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Abstract. This paper introduces a version of the classical traveling salesman problem with time-dependent service times. In our setting, the duration required to provide service to any customer is not fixed but defined as a function of the time at which service starts at that location. The objective is to minimize the total route duration, which consists of the total travel time plus the total service time. The proposed model can handle several types of service time functions, e.g., linear and quadratic functions. We describe basic properties for certain classes of service time functions, followed by the computation of valid lower and upper bounds. We apply several classes of subtour elimination constraints and measure their effect on the performance of our model. Numerical results obtained by implementing different linear and quadratic service time functions on several test instances are presented.

Keywords: Traveling salesman problem, time-dependency, service times, lower and upper bounds.

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1. Introduction

The purpose of this paper is to introduce, model and solve a version of the classical traveling salesman problem (TSP) with time-dependent service times (TSP-TS), an extension of the asymmetric TSP. Öncan et al. [13], and Roberti and Toth [16] present comprehensive reviews of the available mathematical formulations for the asymmetric TSP, some of which will be extended to model the TSP-TS.

In most of the research on the TSP, service times are either ignored, or assumed to be constant and thus accounted for in travel times. However in practice, it can easily be observed that service times vary according to several factors which naturally depend on the time of day (e.g., availability of parking spaces, accessibility to the customer at its location, and so on). In the TSP-TS, the service time required at each customer is not fixed a priori, but depends on the start time of service. The TSP-TS aims to minimize the total route duration including the total travel time and the total service time. This problem can formally be defined on a connected digraph $G = (N, A)$. In this graph, $N = \{0, 1, ..., n, n+1\}$ is the set of nodes and $A = \{(i, j) \mid i, j \in N, i \neq j\}$ is the set of arcs. Nodes 0 and $n+1$ correspond to the starting and ending points of the salesman's tour, respectively. Each node in $N \setminus \{0, n+1\}$ corresponds to a distinct customer. With each arc (i, j) in A is associated a travel time t_{ij}. Each customer i has a service time defined as a continuous function $s_i(b_i)$, where b_i corresponds to the start time of service at that customer location.

In the TSP literature, time-dependency is usually addressed in terms of travel times. The interested reader is referred to Gouveia and Voß [7] who present a classification of formulations proposed for the time-dependent TSP. Picard and Queyranne [14], Vander Wiel and Sahinidis [20], and Bigras et al. [1] consider a time-dependent TSP in which the travel time between any two nodes depends on the time period of the day. It is further assumed that when the salesman starts traversing an arc, no transition from one time period to the next takes place during this travel, in other words there is no transient zone. More specifically, the travel time from node i to node j depends on the time period in which node i is visited. This problem with discrete travel times can be viewed as a single machine scheduling problem with sequence-dependent setup times. Picard and Queyranne [14] provide three integer programming formulations for the time-dependent TSP. The authors analyze the relationships between the relaxations of these models by comparing their lower bounds. It is observed that the shortest path relaxation (related to the first model) is very similar to a formulation proposed by Hadley [8] for the classical TSP. Vander Wiel and Sahinidis [20] propose an algorithm for the time-dependent TSP, based on applying Benders decomposition to a mixed-integer linear programming formulation. The authors also develop a network-based algorithm to identify Pareto-optimal dual solutions of the highly degenerate subproblems. Results indicate that the performance of the algorithm is considerably improved by employing these Pareto-optimal solutions. In Bigras et al. [1], the integer programming formulations of the time-dependent TSP are extended to solve a single machine scheduling problems with sequence-dependent setup times. Two separate objectives are considered: minimizing total flow time and minimizing total tardiness. Instances with 45 and 50 jobs can be solved exactly by the proposed branch-and-bound algorithm.

Cordeau et al. [2] consider a time-dependent TSP in which the predetermined time horizon is partitioned into a number of time intervals, and the average travel speed on each arc during each interval is known. The travel time on each arc is then computed by a procedure introduced by Ichoua et al. [10], and a branch-and-cut procedure is developed to solve the problem. The proposed algorithm is capable of solving instances with up to 40 nodes. In terms of the service cost, Tagmouti et al. [17, 18, 19] consider time-dependency within the scope of the Capacitated Arc Routing Problem (CARP). The classical CARP aims to serve a set of required arcs at minimum cost, using a fleet of capacitated vehicles based at the depot. The three above-mentioned papers focus on a version of the CARP where each arc has a time-dependent service cost but a fixed service time. Tagmouti et al. [17] develop a column generation algorithm, and Tagmouti et al. [18, 19] propose heuristics.

To the best of our knowledge, the TSP-TS has never been considered previously. In contrast to what is done in the papers just mentioned, we can handle several types of service time functions, such as linear and quadratic functions. Moreover, time-dependent service times are included into the model not only through the objective function (e.g., models with time-dependent travel times), but also through the constraints. More specifically, the service time cannot be incorporated into the arc durations.

The remainder of this paper is organized as follows. In Section 2, we describe properties of the service time function and provide the computation of a valid lower bound on the total service time of an optimal solution to our problem. In Section 3, we propose a formulation for the TSP-TS, together with three variants based on different forms of subtour elimination constraints. We also present the computation of a lower bound on the bigM which is employed in our model. Section 4 reports computational results corresponding to different subtour elimination constraints and different service time functions. This section also provides the computation of a valid upper bound on the total route duration of an optimal solution. Finally, our main findings and conclusions are highlighted in Section 5.

2. Service Time Function

In this section, we present the certain properties of the service time function $s_i(b_i)$ at node i.

2.1. First-In-First-Out Property

The first property is related to the First-In-First-Out (FIFO) principle which states that if service at node i starts at a time b_i, any service starting at a later time b'_i at that node cannot be completed earlier than $b_i + s_i(b_i)$.

Proposition 2.1. *$s_i(b_i)$ satisfies the FIFO property if and only if* $\dfrac{ds_i(b_i)}{db_i} \geq -1$.

Proof. ($\rightarrow$ necessity)
If $s_i(b_i)$ satisfies the FIFO property, then

$$b_i + s_i(b_i) \leq b'_i + s_i(b'_i),$$

for all $b'_i > b_i$. The above statement can be rewritten as

$$s_i(b'_i) - s_i(b_i) \geq -(b'_i - b_i),$$

where $b'_i = b_i + \delta$ and $\delta > 0$. The latter inequality yields

$$s_i(b_i + \delta) - s_i(b_i) \geq -\delta,$$

$$\frac{s_i(b_i + \delta) - s_i(b_i)}{\delta} \geq -1,$$

$$\lim_{\delta \to 0} \frac{s_i(b_i + \delta) - s_i(b_i)}{\delta} \geq \lim_{\delta \to 0} -1,$$

$$\frac{ds_i(b_i)}{db_i} \geq -1.$$

($\leftarrow$ sufficiency)
It is given that $s_i(b_i)$ is continuous on $[b_i, b_i']$ where $b_i' > b_i$. From the mean value theorem, we know that there exists at least one point b_i^* in (b_i, b_i') such that

$$\frac{ds_i(b_i^*)}{db_i^*} = \frac{s_i(b_i') - s_i(b_i)}{b_i' - b_i}.$$

If $\dfrac{ds_i(b_i^*)}{db_i^*} \geq -1$ for all b_i^* in (b_i, b_i'), then

$$\frac{s_i(b_i') - s_i(b_i)}{b_i' - b_i} \geq -1,$$

$$-s_i(b_i') + s_i(b_i) \leq b_i' - b_i,$$

$$b_i + s_i(b_i) \leq b_i' + s_i(b_i'),$$

which means that the FIFO property is satisfied. □

At this point, it is worth observing that a TSP solution is not always optimal for the TSP-TS, even when we apply a service time function that satisfies the FIFO property starting from the first customer in the route. To illustrate, suppose that there are three customers (denoted by nodes 1, 2 and 3) and one depot (denoted by nodes 0 and 4), with the travel time matrix of Table 1. The classical TSP has the two optimal solutions $(0, 3, 1, 2, 4)$ and $(0, 2, 1, 3, 4)$, with a total travel time equal to 11.75.

Table 1: Travel time matrix of a small TSP instance

Nodes	0	1	2	3	4
0	0.00	5.00	4.00	4.00	0.00
1	5.00	0.00	2.00	1.75	5.00
2	4.00	2.00	0.00	1.50	4.00
3	4.00	1.75	1.50	0.00	4.00
4	0.00	5.00	4.00	4.00	0.00

Now assume that for the TSP-TS represented by the same graph, the service time function is defined as $s_i(b_i) = b_i^2 - 6b_i + 9$ for all $i \in N \setminus \{0, n+1\}$.

From Proposition 2.1, the FIFO property is satisfied for all $b_i \geq 2.50$. Note that in the TSP solutions, the salesman arrives at the first customer after the earliest time at which the FIFO property starts holding. Moreover, it is obvious that waiting at the depot or at a customer location does not bring any reduction in the total route duration. When we evaluate the two TSP solutions with the time-dependent service times defined above, we observe that the total route durations of the corresponding routes substantially increase to 419.35 and 501.81. The arrival times (AT) and departure times (DT) at each node of the TSP solution $(0, 3, 1, 2, 4)$, which has a lower total route duration, are provided in Table 2 where there is no waiting. This table also gives the total travel time (TT) and the total service time (ST) spent until the departure of the salesman from each node.

Table 2: Details of the TSP solution evaluated with respect to time-dependent service times

Nodes	AT	DT	TT	ST
0	(starting point)	0.00	0.00	0.00
3	4.00	5.00	4.00	1.00
1	6.75	20.81	5.75	15.06
2	22.81	415.35	7.75	407.06
4	419.35	(ending point)	11.75	407.06

The optimal solution of the TSP-TS is $(0, 2, 3, 1, 4)$, with a total route duration of 331.75. Table 3 provides the details of the corresponding route at each node. As in the TSP solutions, waiting is not needed since it does not bring any reduction in the total route duration.

Table 3: Details of the the optimal TSP-TS solution

Nodes	AT	DT	TT	ST
0	(starting point)	0.00	0.00	0.00
2	4.00	5.00	4.00	1.00
3	6.50	18.75	5.50	13.25
1	20.50	326.75	7.25	319.50
4	331.75	(ending point)	12.25	319.50

In a TSP-TS setting, the solution $(0, 2, 3, 1, 4)$ has a lower duration than that of the TSP solutions $(0, 3, 1, 2, 4)$ and $(0, 2, 1, 3, 4)$. This observation implies that a TSP solution may not be optimal for the TSP-TS even when all customers have the same service time function and waiting does not provide any reduction in the total service time. Moreover, it shows that our problem may not correspond to the TSP even when all customers have the same quadratic service time function (with a unique minimum).

2.2. Waiting Property

The second property is related to waiting. If $s_i(b_i)$ does not satisfy the FIFO property when the salesman arrives at customer i, then it may pay to wait at that customer before starting service. We first provide a small example to illustrate this property. The related propositions are then presented.

Suppose that there are three customers (denoted by nodes 1, 2 and 3) and one depot (denoted by nodes 0 and 4). In this setting, $t_{ij} = 0.50$ for all (i, j) in A, and $s_i(b_i) = b_i^2 - 4b_i + 4$ for all $i \in N \setminus \{0, 4\}$. From Proposition 2.1, the FIFO property is satisfied for all $b_i \geq 1.50$. Consider a route that visits customers in the order 1, 2 and 3. Tables 4 and 5 provide the arrival times and the departure times at each node in the current route if waiting is not allowed and if waiting is allowed, respectively. These tables also provide the total travel time and the total service time spent until the departure of the salesman from each node. If there is no waiting, the total service time is equal to 14.79. When waiting is allowed at the first customer, the total service time is reduced to 0.97.

Table 4: Details of the route for the case without waiting

Nodes	AT	DT	TT	ST
0	(starting point)	0.00	0.00	0.00
1	0.50	2.75	0.50	2.25
2	3.25	4.81	1.00	3.81
3	5.31	16.29	1.50	14.79
4	16.79	(ending point)	2.00	14.79

This example shows that when the FIFO property is not always satisfied by the service time function, then the total route duration may be decreased by waiting. The related proposition follows.

Table 5: Details of the route for the case with waiting

Nodes	AT	DT	TT	ST
0	(starting point)	0.00	0.00	0.00
1	0.50	1.75	0.50	0.25
2	2.25	2.31	1.00	0.31
3	2.81	3.47	1.50	0.97
4	3.97	(ending point)	2.00	0.97

Proposition 2.2. *If the salesman arrives at a customer i before b'_i (the earliest time at which the FIFO property starts holding at that customer), waiting at that location to begin service at time b'_i is then beneficial.*

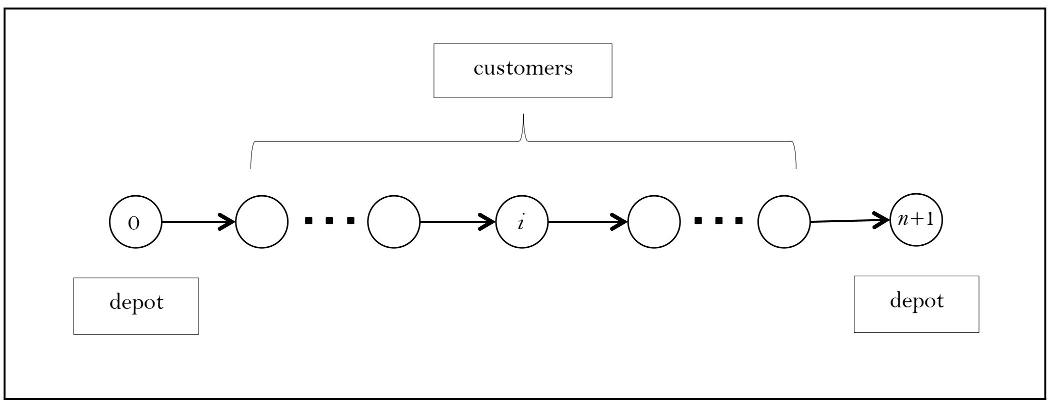

Figure 1: A given route (no waiting)

Proof. Suppose that in the route given by Figure 1, each customer $j \in N \setminus \{0, n+1\}$ has a service time function $s_j(b_j)$ and waiting is not considered. The latter structure implies that the arrival time at any customer i is equivalent to the start time of service at that customer, which is represented by $\widehat{b_i}$.

Suppose that $s_i(b_i)$ satisfies the FIFO property for all b_i where $b'_i \leq b_i \leq \bar{b_i}$ and $\widehat{b_i} < b'_i$. More specifically, $\dfrac{ds_i(b_i)}{db_i} < -1$ for all b_i such that $0 \leq b_i < b'_i$, and therefore such that $b_i \in [\widehat{b_i}, b'_i)$. From the mean value theorem, we know that there exists at least one point b_i^* in $(\widehat{b_i}, b_i')$ such that

$$\frac{ds_i(b_i^*)}{db_i^*} = \frac{s_i(b_i') - s_i(\widehat{b_i})}{b_i' - \widehat{b_i}}.$$

Since $\dfrac{ds_i(b_i^*)}{db_i^*} < -1$ for all $\widehat{b_i} < b_i^* < b_i'$, then

$$\frac{s_i(b_i') - s_i(\widehat{b_i})}{b_i' - \widehat{b_i}} < -1,$$

$$s_i(b_i') - s_i(\widehat{b_i}) < -(b_i' - \widehat{b_i}),$$
$$s_i(b_i') + b_i' < s_i(\widehat{b_i}) + \widehat{b_i}.$$

The last inequality implies that instead of beginning service immediately, waiting at customer i until b_i' (until $s_i(b_i)$ starts satisfying the FIFO property) is beneficial. □

From Proposition 2.2, we distinguish two categories of service times: (i) each customer has a distinct service time function, and (ii) all customers have the same service time function. In Proposition 2.3, we consider the latter case and prove that all the waiting time can be shifted to the depot (instead of spending idle times at customer locations).

Proposition 2.3. *If all customers have the same service time function, then the waiting time required to satisfy the FIFO property can be spent at the depot.*

Proof. Consider a service time function $s_j(b_j)$ for all $j \in N \setminus \{0, n+1\}$, satisfying the FIFO property for all $b_j \geq b_j' \geq 0$. In other words, b_j' is the earliest time at which the FIFO property starts holding at each customer j. Let i be the first customer in the route given by Figure 2. First, the arrival time at customer i is equivalent to the start time of service at that customer, i.e., $\widehat{b_i}$. From Proposition 2.2, we know that it is beneficial to delay service at customer i until b_i'. Since all customers have the same service time function, waiting at the first customer is sufficient to satisfy the FIFO property for all other customers (no additional waiting is needed to satisfy the FIFO along the route). This waiting time can be shifted to the depot. More specifically, waiting is needed only at the depot to satisfy the FIFO property for all customers. □

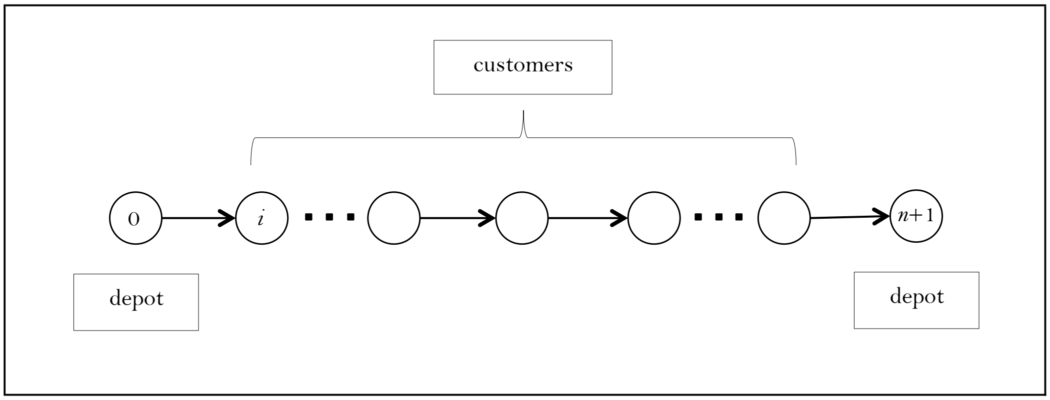

Figure 2: A route where all customers have the same service time function

2.3. Computing A Lower Bound on the Total Service Time

In this section, we derive a lower bound on the total service time of the optimal solution for the case where all customers have the same linear service time function.

Proposition 2.4. *When all customers $i \in N \setminus \{0, n+1\}$ have the same linear service time function, $s_i(b_i) = \beta b_i + \gamma$ where $\beta, \gamma > 0$ and $s_i(b_i) > 0$, the total service time of a route, computed by considering the first $n+1$ smallest travel times, yields a valid lower bound value on the total service time of the optimal solution.*

Proof. Observe that for the service time function defined as $s_i(b_i) = \beta b_i + \gamma$, $\dfrac{ds_i(b_i)}{db_i} = \beta > 0$. In other words, $s_i(b_i)$ is an increasing function when $b_i \in [0, \infty)$, and thus waiting at the depot or at customer locations does not bring any reduction in the total route duration. Therefore, we know that $s_i(b_i)$ satisfies the FIFO property at each customer $i \in N \setminus \{0, n+1\}$ for all $b_i \geq 0$. Moreover, the latest possible departure time from the depot in any route r in G is equal to 0 due to the behaviour of $s_i(b_i)$ for $b_i \in [0, \infty)$. Let (r, i) denote the i^{th} node visited by route r. Note that $(r, 1)$ and $(r, n+2)$ correspond to the depot in each route r.

We have a symmetric travel time matrix in which node 0 and node $n+1$ both correspond to the depot. First define a list L of travel times t_{ij} for all $j > i$ and $j \neq n+1$. When the graph comprises only one customer, the travel time between that customer and the depot needs to be considered twice in the list L (the connection from the depot to the customer, and the connection from the customer to the depot). The list is then sorted in ascending order so that $k < m$ implies $L[k] < L[m]$, where $L[k]$ is the k^{th} element of list L. We then construct a new directed graph $\widetilde{G} = (\widetilde{N}, \widetilde{A})$ where $\widetilde{N} = \{n+2, n+3, ..., 2n+3\}$ is the set of nodes, and $\widetilde{A} = \{(i,j) \mid i \in \widetilde{N} \setminus \{2n+3\}, j = i+1, j \in \widetilde{N} \setminus \{n+2\}\}$ is the set of arcs. In this graph, nodes $n+2$ and $2n+3$ can be viewed as the starting and ending points of each route. Each node $i \in \widetilde{N} \setminus \{n+2, 2n+3\}$ has a service time defined by the original function $s_i(b_i)$. Moreover, the travel time on each arc (i,j) in $\widetilde{A}$, $\widetilde{t}_{ij}$ is equal to $L[i-n-1]$. From the definition of $\widetilde{G}$, it can be observed that only one route (route p) starts and ends at the depot and visits each node $i \in \widetilde{N}$ exactly once. Suppose that the start time of service at the first node of this route (node $n+2$) is set to 0. With respect to b_{n+2} and to the service time spent at node $n+2$, which is equal to 0 by definition, b_{n+3} is computed as $\widetilde{t}_{n+2,n+3}$. We know that $b_{n+3} \leq b_{(r,2)}$ for any possible route r generated for the original problem since $\widetilde{t}_{n+2,n+3} = \min\limits_{(i,j)\in A}\{t_{ij}\}$ and $b_{(r,1)} = 0$ for all r. Similarly, b_{n+4} is equal to $\widetilde{t}_{n+2,n+3} + s_{n+3}(b_{n+3}) + \widetilde{t}_{n+3,n+4}$. It is clear that $b_{n+4} \leq b_{(r,3)}$ for any route r, since

(i) $s_{n+3}(b_{n+3}) \leq s_{(r,2)}(b_{(r,2)})$,

(ii) $\widetilde{t}_{n+2,n+3} + \widetilde{t}_{n+3,n+4} \leq t_{(r,1),(r,2)} + t_{(r,2),(r,3)}$,

(iii) the latest possible departure time from the starting point of any route r is equal to 0.

Reiterating this process, we observe that $b_i \leq b_{(r,i-n-1)}$ at each node $i \in \widetilde{N}$ in route p and for any possible route r generated for the original problem where $(r, i-n-1) \in N$, since

(i) $b_{n+2} = 0$,

(ii) $\sum_{k=n+2}^{i-1} \widetilde{t}_{k,k+1} \leq \sum_{k=n+2}^{i-1} t_{(r,k-n-1),(r,k-n)}$, $i \in \widetilde{N} \setminus \{n+2\}$,

(iii) $\sum_{k=n+2}^{i-1} s_k(b_k) \leq \sum_{k=n+2}^{i-1} s_{(r,k-n-1)}(b_{(r,k-n-1)})$, $i \in \widetilde{N} \setminus \{n+2\}$.

Thus, the total service time of route p yields a valid lower bound on the total service time in an optimal solution to the original problem defined with a linear (increasing) service time function at each customer. □

3. Mathematical Model

We now present mathematical models for the TSP-TS, where b_i is a decision variable corresponding to the start time of service at customer i. If there is no waiting at customer i, then b_i is equal to the arrival time at that customer. Waiting is allowed in this problem setting since it may be helpful to reduce the total service time spent at each customer in the route. In this model, the decision variable x_{ij} takes the value 1 if node j is served immediately after node i, and 0 otherwise.

A mathematical model considering time-dependent service times is then

$$\text{minimize} \sum_{i \in N} \sum_{j \in N} t_{ij} x_{ij} + \sum_{i \in N \setminus \{0, n+1\}} s_i(b_i) \tag{1}$$

$$\text{s.t.} \sum_{j \in N \setminus \{i\}} x_{ij} = 1, \qquad i \in N \setminus \{n+1\}, \tag{2}$$

$$\sum_{i \in N \setminus \{j\}} x_{ij} = 1, \qquad j \in N \setminus \{0\}, \tag{3}$$

$$b_i + s_i(b_i) + t_{ij} - M(1 - x_{ij}) \leq b_j, \qquad i \in N, j \in N, \tag{4}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - 1, \qquad S \subseteq N \setminus \{0, n+1\}, S \neq \emptyset, \tag{5}$$

$$b_i \geq 0, \qquad i \in N, \tag{6}$$

$$x_{ij} \in \{0, 1\}, \qquad i \in N, j \in N. \tag{7}$$

The objective (1) is to minimize the total route duration which consists of the sum of the total travel time and the total service time. Constraints (2) and (3) ensure that each customer is visited exactly once, and impose the degree requirements for the two depot nodes. Constraints (4) link the departure time from a node and the starting time of service at its successor. Constraints (5) are the classical Dantzig, Fulkerson and Johnson (DFJ) subtour elimination constraints [3]. Constraints (6) ensure that the start time of service at each customer is non-negative. Constraints (7) indicate that partial service at customers is not allowed.

In Section 3.1, we present a number of alternative subtour elimination constraints which will be compared in Section 4.1. In Section 3.2, we provide a procedure to compute a lower bound on the bigM.

3.1. Alternative Subtour Elimination Constraints

We consider the three formulations working with different subtour elimination constraints. These are reported by Öncan et al. [13] and Roberti and Toth [16] as the best when the model is solved directly by CPLEX [9].

3.1.1. The Miller, Tucker and Zemlin (MTZ) Formulation

Miller et al. [12] proposed the following MTZ subtour elimination constraints:

$$u_i - u_j + nx_{ij} \leq n - 1, \qquad i, j = 1, ..., n, \tag{8}$$

where u_i, $i \in N \setminus \{0, n+1\}$ is an arbitrary real number identifying the visiting order of customer i in a tour. Note that in the original paper, Miller et al. [12] define the u_i variables without any bound.

3.1.2. The Desrochers and Laporte (DL) Formulation

The MTZ subtour elimination constraints were lifted by Desrochers and Laporte [4], resulting in a stronger LP relaxation. The lifted constraints are stated as follows:

$$u_i - u_j + nx_{ij} + (n-2)x_{ji} \leq n - 1, \qquad i, j = 1, ..., n, \tag{9}$$

$$-u_i + (n-2)x_{i,n+1} + \sum_{j \in N \setminus \{0,n+1\}} x_{ji} \leq -1, \qquad i = 1, ..., n, \tag{10}$$

$$u_i + (n-2)x_{0i} + \sum_{j \in N \setminus \{0,n+1\}} x_{ij} \leq n, \qquad i = 1, ..., n. \tag{11}$$

Desrochers and Laporte [4] also proved that constraints (9) are facet defining.

3.1.3. The Gavish and Graves (GG) Formulation

Several single-commodity, two-commodity and multi-commodity flow formulations were introduced for the asymmetric TSP. The interested reader is referred to Langevin et al. [11] for a comprehensive classification on commodity flow formulations. Gavish and Graves [5] developed a single-commodity flow formulation for which the LP relaxation of the resulting model is stronger than that of the MTZ formulation but weaker than that of the DFJ formulation (see Gouveia [6]). In the GG formulation, subtours are eliminated by using $n(n+1)$ non-negative variables g_{ij}, where $i \in N \setminus \{0, n+1\}$ and

$j \in N \setminus \{0\}$. The GG subtour elimination constraints of this formulation are stated as follows:

$$\sum_{j \in N \setminus \{0\}} g_{ij} \quad - \sum_{j \in N \setminus \{0, n+1\}} g_{ji} = 1, \qquad i = 1, ..., n, \tag{12}$$

$$0 \leq g_{ij} \leq n x_{ij}, \qquad i = 1, ..., n, j = 1, ..., n+1. \tag{13}$$

In (12) and (13), the g_{ij} variables denote the number of arcs on a tour from the depot to arc (i, j). Note that the starting point of each tour is represented by node 0. These two sets of constraints yield a network flow model in which the g_{ij} variables naturally take integer values for fixed values of x_{ij} variables.

3.2. Computing A Lower Bound on the BigM

In this section, we provide the computation of a sufficiently large value for the bigM to be used in constraints (4) of the formulation (1)–(7), where all customers first have the same quadratic service time function (with a unique minimum).

Proposition 3.1. *When all customers have the same quadratic service time function (with a unique minimum), the arrival time of the salesman at the last node in a route, computed by considering the first $n+1$ largest travel times, yields a valid value for M in formulation* (1)–(7)*.*

Proof. Suppose that each customer $i \in N \setminus \{0, n+1\}$ has the same service time function $s_i(b_i) = \alpha {b_i}^2 - \beta b_i + \gamma$, where $\alpha, \beta, \gamma > 0$ and $s_i(b_i) \geq 0$, and that $s_i(b_i)$ has a unique minimum value (the minimum service time) at $b_i = \beta/2\alpha$. Since $\dfrac{ds_i(b_i)}{db_i} \geq 0$ and $\dfrac{d^2 s_i(b_i)}{db_i^2} > 0$ for all $b_i \geq \beta/2\alpha$, $s_i(b_i)$ is an increasing function with an increasing derivative for $b_i \in [\beta/2\alpha, \infty)$. From Proposition 2.1, we know that $s_i(b_i)$ satisfies the FIFO property at each customer $i \in N \setminus \{0, n+1\}$ for all $b_i \geq (\beta-1)/2\alpha$. From Proposition 2.3, we know that for any route r in G (starting and ending at the depot, and visiting each node exactly once), waiting at the depot to arrive at the first customer at $(\beta-1)/2\alpha$ is sufficient to satisfy the FIFO property (no additional waiting is needed for the FIFO along the route r). Moreover, the latest possible arrival time at the first customer in any route r in G is equal to $\beta/2\alpha$ due to the behaviour of $s_i(b_i)$ for $b_i \in [\beta/2\alpha, \infty)$. Let (r, i) denote the i^{th} node visited by route r. Note that $(r, 1)$ and $(r, n+2)$ correspond to the depot in each route r.

We have a symmetric travel time matrix in which nodes 0 and $n+1$ both correspond to the depot. In order to compute a lower bound on the bigM, we first define a list L that contains travel times t_{ij} for all $j > i$ and $j \neq n+1$. When the graph comprises only one customer, the travel time between that customer and the depot needs to be considered twice in the list L, once in each direction. This list is then sorted in decreasing order so that $L[k] > L[m]$ when $k < m$, where $L[k]$ is the k^{th} element of list L. We then construct a new directed graph $\overline{G} = (\overline{N}, \overline{A})$, where $\overline{N} = \{n+2, n+3, ..., 2n+3\}$ is the set of nodes and $\overline{A} = \{(i,j) \mid i \in \overline{N} \setminus \{2n+3\}, j = i+1, j \in \overline{N} \setminus \{n+2\}\}$ is the set of arcs. In this graph, nodes $n+2$ and $2n+3$ can be viewed as the starting and ending points of each route. Each node $i \in \overline{N} \setminus \{n+2, 2n+3\}$ has a service time defined by the original function $s_i(b_i)$. Moreover, the travel time on each arc (i,j) in $\overline{A}$, $\bar{t}_{ij}$ is equal to $L[i-n-1]$. From the definition of $\overline{G}$, it can be observed that there exists only one route (route p) starting and ending at the depot, and visiting each node $i \in \overline{N}$ exactly once. Suppose that the starting time of service at node $n+2$ is set to $\beta/2\alpha$. More specifically, $s_i(b_i)$ satisfies the FIFO property at each node i in route p and waiting along that route does not yield any reduction in the total service time. With respect to b_{n+2} and the service time spent at node $n+2$, which is equal to 0 by definition, b_{n+3} is computed as $(\beta/2\alpha) + \bar{t}_{n+2,n+3}$. We know that $b_{n+3} \geq b_{(r,2)}$ for any possible route r generated for the original problem since $\bar{t}_{n+2,n+3} = \max\limits_{(i,j)\in A}\{t_{ij}\}$ and $(\beta/2\alpha) \geq b_{(r,1)}$ for all r. Similarly, b_{n+4} is equal to $(\beta/2\alpha) + \bar{t}_{n+2,n+3} + s_{n+3}(b_{n+3}) + \bar{t}_{n+3,n+4}$. It is clear that $b_{n+4} \geq b_{(r,3)}$ for any route r, since

(i) $s_{n+3}(b_{n+3}) \geq s_{(r,2)}(b_{(r,2)})$,

(ii) $\bar{t}_{n+2,n+3} + \bar{t}_{n+3,n+4} \geq t_{(r,1),(r,2)} + t_{(r,2),(r,3)}$,

(iii) $\beta/2\alpha$ is the latest possible departure time from the starting point of any route r.

Continuing in this fashion, we observe that $b_i \geq b_{(r,i-n-1)}$ at each node $i \in \overline{N}$ in route p and for any possible route r generated for the original problem where $(r, i-n-1) \in N$, since

(i) $b_{n+2} = \beta/2\alpha$,

(ii) $\sum_{k=n+2}^{i-1} \bar{t}_{k,k+1} \geq \sum_{k=n+2}^{i-1} t_{(r,k-n-1),(r,k-n)}$, $i \in \overline{N} \setminus \{n+2\}$,

(iii) $\sum_{k=n+2}^{i-1} s_k(b_k) \geq \sum_{k=n+2}^{i-1} s_{(r,k-n-1)}(b_{(r,k-n-1)})$, $i \in \overline{N} \setminus \{n+2\}$.

Thus, we can conclude that $b_{2n+3} \geq b_{(r,n+2)}$ for any possible route r generated for the original problem defined on G, where $(r, n+2)$ corresponds to the depot. Note that the departure time of each route r from the starting point is optimized. In other words, the arrival time of the salesman at the end of route p provides a valid and reasonable value for the bigM to be employed in the original problem defined with a quadratic service time function (with a unique minimum) at each customer. $\square$

If we consider a linear service time function, the route constructed by using the first $n+1$ largest travel times is also valid to compute a value for bigM. Suppose that each customer $i \in N \setminus \{0, n+1\}$ has the same linear (increasing) service time function $s_i(b_i) = \beta b_i + \gamma$, where $\beta, \gamma > 0$ and $s_i(b_i) > 0$. We know that $s_i(b_i)$ satisfies the FIFO property at each customer $i \in N \setminus \{0, n+1\}$ for all $b_i \geq 0$ (see the proof of Proposition 2.4). In other words, waiting at the depot or at customer locations does not yield any reduction in the total route duration. It is easy to observe that when we adjust the latest possible departure time from the depot with respect to the linear function considered at each customer, all properties provided in the proof of Proposition 3.1 hold. Thus, one can employ the computation of the bigM when all customers have the same quadratic service time function (with a unique minimum) or the same linear (increasing) service time function.

4. Computational Results

The model just described was coded in C++ and solved by using IBM ILOG CPLEX 12.5 [9]. All experiments were conducted on an Intel(R) Xeon(R) CPU X5675 with 12-Core 3.07 GHz and 96 GB of RAM (by using a single thread). We have experimented with data sets presented in the TSPLIB library of Reinelt [15]. We have focused on the instances with up to 45 nodes defined with a symmetric travel time matrix (burma14, ulysses16, gr17, gr21, ulysses22, gr24, fri26, bayg29, bays29, dantzig42 and swiss42). In addition to these 11 instances, we have generated 11 more instances by selecting (i) the first 30 nodes from dantzig42, swiss42, att48, gr48, hk48 and eil51, (ii) the first 35 nodes from swiss42, gr48 and eil51, (iii) the first 40 nodes from eil51, and (iv) the first 45 nodes from eil51. The original travel times were adjusted in such a way that we have the same average travel time per arc in each instance. Note that in all these instances, there exists one more node for the ending depot, e.g., burma14 includes 15 nodes.

4.1. Effects of Subtour Elimination Constraints

Each instance was solved directly by CPLEX for three types of subtour elimination constraints: (i) MTZ, (ii) DL and (iii) GG. Each corresponding formulation was enhanced by implementing: (i) an upper bound on the total route duration, (ii) a lower bound on the total service time, and (iii) a lower and an upper bound on the start time of service at each customer. A maximum of 7,200 seconds was imposed on the solution time of any instance. A linear service time function $s_i = 10^{-2}b_i + 6(10^{-2})$ was employed for each customer $i \in N \setminus \{0, n+1\}$. Note that this function provides medium service times with respect to other functions employed in Section 4.2.

We first present the above-mentioned four valid bounds, starting with an upper bound on the total route duration of the optimal solution for the case where all customers have the same linear or quadratic service time function. If each customer $i \in N \setminus \{0, n+1\}$ has the same quadratic service time function (with a unique minimum) or the same linear (increasing) service time function, then the total route duration of a route constructed by implementing the nearest-neighbour heuristic provides a valid upper bound value on the optimal total route duration.

Suppose that each customer $i \in N \setminus \{0, n+1\}$ has the same service time function, which is either $s_i(b_i) = \alpha {b_i}^2 - \beta b_i + \gamma$ where $\alpha, \beta, \gamma > 0$ and $s_i(b_i) \geq 0$, or $s_i(b_i) = \beta b_i + \gamma$ where $\beta, \gamma > 0$ and $s_i(b_i) > 0$. When we consider the first case with the quadratic service time function, we know that $s_i(b_i)$ satisfies the FIFO property at each customer $i \in N \setminus \{0, n+1\}$ for all $b_i \geq (\beta - 1)/2\alpha$ (see the proof of Proposition 3.1). Moreover, waiting at the depot to arrive at the first customer at $(\beta - 1)/2\alpha$ is sufficient to satisfy the FIFO property (no additional waiting is needed for the FIFO along the route). If the linear function is employed, then $s_i(b_i)$ satisfies the FIFO property at each customer $i \in N \setminus \{0, n+1\}$ for all $b_i \geq 0$ (see the proof of Proposition 2.4).

The nearest-neighbour heuristic starts building the route by moving to the closest customer to the depot in terms of the travel time. At each iteration, the algorithm selects the closest customer to the last node in the partial route not yet visited. This step is reiterated until all customers are routed, and the resulting route r is a feasible solution to the TSP-TS. Thus, the total route duration of route r, where the departure time from the depot is arranged with respect to the FIFO property of the service time function considered, provides an upper bound on the total duration of an optimal TSP-TS solution.

To implement a lower bound on the total service time, we perform the procedure described in Section 2.3. In addition to the lower and upper bounds (on the total service time and on the total route duration of the optimal solution), we implement two bounds on the start time of service at each customer location. The first related constraint employs an upper bound equal to the bigM used in the formulation (1)–(7). The second related constraint employs a lower bound equal to the earliest time at which the FIFO property starts holding.

Tables 6, 7 and 8 present solutions found within the time limit (7,200 seconds) when MTZ, DL and GG constraints are respectively incorporated into the model (enhanced with the bounds explained above). In these tables, we provide the status of solutions (feasible or optimal), the objective function value (Obj), the percentage of the total travel time in Obj (TT%), the percentage of the total service time in Obj (ST%), and the final optimality gap in percentage (Gap$_f$%). Moreover, we also report average values (Avg) calculated over the instances solved suboptimally and over the instances solved optimally.

The results given in Table 6 indicate that the model with the MTZ subtour elimination constraints cannot provide a feasible solution to three instances within the time limit. Eight instances with up to 31 nodes are solved to optimality, and a feasible solution is provided to the remaining 11 instances. Table 7 shows that the model with the DL subtour elimination constraints performs better than that with the MTZ subtour elimination constraints (a feasible solution is obtained for each instance). Nine instances with up to 31 nodes are solved to optimality. Moreover, the optimal solutions for the eight instances, which are also provided by the model with the MTZ subtour elimination constraints, are obtained within smaller computation times (on average by 11.58%). Table 8 indicates that the best performance is obtained by employing the model with the GG subtour elimination constraints. Optimal solutions are obtained for 13 instances with up to 36 nodes (on average within less than eight minutes). Furthermore, this model decreases the computation time required to solve the eight (nine) instances, which are also provided by the model with the MTZ (DL) subtour elimination constraints, on average by 89.81% (88.22%). According to results given in these three tables, we conclude that the model employing the GG subtour elimination constraints performs much better than those with the MTZ and DL constraints, both in terms of the number of instances solved to optimality and of the required computation time.

Table 6: Solutions with four bounds, MTZ subtour elimination constraints and medium service times, with $s_i(b_i) = 10^{-2}b_i + 6(10^{-2})$ for all $i \in N \setminus \{0, n+1\}$

Name	Status	Obj	TT(%)	ST(%)	Gap_f(%)	Seconds
burma14	Optimal	236.44	93.70	6.30	0.00	1.07
ulysses16	Optimal	279.57	94.51	5.49	0.00	23.97
gr17	Optimal	245.40	94.40	5.60	0.00	62.43
gr21	Optimal	249.32	90.48	9.52	0.00	7.24
ulysses22	Feasible	318.06	92.91	7.09	6.30	-
gr24	Optimal	284.93	89.29	10.71	0.00	3628.16
fri26	Feasible	263.01	89.06	10.94	14.30	-
bayg29	Optimal	371.22	86.74	13.26	0.00	1544.31
bays29	Feasible	331.90	86.95	13.05	3.58	-
att30	Feasible	273.10	86.51	13.49	14.56	-
dantzig30	Feasible	353.77	86.50	13.50	18.90	-
eil30	Optimal	349.16	85.92	14.08	0.00	147.49
gr30	Feasible	305.23	86.61	13.39	8.39	-
hk30	No integer solution					-
swiss30	Optimal	366.78	87.34	12.66	0.00	344.64
eil35	Feasible	397.42	83.79	16.21	1.35	-
gr35	No integer solution					-
swiss35	Feasible	406.92	84.46	15.54	2.65	-
eil40	Feasible	452.89	82.36	17.64	2.72	-
dantzig42	Feasible	294.50	79.80	20.20	13.53	-
swiss42	No integer solution					-
eil45	Feasible	502.52	79.80	20.20	3.97	-
Average	Feasible	354.48	85.34	14.66	8.20	-
Average	Optimal	297.85	90.30	9.70	0.00	719.91

4.2. Effects of the Service Time Function

We have also solved the instances given in the TSPLIB with different service time functions in order to assess the effect of the time-dependent component on the performance of the model. We have applied the GG subtour elimination constraints since it was shown in Section 4.1 that these constraints are the most effective.

Table 7: Solutions with four bounds, DL subtour elimination constraints and medium service times, with $s_i(b_i) = 10^{-2}b_i + 6(10^{-2})$ for all $i \in N \setminus \{0, n+1\}$

Name	Status	Obj	TT(%)	ST(%)	Gap_f(%)	Seconds
burma14	Optimal	236.44	93.70	6.30	0.00	2.64
ulysses16	Optimal	279.57	94.51	5.49	0.00	62.91
gr17	Optimal	245.40	94.40	5.60	0.00	38.71
gr21	Optimal	249.32	90.48	9.52	0.00	11.54
ulysses22	Feasible	318.06	92.91	7.09	7.61	-
gr24	Optimal	284.93	89.29	10.71	0.00	3207.58
fri26	Feasible	263.01	89.06	10.94	11.07	-
bayg29	Optimal	371.22	86.74	13.26	0.00	1241.87
bays29	Optimal	331.90	86.95	13.05	0.00	4515.15
att30	Feasible	273.10	86.51	13.49	9.15	-
dantzig30	Feasible	349.60	86.81	13.19	3.28	-
eil30	Optimal	349.16	85.92	14.08	0.00	171.40
gr30	Feasible	305.23	86.61	13.39	0.40	-
hk30	Feasible	347.35	87.28	12.72	3.38	-
swiss30	Optimal	366.78	87.34	12.66	0.00	355.60
eil35	Feasible	397.62	84.25	15.75	2.48	-
gr35	Feasible	306.91	84.45	15.55	6.15	-
swiss35	Feasible	406.92	84.46	15.54	2.30	-
eil40	Feasible	452.89	82.36	17.64	2.50	-
dantzig42	Feasible	285.07	81.74	18.26	11.57	-
swiss42	Feasible	388.64	81.89	18.11	5.84	-
eil45	Feasible	502.52	79.80	20.20	3.86	-
Average	Feasible	353.61	85.24	14.76	5.35	-
Average	Optimal	301.64	89.92	10.08	0.00	1067.49

4.2.1. Linear Service Times

This section aims to analyze the results obtained by implementing different linear service time functions. Tables 9 and 10 present solutions of the instances where $s_i = 5(10^{-3})b_i + 3(10^{-2})$ for all $i \in N \setminus \{0, n+1\}$ (small service times), and where $s_i = 2(10^{-2})b_i + 1.2(10^{-1})$ for all $i \in N \setminus \{0, n+1\}$ (large service times), respectively.

The results presented in Table 9 indicate that when we apply a small service time function, all instances with up to 46 nodes are solved to optimality within a reasonable amount of time (on average within less than eight minutes). Table 10 shows that six instances with up to 31 nodes are solved optimally by the model employing a large service time function for each cus-

Table 8: Solutions with four bounds, GG subtour elimination constraints and medium service times, with $s_i(b_i) = 10^{-2}b_i + 6(10^{-2})$ for all $i \in N \setminus \{0, n+1\}$

Name	Status	Obj	TT(%)	ST(%)	Gap_f(%)	Seconds
burma14	Optimal	236.44	93.70	6.30	0.00	0.46
ulysses16	Optimal	279.57	94.51	5.49	0.00	7.51
gr17	Optimal	245.40	94.40	5.60	0.00	2.33
gr21	Optimal	249.32	90.48	9.52	0.00	0.82
ulysses22	Optimal	318.06	92.91	7.09	0.00	521.19
gr24	Optimal	284.93	89.29	10.71	0.00	21.27
fri26	Optimal	263.01	89.06	10.94	0.00	93.51
bayg29	Optimal	371.22	86.74	13.26	0.00	129.88
bays29	Optimal	331.90	86.95	13.05	0.00	544.31
att30	Feasible	273.10	86.51	13.49	3.77	-
dantzig30	Feasible	349.60	86.81	13.19	3.18	-
eil30	Optimal	349.16	85.92	14.08	0.00	15.65
gr30	Optimal	305.23	86.61	13.39	0.00	412.90
hk30	Feasible	347.35	87.28	12.72	2.42	-
swiss30	Optimal	366.78	87.34	12.66	0.00	409.23
eil35	Optimal	397.42	83.79	16.21	0.00	3342.23
gr35	Feasible	306.91	84.45	15.55	2.67	-
swiss35	Feasible	406.92	84.46	15.54	1.55	-
eil40	Feasible	452.89	82.36	17.64	1.47	-
dantzig42	Feasible	285.07	81.74	18.26	5.78	-
swiss42	Feasible	388.64	81.89	18.11	5.46	-
eil45	Feasible	502.52	79.80	20.20	3.16	-
Average	Feasible	368.11	83.92	16.08	3.27	-
Average	Optimal	307.57	89.36	10.64	0.00	423.18

tomer. The average computation time required to solve these six instances is approximately 22 minutes. Comparing the solutions given in Table 8 (with a medium service time function) to those in Tables 9 and 10, we observe that the number of instances solved to optimality decreases as the coefficients in the service time function increase. In addition, more computation time is required to obtain an optimal solution for the same instance when using a larger service time function for each customer.

4.2.2. Quadratic Service Times

We now analyze the results obtained by employing a quadratic service time function for each customer. Recall that the lower bound on the total

Table 9: Solutions with four bounds, GG subtour elimination constraints and small service times, with $s_i = 5(10^{-3})b_i + 3(10^{-2})$ for all $i \in N \setminus \{0, n+1\}$

Name	Status	Obj	TT(%)	ST(%)	Gap$_f$(%)	Seconds
burma14	Optimal	228.83	96.81	3.19	0.00	0.18
ulysses16	Optimal	271.74	97.24	2.76	0.00	3.84
gr17	Optimal	238.39	97.18	2.82	0.00	2.05
gr21	Optimal	237.11	95.14	4.86	0.00	0.27
ulysses22	Optimal	306.44	96.43	3.57	0.00	37.72
gr24	Optimal	269.09	94.54	5.46	0.00	1.60
fri26	Optimal	247.99	94.46	5.54	0.00	4.14
bayg29	Optimal	345.49	93.20	6.80	0.00	5.38
bays29	Optimal	309.27	93.31	6.69	0.00	19.35
att30	Optimal	253.85	93.07	6.93	0.00	150.86
dantzig30	Optimal	324.21	92.84	7.16	0.00	247.35
eil30	Optimal	323.40	92.76	7.24	0.00	3.35
gr30	Optimal	283.91	93.11	6.89	0.00	13.48
hk30	Optimal	324.20	93.51	6.49	0.00	176.20
swiss30	Optimal	342.50	93.53	6.47	0.00	5.29
eil35	Optimal	363.39	91.64	8.36	0.00	56.18
gr35	Optimal	281.82	91.97	8.03	0.00	173.19
swiss35	Optimal	373.60	91.99	8.01	0.00	33.69
eil40	Optimal	410.35	90.65	9.35	0.00	367.39
dantzig42	Optimal	257.37	90.53	9.47	0.00	4481.79
swiss42	Optimal	351.15	90.63	9.37	0.00	1060.69
eil45	Optimal	448.11	89.49	10.51	0.00	3125.64
Average	Optimal	308.74	93.37	6.63	0.00	453.17

service time of the optimal solution is valid for the case where all customers have the same linear (increasing) service time function (see Section 2.3). We have thus implemented three bounds (an upper bound on the total route duration of the optimal solution, and a lower and an upper bound on the start time of service at each customer location) for the model with a quadratic service time function. Since the quadratic model is inherently more difficult to solve than the linear one, we focus on the 13 instances for which the optimal solutions can be obtained by the model with a medium linear service time function. Table 11 presents the solutions of these 13 instances where $s_i = 4(10^{-5})b_i^2 - 4(10^{-3})b_i + 10^{-1}$ for all $i \in N \setminus \{0, n+1\}$.

The results presented in Table 11 indicate that nine instances with up to 31 nodes are solved optimally when a quadratic service time function is

Table 10: Solutions with four bounds, GG subtour elimination constraints and large service times, with $s_i = 2(10^{-2})b_i + 1.2(10^{-1})$ for all $i \in N \setminus \{0, n+1\}$

Name	Status	Obj	TT(%)	ST(%)	Gap_f(%)	Seconds
burma14	Optimal	252.62	87.69	12.31	0.00	4.13
ulysses16	Optimal	296.28	89.18	10.82	0.00	58.41
gr17	Optimal	260.34	88.99	11.01	0.00	12.22
gr21	Optimal	275.96	81.74	18.26	0.00	10.75
ulysses22	Feasible	343.58	86.01	13.99	5.19	-
gr24	Optimal	320.42	79.40	20.61	0.00	1880.42
fri26	Feasible	297.39	78.77	21.23	4.67	-
bayg29	Feasible	430.35	74.82	25.18	3.28	-
bays29	Feasible	383.78	75.19	24.81	4.85	-
att30	Feasible	316.51	74.95	25.05	10.78	-
dantzig30	Feasible	404.54	75.52	24.48	9.99	-
eil30	Optimal	408.23	74.22	25.78	0.00	5853.90
gr30	Feasible	353.89	74.70	25.30	7.32	-
hk30	Feasible	400.88	75.78	24.22	8.69	-
swiss30	Feasible	422.54	75.81	24.19	7.23	-
eil35	Feasible	474.90	70.75	29.25	3.94	-
gr35	Feasible	365.75	70.87	29.13	10.66	-
swiss35	Feasible	485.44	70.80	29.20	10.28	-
eil40	Feasible	556.10	67.25	32.75	5.22	-
dantzig42	Feasible	352.36	66.22	33.78	12.72	-
swiss42	Feasible	480.30	66.26	33.74	11.87	-
eil45	Feasible	638.13	63.00	37.00	6.84	-
Average	Feasible	419.15	72.92	27.08	7.72	-
Average	Optimal	302.31	83.54	16.46	0.00	1303.31

considered. This model cannot provide a feasible solution to one instance within the 7,200 seconds time limit, and the remaining three instances can be solved suboptimally. When we compare the solutions given in Table 11 to those of Table 8, we observe that the number of instances solved to optimality decreases when we use a quadratic service time function (even though the proportion of the service time in the total route duration decreases). In addition, the model with a linear service time function requires less computation time to obtain an optimal solution for the same instance (the difference is 66.95% on average).

Table 11: Solutions with three bounds, GG subtour elimination constraints and quadratic service times, with $s_i = 4(10^{-5})b_i^2 - 4(10^{-3})b_i + 10^{-1}$ for all $i \in N \setminus \{0, n+1\}$

Name	Status	Obj	TT(%)	ST(%)	Gap_f(%)	Seconds
burma14	Optimal	224.83	98.53	1.47	0.00	0.77
ulysses16	Optimal	268.14	98.54	1.46	0.00	9.17
gr17	Optimal	234.82	98.66	1.34	0.00	3.94
gr21	Optimal	232.77	96.91	3.09	0.00	2.77
ulysses22	Optimal	301.58	97.98	2.02	0.00	639.74
gr24	Optimal	263.04	96.72	3.28	0.00	47.61
fri26	Optimal	239.08	97.98	2.02	0.00	30.85
bayg29	Feasible	345.11	93.30	6.70	4.34	-
bays29	Feasible	305.46	94.47	5.53	2.69	-
eil30	Feasible	320.74	93.53	6.47	3.26	-
gr30	Optimal	279.94	94.43	5.57	0.00	2247.43
swiss30	Optimal	340.42	94.10	5.90	0.00	1462.91
eil35	No integer solution					-
Average	Feasible	323.77	93.77	6.23	3.43	-
Average	Optimal	264.96	97.09	2.91	0.00	493.91

4.3. Effects of Lower and Upper Bounds

Finally, we analyze the effects of the bounds implemented to enhance the performance of the proposed formulation. We focus on the GG subtour elimination constraints and the linear service times since we observe from Sections 4.1 and 4.2 that the model comprising these constraints and service times performs well on the considered instances. Tables 12, 13 and 14 present the solutions obtained within the 7,200 seconds time limit by employing the basic model (without any additional lower and upper bounds) with the GG subtour elimination constraints where $s_i = 5(10^{-3})b_i + 3(10^{-2})$ for all $i \in N \setminus \{0, n+1\}$, $s_i = 10^{-2}b_i + 6(10^{-2})$ for all $i \in N \setminus \{0, n+1\}$ and $s_i = 2(10^{-2})b_i + 1.2(10^{-1})$ for all $i \in N \setminus \{0, n+1\}$, respectively.

Comparing the results of Table 12 to those of Table 9 (the model with a small service time function and four valid bounds), we observe that the number of instances solved to optimality decreases from 22 (with up to 46 nodes) to 17 (with up to 36 nodes). These 17 instances, where both models can obtain corresponding optimal solutions, are solved by the model with four bounds in smaller computation times (on average by 87.29%). The enhanced model also provides smaller final gaps for the instances which are suboptimally solved by the basic model (on average 99.98%, leading to optimal solutions for all instances).

Comparing the results of Table 13 to those of Table 8 (the model with a medium service time function and four valid bounds), we observe that the number of instances solved to optimality decreases from 13 (with up to 36 nodes) to nine (with up to 31 nodes). These nine instances, where both models can obtain corresponding optimal solutions, are solved by the model with four bounds in smaller computation times (on average by 73.89%). The enhanced model also provides smaller final gaps for the instances which are suboptimally solved by the basic model (on average 76.64%).

Comparing the results of Table 14 to those of Table 10 (the model with a large service time function and four valid bounds), we observe that the number of instances solved to optimality decreases from six (with up to 31 nodes) to four (with up to 22 nodes). These four instances, where both models can obtain corresponding optimal solutions, are solved by the model with four bounds in smaller computation times (on average by 33.61%). The enhanced model also provides smaller final gaps for the instances which are suboptimally solved by the basic model (on average 64.95%).

From the results given in Tables 12, 13 and 14, we conclude that the model proposed in this paper performs much better with the four bounds explained in Section 4.1.

Table 12: Solutions with GG subtour elimination constraints and small service times, with $s_i = 5(10^{-3})b_i + 3(10^{-2})$ for all $i \in N \setminus \{0, n+1\}$

Name	Status	Obj	TT(%)	ST(%)	Gap$_f$(%)	Seconds
burma14	Optimal	228.83	96.81	3.19	0.00	0.38
ulysses16	Optimal	271.74	97.24	2.76	0.00	3.38
gr17	Optimal	238.39	97.18	2.82	0.00	2.36
gr21	Optimal	237.11	95.14	4.86	0.00	0.42
ulysses22	Optimal	306.44	96.43	3.57	0.00	59.44
gr24	Optimal	269.09	94.54	5.46	0.00	7.04
fri26	Optimal	247.99	94.46	5.54	0.00	17.68
bayg29	Optimal	345.49	93.20	6.80	0.00	417.16
bays29	Optimal	309.27	93.31	6.69	0.00	467.83
att30	Optimal	253.85	93.07	6.93	0.00	525.07
dantzig30	Optimal	324.21	92.84	7.16	0.00	1062.66
eil30	Optimal	323.40	92.76	7.24	0.00	1138.93
gr30	Optimal	283.91	93.11	6.89	0.00	62.28
hk30	Optimal	324.20	93.51	6.49	0.00	1743.14
swiss30	Optimal	342.50	93.53	6.47	0.00	21.92
eil35	Feasible	363.39	91.64	8.36	2.78	-
gr35	Optimal	281.82	91.97	8.03	0.00	1109.01
swiss35	Optimal	373.60	91.99	8.01	0.00	270.03
eil40	Feasible	410.35	90.65	9.35	5.95	-
dantzig42	Feasible	257.37	90.53	9.47	3.31	-
swiss42	Feasible	351.15	90.63	9.37	5.06	-
eil45	Feasible	448.11	89.49	10.51	8.00	-
Average	Feasible	377.76	89.90	10.10	6.80	-
Average	Optimal	291.87	94.18	5.82	0.00	406.40

Table 13: Solutions with GG subtour elimination constraints and medium service times, with $s_i = 10^{-2} b_i + 6(10^{-2})$ for all $i \in N \setminus \{0, n+1\}$

Name	Status	Obj	TT(%)	ST(%)	Gap_f(%)	Seconds
burma14	Optimal	236.44	93.70	6.30	0.00	0.80
ulysses16	Optimal	279.57	94.51	5.49	0.00	6.66
gr17	Optimal	245.40	94.40	5.60	0.00	4.51
gr21	Optimal	249.32	90.48	9.52	0.00	1.91
ulysses22	Optimal	318.06	92.91	7.09	0.00	709.47
gr24	Optimal	284.93	89.29	10.71	0.00	122.47
fri26	Optimal	263.01	89.06	10.94	0.00	749.30
bayg29	Feasible	371.22	86.74	13.26	6.40	-
bays29	Feasible	331.90	86.95	13.05	7.09	-
att30	Feasible	273.10	86.51	13.49	6.08	-
dantzig30	Feasible	349.60	86.81	13.19	5.53	-
eil30	Feasible	349.16	85.92	14.08	6.55	-
gr30	Optimal	305.23	86.61	13.39	0.00	2353.52
hk30	Feasible	347.35	87.28	12.72	7.12	-
swiss30	Optimal	366.78	87.34	12.66	0.00	1678.08
eil35	Feasible	397.42	83.79	16.21	11.68	-
gr35	Feasible	306.91	84.45	15.55	9.00	-
swiss35	Feasible	406.92	84.46	15.54	7.59	-
eil40	Feasible	452.89	82.36	17.64	13.71	-
dantzig42	Feasible	285.07	81.74	18.26	13.72	-
swiss42	Feasible	388.64	81.89	18.11	14.05	-
eil45	Feasible	502.52	79.80	20.20	17.52	-
Average	Feasible	384.20	83.35	16.65	11.75	-
Average	Optimal	283.19	90.92	9.08	0.00	625.19

Table 14: Solutions with GG subtour elimination constraints and large service times, with $s_i = 2(10^{-2})b_i + 1.2(10^{-1})$ for all $i \in N \setminus \{0, n+1\}$

Name	Status	Obj	TT(%)	ST(%)	Gap$_f$(%)	Seconds
burma14	Optimal	252.62	87.69	12.31	0.00	4.78
ulysses16	Optimal	296.28	89.18	10.82	0.00	56.14
gr17	Optimal	260.34	88.99	11.01	0.00	16.11
gr21	Optimal	275.96	81.74	18.26	0.00	51.75
ulysses22	Feasible	343.58	86.01	13.99	5.72	-
gr24	Feasible	320.42	79.40	20.61	8.03	-
fri26	Feasible	297.39	78.77	21.23	12.27	-
bayg29	Feasible	430.35	74.82	25.18	18.76	-
bays29	Feasible	383.78	75.19	24.81	17.91	-
att30	Feasible	316.51	74.95	25.05	17.76	-
dantzig30	Feasible	404.54	75.52	24.48	18.64	-
eil30	Feasible	408.23	74.22	25.78	18.80	-
gr30	Feasible	353.89	74.70	25.30	15.66	-
hk30	Feasible	400.88	75.78	24.22	18.12	-
swiss30	Feasible	422.54	75.81	24.19	14.33	-
eil35	Feasible	474.90	70.75	29.25	23.84	-
gr35	Feasible	365.75	70.87	29.13	22.57	-
swiss35	Feasible	485.44	70.80	29.20	21.45	-
eil40	Feasible	556.10	67.25	32.75	27.74	-
dantzig42	Feasible	352.36	66.22	33.78	28.04	-
swiss42	Feasible	480.30	66.26	33.74	28.90	-
eil45	Feasible	638.13	63.00	37.00	33.93	-
Average	Feasible	440.95	71.44	28.56	22.38	-
Average	Optimal	271.30	86.90	13.10	0.00	32.20

5. Conclusions

We have considered a traveling salesman problem with time-dependent service times. Several analytical properties of the service time function were derived and a lower bound on the total service time of the optimal solution was computed. Examples were presented to illustrate properties. We then described a mathematical formulation capable of solving linear and quadratic service time functions. We have developed a procedure to compute a sufficiently large value for the bigM, which was then employed in the proposed mathematical model. In numerical experiments, a lower and an upper bound on the start time of service at each customer were also implemented. Computational results showed that our model (with the four bounds) can successfully solve instances with up to 46 nodes with a linear service time function, and with up to 31 nodes with a quadratic service time function.

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