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Decision Support

Learning target-based preferences through additive models: An application in radiotherapy treatment planning

Luis C. Dias[a,b], Joana Dias[a,b,*], Tiago Ventura[c], Humberto Rocha[a,b], Brígida Ferreira[d], Leila Khouri[c], Maria do Carmo Lopes[c]

[a] University of Coimbra, CeBER, Faculty of Economics, Av. Dias da Silva, 165, 3004–512 Coimbra, Portugal
[b] University of Coimbra, INESCC, Rua Sílvio Lima, 3030-290 Coimbra, Portugal
[c] Medical Physics department, Portuguese Oncology Institute of Coimbra, Coimbra, Portugal
[d] Institute of Biophysics and Biomedical Engineering, University of Lisbon, Lisbon, Portugal

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ABSTRACT

This article presents a new Multi-Criteria Decision Aiding preference disaggregation method based on an asymmetric target-based model. The decision maker's preferences are elicited considering the choices made given a set of comparisons among pairs of solutions (the stimuli). It is assumed that the decision maker has a reference value (target) for the stimulus. Solutions that do not comply with this reference value for some of the criteria dimensions considered will be penalized, and an inferred weight is associated with each dimension to calculate a penalty score for each solution. One of the differentiating features of the proposed model when compared with other existing models is the fact that only solutions that do not meet the target are penalized. The target is not interpreted as an ideal solution, but as a set of threshold values that should be taken into account when choosing a solution. The proposed approach was applied to the problem of choosing radiotherapy treatment plans, using a set of retrospective cancer cases treated at the Portuguese Oncology Institute of Coimbra. Using paired comparison choices made by one radiation oncologist, the preference model was built and was tested with in-sample and out-of-sample data. It is possible to conclude that the preference model is capable of representing the radiation oncologist's preferences, presenting small mean errors and leading, most of the time, to the same treatment plan chosen by the radiation oncologist.

1. Introduction

In many situations a decision-maker (DM) has to take into account, simultaneously, a set of conflicting objectives. In this setting, the objective is no longer to calculate an optimal solution, but to reach a compromise solution that is aligned with the DM's preferences. This compromise solution should be a Pareto-optimal solution (non-dominated solution): no other solution can be better in at least one objective without being worse in some other(s). There are different ways of reaching a compromise solution (Antunes, Alves & Clímaco, 2016; Miettinen, 2012). The elicitation of the DM's preferences can be done *a priori*, *a posteriori* or the search for this compromise solution can be done interactively. The *a priori* methods assume the DM has a value function, which is explicitly constructed using preference information received from the DM. In *a posteriori* methods, a set of Pareto-optimal solutions is calculated first, trying to approximate the diversity of solutions in the Pareto-front. Then, the preference elicitation and choice of a solution take place considering this set of already known solutions. Interactive methods are based on the successive calculation of Pareto-optimal solutions according to the inputs received from the DM. Instead of assuming the existence of a value function, trying to elicit preferences *a priori*, an initial small set of solutions is calculated. By analyzing this set, the DM can have a better understanding of the compromises that exist among the defined objectives and has the possibility of driving the method towards the calculation of new solutions in search areas that are more aligned with his/her preferences.

All these approaches present advantages and disadvantages. The DM may not be able to express any kind of preferences over the set of conflicting objectives, as *a priori* approaches require, before some solutions are calculated. Interactive methods can be compu-

* Corresponding author.
E-mail address: joana@fe.uc.pt (J. Dias).

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tationally very demanding, especially if the problem to be solved in each iteration is non-linear or even non-convex. High computational times (which is usually the case in radiotherapy treatment planning, for instance) are not compatible with an interactive procedure. *A posteriori* methods can be a good alternative if it is possible to find a good set of Pareto-optimal solutions to present to the DM. Even so, when the DM must take into account a significant number of different objectives, choosing one solution out of a set of solutions can be an overwhelming task. This is one of the reasons why *a posteriori* methods can also use discrete Multi-Criteria Decision Aiding / Analysis (MCDA) methods (i.e., methods evaluating a given list of alternatives) to support the DM's choice process. There are many examples of this combined use of approaches. Hadas and Nahum (2016) use both multiobjective and MCDA approaches in the context of the selection of a network of public transport priority lanes. A multiobjective evolutionary algorithm is used to find a set of Pareto-optimal (or near optimal) solutions, followed by MCDA for the selection of the final solution. Cruz-Reyes, Fernandez, Sanchez, Coello and Gomez (2017) consider the inclusion of decision-maker's preferences in a multiobjective evolutionary algorithm by using an MCDA classification method. This combination, which was applied to a project portfolio optimization problem, promotes the definition of regions of interest where the evolutionary search will be more intensive. Laha and Chakraborty (2021) develop a multiobjective optimization model for optimizing the capacity installation of renewable energy. The Pareto-optimal solutions are then ranked using a multimetric sustainability model.

When decisions have to be made repeatedly, in similar situations, and taking into account similar criteria, it is possible to elicit the DM's preferences by the choices he/she has made in a selected number of cases, and then use these elicited preferences in future decision-making situations. A preference model inferred from known judgements made by the DM can be instrumental whatever process is followed for reaching a compromise solution, as discussed above. For an *a priori* approach, it can provide the value function to be optimized. For an *a posteriori* approach, it allows suggesting a choice among several compromise solutions. If the DM does not agree with the suggested choice, then this new information can be used to update the inferred model (Figueira, Greco, Mousseau & Słowiński, 2008). For an interactive approach, if the problem of finding new solutions can be solved in a timespan short enough to allow interactivity, the algorithm can adapt itself throughout successive iterations (Belton et al., 2008).

MCDA preference disaggregation methods (Jacquet-Lagrèze & Siskos, 2001) are promising tools to be used in this context. These methods infer the parameters for a DM's preferences model from a set of judgments provided by him or her. Such judgments are usually a ranking or a classification of a set of examples of alternatives in accordance with the holistic appreciation of the DM, or in accordance with past decisions, or in accordance with already known outcomes. Pioneering preference disaggregation methods include the Euclidean distance model of Srinivasan and Shocker (1973) and the additive value function UTA methods (Jacquet-Lagrèze & Siskos, 1982), which have evolved significantly during the last decades (Ghaderi & Kadziński, 2020; Kadziński, Ghaderi, Wąsikowski & Agell, 2017; Liu, Kadziński, Liao, Mao & Wang, 2020; Matsatsinis, Grigoroudis & Siskos, 2018; Siskos, Grigoroudis & Matsatsinis, 2016; Sobrie, Gillis, Mousseau & Pirlot, 2018) along with other types of MCDA disaggregation models (Angilella, Greco & Matarazzo, 2010; Mousseau & Dias, 2004; Sobrie, Mousseau & Pirlot, 2019).

This work contributes to the literature by developing an asymmetric version of Srinivasan and Shocker (1973)'s seminal target-based model, and applying it to a novel application field for MCDA disaggregation methods, meeting the needs of real-world practitioners. Rather than assuming that downward or upward deviations from the target are equally undesirable using an Euclidean distance, the distance function developed in this work considers a preference direction. It grows quadratically for undesirable deviations (the target is not being achieved) and equals zero if the target is met or if the solution is better than the target. The unknowns are the target value and the weights that the DM attaches to each different dimension (evaluation criterion).

Considering the asymmetry in the Euclidean distance leads to a more complex problem, for which we propose the use of a piece-wise linear approximation. Recent methods capable of inferring a polynomial function, UTA-poly and UTA-splines (Sobrie et al., 2018), do not require the piecewise linear approximation, but these methods were not built for the target-based context considered here and require the use of nonlinear programming. The piecewise linear approximation can nevertheless be improved as much as needed by increasing the number of line segments considered, and requires 0–1 linear programming tools only.

This method was applied to the choice of radiotherapy treatment plans, aiming at learning the preference model of one experienced radiation oncologist (RO) who compared a total of 40 pairs of treatment plans concerning 40 real retrospective cases treated at the Portuguese Oncology Institute of Coimbra. The choice of a given plan is inherently a multicriteria problem, since different and conflicting objectives must be considered simultaneously (the need of sparing healthy tissues is in contradiction with the need of properly irradiating the volumes to treat). Compared to multiobjective optimization applications for radiotherapy treatment planning, prior applications of MCDA to evaluate a discrete set of alternatives in this setting are much less common. The results are compared with two benchmarks based on UTA.

The Euclidean distance model is appealing in the context of preference elicitation since it uses the concept of a distance between stimuli and a multidimensional ideal the DM would like to achieve, with a long tradition in psychology (Srinivasan & Shocker, 1973). Yet, using the Euclidean distance model without any adaptation, despite producing good results (Ventura et al., 2020), has the limitation of treating deviations from the target (the ideal) in a symmetrical way, i.e., positive and negative deviations from the target value are considered to be equally undesirable. This the main motivation for the method proposed in the present work, which does not penalize deviations in those dimensions where the solution is better than the target.

2. Mathematical programming formulations

The following models and mathematical programming formulations for multidimensional analysis of preferences infer a preference model based on a given set of comparisons among pairs of solutions from a given set A. (the stimuli). Following Srinivasan and Shocker's (1973) model, we assume that the DMs have a reference value (target) for the stimulus and we also assume that the penalty for any deviations grows with the square of the difference, weighted by a coefficient reflecting the importance of each dimension. Considering a dimension $g \in G$, the weighted distance of solution y_j ($j \in A$) to a target x is given by $w_g(y_{jg} - x_g)^2$, where w_g denotes a scaling weight attached to dimension g, y_{jg} denotes a stimulus (performance of solution y_j) concerning dimension g, and x_g is the g-dimension value for target x. This model is hereafter referred to as the SSED (Srinivasan and Shocker's Euclidean Distance) model.

Our model differs from the SSED model in a crucial aspect: rather than assuming that any deviations are penalized, considering the target as an ideal point, our model is asymmetrical, assuming the DM seeks to penalize only positive deviations (the excess of the stimulus in relation to the target). The target corresponds to an impact value the DM would not like to exceed, i.e., the DM wishes to minimize the impact of the solution if it lies above the target, being fully satisfied if it lies below the target. However, the following equations and models can be easily adapted to consider the reverse perspective of penalizing negative deviations instead.

Considering a dimension $g \in G$, the weighted disutility or penalty (according to an asymmetric distance) of solution y_j ($j \in A$) to a target x is given by:

$$d_g\left(y_{jg}, x_g, w_g\right) = \begin{cases} w_g\left(y_{jg} - x_g\right)^2, & \text{if } y_{jg} > x_g \\ 0, & \text{if } y_{jg} \le x_g \end{cases} \tag{1}$$

Following the SSED model, the global disutility (unwanted excess of impact) of solution y_j ($j \in A$) to a target x, considering all the dimensions, is given by an additive model:

$$d\left(y_j, x, w\right) = \sum_{g \in G} d_g\left(y_{jg}, x_g, w_g\right) \tag{2}$$

According to this additive model, hereafter referred to as the AED (Asymmetric Euclidean Distance) model, the disutility maintains the nature of a (squared) Euclidean distance, with the aforementioned difference of being asymmetrical.

In a preference disaggregation perspective, the goal is to infer an AED model from observed judgments. Consider that the judgments of a DM (or group of DMs) are represented by a set J such that $(j,k) \in J \Leftrightarrow j,k \in A \wedge j \succ k$ (solution j is preferred to solution k).

To be consistent with a judgment $(j,k) \in J$, the disutility of y_j should be less than the disutility of y_k:

$$(j,k) \in J \Leftrightarrow d\left(y_j, x, w\right) < d(y_k, x, w) \tag{3}$$

Following the method of Srinivasan and Shocker (1973), judgments do not need to be fully consistent, which would be a strong requirement. Thus, an error term z_{jk} can be accepted, but it should be minimized:

$$(j,k) \in J \Leftrightarrow d(y_k, x, w) - d\left(y_j, x, w\right) + z_{jk} > 0 \tag{4}$$

Following Srinivasan and Shocker (1973), the error associated with $(j,k) \in J$ is:

$$s^-_{jk} = \max\left\{0, d\left(y_j, x, w\right) - d(y_k, x, w)\right\} \tag{5}$$

Analogously, one can define a desirable slack for the same pair:

$$s^+_{jk} = \max\left\{0, d(y_k, x, w) - d\left(y_j, x, w\right)\right\} \tag{6}$$

Summing over all the pairs compared by the decision makers results in two indicators:

$$BF = \text{poorness of fit} = \sum_{(j,k) \in J} s^-_{jk}$$

$$GF = \text{goodness of fit} = \sum_{(j,k) \in J} s^+_{jk}$$

Moreover,

$$GF - BF = \sum_{(j,k) \in J} s^+_{jk} - \sum_{(j,k) \in J} s^-_{jk} = \sum_{(j,k) \in J} \left(d(y_k, x, w) - d\left(y_j, x, w\right)\right) \tag{7}$$

A mathematical formulation implementing these ideas, similar to (Srinivasan & Shocker, 1973)'s is:

Minimize

$$\sum_{(j,k) \in J} z_{jk} \tag{8a}$$

Subject to the constraints:

$$\sum_{g \in G} d_g\left(y_{kg}, x_g, w_g\right) - \sum_{g \in G} d_g\left(y_{jg}, x_g, w_g\right) + z_{jk} \ge \varepsilon, \ \forall (j,k) \in J \tag{8b}$$

$$\sum_{(j,k) \in J} \left(\sum_{g \in G} d_g\left(y_{kg}, x_g, w_g\right) - \sum_{g \in G} d_g\left(y_{jg}, x_g, w_g\right)\right) \ge h \tag{8c}$$

$$z_{jk} \ge 0, \ \forall (j,k) \in J \tag{8d}$$

In this formulation, the decision variables are the error terms z_{jk} (for all $(j,k) \in J$), the dimension weights vector $w = \{w_1, \ldots, w_t\}$ and the targets vector $x = \{x_1, \ldots, x_t\}$. Vectors w and x define each disutility function $d(y_j, x, w) = \sum_{g \in G} d_g(y_{jg}, x_g, w_g)$. The objective function (8a) represents the minimization of the sum of the error terms. In constraints (8b), ε denotes an arbitrarily small positive constant to enforce the strict inequality defined in Eq. (4) ($\varepsilon = 0.001$ in the present work). Constraint (8c) represents $GF - BF \ge h$, as suggested by Srinivasan & Shocker, to prevent the calculation of trivial solutions and to require a minimum overall quality in the balance of goodness and poorness of fit (in our experiments, h requires an average difference of 0.01 per pair in J, i.e., $h = 0.01 \times |J|$).

Srinivasan and Shocker solve the nonlinear model (8a-d) by transforming it into a linear program. Unfortunately, the asymmetrical definition of the disutility function (1) in our AED model does not allow the application of similar transformations.

We will use a piece-wise linear approximation to the marginal disutility functions $d_g(y_{jg}, x_g, w_g)$, denoted $d_g^{\sim}(y_{jg}, x_g, w_g)$, that can be made as accurate as desired by increasing the number of breakpoints. Moreover, for the purpose of performing a comparison, we use alternative formulations in which the disutility function is more general, not being constrained to the quadratic shape of eq. (1): in one case we accept any convex and nondecreasing function $d_g^c(y_{jg}, x_g, w_g)$; in an even more general case, we accept any nondecreasing function $d_g^{nd}(y_{jg}, x_g, w_g)$. For all these functions, disutility is zero for any stimulus that does not surpass the respective target.

Let $B_g = \{b_{g,0}, \ldots, b_{g,m(g)}\}$ denote breakpoints for the piecewise disutility function $d_g^{\sim}$, such that all the stimuli, as well as the target, lie in $[b_{g,0}, b_{g,m(g)}]$. These $m(g)+1$ breakpoints are fixed in advance and each dimension can be associated with as many breakpoints as desired. Let $d_{g,i}^{\sim}$ (the same applies ceteris paribus for $d_{g,i}^c$ and $d_{g,i}^{nd}$) denote the disutility corresponding to stimulus $b_{g,i}$. Let $s_{g,i}$ denote the slope of the line segment from point $(b_{g,i-1}, d_{g,i-1}^{\sim})$ to point $(b_{g,i}, d_{g,i}^{\sim})$. Then, if $y_{jg} \in [b_{g,i-1}, b_{g,i}]$, for any $i \in \{1, m(g)\}$, we consider a linear interpolation:

$$d_g^{\sim}\left(y_{jg}, x_g, w_g\right) = d_{g,i-1}^{\sim} + \left(y_{jg} - b_{g,i-1}\right)s_{g,i} \tag{9}$$

Since $d_{g,i}^{\sim} = d_{g,i-1}^{\sim} + (b_{g,i} - b_{g,i-1})s_{g,i}$ and $x_g \ge b_{g,0}$ implies $d_{g,0}^{\sim} = 0$, eq. (9) is equivalent to

$$d_g^{\sim}\left(y_{jg}, x_g, w_g\right) = \sum_{i=1}^{m(g)} \tau_{jg,i} s_{g,i} \tag{10}$$

with

$$\tau_{jg,i} = \begin{cases} b_{g,i} - b_{g,i-1}, & \text{if } y_{jg} \ge b_{g,i} \\ y_{jg} - b_{g,i-1} & \text{if } b_{g,i-1} < y_{jg} < b_{g,i} \\ 0, & \text{if } y_{jg} \le b_{g,i-1} \end{cases} \tag{11}$$

Considering B_g and y_{jg} ($g \in G$, $j \in A$) as given inputs, the positive constants $\tau_{jg,i}$ can be readily determined and $d_{g,i}^{\sim}$ becomes a linear function of $s_{g,i}$, which will be the variables of the linear programs replacing (8a-8d), together with the error terms. The difference of disutility that appears in (8b) and (8c) can then be written as $\sum_{g \in G} \sum_{i=1}^{m(g)} (\tau_{kg,i} - \tau_{jg,i})s_{g,i}$.

The mathematical programs for $d_g^{\sim}$ (AED model), $d_{g,i}^c$ (convex disutilities) and $d_{g,i}^{nd}$ (requiring only monotonicity), presented next, differ in the constraints placed on the slopes $s_{g,i}$.

2.1. Monotonic (nondecreasing) function formulation

The simplest formulation is a linear program that requires the disutility functions, d_g^{nd}, to be monotonic (non-decreasing). This requires only the slope of the function to be non-negative:

Minimize

$$z = \sum_{(j,k)\in J} z_{jk} \tag{12a}$$

Subject to the constraints:

$$\sum_{g\in G}\sum_{i=1}^{m(g)} \left(\tau_{kg,i} - \tau_{jg,i}\right) s_{g,i} + z_{jk} \geq \varepsilon,\ \ \forall (j,k) \in J \tag{12b}$$

$$\sum_{(j,k)\in J} \left(\sum_{g\in G}\sum_{i=1}^{m(g)} \left(\tau_{kg,i} - \tau_{jg,i}\right) s_{g,i}\right) \geq h \tag{12c}$$

$$s_{g,i} \geq 0,\ \ \forall g \in G, i \in \{1, \ldots, m(g)\} \tag{12d}$$

$$z_{jk} \geq 0,\ \ \forall (j,k) \in J \tag{12e}$$

The optimal slopes define unequivocally a piecewise linear function minimizing the sum of the error terms. For each dimension $g \in G$, the disutility function is defined by points $(b_{g,i},\ d_{g,i}^{nd})$, for $i = 0, \ldots, m(g)$, such that $d_{g,0}^{\sim} = 0$ and $d_{g,i}^{nd} = d_{g,i-1}^{nd} + (b_{g,i} - b_{g,i-1})s_{g,i}$. This implicitly defines a target x_g, which is the maximum $b_{g,i}$ such that $d_{g,i}^{nd} = 0$, unless all such values are zero up to $d_{g,m(g)}^{nd} = 0$. The latter situation would correspond to disregarding this dimension in the appreciation of the overall utility. To address this situation and also other cases in which the DM is able to bound the location of the target, two additional constraints can be added:

$$s_{g,i} = 0,\ \ \forall g \in G, i \in \{1, \ldots, LB(g)\} \tag{12f}$$

$$\sum_{g\in G}\sum_{i=1}^{UB(g)+1} \tau_{kg,i} s_{g,i} \geq \varepsilon,\ \ \forall g \in G \tag{12g}$$

Constraints (12f) place a lower bound at breakpoint $b_{g,LB(g)}$ by forcing the slope before this breakpoint to be zero (in practice, the linear program can be simplified by omitting the respective variables). Constraints (12g) place an upper bound at *UB(g)* by requiring that the value function at breakpoint $b_{g,UB(g)+1}$ is at least ε (or any other chosen constant). Due to the linear interpolation, this means that the value function starts increasing after breakpoint $b_{g,UB(g)}$.

The maximum disutility on dimension $g \in G$ is $d_{g,m(g)}^{nd}$. This implicitly defines the weight of this dimension if the disutility functions were normalized so that their maximum value was the same (e.g., 1):

$$\omega_g = \frac{d_{g,m(g)}^{nd}}{\sum_{h\in G} d_{h,m(h)}^{nd}} \tag{13}$$

Linear program (12a-e) uses slopes as decision variables, resembling other UTA formulations in this aspect (Doumpos & Zopounidis, 2007; Ghaderi & Kadziński, 2020; Ghaderi, Ruiz & Agell, 2017). In this kind of disaggregation approaches the error terms can be associated with each alternative (Doumpos & Zopounidis, 2007) or associated with each judgment (Ghaderi et al., 2017). In the present work this is equivalent as the DM does not evaluate the same alternative in different pairs.

2.2. Convex function formulation

The second formulation is a linear program that requires the disutility functions, d_g^c, to be non-decreasing and also convex, therefore being more general than the also convex AED model, but not as general as the previous one. This only requires that the slope of the function does not decrease, replacing constraints (12d) in linear program (12a-e) by the following constraints:

$$s_{g,m(g)} \geq \ldots \geq s_{g,1} \geq 0,\ \ \forall g \in G \tag{14}$$

As in the previous case, the optimal solution implicitly defines target values and weights, and it is again possible to bound the location of the target using the additional constraints (12f-g).

2.3. Asymmetric euclidean distance (AED) formulation

The third formulation is a 0–1 linear program that requires the disutility functions $d_g^{\sim}$, to approximate d_g as defined in (1), with a precision as good as the number of breakpoints used. The piecewise linear function $d_g^{\sim}$ will coincide with d_g in all breakpoints for $i = 0, \ldots, m(g)$:

$$\begin{aligned} d_{g,i}^{\sim} &= d_g^{\sim}\left(b_{g,i}, x_g, w_g\right) = d_g\left(b_{g,i}, x_g, w_g\right) \\ &= \begin{cases} w_g\left(b_{g,i} - x_g\right)^2, & \text{if } b_{g,i} > x_g \\ 0, & \text{if } b_{g,i} \leq x_g \end{cases} \end{aligned} \tag{15}$$

For any i, the slope between $b_{g,i-1}$ and $b_{g,i}$ equals zero if $b_{g,i} \leq x_g$, otherwise it equals $\frac{w_g(b_{g,i}-x_g)^2 - w_g(b_{g,i-1}-x_g)^2}{b_{g,i}-b_{g,i-1}}$, i.e. (after simplifying):

$$s_{g,i} = \begin{cases} w_g\left(b_{g,i} + b_{g,i-1} - 2x_g\right), & \text{if } b_{g,i} > x_g \\ 0, & \text{if } b_{g,i} \leq x_g \end{cases} \tag{16}$$

Let $u \in \{1, \ldots, m(g)\}$ be such that $b_{g,u} = x_g$. It then follows: $s_{g,1} = \ldots = s_{g,u} = 0$ (extending our notation to consider $d_{g,-1} = 0$ if $u = 0$), $s_{g,u+1} - s_{g,u} = w_g(b_{g,u+1} + b_{g,u} - 2x_g) - 0 = w_g(b_{g,u+1} - b_{g,u})$ (since $b_{g,u} = x_g$), and for $k>1$,

$$\begin{aligned} s_{g,u+k} - s_{g,u+k-1} &= w_g\left(b_{g,u+k} + b_{g,u+k-1} - 2x_g\right) \\ &\quad - w_g\left(b_{g,u+k-1} + b_{g,u+k-2} - 2x_g\right) \end{aligned}$$

$$= w_g\left(b_{g,u+k-1} - b_{g,u+k-2}\right) + w_g\left(b_{g,u+k} - b_{g,u+k-1}\right)$$

In summary, for $i > 1$, the difference between consecutive slopes can be written as:

$$s_{g,i} - s_{g,i-1} = \upsilon_{g,i-1} w_g\left(b_{g,i-1} - b_{g,i-2}\right) + \upsilon_{g,i} w_g\left(b_{g,i} - b_{g,i-1}\right) \tag{17}$$

considering binary variables $\upsilon_{g,i}$ defined as:

$$\upsilon_{g,i} = \begin{cases} 1, & \text{if } b_{g,i} > x_g \\ 0, & \text{if } b_{g,i} \leq x_g \end{cases} \tag{18}$$

If the weights w_g are fixed (e.g. all equal to 1), then constraint (17) can be included in a 0–1 linear program without any transformation. Otherwise, constraint (17) can be replaced by constraints (19d-f), for a suitably large positive constant *M*. The full AED formulation is then:

Minimize

$$z = \sum_{(j,k)\in J} z_{jk} \tag{19a}$$

Subject to the constraints:

$$\sum_{g\in G}\sum_{i=1}^{m(g)} \left(\tau_{kg,i} - \tau_{jg,i}\right) s_{g,i} + z_{jk} \geq \varepsilon,\ \ \forall (j,k) \in J \tag{19b}$$

$$\sum_{(j,k)\in J}\left(\sum_{g\in G}\sum_{i=1}^{m(g)}\left(\tau_{kg,i}-\tau_{jg,i}\right)s_{g,i}\right)\geq h \tag{19c}$$

$$s_{g,1}=p_{g,1} \tag{19d}$$

$$s_{g,i}=s_{g,i-1}+p_{g,i-1}+p_{g,i}(i=2,\ldots,m(g)) \tag{19e}$$

$$\left.\begin{array}{c} p_{g,i}\geq w_g\left(b_{g,i}-b_{g,i-1}\right)-M+Mv_{g,i} \\ p_{g,i}\leq w_g\left(b_{g,i}-b_{g,i-1}\right)+M-Mv_{g,i} \\ p_{g,i}\geq 0 \\ p_{g,i}\leq Mv_{g,i} \end{array}\right\}\ \forall g\in G,\ \ i\in\{1,\ldots,m(g)\} \tag{19f}$$

$$v_{g,m(g)}\geq\ldots\geq v_{g,1}\geq 0,\ \ \forall g\in G \tag{19g}$$

$$v_{g,i}\in\{0,1\},\quad \forall g\in G,\ \ i\in\{1,\ldots,m(g)\} \tag{19h}$$

$$w_g\in\left[w_g^{min},w_g^{max}\right],\ \ \forall g\in G \tag{19i}$$

$$z_{jk}\geq 0,\ \ \forall(j,k)\in J \tag{19j}$$

In this formulation, the variables are the error terms z_{jk} (one per judgment), the weights w_g (one per dimension), the slope parcels $p_{g,i}$ (one per dimension and per breakpoint), and the binary variables associated with the slopes $v_{g,i}$ (one per dimension and per breakpoint). The target is implicit in the solution: if $v_{g,u}=0$ and $v_{g,u+1}=1$ then the target is $b_{g,u}$. Constraints (19f) ensure that if $v_{g,i}=1$ then $p_{g,i}=w_g(b_{g,i}-b_{g,i-1})$, and if $v_{g,i}=0$ then $p_{g,i}=0$. Constraints (19g) guarantee that, when the slope becomes greater than 0, it cannot decrease. Constraints (19i) allow controlling the range of weights considered. As in the previous formulations, it is again possible to bound the location of the target using the additional constraints (12f-g).

In this formulation, the weights w_g have the same meaning as the weights in the SSED model, which is different from the weight of the disutility function if normalized so that their maximum value was the same (e.g., 1). The latter normalized weights can nevertheless be computed as $\omega_g=\frac{d^{\sim}_{g,m(g)}}{\sum_{h\in G}d^{\sim}_{h,m(h)}}$. Constraints (19i) allow, if desired, the definition of a lower bound and an upper bound for each weight w_g.

A final note, which applies to all the formulations a) to c), is that the target will coincide with one of the breakpoints. After obtaining an optimal solution, further breakpoints can be added in the vicinity of the target inferred, to allow for a more precise solution. This does not necessarily imply the use of a larger number of breakpoints, because the previous breakpoints below the target (where the function equals zero) will no longer be needed.

3. Application to radiotherapy treatment planning

The calculation of radiotherapy treatment plans is inherently a multiobjective problem (Breedveld, Craft, van Haveren & Heijmen, 2019). In this work, the treatment modality considered is Intensity Modulated Radiation Therapy (IMRT). In IMRT, the radiation is externally delivered to the patient, who is laying on a coach, by a linear accelerator. The head of this linear accelerator has a mutlileaf collimator with a set of right and left leaves that can move, modulating the radiation intensity to conform as well as possible to the volumes to treat (Planning Target Volumes – PTVs). The structures of interest, namely PTVs and organs to spare (Organs at Risk - OARs) are delineated in the patient's medical images (Computed Tomography). Then, the medical prescription defines lower (for the PTVs) and upper (for the OARs) dose radiation bounds, that should be respected as much as possible: it is desirable that the PTVs receive the prescribed radiation dose but, at the same time, it is also desirable that all normal tissues, and especially OARs, receive as low radiation doses as possible. As the radiation is produced externally to the patient body, it must traverse healthy tissues to reach the PTVs, so these objectives are conflicting with each other. The set of objectives that have to be considered in the decision-making process depends heavily on the disease site and the complexity of the case. In head-and-neck cancer cases, for instance, there are many OARs that need to be considered, so the RO has to take into account a large number of objectives in the decision-making process.

Treatment plan optimization is carried out by a planner (usually a medical physicist) that interacts with a Treatment Planning System (TPS). The planning of the treatment requires the definition of all the treatment delivery settings. In IMRT this means deciding how many and which are the radiation incidences, and what should the radiation intensity be from each one of those incidences, defining what is usually known as the fluence map. The planner configures a given objective function in the TPS, that is related with the dosimetric measures established by the medical prescription, by defining some parameters (like weights and lower/upper bounds). Then, the TPS calculates the radiation intensity maps (fluence map optimization) that define the radiation to be delivered, as well as the sequencing process (the definition of the movement of the right and left leaves in the multileaf collimator from each of the radiation incidences). These optimization steps constitute a process usually known as inverse planning optimization. Some examples of different mathematical models and optimization algorithms used can be found in (Dursun, Taşkın & Altınel, 2019; Lim, Kardar, Ebrahimi & Cao, 2020; Lin, Lim & Bard, 2016; Rocha, Dias, Ferreira & Lopes, 2013a, 2013b; Zaghian, Lim & Khabazian, 2018). The planner must try different parameters, in a trial-and-error and time-consuming process, since different parameters will originate different treatment plans, with different characteristics and presenting different compromises between existing objectives. Some of the calculated treatment plans may be clinically unacceptable, by not respecting hard constraints that should be assured (considering tumor control and severe normal tissue complication probabilities, for instance). These unacceptable plans must be discarded. However, those plans that are clinically acceptable are, most of the times, not easily comparable since they will benefit some objectives in detriment of others (they are Pareto-optimal solutions, within the set of treatment plans calculated).

After finding a set of high-quality Pareto-optimal treatment plans, the planner presents these alternatives to the RO, who will choose a single one (the treatment that will be delivered to the patient). Choosing a treatment plan is not an easy task, even if all the treatment plans are clinically acceptable and are of high quality, because many different criteria must be simultaneously taken into account. Different treatment plans will possibly impact in a different way the probability of controlling the disease and the probability of treatment side-effects due to the irradiation of OARs.

Some of the available methods that consider explicitly the multicriteria dimension of radiotherapy treatment planning rely on lexicographic approaches (Breedveld, Storchi, Voet & Heijmen, 2012; van Haveren et al., 2017), where priorities have to be defined *a priori*. Other methods have considered Pareto surface navigation interfaces, where the RO can interact with the system, understanding the existing trade-offs and, in some situations, calculating new treatment plans (Craft & Monz, 2010; Dias, Rocha, Ventura, Ferreira & do Carmo Lopes, 2018; Ehrgott & Winz, 2008; Stubington, Ehrgott, Shentall & Nohadani, 2019). These interactive

Table 1
Groups of structures of interest.

Groups	Structures
PTV	Planning Target Volumes
Critical	Retinas
	Optical Nerves
	Chiasm
	Brainstem
	Spinal cord
	Temporal mandibular joint
	Mandible
Salivary	Parotids
Other	Brain
	Lens
	Pituitary gland
	Oral cavity
	Larynx
	esophagus
	Thyroid
	Lungs

systems seem appealing but, from a practical point of view, they suffer from some important limitations. It is not simple, for the RO, to understand the existing trade-offs, given the large number of existing criteria. Calculating new treatment plans is computationally simple only when the same set of radiation incidences is considered, because a new plan can be calculated by a simple linear combination of the radiation intensities defined by two other plans (Craft & Monz, 2010). To consider an interactive navigation system comprising plans generated with different radiation incidences is much more complex and computationally demanding (Dias et al., 2018). There are no available Pareto interactive based approaches for some treatment modalities, like Volumetric Modulated Arc Therapy, because of the complexity of calculating a Pareto front when the radiation incidences are defined by arcs and not by a set of a small number of incidences.

Most of the times ROs have a set of implicit preferences, that they are not able to express verbally, but that are inherently present in the daily choices they make. It would be extremely beneficial if they could take advantage of a tool that would allow these preferences to be elicited and to be incorporated in the treatment plan selection process. If these preferences could be known it would be possible to better support the RO decision making process, either by incorporating these preferences *a priori* in the inverse optimization procedures and/or by allowing a better selection of a smaller set of plans for the RO to assess. This would decrease the information overload ROs have to deal with. The remainder of this section describes the use of the preference inference mathematical programs presented in Section 2 to address this difficulty in verbalizing explicitly a preference model.

3.1. Experimental setup

In a first stage, a sample of 40 retrospective nasopharynx cancer cases, treated by co-planar IMRT, were considered. For each case, two plans were created. An experienced RO was invited to compare each pair of plans, and to indicate her choice for each case. If the plans were considered equivalent, both plans could be selected. It was also possible to reject plans that would be considered not clinically acceptable. In clinical practice, this choice is usually supported by resorting to the analysis of the dose distribution, dose statistics, the dose volume histogram (that relates radiation dose to tissue volume), for instance. This is a very complex process since the RO has to take simultaneously into account all the structures of interest. For this disease site, many OARs (typically 10–20) and, in many cases, more than one PTV needs to be considered (Table 1).

In a second stage, the described methodology was applied in order to elicit preferences from the paired comparisons made. This elicitation of preferences was based on an existing graphical assessment tool for radiotherapy treatment planning, SPIDERPLAN (Ventura, Lopes, Ferreira & Khouri, 2016), which uses a scoring approach to assess and compare the quality of radiation therapy treatment plans. This graphical tool considers all the structures of interest organized into groups, that can be defined based on the preferences of the RO or the clinical protocol of the health institution. In the described case, the groups followed what is suggested by the Radiation Therapy Oncology Group (RTOG 0615 - Table 1). For each structure, a score is calculated that shows whether the planning goals are or are not being satisfied. If, for a given structure, the planning goal is being reached exactly at the threshold defined then the corresponding score is equal to 1. If it does not comply with the planning goal, then the score is greater than one. If the dose delivered not only fulfills the goal but goes beyond (under) the threshold defined, then the score is less than 1. The dosimetric values that were considered for calculating this score, for each one of the 40 cases, are presented as Supplementary Material.

Suppose that the planning goal for a given PTV is to assure that 98% of the PTV volume receives at least 95% of the prescribed dose (DP_{98} – this corresponds to one point in the dose-volume histogram). Then, the score for this structure will be given by DP_{98}/DD_{98}, where DD_{98} represents the dose delivered to 98% of the PTV, for this treatment plan. This means that the score will be less than or equal to 1 if the planning goal is being achieved and it will be greater than 1 if the goal is not being achieved. For the OARs, the less delivered dose the merrier, so similar scores are considered, but in the opposite direction: DD_{OAR}/DP_{OAR}. The global score for the plan is given by considering a weighted relative sum of the individual scores: the lower the score, the better the plan. Although it is possible to consider different weights for different structures within each group, in this work we assume that all the structures within a group have the same weight (the group is assessed as a whole, the score of a group is a simple average of the scores of its structures).

The fact that complying with a given planning goal corresponds to a score that is less than or equal to 1 does not mean that the target value should be fixed at 1. Actually, the value of 1 is only associated with an admissibility compliance, and not with what the RO would like to obtain. This is particularly important for OARs: whilst a treatment plan achieving the planning goals can be considered as clinically acceptable, the RO will generally prefer plans that spare as most as possible these structures. In some cases, however, significant overlaps between the volumes to treat and structures that should be spared make it impossible to calculate plans with scores less than or equal to 1 for all the groups: there may be structures where it is not possible to comply with the thresholds defined. Once again, compromises have to be accepted.

SPIDERPLAN has already demonstrated to be a valuable tool for the assessment of plan quality (Ventura et al., 2019, 2016; Ventura, Lopes, Rocha, da Costa Ferreira & Dias, 2020). However, weights have to be defined *a priori*, and this can be a very difficult task because they implicitly represent the importance the RO gives to each structure, which is exactly what the RO has difficulties in making explicit. The objective is, thus, to develop a tool that, based on the choices made by the RO in these paired comparisons, allows for a preference elicitation that can be used from that moment forward.

The mathematical formulations presented in Section 2 were applied to this dataset ($|J|$=40). All the experiments used the same parameter values $\varepsilon = 0.001$ and $h = 0.01^*|J|$. The set of breakpoints was always the same for all the dimensions: B_g={$b_{g,0}$,...,

Table 2
Targets (x_g), implicit weights (ω_g), and quadratic weights (w_g) according to each model.

		PTV	Critical	Salivary	Other
Monotonic	x_g	0.5	0.5	0.5	0.9
	ω_g	0.005	0.933	0.005	0.057
Convex	x_g	0.5	0.5	0.5	0.9
	ω_g	0.066	0.021	0.002	0.911
AED ($w_g \in [0.01, 100]$)	x_g	0.8	1	1.2	1.2
	ω_g	0.273	0.027	0.007	0.692
	w_g	0.182	0.026	0.011	1.038
AED ($w_g \in [0.1, 10]$)	x_g	0.8	1	1.2	1.2
	ω_g	0.314	0.120	0.077	0.489
	w_g	0.182	0.100	0.100	0.636
AED (Unweighted)	x_g	0.95	1.1	1.2	1.2
	ω_g	0.345	0.254	0.200	0.200
	w_g	1	1	1	1

Table 3
Mean error and percentage of violations considering the entire sample.

Model	Mean Error	Violations
Monotonic	4.75E-4	5.0%
Convex	6.19E-4	5.0%
AED ($w_g \in [0.01, 100]$)	7.60E-4	5.0%
AED ($w_g \in [0.1, 10]$)	1.95E-3	5.0%
AED (Unweighted)	1.44E-2	7.5%

$b_{g,m(g)}\}$={0, 0.5, 0.8, 0.9, 0.95, 1.0, 1.05, 1.1, 1.2, 1.5, 2}. The breakpoints are not equally distributed; they are concentrated around 1.0, which is the anticipated value for the target. The AED formulation c) (Section 2) was applied three times, considering $w_g = 1,\ \forall g \in G$ (i.e., no weighting), considering $w_g \in [0.01, 100],\ \forall g \in G$, which allows the weights to vary almost freely, and considering $w_g \in [0.1, 10],\ \forall g \in G$, as an intermediate situation. The more general monotonic and convex formulations a) and b) (Section 2) were also applied as benchmarks. The targets were bounded to belong to the interval [0.5, 1.2], by inserting constraints such that the disutility of 0.5 equals zero and the disutility of the first breakpoint after 1.2 (i.e., breakpoint 1.5) is at least $\varepsilon = 0.001$.

The first experiment assessed the mean error (objective function) of the different formulations for the given sample. A second experiment consisted in measuring the mean error based on part of the sample (80%) of the patients, and assessing the mean out-of-sample error corresponding to applying the inferred model to the remaining 20% of the patients. This was done five times, taking 20% of the patients successively. Since the convex and especially the monotonic formulations place less constraints on the value functions, their ability to reproduce the examples is obviously higher (lower in-sample error). However, it remains to be analyzed how they compare in terms of out-of-sample error and in terms of the readability of the resulting value functions. The possibility of easily interpreting the obtained value functions is of the utmost importance for guaranteeing the adoption of this methodology by medical physicists and ROs.

3.2. Results

Fig. 1 depicts the value functions for the four groups of structures according to each model, inferred from the RO's choices. It can be observed that the monotonic model proposes abrupt increases in disutility, and these do not always occur near the target (the point to the right of which disutility is no longer zero). In contrast, disutility increases more gradually in the remaining models. The corresponding targets and weights are presented in Table 2. Targets and weights must be considered jointly: if a group has a large weight but also a target greater than 1, the latter diminishes its importance.

Two fitness measures were considered to assess how well each model could reproduce the DM's choices: the mean error term, i.e. $z/|J|$, and also the percentage of pairs for which the value of the alternative less preferred by the RO is greater than the value of the more preferred alternative by a difference of ε or larger (% violations). The division by $|J|$ allows the comparison with the results of the out-of-sample analyses presented further below, in which one part of the sample is not used. Table 3 presents the results obtained considering the entire set ($|J|$=40). In most of the cases the model is capable of mimicking the choice of the RO, and the mean error obtained is low. Remembering that $\varepsilon = 0.001$ represents an arbitrarily small positive constant to enforce a strict inequality, the mean error is within the order of magnitude of ε for all models except the unweighted AED. Since the objective functions of the mathematical programs are minimizing the mean error, it is natural that the error increases for successively more constrained models.

One of the advantages of performing a preference elicitation is to be able to use the elicited parameters in posterior choices of treatment plans, contributing to an automated selection or, at least, to a definition of a smaller set of treatment plans for the RO to choose from. It is, thus, important to understand how the results obtained by the described methodology behave in data not used in the inference process. With this in mind, the same formulations presented before were used, but considering 80% of the cases only. This can be seen as a learning set. Then, the models obtained are applied to an out-of-sample test set that includes the remaining 20% cases. Results (for the same metrics depicted in Table 3) are presented in Table 4, after repeating the process for five different in-sample and out-of-sample sets.

Concerning in-sample error and violations, these results corroborate the conclusions obtained from the entire sample. From the analysis of these out-of-sample results, the best results are obtained by the convex and the AED ($w_g \in [0.01, 100]$) models.

In summary: The monotonic model is able to reproduce in-sample choices with the lowest error, but this may cause overfitting, since it is not among the best in terms of out-of-sample results and the inferred functions present drastic disutility increases.

The convex model has very good results, being almost as good as the monotonic model in terms of in-sample results, and having more gradual disutility increases. This model was the best in terms of out-of-sample error, but not concerning out-of-sample violations.

The weighted AED model has also very good results, being almost as good as the monotonic and convex models in terms of in-sample results, and having gradual and smoother disutility increases. Model AED ($w_g \in [0.01, 100]$) was the second best in terms of out-of-sample error, and the best concerning out-of-sample violations.

The unweighted AED model performed poorly from every perspective, being too rigid to fit well the DM's judgment.

3.3. Feedback on results

The development of a preference elicitation methodology in the presented context of radiotherapy treatment planning should be done understanding the multidisciplinary nature of the decision-making process and involving, in all the development stages, the stakeholders, namely medical physicists and ROs. The presented methodology can be implemented in real practice only if these stakeholders can understand the process, can easily interpret the results, consider them as trustworthy and accept that the obtained models are indeed representing their preferences and are capable

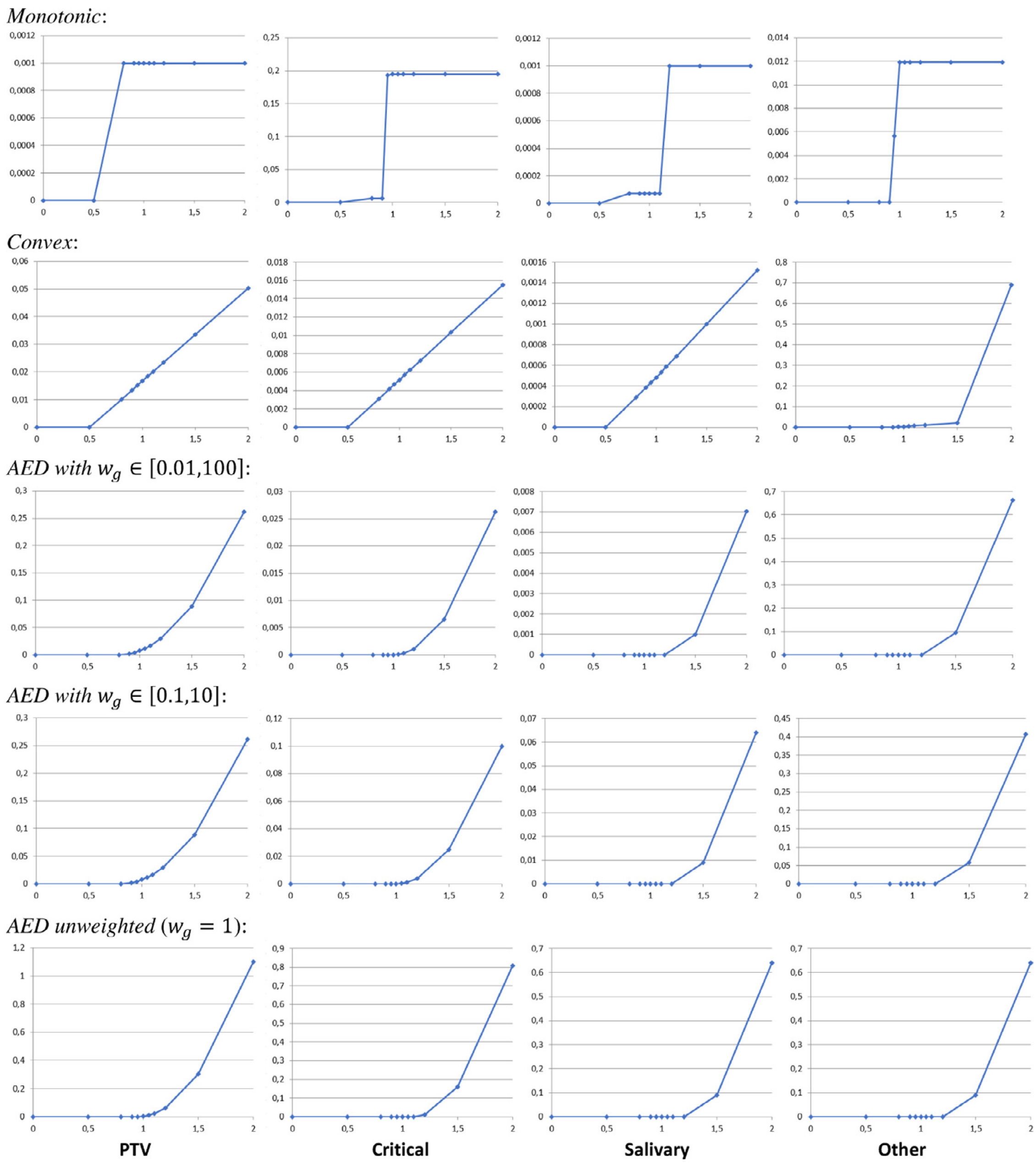

Fig. 1. Inferred value functions for the four groups. (Note: vertical scale changes from graph to graph).

Table 4
Mean error and percentage of violations considering 80% of the cases in the sample and 20% out-of-sample. The values presented are average values calculated over the five experiments.

Model	In-sample Mean error	Mean violations	Out-of-sample Mean error	Mean violations
Monotonic	4.61E-4	4.4%	2.78E-3	10.0%
Convex	5.85E-4	5.6%	9.70E-4	10.0%
AED ($w_g \in [0.01, 100]$)	7.18E-4	6.3%	2.37E-3	5.0%
AED ($w_g \in [0.1, 10]$)	1.93E-3	5.6%	4.42E-3	5.0%
AED (Unweighted)	1.44E-2	7.5%	1.54E-2	10.0%

of mimicking their choices. It is thus crucial to share the results with them to obtain their feedback.

Looking at the inferred value function representations (Fig. 1), the models these stakeholders considered easier to interpret and closer to what they consider to be their own reasoning process are AED with the intervals [0.01, 100] and [0.1, 10]. The value functions inferred by these two models clearly indicate the need of choosing treatment plans that guarantee the compliance with the PTV and critical structures dosimetric thresholds. All the plans that do not comply with these boundaries will be penalized. There is more flexibility in the target definition for Salivary and Other structures, as would be expected. This flexibility is important especially in complicated cases where it might not be possible to simultaneously comply with the desired threshold for all the considered dimensions. The interpretation of the obtained weights, as already mentioned, cannot be done independently of the target values calculated. Although PTV and Critical groups are very important, and can be determinant in the assessment that is made, they do not need to present the largest weights, since the calculated targets are lower than the targets calculated for other groups (meaning that the same value will lead to a higher deviation value for these groups than for the others). The weights associated with each one of the groups do indeed make sense. The larger weight associated with Others group, for instance, can be explained by the large number of structures included in this group. If one of these structures is being more irradiated than the RO considers it should or could be, then this is enough to justify the choice of another plan, affecting the overall importance assigned to this group. The lower weights associated with the Salivary group can also be interpreted by the fact that this group is composed of only two structures (parotids) that, even if over irradiated, will not lead to life-threatening situations for the patient (although considerably affecting quality of life). The analysis of the corresponding target represents this flexibility, but also the clear definition of a tolerance bound, beyond which plans will be heavily penalized.

Both models present very similar in-sample and out-of-sample results. Considering the overall analysis of inferred value function charts, and the corresponding weights and targets, the final choice would be the AED with the [0.1, 10] interval.

4. Discussion and conclusions

In this work we have developed an asymmetric version of (Srinivasan & Shocker, 1973)'s target-based model. This model allows the inference of the DM's preferences, based on a given set of comparisons among pairs of solutions. One of the distinguishing features of the described methodology is the possibility of considering the target as defining desired admissibility thresholds in each of the considered dimensions, instead of an ideal point. Thus, only solutions that present values greater than those targets (assuming minimization criteria) are penalized in the corresponding dimensions.

To assess the feasibility of applying the proposed methodology in a real decision-making situation, an application of the work developed to radiotherapy treatment planning considering a set of retrospective cancer cases treated at the Portuguese Oncology Institute of Coimbra has also been described. The results were compared with two benchmarks based on UTA: a simple formulation to infer a monotonic piecewise linear approximation, and an adaptation of the latter to consider only convex penalty functions. The convex function can be considered an intermediate situation between the formulation allowing any monotonic function and the formulation imposing a specific asymmetric quadratic shape.

It was possible to infer value functions that were easily interpretable and that could indeed represent the reasoning behind the choices made, something that is hard to make in an explicit way. The weighted AED models were the preferred ones, given the achieved goodness of fit and the simple interpretation and legibility of the results. Compared with the unweighted AED, it is possible to conclude that weights can be useful, and contribute to a better acceptance of the model as being an adequate representation of the existing implicit preferences.

The obtained results can be influenced by one important characteristic of the data sample used: all the paired comparisons were made between high quality plans, as usually happens in clinical practice. ROs do choose between high quality treatment plans, since low quality plans are immediately discarded. It is expected that the value functions inferred could be different if clinical plans not complying whatsoever with minimal clinical standards were also amongst the treatment plans considered. Thus, the value functions inferred in this study cannot be interpreted as being representative of the RO's preferences in general. The current inverse treatment planning optimization algorithms available, and the experience of most planner teams, contribute to the generation of sets of high-quality alternative plans. From a preference learning point of view, a greater variation between the plans in terms of quality might be desirable in general, but this would not represent the type of decisions that actually are made in the clinical practice.

The potential advantages of having available such a methodology in the radiotherapy treatment planning context are numerous. It is possible to incorporate the elicited preferences in a score that can drive the inverse treatment planning optimization procedure, reducing the computational time and avoiding the trial-and-error procedure. As this method can also be applied considering the input of multiple DMs (future work that we are planning on doing) it will be possible to guarantee an increased homogeneity in the choice of the treatment plans within a given health institution. It will also be able to support less experienced ROs.

There are many developments that can be considered, based on the achieved results. In the presented application, weights and target values were considered at the group level. It is also possible to have more detailed information considering the individual structures level. It would also be interesting to understand whether the calculated weights and targets allow for an inference of criteria interactions. This is especially interesting for some structures where one of the structures is impaired, but it is still possible to use the other.

Other developments can address some of the limitations of the proposed models, which are inherited from Srinivasan and Shocker's SSED model. In these models, the inference results depend on the number of preference judgments available, and therefore might not be much robust if this number is small. This calls for experimental studies studying the interplay between the characteristics of the problem, namely the number of dimensions considered, and the number and diversity of judgments needed for having stable results. Post-optimization formulations as performed in other methods (e.g., Kadziński et al., 2017; Siskos et al., 2016) can also be considered as future extensions to find extreme parameter values compatible with the provided judgments, even though in our specific application the provided judgments were not perfectly compatible. Other variants of these models can be considered to avoid the use of the parameter h (and corresponding constraint) as done by Srinivasan and Shocker to avoid the trivial solution with all weights equal to zero, by explicitly placing other constraints that can achieve the same purpose.

This methodology can be used in many other settings where some values may act as targets and the main objective of the DM is meeting all the targets as well as possible (Bordley & Kirkwood, 2004). Examples might include satisfying safe operating conditions in a system, meeting management targets concerning several attributes or several projects, or competing in a market without losing to the market leader on important attributes.

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Supplementary materials

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