

Title	A mathematical programming-based solution method for the nonstationary inventory problem under correlated demand
Authors	Xiang, Mengyuan;Rossi, Roberto;Martin-Barragan, Belen;Tarim, S. Armagan
Publication date	2022-04-22
Original Citation	Xiang, M., Rossi, R., Martin-Barragan, B. and Tarim, S. A. (2022) 'A mathematical programming-based solution method for the nonstationary inventory problem under correlated demand', European Journal of Operational Research. doi: 10.1016/j.ejor.2022.04.011
Type of publication	Article (peer-reviewed)
Link to publisher's version	10.1016/j.ejor.2022.04.011
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Download date	2024-04-28 17:44:52
Item downloaded from	https://hdl.handle.net/10468/13214

Appendix A. List of symbols

T	periods in the planning horizon
d_t	random variable
ζ_t	value of random variable d_t
$\tilde{d}_t$	the expected value of random variable d_t
$g(\cdot)$	probability density function
F_t	realised demand set at the beginning of period t
I_t	inventory level at the end of period t
I_0	initial inventory level at the beginning of the planning horizon
Q_t	ordering quantity placed at the beginning of period t
$u(\cdot)$	ordering cost
K	fixed ordering cost
c	proportional ordering cost
h	proportional holding cost
b	proportional penalty cost
$f_t(I_{t-1}, F_t; Q_t)$	immediate cost of period t with opening inventory level I_{t-1} , realised demand set F_t , and order quantity Q_t
$C_t(I_{t-1}, F_t)$	the expected total cost of an optimal policy over period $t, \dots, T$ with opening inventory level I_{t-1} and realised demand set F_t
S_t	order-up-to-level of period t
δ_t	binary variable
$\bar{C}_1(I_0)$	expected total cost over period $1, \dots, T$ under (R, S) policy with initial inventory level I_0
d_{jt}	a random variable denotes the demand over period $j, \dots, t$, i.e. $d_{jt} = d_j + \dots + d_t$
ζ_{jt}	value of random variable d_{jt}
$\tilde{d}_{jt}$	expected value of the convolution $\tilde{d}_j + \dots + \tilde{d}_t$
ω	a random variable
x	a scalar value

$L(x, \omega)$	first order loss function
$\hat{L}(x, \omega)$	complementary of first order loss function
P_{jt}	a binary variable which is set to one if the most recent replenishment up to period t was issued in period j , where $j \leq t$ — if no replenishment occurs before or at period t , then we let $P_{1t} = 1$, this allows us to properly account for demand variance from the beginning of the planning horizon
Ω	support of d_{jt}
W	number of regions in a partition of Ω
i	region index ranging in $1, \dots, W$
Ω_i	the i^{th} subregion of Ω
p_i	$Pr(d_{jt} \in \Omega_i)$
$E[d_{jt} \Omega_i]$	conditional expectation of d_{jt} in Ω_i
$\tilde{H}_t$	the upper bound to the true value of $\sum_{j=1}^t \hat{L}(S_j, d_{jt})P_{jt}$
$\tilde{B}_t$	the upper bound to the true value of $\sum_{j=1}^t L(S_j, d_{jt})P_{jt}$
e_W^{jt}	approximation error
σ_{jt}	the standard deviation of d_{jt}
Z	a standard normal random variable
$\mathcal{N}(\mu, \sigma^2)$	a normal random variable with mean μ and variance σ^2
H	forecast horizon
$D_{s,t}$	demand forecasts made in period s for period t
$\epsilon_{s,t}$	the forecast update made at the end of period s for period t
ϵ_s	forecast update vector generated at the end of period s

Table A.1: A list of symbols

Appendix B. Expected demands of the computational study

This section presents the expected demands for various demand patterns.

Appendix C. Optiamlity gap% for different correlation coefficients

time period	LCY1	LCY2	SIN1	SIN2	EMP1	EMP2	EMP3	EMP4	STA
1	15	3	15	12	3	2	6	9	10
2	16	6	4	7	8	12	7	3	10
3	15	7	4	7	13	14	4	11	10
4	14	11	10	10	22	25	6	11	10
5	11	14	18	13	12	20	8	26	10
6	7	15	4	7	8	13	16	27	10
7	6	16	4	7	11	10	6	11	10
8	3	15	10	12	5	16	24	11	10

Table B.2: Expected demands for various demand patterns

Settings		Mean	Median	IQR	Maximal	Minimal
demand patterns	LCY1	0.38	0.08	0.08	3.02	0.02
	LCY2	0.09	0.09	0.03	0.17	0.04
	SIN1	0.13	0.07	0.05	0.84	0.01
	SIN2	0.05	0.05	0.05	0.10	0.01
	STA	0.07	0.09	0.07	0.18	0.01
	EMP1	0.25	0.08	0.07	1.37	0.00
	EMP2	0.23	0.09	0.20	1.03	0.02
	EMP3	0.07	0.05	0.04	0.21	0.03
	EMP4	0.17	0.07	0.08	0.97	0.01
fc	150	0.24	0.08	0.07	3.02	0.01
	300	0.08	0.07	0.06	0.34	0.00
uc	0	0.23	0.08	0.03	3.02	0.01
	1	0.09	0.07	0.07	0.69	0.00
pc	5	0.07	0.08	0.06	0.17	0.02
	10	0.15	0.07	0.07	0.97	0.00
	20	0.26	0.08	0.05	3.02	0.01
cv	0.1	0.07	0.07	0.06	0.42	0.00
	0.2	0.25	0.08	0.07	3.02	0.01
rho	-0.75	0.07	0.08	0.05	0.17	0.00
	-0.25	0.10	0.07	0.06	1.03	0.01
	0.25	0.29	0.07	0.09	3.02	0.01
	0.75	0.16	0.08	0.11	0.84	0.01
Overall		0.16	0.07	0.07	3.02	0.01

Table C.3: Optimality gaps % for different pivot parameters