Accepted Manuscript

Combined social networks and data envelopment analysis for ranking

Clara Simon de Blas , Jose Simon Martin ,
Daniel Gomez Gonzalez

PII: S0377-2217(17)30938-4
DOI: 10.1016/j.ejor.2017.10.025
Reference: EOR 14749

To appear in: *European Journal of Operational Research*

Received date: 15 July 2016
Revised date: 6 October 2017
Accepted date: 13 October 2017

Please cite this article as: Clara Simon de Blas , Jose Simon Martin , Daniel Gomez Gonzalez , Combined social networks and data envelopment analysis for ranking, *European Journal of Operational Research* (2017), doi: 10.1016/j.ejor.2017.10.025

Highlights

- A network representation of decision making units based on data envelopment analysis is proposed.
- A novel decision making units ranking method is provided based on centrality measures.
- A detection method of self evaluators is developed.
- A detection of non benchmark influence decision making units in data envelopment analysis.
- We present a discussion with other network based ranking methods.

Combined social networks and data envelopment analysis for ranking

Clara Simon de Blas[a,*], Jose Simon Martin[b] , Daniel Gomez Gonzalez[c]

[a]Departament of Computer's Sciences, Technical School of Computer Engineering, Rey Juan Carlos University, Madrid City, Spain
[b]University Institute of Health Care Evaluation. Universidad Complutense de Madrid. School of Medicine. Madrid 28040. Spain.
[c]Department of Statistics and Operations Research. School of Statistics Studies. Madrid 28040. Spain.
*Corresponding author. E-mail address: clara.simon@urjc.es

Abstract: In this work, we propose a method for ranking efficient decision-making units (DMUs) that uses measures of dominance derived from social network analysis in combination with data envelopment analysis (DEA). For this purpose, a directed and weighted graph is constructed, in which the nodes represent the system's DMUs and the edges represent the relationships between them. The objective is to identify and rank the most important nodes by taking into account the influence or dominance relations between the DMUs. The method uses a weighted HITS algorithm to identify the hubs and the authorities in the network by assigning to each DMU two numbers, the authority weight and the hub weight. Additionally, this method allows for the identification of DMUs whose exclusion from the DEA analysis does not modify the efficiency values obtained for the remaining DMUs.

Keywords: Data envelopment analysis, Social networks, Rankings, Benchmarking, PageRank.

1. Introduction

Data Envelopment Analysis (also known by its initials DEA) is a common method, used by in a large number of studies on benchmarking, for assessing the efficiency of organizations, introduced by Charnes, Cooper, and Rhodes (1978), DEA is based on a methodology that measures the relative efficiencies of a given group of organizations, classifying them as efficient and inefficient. The criterion used for this classification is determined by the location of each organization with respect to the efficient production frontier. This frontier is formed by the units that represent best management practices, in regards to outputs and the resources used in their production. Units identified as inefficient can be compared to one or more units located on the frontier, which can be considered as benchmarks for guiding improvement efforts (Bergendahl, 1998).

However, an important problem in the application of DEA for ranking is that usually all efficient units are given the same efficiency score of one (Cook & Seiford, 2009), there being, to date, no agreement over the best methodology for ordering or classifying efficient units by importance.

Some, standard measures used in DEA for ranking efficient and inefficient units are worth mentioning: counting the number of times a particular efficient company acts as a reference DMU (Zhu, 2000), calculating a super- efficiency measure (Andersen & Petersen, 1993; Bardhan et al 1996; Tone, 2002; Chen, 2005), calculating a peer index (Torgersen, Forsund, & Kittelsen, 1996, Zhu, 2000), weight restrictions (Dyson & Thanassoulis, 1988; Charnes et al., 1990; Thompson et al. 1986; Wong & Beasley, 1990), value efficiency analysis (Halme et al., 2000), cross-evaluation (Doyle & Green, 1994), multiple objective linear programming (Li & Reeves, 1999), multivariate statistical techniques (Friedman & Sinuany-Stern, 1997, Sinuany-Stern, Mehrez & Barboy, 1994), and conditional directional distance (Daraio, Bonaccorsi & Simar, 2015). For a good summary of such ranking techniques see: Angulo-Meza and Lins (2002); Adler, Friedman, and Sinuany-Stern (2002); Lu and Lo (2009); and Lotfi et al. (2013).

Since DEA is applied to DMUs groups with homogenous characteristics, it has been considered convenient to use Social Network Analysis (SNA) to identify the importance of each DMU within the network. Nevertheless, the use of SNA analysis in DEA ranking problems has been limited by the way in which the DEA network is modeled or the centrality measure considered. Liu, Lu, Yang, and Chuang (2009), use the eigenvector centrality measure proposed by Bonacich and Lloyd (2001) on a social network, that is built by adding the lambda values associated with the reference DMUs, obtained by applying DEA to all of the possible input and output combinations. Liu and Lu (2010) modified this approach to correct the convergence of the proposed algorithm and Leen and Chun (2015) use PageRank (Brin & Page, 1998), the algorithm behind Google's search engine, for determining the influence and rank of efficient DMUs.

In this paper, we propose a method for the ranking of DMUs that uses the bidimensional dominance measure proposed by Kleinberg (1999) and known as hubs (inefficient DMUs) and authorities (efficient DMUs), in combination with DEA. The network proposed in this paper to represent the DEA production model, is a directed dominance network, with no cycles between nodes. The relations between DMUs with respect to the efficiency frontier yields to a graph where nodes are non-strongly connected, so that all nodes have zero eigenvector centrality. Among standard eigenvector centrality alternatives, Katz centrality (Katz, 1953) and PageRank centrality are worth mentioning. Katz centrality assigns each node an initial amount of centrality and PageRank, introduces a stochastic adjustment that randomly allows connections between nodes to prevent a solution where all nodes have zero centrality. Since DMUs connections are strictly hierarchical, randomly created connections are implausible and there is no initial centrality to be assigned to each DMU in the network, both methodologies have been found inadequate.

The main objective of this work is to adapt the measure proposed by Kleinberg (1999) to establish a ranking between DMUs with respect to the efficiency frontier and to compare the results with the rankings obtained by means of previously suggested SNA measures, cross-efficiency, and super- efficiency.

The method proposed in this paper has the advantage that the ranking applies to both efficient and inefficient units, taking into account the quality of the DMUs in the network and can be implemented using either efficiency, or super-efficiency scores, regardless of the production model used (CCR or VRS). In addition, the suggested network has a direct interpretation for managers. The novelty of this method is that it can detect self-evaluators and identify the presence of inefficient DMUs whose exclusion from the analysis does not affect the efficiency scores of the remaining units.

This paper is organized as follows. In section 2, we briefly review the basic model of the DEA, centrality measures in social network analysis, and we describe our proposed ranking method. In section 3, we analyze the different ranking methods based on network and centrality measures. Illustrative examples follow in section 4. Finally, the paper concludes in section 5 with a discussion of the merits and limitations of the network-based approach.

2. Methods

2.1 The DEA model

To determine the best practice frontier, an input-oriented constant return scale (CRS) was assumed, and the model below was formulated (based on Charnes et al. (1978)).

The Primal DMU_k Model:

$$max \sum_{r=1}^{s} u_r y_{rk},$$

$$\begin{cases} \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0 \quad \forall \text{ j=1,...,n}, \\ \sum_{i=1}^{m} v_i x_{ij} = 1, \qquad\qquad (Eq. 1) \\ u_r, v_s \geq 0 \quad \forall \text{ r=1,...,s; i=1,..., m.} \end{cases}$$

The Dual Model for $DMU_{0:}$

$$min\theta_k,$$

$$\begin{cases} \theta_k x_{ik} - \sum_{j=1}^{n} \lambda_{kj} x_{ij} \geq 0 \quad \forall \text{ i=1,...,m}, \\ \sum_{j=1}^{n} \lambda_{kj} y_{rj} \geq y_{rk} \quad \forall \text{ r=1,...,s}, \qquad (Eq. 2) \\ \lambda_{kj} \geq 0 \quad \forall \text{ j=1,...,n}, \end{cases}$$

where $X=(x_{ij})_{i=1,\ldots,m;j=1,\ldots,n}$ is the set of inputs of the reference units and $v=(v_i)_{i=1,\ldots,n}$ gives the weights; $Y=(y_{rj})_{r=1,\ldots,s;j=1,\ldots,n}$ is the set of outputs of the reference units and $u=(u_i)_{i=1,\ldots,n}$ gives the weights in the primal problem; θ is a real variable; and $\lambda = (\lambda_{kj})_{k=1,\ldots,n;\, j=1,\ldots,n}$ is the set of weights of the reference units for each DMU in the dual problem.

From the solution to this problem and its dual, we obtain, for each reference DMU unit:

- the efficiency score of the reference unit Σ_k and,
- for each inefficient reference unit, the set of reference units (peer group: $\{j=1,\ldots,n/\varsigma_{kj} >0\}$) and the corresponding weights (lambda weights ς_{kj}).

For the case in which the lambda weights sum to one, the model is known as VRS (Banker et al., 1984).

As reference methods for ranking the DMUs we compare our results with the classical approach obtained by super-efficiency and cross-efficiency.

Super-efficiency

This method was proposed by Andersen and Petersen (1993). The core idea of this model is to exclude the DMU under evaluation from the reference set. This allows a DMU to be located on the efficient frontier, i.e. to be super-efficient. Therefore, the super-efficiency score for an efficient DMU can, in principle, take any value greater than or equal to one.

Chen (2004) proposed a method for ranking efficient units based on a modification of the calculation of super-efficiency for the VRS case, avoiding the infeasible cases, which captures input deficits and output excesses.

Cross-efficiency

Sexton, Silkman and Hogan (1986) developed a method where each unit is evaluated with the optimal efficiency weights of the other DMUs, to compute a cross-efficiency matrix. Once the matrix is obtained, each unit can be self-evaluated with their optimal efficiency weights and yields the evaluation acquired with the optimal efficiency weights of the remaining DMUs in the reference set (peer evaluation). For further details, see Doyle and Green (1994).

2.2 Ranking nodes in social network

Given a network, one of the most important problems in Social Network Analysis (SNA) is the identification of important key nodes. A common approach for ranking nodes in a network is to use centrality measures (Freeman, 1978). The idea is that the centrality measure of a node reflects the node's importance.

In this paper, we focus on centrality measures for dominance (or reference) networks where relationships between nodes are weighted and directed through a dominance relation. As we will see later with more details, the edge (i,j) is present in the network if node j is a reference for node i. In the sequel, we will refer only to influence measures for nodes in weighted and directed (dominance) networks.

As it is pointed in Borgatti (2005), the most popular and most used centrality-influence measures are those that are based on eigenvector calculations. Considering the eigenvector centrality defined by Bonacich and Lloyd (2001), the definition of the eigenvector centrality of a node is based on the idea that connections to important nodes contribute more to the centrality of node i than an equal number of connections of node i to low importance centrality nodes. The centrality of node i (x_i) is computed as the sum of the centrality values of its neighbors multiplied by a constant α. From a mathematical point of view we have

$$x_i = \alpha \sum_{j \in N_i} x_j, \qquad \text{(Eq. 3)}$$

where N_i is the set of nodes neighboring node i.

Node j is a neighbor of node i if there is a directed edge (j, i) connecting j and i (i.e. i is a reference for j). In an eigenvector expression,

$$\forall\, i = 1, \dots, n;\ x_i = \sum_{j=1}^{n} \alpha A_{ji} x_j\ , \qquad \text{(Eq. 4)}$$

where A is the adjacency matrix of the network.

The problem is reduced to the calculation of the eigenvector associated to the eigenvalue α. The power iteration algorithm, defined by Mises, Pollaczek-Geiringer and Praktische (1929), is probably the most well-known algorithm for calculating a dominant eigenvector. The procedure starts with a vector b_0 which may be an approximation to the dominant eigenvector or a random vector of a given matrix A. The method is described by the recurrence relation

$$b_{k+1} = \frac{Ab_k}{\|Ab_k\|} \quad \text{(Eq.5)}$$

If we assume A has an eigenvalue that is strictly greater in magnitude than its other eigenvalues and the starting vector b_0 has a nonzero component in the direction of an eigenvector associated with the dominant eigenvalue, then a subsequence b_k converges to an eigenvector associated with the dominant eigenvalue. It is important to know that the convergence of this algorithm depends strongly on the properties of matrix A and, in general, the power iteration method and eigenvector centrality method do not deal well with sparse and asymmetric adjacency matrices.

With the aim of remedying some of these deficiencies, but following a similar approach, other centrality measures have been defined, such as alpha centrality, PageRank, and hubs and authorities.

The hubs and authorities centrality measure is adequate for those networks in which there are two types of nodes, as is the case in DEA networks, where there are efficient and inefficient nodes and, in addition to the importance of the position that a node has in the network, the evaluation of its importance should be made in a separate way as is done in the HITS algorithm. For this reason we think that the hubs and authorities measure has the advantages of both the eigenvector and PageRank centrality measures.

2.3 The network-based approach for ranking efficient units

Once the DEA model has been solved, the efficient units are ranked using the following steps:

1. Build the DMU network.
2. Compute the authority and hub weights of the network by means of a modified weighted HITS algorithm (Kleinberg, 1999).

Step 1: The DMU network

To sort the efficient DMUs and visualize the relationships between the efficient and inefficient DMUs, a network representing the reference relationships between these DMUs is defined. The DMU's reference set contains the group of units that provides better results. Moreover, the reference groups comprise the sets of units that an inefficient unit should take as its target to achieve efficiency. Therefore, the relationships between the DMUs are directional and will be represented in the graph by directed edges (for example, if the DMU j is a reference for DMU i, then this will be expressed in the graph as i-> j).

To identify the efficient and inefficient DMUs, it is possible to build the network in the following way.

Let G = (V, E) be a graph, where the set V of nodes comprises the set of DMUs. In general, the outcome of a DEA model is a partition of V into two sets of nodes, one (V_E) containing the efficient units and the other (V_{NE}) containing the inefficient units. There are case studies where all DMUs in the reference set turn out to be efficient units and we have to resort to a super-efficiency model to distinguish the DMUs that can serves as benchmarks for management practices.

In the case a CCR and VRS DEA model is considered, we can define the DEA network as a bipartite digraph $V=(V_E U V_{NE}, E)$ in which there are no edges between nodes of the same group. Let E be the set of edges representing the reference relations between efficient and inefficient units. As relations represent which units are references for others and it is clear that (i, j) is not the same as (j, i), a directed network is necessary. The weight $\varsigma_{(i,\ j)}$ of the edge (i, j) in the graph E is the lambda weight obtained when solving the dual DEA problem. An edge (i, j) belongs to the set E if the DMU j is a reference for the DMU i, i.e., E = {(i, j) / $\varsigma_{(i,\ j)} > 0$ with $i \in V_{NE},\ j \in V_E$ }.

Note that, for a pair (i, j), the associated weight $\varsigma_{(i,\ j)}$ represents the amount by which the inefficient unit i should be improved to look similar to the unit j and so to achieve efficiency.

Therefore, an edge (i, j) with a large weight represents a node/unit j with significant influence on node/unit i. For example, an isolated DMU unit in the network will not be a reference for any other DMU; therefore, its importance in the network is very low. In contrast, a unit with many relationships and a high reference score will have a significant degree of influence on the network, and its importance will be greater.

Step 2: The identification of authorities and hubs

The objective of this stage is to provide a method for identifying the most important DMU units (now nodes in the network) in a DEA problem. To this end, we use a rank method based on centrality/power measures that takes into account the influence or dominance relationships defined in Step 1.

The hubs and authorities centrality measure seems adequate for DEA networks since these networks distinguish efficient and inefficient units. There exist two classes of nodes in the network and it is necessary to rank them in separate ways, and this can be achieved using the hubs and authorities centrality measure.

The work developed by Kleinberg (1999) aimed to determine the important or featured nodes in a network of web sites, as well as in a network of scientific citations of scholarly articles. It fits our objective of ranking efficient units. To relate concepts from the study of social networks with DEA, we must consider that the authorities in a network of DMUs constitute a small group of network members whose management practices in their processes serve as an example for the other members, and that a hub is a key member in the benchmark set of the network.

The idea on which the HITS algorithm (Kleinberg 1999) is based is the association of two values with each DMU, namely, the authority weight a_i and the hub weight h_i. We consider DMUs with higher a_i values to be authorities, while DMUs with higher h_i values are regarded as hubs. A good hub increases the authority weight of the DMUs that it references. A good authority increases the hub weights of the DMUs that point to it. The idea is to apply the two operations iteratively until the values of authority and hub weights are balanced. (See Figure 1)

<Figure 1 here>

The authority power of node i is calculated as the sum of the hub power of the nodes for which it is a reference. The hub power of node k will be the sum of the authority powers of the nodes in its reference set.

Formally, if we denote by **a** the n-dimensional vector associated with the authority powers, by **h** the n-dimensional vector associated with the hub powers, and by A_{ij} the weight that represents the amount by which the inefficient unit i should be improved so as to look similar to the unit j and to achieve efficiency, we have that

$$a_i = \alpha \sum_{j=1}^{n} A_{ji} h_j, \qquad h_i = \alpha \sum_{j=1}^{n} A_{ij} a_j. \qquad \text{(Eq.6)}$$

We can see that the authority value of node i is the sum of the hub value of the nodes that have node i as a reference. On the other hand, the hub value of node i is the sum of the authority values of the nodes in the reference set of node i.

The previous expression can be rewritten as

$$a = \alpha A^t h \qquad h = \alpha A a \qquad \text{(Eq.7)}$$

This is equivalent to

$$a = \alpha A^t A\, a,$$

$$h = \alpha A A^t h. \quad \text{(Eq. 8)}$$

Consequently, in order to find the authority and hub values we have to compute the eigenvector of A associated to its dominant eigenvalue.

The main difference in terms of the eigenvector calculation with the classical eigenvector centrality measure is that now we can work with the matrices A^tA and AA^t. These two matrices have good properties since they are symmetric and positive semidefinite.

For a VRS or CCR DEA model, the matrix A represents the degree to which inefficient units have to emulate efficient units to achieve better management performance. Taking into account that our aim is to consider all the information given by a solution of a DEA model we would like to incorporate also the information given by the vector Θ in the HITS algorithm (Kleinberg 1999). In order to incorporate the efficiency scores of the units in the calculation of the authority and hub values, equation (8) could be rewritten as

$$a_i = \alpha \sum_{j=1}^{n} A_{ji} h_j \theta_j, \qquad h_i = \alpha \sum_{j=1}^{n} A_{ij} a_j \theta_j. \qquad (Eq. 9)$$

This modification has the advantage of taking into account the efficiency score discriminating above the inefficient units, as better or worst hubs.

For super-efficient models, these approaches take into account that in this case the set of inefficient units may be empty, and every DMU may play the role of a reference for the others. In this case, other centrality measures may apply, but the modification of the HITS algorithm proposed above has the advantage that it considers the super-efficiency scores in the network.

3. Analysis of centrality measures used for DEA ranking.

In this section, we compare our proposed method with other network-based approaches for DEA ranking.

Other centrality or power measures can be used for the same purpose but they do not have the same good properties. For example, if we apply the degree centrality measure that counts how many times an efficient DMU occurs in the reference sets of inefficient DMUs, we can assign more importance to a DMU regardless of the weights that it has in the reference sets. If we only consider the sum of the DMU weights that are referenced by an efficient unit, we can assign more importance to a DMU regardless of the number of times that it is referenced by an inefficient unit. Also, observe that classical centrality measures, such as closeness, betweenness, and flow centrality, do not make sense for the networks considered here. The weighted in-degree centrality measure for digraphs could be used to rank the efficient DMUs (i.e the nodes of V_E). For a node y in V_E, the in-degree centrality is computed as

$$y_j = \sum_{i=1}^{n} A_{ij}, \text{ where A is the weighted adjacency matrix of the network. (Eq.10)}$$

Note that this definition ranks the efficient nodes of the DEA network and this ranking coincides with that used in Torgersen, Forsund & Kittelsen (1996) and Zhu (2000). Now we will analyze some spectral centrality measures for the DEA network-

It can be proved, that the only solution of the eigenvector centrality measure proposed by Bonacich and Lloyd (1972), computed for the weighted DEA network in the CCR and VRS models, is the zero vector since there is no "in-relation" (see Bonacich and Lloyd (2001)).

The measure of eigenvector centrality proposed by Bonacich and Lloyd (1972) and applied by Liu et al. (2009) and Liu and Lu (2010) is intended for communication networks, and propagation and influence type processes (see Borgatti (2005) and Gomez, Figueira, and Eusebio (2013) for more details), but is unable to represent the bipartite nature of a DEA classification of DMUs into efficient and inefficient units and has additional weaknesses, as eigenvector centrality has convergence problems. Given that the network built from the values obtained by the DEA is a special type of dominance network and not a communication network, the adjacency matrix associated with the network is sparse, and therefore the maximum eigenvalue generally is zero, with multiplicity greater than one. This means that there is no unique eigenvector associated with the largest eigenvalue of the adjacency matrix (Ker(A-ςI)), and the centrality measure proposed by Bonacich and Lloyd (1972) performs badly when applied to DMU ranking.

For this reason, Liu et al. (2009) saw the need to build a new network, by adding different DEA executions and considering various combinations of inputs and outputs, to determine the sets of edges and their weights. Although the resulting matrix is denser than the original, the process of working on multiple DEA specifications is analogous to a contest with multiple rounds where contest winners are judged by aggregating scores from multiple rounds. However, some of these combinations can be unrealistic in the management of the production processes or services, although, as pointed out by Cook, Tone and Zhu (2014), they can be used for operations benchmarking. For instance, we can consider an example from library management (Simon, Simon & Arias 2011), with three basic inputs: library personnel, library expenditures on information resources and library surface area; and three final outputs: circulation, interlibrary loans and documents downloads. The resulting combinations, using library surface area as an input and any subset of the three final outputs, can be worthless for the Library manager. A weighted multiple DEA specifications can solve this problem by, assigning low weights to unrealistic production models. Furthermore, as indicated by Serrano Cinca and Mar Molinero (2004), we must consider that listing all of the possible DEA models that can be derived from the possible inputs and outputs combinations does contains a great deal of information, but that it also is redundant, since some DEA models may be equivalent, and some may contain independent information. In addition, for cases in which the number of inputs and outputs used is large, the calculation time for the new network is significantly higher than for the original network. Despite this, comparing many input/output combinations is a good strategy when a DEA problem is intended for benchmarking as these comparisons can be useful as a reference of good benchmarking practices for certain input and output variables considered in the production model. In the example provided by Liu et al. (2009), the associated adjacency matrix corresponds to a connected graph with high density. In this case, a unique eigenvector is associated with the largest eigenvalue of the adjacency matrix. However, when the relations between the reference DMUs with respect to the efficient units in the original production model are represented in the network, the associated adjacency matrix comes from a graph that is, in the majority of cases, disconnected, asymmetric, and not very dense. This leads to small eigenvalues and multiple eigenvectors associated with the largest eigenvalue.

Additionally, the eigenvalue centrality measure does not always converge. There are situations in which the iteration process diverges or loops endlessly. However, as is noted by Liu et al. (2009), various methodologies exist for addressing this problem.

Remark. The Alpha centrality computed for the weighted DEA network in the CCR and VRS models ranks the nodes by the “in-degree” centrality. Furthermore, for any positive alpha value, this measure is proportional to the “in-degree” centrality measure and also ranks the efficient units as in Torgersen, Forsund and Kittelsen (1996) and Zhu (2000).

The eigenvector-like centrality measure proposed (PageRank, Brin & Page, 1998) by Leem and Chun (2015) for determining the influence and rank of efficient DMUs, also has some deficiencies. In the case where most DMUs are efficient and some are not references for any inefficient DMU, the PageRank centrality for all DMUs remains the same and thus does not provide information suitable for discriminating among them or ranking them (Leem & Chun, 2015). Consequently, the PageRank measure does not discriminate among the inefficient units, assigning them all the same centrality value.

Note that the aforementioned measures do not rank the inefficient nodes whereas, the hubs and authorities algorithm distinguishes between units that are authorities (efficient units in the VRS and CCR models) and units that contribute to the quality of authorities (inefficient units), ranking both classes of nodes.

4. Results

To compare the proposed methodology with existing methodologies for DMU rankings in DEA models, we selected two classical examples from the literature. The first example considered is a VRS input-oriented model proposed by Chen (2004). The second production model CRS is described in Serrano-Cinca, Fuertes Callén, and Mar Molinero (2005).
In both examples, super-efficiency values are calculated using a reference ranking method. Furthermore, in the example described by Serrano-Cinca, Fuertes Callén, and Mar Molinero (2005) cross-efficiency values are computed using for the DMU ranking the score obtained by each unit when using the optimal efficiency weights of the remaining units (peer evaluation).

4.1 Example 1

Chen (2004) gave an example in which 15 cities in the USA are compared with 3 inputs and 3 outputs: high-end housing price (I_1), lower-end housing monthly rental (I_2), number of violent crimes (I_3), median household income (O_1), number of population with bachelor's degrees (O_2), and number of individuals with doctoral degrees (O_3). The data and the results of the VRS model are shown in Table 1.

<Table 1 here>

The graph associated with this example is shown in Figure 2. It is composed of 15 vertices. Seattle, Denver, Philadelphia, Minneapolis, Raleigh, St. Louis, Washington, Pittsburgh, Boston, and Milwaukee represent efficient units. The determination of the graph's edges is given by the reference relations of the efficient and inefficient units, as described above.

<Figure 2 here>

Columns 2, 3 and 4 of Table 2 show the authorities and hubs weight and partition obtained by applying the proposed weighted HITS algorithm (equation 9). The column DMUs Network reports the ranking of DMUs obtained by the proposed approach in which first the authority DMUs are ranked by decreasing authority weight and then the hub DMUs are ranked by decreasing hub weight. Column 6 adds the ranking obtained applying the proposed weighted HITS algorithm to the network proposed by Liu and Lu (2010); column 7 shows the ranking obtained applying the centrality measure proposed by Liu and Lu (2010); columns 8 and 9 exhibit the rankings obtained applying the algorithms proposed by Chen (2004) to compute the super-efficiency scores in a VRS model; and, finally, column 10 presents the ranking obtained using the PageRank algorithm (equation 7) proposed by Leem and Chun (2015).

<Table 2 here>

As shown in Table 2, the DMUs corresponding to Seattle, Denver, Philadelphia, St. Louis, Washington, and Pittsburgh are good authorities, while the DMUs corresponding to Boston, Minneapolis, Raleigh, and Milwaukee are not, because the latter are not found in any of the reference sets of inefficient units (see Table 1) .

It should be noted that, according to our approach, Denver and Pittsburgh occupy the second and third positions in the ranking, respectively, which are greater than the positions given them by the method proposed by Chen (2004), given that the inefficient DMUs of their reference sets represent "good" hubs and provide a good example of management for the others. PageRank fails to detect the link strength and authority position and ranks Denver above St. Louis. The number of link connections prevails over the dominance structure underlying the DEA model.

Even though Minneapolis, Raleigh, Boston, and Milwaukee are efficient DMUs, they are not in the reference set of any inefficient DMUs and therefore should not occupy top positions in the ranking of DMUs with good management practices. PageRank also identifies these DMUs, as these units lack connections with the remaining units of the network.

The differences with the network proposed by Liu and Lu (2010) when applying the Hits algorithm to the network proposed by Liu and Lu (2010) are related to the importance of DMUs in the multiple context case. DMUs such as Philadelphia acquire a higher importance in cases in which some inputs/outputs of the production model are omitted. The same can be observed when considering the eigenvector associated to the largest eigenvalue of the adjacency matrix of the graph proposed by Liu and Lu (2010; whereas Boston is a self-evaluator in the original DEA network, it acquires higher importance in the multiple context case.

When the DMUs that have not been identified either as a hub or an authority are removed from the network (Boston, Minneapolis, Raleigh, and Milwaukee), the efficiency scores assigned by the DEA model remain unchanged because these DMUs do not influence the rest of the DMUs (see Table 3).

<Table 3 here>

4.2 Example 2

In this example, 40 internet companies are compared with respect to 3 inputs and 2 outputs: employees (I_1), expenses (I_2), assets (I_3), visitors (O_1), and revenues (O_2).

Table 4 shows the data and the results obtained applying the CCR and super-efficiency models.

<Table 4 here>

The graph associated with this example is shown in Figure 3. It is composed of 40 vertices, where the vertices ASKJ, BOUT, BUYX, LFMN, MQST, ONHN, SWBD, TVLY, UBID, and UPRO represent efficient units. The determination of the graph's edges is given by the reference relationships of the efficient and inefficient units, as described above.

<Figure 3 here>

Columns 2, 3 and 4 of Table 5 show the authorities and hubs weight and partition obtained by applying the proposed weighted HITS algorithm (equation 9). The column DMUs Network reports the ranking of DMUs obtained by the proposed approach in which first the authority DMUs are ranked by decreasing authority weight and then the hub DMUs are ranked by decreasing hub weight. Column 6 adds the ranking obtained applying the cross-efficiency scores method; column 7 shows the ranking obtained applying the proposed weighted HITS algorithm to the network proposed by Liu and Lu (2010); column 8 exhibits the ranking obtained applying the centrality measure proposed by Liu and Lu (2010); and, finally, column 9 presents the ranking obtained by the PageRank algorithm (equation 7) proposed by Leem and Chun, (2015).

<Table 5 here>

The DMUs UBID, BUYX, and ASKJ occupy the first positions in the ranking obtained by the method proposed in this work, given that the inefficient DMUs of their reference sets represent "good" hubs and provide good examples of management for the others. PageRank fails to detect the link strength and authority position by assigning higher ranks to efficient units such as UPRO than to TVLY and LFMN. The number of link connections prevails over the dominance structure underlying the DEA model.

The differences with the network proposed by Liu and Lu (2010) when applying Hits algorithm to the network proposed by Liu and Lu (2010) are related to the importance of DMUs in the multiple context case. DMUs such as AHWYQ and SWBD acquire a higher importance in the case in which some inputs/outputs of the production model are omitted.

It is possible to have a case where an inefficient DMU A has a higher efficiency score than another DMU B and the proposed method ranks B higher than the A. For instance, Amazon (AMZN) has an efficiency score of 0.39 and Egghead (EGGS) a score of 0.88. Using the proposed method, AMZN is ranked 11th and EGGS 15th, due to the quality of the peers in their reference sets. UBID, which is in the AMZN reference set, had reported revenue of $204,295 in 1999, placing it in the top 15 percent of firms, whereas TVLY, in the EGGS reference set, had reported revenue of $64,187 in 1999 and only made it into the top 30 percent.

After removing the DMUs that have not been identified as hubs or authorities, the efficiency scores obtained by the DEA model are unaltered, given that these DMUs do not influence the management of the other DMUs (see Table 6).

<Table 6 here>

5. Conclusions

The methodology proposed in this paper for ranking efficient and inefficient DMUs based on dominance measures derived from social network analysis has several advantages. First, the network that is built represents graphically the relationships between an inefficient DMU and its reference set. This simplifies the interpretation of the role played by each DMU within the set. Furthermore, allows managers to visualize the network relations between DMUs.

Second, the modified bidimensional hubs and authorities measure provides a basis for ranking efficient and inefficient units. The values of hubs and authorities reflect the following desirable properties for ranking methods:

- Property 1. Efficient DMU's are always ranked above inefficient units. *(Since the set of authorities coincides with the set of efficient units)*
- Property 2. The ranking of an efficient unit is higher when an inefficient unit that contains the efficient DMU in its reference set represents a good benchmark (*the ranking of an efficiency unit is larger when the inefficient units that contain the efficient DMU in their reference sets represent "good" hubs, i.e., their reference sets contains efficient DMUs that are also good examples of management practices for the other DMUs*).
- Property 3. An efficient DMU that is not a reference for any inefficient DMU must be ranked after the remaining efficient DMUs. In this situation, as noted by Charnes et al. (1978), the efficient unit does not act as a benchmark for any inefficient DMU except itself, which is the case of a self-evaluator. *(The efficient DMUs that are not references for any inefficient DMU have an authority weight equal to zero.)*
- Property 4. The ranking method can be used independently of the DEA model assumption (CRS, VRS, super-efficiency, etc.)

Third, this methodology allows us to identify the presence of inefficient DMUs that are not classified as hubs. It is worth noting that this method is robust with respect to excluding such DMUs from the DEA analysis, because the efficiency scores obtained for the other DMUs are not modified.

In the case in which multiple DEA specifications can be considered as productions models without considering all input/output variables, the application of the HITS algorithm enriches the information of the contribution of each DMU. However, we suggest as a future line of research considering a weighted multiple DEA specifications network, assigning low weights to unrealistic production models.

Acknowledgements

The authors would like to thank anonymous referees for their constructive comments that have much improved the readability of this article.

References

Adler, N., Friedman, L., & Sinuany-Stern, Z. (2002). Review of ranking methods in the data envelopment analysis context. *European Journal of Operational Research 140*, 249–265

Andersen, P., & Petersen, N. C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science 39*, 1261–1264.

Angulo-Meza, L., & Lins, MPE. (2002). Review of methods for increasing discrimination in data envelopment analysis. *Annals of Operations Research 116*, 225–42.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Models for estimating technical and scale efficiencies. *Management Science 30*, 1078–1092.

Bardhan, I., Bowlin, W. F., Cooper, W. W., & Sueyoshi, T. (1996). Models for efficiency dominance in data envelopment analysis. Part I: Additive models and MED measures. *Journal of the Operations Research Society of Japan 39*, 322–332.

Bergendahl, G. (1998). DEA and benchmarks - an application to Nordic banks. *Annals of Operations Research* 82, 233-249.

Bonacich, P., & Lloyd, P. (2001). Eigenvector-like measures of centrality for asymmetric relations. *Social Networks 23*, 191–201.

Borgatti, S. P. (2005). Centrality and network flow. *Social Networks 27*, 55–71.

Brin, S., & Page, L. (1998). The anatomy of a largescale hypertextual web search engine. *Computer Networks and ISDN Systems 33*, 107-117.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research 2*, 429–444.

Charnes, A., Cooper, W.W., Huang, Z. M. & Sun, D.B. (1990). Polyhedral cone-ratio DEA models with an illustrative application to large commercial banks. *Journal of Econometrics 46*, 73–91.

Chen, Y. (2004). Ranking efficient units in DEA. *Omega 32*, 213–219.

Chen, Y. (2005). Measuring super-efficiency in DEA in the presence of infeasibility. *European Journal of Operational Research 161*, 545–551.

Cook, W. D, & Seiford, L.M. (2009). Data envelopment analysis (DEA)—thirty years on. *European Journal of Operational Research 192*, 1–17.

Cook, W. D., Tone, K., & Zhu, J. (2014). Data envelopment analysis: Prior to choosing a model. *Omega 44*, 1-4.

Daraio, C., Bonaccorsi, A., & Simar, L. (2015). Rankings and university performance: A conditional multidimensional approach. *European Journal of Operational Research 244*, 918–930.

Doyle, J. R., & Green, R. H. (1994). Efficiency and cross-efficiency in DEA: Derivations, meanings and uses. *Journal of the Operational Research Society 45*, 567–578.

Dyson, R. G., & Thanassoulis, E. (1988). Reducing weight flexibility in data envelopment analysis. *Journal of the Operational Research Society 39*, 563–576.

Freeman, L.C. (1978). Centrality in Social Networks. Conceptual clarification. *Social Networks 1*, 215-239.

Friedman, L., & Sinuany-Stern, Z. (1997). Scaling units via the canonical correlation analysis and the data envelopment analysis. *European Journal of Operational Research 100*, 629–637.

Gomez, D., Figueira, J. R., & Eusebio, A. (2013). Modeling centrality measures in social network analysis using bi-criteria network flow optimization problems. *European Journal of Operational Research 226*, 354–365.

Halme, M., Joro, T., Korhonen, P., Salo, S., & Wallenius, J. (2000). Value efficiency analysis for incorporating preference information in DEA. *Management Science 45*, 103–115.

Katz, L. (1953). A New Status Index Derived from Sociometric Analysis. Psychometrika, 39–43.

Kleinberg, J. (1999). Authoritative sources in a hyperlinked environment. *JACM* 46, 604–632.

Leem, B., & Chun, H. (2015). Measuring the Influence of Efficient Ports using Social Network Metrics. *International Journal of Engineering Business Management* 7, 1-8.

Li, X. B., & Reeves, G. R. (1999). A multiple criteria approach to data envelopment analysis, *European Journal of Operational Research 115*, 507–517.

Liu, J. S., Lu, W.M., Yang, C., & Chuang, M. (2009). A network-based approach for increasing discrimination in data envelopment analysis. *Journal of the Operational Research Society 60*, 1502–1510.

Liu, J.S., & Lu, W. M. (2010). DEA and ranking with the network-based approach: a case of R&D performance. *Omega 38*, 453–464.

Lotfi, F. H, Jahanshahloo, G. R, Khodabakhshi, M, Rostamy-Malkhlifeh, M, Moghaddas, Z, & Vaez-Ghasemi, M. (2013). A Review of Ranking Models in Data Envelopment Analysis. Journal of Applied Mathematics 2013, Article ID 492421, 20 pages.

Lu, W. M., & Lo, S. F. (2009). An interactive benchmark model ranking performers—application to financial holding companies. *Mathematical and Computer Modelling* 49, 172-179.

Mises, R. & Pollaczek-Geiringer, H., Praktische. (1929). Verfahren der Gleichungsauflösung. *ZAMM - Zeitschrift für Angewandte Mathematik und Mechanik* 9, 152-164.

Serrano Cinca, C., Fuertes Callén, Y., & Mar Molinero, C. (2005). Measuring DEA efficiency in Internet companies. *Decision Support Systems 38*, 557–573.

Serrano Cinca, C., & Mar Molinero, C. (2004). Selecting DEA specifications and ranking units via PCA. *Journal of the Operational Research Society 55*, 521–528.

Sexton, T. R., Silkman, R. H., & Hogan, A. J. (1986). Data envelopment analysis: Critique and extensions. *New Directions for Program Evaluation* 32, 73–105.

Simon, J., Simon, C.,& Arias, A. (2011). Changes in productivity of Spanish university libraries. *Omega 39 (5)*, 578-588.

Sinuany-Stern, Z., Mehrez, A., & Barboy, A. (1994). Academic departments efficiency via data envelopment analysis. *Computers and Operations Research 21*, 543–556.

Thompson, R.G., Singleton Jr., F.D., Thrall, R.M., & Smith, B.A. (1986). Comparative site evaluations for locating a high-energy physics lab in Texas. *Interfaces 16*, 35–49.

Tone, K. (2002). A slack-based measure of super-efficiency in data envelopment analysis. *European Journal of Operational Researc*, *143*, 32–41.

Torgersen, A. M., Forsund, F. R., & Kittelsen, S.A. (1996). Slack-adjusted efficiency measures and ranking of efficient units. *The Journal of Productivity Analysis 7*, 379–398.

Wong, Y.H., & Beasley, J.E. (1990). Restricting weight flexibility in DEA. *Journal of the Operational Research Society 41*, 829–835.

Zhu, J. (2000). Multi-factor performance measure model with an application to Fortune 500 companies. *European Journal of Operational Research 123*, 105–124.

Figure 1. The basic operations. Source: Kleinberg (1999), p. 612

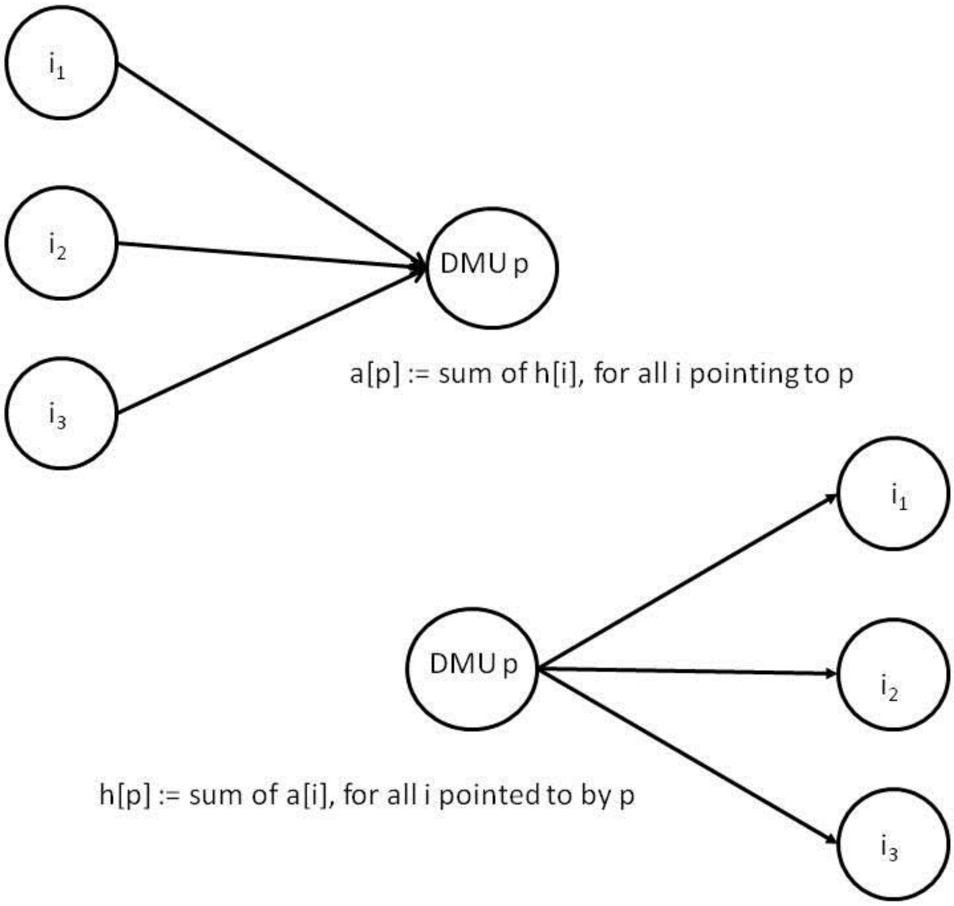


Figure 2. An example network of USA cities

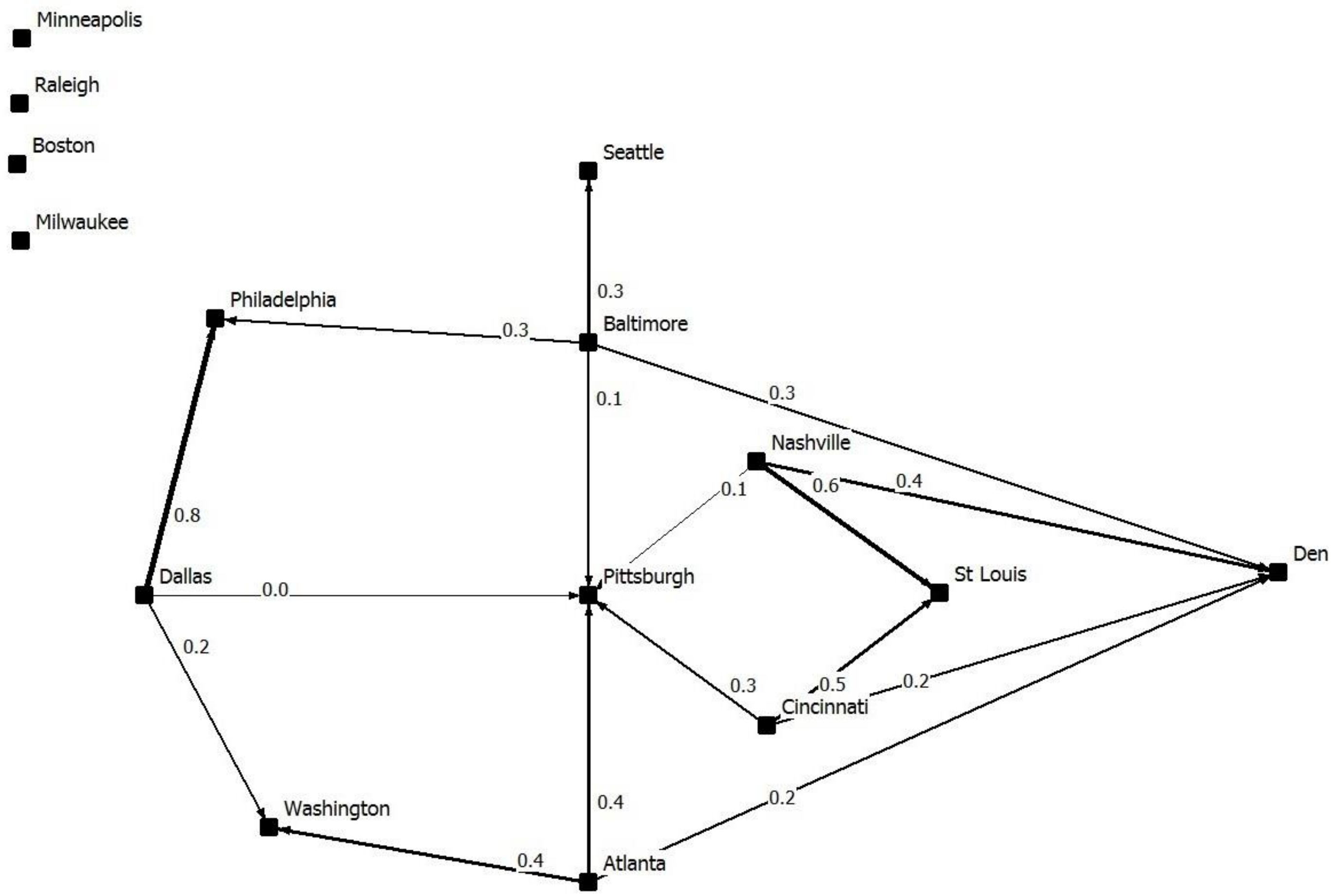
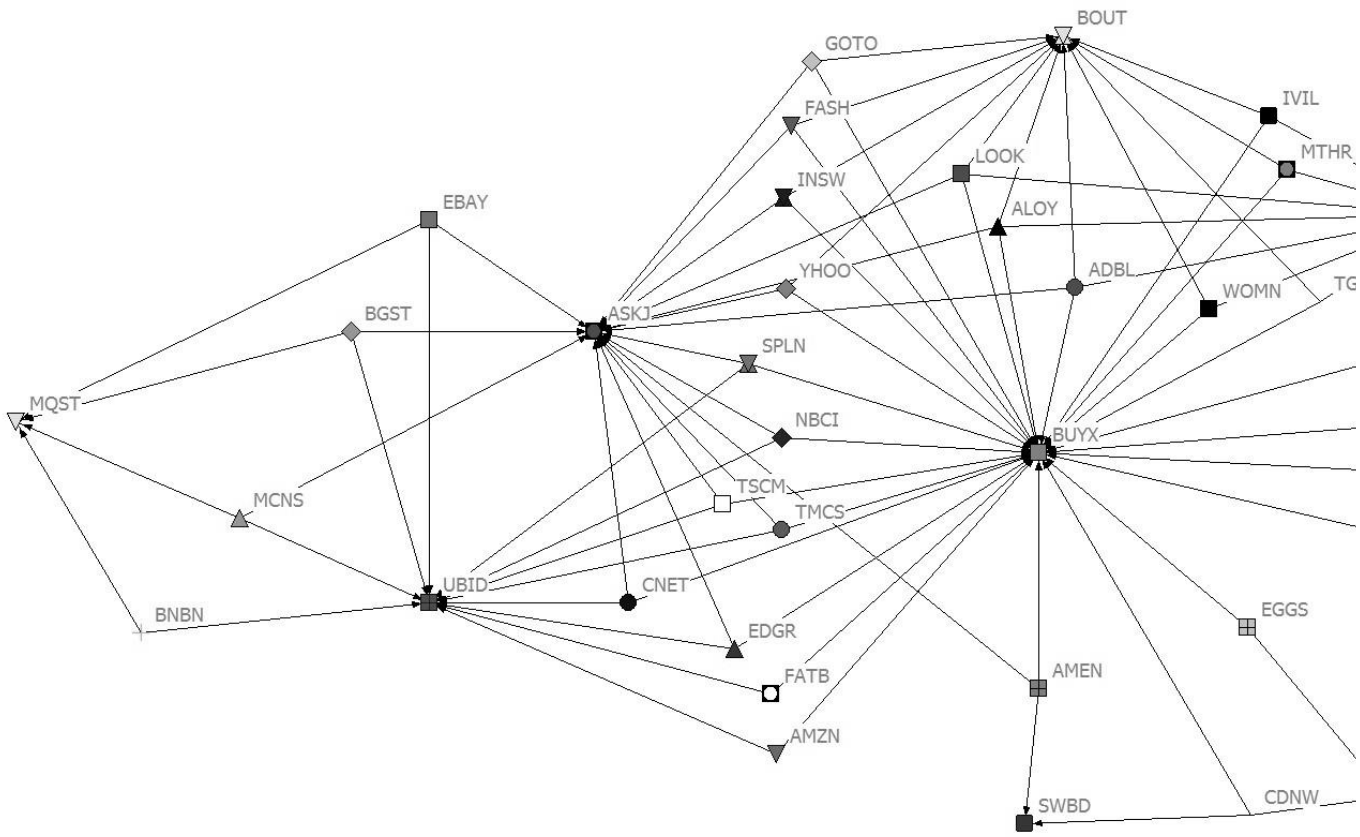


Figure 3. An example network of the Internet Companies



TABLES

Table 1. Data of the USA cities in the example and the VRS efficiency results

DMU	I_1	I_2	I_3	O_1	O_2	O_3	VRS efficiency score	Super efficiency	Peers VRS	Lambda weights VRS
Seattle	586	581	1193.06	46928	0.6534	9.878	1	1.44		
Denver	475	558	1131.64	42879	0.5529	5.301	1	1.015		
Philadelphia	201	600	3468.00	43576	1.135	18.2	1	Infeasible		
Minneapolis	299	609	1340.55	45673	0.729	7.209	1	1.22		
Raleigh	318	613	634.70	40990	0.319	4.94	1	1.16		
St. Louis	265	558	657.50	39079	0.515	8.5	1	1.51		
Cincinnati	467	580	882.40	38455	0.3184	4.48	0.95	0.94	2, 6, 9	0.21, 0.47, 0.31
Washington	583	625	3286.70	54291	1.7158	15.41	1	Infeasible		
Pittsburgh	347	535	917.04	34534	0.4512	8.784	1	1.04		
Dallas	296	650	3714.30	41984	1.2195	8.82	0.93	0.92	3, 8, 9	0.78, 0.18, 0.03
Atlanta	600	740	2963.10	43249	0.9205	7.805	0.77	0.77	2, 8, 9	0.20, 0.35, 0.44
Baltimore	575	775	3240.75	43291	0.5825	10.5	0.74	0.73	1, 2, 3, 9	0.32, 0.29, 0.253, 0.12
Boston	351	888	2197.12	46444	1.04	18.208	1	Infeasible		
Milwaukee	283	727	778.35	41841	0.321	4.665	1	1.06		
Nashville	431	695	1245.75	40221	0.2365	3.575	0.80	0.80	2, 6, 9	0.36, 0.58, 0.05

Table 2. The authority weights and ranking by DMU network, Liu network with the proposed method, Liu, Chen and Cross Efficiency methods for the example in Chen (2004).

DMU	Authority weight	Hub weight	Hub and authority partition	DMUs Network	Liu with Hits algorithm	Liu	$Chen_a$	$Chen_b$	Page Rank
Seattle	0.0965		Authority	6	7	8	3	3	6
Denver	0.4844		Authority	2	6	5	10	10	1
Philadelphia	0.4029		Authority	4	1	1	2	2	3
Minneapolis	0		Authority	7	10	9	6	5	7
Raleigh	0		Authority	7	5	7	7	7	7
St. Louis	0.6165		Authority	1	3	2	1	1	2
Cincinnati		0.5393	Hub	12	13	-	-	-	-
Washington	0.1944		Authority	5	4	4	4	4	5
Pittsburgh	0.4193		Authority	3	2	3	9	9	4
Dallas		0.3737	Hub	13	11	-	-	-	-
Atlanta		0.3611	Hub	14	15	-	-	-	-
Baltimore		0.3366	Hub	15	14	-	-	-	-
Boston	0		Authority	7	9	6	5	6	7
Milwaukee	0		Authority	7	8	10	8	8	7
Nashville		0.5708	Hub	11	12	-	-	-	-

$Chen_a$: super efficiency model input oriented and $Chen_b$: super efficiency model output oriented

Table 3. The VRS and super-efficiency results upon the removal of DMUs that are not identified as Hubs or Authorities.

DMU	VRS efficiency score	Super-efficiency
Seattle	1	1.44
Denver	1	1.015
Philadelphia	1	Infeasible
St. Louis	1	1.51
Cincinnati	0.95	0.94
Washington	1	Infeasible
Pittsburgh	1	1.04
Dallas	0.93	0.92
Atlanta	0.77	0.77
Baltimore	0.74	0.73
Nashville	0.80	0.80

Table 4. Data of Internet Companies, CCR and super-efficiency results

DMU	I_1	I_2	I_3	O_1	O_2	CCR efficiency score	Super-efficiency score	Peers CCR	Lambda weights CCR
ADBL	60	16321	39926	199	1743	0.08	0.08	6. 9. 10. 38	0.0002; 0.011; 0.002; 0.006
AHWYQ	41	14675	15795	904	2060	0.36	0.36	10. 23. 30. 38	0.002; 0.029; 0.013; 0.067
ALOY	120	36275	57668	1416	33864	0.36	0.36	6. 9. 10. 38	0.028; 0.044; 0.053; 0.041
AMEN	81	18196	19784	322	6899	0.15	0.15	6. 10. 32	0.021; 0.011; 0.006
AMZN	7600	896400	2471551	14812	1639839	0.39	0.39	10; 37	1.277; 4.284
ASKJ	416	62850	75764	13113	22027	1.00	1.09		
BGST	200	20150	29852	1220	10658	0.36	0.36	6. 27. 37	0.064; 0.042; 0.038
BNBN	1237	165273	679518	5401	202567	0.34	0.34	27. 37	0.442; 0.914
BOUT	300	67821	242081	13116	26962	1.00	1.01		
BUYX	230	121498	119606	2378	596848	1.00	2.69		
CDNW	537	150679	118802	6654	147189	0.43	0.43	10. 32. 36	0.194; 2.116; 0.213
CNET	671	125878	1230311	10587	112345	0.54	0.54	6. 10. 37	0.775; 0.152; 0.022
EBAY	1212	170509	969825	14032	224724	0.58	0.58	6. 27. 37	0.581; 0.588; 0.935
EDGR	45	8158	37739	182	4731	0.20	0.20	6. 10. 37	0.01; 0.003; 0.015
EGGS	560	190305	129130	1858	541208	0.81	0.81	10. 36	0.759; 1.370
FASH	43	10668	43541	346	3690	0.22	0.22	6. 9. 10	0.005; 0.021; 0.005
FATB	349	39073	48465	187	35338	0.19	0.19	10. 37	0.051; 0.024
GOTO	317	54200	129512	6928	26809	0.67	0.67	6. 9. 10	0.005; 0.021; 0.005
HITS	73	26575	165400	438	1044	0.10	0.10	10. 23. 38	0.01; 0.024; 0.031
INSW	297	60223	118281	2175	21841	0.23	0.23	6. 9. 10	0.119; 0.041; 0.03
IVIL	396	123062	312748	3707	36576	0.21	0.21	9. 10. 38	0.081; 0.053; 0.286
KOOP	185	66910	99720	4845	9431	0.42	0.42	10. 23. 38	0.004; 0.259; 0.342
LFMN	90	45192	76857	7034	14019	1.00	1.07		
LOOK	535	109065	161519	8470	48865	0.45	0.45	6.9. 10. 38	0.379; 0.112; 0.059; 0.215
MCNS	294	34188	222781	607	6362	0.11	0.11	6. 27. 37	0.01; 0.065; 0.019
MKTW	202	77324	156855	3367	24935	0.30	0.30	10. 23. 38	0.034; 0.18; 0.229
MQST	335	32843	65010	6528	34487	1.00	1.07		
MTHR	190	55380	71374	2434	5769	0.24	0.24	9. 10. 38	0.025; 0.004; 0.238
NBCI	635	115154	2494096	2174	36100	0.14	0.14	6. 10. 37	0.144; 0.028; 0.08
ONHN	101	51375	32720	7755	3767	1.00	1.32		
SPLN	453	83908	271461	5572	60278	0.43	0.43	6. 10. 37	0.405; 0.075; 0.031
SWBD	52	14990	12195	2577	8304	1.00	1.03		
TGLO	220	62464	138843	5564	18641	0.50	0.50	9. 10. 38	0.116; 0.018; 0.453
TMCS	1172	230098	804669	4223	105303	0.16	0.16	6. 10. 37	0.259; 0.091; 0.221
TSCM	253	43530	143550	962	14316	0.16	0.16	6. 10. 37	0.064; 0.008; 0.04
TVLY	84	44563	9639	3479	64187	1.00	2.72		
UBID	281	45827	79266	2749	204925	1.00	1.08		
UPRO	157	46773	42816	8821	10392	1.00	1.10		
WOMN	277	88216	172539	4256	30023	0.31	0.31	9. 10. 38	0.021; 0.042; 0.439
YHOO	1992	440647	1520129	39569	591786	0.66	0.66	6. 9. 10	2.166; 0.692; 0.88

Table 5. The authority weights and ranking by DMU network, Cross Efficiency method, Liu network with the proposed method, Liu and Page Rank methods for the example in Liu et al. (2009)

DMU	Authority weight	Hub weight	Partition of hubs and authorities	DMUs Network	Cross Efficiency	Liu with Hits algorithm	Liu	Page Rank
ADBL		0.0002	Hub	38	40	29	-	-
AHWYQ		0.0002	Hub	39	22	4	4	-
ALOY		0.0049	Hub	27	21	22	-	-
AMEN		0.0015	Hub	34	35	18	-	-
AMZN		0.9420	Hub	11	24	20	-	-
ASKJ	0.1633		Authority	3	2	8	11	2
BGST		0.0103	Hub	24	23	17	-	-
BNBN		0.1875	Hub	13	25	34	-	-
BOUT	0.0355		Authority	6	5	13	12	4
BUYX	0.3177		Authority	2	3	2	3	1
CDNW		0.0191	Hub	20	20	33	-	-
CNET		0.0419	Hub	17	16	39	-	-
EBAY		0.2137	Hub	12	14	40	-	-
EDGR		0.0035	Hub	30	32	5	5	-
EGGS		0.0629	Hub	15	13	19	-	-
FASH		0.0007	Hub	35	30	11	9	-
FATB		0.0083	Hub	25	33	23	-	-
GOTO		0.0175	Hub	22	11	37	-	-
HITS		0.0001	Hub	40	39	27	-	-
INSW		0.0065	Hub	26	29	16	-	-
IVIL		0.0043	Hub	29	31	28	-	-
KOOP		0.0004	Hub	37	19	31	-	-
LFMN	0.0001		Authority	9	8	9	7	6
LOOK		0.0182	Hub	21	17	38	-	-
MCNS		0.0049	Hub	28	38	26	-	-
MKTW		0.0024	Hub	32	27	24	-	-
MQST	0.0538		Authority	4	7	7	8	7
MTHR		0.0006	Hub	36	28	15	-	-
NBCI		0.0229	Hub	19	37	21	-	-
ONHN	0		Authority	10	9	10	10	10
SPLN		0.0255	Hub	18	18	35	-	-
SWBD	0.0093		Authority	7	4	1	1	9
TGLO		0.0023	Hub	33	15	36	-	-
TMCS		0.0594	Hub	16	34	32	-	-
TSCM		0.0108	Hub	23	36	25	-	-
TVLY	0.0380		Authority	5	10	3	2	8
UBID	0.9310		Authority	1	6	6	6	3
UPRO	0.0018		Authority	8	1	12	-	5
WOMN		0.0032	Hub	31	26	30	-	-
YHOO		0.1411	Hub	14	12	14	13	-

Table 6. The CCR and super-efficiency results when removing DMUs that are not identified as Hubs or Authorities.

DMU	CCR efficiency score	Super-efficiency score
ADBL	0.08	0.08
AHWYQ	0.36	0.36
ALOY	0.39	0.39
AMEN	1.00	1.09
AMZN	0.36	0.36
ASKJ	0.34	0.34
BGST	1.00	1.01
BNBN	1.00	2.69
BOUT	0.43	0.43
BUYX	0.54	0.54
CDNW	0.58	0.58
CNET	0.20	0.20
EBAY	0.81	0.81
EDGR	0.19	0.19
EGGS	0.67	0.67
FASH	0.23	0.23
FATB	0.21	0.21
GOTO	1.00	1.07
HITS	0.45	0.45
INSW	0.11	0.11
IVIL	1.00	1.07
KOOP	0.14	0.14
LFMN	1.00	1.32
LOOK	0.43	0.43
MCNS	1.00	1.03
MKTW	0.16	0.16
MQST	0.16	0.16
MTHR	1.00	2.72
NBCI	1.00	1.08
ONHN	1.00	1.10
SPLN	0.66	0.66