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Electronic Notes in Discrete Mathematics 25 (2006) 113–116
Electronic Notes in DISCRETE MATHEMATICS
www.elsevier.com/locate/endm

Adjacent-Resource Scheduling

Why spatial resources are so hard to incorporate

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Keywords: RCPSP, Spatial Resource, Complexity, Online scheduling

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1 Introduction

In many practical settings of project scheduling one has to deal with spatial resources. For example, when building a ship an amount of space is required on the dry dock, the same for any other kind of assembly which has to be preformed in some kind of space. We investigate the impact of spatial resources on the complexity of resource constraint project scheduling problems (RCPSP). To get an insight in the problems caused by spatial resources we look at some polynomial solvable machine scheduling problems, which are special cases of project scheduling problems. We show that the most elementary machine scheduling problems with the addition of a spatial resource become NP-hard.
The concept of spatial resources is defined by de Boer [2], as a resource used by a group of activities, such that the resource is occupied from the start of the first activity till the completion of the last activity of the group. In addition to this, spatial resources are not divisable and distributable like normal renewable resources. A group requires to be assigned to adjacent resource units, and it is not allowed to shift this assignment to other resource units.

doi:10.1016/j.endm.2006.06.083

For example, when a ship has to be docked for repairs it needs adjacent parts of the dock, and it is not an option to shift the ship during the repears. Duin and van der Sluis [5] study the adjacency requirement for check-in counters at airports. They show that minimizing the number of counters, given the number of counters needed at each time slot for each flight, is NP-hard.
In this paper, firstly, we show that the complexity of elementary machine scheduling problems, with the addition of one spatial resource become NP-hard. Secondly, we look at online scheduling with one spatial resource as the only resource. Finally, we discuss some possible directions of future research.

2 Machine scheduling with one spatial resource

Intuitively it seems clear that scheduling with the addition of spatial resources becomes harder. Spatial resources add an extra dimension to the problem, first there was only time, now there is space as well. In this section, we show that this intuition is correct by means of adding one spatial resource to machine scheduling problems.
We use the classification scheme $\alpha|\beta|\gamma$, introduced by Graham et al. [6] for machine scheduling, which is extended for project scheduling (see for an overview Demeulemeester and Herroelen [4]). In the α field an S denotes the presence of a spatial resource. With $q_j \in [0,1]$ or $q_j \in \{0,1\}$ in the β field we denote that the amount required for job j is between 0 and 1 or equal to 0 or 1, respectively. The capacity of the spatial resource is normalized to 1.
Parallel machine scheduling problems with unit processing times and a makespan ($P|p_j = 1|C_{max}$) or a sum of completion times ($P|p_j = 1|\sum C_j$) objective, are known to be polynomial solvable. If we add a spatial resource we can prove the following by reductions from $3 - Partition$:

Theorem 2.1 *$P3, S|p_j = 1, q_j \in [0,1]|C_{max}$ is strongly NP-hard.*

Corollary 2.2 *$P3, S|p_j = 1, q_j \in [0,1]|\sum C_j$ is strongly NP-hard.*

With a spatial resource added to flow shops or open shops we can derive similar statements:

Theorem 2.3 *$F2, S|q_j \in \{0,1\}|C_{max}$ is strongly NP-hard.*

Corollary 2.4 *$F2, S|pmtn, q_j \in \{0,1\}|C_{max}$ is strongly NP-hard.*

Theorem 2.5 *$O3, S|p_{ij} = 1, q_j \in [0,1]|C_{max}$ is strongly NP-hard.*

Corollary 2.6 *$O3, S|p_{ij} = 1, q_j \in [0,1]|\sum C_j$ is strongly NP-hard.*

Theorem 2.7 *$O2, S|q_j \in \{0,1\}|C_{max}$ is strongly NP-hard.*

We have proven that basic scheduling problems on more than one machine are strongly NP-hard when spatial resources are added. With the reduction schemes from Brucker [3], we see immediately that virtually every scheduling problem becomes strongly NP-hard with the presence of spatial resources.

3 Online scheduling with one spatial resource

When one spatial resource is the only resource in the RCPSP, we have a problem very similar to the strip packing problem (first studied in [1]). Duin and van der Sluis [5] prove that even when all start times of the jobs are fixed, this problem is still strongly NP-hard. Since for the RCPSP without spatial resources it is common to use a serial planning heuristic, we are interested what an online algorithm can do in this situation. If we can find a good online algorithm, it can serve as a placement rule within the existing serial planning heuristics. Because of the complexity results we focus on problems with a small fixed number of spatial units (the spatial resource is defined in Q equally sized units), in particular 3. We study the following problem. Each job j has a processing time of p_j and a spatial requirement of $q_j \in \mathbb{N}$. There is a 1-dimensional space available for the jobs which has a capacity of $Q \in \mathbb{N}$ resource units. The spatial units assigned to a job must be connected. Once a job is scheduled, it cannot be preempted or moved within the spatial resource. As soon as we start working on the most recent scheduled job, we are given a new job to schedule and its characteristics become known. The objective is to minimize the makespan. All parameters are integers.

Theorem 3.1 *No online algorithm for the online scheduling problem with* 3 *resource units can be* ρ*-competitive with* $\rho < \frac{2\sqrt{5}}{1+\sqrt{5}}$ *(*≈ 1.38*).*

Conjecture 3.2 *There exists a* $\frac{2\sqrt{5}}{1+\sqrt{5}}$*-competitive algorithm for the online scheduling problem with* 3 *resource units.*

We expect there does not exist a reasonable competitive ratio for arbitrary values of Q. Therefore, combining a simple placement rule with existing serial planning techniques for RCPSP will not suffice.

4 Future Research

This reseach has shown us that it is very complicated to incorporate spatial resources. In future research we will focus on the development of heuristic methods to tackle the RCPSP with spatial resources. Currently we are working on an approach that solves the groups to spatial resource assignment prior to the scheduling of the activities.

References

[1] B.S. Baker, E.G. Coffman and R.L. Rivest. Orthogonal packings in two dimensions. *SIAM, J. Comput. 9 (1980) 846-855*

[2] R. de Boer. Resource-constrained Multi-Project Management, A Hierarchical Decision Support System. *PhD thesis, University of Twente, 1998*

[3] P. Brucker. Scheduling Algorithms. *Springer (2004)*

[4] E.L. Demeulemeester and W.S. Herroelen. Project Scheduling, a research handbook. *Kluwer (2002)*

[5] C.W. Duin and E. van der Sluis. On the complexity of Adjacent Resource Scheduling. *Journal of Scheduling (2005) (to appear)*

[6] R.L. Graham, E.L. Lawler, J.K. Lenstra and A.H.G. Rinnooy Kan. Optimization and approximation in deterministic sequencing and scheduling: A Survey. *Annals of Discrete Mathematics 5 (1979) 287-326.*