}^s, \bar{p}_{ij}^s, q_j^s$ and \bar{q}_j^s iii. Create variables r_{jl}^s, \bar{r}_{jl}^s and e_i^s (if $\neq 0$) iv. Create variables h_j^s and v^s (d) Constraint creation <ol style="list-style-type: none"> i. For each $j \in J$: Create constraints 3a, 4a and 4b ii. For each (s, j, t): Create constraint 3b iii. For each $a \in A$: Create constraint 5a iv. Create constraints 3c and 5b v. For each demand: Create constraints 6–9 vi. For each (s, t): Create constraints 12a, 13 and 26 vii. For each (s, j, t): Create constraints 16, 21–25 (e) Create objective function with the multiplication of w_{operator} and w_{users}, and 20 and 2 respectively (f) Solve the model and, calculate net users' and operator's benefit |
|---|--|
-

Fig. 7. Summary of the methodology for the entire approach with the weights w_{operator} and w_{users} for the users' and operator's benefit respectively.

the coordinates 43.73N–7.19E and the model selects to operate a station there in all efficient solutions.

From the efficient frontier shown in Fig. 9, it can be seen that the operator should sacrifice some of its net revenue in order to improve total users' benefit and vice versa. Although the revenue and subsidy of the served demand is higher when more demand is served, the rate of increase of the operational costs (e.g. vehicle operating cost, relocation cost) is lower than the rate of increase of the associated benefits. Both the number of vehicles in the system and the increase of relocation operations decrease the utilization of the vehicles.

Another interesting result is associated with the selection of common stations in determining the efficient frontier. It is observed that (in addition to 42 already operating stations) all seven efficient solutions select stations among a set of 46 candidate locations. More specifically, 13 of these stations appear in all solutions; 5, 7 and 3 in six, five and four solutions (out of seven) respectively. This result suggests that from station location point of view, the efficient station locations are not in conflict when considering the user and the operator objectives and the solution is robust. Since there is no conflict in station locations, these 28 stations are assumed to be operating stations in addition to already operating 42 stations in the further analysis.

After deciding about the number and location of the stations (strategic decision), we perform further analysis in order to explore if different demand levels, coverage distances and subsidy amounts influence the solution.

4.2. Effect of demand

Firstly, we examine the effect of demand by using five different levels and equal weight for the users' and the operator's objectives. The results of these runs are demonstrated in Fig. 10. In Fig. 10, there are two sets of bar charts for each level of demand. These bar charts correspond to different number of available vehicles, bounded vs. relaxed. Bounded is referred to the cases where the number of vehicles is forced to be less than or equal the corresponding number of the baseline scenario. Please note that, in the relaxed case there is no such constraint. Moving from left to right we generate for both cases (bounded and relaxed), alternative demand levels by increasing the baseline demand by 50% up to the level of 200%. The table at the bottom of the graph, summarizes the total number of trip requests, the number of lost demand and their percentage.

For the relaxed case, the operator's benefits for increasing levels of demand are increasing faster than the users' benefits. In the bounded case we observe the same pattern. As the demand increases, net benefits are increasing since the model can select to serve the most profitable customers from a larger pool of candidate customers. In the bounded case, the slopes of users' and operator's benefits curves decreases as the demand increases. This is because of the limitation on the number of vehicles. This is an expected result since the model does not penalizes lost demand while at the same time increases the value of the objective function from the served demand. Note that, this increase of demand results

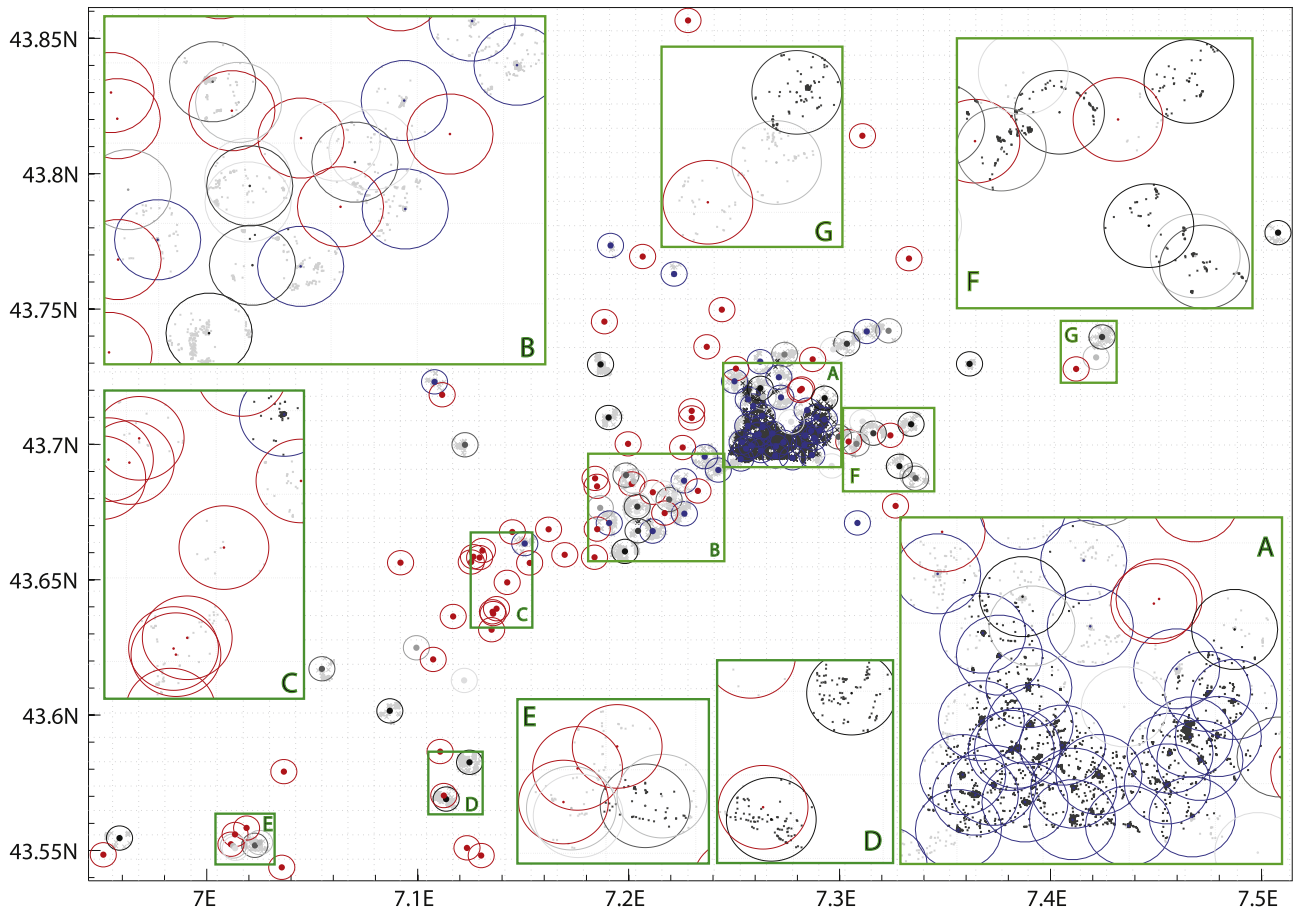


Fig. 8. The origin and destinations of the divided trips, the operating (blue) and candidate (gray, black and red) stations and their catchment areas. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 1

Some values of the parameters used in the model.

Fixed vehicle cost (euro per day)	20
Vehicle operating cost (euro per kilometer)	0.01
Average number of trips per scenario	155.2
Average trip length (kilometer)	30
Max accessibility distance (kilometer)	0.5
Minimum coverage (%)	20
Subsidy (euro per hour)	5
Revenue per unit time (euro per hour)	8
Accessibility cost (euro per kilometer)	5
Utility (euro per hour)	12
Relocation speed (kilometer per hour)	30
Relocation personnel cost (euro per hour)	18

to a higher density of demand, a fact that gives more flexibility to the model to select customers leading to improved objective function values. For the 50% increased demand, the benefit lost for both the operator and the users are almost imperceptible. However, the difference between the relaxed and bounded cases becomes significant with a demand increase of 100%. This means that in case of significant increase in demand, the system has to be redesigned in some aspects to improve the quality of service and revenues.

Another important finding is the relationship of costs, benefits and revenues. Since the rental fee is a cost for the users and a benefit for the operator, it has no effect in our objective function for this specific example since equal weights are used for the users' and the operator's objectives. The subsidy and the users utility are the only two values contributing to the increase of the value of the

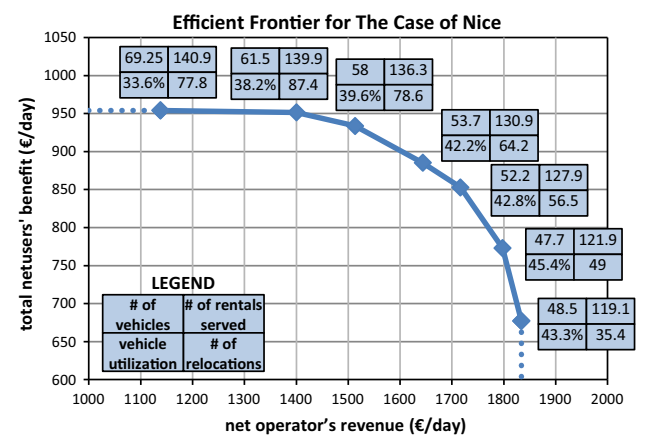


Fig. 9. The efficient frontier for the case of Nice, France.

objective function and consequently more orders (customers) are served.

In the calculation of the required relocation personnel, it is observed that relocation cost is not significantly affecting operator's income. In the most congested system, not more than 35 hours of relocation personnel are required which corresponds to a cost of 615 euro, about 12% of the rental charge. This finding suggests that relocation operations do not significantly increase the operator's cost.

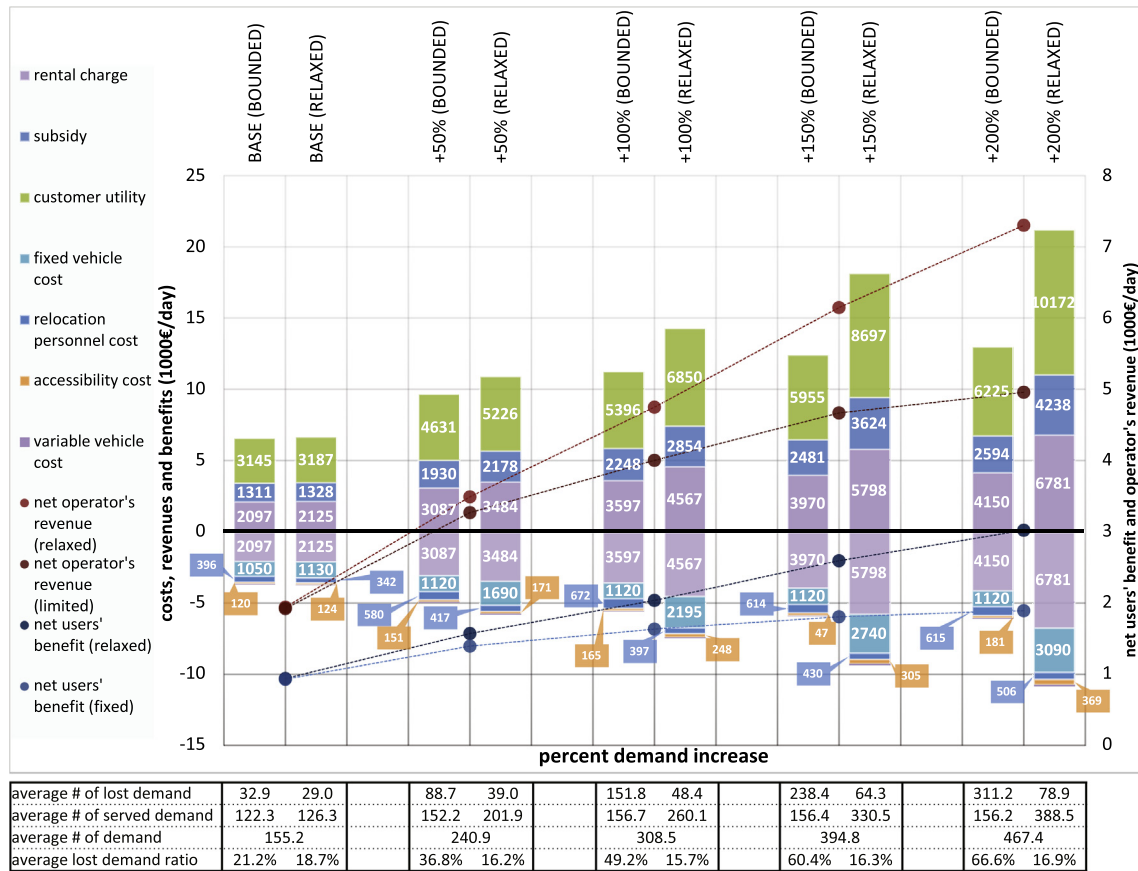


Fig. 10. The costs, benefits and revenues with the increased demand.

The accessibility cost is not significant because both the accessibility cost per kilometer (5 euro per kilometer) and maximum accessibility distance (0.5 kilometer) are substantially lower compared to other costs (e.g. utility: 20 euro per hour, relocation personnel cost: 18 euro per hour).

Another important finding is related to the change in the percentage of unserved requests. The unsatisfied demand is increasing with the total number of trips. In the “relaxed” cases the percentage of lost demand is decreasing until +100% demand. This may be due to the fact that the cost of unserved demand due to shortage of vehicles is less than the cost of acquiring extra vehicles to serve the lost demand. However, we observe an increase in the percentage of lost demand when demand is more than doubled. From a detailed observation of the results, it can be inferred that the concentration of demand during specific intervals at specific geographical locations is high. During these intervals the model either prefers not to serve additional “orders”, since the cost is more than the benefit or cannot manage to serve extra demand since it reaches its limitations in busy time intervals. On the other hand in bounded cases, when demand increases more than 100%, the number of demand served does not change. A careful look at the results shows that, the bounded system reaches its limitations and cannot serve more customers without increasing system resources (e.g. the number of vehicles).

4.3. Effect of accessibility distance

The effect of maximum accessibility distance was also investigated for two different levels of demand (e.g. base and +100%). Six different accessibility distances from 500 to 1000 meters in every 100 meters intervals were tested. The demand generated for the 500 meters accessibility distance is used for all 6 cases to

test only the effect of flexibility. Fig. 11 shows the value of the objective function components (left axis) and the operator's and users' net benefits (right axis) as a function of maximum coverage distance.

In both graphs shown in Fig. 11, it can be seen that the maximum accessibility distance decreases the net users' benefit slightly (around 1%) while operator's revenue is improved 1–4% for each accessibility distance increment. However, the same trend is not followed by the demand served. These two results are the consequence of the flexibility introduced to the system. The average number of accessible stations for the covered origin or destination points increases from 2.30 to 6.65. The increase of the number of accessible stations, results to an expanded feasible region and leads to an improvement of the operator's revenue. Since accessibility cost is low (5 euro per kilometer) compared to operational costs of the operator, the model leads to a choice that decreases the operational cost when accessibility distance is increased.

This analysis shows the importance of station accessibility. In our model, the effect of other public transportation systems to accessibility distance is not taken into consideration. It is assumed that the users can reach stations that are close enough to walk, while they might be more options in multimodal transport networks. This underlines the nature of the car-sharing systems that work as systems complimentary to public transportation, which contribute to the improvement of the overall mobility.

4.4. Effect of subsidy

The effect of subsidy on car-sharing system performance was also studied. Three different levels of subsidy (0, 2.5 and 5 euro per hour) were investigated for three different levels of demand

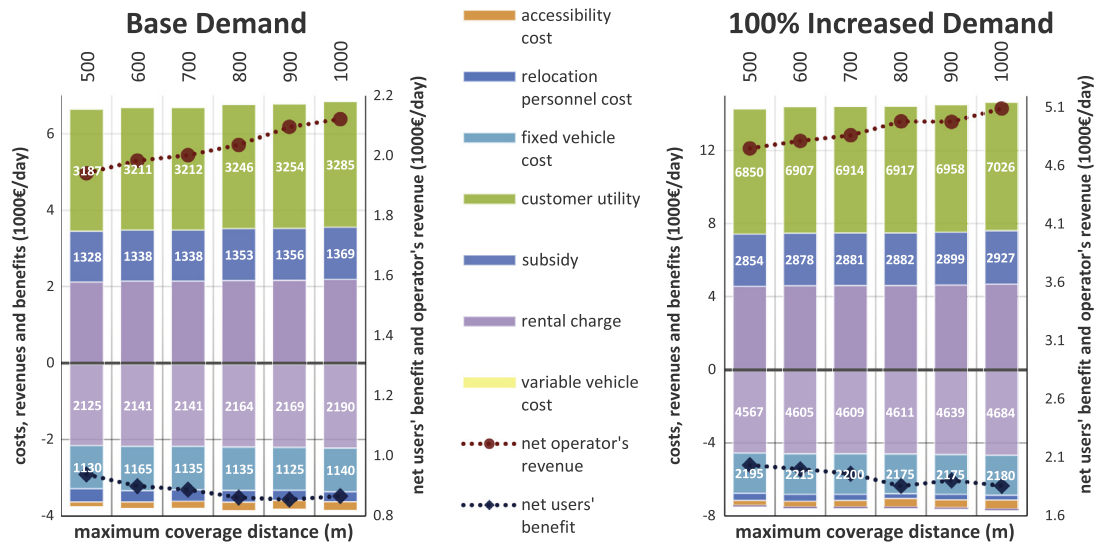


Fig. 11. The costs, benefits and revenues for different maximum accessibility distances.

(50% decreased, base and 100% increased demand). Alternatively, if an exact model of demand sensitivity to pricing exists, a similar analysis could be made. The results of this analysis are shown in Fig. 12. The value of the objective function components (left axis) and the operator's and users' net benefits (right axis) are shown for different levels of subsidy.

The results of this analysis suggest that, the percent of demand served increased by 4–11%. Unprofitable demand in low or no-subsidy becomes profitable for the operator. Although 5–15% increase of the operator's cost (fix and variable vehicle, and relocation personnel costs) is required, the extra revenues generated outweigh the extra costs.

Note that, the increase in subsidy results in increase in the number of vehicles. However, it is not the case for the relocation personnel. Since increased subsidy enables operator to have more vehicles, the system becomes less dependent on relocation operations.

Another important finding of the analysis of subsidy levels relates to the effect of demand balance between demand level

and subsidy on net revenues. If the net revenues of the operator for the same subsidy amount with different demand levels are compared, it can be observed that the increase in the profit is faster than the increase in demand. The operator earns more than double with double demand. This is something expected: Increase in demand makes the system more efficient and profitable and as a result the level of subsidy can decrease.

5. Practical considerations

In this paper we have developed and implemented a methodological framework for optimizing one-way car sharing systems with reservations. The implementation of the proposed methodological framework to a given problem setting requires the consideration of a number of practical issues. A central issue related to strategic decision-making for car-sharing systems is the estimation of the spatial and temporal distribution of the demand. The expected

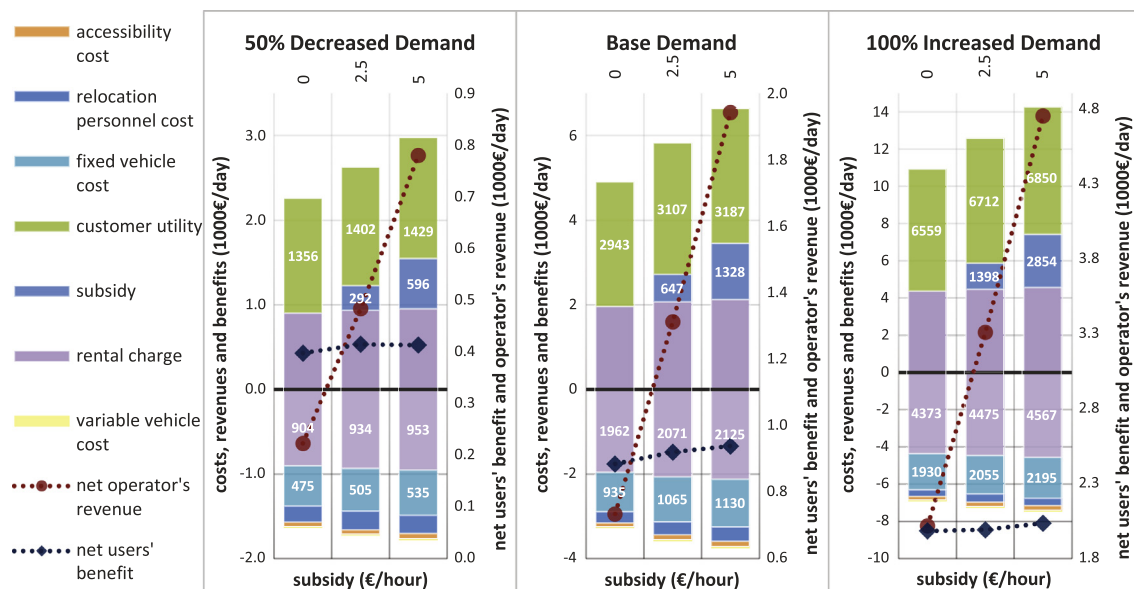


Fig. 12. The costs, benefits and revenues for different subsidy levels.

demand for car-rentals expressed in terms of origin–destination matrices for different hours of the day, days of the week, and months of the year constitute essential inputs to the proposed model. Two different cases have to be considered, (i) the development of a one-way car sharing system from scratch and (ii) the transformation of an existing two-way system to one-way system. While our methodology and problem formulation is general, implementation issues might be different. An important aspect of this analysis is the associated demand for the service. In our analysis, we converted with a simplistic approach a two-way to a one-way demand. While this might be a reasonable approximation, given the available data, it might not integrate induced demand because of the improved quality of service. Therefore, at the strategic planning phase of a car-sharing system, it is essential to conduct thorough surveys that will try to forecast (as accurately as possible) the expected demand. These demand forecasting models should incorporate potential characteristics of the car-sharing system that will offer alternative levels of service to its users, at different pricing levels. While pricing is not carefully analyzed due to lack of data, an elastic demand formulation could be a future extension of this work.

Another practical issue related to the implementation of the proposed model relates to the identification of the candidate locations where potential stations of the car sharing system can be established. The definition of the candidate station locations should consider the spatial distribution of the demand, the contribution of various locations to increasing the accessibility of key activity centers, the improvement of the connectivity of the public urban transport system, and the availability of physical space for establishing the required infrastructure for parking and recharging the vehicles of the car sharing system. Therefore, it is very important to integrate the design of the car-sharing system with the urban and public transport planning activities of a given municipality. In case (ii) of transforming a two-way to one-way system, we consider that the existing stations will not close (due to high cost), but such an analysis would be possible if an operator is willing to make such a decision.

Close cooperation of the relevant agencies, in charge of these planning activities, is a must for ensuring the acceptance and implementation of strategic planning outcome. Other practical issues associated with the implementation of a car-sharing system include the business model that will be used to distribute benefits and costs associated with the establishment and operation of the car sharing system. Another important practical issue is the hiring of personnel for relocation. Nevertheless, our analysis shows that such an addition will be very beneficial both for the operator (increased benefit) and the user (better quality of service and less loss demand). Budget or subsidy constraints can also be easily integrated in the model if specified by the operator. It would be also necessary that the operator is aware of the location and the status of each vehicle in the system. To the best of our knowledge, most of the existing systems provide such information through GPS technology. These practical considerations can be further explored in future research activities.

6. Concluding remarks

A multi-objective model for supporting strategic and tactical planning decisions for car-sharing systems was developed and tested in a large-scale real-world setting. The model considers simultaneously the net benefits of both the operator and the users. The proposed model closes a gap in the existing literature by considering simultaneously decisions associated with the allocation of strategic assets, i.e. stations and vehicles of car-sharing systems and the allocation of personnel for relocation operations (tactical decision). The model provides decision makers with ample opportunities to perform sensitivity analysis for relevant model parameters. This feature is particularly useful for cost values that are

difficult to establish empirically (e.g. utility gain of satisfied customers, population coverage, station accessibility cost). Furthermore, the multi-objective nature of the model allows the decision maker to examine the trade-off between operator's profit and users' level of service. This last feature is of particular importance if we consider that car-sharing systems are subsidized with public funds. The results obtained from the application of the model to a case resembling real world decision making requirements, provides useful information regarding the system performance.

Although the model provides satisfactory results for the case under consideration, it should be pointed out that the results are dependent on the model parameters used and cannot be directly generalized. However the proposed model can be utilized in different settings without difficulty. The value of the research presented herein stems from the innovative model proposed and its use for supporting strategic and tactical decision for car-sharing systems.

Research work under way involves the integration of the proposed model with a simulation model that will provide a more realistic representation of the relocation operation costs by looking on operational decisions. Modeling the operational problem and assigning the vehicle rosters while taking their electrical charge level into consideration is another future work directions. A field implementation of the proposed framework for one-way car-sharing is under preparation. Operational problem will consider different sources of uncertainties, such as last minute reservations, deviations from scheduled pick up and drop off times, level of charging and others. An operational model can also influence or redirect demand with pricing strategies, by giving for example the flexibility to choose the exact station or location (multiple stations) to the users.

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