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# An Improved Method for Pricing and Hedging Long Dated American Options

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## Abstract

The majority of quasi-analytic pricing methods for American options are efficient near maturity but are prone to larger errors when time-to-maturity increases. We introduce a new methodology to increase the accuracy of almost any existing quasi-analytic approach in pricing long-maturity American options. The new methodology, called the “extension-method”, relies on an approximation of the optimal exercise price near the beginning of the contract combined with existing pricing approaches so that the maturity range for which small errors are attainable is extended. Our method retains the quasi-analytic nature of the methods it improves. Generic quasi-analytic formulae for the price of an American put as well as for its hedging parameter are derived. Our scenarios-based numerical study indicates that our method considerably improves both the pricing and the hedging performance of a number of established approaches for a wide range of maturities. The superiority of this approach is illustrated with real financial data by considering S&P 100<sup>TM</sup> LEAPS<sup>®</sup> options traded from January 2008 to May 2015.

*Keywords:* American options, Optimal exercise price, Quasi-analytic method, Delta-hedging, LEAPS

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## **An Improved Method for Pricing and Hedging Long Dated American Options**

### **1. Introduction**

The problem of pricing American options has been widely examined in the last 40 years. The main challenge is due to the fact that the American optionality requires the selection of the optimal exercise price (henceforth, OEP) together with the valuation of the contingent claim. Several types of approximation approaches have been proposed in the literature to solve this problem. Within the broad class of approximation methods, in this paper we focus on the quasi-analytic methods consisting of analytic formulae that require at most a reasonably small number of numerical solutions of integral equations. The first method in this subclass is described in Geske and Johnson (1984), who used a portfolio of compound European options to replicate the early exercise feature of American options. Bunch and Johnson (1992) improved the efficiency of the Geske-Johnson method by optimally locating the exercise points and showed that most of the time only two – and in a few cases for deep-in-the-money options only three – early-exercise dates including maturity are required. The quadratic approximation in Barone-Adesi and Whaley (1987) gives an approximated solution of the Black-Scholes partial differential equation in closed form. This method, extremely fast and accurate for very short and very long maturities, has been refined by Ju and Zhong (1999) including a second-order extension that improves accuracy for middle-term maturities. Subsequently, Li (2010a) further refined the Ju-Zhong method by a more careful use of the smooth pasting condition for American options. Their method results in a more precise estimation of the OEP. However, the approximations in Barone-Adesi and Whaley (1987), Ju

and Zhong (1999) and Li (2010a) have the limitation that the pricing error cannot be controlled, that is, these methods are not convergent to the “true” price because they cannot be made more precise by including additional terms. Ju (1998) proposed a piecewise exponential function for the OEP.

An important step in the American option pricing literature was the result of Kim (1990) and Carr et al. (1992), who derived an implicit-form integral equation for the OEP. Hence, the pricing of American options can be reduced to identifying the OEP efficiently. Ibáñez (2003) modified Kim’s method to guarantee that the prices monotonically converge to the true prices when the number of time steps increases. Kim et al. (2013), based on an idea from Little et al. (2000), transformed the integral equation into a numerical functional form with respect to the optimal exercise boundary, and subsequently constructed an iterative method to calculate the boundary as a fixed point of the functional. Recently, Broadie and Detemple (1996), Laprise et al. (2006) and Chung et al. (2010) proposed tight quasi-analytic bounds for American options. Additionally, Chockalingam and Muthuraman (2015) employed the approximate moving boundaries method which iteratively finds an approximation of the OEP and Chockalingam and Feng (2015) extended on Ibáñez and Paraskevopoulos (2011) to investigate the cost of a suboptimal OEP.

Almost all the methods may produce large pricing errors for long-maturity options since the convergence to the “true” price depends on the decrease of the size of the time-step (i.e. early exercise dates) or, equivalently, on the increase in the number of iterations. However, an increase in the number of iterations makes these methods rapidly inefficient. In Table 1 – see rows ‘Std’, which contain the results for the ‘standard’ versus rows ‘Ext’, the ‘extended’ version of the methods – the performance of several pricing methods is reported with respect to the mean absolute percentage error, MAPE, for

maturities ranging from less than six months to between four and a half and five years. All the methods considered were selected because they perform very well for short maturities as reported by several other studies.<sup>1</sup>

In this paper, we propose a quasi-analytic approach that aims to improve the performance of existing methods in pricing and hedging long-maturity options. The new approach, which we call the “extension method”, relies on the fact that the OEP is independent of the current underlying asset price. The state space is divided into the continuation and the exercise regions, which are precisely separated by the OEP. In the following, in a novel way each option’s time-to-maturity is divided into two components according to the closeness to the maturity date. We use a constant approximation function<sup>2</sup> for the first part of the option life and existing pricing methods (with their associated estimation for the OEPs) for the second part (see Figure 1). The division of time to maturity and OEP profile is marked by a time-point  $t_x$ . The value of  $t_x$  is determined by performing several empirical trials and, although it is dependent on the quasi-method, our results in Figure 2 suggest that  $t_x/T$  is around 0.5, although for Ju and Zhong (1999)  $t_x/T = 0.3$  and for Ibáñez (2003)  $t_x/T = 0.35$ .

Under the proposed extension methodology, the option price is equal to the sum of the expected discounted-payoff from the first part of the option life and the expected discounted-payoff from the second part, conditioned on not exercising the option in the first part. We derive formulae for the American put price and also for the corresponding hedging parameters.<sup>3</sup> We

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<sup>1</sup>The comparison is done by using the studies reported in Table 1.

<sup>2</sup>The fact that the OEP becomes constant for long maturities helps to iterative methods as well; since the boundary at time  $t_n$  is a good starting point for the boundary at  $t_{n-1}$ . We thank an anonymous referee for making this point.

<sup>3</sup>The formulae are given in the on-line appendix.

also prove the convergence of the American put option price obtained with our proposed extension method to the perpetual put price when maturity increases infinitely. An extensive scenario-based study is carried-out showing that, when compared with established quasi-analytic methods, the new method leads to sizeable improvements in pricing and hedging American options, especially for longer maturities where existing methods generally fail. Then, we show that the extension method also improves the existing methodologies when applied to real data, LEAPS<sup>®</sup> options on the S&P 100<sup>TM</sup> index between 2 January 2008 and 29 May 2015.

The remainder of the paper is organized as follows. Section 2 describes the modelling framework. The main theoretical results are discussed in Section 3 where the closed-form pricing and hedging formulae are derived. Section 4 is a numerical scenarios-based study of the pricing and hedging performance of the extension method. Section 5 reports the pricing performance over the S&P 100<sup>TM</sup> LEAPS<sup>®</sup> options and Section 6 concludes.

## 2. Modelling framework

All modelling referring to American option pricing in this paper is done assuming that, under the risk-neutral measure  $\mathbb{Q}$ , the dynamics of the underlying stock  $S$  is given by:

$$dS_t = (r - \delta)S_t dt + \sigma S_t dW_t, t \geq t_0 \quad (1)$$

where  $r$  is the risk-free rate and  $\delta$  is the annual dividend yield with continuous compounding. For simplicity, the difference  $r - \delta$  is denoted henceforth by  $b$  and  $\{W_t\}_{t \geq t_0}$  refers to a Wiener process under the martingale measure  $\mathbb{Q}$ . Without any loss of generality, we only consider the case of American put

options.<sup>4</sup> The OEP of the American put option with maturity  $T$  and strike price  $K$  is a continuous function, see Jacka (1991), non-decreasing with respect to time, with limiting value for  $t \rightarrow T$  equal to  $\min\{K, rK/\delta\}$  when dividends are paid at the rate  $\delta$ , and bounded below by the optimal exercise price of the perpetual put option,  $S_f^\infty$ , where

$$S_f^\infty = \frac{\beta}{\beta - 1}K, \quad (2)$$

with  $\beta = \left(\frac{1}{2} - \frac{b}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{b}{\sigma^2}\right)^2 + 2\frac{r}{\sigma^2}}$ . Ekstrom (2004) proved that the OEP is convex and Xie et al. (2011) showed that the decay rate of the OEP to the flat function in (2) is more than exponential in time-to-maturity. Hence, for long maturities, the OEP is well approximated by a flat function.

The key to our approach is to divide the option life into two parts, one closest to the beginning of the contract and one to maturity, and then use existing pricing methods and their corresponding estimation approaches for the OEP in the second part, while considering a flat approximation of the OEP for the first part. The advantage is threefold: the estimation of the OEP near maturity is precise, the existing methods are used where they have better performance (comparative advantage), and very low computational effort is required near the beginning of the contract. The extension method is based on the property discussed in Geske and Johnson (1984) and Kim (1990) that, under the Black-Scholes model, the optimal exercise price does not depend on the spot price. Thus, it is possible to employ the OEP of a shorter maturity option to build part of the optimal exercise price of an American option on

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<sup>4</sup>All the formulae and propositions can be equivalently derived for American call options. Additionally, one can price and hedge call options by using the put-call symmetry in McDonald and Schroder (1998).

the same asset, with the same strike price but with longer maturity.

Figure 1 depicts the intuition behind our method. For a given set of parameter values,<sup>5</sup> the figure plots the OEPs for two American put options, written on the same underlying asset, with maturities  $t_1$  and  $T$ , where  $t_1 < T$ . Assume that we are now at time  $t_0$  and consider the intermediary time point  $t_x = t_0 + (T - t_1)$ .<sup>6</sup> The two options have identical characteristics apart from their different maturity dates, and because the OEP does not depend on the prevailing spot price, the optimal exercise prices for the two options will coincide whenever the options have the same time to maturity. For any time  $t$  in the interval  $[t_x, T]$ , the long-maturity option's OEP will be the same as the short-maturity option's OEP, which is defined on  $[t_0, t_1]$ . The continuous line represents the OEP of the option with maturity  $T$  and the dash-dot lines represent the optimal exercise of the option with maturity  $t_1$ . The left-most dash-dot line is the "original" function and the other is its translation over the continuous line to show they coincide on the interval  $[t_x, T]$ .<sup>7</sup>

### 3. The extension method

The first step in our new method is to split the option life into two parts: for the first part (i.e. the one closest to the beginning of the contract), we approximate the OEP as a constant  $A$ , while the pricing method that we are extending provides the OEP and the pricing formula for the part closest to maturity. Consequently, the OEP  $S_f^{(E)}(\cdot)$  of an American put option, with

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<sup>5</sup>The parameter values used are  $\sigma = 20\%$ ,  $\delta = 5\%$ ,  $r = 8\%$  and  $K = 100$ .

<sup>6</sup>In Figure 1, we assume  $t_0 = 0$ ,  $t_1 = 1$  year,  $T = 2.5$  years and  $t_x = 1.5$  years.

<sup>7</sup>The OEPs are calculated by the integral method in Kim (1990).

maturity  $T$ , starting life at  $t_0$ , is given by:

$$S_f^{(E)}(t) = \begin{cases} \Lambda & \text{for } t \in [t_0, t_x) \\ S_f(t - (T - t_1)) & \text{for } t \in [t_x, T] \end{cases} \quad (3)$$

where  $t_x \in [t_0, T]$  is the break-point (i.e. the time point separating the option life in two parts, as explained above).<sup>8</sup>  $S_f(\cdot)$  is the optimal exercise price of the shorter maturity option (written on the same underlying asset, with the same strike price). As illustrated in Figure 1, we can think of the shorter maturity option as either starting life at  $t_0$  and having maturity date  $t_1$ , or as starting life at  $t_x$  and having maturity date  $T$ . In either case, at the onset, the shorter maturity option has  $t_1 - t_0 = T - t_x$  time (years) to maturity.  $S_f(\cdot)$  will be estimated via an existing quasi-analytic method in the literature which we are extending to price the long-maturity option.

With the OEP given in (3), the price of the American put option is calculated as the sum of the expected discounted payoff between  $t_0$  and  $t_x$ , assuming that the option is exercised as soon as the spot price hits  $\Lambda$  (see formulae (A.3) and (A.4) in the appendix), and the expected discounted payoff from the short-maturity American option (between  $t_x$  and  $T$ ) conditioned on not hitting  $\Lambda$  between  $t_0$  and  $t_x$  (see formula (A.3)). Proposition 3.1 derives the pricing formula of the extension method, where the following notation will be used:  $P_{t_x}(S_{t_x}, T, K)$  is the price of the option with time to maturity  $T - t_x$  (short-maturity option) at time  $t_x$  when the underlying

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<sup>8</sup>Selecting  $t_x \rightarrow t_0$ , the option price obtained with our proposed extension method converges to the price obtained via the method we are extending (i.e. what we call the corresponding standard method) and when  $t_x \rightarrow T$ , the extension method price converges to the price of the method in Bjerksund and Stensland (1993). Figure 2 plots for each of the methods we extend on the mean absolute percentage error as a function of  $t_x$ .

asset price is  $S_{t_x}$  and the OEP is given by  $S_{f_x}(\cdot)$  – a function defined on  $[t_x, T]$ , which is the translation of the function  $S_f(\cdot)$ , the latter being defined on  $[t_0, t_1]$  – and  $P_{t_0}^{(E)}(S_{t_0}, T, K|t_x, \Lambda)$  is the price of the option with time to maturity  $T - t_0$  (long-maturity option), at time  $t_0$ , when the underlying asset price is  $S_{t_0}$  and when the optimal exercise price is given by (3). We also use the simplified notation  $\varphi(\gamma, H)$  to denote the expectation term  $\varphi_{t_0}^P(S_{t_0}, t_x|\gamma, H, \Lambda) = E_{t_0} [e^{-rt_x} S_{t_x}^\gamma I(S_{t_x} > H) I(\inf_{t \in [t_0, t_x]} S_t > \Lambda)]$ , which is given in Appendix A (see equation A.2), together with its derivation and the expression for  $f_0(\cdot)$  – the probability density function of an arithmetic Brownian motion with positive initial value  $z_{t_0}$ , drift parameter  $b_1 = b - \frac{1}{2}\sigma^2$ , volatility parameter  $\sigma$  and an absorbing barrier at 0.

**Proposition 3.1.** *Assuming Black-Scholes dynamics, the price of an American put option with strike price  $K$  and maturity  $T$  at time  $t_0$ , based on the extension of the standard method with pricing function  $P_{t_x}(S_{t_x}, T, K)$ , is:*

$$P_{t_0}^{(E)}(S_{t_0}, T, K|t_x, \Lambda) = e^{rt_0} \left\{ \alpha(\Lambda) \left[ S_{t_0}^\beta e^{-rt_0} - \varphi(\beta, \Lambda) \right] - \varphi(1, \Lambda) + \varphi(1, S_f^{(E)}(t_x)) \right. \\ \left. + K \left[ \varphi(0, \Lambda) - \varphi(0, S_f^{(E)}(t_x)) \right] \right\} + \int_B^{+\infty} g(z) dz, \quad (4)$$

where  $B = \ln \frac{S_f^{(E)}(t_x)}{\Lambda}$ ,  $\alpha(\Lambda) = (K - \Lambda)\Lambda^{-\beta}$ ,  $\beta = \left(\frac{1}{2} - \frac{b}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{b}{\sigma^2}\right)^2 + 2\frac{r}{\sigma^2}}$  and  $g(z) = e^{-r(t_x - t_0)} P_{t_x}(\Lambda e^z, T, K) f_0(z)$ .

*Proof.* See Appendix A. □

The proof of this proposition is similar to the proof for the pricing formula in Bjerksund and Stensland (2002). Thus, when the OEP for the short-maturity option is flat, the method described in Bjerksund and Stensland (1993) applies and our pricing formula becomes the pricing formula in Bjerksund and Stensland (2002). For this reason our method can be seen as a

generalization of Bjerksund and Stensland (2002) that combines any quasi-analytic pricing formula for the short-maturity American put option and a flat approximation of the OEP near the beginning of the contract.

Moreover, the following proposition shows that asymptotically our pricing formula converges to the perpetual put option price.

**Proposition 3.2.** *For any  $t_x \in (t_0, T]$ , any  $0 < \Lambda < S_{f_x}(t_x)$  and any pricing formula for the short-maturity option  $P_{t_x}(S_{t_x}, T, K)$ , when  $T \rightarrow +\infty$ , the price*

$$P_{t_0}^{(E)}(S_{t_0}, T, K | t_x, \Lambda)$$

*given in Proposition 3.1 converges to the price of a perpetual option written on the same underlying asset, with the same strike price and which is exercised as soon as the underlying asset price hits  $\Lambda$ .*

The proof is given in Appendix B.

The calculation of the delta parameter is equally important in financial markets. The following result provides an analytic formula for the calculation of the delta parameter<sup>9</sup> of an American put option by the extension method, relying on the independence of the OEP from the current asset price.

**Proposition 3.3.** *Under the same conditions as stated in Proposition 3.1, the delta parameter is given by the following formula*

$$\begin{aligned} \Delta_{t_0} = & e^{rt_0} \left\{ \alpha(\Lambda) \left[ \beta S_{t_0}^{\beta-1} e^{-rt_0} - \varphi'(\beta, \Lambda) \right] - \varphi'(1, \Lambda) + \varphi'(1, S_f^{(E)}(t_x)) \right. \\ & \left. + K \left[ \varphi'(0, \Lambda) - \varphi'(0, S_f^{(E)}(t_x)) \right] \right\} + \int_B^{+\infty} g'(z) dz, \end{aligned} \quad (5)$$

where  $g'(z) = e^{-r(t_x-t_0)} P_{t_x}(\Lambda e^z, T, K) f'_0(z)$ .

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<sup>9</sup>In the on-line appendix, we provide formulae for the other hedging parameters.

*Proof.* Here we use the simplified notation  $\varphi'(\gamma, H)$  to denote the partial derivative  $\varphi'_{t_0, S_{t_0}}(S_{t_0}, t_x | \gamma, H, \Lambda) = \frac{\partial \varphi_{t_0}^P(S_{t_0}, t_x | \gamma, H, \Lambda)}{\partial S_{t_0}}$ , and where  $f'_0(z) = \frac{\partial f_0(z)}{\partial S_{t_0}}$ . The result is an application of the Leibniz's derivation formula to function (4), taking into account the results  $\frac{\partial B}{\partial S_{t_0}} = 0$ ,  $\frac{\partial P_{t_x}(\Lambda e^z, T, K)}{\partial S_{t_0}} = 0$ ,  $\frac{\partial \Lambda}{\partial S_{t_0}} = 0$  that follow from the independence of the OEP from the current asset price.  $\square$

The pricing formula in Proposition 3.1 and the delta parameter in Proposition 3.3 work under any specification for the pricing formula of the short-maturity option. Choosing one or another pricing formula for this option only changes the last addend of formulae (4) and (5) (i.e. the two integrals).

#### 4. Scenarios-based study

The extension method is applied to:<sup>10</sup> the compound-option method in Geske and Johnson (1984) with two and three exercise dates, the quadratic method in Barone-Adesi and Whaley (1987), the integral method in Kim (1990) with two and three exercise dates, many variants of the improved integral-method in Ibáñez (2003), the improved quadratic method in Ju and Zhong (1999), the static-replicating portfolio method in Chung and Shih (2009) with two and three exercise dates, and the interpolation method in Li (2010b). Comparing the performance of each standard method with its extended version, we highlight how the performance improves considerably when the extension method is employed. The focus will be on the improvement in accuracy since the computational effort required by the extension method is only slightly greater than the standard methods and, in most cases, the additional computational time is negligible.<sup>11</sup>

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<sup>10</sup>A brief description of the standard methods is provided in the on-line appendix.

<sup>11</sup>For the methods developed by Barone-Adesi and Whaley (1987), Kim (1990), Chung and Shih (2009) and Ibáñez (2003) the integrals in (4) and (5) can be calculated analytically

The numerical study will be carried out for a flat approximation  $\Lambda$  fixed equal to  $S_{f_x}(t_x)$  in order to ensure the continuity of the OEP. However, for the integral method and the compound-option method, because when few steps are considered the value of  $S_{f_x}(t_x)$  can be quite unreliable,  $\Lambda$  is fixed to be equal to the OEP at time  $t_x$  calculated by the quadratic method in Barone-Adesi and Whaley (1987). Moreover, without loss of generality, we fix  $t_0 = 0$ , we express  $t_x$  as a percentage of time-to-maturity  $t_x = \vartheta T$  and consider 10 ratios  $\vartheta \in \{0.05 + 0.1j | j = 0, \dots, 9\}$  to study the performance of the extension method under different approximations of the OEP.

#### *4.1. Pricing performance*

The pricing-performance study is constructed from a total of 10,000 randomly generated options scenarios. In particular, the parameters in (1) and the options characteristics are drawn as in Broadie and Detemple (1996): the volatility  $\sigma$  is distributed uniformly between 0.1 and 0.6; the initial asset price  $S_{t_0}$  is fixed at 100; the strike price  $K$  is distributed uniformly between 70 and 130; the dividend rate  $\delta$  is distributed uniformly between 0.0 and 0.1 with probability 0.8 and equal to 0.0 with probability 0.2, and the risk-free interest rate is uniformly distributed between 0.0 and 0.1. Given the importance of time-to-maturity to establish the usefulness of the extension method, we divide the simulated scenarios into 10 sets with equal cardinality. The sets are labeled  $A, \dots, J$  and divide the options scenarios according to their time-to-maturity in ranges of 6 months. So, for example, set  $A$  contains scenarios with maturity between 1 day and 6 months, set  $B$  scenarios with maturities between 6 months and 1 year, and so on up to set  $J$  which

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while the others require a numerical calculation of the integrals that is much faster than the numerical solution of integral equations.

contains scenarios with maturities between 4.5 years and 5 years.

We use the mean absolute percentage error (MAPE) to compare each method with its “extended” version. The “exact” fair price (benchmark) is the binomial tree price with 15,000 time-steps. As in Broadie and Detemple (1996), options with benchmark prices less than 0.5 are discarded. The models performance for different maturities is summarized in Table 1. In the on-line appendix (Tables 5-7), we present the result classified per moneyness.<sup>12</sup> For each standard model considered, applying the extension method increases its pricing performance. Table 1 shows that the extension method has the advantage of levelling out the performance of quasi-analytic methods across maturities, shrinking the range of MAPEs. The extension method reduces to about half the MAPEs of the standard methods for all options scenarios with maturities longer than six months. For some methods (Barone-Adesi and Whaley (1987) and Chung and Shih (2009) with two and three exercise-dates) the extension method achieves reductions in MAPE of over 80%. The extension method also works efficiently for options with maturities less than 6 months. An application of the modified Diebold-Mariano test<sup>13</sup> shows that in the majority of cases these reductions of MAPE are statistically significant at the 99% confidence level.

Our results are in line with several related papers. AitSahlia and Carr (1997) confirm that the Barone-Adesi-Whaley method outperforms for both short and long maturities the compound-option method by Geske and Johnson (1984) but is outperformed by Kim’s method. Li (2010b) shows that his method outperforms the Barone-Adesi-Whaley method for medium-long

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<sup>12</sup>All results for the extended versions of the methods are in relation to the ratio  $t_x/T$  which has the lowest MAPE linked to it. Figure 2 plots the results for the other ratios.

<sup>13</sup>We thank an anonymous referee for suggesting the modified Diebold-Mariano test.

maturities but does not perform as well for short maturities. Additionally, the improved quadratic method by Ju and Zhong (1999) and Ibáñez (2003) are by far the best performing standard method among the ones we consider in this paper. Although the pricing errors of the last two methods are already small compared to the others, our extension method improves on them with a MAPE reduction over the entire set of options scenarios to about a half. Competitive results are also obtained via the standard Chung-Shih method. This method, when combined with our extension method, returns the best performance across maturities and moneyness.

The results of the modified Diebold-Mariano test show that for maturities longer than one year the methods have significantly different MAPE, the extended methods performing significantly different among each other regarding option pricing.<sup>14</sup> Figure 2 plots the MAPE (cumulative for the scenarios in A-J) of each standard method against the one of the corresponding extended method for different ratios  $t_x/T$ . For any ratio  $t_x/T$  below 0.7, each extended version produces MAPEs that are sensibly smaller than the standard version. Figure 2 also illustrates that for most of the methods investigated in this paper, the optimal ratio is about 0.5. Hence, if additional information about the optimal ratio is missing, one can simply fix  $t_x/T = 0.5$  and still realize significant improvements relative to the standard methods.

#### *4.2. Hedging performance*

The numerical study on hedging performance is based on the implementation of delta-hedging strategies and an analysis of the hedging errors. We analyze a set of 15 option scenarios with strike price  $K = 100$ , maturity  $T$  (in years)

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<sup>14</sup>In the on-line appendix, we also present the pricing performance under other error measures and the results are aligned with those presented here.

in the set  $\{1, 2, 3, 4, 5\}$ , written on underlying assets with volatility  $\sigma = 40\%$  and dividend yield  $\delta = 0.04$ . Three different initial spot price  $S_0$  are considered in the set  $\{90, 100, 110\}$  are considered, with  $r = 5\%$ . For each  $\mu$  we simulate 1,000 paths, and along each path we set-up the delta-hedging strategy deriving from the selected method.

Table 2 presents the average quadratic hedging error over the 15 option scenarios. The hedging error is defined as the difference in value between the option pay-off at the exercise-date and the hedging portfolio.<sup>15</sup> For this exercise, we consider the fair price for each option to be the 15,000 time-step binomial-tree price. The results indicate an improved hedging performance when compared to the standard methodologies. This conclusion is robust to changes in model parameters, choice of standard method which is extended, and to option maturity. The average improvement of the extension method over the standard methods (above 38% reduction of the average quadratic hedging error) is economically and statistically significant. An application of the modified Diebold-Mariano test shows that all the differences in terms of average quadratic hedging error between each standard method and our extended versions are significant at the 99% confidence level.

#### *4.3. Model error comparison: performance of the extension method under a stochastic volatility model with jumps*

There is a vast literature highlighting that the Black-Scholes model has several limitations in relation to the observed empirical characteristics of returns in financial markets. In this section, we consider a more realistic data gener-

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<sup>15</sup>For more details on the average quadratic hedging error measure see, among others, Remillard et al. (2012).

ation process,<sup>16</sup> the stochastic volatility model of the Heston type combined with the Merton jump-diffusion model as introduced by Bates (1996):

$$\begin{aligned} S_t &= S_0 e^{X_t}, & dX_t &= \left( \alpha - \frac{1}{2} Y_t \right) dt + \sqrt{Y_t} dW_t^1 + dZ_t, & X_0 &= 0 \\ dY_t &= \epsilon(\eta - Y_t)dt + \theta\sqrt{Y_t}dW_t^2 \end{aligned} \quad (6)$$

for  $\alpha \in \Re$ ,  $\epsilon \geq 0$ ,  $\eta \geq 0$  and<sup>17</sup>  $\theta \geq 0$ . Above,  $W_t^1$  and  $W_t^2$  are standard Wiener processes having constant correlation  $\rho$  and  $Z$  is a compound Poisson process with intensity  $\lambda$  where the jumps  $J$  are Gaussian distributed with mean  $\gamma$  and standard deviation  $\delta_J$ .

In order to evaluate the extension method, we consider the option scenarios in Chiarella et al. (2009), Toivanen (2010) and Ballestra and Sgarra (2010). Test-1 and Test-2 (Table 1 and 2 in Ballestra and Sgarra (2010), respectively) are made of five American call options each with  $K = 100$ ,  $T = 0.5$  year,  $r = 0.03$ ,  $\delta = 0.05$  and five underlying spot rates  $S_0 = \{80, 90, 100, 110, 120\}$ . The parameters of model (6) are given as:  $\epsilon = 2$ ,  $\eta = 0.04$ ,  $\theta = 0.4$ ,  $\lambda = 5$ ,  $\delta_J = 0.1$  and  $\gamma = -\frac{\delta^2}{2}$ . Furthermore, options in Test-1 have  $\rho = 0.5$  while in Test-2 have  $\rho = -0.5$ . Test-3 (Table 3 in Ballestra and Sgarra (2010)) is made of five American call option scenarios with  $K = 100$ ,  $T = 0.5$  year,  $r = 0.03$ , and  $\delta = 0.05$ . The parameters of model (6) are given as:  $\epsilon = 2$ ,  $\eta = 0.04$ ,  $\theta = 0.25$ ,  $\rho = -0.5$ ,  $\lambda = 0.2$ ,  $\delta_J = 0.4$ , and  $\gamma = -0.5$ . Test-4 (Table 4 in Ballestra and Sgarra (2010)) is made of five American put option scenarios with  $K = 100$ ,  $T = 5$  year,

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<sup>16</sup>We thank an anonymous referee for suggesting to evaluate the performance of the newly introduced extension method under model error and to use a more realistic data generating process.

<sup>17</sup>The parameter  $\theta$  is the diffusion coefficient of the Feller process followed by  $Y$  and if  $\theta < 2\epsilon\eta$  then the process  $Y$  stays always positive.

$r = 0.0319$ , and  $\delta = 0$ . The parameters are  $\epsilon = 6.21$ ,  $\eta = 0.019$ ,  $\theta = 0.61$ ,  $\rho = -0.7$ ,  $\lambda = 0.5$ ,  $\delta_J = 0.2$  and  $\gamma = -\frac{\delta_J^2}{2}$ .

The comparison in this section follows the model error comparison in Psychoyios and Skiadopoulos (2006). For each of the 20 option scenarios, we calculate the implied volatility.<sup>18</sup> We then priced the 20 option scenarios by using the seven standard methods as above and their extended version. The “benchmark” prices are those in Chiarella et al. (2009), Toivanen (2010) and Ballestra and Sgarra (2010) which were obtained by using finite difference approximations and Monte Carlo simulations on an extremely fine mesh.

Table 3 shows the results. In most of the cases, the extension method outperforms the standard methods. Exceptions are Li (2010b), Ju and Zhong (1999) and Ibáñez and Paraskevopoulos (2011) with Richardson extrapolation with  $ns = 22$  where the standard method and the corresponding extended version have virtually the same performance.

## 5. Empirical study: the S&P 100<sup>TM</sup> LEAPS<sup>®</sup> case

The new method is also relevant with real market data. In the following we employ a methodology similar to that used in Linaras and Skiadopoulos (2005) for the S&P 100<sup>TM</sup> index that has exchange-listed options of both American and European exercise type.

### 5.1. Dataset

We consider both American (ticker OEX<sup>®</sup>) and European (ticker XEO<sup>®</sup>) LEAPS<sup>®</sup> options written on the S&P 100<sup>TM</sup> index traded between the 2 January 2008 and 29 May 2015, covering 1,826 days, spanned over the eight

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<sup>18</sup>We used the binomial tree method with 15,000 steps to recover the volatility.

years.<sup>19</sup> We collected ask and bid prices, open interest, and volume together with the contract specifications (strike, maturity date and exercise style). Additionally, we used the Libor as a proxy of the risk-free rate. We collected the last prices of the S&P 100<sup>TM</sup> index and the US dollar Libor for one week, one month, three months, six months and one year. Since S&P 100<sup>TM</sup> index is a dividend-paying asset, we proceed by estimating the dividend yield over the life of each contract. As in Linaras and Skiadopoulos (2005), we imply the dividend yield for each trading day and maturity from the at-the-money put-call parity. Then, we imply from the market prices the volatility of the underlying asset. We follow Brandt and Wu (2002), Panigirtzoglou and Skiadopoulos (2004) and Linaras and Skiadopoulos (2005), employing out-of-the-money and at-the-money XEO<sup>®</sup> (European) option prices to build the volatility surfaces.

We filtered the data by a methodology similar to the one outlined in Linaras and Skiadopoulos (2005). From the XEO<sup>®</sup> (European), we discarded options with (1) zero volume and zero open interest, (2) a premium smaller than \$0.5 or (3) a negative yield since it corresponds to the existence of arbitrage. We also discard options whose prices do not satisfy the static bounds. Additionally, we retain only options with maturities greater than 250 trading days. Table 4 summarizes the statistics of the retained options.

## 5.2. Results

For comparison, we employ the six error measures in Brandt and Wu (2002) and Linaras and Skiadopoulos (2005): the mean valuation error (MVE) that measures the average difference between each model price and the OEX<sup>®</sup> mid-price; the root mean squared valuation error (RMSVE) that calculates

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<sup>19</sup>We retrieved the data from the Chicago Board of Exchange Trade (CBOE).

the square-root of the average squared difference between the model price and the OEX<sup>®</sup> mid price; the frequency in bid-ask (FIBA) that is the percentage of time the model price is included within the market bid-ask spread; the mean outside error (MOE) that is the average error outside of the bid-ask spread; the root mean squared outside error (RMSOE) that quantifies the variability of the errors outside the bid-ask spread; and mean relative outside error (MROE) that is the average outside error divided by the market price, which corresponds to the MOE but in percentage terms. The error outside the spread employed in MOE, RMSOE and MROE is defined as the model price minus the bid quote (ask quote) if the model price is below (above) the bid price (ask price) and it is fixed to zero for the cases when the model price falls within the bid-ask spread.

We fix the ratios  $\varphi = t_x/T$  to the values that correspond to the minimum MAPE in Figure 2, for any of the considered methods. Table 5 summarizes the comparison results for maturities of more than 500 trading days. In the vast majority of cases, the extension method outperforms the standard methods. These improvements are in the majority of cases also economically significant, well above the minimum tick in the S&P 100<sup>TM</sup> market (0.10). An application of the modified Diebold-Mariano test shows that in many cases the differences are statistically significant. Overall, the extension method prices are closer to the benchmark OEX<sup>®</sup> mid-price, more condensed around it and the fewer prices outside the bid-ask spread are more symmetric and smaller in size. The analysis of the S&P 100<sup>TM</sup> LEAPS<sup>®</sup> indicates the superiority of the extended methods. Additionally, Table 5 shows that the advantages of the extension method increased during the most recent period (2012-2015). In the on-line appendix, we provide additional results classified for time to maturities and moneyness. In Table 5,

we also price the S&P 100<sup>TM</sup> LEAPS<sup>®</sup> by using the standard binomial tree in Cox et al. (1979) and the implied tree method in Derman et al. (1995). While the former has a similar performance to that of the numerical methods analysed, the latter is inferior to the other methods considered here.

## 6. Conclusion

Most quasi-analytic methods currently used for pricing and hedging American options perform well for short-maturity options but not so well for long-maturity ones. We proposed here a technique that can improve the pricing and hedging performance of any quasi-analytic method for long-maturity options. Our scenarios-based study shows that, for each selected method, remarkable improvements can be obtained for hedging and pricing at the cost of negligible computational time.

We also evaluate the newly proposed methodology applied to the LEAPS<sup>®</sup> options on the S&P 100<sup>TM</sup> stock index traded from January 2008 to May 2015. The extension method outperforms existing methodologies using real financial data, making this methodology preferable to existing ones.

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## Appendices

### A. Proof Proposition 3.1

Given the optimal exercise price in (3) for any  $0 < \Lambda \leq S_{f_x}(t_x)$ , we define

$$t^* = \inf \left\{ \inf_{t \in [t_0, \infty)} \{S_t \leq S_f^{(E)}(t)\}, T \right\} = \inf \{t_0^*(\Lambda), t_x^*(S_{f_x}(t)), T\}$$

where  $t_u^*(x) = \inf_{t \in [t_u, \infty)} \{S_t \leq x\}$ . Consequently, the American put price is:

$$\begin{aligned} P_{t_0}^{(E)}(S_{t_0}, T, K | t_x, \Lambda) &= E_{t_0} [e^{-r(t^*-t_0)}(K - S_{t^*})^+] = e^{rt_0} E_{t_0} [e^{-rt^*}(K - S_{t^*})^+] \\ &= e^{rt_0} \left\{ E_{t_0} [e^{-rt^*}(K - \Lambda)I(t_0 \leq t^* < t_x)] + E_{t_0} [e^{-rt^*}(K - S_{t_x})I(t^* = t_x)] \right\} \\ &\quad + E_{t_0} [e^{-r(t^*-t_0)}(K - S_f^{(E)}(t^*))^+ I(t_x < t^* \leq T)] \end{aligned} \quad (\text{A.1})$$

where  $I(\cdot)$  is the indicator function. The price of a perpetual put option starting at time  $u$  is  $E_u [e^{-r(t_u^*(x)-u)}(K - x)] = \alpha(x)S_u^\beta$  where  $\alpha(x)$  and  $\beta$  are given in Proposition 3.1. We prove here the result for  $\gamma > 0$ , the case when  $\gamma = 0$  being trivial. A helpful result is the following expectation

$$\begin{aligned} \varphi_{t_0}^P(S_{t_0}, t_x | \gamma, H, \Lambda) &= E_{t_0} \left[ e^{-rt_x} S_{t_x}^\gamma I(S_{t_x} > H) I \left( \inf_{t \in [t_0, t_x)} S_t > \Lambda \right) \right] \\ &= \Lambda^\gamma E_{t_0} \left[ e^{-rt_x} e^{zt_x} I(z_{t_x} > B_H) I \left( \inf_{t \in [t_0, t_x)} z_t > 0 \right) \right] \end{aligned}$$

$$= e^{\lambda t_x} S_{t_0}^\gamma \left[ N(d_{\varphi,1}(H)) - \left( \frac{\Lambda}{S_{t_0}} \right)^\kappa N(d_{\varphi,2}(H)) \right] \quad (\text{A.2})$$

where  $B_H = \gamma \ln \frac{H}{\Lambda}$ ,  $z_{t_x} = \gamma \ln \frac{S_{t_x}}{\Lambda}$ ,  $N(\cdot)$  is the standard normal cumulative distribution function,  $d_{\varphi,1}(H) = \frac{\ln \frac{S_{t_0}}{H} + (b + (\gamma - \frac{1}{2})\sigma^2)(t_x - t_0)}{\sigma \sqrt{t_x - t_0}}$ ,  $d_{\varphi,2}(H) = \frac{\ln \frac{\Lambda^2}{S_{t_0} H} + (b + (\gamma - \frac{1}{2})\sigma^2)(t_x - t_0)}{\sigma \sqrt{t_x - t_0}}$ ,  $\lambda = -r + \gamma b + \frac{1}{2}\gamma(\gamma - 1)\sigma^2$  and  $\kappa = \frac{2b}{\sigma^2} + (2\gamma - 1)$ . The first expectation in equation (A.1) indicates the expected payoff from exercising the option between  $t_0$  and  $t_x$  and is

$$\begin{aligned} E_{t_0} [e^{-rt^*} (K - \Lambda) I(t_0 \leq t^* < t_x)] &= E_{t_0} \left[ e^{-rt_0^*(\Lambda)} (K - \Lambda) I \left( \inf_{t \in [t_0, t_x]} S_t < \Lambda \right) \right] \\ &= \alpha(\Lambda) S_{t_0}^\beta e^{-rt_0} - \alpha(\Lambda) E_{t_0} \left[ e^{-rt_x} S_{t_x}^\beta I(S_{t_x} \geq \Lambda) I \left( \inf_{t \in [t_0, t_x]} S_t > \Lambda \right) \right] \\ &= \alpha(\Lambda) S_{t_0}^\beta e^{-rt_0} - \alpha(\Lambda) \varphi_{t_0}^P(S_{t_0}, t_x | \beta, \Lambda, \Lambda) \end{aligned} \quad (\text{A.3})$$

The second expectation in equation (A.1) can be calculated as:

$$\begin{aligned} E_{t_0} [e^{-rt^*} (K - S_{t_x}) I(t^* = t_x)] &= E_{t_0} \left[ e^{-rt_x} (K - S_{t_x}) I(\Lambda \leq S_{t_x} \leq S_f^{(E)}(t_x)) I \left( \inf_{t \in [t_0, t_x]} S_t > \Lambda \right) \right] \\ &= E_{t_0} \left[ e^{-rt_x} (K - S_{t_x}) \left[ I(S_{t_x} \geq \Lambda) - I(S_{t_x} \geq S_f^{(E)}(t_x)) \right] I \left( \inf_{t \in [t_0, t_x]} S_t > \Lambda \right) \right] \\ &= K \left[ \varphi_{t_0}^P(S_{t_0}, t_x | 0, \Lambda, \Lambda) - \varphi_{t_0}^P(S_{t_0}, t_x | 0, S_f^{(E)}(t_x), \Lambda) \right] \\ &\quad - \left[ \varphi_{t_0}^P(S_{t_0}, t_x | 1, \Lambda, \Lambda) - \varphi_{t_0}^P(S_{t_0}, t_x | 1, S_f^{(E)}(t_x), \Lambda) \right]. \end{aligned} \quad (\text{A.4})$$

The third expectation in equation (A.1) corresponds to the expected payoff from exercise the option in the interval  $(t_x, T]$  and it is

$$\begin{aligned} E_{t_0} [e^{-rt^*} (K - S_f^{(E)}(t^*))^+ I(t_x < t^* \leq T)] &= \\ &= e^{-rt_x} E_{t_0} \left[ P_{t_x}(\Lambda e^{z_{t_x}}, T, K) I(z_{t_x} > B) I \left( \inf_{t \in [t_0, t_x]} z_t > 0 \right) \right] \end{aligned} \quad (\text{A.5})$$

$$= e^{-rt_x} \int_B^{+\infty} P_{t_x}(Ae^z, T, K) f_0(z) dz \quad (\text{A.6})$$

where

$$f_0(z) = \frac{n \left( \frac{z - z_{t_0} - b_1(t_x - t_0)}{\sigma \sqrt{t_x - t_0}} \right) - e^{-\frac{2b_2 z_{t_0}}{\sigma^2}} n \left( \frac{z + z_{t_0} - b_1(t_x - t_0)}{\sigma \sqrt{t_x - t_0}} \right)}{\sigma \sqrt{t_x - t_0}}.$$

By replacing the three expectations in equations (A.3), (A.4) and (A.6) within equation (A.1), we get the pricing formula (4).

## B. Proof Proposition 3.2

The proof consists in showing that  $\lim_{T \rightarrow +\infty} P_{t_0}^{(E)}(S_{t_0}, T, K | t_x, \Lambda) = \alpha(\Lambda) S_{t_0}^\beta$ , for any selection of  $t_x \in (t_0, T]$  and  $\Lambda$ . For  $t_x = t_0 + \vartheta(T - t_0)$  with  $\vartheta \in (0, 1]$ , we prove that this limit is independent of  $\vartheta$ . We discuss separately the cases for different values of  $\gamma$ . When  $\gamma = \beta$ , for positive risk-free rate  $r$ ,  $\gamma = \beta = \left(\frac{1}{2} - \frac{b}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{b}{\sigma^2}\right)^2 + 2\frac{r}{\sigma^2}} < \frac{1}{2} - \frac{b}{\sigma^2}$  and, since  $b + (\gamma - \frac{1}{2})\sigma^2 < 0$ ,  $\lim_{T \rightarrow +\infty} d_{\varphi,1}(H) = \lim_{T \rightarrow +\infty} d_{\varphi,2}(H) = -\infty$  and  $\lim_{T \rightarrow +\infty} \varphi_{t_0}^P(S_{t_0}, t_x | \beta, \Lambda, \Lambda) = 0$  for any  $\vartheta$ . We note that this result also holds when  $\lambda > 0$ . On the other hand, for  $\gamma = 0$  or  $\gamma = 1$ ,  $\lambda = -r + \gamma b + \frac{1}{2}\gamma(\gamma - 1)\sigma \leq 0$  and  $\lim_{T \rightarrow +\infty} d_{\varphi,1}(H) = \lim_{T \rightarrow +\infty} d_{\varphi,2}(H) = \mathbf{v}$ , with  $\mathbf{v}$  independent from  $H$ . Therefore, the limit  $\lim_{T \rightarrow +\infty} \varphi_{t_0}^P(S_{t_0}, t_x | \gamma, H, \Lambda)$  is finite for any positive and finite  $H$  and, since it does not depend on the selection of  $H$ , for any finite  $H_1$  and  $H_2$ ,  $\lim_{T \rightarrow +\infty} [\varphi_{t_0}^P(S_{t_0}, t_x | \gamma, H_1, \Lambda) - \varphi_{t_0}^P(S_{t_0}, t_x | \gamma, H_2, \Lambda)] = 0$   $\lim_{T \rightarrow +\infty} \int_B^{+\infty} g(z) dz = \lim_{T \rightarrow +\infty} \int_B^{+\infty} e^{-r(t_x - t_0)} P_{t_x}(Ae^z, T, K) f_0(z) dz = 0$  since  $\lim_{T \rightarrow +\infty} f_0(z) = 0$ ,  $\lim_{T \rightarrow +\infty} e^{-r(t_x - t_0)} = \lim_{T \rightarrow +\infty} e^{-r\vartheta(T - t_0)} = 0$ , and  $0 \leq P_{t_x}(Ae^z, T, K) \leq K$  following the non-arbitrage condition. Since the quantities  $\alpha(\Lambda)$  and  $\beta$  are time invariant the result follows.

**Table 1:** Scenarios-based pricing performances of quasi-analytic standard methods and our extended versions: Mean Absolute Percentage Error.

	Maturities									
	A	B	C	D	E	F	G	H	I	J
	Geske and Johnson (1984) - 2 time steps									
Std.	0.389%	0.871%	1.343%	1.893%	2.084%	2.622%	3.131%	3.587%	3.984%	4.350%
Ext.	0.244%**	0.433%**	0.552%**	0.653%**	0.682%**	0.751%**	0.816%**	0.823%**	0.845%**	0.843%**
	Geske and Johnson (1984) - 3 time steps									
Std.	0.276%	0.608%	0.930%	1.295%	1.417%	1.788%	2.112%	2.376%	2.648%	2.902%
Ext.	0.195%**	0.328%**	0.436%**	0.511%**	0.523%**	0.573%**	0.617%**	0.623%**	0.650%**	0.668%**
	Barone-Adesi and Whaley (1987)									
Std.	0.155%	0.395%	0.632%	0.900%	1.202%	1.429%	1.696%	1.945%	2.252%	2.380%
Ext.	0.119%	0.140%	0.192%*	0.237%**	0.239%**	0.268%**	0.288%**	0.298%**	0.337%**	0.337%**
	Li (2010b)									
Std.	0.169%	0.293%	0.515%	0.669%	0.879%	1.053%	1.288%	1.532%	1.821%	1.965%
Ext.	0.087%	0.131%**	0.205%**	0.247%**	0.256%*	0.320%**	0.359%**	0.393%**	0.395%**	0.41%**
	Kim (1990) - 2 time steps									
Std.	0.248%	0.404%	0.578%	0.713%	0.794%	0.899%	0.988%	1.182%	1.291%	1.488%
Ext.	0.180%**	0.271%**	0.348%**	0.404%**	0.450%**	0.476%**	0.500%**	0.503%**	0.535%**	0.534%**
	Kim (1990) - 3 time steps									
Std.	0.125%	0.198%	0.275%	0.346%	0.358%	0.445%	0.518%	0.649%	0.678%	0.862%
Ext.	0.128%*	0.129%*	0.158%**	0.177%**	0.189%**	0.203%**	0.217%**	0.229%**	0.249%**	0.256%**
	Chung and Shih (2009) - 2 time steps									
Std.	0.102%	0.146%	0.155%	0.189%	0.182%	0.281%	0.400%	0.566%	0.688%	0.867%
Ext.	0.063%**	0.084%	0.079%*	0.069%*	0.058%**	0.051%**	0.057%**	0.069%**	0.069%**	0.069%**
	Chung and Shih (2009) - 3 time steps									
Std.	0.067%	0.086%	0.091%	0.115%	0.112%	0.173%	0.243%	0.341%	0.412%	0.517%
Ext.	0.040%	0.049%	0.046%**	0.040%*	0.035%**	0.036%**	0.038%**	0.044%**	0.056%**	0.062%**
	Ju and Zhong (1999)									
Std.	0.051%	0.108%	0.141%	0.167%	0.180%	0.205%	0.234%	0.260%	0.290%	0.305%
Ext.	0.044%	0.076%	0.096%*	0.105%**	0.105%*	0.106%**	0.112%**	0.108%**	0.108%**	0.108%**
	Ibanez (2003) - 4 time steps									
Std.	0.262%	0.609%	0.702%	0.880%	0.974%	1.141%	1.254%	1.357%	1.382%	1.561%
Ext.	0.211%	0.322%**	0.443%**	0.558%**	0.610%**	0.700%**	0.782%**	0.832%**	0.910%**	0.960%**
	Ibanez (2003) - Richardson extrapolation with $ns = 4$									
Std.	0.236%	0.549%	0.629%	0.787%	0.870%	1.015%	1.113%	1.203%	1.219%	1.377%
Ext.	0.050%	0.076%**	0.101%**	0.119%**	0.127%**	0.138%**	0.148%**	0.151%**	0.158%**	0.167%**
	Ibanez (2003) - Richardson extrapolation with $ns = 8$									
Std.	0.126%	0.290%	0.325%	0.402%	0.440%	0.505%	0.550%	0.587%	0.584%	0.652%
Ext.	0.048%	0.081%**	0.110%**	0.130%**	0.140%**	0.155%**	0.169%**	0.176%**	0.188%**	0.199%**
	Ibanez (2003) - Richardson extrapolation with $ns = 22$									
Std.	0.053%	0.111%	0.122%	0.150%	0.163%	0.185%	0.203%	0.216%	0.212%	0.233%
Ext.	0.033%	0.047%	0.061%**	0.074%**	0.081%**	0.089%**	0.097%**	0.100%**	0.108%**	0.115%**

Note: This table presents the Mean Absolute Percentage Error (MAPE) for the best seven quasi-analytic methods (indicated as 'Std' for standard) previously reported, see Figures 3 and 4 in Broadie and Detemple (1996, p. 1227), Tables 2a-2e and 3a-3e in AitSahlia and Carr (1997, pp. 76-85) (the results are for options on a non-dividend paying asset for short maturities and on a dividend-paying asset for long maturities), Exhibits 3 and 5 in Ju and Zhong (1999), Tables 3, 4 and 5 in Li (2010b, pp. 91-93), Figures 4. and 5. in Kallast and Kivinukk (2003, pp. 373-374) and Tables 4 and 5 in Kim et al. (2013, p. 7). are compared with our extended versions (indicated as 'Ext'), i.e. when the extension method in Proposition 3.1 is applied to them. Ten ranges of maturity (in years) are considered: (0; 0.5] (*A*), (0.5; 1] (*B*), (1; 1.5] (*C*), (1.5; 2] (*D*), (2; 2.5] (*E*), (2.5; 3] (*F*), (3; 3.5] (*G*), (3.5; 4] (*H*), (4; 4.5] (*I*), (4.5; 5] (*J*). The results are based on the scenarios in Section 4. - The entries for the extension method are calculated for the ratio  $t_x/T$  corresponding to the minimum of the solid lines in Figure 2. The symbols \* and \*\* indicate the results of the modified Diebold-Mariano test and they show that the values in the pair (Std,Ext) are significantly different at the 95% confidence level, and 99% confidence level, respectively. The benchmark prices are obtained by the binomial tree model with 15000 steps.

**Table 2:** Scenarios-based hedging performances for different maturities

	In-the-money options					At-the-money options					Out-the-money options				
	1 (year)	2	3	4	5	1	2	3	4	5	1	2	3	4	5
	Geske and Johnson (1984) - 2 time steps														
Std.	1.013	1.169	1.207	1.246	1.187	0.943	1.084	1.135	1.211	1.215	0.900	1.004	1.098	1.197	1.233
Ext.	0.676**	0.748**	0.820**	0.963**	1.074**	0.629**	0.685**	0.791**	0.954**	1.079**	0.603**	0.636**	0.756**	0.930**	1.057**
	Geske and Johnson (1984) - 3 time steps														
Std.	1.007	1.182	1.209	1.266	1.256	0.939	1.086	1.156	1.252	1.285	0.914	1.020	1.109	1.222	1.280
Ext.	0.676**	0.747**	0.819**	0.962**	1.073**	0.629**	0.684**	0.790**	0.952**	1.077**	0.603**	0.635**	0.755**	0.928**	1.055**
	Barone-Adesi and Whaley (1987)														
Std.	0.995	1.155	1.239	1.350	1.456	0.977	1.101	1.211	1.351	1.450	0.938	1.065	1.197	1.361	1.437
Ext.	0.632**	0.652**	0.701**	0.846**	0.976**	0.586**	0.584**	0.672**	0.837**	0.964**	0.575**	0.563**	0.651**	0.827**	0.942**
	Li (2010b)														
Std.	0.997	1.147	1.237	1.346	1.444	0.976	1.101	1.212	1.348	1.442	0.939	1.046	1.193	1.346	1.426
Ext.	0.634**	0.654**	0.704**	0.848**	0.971**	0.586**	0.591**	0.673**	0.838**	0.964**	0.575**	0.564**	0.650**	0.827**	0.943**
	Kim (1990) - 2 time steps														
Std.	1.000	1.169	1.177	1.220	1.194	0.941	1.079	1.128	1.209	1.225	0.908	1.003	1.080	1.184	1.226
Ext.	0.632**	0.653**	0.705**	0.853**	0.985**	0.586**	0.585**	0.675**	0.843**	0.971**	0.575**	0.565**	0.655**	0.832**	0.949**
	Kim (1990) - 3 time steps														
Std.	0.998	1.166	1.185	1.245	1.265	0.956	1.057	1.140	1.243	1.288	0.931	1.023	1.110	1.224	1.284
Ext.	0.623**	0.596**	0.617**	0.731**	0.841**	0.592**	0.548**	0.580**	0.718**	0.821**	0.575**	0.517**	0.568**	0.718**	0.798**
	Chung and Shih (2009) - 2 time steps														
Std.	1.009	1.156	1.245	1.379	1.497	0.990	1.099	1.254	1.392	1.493	0.940	1.073	1.220	1.397	1.467
Ext.	0.622**	0.638**	0.707**	0.849**	0.989**	0.593**	0.584**	0.662**	0.835**	0.969**	0.575**	0.556**	0.650**	0.832**	0.943**
	Chung and Shih (2009) - 3 time steps														
Std.	1.012	1.161	1.246	1.369	1.490	0.988	1.098	1.243	1.384	1.484	0.940	1.062	1.210	1.395	1.462
Ext.	0.623**	0.640**	0.707**	0.849**	0.988**	0.593**	0.586**	0.663**	0.834**	0.969**	0.576**	0.558**	0.651**	0.832**	0.944**
	Ju and Zhong (1999)														
Std.	0.996	1.157	1.242	1.359	1.460	0.977	1.102	1.214	1.354	1.454	0.939	1.066	1.199	1.362	1.438
Ext.	0.623**	0.596**	0.617**	0.730**	0.839**	0.592**	0.548**	0.579**	0.717**	0.820**	0.575**	0.517**	0.568**	0.717**	0.797**
	Ibanez (2003) - 8 time steps														
Std.	1.014	1.195	1.260	1.350	1.421	0.968	1.105	1.208	1.329	1.422	0.953	1.051	1.192	1.331	1.423
Ext.	0.630**	0.603**	0.613**	0.729**	0.837**	0.590**	0.546**	0.582**	0.719**	0.824**	0.569**	0.521**	0.569**	0.715**	0.804**

Note: - This table presents the average quadratic hedging error for seven quasi-analytic methods (indicated as 'Std' for standard) and our extended versions (indicated as 'Ext'). The results are based on three sets of 1,000 simulated paths of the underlying asset prices. The parameters are  $r = 0.05$ ,  $\delta = 0.04$ ,  $K = 100$ ,  $\sigma = 0.4$  and  $S_0 = \{90, 100, 110\}$ . The analysis is based on monthly hedging rolling frequency. The results are presented for five different time-to-maturities from one to five years. The symbols \* and \*\* indicate the results of the modified Diebold-Mariano test and they show that the values in the pair (Std,Ext) are significantly different at the 95% confidence level, and 99% confidence level, respectively. The benchmark prices are obtained by the binomial tree model with 15000 steps.

**Table 3:** Pricing performances of quasi/analytic standard methods and our extended version: Stochastic volatility with jumps model

	$S_0$	Scenarios																				MAPE	RMSE
		Test-1					Test-2					Test-3					Test-4						
		80	90	100	110	120	80	90	100	110	120	80	90	100	110	120	80	90	100	110	120		
	Exact	1.4843	3.7145	7.7027	13.6722	21.3653	1.2359	3.3532	7.5970	13.8830	21.7186	0.3285	2.1094	6.7116	13.7493	22.1433	21.3053	15.6365	11.5887	8.6680	6.5464		
Geske and Johnson (1984) 2 time steps	Std.	1.4737	3.6832	7.6341	13.5544	21.1654	1.2265	3.3247	7.5297	13.7658	21.5286	0.3255	2.0885	6.6475	13.6300	21.9616	20.0000	14.2843	10.7048	8.0413	6.0808	0.0249%	0.0377%
	Ext.	1.4776	3.6976	7.6652	13.5869	21.1246	1.2297	3.3381	7.5609	13.7990	21.5021	0.3262	2.0982	6.6777	13.6627	21.9480	20.9597	15.3773	11.3671	8.4745	6.3789	0.0097%**	0.0116%*
Geske and Johnson (1984) 3 time steps	Std.	1.4758	3.6913	7.6532	13.5917	21.2430	1.2282	3.3322	7.5487	13.8026	21.6002	0.3258	2.0939	6.6659	13.6670	22.0279	20.0000	14.7945	11.0090	8.2335	6.2112	0.018%	0.0273%
	Ext.	1.4785	3.7004	7.6740	13.6212	21.2439	1.2305	3.3406	7.5695	13.8326	21.6090	0.3264	2.1000	6.6860	13.6968	22.0436	21.0849	15.4321	11.3919	8.4879	6.3888	0.0075%**	0.0097%*
Barone-Adesi and Whaley (1987)	Std.	1.4952	3.7255	7.7025	13.6467	21.3042	1.2458	3.3650	7.5977	13.8585	21.6603	0.3349	2.1207	6.7124	13.7224	22.0882	21.2311	15.6558	11.7004	8.8394	6.7479	0.0063%	0.0101%
	Ext.	1.4871	3.7140	7.6919	13.6471	21.3063	1.2384	3.3539	7.5873	13.8581	21.6650	0.3304	2.1114	6.7030	13.7225	22.0943	21.2304	15.6337	11.6221	8.7243	6.6164	0.0026%*	0.0036%*
Li (2010b)	Std.	1.4905	3.7204	7.6971	13.6415	21.2995	1.2411	3.3598	7.5923	13.8532	21.6555	0.3304	2.1152	6.7069	13.7171	22.0835	21.0936	15.4723	11.5011	8.6370	6.5487	0.0035%	0.0045%
	Ext.	1.4861	3.7135	7.6902	13.6440	21.3172	1.2373	3.3532	7.5855	13.8550	21.6723	0.3291	2.1102	6.7009	13.7194	22.0986	21.1144	15.5233	11.5143	8.6218	6.5202	0.0027%*	0.0036%*
Kim (1990) 2 time steps	Std.	1.4747	3.6904	7.6629	13.6249	21.2769	1.2272	3.3313	7.5585	13.8351	21.6376	0.3256	2.0931	6.6768	13.7000	22.0675	20.4384	15.5202	11.6354	8.7054	6.5383	0.0069%	0.0105%
	Ext.	1.4777	3.6996	7.6744	13.6285	21.2915	1.2298	3.3400	7.5699	13.8392	21.6482	0.3260	2.0998	6.6870	13.7038	22.0753	21.1868	15.5333	11.4951	8.5870	6.4769	0.005%	0.0055%
Kim (1990) 3 time steps	Std.	1.4778	3.7000	7.6800	13.6549	21.3536	1.2299	3.3404	7.5755	13.8647	21.7066	0.3263	2.1000	6.6929	13.7299	22.1298	21.2386	15.9015	11.8050	8.7874	6.5930	0.0051%	0.0073%
	Ext.	1.4801	3.7062	7.6893	13.6526	21.3139	1.2319	3.3463	7.5847	13.8635	21.6731	0.3267	2.1044	6.7011	13.7280	22.1026	21.2334	15.6297	11.5995	8.6798	6.5525	0.0021%*	0.0023%*
Chung and Shih (2009) 2 time steps	Std.	1.4788	3.7016	7.6812	13.6516	21.3562	1.2308	3.3419	7.5767	13.8618	21.7064	0.3266	2.1012	6.6934	13.7267	22.1281	21.4926	15.7831	11.6701	8.7043	6.5557	0.0035%	0.0043%
	Ext.	1.4804	3.7068	7.6914	13.6624	21.3512	1.2321	3.3468	7.5868	13.8731	21.7056	0.3268	2.1046	6.7028	13.7377	22.1311	21.3080	15.6484	11.5965	8.6696	6.5419	0.0014%**	0.0018%*
Chung and Shih (2009) 3 time steps	Std.	1.4806	3.7067	7.6912	13.6635	21.3618	1.2323	3.3467	7.5866	13.8740	21.7138	0.3271	2.1046	6.7027	13.7387	22.1373	21.4361	15.7298	11.6369	8.6881	6.5502	0.0021%	0.0027%
	Ext.	1.4818	3.7101	7.6964	13.6671	21.3522	1.2333	3.3499	7.5917	13.8779	21.7075	0.3272	2.1070	6.7075	13.7424	22.1337	21.3010	15.6434	11.5937	8.6689	6.5430	0.0009%**	0.0012%*
Ju and Zhon (1999)	Std.	1.4844	3.7096	7.6874	13.6502	21.3444	1.2360	3.3497	7.5827	13.8607	21.6957	0.3294	2.1079	6.6991	13.7255	22.1183	21.2772	15.5967	11.5563	8.6523	6.5470	0.0014%	0.0016%
	Ext.	1.4834	3.7093	7.6904	13.6569	21.3481	1.2349	3.3494	7.5857	13.8674	21.7010	0.3287	2.1074	6.7019	13.7322	22.1251	21.2761	15.6090	11.5651	8.6535	6.5415	0.0012%*	0.0012%*
Ibanez (2003) Richardson ext. $ns = 4$	Std.	1.4781	3.6989	7.6715	13.6246	21.3005	1.2302	3.3394	7.5669	13.8353	21.6541	0.3264	2.0991	6.6835	13.6999	22.0789	20.7927	15.3657	11.3638	8.4737	6.3830	0.0084%	0.0115%
	Ext.	1.4827	3.7108	7.6959	13.6650	21.3478	1.2342	3.3506	7.5912	13.8758	21.7039	0.3278	2.1076	6.7068	13.7403	22.1308	21.2385	15.5900	11.5456	8.6281	6.5101	0.0017%**	0.0022%*
Ibanez (2003) Richardson ext. $ns = 8$	Std.	1.4808	3.7060	7.6859	13.6479	21.3311	1.2325	3.3460	7.5813	13.8587	21.6845	0.3272	2.1041	6.6973	13.7232	22.1094	21.1185	15.5042	11.4667	8.5632	6.4583	0.0043%	0.0058%
	Ext.	1.4827	3.7107	7.6951	13.6624	21.3479	1.2342	3.3505	7.5904	13.8732	21.7023	0.3278	2.1076	6.7061	13.7377	22.1279	21.2483	15.5848	11.5384	8.6224	6.5063	0.0018%**	0.0024%*
Ibanez (2003) Richardson ext. $ns = 22$	Std.	1.4830	3.7112	7.6960	13.6636	21.3522	1.2344	3.3510	7.5913	13.8745	21.7057	0.3279	2.1079	6.7069	13.7390	22.1307	21.2550	15.5851	11.5411	8.6265	6.5105	0.0016%	0.0022%*
	Ext.	1.4835	3.7125	7.6984	13.6673	21.3550	1.2349	3.3522	7.5937	13.8782	21.7092	0.3280	2.1089	6.7092	13.7427	22.1347	21.2704	15.6079	11.5606	8.6423	6.5233	0.001%*	0.0014%*

Note: This table presents the prices of the 20 option scenarios in Ballestra and Sgarra (2010) (Tables 1-4), under a stochastic volatility with jumps model for seven quasi-analytic methods (indicated as 'Std' for standard) and our extended versions (indicated as 'Ext'). The options scenarios are summarized in Section 4.3. The symbols \* and \*\* indicate the results of the modified Diebold-Mariano test for the MAPE and RMSE measures only and they show that the values in the pair (Std,Ext) are significantly different at the 95% confidence level, and 99% confidence level, respectively. The "benchmark" prices are those in Chiarella et al. (2009), Toivanen (2010) and Ballestra and Sgarra (2010) which were obtained by using finite difference approximations and Monte Carlo simulations on an extremely fine mesh.

**Table 4:** Summary statistics for the OEX<sup>®</sup>(American) LEAPS<sup>®</sup> on S&P 100<sup>TM</sup>

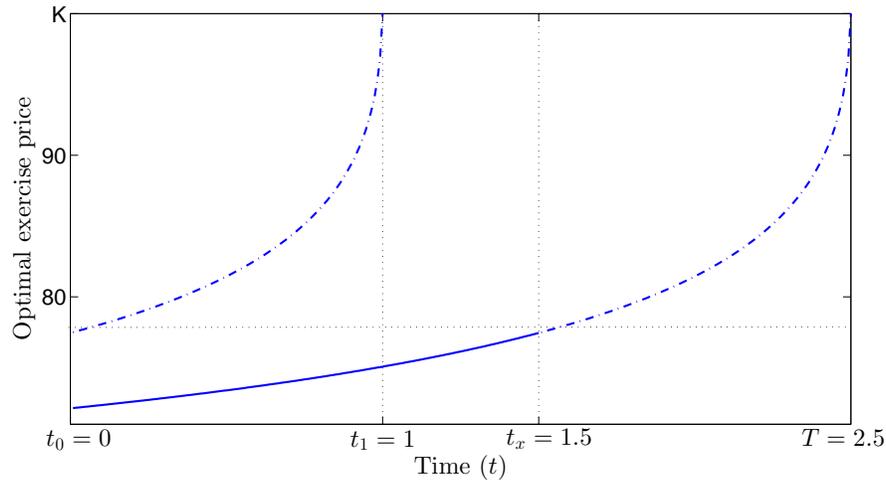
	From January 2008 to December 2011				From January 2012 to May 2015			
	Medium Maturities		Long Maturities		Medium Maturities		Long Maturities	
	Call	Put	Call	Put	Call	Put	Call	Put
Retained options	8,140	10,618	5,252	6,148	15,355	7,477	10,327	4,677
Average bid/ask spread	-7.258	-4.587	-7.410	-5.215	-3.324	-3.172	-3.587	-3.359
Average % bid/ask spread	-0.064	-0.248	-0.087	-0.209	-0.152	-0.135	-0.093	-0.116
Average mean price	148.811	35.859	98.265	43.605	79.600	38.584	84.447	41.942
Average time-to-maturity (years)	1.514	1.526	2.496	2.471	1.517	1.503	2.429	2.458
Subtotal (% of options)	11.97%	15.62%	7.72%	9.04%	22.58%	11.00%	15.19%	6.88%

**Table 5:** Empirical pricing performances on S&P 100<sup>TM</sup> LEAPS<sup>®</sup>

	From January 2008 to December 2011						From January 2012 to May 2015					
	MRE	RMSVE	FIBA (%)	MOE	RMSOE	MROE	MRE	RMSVE	FIBA (%)	MOE	RMSOE	MROE
	Geske and Johnson (1984) - 2 time steps											
Std.	-0.743	3.410	70.6%	-1.402	4.813	-0.025	-0.551	1.988	93.6%	-1.154	5.627	-0.022
Ext.	-0.274**	3.167*	77.6%**	-0.689**	4.541**	-0.013**	0.017**	1.940*	92.1%**	-0.109**	5.579**	-0.010**
	Geske and Johnson (1984) - 3 time steps											
Std.	-0.431	3.230	74.9%	-0.958	4.612	-0.018	-0.180	1.861	93.3%	-0.465	5.534	-0.015
Ext.	-0.212**	3.161**	78.3%**	-0.577**	4.531**	-0.011*	0.082**	1.969*	91.8%**	0.011**	5.595*	-0.009**
	Barone-Adesi and Whaley (1987)											
Std.	0.573	3.128	84.2%	0.768	4.459	0.017	0.678	2.261	88.0%	1.091	5.772	0.006
Ext.	0.143**	3.107**	82.7%**	0.006**	4.453	0.001**	0.408**	2.102*	89.8%**	0.601**	5.671*	-0.002**
	Li (2010b)											
Std.	0.478	3.119	85.1%	0.593	4.447	0.011	0.563	2.169	89.1%	0.891	5.711	0.003
Ext.	0.082**	3.125**	82.1%**	-0.093**	4.475*	-0.002**	0.324**	2.058*	90.4%**	0.453**	5.643**	-0.003
	Kim (1990) - 2 time steps											
Std.	-0.375	3.314	76.2%	-0.821	4.686	-0.016	0.139	2.050	90.5%	0.127	5.636	-0.008
Ext.	-0.107**	3.102**	80.2%**	-0.446**	4.459*	-0.010*	0.236**	1.995*	90.8%**	0.292**	5.607*	-0.007
	Kim (1990) - 3 time steps											
Std.	0.015	3.100	82.4%	-0.246	4.441	-0.008	0.486	2.193	88.9%	0.727	5.728	-0.002
Ext.	0.060*	3.074*	83.2%**	-0.170**	4.415**	-0.006	0.419*	2.106**	89.5%**	0.617**	5.674*	-0.003
	Chung and Shih (2009) - 2 time steps											
Std.	0.189	3.054	85.8%	0.048	4.378	-0.005	0.500	2.168	89.0%	0.753	5.714	-0.002
Ext.	0.136	3.075	84.2%	-0.039	4.410	-0.004	0.460	2.138	89.4%	0.689*	5.693	-0.002
	Chung and Shih (2009) - 3 time steps											
Std.	0.216	3.059	85.9%	0.098	4.383	-0.004	0.531	2.188	88.9%	0.809	5.726	-0.001
Ext.	0.146**	3.077*	84.1%**	-0.018**	4.412**	-0.004	0.473**	2.148**	89.4%**	0.713**	5.699*	-0.002
	Ju and Zhong (1999)											
Std.	0.159	3.062	85.3%	0.008	4.394	-0.002	0.444	2.087	89.7%	0.671	5.663	-0.002
Ext.	0.117**	3.080*	83.9%**	-0.069	4.417**	-0.004	0.434	2.11	89.6%	0.646*	5.675	-0.002
	Ibanez (2003) - Richardson extrapolation with $ns = 22$											
Std.	0.103	3.071	84.5%	-0.065	4.404	-0.004	0.451	2.160	89.0%	0.666	5.709	-0.003
Ext.	0.067**	2.930	84.0%**	-0.123	4.287	-0.005	0.449	2.157	89.3%	0.665**	5.707*	-0.003
	Derman et.al. (1995)											
50 steps	-2.401	9.286	31.7%	-2.343	10.607	-0.097	-12.772	23.527	22.4%	-14.075	26.245	-0.360
100 steps	-1.948	9.387	31.8%	-1.763	10.683	-0.080	-10.900	22.659	23.2%	-12.006	25.335	-0.304
	Cox, Ross and Rubinstein (1979)											
2000 steps	0.231	3.066	85.6%	0.130	4.392	-0.002	0.552	2.197	88.9%	0.852	5.731	-0.001

Note: The six error measures in Linaras and Skiadopoulou (2005), MRE, RMSVE, FIBA (%), MOE, RMSOE and MROE, are calculated for all seven quasi-analytic methods (labelled 'Std.' for standard) and our extended versions (indicated as 'Ext.'). using the S&P 100<sup>TM</sup> LEAPS<sup>®</sup> prices between January 2012 and May 2015, for maturities above 500 trading days. The symbols \* and \*\* indicate the results of the modified Diebold-Mariano test and they show that the values in the pair (Std,Ext) are significantly different at the 95% confidence level, and 99% confidence level, respectively. The benchmark prices are obtained by the binomial tree model with 15000 steps. The implied tree method by Derman et al. (1995) with 50 and 100 time steps and the Cox et al. (1979) with 2000 time steps are also reported.

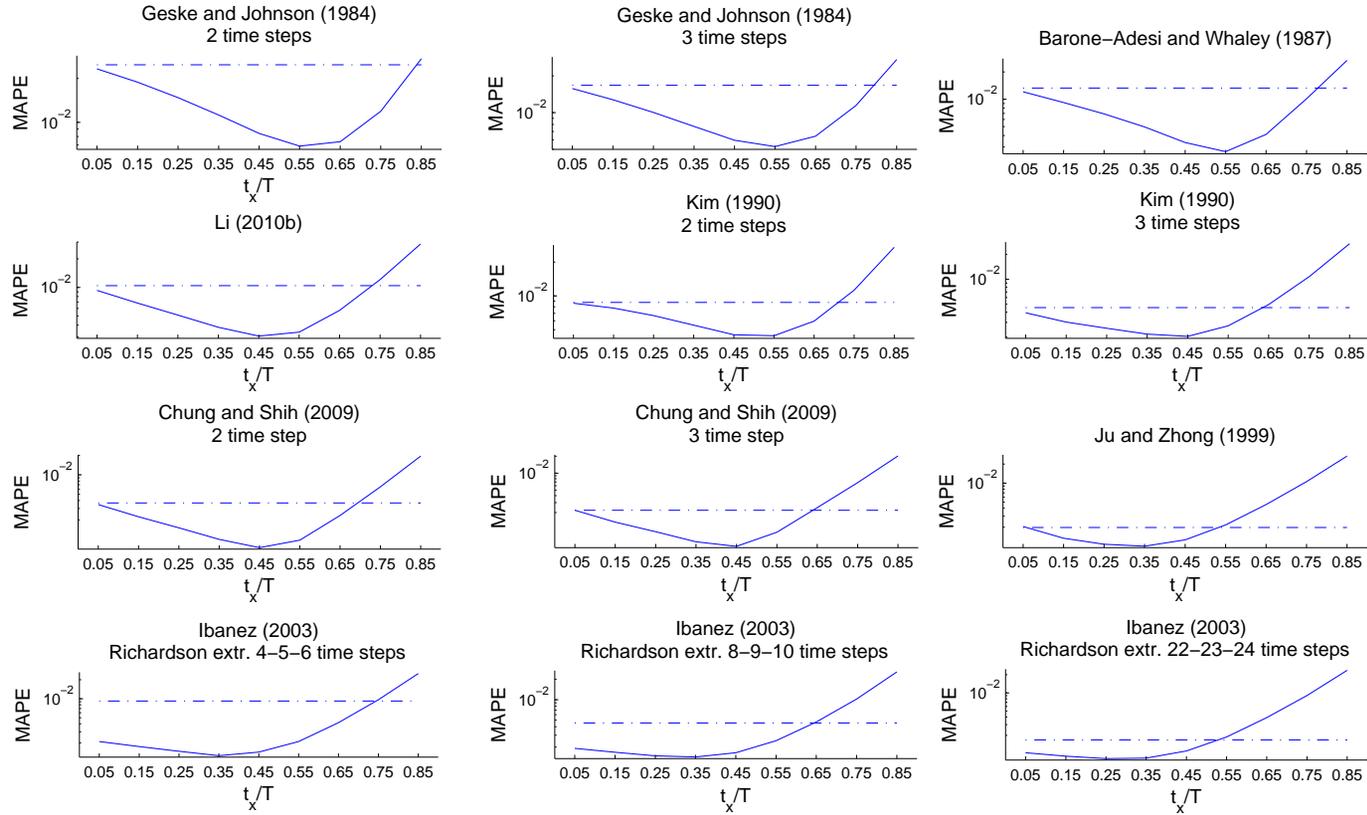
**Figure 1:** Example of extension method mechanism



Note: The optimal exercise prices of two American put options are considered in the figure. The two options are written on the same underlying asset with  $\sigma = 20\%$ ,  $\delta = 5\%$ ,  $r = 8\%$  and  $K = 100$ . One option has maturity  $t_1 = 1$  year and the other  $T = 2.5$  years. The continuous line represents the optimal exercise price of the option with maturity  $T$  and the dash-dot lines represent the optimal exercise of the option with maturity  $t_1$ . In particular, the left-most dash-dot line is the ‘original’ function and the other is its translation over the continuous line to show they coincide in the interval  $[t_x, T]$  where  $t_x = t_0 + (T - t_1) = 1.5$  years represents the size of the translation. The OEPs are calculated by the integral method in Kim (1990).

**Figure 2:** Scenarios-based pricing performance of quasi-analytic standard methods and our extended versions: Mean Average Percentage Error as a function of the ratio  $t_x/T$

CC



Note: This figure shows the ranges of ratios  $t_x/T$  for which the extended version (solid lines) outperforms the standard version (dash-dot lines) for each method. The results are shown for all maturities ( $\leq 5$  years).