Tang, Guolin, Yang, Yongxuan, Gu, Xiaowei, Chiclana, Francisco, Liu, Peide and Wang, Fubin (2022) ***A new integrated multi-attribute decision-making approach for mobile medical app evaluation under q-rung orthopair fuzzy environment.*** **Expert Systems with Applications, 200 . ISSN 0957-4174.**

A new integrated multi-attribute decision-making approach for mobile medical app evaluation under *q*-rung orthopair fuzzy environment

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Abstract: Mobile medical app evaluation can be modelled as a multi-attribute decision-making (MADM) problem with multiple assessment attributes. Due to the increasing complexity and high uncertainty of decision environments, numerical numbers and/or traditional fuzzy sets may not be appropriate to model attribute information of mobile medical apps. In addition, heterogeneous relationships are often observed among different attributes in various practical decision situations. To deal with these issues, a q-rung orthopair fuzzy (q-ROF) MADM approach, which is a very powerful tool for describing vague information occurring in real decision circumstances, is proposed to handle decision-making problems in medical app evaluation. In particular, q-rung orthopair fuzzy numbers (q-ROFNs) are first applied to better express the preference information and expert assessment information. Then, q-ROFNs are extended by combining with Zhenyuan integral, resulting in the q-ROF Zhenyuan integral (q-ROFZI). This integral can capture complementary, redundant and/or independent characteristics among the attributes and is superior to existing operators on q-ROFNs. Next, based on the best-worst method (BWM) and Shapley value, two optimization models are constructed to objectively identify optimal fuzzy measures on the attribute set. Finally, a novel integrated q-ROF MADM approach is proposed and its computation procedure is presented and illustrated with its application to the problem of mobile medical app evaluation. A comparative analysis is carried out to demonstrate the validity, rationality, robustness and superiority of the developed method.
Keywords： Multi-attribute decision-making, Mobile medical app, q-Rung orthopair fuzzy numbers, Zhenyuan integral, Best-worst method.

1. Introduction

While providing convenience for personal health, a mobile medical app is also accompanied by corresponding management problems, such as insufficient user privacy protection, difficult verification of information authenticity, high risk of telemedicine diagnosis and false medical advertisements, etc., which lead to poorer user experience and seriously impair the reputation of doctors and hospitals. In order to enhance the competitiveness and better serve users, mobile medical app evaluation is necessary. Mobile medical app evaluation can be modelled as a multi-attribute decision-making (MADM) problem with multiple assessment attributes, including functionality, safety, interface and reliability. In recent years, several researched have applied MADM-based methodologies to mobile medical app evaluation and selection issues. For example, Li (2018) used the Delphi method and AHP (Analytic Hierarchy Process) model to evaluate mobile medical app. Li et al. (2021) integrated the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) and AHP model for the medical and health app user experience evaluation. Considering the complexity of objective things and disturbances from internal or external uncertainty and ambiguity, it is more realistic for expert to describe assessment information using fuzzy numbers (FNs) (Dixit et al., 2020; Venugopal et al., 2022; Sirbiladze, 2021). Based on this observation, Xu (2018) proposed an integrated approach for evaluating mobile medical app based on the AHP model and fuzzy comprehensive evaluation method.

In the existing fuzzy mobile medical app evaluation research, the traditional FNs serve as a tool to represent uncertain information. However, in the real word, for mobile medical app evaluation problems, most of the assessment detailed information is unknown and there exists a high degree of uncertainty. Consequently, it is insufficient to describe attribute values of alternatives using traditional FNs and their extensions, such as intuitionistic fuzzy numbers (IFNs) (Atanassov, 1986; Atanassov, 1989) and Pythagorean fuzzy numbers (PFNs) (Yager & Abbasov, 2013). On the other hand, q-rung orthopair fuzzy numbers (q-ROFNs) (Yager, 2017) are viewed as a feasible tool to cope with high uncertainty because they can maximize the accuracy and integrity of fuzzy information, making them appropriate for managing the diversity and uncertainty of decision maker' assessment information in mobile medical app evaluation. The concept of q-ROFNs was proposed by Yager (2017), whose membership function (MF), $t(QR)$, and non-membership function (NMF), $f(QR)$, meet the condition $(t(QR))^{q} + (f(QR))^{q} \leq 1$. When $q = 1$, q-ROFNs reduce to IFNs, while q-ROFNs reduce to PFNs given $q = 2$. In other words, IFNs and PFNs are special cases of q-ROFNs. Furthermore, q-ROFNs provide greater range for decision makers (DMs) to describe uncertain information because the value of the flexible parameter q can be dynamically changed to conform the information expression range. Therefore, compared with IFNs and PFNs, q-ROFNs are more flexible and fit for depicting the uncertain information in actual management decision-making problems.

Due to their effectiveness and flexibility, q-ROFNs have been applied widely in MADM problems, and a variety of q-ROF MADM methods have been developed. Existing q-ROF MADM methods can be roughly divided into two categories: extended traditional decision-making methods (Alkan & Kahraman, 2021; Arya & Kumar, 2020; Khan et al., 2021; Wang et al., 2020; Wang et al., 2020a) and aggregation operators' methods (Darko & Liang, 2020; Garg & Chen, 2020; Jin et al., 2021; Khan et al. 2019; Yang et al., 2021). The first category is inferior to the second because it only ranks alternatives. In contrast, the second category provides comprehensive values of alternatives along with a ranking of alternatives. Thus, the second category is now gaining increasing attention in the field of MADM. As for the second category, some q-ROF aggregation operators have been developed to aggregate attribute values with their related weights to rank the alternatives. For example, Darko & Liang (2020) studied several q-ROF Hamacher aggregation (Wq-ROFHA) operators for MADM problems; Garg & Chen (2020) investigated some weighted averaging neutral aggregation operators and applied them to handle MADM problems. These operators assume that all q-ROFNs are independent. The interaction among the aggregated arguments, which exists in reality, is neglected. Obviously, this is irrational for real-world decision-making problems. In order to capture the interaction among q-ROFNs, Wei et al. (2018) and Liu & Wang (2019) extended Heronian mean (HM) operator and Bonferroni mean (BM) operator to q-ROFNs, respectively, and proposed a family of q-ROF HM operators and a series of q-ROF BM operators for solving MADM problems. Nevertheless, these extended HM and BM operators can reflect interrelationships between two attributes. In order to consider interrelationships among multiple attributes, Liu & Wang (2020) and Xing et al. (2020) presented some q-ROF Maclaurin symmetric mean (MSM) operators and q-ROF Hamy mean (HAM) operators for handling MADM problems by utilizing MSM and HAM, respectively.

However, the above mentioned q-ROF aggregation operators only capture the homogeneous relationship of the attributes, i.e., they assume that each attribute is associated with other attributes. Nevertheless, in real situations, attributes do not always exhibit a homogeneous relationship. For example, when evaluating a mobile medical app, the following attributes are considered: functionality, safety, interface and reliability. Here, safety and interface could be considered as positively interactive attributes, while safety and reliability could be considered as negatively interactive attributes. In order to reflect heterogeneous relationships among attributes, Liang et al. (2019) and Yang & Pang (2019) extended Choquet integral and partitioned Bonferroni mean operator (PBM) to q-ROFNs, respectively, and proposed a q-ROF Choquet integral (q-ROFCI) (Liang et al., 2019) and a q-ROF weighted PBM (q-ROFWPBM) operator (Yang & Pang, 2019) for handling heterogeneous MADM problems. It should be noted that q-ROFCI can handle heterogeneous interrelationships in a more comprehensive manner than the q-ROFWPBM operator because it captures complementary, redundant or independent characteristics among the attributes. A q-ROFWPBM operator, however, assumes that all attributes are partitioned into several clusters, where attributes within the same cluster are relevant, but attributes in different clusters are seen as irrelevant.

The Zhenyuan integral, which is a nonlinear integral derived from fuzzy measure (Sugeno, 1974), was introduced by Wang et al. (2000) and it has become one of the most famous information aggregation operators in modern information fusion theory. In comparison with the Choquet integral, the prominent characteristic of the Zhenyuan integral lies in the fact that it considers the overall heterogeneous interaction among attributes, whereas the Choquet integral only captures the interaction between adjacent combinations of attributes. Therefore, Zhenyuan integral is a more powerful tool to aggregate information by considering both the relative importance of decision attributes and their heterogeneous relationships. Based on Zhenyuan integral, Zeng & Mu (2018) presented the intuitionistic fuzzy Zhenyuan averaging (IFZA) operator for handling information technology improvement project selection; Mu & Zeng (2019) introduced the Atanassov intuitionistic fuzzy Zhenyuan averaging (AIFZA) operator and the Atanassov intuitionistic fuzzy Zhenyuan geometric (AIFZG) operator to assess the complicated intuitionistic fuzzy problems; Mu et al. (2018) proposed the interval-valued intuitionistic fuzzy Zhenyuan averaging (IVIFZA) operator and the interval-valued intuitionistic fuzzy Zhenyuan geometric (IVIFZG) operator for MADM problems. However, to the authors' best knowledge, very few scholars have combined the Zhenyuan integral with IFNs and interval-valued IFNs, and no attempt has been made so far to integrate the Zhenyuan integral and q-ROFNs. Hence, the integration between Zhenyuan integral and q-ROFNs is the main focus of the present paper.

1.1 Motivation and novelty of the new model

As described above, the mobile medical app evaluation process involves both uncertain information aggregation and interactive characteristics of the attributes. Thus, fuzzy sets and their extensive forms, such as IFNs and PFNs, may be insufficient in handling real-world situations due to the increasing uncertainty of the mobile medical app evaluation problems, while the q-ROFNs can be used instead. When assessing a mobile medical app, safety and interface could be considered as positively interactive attributes, while safety and reliability could be considered as negatively interactive attributes. Redundant and complementary relationships are often observed among these attributes. Zhenyuan integral, on the other hand, is well-known for its strong capability of modelling heterogeneous relationships of the attributes. Hence, in this work, q-ROFNs and Zhenyuan integral are integrated together (herein named as q-ROFZI) to address the problems of mobile medical app evaluation under q-ROF settings. In addition, the lack of prior knowledge and limited human expertise in regard to mobile medical app evaluation problems implies that the information related to attribute weights is not always known in advance. In such cases, the weights of attributes should be identified first. The best-worst method (BWM), initially introduced by Rezaei (2015), is a new weighting models with two core merits: 1) it reduces the number of pairwise comparisons between attributes, and; 2) it maintains the consistency between judgements. Thus, two optimal models are constructed by integrating the BWM and Shapley value (herein named as q-ROFBWM) to obtain the weights of attributes via optimal fuzzy measures.

Main innovations of this paper are summarized as follows.

- We use q-ROFNs in the integrated MADM for real-life mobile medical app evaluation problems within the context of q-ROFNs to attain greater flexibility in representing judgement for DM.

- We propose the q-ROF Zhenyuan integral (q-ROFZI), which is an integration of q-ROFNs and Zhenyuan integral, to capture the positively interactive, negatively interactive or independent characteristics of the attributes and effectively aggregate q-ROF information.
- We design two linear mathematical programming models for optimal fuzzy measures on the attribute set, based on the BWM and Shapley value, which can objectively obtain the weights of evaluation attributes.
- We further propose a new q-ROF MADM approach to solving the problems and provide an in-depth discussion against existing methods to demonstrate its validity, reliability and superiority, serving as a good example for users to evaluate a mobile medical app.

1.2 Organization of paper

The remainder of this paper is organized as follows: Section 2 briefly recalls some basic concepts and properties of q-ROFNs, fuzzy measure and Zhenyuan integral. In Section 3, based on q-ROFNs and Zhenyuan integral, a new q-ROF Zhenyuan integral is presented, and some desirable properties are studied. Section 4 builds two weight optimization models based on the BWM and Shapley value. Then a novel method for MADM is developed by integrating Zhenyuan integral and BWM within the q-ROF setting. In Section 5, the new approach is applied to solve a mobile medical app evaluation problem. Also, comparison analyses are presented to ascertain the effectiveness and superiority of the developed approach. Section 6 concludes this study and elaborates on future research studies.

2. Preliminaries

2.1 *q*-ROFNs

Definition 1 (Yager, 2017)**.** Let X denote the universe of discourse. A q-rung orthopair fuzzy set (q-ROFS) Q in X is represented as the set

$$Q=\left\{\left\langle x,u_Q(x),v_Q(x)\right\rangle \mid x\in X\right\} \tag{1}$$

where $u_Q:Q\to[0,1]$ is the MF and $v_Q:X\to[0,1]$ is the NMF of elements of X to the set A, respectively, subject to the restriction $0\le\left(u_Q(x)\right)^q+\left(v_Q(x)\right)^q\le 1$ $(q\ge 1)$. The degree of indeterminacy of element $x\in X$ is $\pi_Q(x)=\left(1-\left(u_Q(x)\right)^q-\left(v_A(x)\right)^q\right)^{\frac{1}{q}}$. For convenience, we call $Q=(u,v)_q$ a q-ROFN.

Definition 2 (Liu & Wang, 2018)**.** Given two q-ROFNs $Q_1=(u_1,v_1)_q$ and $Q_2=(u_2,v_2)_q$, $\forall\lambda>0$, their operational laws are as follows:

(1) $Q_1\oplus Q_2=\left(\left(u_1^q+u_2^q-u_1^qu_2^q\right)^{\frac{1}{q}},v_1v_2\right)_q$;

(2) $\lambda Q_1=\left(\left(1-\left(1-u_1^q\right)^{\lambda}\right)^{\frac{1}{q}},v_1^{\lambda}\right)_q$;

Definition 3 (Liu & Wang, 2018)**.** Assume that $Q=(u,v)_q$ is a q-ROFN, its score function is formulated as:

$$S(Q) = u^q - v^q \tag{2}$$

Definition 4 (Liu & Wang, 2018)**.** Assume that $Q = (u, v)_q$ is a q-ROFN, its accuracy function is formulated as:

$$H(Q) = u^q + v^q \tag{3}$$

Notice that $S(Q) \in [-1,1]$, while $H(Q) \in [0,1]$. For any two q-ROFNs $Q_1 = (u_1, v_1)_q$ and $Q_2 = (u_2, v_2)_q$, the following comparison rules (Liu & Wang, 2018) are defined:

(1) If $S(Q_1) > S(Q_2)$, then $Q_1 > Q_2$;

(2) If $S(Q_1) = S(Q_2)$, then

If $H(Q_1) > H(Q_2)$, then $Q_1 > Q_2$;

If $H(Q_1) = H(Q_2)$, then $Q_1 = Q_2$.

The following proposition derives from Definitions 1 and 4.

Proposition 1. Given two q-ROFNs $Q_1 = (u_1, v_1)_q$ and $Q_2 = (u_2, v_2)_q$, $Q_1 = Q_2$ if and only if there are $u_1 = u_2$ and $v_1 = v_2$.

2.2 Fuzzy measure and Shapley value

Definition 5 (Sugeno, 1974)**.** Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set, and $P(X)$ the power set of X. A fuzzy measure on X is a function $\mu : P(X) \to [0,1]$ that verifies the following properties:

(1) $\mu(\varnothing) = 0, \mu(X) = 1$;

(2) If $A, B \subseteq X$ and $A \subseteq B$, then $\mu(A) \leq \mu(B)$.

Fuzzy measures consider the weight of any combination of elements $x_i \in X$ and also express the relationship of such elements. In game theory, the generalized Shapley index measures the game power or strength of each coalition rather than the power of each of the players.

Definition 6 (Marichal, 2000)**.** The generalized Shapley index is

$$\varphi(\mu(S)) = \sum_{T \subset X \setminus S} \frac{(n-t-s)!t!}{(n-s+1)!} (\mu(S \cup T) - \mu(T)), \forall S \subset X \tag{4}$$

where n, t and s are the cardinalities of sets X, T and S, respectively.

Note that if there is only one element in S, (4) becomes the Shapley value:

$$\varphi(\mu(\{x_i\})) = \sum_{T \subseteq X \setminus \{x_i\}} \frac{(n-t-1)!t!}{n!} (\mu(\{x_i\} \cup T) - \mu(T)), \forall x_i \in X \tag{5}$$

From (4), we observe that the generalized Shapley index is an expected value of all marginal contributions between the coalition S and every coalition in $X \setminus S$, and the Shapley value is an expected value of all marginal contributions between the element x_i and every coalition in $X \setminus \{x_i\}$.

Property 1 (Meng & Tang, 2013)**.** Assume that μ is a fuzzy measure on $X = \{x_1, x_2, ..., x_n\}$, and φ is the Shapley value for the fuzzy measure μ on the set X. Then, we have $\varphi\left(\mu\left(\{x_i\}\right)\right) \geq 0$ and $\sum_{i=1}^{n} \varphi\left(\mu\left(\{x_i\}\right)\right) = 1$ for any $x_i \in X$.

It should be noted that when the fuzzy measure μ is defined on a criteria set X, $\mu(S)$ can be treated as the weight of the criteria set $S\left(S \subset X\right)$. Thus, in addition to the traditional weight on a single element $x_i (x_i \in X)$, weights on any combination of x_i can also be denoted by the fuzzy measure μ. In order to capture the influence from other attributes $T\left(T \subset X \setminus S\right)$ on the criteria set S, the generalized Shapley index $\varphi\left(\mu\left(S\right)\right)$ and the generalized Banzhaf index $\varphi'\left(\mu\left(S\right)\right)$ can be used. The generalized Shapley index $\varphi\left(\mu\left(S\right)\right)$ and the generalized Banzhaf index $\varphi'\left(\mu\left(S\right)\right)$ both verify some desirable properties, such as the monotonicity and the boundedness. However, the Shapley value $\varphi\left(\mu\left(\{x_i\}\right)\right)$ meet the constraint $\sum_{i=1}^{n} \varphi\left(\mu\left(\{x_i\}\right)\right) = 1$ whilst the Banzhaf value $\varphi'\left(\mu\left(\{x_i\}\right)\right)$ does not satisfy this, which implies that $\left\{\varphi\left(\mu\left(\{x_1\}\right)\right), \varphi\left(\mu\left(\{x_2\}\right)\right), ..., \varphi\left(\mu\left(\{x_n\}\right)\right)\right\}$ is a better weight vector for this study.

2.3 Zhenyuan integral

Definition 7 (Wang et al., 2000)**.** Given a set function $\mu : \partial \to [0, +\infty)$ satisfying $\mu(\varnothing)=0$, a set $D \in \partial$, and a function $h : X \to [0, +\infty)$, the Zhenyuan integral (ZI) of h with respect to μ on D, in symbol $\int_D h d\mu$, is defined as

$$\int_D h d\mu = \sup\left\{\sum_{i=1}^{k} \lambda_i \mu(G_i) \mid h \geq \sum_{i=1}^{k} \lambda_i \gamma_{G_i}, k \geq 0, G_i \in \partial \cap D, \lambda_i \geq 0, i = 1, 2, \ldots, k\right\}$$

where γ is the symbol of the characteristic function and $\partial \cap D = \left\{G \cap D \mid G \subseteq \partial\right\}$.

When X is finite, namely, $X = \left\{x_1, x_2, \ldots, x_n\right\}$, we take its power set $P\left(X\right)$ as ∂. In this situation, any function defined on X is measurable. Besides, because all singletons are included in the power set $P\left(X\right)$, the supremum must be reached when the equality $h = \sum_{i=1}^{k} \lambda_i \gamma_{G_i}$ holds. Thus, the expression of the ZI can be degenerated to the following form:

$$\int_D h d\mu = \max\left\{\sum_{i=1}^{2^n - 1} \lambda_i \mu\left(G_i \cap D\right) \mid h = \sum_{i=1}^{2^n - 1} \lambda_i \gamma_{G_i \cap D}, \lambda_i \geq 0\right\}$$

where $G_i = \left\{ x_j \mid \frac{i}{2^j} - \left[\frac{i}{2^j} \right] \geq \frac{1}{2}, 1 \leq j \leq n \right\} \subseteq X, i = 1, 2, \ldots, 2^n - 1$.

To illustrate the difference between Choquet integral and Zhenyuan integral, the following example is provided.

Example 1. Assume that a person can invest 0.4 billion yuan in the computer industry (x_1), 0.9 billion yuan in the education industry (x_2) and 0.2 billion yuan in the entertainment industry (x_3). He can make a portfolio investment or make a separate investment. The income of the portfolio is listed in Table 1.

Table 1. Income per billion yuan of investment

Investment portfolio	Income	Investment portfolio	Income
$\varnothing$	0	$\{x_2, x_3\}$	0.9
$\{x_1\}$	0.2	$\{x_1, x_2\}$	0.7
$\{x_2\}$	0.3	$\{x_1, x_3\}$	0.8
$\{x_3\}$	0.5	$\{x_1, x_2, x_3\}$	1

The rates of income could be considered as a nonnegative set function μ defined on the power set of $X = \{x_1, x_2, x_3\}$. Asumme that h is an investment function on set $\{x_1, x_2, x_3\}$ with its values $h(x_1)$=0.4, $h(x_2)$=0.9 and $h(x_3)$=0.2. It is known that the person can arrange his investment decision in any combination, investing either individually or cooperatively. To begin with, based on the Choquet integral, the general income (GI) can be calculated as follows:

$$\begin{aligned}
GI &= \sum_{i=1}^{3} h\left(x_{(i)}\right)\left(\mu\left(X_{(i)}\right) - \mu\left(X_{(i+1)}\right)\right) \\
&= h\left(x_{(3)}\right)\left(\mu\left(\{x_1, x_2, x_3\}\right) - \mu\left(\{x_1, x_2\}\right)\right) + h\left(x_{(1)}\right)\left(\mu\left(\{x_1, x_2\}\right) - \mu\left(\{x_2\}\right)\right) + h\left(x_{(2)}\right)\left(\mu\left(\{x_2\}\right)\right) \\
&= 0.2 \times (1\text{-}0.7) + 0.4 \times (0.7\text{-}0.3) + 0.9 \times 0.3 \\
&= 0.49
\end{aligned}$$

Then, based on the Zhenyuan integral, the GI' can be calculated as follows:

$$\begin{aligned}
GI' = \max \{ &\lambda_1 \mu(\{x_1\}) + \lambda_2 \mu(\{x_2\}) + \lambda_3 \mu(\{x_1, x_2\}) + \lambda_4 \mu(\{x_3\}) \\
&+ \lambda_5 \mu(\{x_1, x_3\}) + \lambda_6 \mu(\{x_2, x_3\}) + \lambda_7 \mu(\{x_1, x_2, x_3\}) \}
\end{aligned}$$

$$s.t. \begin{cases} \lambda_1 + \lambda_3 + \lambda_5 + \lambda_7 = 0.4; \\ \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7 = 0.9; \\ \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 = 0.2; \\ \lambda_i \geq 0 (i = 1, 2, ..., 7). \end{cases}$$

According to the rates of income listed in Table 1, above model is transformed into the following mathematical programming model:

$$GI' = \max \{0.2\lambda_1 + 0.3\lambda_2 + 0.7\lambda_3 + 0.5\lambda_4 + 0.8\lambda_5 + 0.9\lambda_6 + 1\lambda_7\}$$

$$s.t. \begin{cases} \lambda_1 + \lambda_3 + \lambda_5 + \lambda_7 = 0.4; \\ \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7 = 0.9; \\ \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 = 0.2; \\ \lambda_i \geq 0 (i = 1, 2, ..., 7). \end{cases} \tag{6}$$

Solving model (6) by using Matlab R2017b software, we derive $GI' = 0.55$.

Through the above example, we can see that the obtained result of Zhenyuan integral is larger than the Choquet integral. The main reason for this is that the Choquet integral omits the relationship between some elements, such as the relation of $\{x_1, x_3\}$ and $\{x_2, x_3\}$, producing a biased result, whereas the Zhenyuan integral can precisely reflect the total interaction among (combinations of) attributes. Therefore, in this study, Zhenyuan integral is extended to accommodate the q-ROF environment, leading to the proposed q-ROF Zhenyuan integral detailed in the next section.

3. The proposed methodology

This section describes in detail the innovative methodology that integrates ZI, BWM and Shapley value to address q-ROF MADM problems with interactive criteria and incomplete weights.

In a general q-ROF MADM problem, there are m alternatives $A_i\left(i=1,2,...,m\right)$ and n main-attributes $c_j\left(j=1,2,...,n\right)$, each one composed of n_j sub-attributes $c_{jk_j}\left(k_j=1,2,...,n_j\right)$. The performance assessment matrix is denoted by $Q=[Q_{ijk_j}]_{m\times n\times n_j}$, where $Q_{ijk_j}=\left(u_{ijk_j},v_{ijk_j}\right)_q$ is the q-ROFN provided by DM for the alternative A_i regarding the sub-attribute c_{jk_j}, subject to $0\le u_{ijk_j}\le 1$, $0\le v_{ijk_j}\le 1$ and $\left(u_{ijk_j}\right)^q+\left(v_{ijk_j}\right)^q\le 1\ \left(q\ge 1\right)$.

The proposed methodology is composed of the three phases (see Fig. 1): problem structuring, determining optimal fuzzy measures using the q-ROFBWM, and ranking alternatives using the q-ROFZI.

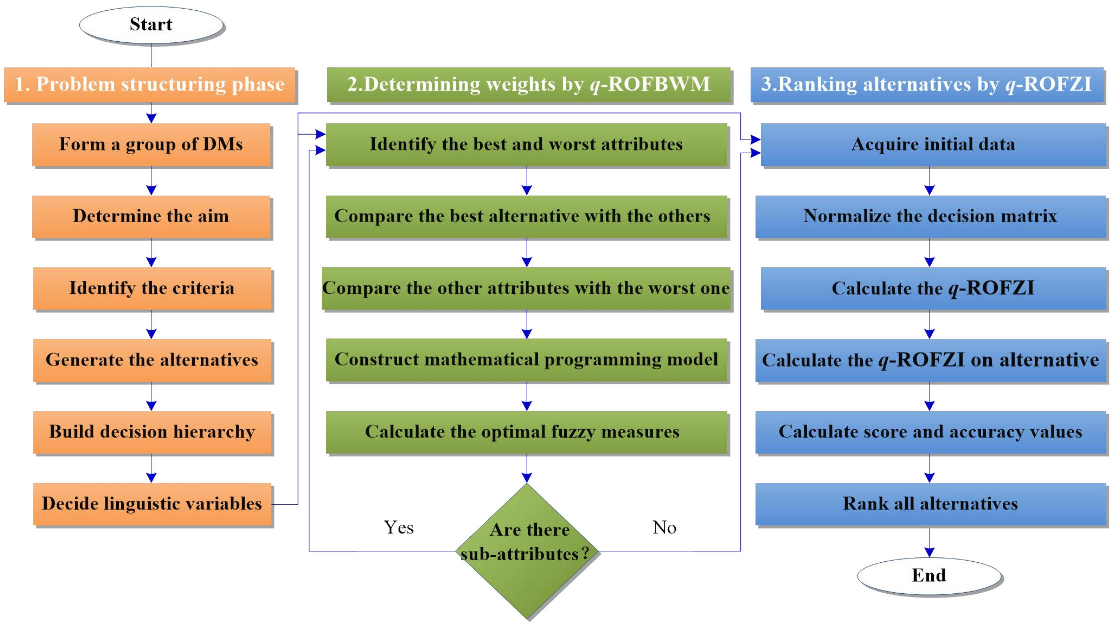

Fig. 1. Flowchart of the proposed MADM method.

3.1 Problem structuring

In this phase, the purpose and the scope of this study, alternatives, main-attributes and sub-attributes are identified. Meanwhile, hierarchical representation of the attributes along with the goal statement at the top of the hierarchy is identified. Additionally, the linguistic term sets and their corresponding q-ROFNs are determined.

3.2 The q-ROFBWM for identifying optimal fuzzy measures

In recent years, a number of models have been established to identify attribute weights (Chen et al., 2021; Salimi et al., 2020; Wei, 2011; Zhang et al., 2021). However, they are all based on the assumption that the attributes are independent. It is well known that the attributes used in many assessments are interactive among each other. In order to take the interactions among the attributes into consideration, two optimization models are constructed to determine the optimal fuzzy measures on the attribute set based on BWM and the Shapley value.

BWM is a new procedure to identify the weights of attributes (Rezaei, 2015), which makes reference comparisons. Reference comparisons essentially are pairwise comparisons of attributes with the best and worst attributes (see Fig. 2). Then, an optimization model is established based on the reference comparisons. Eventually, through solving the optimization model, the attribute weights are derived. Compared with AHP, it needs fewer pairwise comparisons, but provides more reliable and precise outcome. Thus, it has received sustained attention in many decision-making fields. However, BWM is based on the assumption that the attributes are independent and fails to model the interactive relationship among the attributes. In addition, BWM is unfit q-ROF environments, which further confines the limited usage range of the BWM. In contrast, the Shapley value is an expected value of the global interaction between an attribute c_j and all coalitions in $C\backslash c_j$.

Considering their prominent features, BWM and the Shapley value, are combined and extended to q-ROF environments for computing optimal fuzzy measures on the attribute set, leading to the q-ROFBWM as outlined below.

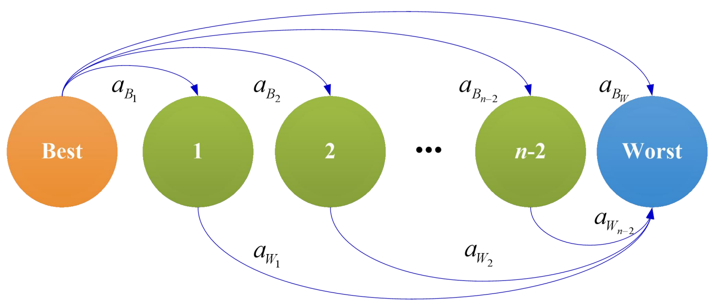

Fig. 2. Reference comparisons in BWM.

Step 1 (Identification of the best and worst attributes): The best attribute c_B and the worst attribute c_W are identified by the DM, based on the decision main-attributes $\{c_1, c_2, ..., c_n\}$ formed in Section 3.1.

Step 2 (Comparison of the best attribute with the others): The best main-attribute is compared with the other main-attributes utilizing a linguistic term set. Then, the best-to-others vector $A_B = \left(a_{B_1}, a_{B_2}, \ldots, a_{B_n}\right)$ is derived, where $a_{B_j} = \left(u_{a_{B_j}}, v_{a_{B_j}}\right)_q$ $(j = 1, 2, ..., n)$ is the q-ROF preference of the best main-attribute c_B over the main-attribute c_j.

Step 3 (Comparison of the other attributes with the worst one): The other main-attributes are compared with the worst main-attribute utilizing a linguistic term set. Then, the others-to-worst vector $A_W = \left(a_{W_1}, a_{W_2}, a_{W_3}, \ldots, a_{W_n}\right)$ is derived, where $a_{W_j} = \left(u_{a_{W_j}}, v_{a_{W_j}}\right)_q$ $(j = 1, 2, ..., n)$ is the q-ROF preference of the main-attribute c_j over the worst main-attribute c_W.

Step 4 (Calculation of optimal measures): To determine the optimal fuzzy measures on the main-attribute set, the pairs $\varphi\left(\mu\left(\{c_B\}\right)\right) - S\left(a_{B_j}\right)\varphi\left(\mu\left(\{c_j\}\right)\right)$ and $\varphi\left(\mu\left(\{c_j\}\right)\right) - S\left(a_{W_j}\right)\varphi\left(\mu\left(\{c_W\}\right)\right)$ are first derived, where $\varphi\left(\mu\left(\{c_\bullet\}\right)\right)(\bullet = B, W, j; j = 1, 2, ..., n)$ is the Shapley value of the fuzzy measure

$\mu(\{c_\bullet\})$ on the main-attribute $c_\bullet$ while $S(a_\circ)$ $(\circ = B_j, W_j; j = 1,2,...,n)$ is the score value of q-ROFN $a_\circ$. Then, we aim to minimize the maximum of $\left|\varphi(\mu(\{c_B\})) - S(a_{B_j})\varphi(\mu(\{c_j\}))\right|$ and $\left|\varphi(\mu(\{c_j\})) - S(a_{W_j})\varphi(\mu(\{c_W\}))\right|$ for every j (j=1,2,...,n).

According to the properties of fuzzy measures and the score function of q-ROFN, if the weight information is fully unknown, the following weight optimization model is established:

$$\min \max_j \left\{\left|\varphi(\mu(\{c_B\})) - \left(u_{a_{B_j}}^q - v_{a_{B_j}}^q\right)\varphi(\mu(\{c_j\}))\right|, \left|\varphi(\mu(\{c_j\})) - \left(u_{a_{W_j}}^q - v_{a_{W_j}}^q\right)\varphi(\mu(\{c_W\}))\right|\right\}$$
$$s.t.\begin{cases} \mu(\varnothing) = 0, \mu(C) = 1; \\ \mu(S) \le \mu(T), \forall S, T \subseteq C, S \subseteq T; \\ \varphi(\mu(\{c_j\})) = \sum_{T \subseteq C \setminus \{c_j\}} \frac{(n-t-1)!t!}{n!}\left(\mu(\{c_j\} \cup T) - \mu(T)\right), \forall c_j \in C. \end{cases} \tag{7}$$

Model (7) can be rewritten as follows:

$$\min\ k$$
$$s.t.\begin{cases} \left|\varphi(\mu(\{c_B\})) - u_{a_{B_j}}^q \varphi(\mu(\{c_j\})) + v_{a_{B_j}}^q \varphi(\mu(\{c_j\}))\right| \le k; \\ \left|\varphi(\mu(\{c_j\})) - u_{a_{W_j}}^q \varphi(\mu(\{c_W\})) + v_{a_{W_j}}^q \varphi(\mu(\{c_W\}))\right| \le k; \\ \mu(\varnothing) = 0, \mu(C) = 1; \\ \mu(S) \le \mu(T), \forall S, T \subseteq C, S \subseteq T; \\ \varphi(\mu(\{c_j\})) = \sum_{T \subseteq C \setminus \{c_j\}} \frac{(n-t-1)!t!}{n!}\left(\mu(\{c_j\} \cup T) - \mu(T)\right), \forall c_j \in C. \end{cases} \tag{8}$$

By solving model (8), the consistency value k^* and optimal fuzzy measures μ^* on the main-attribute set are derived. A consistency value k^* close to zero implies a high level of consistency (Rezaei, 2016). The optimal fuzzy measures can be regarded as the main-attribute weights:

$$I_{MA} = \left(\mu^*(\{c_1\}), \mu^*(\{c_2\}), ..., \mu^*(\{c_1, c_2, ..., c_n\})\right) \tag{9}$$

In the same way, sub-attribute weights $I_{SA_{jk_j}}$ $(k_j = 1,2,...,n_j; j = 1,2,...,n)$ are identified:

$$I_{SA_{k_j}} = \left(\mu^*(\{c_{jk_1}\}), \mu^*(\{c_{jk_2}\}), ..., \mu^*(\{c_{jk_1}, c_{jk_2}, ..., c_{jk_n}\})\right) \tag{10}$$

According to the properties of fuzzy measures and the score function of q-ROFN, if the weight information is incompletely known, the following weight optimization model is established:

$$\min\ k$$

$$s.t.\begin{cases} \left|\varphi\left(\mu\left(\{c_B\}\right)\right)-u_{a_{Bj}}^{q}\varphi\left(\mu\left(\{c_j\}\right)\right)+v_{a_{Bj}}^{q}\varphi\left(\mu\left(\{c_j\}\right)\right)\right|\leq k; \\ \left|\varphi\left(\mu\left(\{c_j\}\right)\right)-u_{a_{Wj}}^{q}\varphi\left(\mu\left(\{c_W\}\right)\right)+v_{a_{Wj}}^{q}\varphi\left(\mu\left(\{c_W\}\right)\right)\right|\leq k; \\ \mu(\{c_j\})\in H_{c_j}\ (j=1,2,...,n); \\ \mu(\varnothing)=0, \mu(C)=1; \\ \mu(S)\leq\mu(T), \forall S,T\subseteq C, S\subseteq T; \\ \varphi\left(\mu\left(\{c_j\}\right)\right)=\sum_{T\subseteq C\backslash\{c_j\}}\frac{(n-t-1)!t!}{n!}\left(\mu\left(\{c_j\}\cup T\right)-\mu\left(T\right)\right)(j=1,2,...,n). \end{cases} \tag{11}$$

where H_{c_j} is the range of the provided weight information of main-attribute c_j. By solving model (11), the optimal fuzzy measures μ^* on the main-attribute set and the consistency value k^* are derived. The optimal fuzzy measures can be regarded as the main-attribute weights:

$$I_{MA}=\left(\mu^*\left(\{c_1\}\right),\mu^*\left(\{c_2\}\right),....,\mu^*\left(\{c_1,c_2,...,c_n\}\right)\right) \tag{12}$$

In the same way, sub-attribute weights $I_{SA_{jk_j}}\ (k_j=1,2,...,n_j; j=1,2,...,n)$ are identified:

$$I_{SA_{k_j}}=\left(\mu^*\left(\{c_{jk_1}\}\right),\mu^*\left(\{c_{jk_2}\}\right),....,\mu^*\left(\{c_{jk_1},c_{jk_2},...,c_{jk_n}\}\right)\right) \tag{13}$$

3.3 The *q*-ROFZI for ranking alternatives

In this phase, all alternatives are ranked by considering the optimal fuzzy measures derived from *q*-ROFBWM. Before introducing the main steps to rank alternatives using *q*-ROFZI, its concept is defined below and its desirable properties and special cases explored.

(1) *q*-Rung orthopair fuzzy Zhenyuan integral

Definition 8. Let $Q_i=\left(u_i,v_i\right)_q\left(i=1,2,\ldots,n\right)$ be a collection of *q*-ROFNs, $C=\{c_1,c_2,...,c_n\}$ a set of attributes and μ a fuzzy measure on C. The *q*-ROFZI is expressed as follows:

$$q-ROFZI(Q_1,Q_2,...,Q_n)=\max\left\{\bigoplus_{j=1}^{2^n-1}\lambda_j\mu(G_j)\,|\,Q_i=\bigoplus_{j=1}^{2^n-1}\lambda_j\chi_{G_j}\right\} \tag{14}$$

where $\lambda_j=\left(u_{\lambda_j},v_{\lambda_j}\right)_q$ denotes a *q*-ROFN, χ is the characteristic function and $G_j=\left\{c_i\,|\,\frac{j}{2^i}-\left[\frac{j}{2^i}\right]\geq\frac{1}{2},1\leq i\leq n\right\}\subseteq C\ \left(j=1,2,...,2^n-1\right)$.

Theorem 1. The aggregation output of the set $Q_i=\left(u_i,v_i\right)_q\left(i=1,2,\ldots,n\right)$ of *q*-ROFNs by the *q*-ROFZI is the following *q*-ROFN:

$$q-ROFZI(Q_1,Q_2,...,Q_n)=$$
$$\max\left\{\left(\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\mu(G_j)}\right)_q\right\} \tag{15}$$
$$s.t.\left\{Q_i=\left(\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\chi_{G_j}}\right)^{\frac{1}{q}},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\chi_{G_j}}\right)_q\right.$$

Proof. The following formula is proved first by induction on n.

$$\overset{2^n-1}{\underset{j=1}{\oplus}}\lambda_j\mu(G_j)=\left(\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\mu(G_j)}\right)_q \tag{16}$$

(i) When n=2, according to the operational rules of q-ROFNs, we obtain

$$\lambda_j\mu(G_j)=\left(\left(1-\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},v_{\lambda_j}^{\mu(G_j)}\right)_q$$

for j=1, 2, 3.

Then, we obtain

$$\overset{3}{\underset{j=1}{\oplus}}\lambda_j\mu(G_j)$$
$$=\left(\left(1-\left(1-u_{\lambda_1}^q\right)^{\mu(G_1)}\right)^{\frac{1}{q}},v_{\lambda_1}{}^{\mu(G_1)}\right)_q\oplus\left(\left(1-\left(1-u_{\lambda_2}^q\right)^{\mu(G_2)}\right)^{\frac{1}{q}},v_{\lambda_2}{}^{\mu(G_2)}\right)_q\oplus\left(\left(1-\left(1-u_{\lambda_3}^q\right)^{\mu(G_3)}\right)^{\frac{1}{q}},v_{\lambda_3}{}^{\mu(G_3)}\right)_q$$
$$=\left(\left(1-\left(1-u_{\lambda_1}^q\right)^{\mu(G_1)}\left(1-u_{\lambda_2}^q\right)^{\mu(G_2)}\right)^{\frac{1}{q}},v_{\lambda_1}{}^{\mu(G_1)}v_{\lambda_2}{}^{\mu(G_2)}\right)_q\oplus\left(\left(1-\left(1-u_{\lambda_3}^q\right)^{\mu(G_3)}\right)^{\frac{1}{q}},v_{\lambda_3}{}^{\mu(G_3)}\right)_q$$
$$=\left(\left(1-\left(1-u_{\lambda_1}^q\right)^{\mu(G_1)}\left(1-u_{\lambda_2}^q\right)^{\mu(G_2)}\left(1-u_{\lambda_3}^q\right)^{\mu(G_3)}\right)^{\frac{1}{q}},v_{\lambda_1}{}^{\mu(G_1)}v_{\lambda_2}{}^{\mu(G_2)}v_{\lambda_3}{}^{\mu(G_3)}\right)_q$$
$$=\left(\left(1-\prod_{j=1}^{3}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{3}v_{\lambda_j}{}^{\mu(G_j)}\right)_q$$

Thus, (16) is correct for n=2.

(ii) Assume that (16) is also correct for n=k, i.e.

$$\overset{2^k-1}{\underset{j=1}{\oplus}}\lambda_j\mu(G_j)$$
$$=\left(\left(1-\prod_{j=1}^{2^k-1}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{2^k-1}v_{\lambda_j}^{\mu(G_j)}\right)_q$$

(iii) When n=k+1, we obtain

$$\overset{2^{k+1}-1}{\underset{j=1}{\oplus}}\lambda_j\mu(G_j)=\overset{2^k-1}{\underset{j=1}{\oplus}}\lambda_j\mu(G_j)\oplus\overset{2^{k+1}-2}{\underset{j=2^k}{\oplus}}\lambda_j\mu(G_j)\oplus\lambda_{2^{k+1}-1}\mu(G_{2^{k+1}-1})$$

$$=\left(\left(1-\prod_{j=1}^{2^k-1}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{2^k-1}v_{\lambda_j}^{\mu(G_j)}\right)_q\oplus\left(\left(1-\prod_{j=2^k}^{2^{k+1}-2}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=2^{k+1}-2}^{2^{k+1}-2}v_{\lambda_j}^{\mu(G_j)}\right)_q$$

$$\oplus\left(\left(1-\left(1-u_{\lambda_{2^{k+1}-1}}^q\right)^{\mu(G_{2^{k+1}-1})}\right)^{\frac{1}{q}},v_{\lambda_{2^{k+1}-1}}^{\mu(G_{2^{k+1}-1})}\right)_q$$

$$=\left(\left(1-\prod_{j=1}^{2^{k+1}-2}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{2^{k+1}-2}v_{\lambda_j}^{\mu(G_j)}\right)_q\oplus\left(\left(1-\left(1-u_{\lambda_{2^{k+1}-1}}^q\right)^{\mu(G_{2^{k+1}-1})}\right)^{\frac{1}{q}},v_{\lambda_{2^{k+1}-1}}^{\mu(G_{2^{k+1}-1})}\right)_q$$

$$=\left(\left(1-\prod_{j=1}^{2^{k+1}-1}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{2^{k+1}-1}v_{\lambda_j}^{\mu(G_j)}\right)_q$$

Thus, (16) is correct for $n=k+1$, and we conclude that (16) holds for all n.

Analogously, we have $Q_i=\overset{2^n-1}{\underset{j=1}{\oplus}}\lambda_j\chi_{G_j}=\left(\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\chi_{G_j}}\right)^{\frac{1}{q}},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\chi_{G_j}}\right)_q$. Therefore, we obtain

$$q-ROFZI(Q_1,Q_2,\ldots,Q_n)$$
$$=\max\left\{\left(\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\mu(G_j)}\right)_q\right\}$$
$$s.t.\left\{Q_i=\left(\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\chi_{G_j}}\right)^{\frac{1}{q}},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\chi_{G_j}}\right)_q\right.$$

Additionally, we illustrate that the aggregated result of (15) is a q-ROFN. According to Definition 1, it is easy to prove the following inequalities:

$$0\le\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}}\le 1\quad\text{and}\quad 0\le\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\mu(G_j)}\le 1.$$

Since $0\le u_{\lambda_j}^q+v_{\lambda_j}^q\le 1$, it is $0\le u_{\lambda_j}^q\le 1-v_{\lambda_j}^q$. Thus, we derive

$$0\le 1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}+\prod_{j=1}^{2^n-1}\left(1-\left(1-v_{\lambda_j}^q\right)\right)^{\mu(G_j)}\le 1-\prod_{j=1}^{2^n-1}\left(1-\left(1-v_{\lambda_j}^q\right)\right)^{\mu(G_j)}+\prod_{j=1}^{2^n-1}\left(1-\left(1-v_{\lambda_j}^q\right)\right)^{\mu(G_j)}=1.$$

Therefore, (15) is still a q-ROFN, and the proof of Theorem 1 is completed.

To illustrate how to utilize the developed integral, the following example is provided.

Example 2. Let $Q_1=(0.7,0.6)_q$, $Q_2=(0.6,0.5)_q$ and $Q_3=(0.9,0.5)_q$ be q-ROFNs on the set of criteria $C=\{c_1,c_2,c_3\}$, with following fuzzy measure: $\mu(\{c_1\})=\mu(\{c_2\})=0.5$, $\mu(\{c_3\})=0.3$,

$\mu(\{c_1,c_2\})=0.8$, $\mu(\{c_1,c_3\})=0.7$, $\mu(\{c_2,c_3\})=0.7$ and $\mu(\{c_1,c_2,c_3\})=1$. The computation of $q-ROFZI(Q_1,Q_2,Q_3)$ by formula (15) is as follows:

According to Definition 8, $q-ROFZI(Q_1,Q_2,Q_3)$ is written as a programming model:

$$\max\left\{\mathop{\oplus}_{j=1}^{7}\lambda_j\mu(G_j)\right\}$$
$$s.t.\begin{cases}\lambda_1\oplus\lambda_3\oplus\lambda_5\oplus\lambda_7=Q_1;\\ \lambda_2\oplus\lambda_3\oplus\lambda_6\oplus\lambda_7=Q_2;\\ \lambda_4\oplus\lambda_5\oplus\lambda_6\oplus\lambda_7=Q_3.\end{cases}\tag{17}$$

where $\lambda_j=\left(u_{\lambda_j},v_{\lambda_j}\right)_q$ denotes a q-ROFN and $G_j=\left\{c_i \mid \frac{j}{2^i}-\left[\frac{j}{2^i}\right]\geq\frac{1}{2},1\leq i\leq 3\right\}\subseteq C$ $(j=1,2,...,7)$.

Then, according to the operational laws of q-ROFNs, model (17) is rewritten as follows:

$$\max\left\{\left(\left(1-\prod_{j=1}^{7}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{7}v_{\lambda_j}^{\mu(G_j)}\right)_q\right\}$$
$$s.t.\begin{cases}\left(\left(1-\left(1-u_{\lambda_1}^q\right)\left(1-u_{\lambda_3}^q\right)\left(1-u_{\lambda_5}^q\right)\left(1-u_{\lambda_7}^q\right)\right)^{\frac{1}{q}},v_{\lambda_1}v_{\lambda_3}v_{\lambda_5}v_{\lambda_7}\right)_q=(u_1,v_1)_q;\\ \left(\left(1-\left(1-u_{\lambda_2}^q\right)\left(1-u_{\lambda_3}^q\right)\left(1-u_{\lambda_6}^q\right)\left(1-u_{\lambda_7}^q\right)\right)^{\frac{1}{q}},v_{\lambda_2}v_{\lambda_3}v_{\lambda_6}v_{\lambda_7}\right)_q=(u_2,v_2)_q;\\ \left(\left(1-\left(1-u_{\lambda_4}^q\right)\left(1-u_{\lambda_5}^q\right)\left(1-u_{\lambda_6}^q\right)\left(1-u_{\lambda_7}^q\right)\right)^{\frac{1}{q}},v_{\lambda_4}v_{\lambda_5}v_{\lambda_6}v_{\lambda_7}\right)_q=(u_3,v_3)_q;\\ 0\leq u_{\lambda_j}^q+v_{\lambda_j}^q\leq 1, j=1,2,\ldots,7.\end{cases}\tag{18}$$

It is worth mentioning that both the objective function and constraints of model (18) contain the q-ROFNs. Thus, model (18) is called a q-ROF mathematical programming. As no approach exists for addressing such a type of q-ROF programming model, we herein propose an approach to solving this mathematical programming problem.

According to the inclusion relationship of q-RONs and Proposition 1, model (18) is equivalently transformed into the following bi-objective q-ROF mathematical programming:

$$\max\left\{\left(1-\prod_{j=1}^{7}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}}\right\}$$
$$\min\left\{\prod_{j=1}^{7}v_{\lambda_j}^{\mu(G_j)}\right\}\tag{19}$$
$$s.t.\begin{cases}\left(1-u_{\lambda_1}^q\right)\left(1-u_{\lambda_3}^q\right)\left(1-u_{\lambda_5}^q\right)\left(1-u_{\lambda_7}^q\right)=1-u_1{}^q,\\ \left(1-u_{\lambda_2}^q\right)\left(1-u_{\lambda_3}^q\right)\left(1-u_{\lambda_6}^q\right)\left(1-u_{\lambda_7}^q\right)=1-u_2{}^q,\\ \left(1-u_{\lambda_4}^q\right)\left(1-u_{\lambda_5}^q\right)\left(1-u_{\lambda_6}^q\right)\left(1-u_{\lambda_7}^q\right)=1-u_3{}^q,\\ v_{\lambda_1}v_{\lambda_3}v_{\lambda_5}v_{\lambda_7}=v_1,\\ v_{\lambda_2}v_{\lambda_3}v_{\lambda_6}v_{\lambda_7}=v_2,\\ v_{\lambda_4}v_{\lambda_5}v_{\lambda_6}v_{\lambda_7}=v_3,\\ 0\leq u_{\lambda_j}^q+v_{\lambda_j}^q\leq 1, j=1,2,\ldots,7.\end{cases}$$

Notice that the first objective $\max\left\{\left(1-\prod_{j=1}^{7}\left(1-u_{\lambda_j}^{q}\right)^{\mu(G_j)}\right)^{\frac{1}{q}}\right\}$ is equivalent to $\min\left\{\left(\prod_{j=1}^{7}\left(1-u_{\lambda_j}^{q}\right)^{\mu(G_j)}\right)^{\frac{1}{q}}\right\}$, which is equivalent to $\min\left\{\sum_{j=1}^{7}\mu\left(G_j\right)\ln\left(1-u_{\lambda_j}^{q}\right)\right\}$ because $0\le u_{\lambda_j}<1$, $q\ge 1$ and q is constant. The second objective $\min\left\{\prod_{j=1}^{7}v_{\lambda_j}^{\mu(G_j)}\right\}$ is equivalent to $\min\left\{\sum_{j=1}^{7}\mu\left(G_j\right)\ln v_{\lambda_j}\right\}$.

Considering that $0\le u_{\lambda_j}<1$ and $q\ge 1$, the first constraint in model (19) can be rewritten as $\ln\left(1-u_{\lambda_1}^{q}\right)+\ln\left(1-u_{\lambda_3}^{q}\right)+\ln\left(1-u_{\lambda_5}^{q}\right)+\ln\left(1-u_{\lambda_7}^{q}\right)=\ln\left(1-u_1{}^{q}\right)$. Similarly, the second and third constraints in model (19) can be rewritten as

$$\ln\left(1-u_{\lambda_2}^{q}\right)+\ln\left(1-u_{\lambda_3}^{q}\right)+\ln\left(1-u_{\lambda_6}^{q}\right)+\ln\left(1-u_{\lambda_7}^{q}\right)=\ln\left(1-u_2{}^{q}\right),$$
$$\ln\left(1-u_{\lambda_4}^{q}\right)+\ln\left(1-u_{\lambda_5}^{q}\right)+\ln\left(1-u_{\lambda_6}^{q}\right)+\ln\left(1-u_{\lambda_7}^{q}\right)=\ln\left(1-u_3{}^{q}\right).$$

As there is $0<v_{\lambda_j}\le 1$, the fourth constraint in model (19) can be rewritten as $\ln\left(v_{\lambda_1}\right)+\ln\left(v_{\lambda_3}\right)+\ln\left(v_{\lambda_5}\right)+\ln\left(v_{\lambda_7}\right)=\ln\left(v_1\right)$. Similarly, the fifth and sixth constraints in model (19) can be rewritten as

$$\ln\left(v_{\lambda_2}\right)+\ln\left(v_{\lambda_3}\right)+\ln\left(v_{\lambda_6}\right)+\ln\left(v_{\lambda_7}\right)=\ln\left(v_2\right),$$
$$\ln\left(v_{\lambda_4}\right)+\ln\left(v_{\lambda_5}\right)+\ln\left(v_{\lambda_6}\right)+\ln\left(v_{\lambda_7}\right)=\ln\left(v_3\right).$$

Constraint $0\le u_{\lambda_j}^{q}+v_{\lambda_j}^{q}\le 1$ in model (19) is equivalent to $0<v_{\lambda_j}^{q}\le 1-u_{\lambda_j}^{q}$ and $0<1-u_{\lambda_j}^{q}\le 1$, which is further equivalent to $q\ln\left(v_{\lambda_j}\right)\le\ln\left(1-u_{\lambda_j}^{q}\right)$ and $\ln\left(1-u_{\lambda_j}^{q}\right)\le 0$.

Denoting $x_j=\ln\left(1-u_{\lambda_j}^{q}\right)\left(j=1,2,...,7\right)$, $y_j=\ln\left(v_{\lambda_j}\right)$ and q=3; using the linear weighted sum method, model (19) can be converted into the below single-objective linear programming model:

$$\min\left\{\sum_{j=1}^{7}\mu\left(G_j\right)x_j+\sum_{j=1}^{7}\mu\left(G_j\right)y_j\right\}$$
$$s.t.\begin{cases}x_1+x_3+x_5+x_7=\ln(1-u_1^{q}),\\ x_2+x_3+x_6+x_7=\ln(1-u_2^{q}),\\ x_4+x_5+x_6+x_7=\ln(1-u_3^{q}),\\ y_1+y_3+y_5+y_7=\ln(v_1),\\ y_2+y_3+y_6+y_7=\ln(v_2),\\ y_4+y_5+y_6+y_7=\ln(v_3),\\ x_j-3y_j\ge 0,\\ x_j\le 0, j=1,2,3,\ldots,7.\end{cases} \tag{20}$$

Furthermore, by substituting the values of Q_i and $\mu\left(G_j\right)$ into model (20), this becomes

$$\min\{0.5x_1+0.5x_2+0.8x_3+0.3x_4+0.7x_5+0.7x_6+x_7$$
$$+(0.5y_1+0.5y_2+0.8y_3+0.3y_4+0.7y_5+0.7y_6+y_7)\} \tag{21}$$
$$s.t.\begin{cases} x_1+x_3+x_5+x_7=\ln(1-0.7^3),\\ x_2+x_3+x_6+x_7=\ln(1-0.6^3),\\ x_4+x_5+x_6+x_7=\ln(1-0.9^3),\\ y_1+y_3+y_5+y_7=\ln(0.6),\\ y_2+y_3+y_6+y_7=\ln(0.5),\\ y_4+y_5+y_6+y_7=\ln(0.5),\\ x_j-3y_j\geq 0,\\ x_j\leq 0, j=1,2,3,\dots 7. \end{cases}$$

Solving this model using Matlab R2017b software, the following solution is derive

$x_1=-0.4201, x_2=-0.2433, x_3=0, x_4=-1.3056, x_5=0, x_6=0, x_7=0,$

$y_1=-0.5108, y_2=-0.6931, y_3=0, y_4=-0.6931, y_5=0, y_6=0, y_7=0.$

Therefore, we have

$$q-ROFZI(Q_1,Q_2,Q_3)=\left(\left(1-e^{\sum_{j=1}^{7}x_j\mu(G_j)}\right)^{\frac{1}{q}}, e^{\sum_{j=1}^{7}y_j\mu(G_j)}\right)_3=(0.8015, 0.4449)_3.$$

In the following, some special cases of the q-ROFZI are derived by using different fuzzy measures or assigning various values to parameter q.

Corollary 1. Let μ be a fuzzy measure on $C=\{c_1,c_2,...,c_n\}$, $R,S\subseteq C$ and $R\cap S=\varnothing$. If $\mu(R\cap S)=\mu(R)+\mu(S)$, then the q-ROFZI becomes the q-ROF weighted averaging (q-ROFWA) operator as

$$q-ROFWAI(Q_1,Q_2,...,Q_n)=\left(\left(1-\prod_{i=1}^{n}\left(1-u_i^q\right)^{\mu(\{c_i\})}\right)^{\frac{1}{q}}, \prod_{i=1}^{n}v_i^{\mu(\{c_i\})}\right)_q. \tag{22}$$

Proof. Based on Definition 5, it is $\forall G_j\subseteq C, \mu(G_j)=\sum_{c_i\in G_j}\mu(\{c_i\})$. Thus:

$$\begin{aligned} & q-ROFZI(Q_1,Q_2,...,Q_n)\\ &=\max\left\{\overset{2^n-1}{\underset{j=1}{\oplus}}\lambda_j\mu(G_j)\mid Q_i=\overset{2^n-1}{\underset{j=1}{\oplus}}\lambda_j\chi_{G_j}\right\}\\ &=\max\left\{\overset{n}{\underset{i=1}{\oplus}}\left(\overset{2^n-1}{\underset{j=1}{\oplus}}\lambda_j\chi_{G_j}\right)\mu(\{c_j\})\mid Q_i=\overset{2^n-1}{\underset{j=1}{\oplus}}\lambda_j\chi_{G_j}\right\}\\ &=\max\left\{\overset{n}{\underset{i=1}{\oplus}}Q_i\mu(\{c_j\})\right\}\\ &=\left(\left(1-\prod_{i=1}^{n}\left(1-u_i^q\right)^{\mu(\{c_i\})}\right)^{\frac{1}{q}}, \prod_{i=1}^{n}v_i^{\mu(\{c_i\})}\right)_q\\ &=q-ROFWA(Q_1,Q_2,...,Q_n) \end{aligned}$$

Corollary 2. If q =1, then the q-ROFZI becomes the Atanassov intuitionistic fuzzy Zhenyuan averaging (AIFZA) operator

$$AIFZA(Q_1,Q_2,...,Q_n)$$
$$=\max\left\{\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}\right)^{\mu(G_j)},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\mu(G_j)}\right)_q\right\}$$
$$s.t.\left\{Q_i=\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}\right)^{\chi_{G_j}},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\chi_{G_j}}\right)_q\right.$$

Corollary 3. If q =2, then the q-ROFZI becomes the Pythagorean fuzzy Zhenyuan averaging integral (PFZAI)

$$PFZAI(Q_1,Q_2,...,Q_n)$$
$$=\max\left\{\left(\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^2\right)^{\mu(G_j)}\right)^{\frac{1}{2}},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\mu(G_j)}\right)_q\right\}$$
$$s.t.\left\{Q_i=\left(\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^2\right)^{\chi_{G_j}}\right)^{\frac{1}{2}},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\chi_{G_j}}\right)_q\right.$$

In the following, some describable properties of the q-ROFZI operator are investigated.

Property 2. Let $Q=\{Q_1,Q_2,...,Q_n\}$ and $Q'=\{Q_1',Q_2',...,Q_n'\}$ be two sets of q-ROFNs on X, and μ a fuzzy measure on X. If $Q_i\le Q_i'$ for all i, that is, $u_i\le u_i'$ and $v_i\ge v_i'$, then

$$q-ROFZI\left(Q_1,Q_2,...,Q_n\right)\le q-ROFZI\left(Q_1',Q_2',...,Q_n'\right).$$

Proof. According to Proposition 1 and Theorem 1, it is

$$u_i=\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\chi_{G_j}}\right)^{\frac{1}{q}},\ v_i=\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\chi_{G_j}},\ u_i'=\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}'^q\right)^{\chi_{G_j}}\right)^{\frac{1}{q}}\text{ and }v_i'=\prod_{j=1}^{2^n-1}v_{\lambda_j}'^{\chi_{G_j}}.$$

Since $u_i\le u_i'$ and $v_i\le v_i'$, it is

$$u_i=\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\chi_{G_j}}\right)^{\frac{1}{q}}\le u_i'=\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}'^q\right)^{\chi_{G_j}}\right)^{\frac{1}{q}}\text{ and }v_i=\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\chi_{G_j}}\ge v_i'=\prod_{j=1}^{2^n-1}v_{\lambda_j}'^{\chi_{G_j}}.$$

Thus,

$$\max\left\{\left(\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{2^n-1}v_{\lambda_j}^{\mu(G_j)}\right)_q\right\}\le\max\left\{\left(\left(1-\prod_{j=1}^{2^n-1}\left(1-u_{\lambda_j}'^q\right)^{\mu(G_j)}\right)^{\frac{1}{q}},\prod_{j=1}^{2^n-1}v_{\lambda_j}'^{\mu(G_j)}\right)_q\right\}.$$

Therefore, it is $q-ROFZI\left(Q_1,Q_2,...,Q_n\right)\le q-ROFZI\left(Q_1',Q_2',...,Q_n'\right)$.

Property 3. Let $Q=\{Q_1,Q_2,...,Q_n\}$ be a set of q-ROFNs on X and μ a fuzzy measure on X. Then

$$q-ROFZI\left(Q_1,Q_2,...,Q_n\right)=q-ROFZI\left(Q_1',Q_2',...,Q_n'\right),$$

where $\{Q_1',Q_2',...,Q_n'\}$ is a permutation of $\{Q_1,Q_2,...,Q_n\}$.

Proof. This is obvious from Theorem 1.

Property 4. Let $Q=\{Q_1,Q_2,...,Q_n\}$ be a set of q-ROFNs on X and μ a fuzzy measure on X.

If $t > 0$, then

$$q-ROFZI\left(tQ_1, tQ_2, ..., tQ_n\right) = tq-ROFZI\left(Q_1', Q_2', ..., Q_n'\right).$$

Proof. Based on the operational rules of q-ROFNs and Definition 8, it is

$$\begin{aligned}
&q-ROFZI(tQ_1, tQ_2, ..., tQ_n) \\
&= \max\left\{ \bigoplus_{j=1}^{2^n-1} \lambda_j \mu(G_j) \,|\, tQ_i = \bigoplus_{j=1}^{2^n-1} \lambda_j \chi_{G_j} \right\} \\
&= \max\left\{ \bigoplus_{j=1}^{2^n-1} \lambda_j \mu(G_j) \,|\, Q_i = \bigoplus_{j=1}^{2^n-1} \frac{\lambda_j}{t} \chi_{G_j} \right\} \\
&= \max\left\{ \bigoplus_{j=1}^{2^n-1} t\gamma_j \mu(G_j) \,|\, Q_i = \bigoplus_{j=1}^{2^n-1} \gamma_j \chi_{G_j} \right\} \\
&= t\max\left\{ \bigoplus_{j=1}^{2^n-1} \gamma_j \mu(G_j) \,|\, Q_i = \bigoplus_{j=1}^{2^n-1} \gamma_j \chi_{G_j} \right\} \\
&= tq-ROFZAI(tQ_1, tQ_2, ..., tQ_n)
\end{aligned}$$

(2) The main steps using *q*-ROFZI to rank alternatives

This part describes the main steps of using the proposed q-ROFZI within q-ROF environments to rank the alternative A_i (i = 1, 2, . . ., m).

Step 1 (Initial Data Acquisition): The assessment data of sub-attributes provided by DM are in the form of the fuzzy linguistic values. Then these values are converted into q-ROFNs.

Step 2 (Standardization): If the sub-attributes are of benefit type, there is no need to standardize the assessment data. However, if there is sub-attributes of cost type, the following formula is used to standardize them.

$$Q_{ijk_j} = \begin{cases} \left(u_{ijk_j}, v_{ijk_j}\right)_q & \text{for benefit sub-attribute } c_{jk_j}; \\ \left(v_{ijk_j}, u_{ijk_j}\right)_q & \text{for cost sub-attribute } c_{jk_j}. \end{cases} \tag{23}$$

Step 3 (Calculating the q-ROFZI): The proposed q-ROFZI in (15) is used to aggregate assessment data Q_{ijk_j} of sub-attributes, and derive integrated values Q_{ij} of main-attributes.

Step 4 (Calculating the q-ROFZI on alternatives): The proposed q-ROFZI in (15) is used to aggregate integrated values Q_{ij} of main-attributes, and derive overall values Q_i of each alternative $A_i\left(i = 1, 2, ..., m\right)$.

Step 5 (Calculation of score values and accuracy values): The score value $S(Q_i)$ and the accuracy value $H(Q_i)$ of each alternative A_i (i=1,2,...,m) are calculated based on (2) and (3), respectively.

Step 6 (Ranking the alternatives): On the basis of the yielded results of *Step 5* and the comparison rules of q-ROFNs, the ranking of alternatives A_1, A_2,..., and A_m is derived and the best alternative is determined.

4. Case study: mobile medical app evaluation

This section provides an illustrative case to verify the application of the developed methodology for evaluating mobile medical app problems. The mobile medical app market has developed rapidly and has reached a certain scale thanks to the wide popularization of mobile internet and smart phones in people's daily life, the continuous upgrading and application of various advanced technologies, as well as the reduction of the development cost and threshold of mobile applications. According to the statistics, there are thousands of mobile medical apps with various functions in the current market. According to usage, these apps can be classed into platform mobile medical apps and functional mobile medical apps. Platform mobile medical apps, such as ChunYuYiSheng, PAGoodDoctor, GoodDoctorOnline, WeDoctor and ThumbDoctor, integrate the following functions: online consultation, appointment registration, health consultation, disease analysis, dietary recommendation, etc.; while functional mobile medical apps, such as MeetYou, BaoBaoZhiDao, Welltang, are mainly based on a particular function, such as female menstrual period, infant development and diabetes mellitus, which makes them suitable only for specific people or usage environment.

Although a number of mobile medical apps have emerged in the market, which has brought convenience to people's daily life, there are still some deficiencies in these apps, such as insufficient user privacy protection, difficult verification of information authenticity, high risk of telemedicine diagnosis and false medical advertisements. To better understand the mobile medical app market, eight popular Chinese mobile medical apps are considered for evaluation, herein anonymously denoted as A_1, A_2, A_3, A_4, A_5, A_6, A_7 and A_8. The procedure of the developed methodology to handle this case study is illustrated next.

4.1 Problem structuring

The aim of this MADM problem is described as "evaluating Chinese mobile medical apps and selecting the optimal one(s)", which is at the top level of the decision hierarchy. Four main-attributes of the mobile medical app evaluation are identified by reviewing the related literature (Li, 2018; Li et al., 2021; Yu, 2018): functionality (c_1), safety (c_2), interface (c_3) and reliability (c_4) . They are placed in the second level of the decision hierarchy. A total of 14 sub-attributes is placed in the third level of the decision hierarchy. In the fourth level, 8 alternatives are provided. The complete decision hierarchy is formed as depicted in Fig. 3.

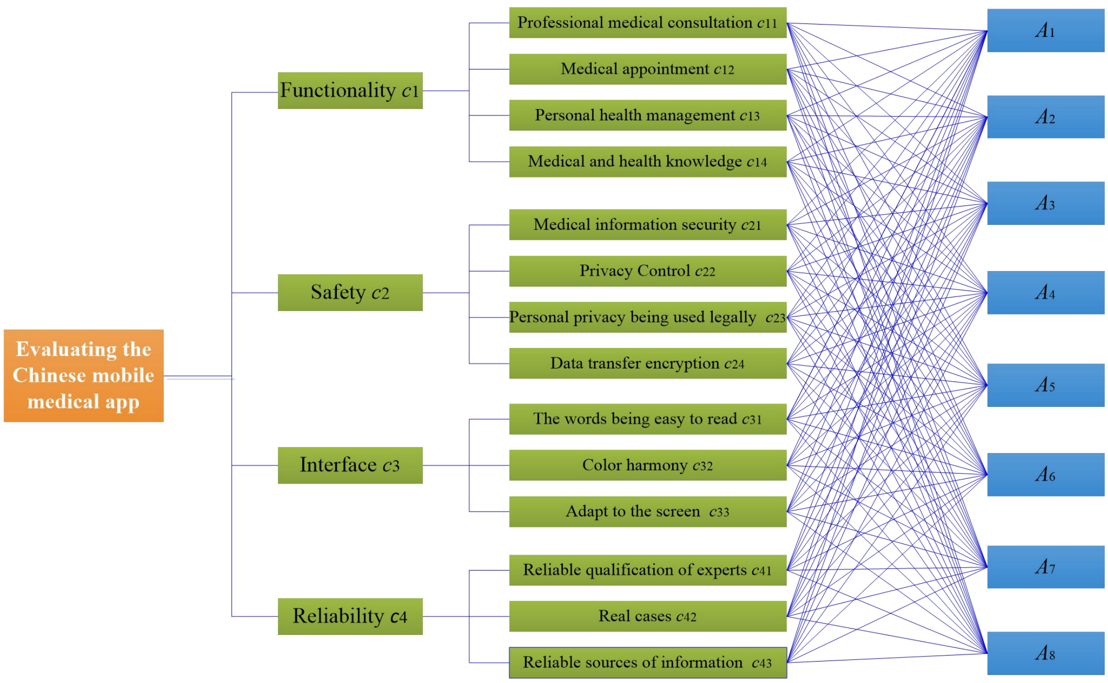

Fig. 3. Chinese mobile medical evaluation framework

Once the decision hierarchy is built, the linguistic variables and their corresponding q-ROFNs are determined. Linguistic terms for pair-wise comparison of attributes are listed in Table 2. Linguistic terms for evaluating alternatives are identified and listed in Table 3.

Table 2. Linguistic terms for pair-wise comparison of attributes

Linguistic terms	q-ROFNs
Absolutely More Important (*AMI*)	$(0.95, 0.5)_q$
Intermediate Value (*IV8*)	$(0.9, 0.5)_q$
Very Strong More Important (*VSMI*)	$(0.85, 0.55)_q$
Intermediate Value (*IV6*)	$(0.8, 0.55)_q$
Strongly More Important (*SMI*)	$(0.75, 0.6)_q$
Intermediate Value (*IV4*)	$(0.7, 0.6)_q$
Moderately More Important (*MMI*)	$(0.65, 0.55)_q$
Intermediate Value (*IV2*)	$(0.6, 0.55)_q$
Equally Important (*EI*)	$(0.5, 0.5)_q$

Table 3. Linguistic terms for evaluating the alternatives

Linguistic terms	q-ROFNs
Absolutely Good (*AG*)	$(0.98, 0.2)_q$
Very Good (*VG*)	$(0.9, 0.6)_q$
Good (*G*)	$(0.8, 0.7)_q$
Medium Good (*MG*)	$(0.6, 0.55)_q$
Medium (*M*)	$(0.5, 0.5)_q$
Medium Bad (*MB*)	$(0.55, 0.6)_q$
Bad (*B*)	$(0.7, 0.8)_q$
Very Bad (*VB*)	$(0.6, 0.9)_q$
Absolutely Bad (*AB*)	$(0.2, 0.98)_q$

4.2 Identifying optimal fuzzy measures

Step 1 (Identification of the best and worst attributes): Among the four main-attributes c_1, c_2, c_3 and c_4, it is assumed that decision makers, according to their cognition and experience, determine the best and the worst attributes as c_1 and c_3 , respectively.

Step 2 (Comparison of the best attribute with the others): Table 4 shows the best main-attribute c_1 comparison with the other main-attributes c_j (j=1,2,3) with the linguistic terms set of Table 2.

Table 4. Linguistic evaluations of most important main-attribute versus all other main-attributes

Best attribute	Other attributes			
	c_1	c_2	c_3	c_4
c_1	*EI*	*MMI*	*IV6*	*IV2*

Step 3 (Comparison of the other attributes with the worst one): Table 5 shows the other main-attributes c_j (j=1,2,4) comparison with the worst main-attribute c_3 with the linguistic terms set of Table 2.

Table 5. Linguistic evaluations of all other main-attributes versus the least important main-attribute

Worst attribute	Other attributes			
	c_1	c_2	c_3	c_4
c_3	*IV6*	*IV2*	*EI*	*MMI*

Step 4 (Calculation of optimal measures): We invite four top medical experts to provide the attribute weight information. For example, when the DMs provide the weight of functionality c_1, they think that its weights are 0.3, 0.35, 0.35 and 0.4, respectively. Then, we obtain the value range of the weight of functionality c_1, denoted by $0.3 \le \mu(\{c_1\}) \le 0.4$. In this way, the weights of other main-attributes are obtained, which are: $0.1 \le \mu(\{c_2\}) \le 0.2$, $0.05 \le \mu(\{c_3\}) \le 0.15$, $0.2 \le \mu(\{c_4\}) \le 0.3$. Based on model (11), optimal fuzzy measures on the main-attribute set are identified by solving the below linear constrained optimization model (without loss of generality, we set parameter q=3).

$$\min\ k$$
$$s.t.\begin{cases}\left|\varphi(\mu(\{c_1\}))-0.1250\varphi(\mu(\{c_1\}))+0.1250\varphi(\mu(\{c_1\}))\right|\le k;\\ \left|\varphi(\mu(\{c_1\}))-0.2746\varphi(\mu(\{c_2\}))+0.1664\varphi(\mu(\{c_2\}))\right|\le k;\\ \left|\varphi(\mu(\{c_1\}))-0.5120\varphi(\mu(\{c_3\}))+0.1664\varphi(\mu(\{c_3\}))\right|\le k;\\ \left|\varphi(\mu(\{c_1\}))-0.2160\varphi(\mu(\{c_4\}))+0.1664\varphi(\mu(\{c_4\}))\right|\le k;\\ \left|\varphi(\mu(\{c_1\}))-0.5120\varphi(\mu(\{c_3\}))+0.1664\varphi(\mu(\{c_3\}))\right|\le k;\\ \left|\varphi(\mu(\{c_2\}))-0.2160\varphi(\mu(\{c_3\}))+0.1664\varphi(\mu(\{c_3\}))\right|\le k;\\ \left|\varphi(\mu(\{c_3\}))-0.1250\varphi(\mu(\{c_3\}))+0.1250\varphi(\mu(\{c_3\}))\right|\le k;\\ \left|\varphi(\mu(\{c_4\}))-0.2746\varphi(\mu(\{c_3\}))+0.1664\varphi(\mu(\{c_3\}))\right|\le k;\\ \varphi(\mu(\{c_j\}))=\sum_{T\subseteq C\backslash\{c_j\}}\frac{(4-t-1)!t!}{4!}(\mu(\{c_j\}\cup T)-\mu(T))(j=1,2,3,4);\\ 0.3\le\mu(\{c_1\})\le0.4, 0.1\le\mu(\{c_2\})\le0.2, 0.05\le\mu(\{c_3\})\le0.15, 0.2\le\mu(\{c_4\})\le 0.3;\\ \mu(\varnothing)=0,\mu(C)=1;\\ \mu(S)\le\mu(T),\forall S,T\subseteq C,S\subseteq T.\end{cases}$$

Using Matlab R2017b software, the following solution is obtained:

$$\mu^*(\{c_1\})=0.3,\mu^*(\{c_2\})=0.2,\mu^*(\{c_3\})=0.15,\mu^*(\{c_4\})=0.3,\mu^*(\{c_1,c_2\})=0.3,\mu^*(\{c_1,c_3\})=0.3,$$
$$\mu^*(\{c_1,c_4\})=0.3,\mu^*(\{c_2,c_3\})=0.5946,\mu^*(\{c_2,c_4\})=0.3376,\mu^*(\{c_3,c_4\})=0.4054,$$
$$\mu^*(\{c_1,c_2,c_3\})=1,\mu^*(\{c_1,c_2,c_4\})=1,\mu^*(\{c_1,c_3,c_4\})=1,\mu^*(\{c_2,c_3,c_4\})=1.$$

The linguistic evaluations of the best sub-attribute versus all the other sub-attributes and the linguistic evaluations of the other sub-attributes versus the worst sub-attribute are given in Table 6.

Table 6. Linguistic evaluations of pair-wise comparison of sub-attributes

Best attribute	Other attributes				Worst attribute	Other attributes			
	c_{11}	c_{12}	c_{13}	c_{14}		c_{11}	c_{12}	c_{13}	c_{14}
c_{11}	*EI*	*MMI*	*VSMI*	*SMI*	c_{13}	*VSMI*	*MMI*	*EI 1*	*IV2*
	c_{21}	c_{22}	c_{23}	c_{24}		c_{21}	c_{22}	c_{23}	c_{24}
c_{21}	*EI*	*MMI*	*SMI*	*IV8*	c_{24}	*IV8*	*MMI*	*IV2*	*EI*
	c_{31}	c_{32}	c_{33}			c_{31}	c_{32}	c_{33}	
c_{32}	*MMI*	*EI*	*IV2*		c_{31}	*EI*	*MMI*	*IV2*	

	c_{41}	c_{42}	c_{43}		c_{41}	c_{42}	c_{43}
c_{41}	*EI*	*IV4*	*VSMI*	c_{43}	*VSMI*	*IV2*	*EI*

The importance of the sub-attributes is as follows:

$0.4 \le \mu(\{c_{11}\}) \le 0.5,\ 0.15 \le \mu(\{c_{12}\}) \le 0.25,\ 0.05 \le \mu(\{c_{13}\}) \le 0.1,\ 0.05 \le \mu(\{c_{14}\}) \le 0.15;$

$0.35 \le \mu(\{c_{21}\}) \le 0.5,\ 0.15 \le \mu(\{c_{22}\}) \le 0.3,\ 0.05 \le \mu(\{c_{23}\}) \le 0.2,\ 0.05 \le \mu(\{c_{24}\}) \le 0.15;$

$0.1 \le \mu(\{c_{31}\}) \le 0.3,\ 0.3 \le \mu(\{c_{32}\}) \le 0.6,\ 0.2 \le \mu(\{c_{33}\}) \le 0.35;\ 0.2 \le \mu(\{c_{41}\}) \le 0.7,$

$0.05 \le \mu(\{c_{42}\}) \le 0.25,\ 0.05 \le \mu(\{c_{43}\}) \le 0.15,\ 0.5 \le \mu(\{c_{41}, c_{42}\}) \le 0.65,$

$0.5 \le \mu(\{c_{41}, c_{43}\}) \le 0.65,\ 0.3 \le \mu(\{c_{42}, c_{43}\}) \le 0.4.$

Similar to the above procedure, the optimal fuzzy measures on the sub-attribute sets are derived and listed in Table 7.

Table 7. Fuzzy measures on the sub-attribute sets

Sub-attribute set	FM	Sub-attribute set	FM	Sub-attribute set	FM	Sub-attribute set	FM
$\{c_{11}\}$	0.4	$\{c_{21}\}$	0.35	$\{c_{31}\}$	0.3	$\{c_{41}\}$	0.2
$\{c_{12}\}$	0.25	$\{c_{22}\}$	0.3	$\{c_{32}\}$	0.35	$\{c_{42}\}$	0.25
$\{c_{13}\}$	0.1	$\{c_{23}\}$	0.2	$\{c_{33}\}$	0.35	$\{c_{43}\}$	0.15
$\{c_{14}\}$	0.15	$\{c_{24}\}$	0.15	$\{c_{31},c_{32}\}$	0.65	$\{c_{41},c_{42}\}$	0.5
$\{c_{11},c_{12}\}$	0.4	$\{c_{21},c_{22}\}$	0.35	$\{c_{31},c_{33}\}$	0.65	$\{c_{41},c_{43}\}$	0.5
$\{c_{11},c_{13}\}$	0.4	$\{c_{21},c_{23}\}$	0.35	$\{c_{32},c_{33}\}$	0.7	$\{c_{42},c_{43}\}$	0.4
$\{c_{11},c_{14}\}$	0.4	$\{c_{21},c_{24}\}$	0.35	$\{c_{31},c_{32},c_{33}\}$	1	$\{c_{41},c_{42},c_{43}\}$	1
$\{c_{12},c_{13}\}$	0.7111	$\{c_{22},c_{23}\}$	0.4607				
$\{c_{12},c_{14}\}$	0.6530	$\{c_{22},c_{24}\}$	0.5244				
$\{c_{13},c_{14}\}$	0.8889	$\{c_{23},c_{24}\}$	0.7089				
$\{c_{11},c_{12},c_{13}\}$	1	$\{c_{21},c_{22},c_{23}\}$	1				
$\{c_{11},c_{12},c_{14}\}$	1	$\{c_{21},c_{22},c_{24}\}$	1				
$\{c_{11},c_{13},c_{14}\}$	1	$\{c_{21},c_{23},c_{24}\}$	1				
$\{c_{12},c_{13},c_{14}\}$	1	$\{c_{22},c_{23},c_{24}\}$	1				
$\{c_{11},c_{12},c_{13},c_{14}\}$	1	$\{c_{21},c_{22},c_{23},c_{24}\}$	1				

4.3 Ranking alternatives

Table 8. The linguistic decision making matrix *Q*

	c_{11}	c_{12}	c_{13}	c_{14}	c_{21}	c_{22}	c_{23}	c_{24}	c_{31}	c_{32}	c_{33}	c_{41}	c_{42}	c_{43}
A_1	*G*	*MG*	*VG*	*M*	*G*	*MG*	*G*	*M*	*G*	*M*	*VG*	*G*	*AG*	*G*
A_2	*MG*	*MB*	*M*	*G*	*VG*	*G*	*M*	*G*	*M*	*M*	*G*	*G*	*VG*	*MG*
A_3	*MB*	*MG*	*MG*	*MG*	*M*	*B*	*VG*	*M*	*MG*	*M*	*VG*	*MG*	*VG*	*G*
A_4	*MB*	*MG*	*G*	*VG*	*MB*	*VB*	*MB*	*MG*	*M*	*MG*	*G*	*M*	*G*	*M*
A_5	*MG*	*M*	*VG*	*M*	*B*	*G*	*G*	*M*	*MB*	*G*	*G*	*B*	*VB*	*G*
A_6	*VB*	*G*	*MB*	*VB*	*MG*	*M*	*MG*	*B*	*M*	*G*	*MB*	*G*	*B*	*B*
A_7	*B*	*VB*	*VG*	*MG*	*VG*	*MB*	*G*	*VB*	*MG*	*MB*	*VG*	*MB*	*M*	*MG*
A_8	*M*	*MB*	*M*	*M*	*G*	*VG*	*M*	*VB*	*B*	*VG*	*VB*	*B*	*VG*	*MB*

Step 1 (Initial Data Acquisition): The DMs use linguistic terms in Table 3 to assess the mobile medical app A_i (i = 1,2,...,8) based on the sub-attributes c_{jk_j} (j=1,2,3,4; k_1, k_2=1,2,3,4; k_3, k_4=1,2,3), that is, they give the assessment results of the attributes by using the linguistic term set S={*AG*, *VG*, *G*, *MG*, *M*, *MB*, *B*, *VB*, *AB*}={Absolutely Good, Very Good, Good, Medium, Medium Bad, Bad, Very Bad, Absolutely Bad}. For example, when the DMs give their assessment value of the mobile medical app

A_1 about professional medical consultation c_{11}, these DMs think that the assessment result of attribute c_{11} corresponding to alternative A_1 is "Very Good", "Good", "Good" and "Medium Good", respectively. Because the attribute values are not different from each other, the DMs need to reconfirm the attribute values until all the DMs reach an agreement. Suppose that all DMs think its attribute value is "Good". Thus, the evaluation value of the mobile medical app A_1 about the attribute c_{11} is obtained, denoted as "*G*". In this way, all the evaluation values from the DMs are derived to construct the linguistic decision matrix as tabulated in Table 8.

According to the transformation rules in Table 3, Table 8 values are converted into q-ROFNs given by Table 9.

Table 9. The q-ROF decision making matrix Q

	c_{11}	c_{12}	c_{13}	c_{14}	c_{31}	c_{32}	c_{33}
A_1	$(0.8,0.7)_q$	$(0.6,0.55)_q$	$(0.9,0.6)_q$	$(0.5,0.5)_q$	$(0.8,0.7)_q$	$(0.5,0.5)_q$	$(0.9,0.6)_q$
A_2	$(0.6,0.55)_q$	$(0.55,0.6)_q$	$(0.5,0.5)_q$	$(0.8,0.7)_q$	$(0.5,0.5)_q$	$(0.5,0.5)_q$	$(0.8,0.7)_q$
A_3	$(0.55,0.6)_q$	$(0.6,0.55)_q$	$(0.6,0.55)_q$	$(0.6,0.55)_q$	$(0.6,0.55)_q$	$(0.5,0.5)_q$	$(0.9,0.6)_q$
A_4	$(0.55,0.6)_q$	$(0.6,0.55)_q$	$(0.8,0.7)_q$	$(0.9,0.6)_q$	$(0.5,0.5)_q$	$(0.6,0.55)_q$	$(0.8,0.7)_q$
A_5	$(0.6,0.55)_q$	$(0.5,0.5)_q$	$(0.9,0.6)_q$	$(0.5,0.5)_q$	$(0.55,0.6)_q$	$(0.8,0.7)_q$	$(0.8,0.7)_q$
A_6	$(0.6,0.9)_q$	$(0.8,0.7)_q$	$(0.55,0.6)_q$	$(0.6,0.9)_q$	$(0.5,0.5)_q$	$(0.8,0.7)_q$	$(0.55,0.6)_q$
A_7	$(0.7,0.8)_q$	$(0.6,0.9)_q$	$(0.9,0.6)_q$	$(0.6,0.55)_q$	$(0.6,0.55)_q$	$(0.55,0.6)_q$	$(0.9,0.6)_q$
A_8	$(0.5,0.5)_q$	$(0.55,0.6)_q$	$(0.5,0.5)_q$	$(0.5,0.5)_q$	$(0.7,0.8)_q$	$(0.9,0.6)_q$	$(0.6,0.9)_q$
	c_{21}	c_{22}	c_{23}	c_{24}	c_{41}	c_{42}	c_{43}
A_1	$(0.8,0.7)_q$	$(0.6,0.55)_q$	$(0.8,0.7)_q$	$(0.5,0.5)_q$	$(0.8,0.7)_q$	$(0.98,0.2)_q$	$(0.8,0.7)_q$
A_2	$(0.9,0.6)_q$	$(0.8,0.7)_q$	$(0.5,0.5)_q$	$(0.8,0.7)_q$	$(0.8,0.7)_q$	$(0.9,0.6)_q$	$(0.6,0.55)_q$
A_3	$(0.5,0.5)_q$	$(0.7,0.8)_q$	$(0.9,0.6)_q$	$(0.5,0.5)_q$	$(0.6,0.55)_q$	$(0.9,0.6)_q$	$(0.8,0.7)_q$
A_4	$(0.55,0.6)_q$	$(0.6,0.9)_q$	$(0.55,0.6)_q$	$(0.6,0.55)_q$	$(0.5,0.5)_q$	$(0.8,0.7)_q$	$(0.5,0.5)_q$
A_5	$(0.7,0.8)_q$	$(0.8,0.7)_q$	$(0.8,0.7)_q$	$(0.5,0.5)_q$	$(0.7,0.8)_q$	$(0.6,0.9)_q$	$(0.8,0.7)_q$
A_6	$(0.6,0.55)_q$	$(0.5,0.5)_q$	$(0.6,0.55)_q$	$(0.7,0.8)_q$	$(0.8,0.7)_q$	$(0.7,0.8)_q$	$(0.7,0.8)_q$
A_7	$(0.9,0.6)_q$	$(0.55,0.6)_q$	$(0.8,0.7)_q$	$(0.6,0.9)_q$	$(0.55,0.6)_q$	$(0.5,0.5)_q$	$(0.6,0.55)_q$
A_8	$(0.8,0.7)_q$	$(0.9,0.6)_q$	$(0.5,0.5)_q$	$(0.6,0.9)_q$	$(0.7,0.8)_q$	$(0.9,0.6)_q$	$(0.55,0.6)_q$

Step 2 (Standardization): Since all the sub-attributes are of benefit type, there is no need to normalize the decision making matrix Q.

Step 3 (Calculating the q-ROFZI): The proposed q-ROFZI in (15) is used to calculate the integrated values Q_{ij} of alternative A_i (i = 1,2,...,8) according to the main-attribute c_j (j=1,2,3,4). Taking Q_{11} as an example, model (24) is derived.

$$\begin{aligned}
&\max\{0.4\lambda_1\oplus 0.25\lambda_2\oplus 0.4\lambda_3\oplus 0.1\lambda_4\oplus 0.4\lambda_5\oplus 0.7111\lambda_6\\
&\quad\oplus\lambda_7\oplus 0.15\lambda_8\oplus 0.4\lambda_9\oplus 0.6530\lambda_{10}\oplus 0.8889\lambda_{11}\oplus\lambda_{12}\oplus\lambda_{13}\oplus\lambda_{14}\oplus\lambda_{15}\}\\
&s.t.\begin{cases}
\lambda_1\oplus\lambda_3\oplus\lambda_5\oplus\lambda_7\oplus\lambda_9\oplus\lambda_{11}\oplus\lambda_{13}\oplus\lambda_{15}=(0.8,0.7)_q;\\
\lambda_2\oplus\lambda_3\oplus\lambda_6\oplus\lambda_7\oplus\lambda_{10}\oplus\lambda_{11}\oplus\lambda_{14}\oplus\lambda_{15}=(0.6,0.55)_q;\\
\lambda_4\oplus\lambda_5\oplus\lambda_6\oplus\lambda_7\oplus\lambda_{12}\oplus\lambda_{13}\oplus\lambda_{14}\oplus\lambda_{15}=(0.9,0.6)_q;\\
\lambda_8\oplus\lambda_9\oplus\lambda_{10}\oplus\lambda_{11}\oplus\lambda_{12}\oplus\lambda_{13}\oplus\lambda_{14}\oplus\lambda_{15}=(0.5,0.5)_q.
\end{cases}
\end{aligned}\tag{24}$$

where $\lambda_j=\left(u_{\lambda_j},v_{\lambda_j}\right)_q\ (j=1,2,...,15)$ denotes a q-ROFN.

Then, according to the operational laws of q-ROFNs, model (24) is rewritten as follows:

$$\max\Bigg\{\Bigg(\Big(1-\left(1-u_{\lambda_1}^q\right)^{0.4}\left(1-u_{\lambda_2}^q\right)^{0.25}\left(1-u_{\lambda_3}^q\right)^{0.4}\left(1-u_{\lambda_4}^q\right)^{0.1}\left(1-u_{\lambda_5}^q\right)^{0.4}\left(1-u_{\lambda_6}^q\right)^{0.7111}\left(1-u_{\lambda_7}^q\right)$$

$$\left(1-u_{\lambda_8}^q\right)^{0.15}\left(1-u_{\lambda_9}^q\right)^{0.4}\left(1-u_{\lambda_{10}}^q\right)^{0.6530}\left(1-u_{\lambda_{11}}^q\right)^{0.8889}\left(1-u_{\lambda_{12}}^q\right)\left(1-u_{\lambda_{13}}^q\right)\left(1-u_{\lambda_{14}}^q\right)\left(1-u_{\lambda_{15}}^q\right)\Big)^{\frac{1}{q}},$$

$$v_{\lambda_1}^{0.4}v_{\lambda_2}^{0.25}v_{\lambda_3}^{0.4}v_{\lambda_4}^{0.1}v_{\lambda_5}^{0.4}v_{\lambda_6}^{0.7111}v_{\lambda_7}^{1}v_{\lambda_8}^{0.15}v_{\lambda_9}^{0.4}v_{\lambda_{10}}^{0.6530}v_{\lambda_{11}}^{0.8889}v_{\lambda_{12}}^{1}v_{\lambda_{13}}^{1}v_{\lambda_{14}}^{1}v_{\lambda_{15}}^{1}\Bigg)\Bigg\}$$

$$s.t.\begin{cases}\left(\left(1-\left(1-u_{\lambda_1}^q\right)\left(1-u_{\lambda_3}^q\right)\left(1-u_{\lambda_5}^q\right)\left(1-u_{\lambda_7}^q\right)\left(1-u_{\lambda_9}^q\right)\left(1-u_{\lambda_{11}}^q\right)\left(1-u_{\lambda_{13}}^q\right)\left(1-u_{\lambda_{15}}^q\right)\right)^{\frac{1}{q}},v_{\lambda_1}v_{\lambda_3}v_{\lambda_5}v_{\lambda_7}v_{\lambda_9}v_{\lambda_{11}}v_{\lambda_{13}}v_{\lambda_{15}}\right)_q=(0.8,0.7)_q;\\ \left(\left(1-\left(1-u_{\lambda_2}^q\right)\left(1-u_{\lambda_3}^q\right)\left(1-u_{\lambda_6}^q\right)\left(1-u_{\lambda_7}^q\right)\left(1-u_{\lambda_{10}}^q\right)\left(1-u_{\lambda_{11}}^q\right)\left(1-u_{\lambda_{14}}^q\right)\left(1-u_{\lambda_{15}}^q\right)\right)^{\frac{1}{q}},v_{\lambda_2}v_{\lambda_3}v_{\lambda_6}v_{\lambda_7}v_{\lambda_{10}}v_{\lambda_{11}}v_{\lambda_{14}}v_{\lambda_{15}}\right)_q=(0.6,0.55)_q;\\ \left(\left(1-\left(1-u_{\lambda_4}^q\right)\left(1-u_{\lambda_5}^q\right)\left(1-u_{\lambda_6}^q\right)\left(1-u_{\lambda_7}^q\right)\left(1-u_{\lambda_{12}}^q\right)\left(1-u_{\lambda_{13}}^q\right)\left(1-u_{\lambda_{14}}^q\right)\left(1-u_{\lambda_{15}}^q\right)\right)^{\frac{1}{q}},v_{\lambda_4}v_{\lambda_5}v_{\lambda_6}v_{\lambda_7}v_{\lambda_{12}}v_{\lambda_{13}}v_{\lambda_{14}}v_{\lambda_{15}}\right)_q=(0.9,0.6)_q;\\ \left(\left(1-\left(1-u_{\lambda_8}^q\right)\left(1-u_{\lambda_9}^q\right)\left(1-u_{\lambda_{10}}^q\right)\left(1-u_{\lambda_{11}}^q\right)\left(1-u_{\lambda_{12}}^q\right)\left(1-u_{\lambda_{13}}^q\right)\left(1-u_{\lambda_{14}}^q\right)\left(1-u_{\lambda_{15}}^q\right)\right)^{\frac{1}{q}},v_{\lambda_8}v_{\lambda_9}v_{\lambda_{10}}v_{\lambda_{11}}v_{\lambda_{12}}v_{\lambda_{13}}v_{\lambda_{14}}v_{\lambda_{15}}\right)_q=(0.5,0.5)_q;\\ 0\le u_{\lambda_j}^q+v_{\lambda_j}^q\le 1, j=1,2,\ldots,15.\end{cases}\tag{25}$$

According to the inclusion relationship of q-RONs and Proposition 1, model (25) is equivalently transformed into the following bi-objective q-ROF mathematical programming:

$$\max\Bigg\{\Big(1-\left(1-u_{\lambda_1}^q\right)^{0.4}\left(1-u_{\lambda_2}^q\right)^{0.25}\left(1-u_{\lambda_3}^q\right)^{0.4}\left(1-u_{\lambda_4}^q\right)^{0.1}\left(1-u_{\lambda_5}^q\right)^{0.4}\left(1-u_{\lambda_6}^q\right)^{0.7111}\left(1-u_{\lambda_7}^q\right)$$

$$\left(1-u_{\lambda_8}^q\right)^{0.15}\left(1-u_{\lambda_9}^q\right)^{0.4}\left(1-u_{\lambda_{10}}^q\right)^{0.6530}\left(1-u_{\lambda_{11}}^q\right)^{0.8889}\left(1-u_{\lambda_{12}}^q\right)\left(1-u_{\lambda_{13}}^q\right)\left(1-u_{\lambda_{14}}^q\right)\left(1-u_{\lambda_{15}}^q\right)\Big)^{\frac{1}{q}}\Bigg\}$$

$$\min\left\{v_{\lambda_1}^{0.4}v_{\lambda_2}^{0.25}v_{\lambda_3}^{0.4}v_{\lambda_4}^{0.1}v_{\lambda_5}^{0.4}v_{\lambda_6}^{0.7111}v_{\lambda_7}^{1}v_{\lambda_8}^{0.15}v_{\lambda_9}^{0.4}v_{\lambda_{10}}^{0.6530}v_{\lambda_{11}}^{0.8889}v_{\lambda_{12}}^{1}v_{\lambda_{13}}^{1}v_{\lambda_{14}}^{1}v_{\lambda_{15}}^{1}\right\}$$

$$s.t.\begin{cases}\left(1-\left(1-u_{\lambda_1}^q\right)\left(1-u_{\lambda_3}^q\right)\left(1-u_{\lambda_5}^q\right)\left(1-u_{\lambda_7}^q\right)\left(1-u_{\lambda_9}^q\right)\left(1-u_{\lambda_{11}}^q\right)\left(1-u_{\lambda_{13}}^q\right)\left(1-u_{\lambda_{15}}^q\right)\right)^{\frac{1}{q}}=0.8;\\ \left(1-\left(1-u_{\lambda_2}^q\right)\left(1-u_{\lambda_3}^q\right)\left(1-u_{\lambda_6}^q\right)\left(1-u_{\lambda_7}^q\right)\left(1-u_{\lambda_{10}}^q\right)\left(1-u_{\lambda_{11}}^q\right)\left(1-u_{\lambda_{14}}^q\right)\left(1-u_{\lambda_{15}}^q\right)\right)^{\frac{1}{q}}=0.6;\\ \left(1-\left(1-u_{\lambda_4}^q\right)\left(1-u_{\lambda_5}^q\right)\left(1-u_{\lambda_6}^q\right)\left(1-u_{\lambda_7}^q\right)\left(1-u_{\lambda_{12}}^q\right)\left(1-u_{\lambda_{13}}^q\right)\left(1-u_{\lambda_{14}}^q\right)\left(1-u_{\lambda_{15}}^q\right)\right)^{\frac{1}{q}}=0.9;\\ \left(1-\left(1-u_{\lambda_8}^q\right)\left(1-u_{\lambda_9}^q\right)\left(1-u_{\lambda_{10}}^q\right)\left(1-u_{\lambda_{11}}^q\right)\left(1-u_{\lambda_{12}}^q\right)\left(1-u_{\lambda_{13}}^q\right)\left(1-u_{\lambda_{14}}^q\right)\left(1-u_{\lambda_{15}}^q\right)\right)^{\frac{1}{q}}=0.5;\\ v_{\lambda_1}v_{\lambda_3}v_{\lambda_5}v_{\lambda_7}v_{\lambda_9}v_{\lambda_{11}}v_{\lambda_{13}}v_{\lambda_{15}}=0.7;\\ v_{\lambda_2}v_{\lambda_3}v_{\lambda_6}v_{\lambda_7}v_{\lambda_{10}}v_{\lambda_{11}}v_{\lambda_{14}}v_{\lambda_{15}}=0.55;\\ v_{\lambda_4}v_{\lambda_5}v_{\lambda_6}v_{\lambda_7}v_{\lambda_{12}}v_{\lambda_{13}}v_{\lambda_{14}}v_{\lambda_{15}}=0.6;\\ v_{\lambda_8}v_{\lambda_9}v_{\lambda_{10}}v_{\lambda_{11}}v_{\lambda_{12}}v_{\lambda_{13}}v_{\lambda_{14}}v_{\lambda_{15}}=0.5;\\ 0\le u_{\lambda_j}^q+v_{\lambda_j}^q\le 1, j=1,2,\ldots,15.\end{cases}\tag{26}$$

Considering that $0\le u_{\lambda_j}<1$, $q\ge 1$ and q is constant, the first objective in model (26) is equivalent to

$$\min\Big\{0.4\ln\left(1-u_{\lambda_1}^q\right)+0.25\ln\left(1-u_{\lambda_2}^q\right)+0.4\ln\left(1-u_{\lambda_3}^q\right)+0.1\ln\left(1-u_{\lambda_4}^q\right)+0.4\ln\left(1-u_{\lambda_5}^q\right)+0.7111\ln\left(1-u_{\lambda_6}^q\right)$$

$$+\ln\left(1-u_{\lambda_7}^q\right)+0.15\ln\left(1-u_{\lambda_8}^q\right)+0.4\ln\left(1-u_{\lambda_9}^q\right)+0.653\ln\left(1-u_{\lambda_{10}}^q\right)+0.8889\ln\left(1-u_{\lambda_{11}}^q\right)+\ln\left(1-u_{\lambda_{12}}^q\right)$$

$$+\ln\left(1-u_{\lambda_{13}}^q\right)+\ln\left(1-u_{\lambda_{14}}^q\right)+\ln\left(1-u_{\lambda_{15}}^q\right)\Big\}$$

As there is $0<v_{\lambda_j}\le 1$, the second objective in model (26) is equivalent to

$$
\begin{aligned}
\min\Big\{&0.4\ln\left(1-u_{\lambda_1}^q\right)+0.25\ln\left(1-u_{\lambda_2}^q\right)+0.4\ln\left(1-u_{\lambda_3}^q\right)+0.1\ln\left(1-u_{\lambda_4}^q\right)+0.4\ln\left(1-u_{\lambda_5}^q\right)+0.7111\ln\left(1-u_{\lambda_6}^q\right)\\
&+\ln\left(1-u_{\lambda_7}^q\right)+0.15\ln\left(1-u_{\lambda_8}^q\right)+0.4\ln\left(1-u_{\lambda_9}^q\right)+0.653\ln\left(1-u_{\lambda_{10}}^q\right)+0.8889\ln\left(1-u_{\lambda_{11}}^q\right)+\ln\left(1-u_{\lambda_{12}}^q\right)\\
&+\ln\left(1-u_{\lambda_{13}}^q\right)+\ln\left(1-u_{\lambda_{14}}^q\right)+\ln\left(1-u_{\lambda_{15}}^q\right)\Big\}
\end{aligned}
$$

and, the first constraint in model (26) can be rewritten as

$$\ln\left(1-u_{\lambda_1}^q\right)+\ln\left(1-u_{\lambda_3}^q\right)+\ln\left(1-u_{\lambda_5}^q\right)+\ln\left(1-u_{\lambda_7}^q\right)+\ln\left(1-u_{\lambda_9}^q\right)+\ln\left(1-u_{\lambda_{11}}^q\right)+\ln\left(1-u_{\lambda_{13}}^q\right)+\ln\left(1-u_{\lambda_{15}}^q\right)=\ln\left(1-0.8^q\right).$$

Similarly, the second, third, fourth, fifth, sixth, seventh and eighth constraints in model (26) can be rewritten as

$$\ln\left(1-u_{\lambda_2}^q\right)+\ln\left(1-u_{\lambda_3}^q\right)+\ln\left(1-u_{\lambda_6}^q\right)+\ln\left(1-u_{\lambda_7}^q\right)+\ln\left(1-u_{\lambda_{10}}^q\right)+\ln\left(1-u_{\lambda_{11}}^q\right)+\ln\left(1-u_{\lambda_{14}}^q\right)+\ln\left(1-u_{\lambda_{15}}^q\right)=\ln\left(1-0.6^q\right)$$

$$\ln\left(1-u_{\lambda_4}^q\right)+\ln\left(1-u_{\lambda_5}^q\right)+\ln\left(1-u_{\lambda_6}^q\right)+\ln\left(1-u_{\lambda_7}^q\right)+\ln\left(1-u_{\lambda_{12}}^q\right)+\ln\left(1-u_{\lambda_{13}}^q\right)+\ln\left(1-u_{\lambda_{14}}^q\right)+\ln\left(1-u_{\lambda_{15}}^q\right)=\ln\left(1-0.9^q\right);$$

$$\ln\left(1-u_{\lambda_8}^q\right)+\ln\left(1-u_{\lambda_9}^q\right)+\ln\left(1-u_{\lambda_{10}}^q\right)+\ln\left(1-u_{\lambda_{11}}^q\right)+\ln\left(1-u_{\lambda_{12}}^q\right)+\ln\left(1-u_{\lambda_{13}}^q\right)+\ln\left(1-u_{\lambda_{14}}^q\right)+\ln\left(1-u_{\lambda_{15}}^q\right)=\ln\left(1-0.5^q\right);$$

$$\ln v_{\lambda_1}+\ln v_{\lambda_3}+\ln v_{\lambda_5}+\ln v_{\lambda_7}+\ln v_{\lambda_9}+\ln v_{\lambda_{11}}+\ln v_{\lambda_{13}}+\ln v_{\lambda_{15}}=\ln 0.7\ ;$$

$$\ln v_{\lambda_2}+\ln v_{\lambda_3}+\ln v_{\lambda_6}+\ln v_{\lambda_7}+\ln v_{\lambda_{10}}+\ln v_{\lambda_{11}}+\ln v_{\lambda_{14}}+\ln v_{\lambda_{15}}=\ln 0.55;$$

$$\ln v_{\lambda_4}+\ln v_{\lambda_5}+\ln v_{\lambda_6}+\ln v_{\lambda_7}+\ln v_{\lambda_{12}}+\ln v_{\lambda_{13}}+\ln v_{\lambda_{14}}+\ln v_{\lambda_{15}}=\ln 0.6;$$

$$\ln v_{\lambda_8}+\ln v_{\lambda_9}+\ln v_{\lambda_{10}}+\ln v_{\lambda_{11}}+\ln v_{\lambda_{12}}+\ln v_{\lambda_{13}}+\ln v_{\lambda_{14}}+\ln v_{\lambda_{15}}=\ln 0.5.$$

Constraint $0\le u_{\lambda_j}^q+v_{\lambda_j}^q\le 1$ in model (26) is equivalent to $0<u_{\lambda_j}^q\le 1$ and $v_{\lambda_j}^q\le 1-u_{\lambda_j}^q$, and therefore, equivalent to $q\ln\left(v_{\lambda_j}\right)\le\ln\left(1-u_{\lambda_j}^q\right)$ and $\ln\left(1-u_{\lambda_j}^q\right)\le 0$.

Denoting $x_j=\ln\left(1-u_{\lambda_j}^q\right)(j=1,2,\ldots,15)$, $y_j=\ln\left(v_{\lambda_j}\right)$ and q=3, using the linear weighted sum method, model (26) can be converted into the below single-objective linear programming model:

$$
\begin{aligned}
\min\{&0.4x_1+0.25x_2+0.4x_3+0.1x_4+0.4x_5+0.7111x_6+x_7+0.15x_8+0.4x_9+0.653x_{10}+0.8889x_{11}\\
&+x_{12}+x_{13}+x_{14}+x_{15}-\left(0.4y_1+0.25y_2+0.4y_3+0.1y_4+0.4y_5+0.7111y_6+y_7+0.15y_8+0.4y_9\right.\\
&\left.+0.653y_{10}+0.8889y_{11}+y_{12}+y_{13}+y_{14}+y_{15}\right)\}
\end{aligned}
$$

$$
s.t.\begin{cases}
x_1+x_3+x_5+x_7+x_9+x_{12}+x_{13}+x_{15}=\ln\left(1-0.8^3\right);\\
x_2+x_3+x_6+x_7+x_{10}+x_{12}+x_{14}+x_{15}=\ln\left(1-0.6^3\right);\\
x_4+x_5+x_6+x_7+x_{11}+x_{13}+x_{14}+x_{15}=\ln\left(1-0.9^3\right);\\
x_8+x_9+x_{10}+x_{11}+x_{12}+x_{13}+x_{14}+x_{15}=\ln\left(1-0.5^3\right);\\
y_1+y_3+y_5+y_7+y_9+y_{12}+y_{13}+y_{15}=\ln 0.7;\\
y_2+y_3+y_6+y_7+y_{10}+y_{12}+y_{14}+y_{15}=\ln 0.55;\\
y_4+y_5+y_6+y_7+y_{11}+y_{13}+y_{14}+y_{15}=\ln 0.6;\\
y_8+y_9+y_{10}+y_{11}+y_{12}+y_{13}+y_{14}+y_{15}=\ln 0.5;\\
x_j-3y_j\ge 0\ \left(j=1,2,\ldots,15\right);x_j\le 0\left(j=1,2,\ldots,15\right).
\end{cases}
\tag{27}
$$

Using Matlab R2017b software, the following solution is obtained:

$x_1 = -0.7174, x_2 = 0, x_3 = 0, x_4 = -0.9288, x_5 = 0, x_6 = -0.2433, x_7 = 0, x_8 = 0, x_9 = 0,$
$x_{10} = 0, x_{11} = -0.1335, x_{12} = 0, x_{13} = 0, x_{14} = 0, x_{15} = 0, y_1 = -0.3567, y_2 = 0,$
$y_3 = 0, y_4 = -0.3096, y_5 = 0, y_6 = -0.0811, y_7 = 0, y_8 = -0.0563, y_9 = 0,$
$y_{10} = -0.5167, y_{11} = -0.1201, y_{12} = 0, y_{13} = 0, y_{14} = 0, y_{15} = 0.$

Therefore, it is

$$Q_{11} = q - ROFZI(Q_{11_1}, Q_{12_1}, Q_{13_1}, Q_{14_1})$$
$$= \left(\left(1 - e^{0.4x_1 + 0.25x_2 + 0.4x_3 + 0.1x_4 + 0.4x_5 + 0.7111x_6 + x_7 + 0.15x_8 + 0.4x_9 + 0.653x_{10} + 0.8889x_{11} + x_{12} + x_{13} + x_{14} + x_{15}}\right)^{\frac{1}{3}}, \right.$$
$$\left. e^{0.4y_1 + 0.25y_2 + 0.4y_3 + 0.1y_4 + 0.4y_5 + 0.7111y_6 + y_7 + 0.15y_8 + 0.4y_9 + 0.653y_{10} + 0.8889y_{11} + y_{12} + y_{13} + y_{14} + y_{15}} \right)_3$$
$$= (0.7879, 0.5046)_3.$$

Similarly, the following integrated values are obtained:

$Q_{12} = (0.7581, 0.5471)_3, Q_{13} = (0.8004, 0.5896)_3, Q_{14} = (0.8931, 0.5118)_3, Q_{21} = (0.6885, 0.4625)_3,$
$Q_{22} = (0.8533, 0.5564)_3, Q_{23} = (0.6594, 0.5625)_3, Q_{24} = (0.7753, 0.6938)_3, Q_{31} = (0.6659, 0.4126)_3,$
$Q_{32} = (0.7408, 0.5587)_3, Q_{33} = (0.7596, 0.5484)_3, Q_{34} = (0.7607, 0.6909)_3, Q_{41} = (0.8443, 0.4952)_3,$
$Q_{42} = (0.6296, 0.5518)_3, Q_{43} = (0.6794, 0.5816)_3, Q_{44} = (0.6247, 0.6211)_3, Q_{51} = (0.6977, 0.4470)_3,$
$Q_{52} = (0.7780, 0.6321)_3, Q_{53} = (0.7530, 0.6684)_3, Q_{54} = (0.6794, 0.8317)_3, Q_{61} = (0.7184, 0.7408)_3,$
$Q_{62} = (0.6664, 0.4950)_3, Q_{63} = (0.6685, 0.5996)_3, Q_{64} = (0.7249, 0.7789)_3, Q_{71} = (0.7788, 0.6758)_3,$
$Q_{72} = (0.8210, 0.6204)_3, Q_{73} = (0.7650, 0.5845)_3, Q_{74} = (0.5357, 0.5681)_3, Q_{81} = (0.5804, 0.3602)_3,$
$Q_{82} = (0.8142, 0.6214)_3, Q_{83} = (0.7870, 0.7538)_3, Q_{84} = (0.7410, 0.7345)_3.$

Step 4 (Calculating the q-ROFZI on alternatives): By applying the proposed q-ROFZI in (15), the following overall values Q_i of each alternative $A_i\ (i = 1, 2, ..., 8)$ are derived:

$$Q_1 = (0.8679, 0.4380)_3, Q_2 = (0.8052, 0.4781)_3, Q_3 = (0.7891, 0.4477)_3, Q_4 = (0.7690, 0.4645)_3,$$
$$Q_5 = (0.7851, 0.5469)_3, Q_6 = (0.7505, 0.5763)_3, Q_7 = (0.8010, 0.5233)_3, Q_8 = (0.8003, 0.5145)_3.$$

Step 5 (Calculation of score values and accuracy values): The following score value $S(Q_i)$ of the overall value $Q_i\ (i = 1, 2, ..., 8)$ are derived:

$$S(Q_1) = 0.5698, S(Q_2) = 0.4128, S(Q_3) = 0.4017, S(Q_4) = 0.3545,$$
$$S(Q_5) = 0.3204, S(Q_6) = 0.2314, S(Q_7) = 0.3705, S(Q_8) = 0.3763.$$

There is no need to compute the accuracy values $H(Q_i)$ because the score values $S(Q_i)$ are sufficient to derive the ranking of the q-ROFNs in this practical instance .

Step 6 (Ranking the alternatives): The final ordering of the eight assessed Apps is: $A_1 > A_2 > A_3 > A_8 > A_7 > A_4 > A_5 > A_6$, which makes A_1 the optimal App.

4.4 Sensitive analysis

The proposed methodology involves parameter q, which depends on the preference of decision-maker and the decision-making environment. Therefore, it is necessary to explore its effect on the decision-making results and then determine its value in actual situations. We set various values of q of the proposed methodology to solve the above practical case. The decision results are depicted in Figs. 4 and 5. The following conclusions are drawn:

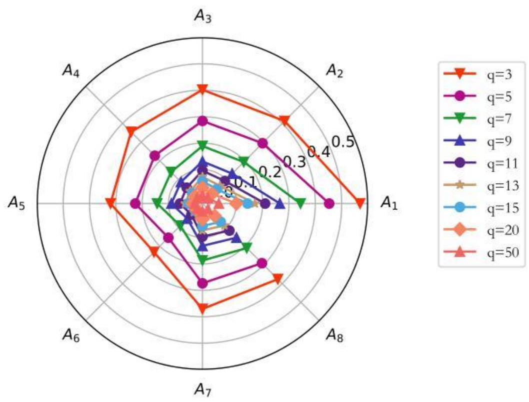

Fig. 4 The effect of q on the score values

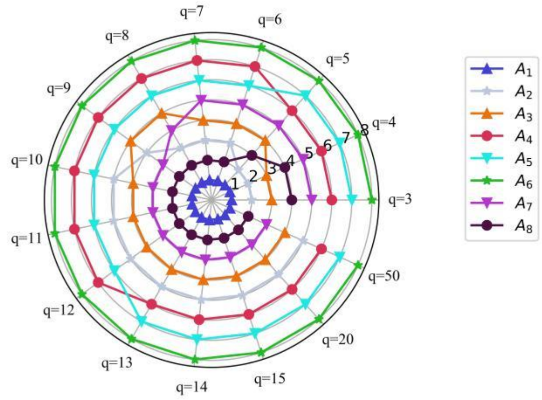

Fig. 5. The effect of q on the ranking orderings

- The score values, for each of the considered alternatives, obtained by our proposed MADM method decrease when the value of the parameter q increases. In addition, all score values obtained by our proposed method approach 0 as the parameter q increases. Thus, the parameter q can be viewed as a form of ''DM's attitude''. The bigger the value of parameter q is, the more pessimistic the DM is; the smaller the value of q is, the more optimistic the DM is.
- With the change of parameter q in our proposed MADM method, the ranking orderings change accordingly. When the parameter q=3 or 4, the ranking ordering is: $A_1>A_2>A_3>A_8>A_7>A_4>A_5>A_6$; when the parameter q=5, the ranking ordering is: $A_1>A_2>A_8>A_3>A_7>A_4>A_5>A_6$; when the parameter q=6 or 7, the ranking ordering is: $A_1>A_8>A_2>A_3>A_7>A_5>A_4>A_6$; when the parameter q=8, the ranking ordering is: $A_1>A_8>A_2>A_7>A_3>A_5>A_4>A_6$; when the parameter q=9, the ranking ordering is: $A_1>A_8>A_7>A_2>A_3>A_5>A_4>A_6$; when the parameter q=10, 11 or 12, the ranking ordering is: $A_1>A_8>A_7>A_3>A_2>A_5>A_4>A_6$; when the parameter q=13, 14, 15, 20 or 50, the ranking ordering is: $A_1>A_8>A_7>A_3>A_2>A_4>A_5>A_6$. Therefore, the parameter q or DM's attitude influences the ranking ordering of alternatives, which is expected.
- Although the ranking orderings vary for different q values, A_1 is always found to be the best App while A_6 is always classified as the worst App. In addition, when $q \geq 13$, the ranking ordering remains unchanged. Thus, it can be concluded that our proposed MADM method has a certain degree of robustness, especially when the value of the parameter q is relatively large.

As the ranking results rely on the parameter q to some extent, how to identify a suitable value of the parameter q is the key question when solving this problem. There are usually two ways to do this (Tang et al., 2020). The first one relies on DM's preferences. The other one is from the viewpoint of simplification, that is, the minimum integer q satisfying the restriction $(u_Q)^q + (v_Q)^q \leq 1$ should be taken. For example, let the evaluating value given by a DM be (0.9, 0.6); then, the parameter q can set to be 3 since $0.9^2 + 0.6^2 > 1$ and $0.9^3 + 0.6^3 < 1$.

4.5 Validity test of the proposed methodology

To illustrate the effectiveness of the proposed MADM method, the validity verification approach presented by Wang & Triantaphyllou (2008) is employed, which is composed of the following three test criteria.

Test criterion 1: An effective MADM method should not change the best alternative while substituting a worse alternative for another worse alternative without altering attributes' weights.

Utilizing test criterion 1, the non-optimal mobile medical app $A_{\sim}$={M, M, MB, MG, M, B, VG, MB, *MG*, *M*G, *G*, *MG*, *G*, *G*} is substituted for the worse mobile medical app A_3={*MB*, *MG*, *MG*, *MG*, *M*, *B*, *VG*, *M*, *MG*, *M*, *VG*, *MG*, *VG*, *G*} in the original decision matrix. When the BIT2FAC operator is utilized in Step 3 to this modified data, the following collective values of all mobile medical apps are derived:

$$Q_1=(0.8679,0.4380)_3, Q_2=(0.8052,0.4781)_3, Q_3=(0.7521,0.4618)_3, Q_4=(0.7690,0.4645)_3,$$
$$Q_5=(0.7851,0.5469)_3, Q_6=(0.7505,0.5763)_3, Q_7=(0.8010,0.5233)_3, Q_8=(0.8003,0.5145)_3.$$

The score values $S(Q_i)$ of all mobile medical apps are:

$$S(Q_1)=0.5698, S(Q_2)=0.4128, S(Q_3)=0.3269, S(Q_4)=0.3545,$$
$$S(Q_5)=0.3204, S(Q_6)=0.2314, S(Q_7)=0.3705, S(Q_8)=0.3763.$$

Thus, the ranking ordering is $A_1>A_2>A_8>A_7>A_4>A_3>A_5>A_6$, i.e., the optimal mobile medical app is still A_1. Therefore, our proposed MADM method meets test criterion 1.

Test criterion 2: An effective MADM approach should meet the transitive property.

Test criterion 3: While a MADM problem is decomposed into several sub-problems, and the same method is utilized to deal with these sub-problems to offer the ranking of the alternatives, the combined ranking of the alternatives should be consistent with the initial MADM problem.

Utilizing test criteria 2 and 3, the original MADM problem is decomposed into three smaller MADM problems {A_1, A_3, A_4, A_7, A_8}, {A_2, A_3, A_4, A_5, A_8} and {A_1, A_2, A_5, A_6, A_7}. Based on the process of our proposed MADM method, the ranking orderings obtained for these three sub-problems are $A_1>A_3>A_8>A_7>A_4$, $A_2>A_3>A_8>A_4>A_5$ and $A_1>A_2>A_7>A_5>A_6$, respectively. If the orderings of the sub-problems are combined together, the following combined ranking ordering is obtained $A_1>A_2>A_3>A_8>A_7>A_4>A_5>A_6$, which is consistent with the ranking of the original MADM problem and satisfies the transitive property. Therefore, our proposed MADM method meets test criteria 2 and 3.

5. Comparative analysis with the previous methods

To further explain the reliability and superiority of the proposed methodology, a comparison with the previous methods is carried out. The comparison process is based on two aspects: analysis of the relevant weight calculation model and analysis of the relevant information aggregation operators.

5.1 Analysis of the relevant weight calculation model

(1) Comparison with existing fuzzy AHP models

AHP, initially introduced by Saaty (1980), is one of the most typical MADM models that support DM to determine optimal attribute weights in a decision hierarchy. Since the AHP model is unable to model the ambiguity of human subjective consciousness, it has been extended to a variety of fuzzy environments (Mathew et al., 2020; Bakioglu & Atahan, 2021; Lahane & Kant, 2021; Dogan, 2021; Zhang et al., 2020; Buyukozkan & Gocer, 2021). Different from these fuzzy AHP models, the proposed q-ROFBWM uses q-ROFNs to conduct the pairwise comparisons, which are a powerful tool for characterizing imprecise and uncertain information. In addition, the proposed q-ROFBWM only implements reference comparisons, which means it only needs to identify the preference of the best attribute over all the other attributes and the preference of all the attributes over the worst attribute utilizing q-ROFNs. If n attributes are assessed, $2n$-3 pairwise comparisons in total are required in the proposed q-ROFBWM, while the existing fuzzy AHP models require $(n-1)n/2$ pairwise comparisons. The difference in the number of pairwise comparisons required is depicted in Fig. 6. When the number of attributes increases, the DM may provide conflicting logical judgments in the pairwise comparisons between attributes. Therefore, compared with these fuzzy AHP models, the proposed q-ROFBWM not only improves the accuracy to deal with uncertain information, but also reduces the number of times of pairwise comparisons and helps to maintain the consistency of judgments.

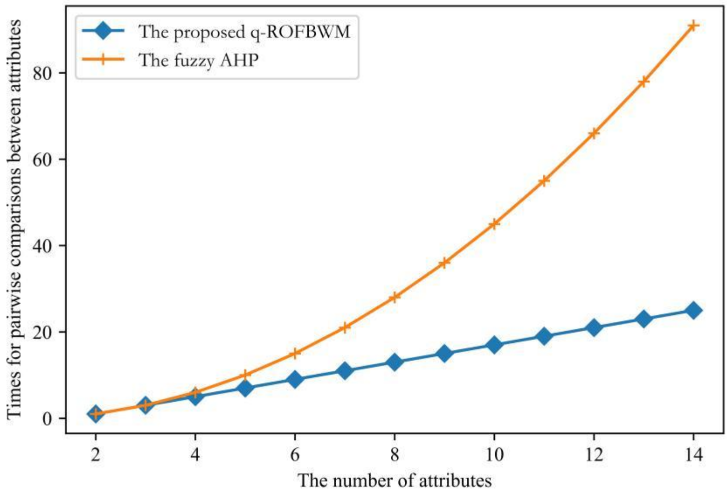

Fig. 6. Times for pairwise comparisons of utilizing fuzzy AHP and proposed q-ROFBWM.

(2) Comparison with the existing *q*-ROF BWM

By employing q-ROFNs to represent the preference information between attributes, Mi et al. (2019) developed a q-ROF BWM to compute the optimal weights of attributes. Since our proposed BWM also utilizes q-ROFNs to express pairwise comparisons between attributes, their output weights, as shown in Fig. 7, are compared to demonstrate the effectiveness and superiority of the proposed q-ROFBWM. From Fig. 7d, we observe that when the interface sub-attributes are independent, their weights derived by our proposed q-ROFBWM and the existing q-ROF BWM (Mi et al., 2019) are very close. This implies that our proposed q-ROFBWM is effective. However, the interactive relationship between other sub-attributes and main-attributes implies that their weights calculated by these two methods are different. This is clear for example in Fig. 7c, where weights of safety (c_2) sub-attributes for our proposed q-ROFBWM and the existing q-ROF BWM are depicted. For the safety sub-attributes, although the weights of $\{c_{21}\}$, $\{c_{22}\}$, $\{c_{23}\}$ and $\{c_{24}\}$ are identical for these two models, the weights of their different combination are quite distinct. The reason for this is explained as follows. The existing q-ROF BWM is based on the assumption that the safety sub-attributes are independent of one another and their effects are considered as additive. For actual mobile medical app evaluation problems, there exists some degree of interactive features among safety sub-attributes. Hence, such assumption is too strong to match decision behaviors in the actual decision-making problems. Our proposed q-ROFBWM is based on fuzzy measures. Hence, it can reflect the redundant characteristics between c_{21} and c_{22}, c_{21} and c_{23}, c_{21} and c_{24}, and c_{22} and c_{23}, and also capture the complementary relationships between c_{22} and c_{24}, c_{23} and c_{24}, c_{21}, c_{22} and c_{23}, c_{21}, c_{22} and c_{24}, c_{21}, c_{23} and c_{24}, and c_{22}, c_{23} and c_{24}. Thus, the weights of $\{c_{21},c_{22}\}$, $\{c_{21},c_{23}\}$, $\{c_{21},c_{24}\}$ and $\{c_{22},c_{23}\}$ calculated by our proposed BWM are smaller than those derived by the existing q-ROF BWM, whereas the weights of $\{c_{22},c_{24}\}$, $\{c_{23},c_{24}\}$, $\{c_{21},c_{22},c_{23}\}$, $\{c_{21},c_{22},c_{24}\}$, $\{c_{21},c_{23},c_{24}\}$ and $\{c_{22},c_{23},c_{24}\}$ calculated by our proposed BWM are greater than those derived by the existing q-ROF BWM. Therefore, the application scope of our proposed BWM is wilder than the existing q-ROF BWM because it is highly suitable for MADM problems involving independent and dependent attributes.

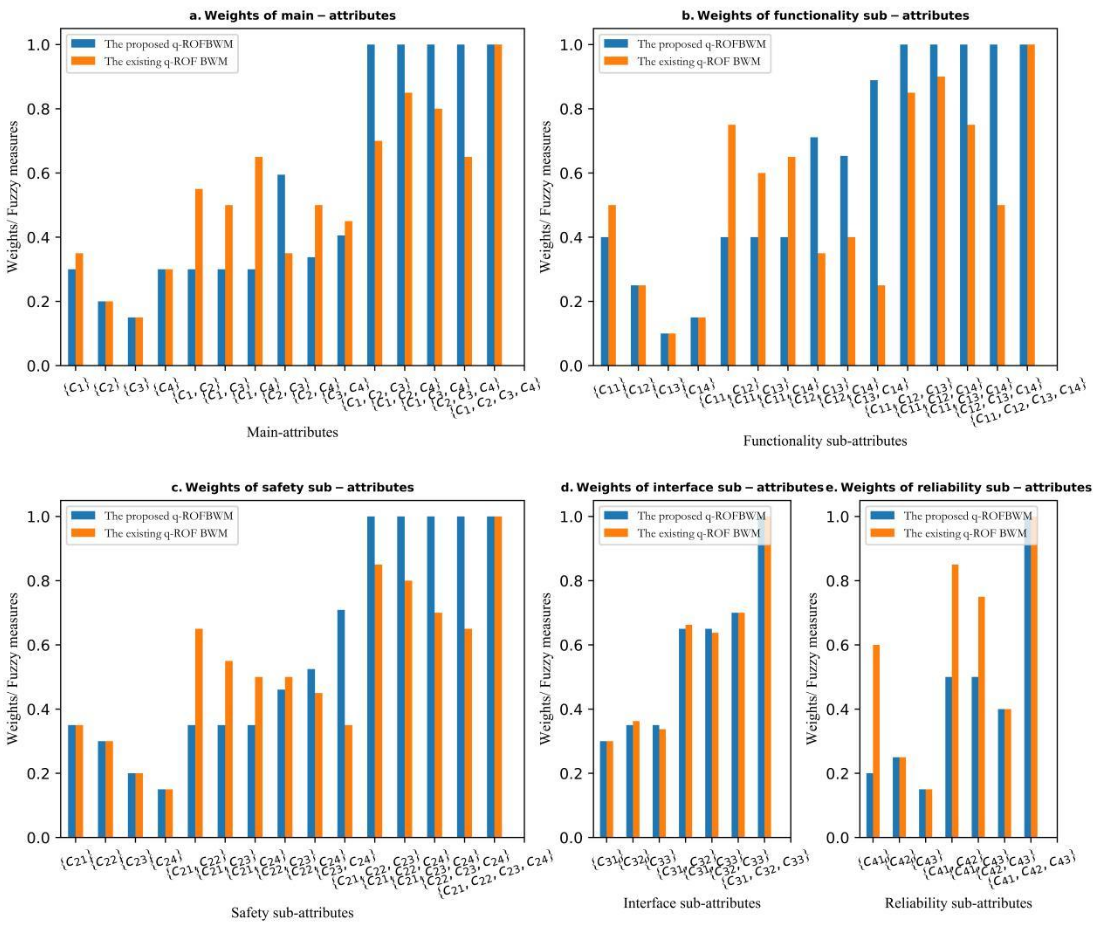

Fig. 7. Results comparison of the proposed q-ROFBWM and the q-ROF BWM (Mi et al., 2019)

(3) Comparison with other fuzzy BWM

The identification of the best and worst attributes will have a great effect on the weights determined by the BWM. In practical situations, the DM mainly determines these attributes intuitively and fails to identify them objectively. It would be risky when the weights of attribute are pretty close. This implies that the best or worst attribute might be indistinguishable from other attributes, and the DM would select the non-best attribute and non-worst attribute, which are not conformed with the real case. In such a situation, once the order of importance of the attributes is previously specified, the weights of the attributes derived by the BWM will still follow the identical previously specified order, and thus the attributes weights will have deviations. We compare the outcomes derived by the proposed BWM with those derived by existing fuzzy BWM, including Z-number BWM (ZBWM) (Liu et al., 2021), interval-valued intuitionistic uncertain linguistic BWM (IVIULBWM) (Liu et al., 2019), interval-valued intuitionistic fuzzy BWM (IVIFBWM) (Wang et al., 2021), and intuitionistic fuzzy BWM (IFBWM) (Wan & Dong, 2021), as tabulated in Table 10.

From Table 10, we observe that although the DM has previously specified the best and worst attributes from the attribute set, the proposed q-ROFBWM can revise the deviations noted above and obtain the right order of the weights, whereas the weights derived by these existing BWM still follow the previously specified order. The main reason for this is that the proposed q-ROFBWM is based on the Shapley value, which can capture the interaction between attributes and revise the weights of criteria. Nevertheless, these exiting BWM presume that the attributes are independent and neglect the effect of interaction among the attributes on the importance of the attributes, which is not realistic in many real world applications. Due to some inherent interaction between diverse attributes, exiting BWM do not work well in many decision making problems.

Table 10. Comparing with different BWM

Method	Properties			
	Best and worst attributes specified by DM	The best and worst attributes derived by BWM	Can revise the best and worst attributes	Can capture the interaction among the attributes
The proposed q-ROFBWM	c_{1B}=c_{11}, c_{2B}=c_{21}, c_{3B}=c_{32}, c_{4B}=c_{41}, c_B=c_1; c_{1W}=c_{13}, c_{2W}=c_{24}, c_{3W}=c_{31}, c_{4W}=c_{43}, c_W=c_3.	c_{1B}=c_{12}, c_{2B}=c_{21}, c_{3B}=c_{32}, c_{4B}=c_{41}, c_B=c_1; c_{1W}=c_{13}, c_{2W}=c_{24}, c_{3W}=c_{31}, c_{4W}=c_{43}, c_W=c_3.	Yes	Yes
Liu et al. (2021)'s ZBWM	c_{1B}=c_{11}, c_{2B}=c_{21}, c_{3B}=c_{32}, c_{4B}=c_{41}, c_B=c_1; c_{1W}=c_{13}, c_{2W}=c_{24}, c_{3W}=c_{31}, c_{4W}=c_{43}, c_W=c_3.	c_{1B}=c_{11}, c_{2B}=c_{21}, c_{3B}=c_{32}, c_{4B}=c_{41}, c_B=c_1; c_{1W}=c_{13}, c_{2W}=c_{24}, c_{3W}=c_{31}, c_{4W}=c_{43}, c_W=c_3.	No	No
Liu et al. (2019)'s IVIULBWM	c_{1B}=c_{11}, c_{2B}=c_{21}, c_{3B}=c_{32}, c_{4B}=c_{41}, c_B=c_1; c_{1W}=c_{13}, c_{2W}=c_{24}, c_{3W}=c_{31}, c_{4W}=c_{43}, c_W=c_3.	c_{1B}=c_{11}, c_{2B}=c_{21}, c_{3B}=c_{32}, c_{4B}=c_{41}, c_B=c_1; c_{1W}=c_{13}, c_{2W}=c_{24}, c_{3W}=c_{31}, c_{4W}=c_{43}, c_W=c_3.	No	No
Wang et al. (2021)'s IVIFBWM	c_{1B}=c_{11}, c_{2B}=c_{21}, c_{3B}=c_{32}, c_{4B}=c_{41}, c_B=c_1; c_{1W}=c_{13}, c_{2W}=c_{24}, c_{3W}=c_{31}, c_{4W}=c_{43}, c_W=c_3.	c_{1B}=c_{11}, c_{2B}=c_{21}, c_{3B}=c_{32}, c_{4B}=c_{41}, c_B=c_1; c_{1W}=c_{13}, c_{2W}=c_{24}, c_{3W}=c_{31}, c_{4W}=c_{43}, c_W=c_3.	No	No
Wan & Dong (2021)'s IFBWM	c_{1B}=c_{11}, c_{2B}=c_{21}, c_{3B}=c_{32}, c_{4B}=c_{41}, c_B=c_1; c_{1W}=c_{13}, c_{2W}=c_{24}, c_{3W}=c_{31}, c_{4W}=c_{43}, c_W=c_3.	c_{1B}=c_{11}, c_{2B}=c_{21}, c_{3B}=c_{32}, c_{4B}=c_{41}, c_B=c_1; c_{1W}=c_{13}, c_{2W}=c_{24}, c_{3W}=c_{31}, c_{4W}=c_{43}, c_W=c_3.	No	No

For computational complexity, our proposed q-ROFBWM, the existing q-ROF BWM and other fuzzy BWMs are all based on the BWM, so they have similar computational complexity. They require to implement reference comparisons. This results in fewer comparisons with regard to existing fuzzy AHP models. It contributes to less data collection, computation, and analysis time. Therefore, in our proposed q-ROFBWM, the existing q-ROF BWM and other fuzzy BWMs, there are $2n-3$ comparisons which are far less than comparisons in AHP ($n(n-1)/2$).

5.2 Analysis of the relevant information aggregation operators

(1) Comparison with the existing fuzzy Zhenyuan integral

The Zhenyuan integral, proposed by Wang et al. (2000), generalizes the ordered weighted average operator and weighted average operator. However, its arguments are crisp values. In practice, we usually encounter cases where the aggregated arguments cannot be denoted as crisp values. Mu & Zeng (2019), and Mu et al. (2018) combined the Zhenyuan integral with IFNs and IVIFNs, respectively, and presented the AIFZA operator and IVIFZA operator, respectively. In comparison with these existing fuzzy Zhenyuan integral, our newly proposed Zhenyuan integral is based on q-ROFNs. For IFNs, the sum of MF u_Q and NMF v_Q is in [0,1], while for IVIFNs the sum of upper MF u_Q and upper NMF v_Q is in [0,1]. This means that both IFNs and IVIFNs fail to express fuzzy information that do not verify their respective unit interval constraints. On the other hand, q-ROFNs require that the sum of the qth power of the MF u_Q and the qth power of the NMF v_Q is within [0,1], where $q \geqslant 1$. In addition, by adjusting the parameter q, q-ROFNs are more flexible to represent fuzzy information. The increase of the parameter q value translates into a wider scope of the fuzzy information it can capture, as depicted in Fig. 8. Theoretically speaking, it is apparent that our proposed q-ROFZI has a more powerful capability to describe uncertain information of MADM problems than the existing fuzzy Zhenyuan integral (Mu & Zeng, 2019; Mu et al., 2018).

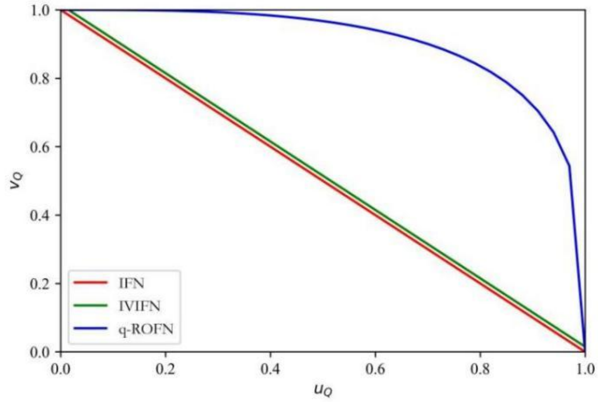

Fig. 8. Comparison of the space ranges of IFN, IVIFN and q-ROFN.

Clearly, the existing fuzzy Zhenyuan integrals (Mu & Zeng, 2019; Mu et al., 2018) fail to handle q-ROFNs in general. Thus, before applying these existing fuzzy Zhenyuan integrals to solve the above q-ROFNs application example, it is necessary to transform q-ROFNs into IFVs and IVIFNs according to Zhang *et al*'s transformation rule (Zhang et al., 2019), which may lead to distortion of information. The experimental results obtained for these integrals are listed in Table 11. The ranking ordering obtained by our proposed q-ROF Zhenyuan integral is slightly different to that obtained by the existing fuzzy Zhenyuan integral (Mu & Zeng, 2019; Mu et al., 2018), although the best and worst mobile medical apps are the same for the three Zhenyuan integrals. The comparative results verify the satisfaction performance of our proposed q-ROFZI. All in all, from theoretical analysis and numerical analysis, it can be verified that our proposed q-ROFZI is superior in expressing uncertain information.

Table 11. Ranking results from different fuzzy Zhenyuan integral for above application example

Method	Overall values	Ranking ordering
Mu & Zeng (2019)'s IFZA	Q_1=(0.6994,0.1887), Q_2=(0.6264,0.2184), Q_3=(0.6036,0.2056), Q_4=(0.5852,0.2164), Q_5=(0.6065, 0.2558), Q_6=(0.5631,0.2868), Q_7=(0.6214,0.2397), Q_8=(0.5947,0.2495).	A_1>A_2>A_3>A_7 >A_4>A_5>A_8>A_6
Mu et al. (2018)'s IVIFZA	Q_1=([0.6994,0.6994], [0.1887,0.1887]), Q_2=([0.6264,0.6264], [0.2184,0.2184]), Q_3=([0.6036,0.6036], [0.2056,0.2056]), Q_4=([0.5852,0.5852], [0.2164,0.2164]), Q_5=([0.6065,0.6065], [0.2558,0.2558]), Q_6=([0.5631,0.5631], [0.2868,0.2868]), Q_7=([0.6214,0.6214], [0.2397,0.2397]), Q_8=([0.5947,0.5947], [0.2495,0.2495]).	A_1>A_2>A_3>A_7 >A_4>A_5>A_8>A_6
Our proposed q-ROFZI	Q_1=(0.8679,0.4380)$_3$, Q_2=(0.8052,0.4781)$_3$, Q_3=(0.7891,0.4477)$_3$, Q_4=(0.7690,0.4645)$_3$, Q_5=(0.7851,0.5469)$_3$, Q_6=(0.7505,0.5763)$_3$, Q_7=(0.8010,0.5233)$_3$, Q_8=(0.8003,0.5145)$_3$.	A_1>A_2>A_3>A_8 >A_7>A_4>A_5>A_6

(2) Comparison with the existing q-ROF aggregation operators

In order to further demonstrate the functional superiority of our proposed q-ROFZI, it is compared with the existing q-ROF aggregation operators, including the Wq-ROFHA operator (Darko & Liang, 2020), the q-ROF weighted Archimedean BM (q-ROFWABM) operator (Liu & Wang, 2019), the q-ROF weighted generalized MSM (q-ROFWGMSM) operator (Liu & Wang, 2020), the q-ROFWPBM operator (Yan & Pang, 2019) and the q-ROFCI (Liang et al. 2019).

The relationships among assessment attributes are commonly categorized into two types: homogeneous relationship and heterogeneous relationship. The first type is further divided into independent relationship and interdependent relationship, where the interdependent relationship may exist between any two attributes or involve multiple attributes. The second type can be divided into the partitioned relevant relationship and whole relevant relationship. The partitioned relevant relationship means that all attributes are partitioned into several clusters, where criteria in the same cluster are relevant, while attributes in various clusters are irrelevant. The whole relevant relationship includes the interaction of two combinations of attributes and the overall interaction between attributes, where the interaction mainly refers to redundant relationship, complementary relationship and independence. An attribute relationship flow diagram is depicted is Fig. 9. One can easily observe that our proposed q-ROFZI is superior to the existing q-ROF aggregation operators because it can fully capture the heterogeneous relationship among attributes.

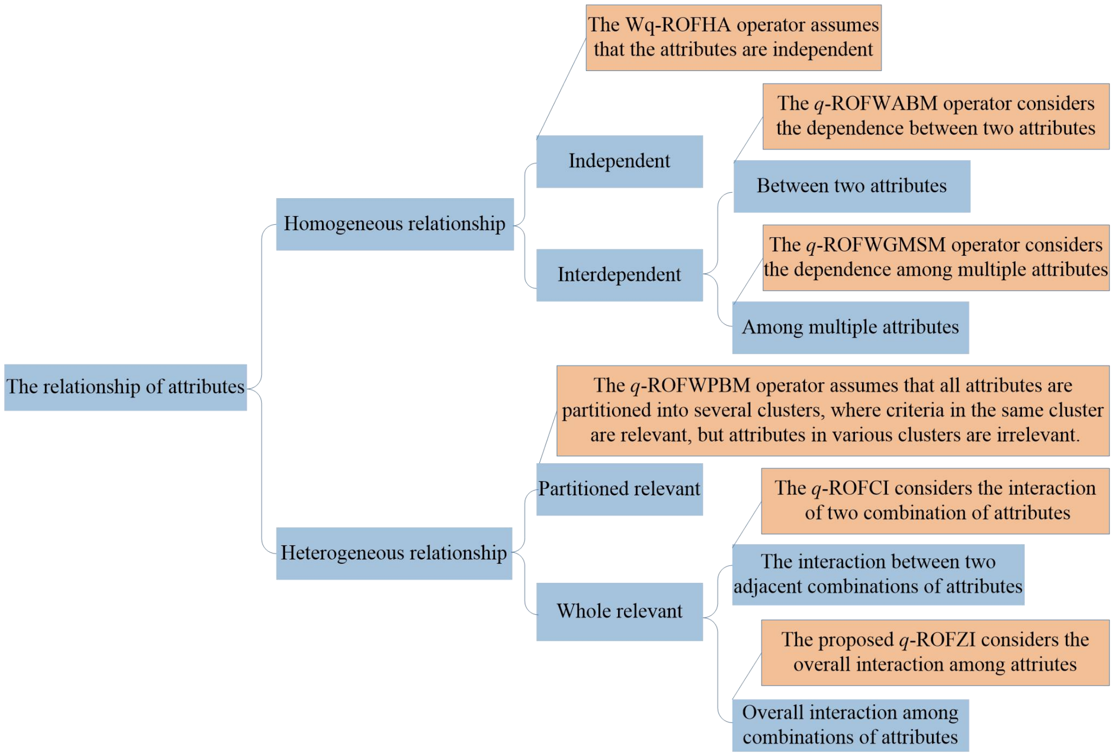

Fig. 9. Relationships of attributes

To demonstrate the superiority of the proposed q-ROFZI, it is further compared with the aforementioned q-ROF aggregation operators. The ranking orderings of the mobile medical apps for different q-ROF aggregation operators are listed in Table 12.

Table 12. Ranking results from different q-ROF operators for above application example

Method	Score value	Ranking ordering
Darko & Liang (2020)'s Wq-ROFHA operator	$S(Q_1)$=0.3763, $S(Q_2)$=0.1763, $S(Q_3)$=0.1490, $S(Q_4)$=0.0364, $S(Q_5)$=-0.0156, $S(Q_6)$=-0.1273, $S(Q_7)$=0.0159, $S(Q_8)$=0.1054.	A_1>A_2>A_3>A_8 >A_4>A_7>A_5>A_6
Liu & Wang (2019)'s q-ROFWABM operator	$S(Q_1)$=0.1557, $S(Q_2)$=0.0808, $S(Q_3)$=0.0572, $S(Q_4)$=-0.0278, $S(Q_5)$=-0.0456, $S(Q_6)$=-0.1103, $S(Q_7)$=-0.0320, $S(Q_8)$=-0.0390.	A_1>A_2>A_3>A_4 >A_7>A_8>A_5>A_6
Liu & Wang (2020)'s q-ROFWGMSM operator	$S(Q_1)$=0.1967, $S(Q_2)$=0.1031, $S(Q_3)$=0.1188, $S(Q_4)$=0.0167, $S(Q_5)$=0.0021, $S(Q_6)$=-0.0792, $S(Q_7)$=0.0178, $S(Q_8)$=-0.0321.	A_1>A_3>A_2>A_7 >A_4>A_5>A_8>A_6
Yang & Pang (2019)'s q-ROFWPBM operator	$S(Q_1)$=0.5698, $S(Q_2)$=0.4128, $S(Q_3)$=0.4017, $S(Q_4)$=0.3545, $S(Q_5)$=0.3204, $S(Q_6)$=0.2314, $S(Q_7)$=0.3705, $S(Q_8)$=0.3763.	A_1>A_3>A_2>A_8 >A_4>A_7>A_5>A_6
Liang et al (2019)'s q-ROFCI	$S(Q_1)$=-0.4433, $S(Q_2)$=-0.5315, $S(Q_3)$=-0.5262, $S(Q_4)$=-0.5778, $S(Q_5)$=-0.6040, $S(Q_6)$=-0.6443, $S(Q_7)$=-0.5803, $S(Q_8)$=-0.5721.	A_1>A_2>A_3>A_8 >A_7>A_4>A_5>A_6
Our proposed q-ROFZI	$S(Q_1)$=0.3437, $S(Q_2)$=0.1938, $S(Q_3)$=0.1638, $S(Q_4)$=0.0686, $S(Q_5)$=0.0245, $S(Q_6)$=-0.0488, $S(Q_7)$=0.0817, $S(Q_8)$=0.1249.	A_1>A_2>A_3>A_8 >A_7>A_4>A_5>A_6

From Table 12, one can observe that, the ranking ordering obtained by our q-ROFZI is the same as that of Liang et al (2019)'s q-ROFCI, but different to the ranking ordering obtained utilizing Darko & Liang (2020)'s Wq-ROFHA operator, Liu & Wang (2019)'s q-ROFWABM operator, Liu & Wang (2020)'s q-ROFWGMSM operator, and Yang & Pang (2019)'s q-ROFWPBM operator, whereas the best and worst mobile medical apps are the same for these six aggregation operators. The main reasons for these q-ROF aggregation operators to obtain different ranking orderings of mobile medical apps are explained as follows:

- Darko & Liang (2020)'s Wq-ROFHA operator assumes that the attributes are independent, which is inconsistent with the real interdependent relationship among the attributes in this application example. For example, medical appointment and medical and health knowledge are positively interactive. Generally speaking, the medical appointment and medical and health knowledge are complementary; hence, the comprehensive weight of the two attributes considered together should be greater than the sum of the weight of the two attributes when considered alone. Therefore, the ranking ordering obtained by the Darko & Liang (2020)'s Wq-ROFHA operator is unfit for this purpose because it neglects the dependent relationship of the attributes.
- Although Liu & Wang (2019)'s q-ROFWABM operator and Liu & Wang (2020)'s q-ROFWGMSM operator can capture the homogeneous relationship by aggregated arguments, they fail to describe the interdependent relationship by the attributes' weights; i.e., they assume the following equalities: $\mu(\kappa)=\sum_{c_j\in\kappa}\mu(\{c_j\})$ and $\mu(\gamma)=\sum_{c_{k_j}\in\gamma}\mu(\{c_{k_j}\})$, which are incorrect in this application example. Therefore, the ranking orderings obtained by Liu & Wang (2019)'s q-ROFWABM operator and Liu & Wang (2020)'s q-ROFWGMSM operator are also unfit for this purpose.

- Yang & Pang (2019)'s q-ROFWPBM operator can reflect the heterogeneous relationship by aggregated arguments. However, it is assumed that all attributes are partitioned into several clusters, where criteria in the same cluster are relevant, but attributes in various clusters are irrelevant. Specifically, for three attributes, it can model three situations: (1) the three attributes are independent; (2) the three attributes are dependent; (3) two attributes are dependent and they have no relationship with the other attribute. As for the reliability sub-attributes c_{41}, c_{42} and c_{43}, it is known that relevant relationship exists between c_{41} and c_{42}, and between c_{41} and c_{43}, respectively, while c_{42} and c_{43} are independent. Thus, Yang & Pang (2019)'s q-ROFWPBM operator fails to address this situation. Therefore, the ranking ordering obtained by Yang & Pang (2019)'s q-ROFWPBM operator is still unfit for this purpose.
- It is worth noting that both our proposed q-ROFZI and Liang et al (2019)'s q-ROFCI are based on fuzzy measures; i.e., they verify $\mu(\{c_{k_{j1}}, c_{k_{j2}}\}) < \mu(\{c_{k_{j1}}\}) + \mu(\{c_{k_2}\})$, $\mu(\{c_{k_{j3}}, c_{k_{j4}}\}) = \mu(\{c_{k_3}\}) + \mu(\{c_{k_4}\})$ and $\mu(\{c_{k_{j5}}, c_{k_{j6}}\}) > \mu(\{c_{k_5}\}) + \mu(\{c_{k_6}\})$. Thus, they can model the redundant, complementary and independent relationship of attributes. Therefore, our proposed q-ROFZI and Liang et al (2019)'s q-ROFCI obtain a realistic ranking ordering, which is not the case with the q-ROF aggregation operators in (Darko & Liang, 2020; Liu & Wang, 2019; Liu & Wang, 2020; Yang & Pang, 2019).

To summarize, we have verified our proposed q-ROFZI is superior to Darko & Liang (2020)'s Wq-ROFHA operator, Liu & Wang (2019)'s q-ROFWABM operator, Liu & Wang (2020)'s q-ROFWGMSM operator, and Yang & Pang (2019)'s q-ROFWPBM operator in modeling the dependent relationship among the attributes. However, because the ranking ordering from our proposed q-ROFZI and Liang et al (2019)'s q-ROFCI is the same, the superiority of our proposed q-ROFZI is not apparent. Thus, we conduct a simulation test to confirm the superiority of our proposed q-ROFZI in reflecting the heterogeneous relationship among the attributes. We randomly generate 1000 fuzzy measures on the attribute set using Matlab R2017b software and derive ranking orderings where each alternative appears in different ranking positions. The fluctuation scenario is displayed in Figs. 10 and 11. From Figs. 10 and 11, the fluctuation of each alternative in different ranking positions generated by Liang et al (2019)'s q-ROFCI is weaker than that generated by our proposed q-ROFZI. It is evident that Liang et al (2019)'s q-ROFCI is less sensitive to changes in fuzzy measures, expressing the heterogeneous relationship among the attributes, as opposed to our proposed q-ROFZI. Indeed, the ranking ordering should be modified to a certain extent with changes in the importance of some subsets of attributes, especially if relatively significant changes occur. Clearly, Liang et al (2019)'s q-ROFCI induces the loss of information because it omits the relationship between some subsets of attributes, so the ranking orderings are less robust. Therefore, our proposed q-ROFZI can capture the total heterogeneous relationship among attributes in a more comprehensive and accurate way.

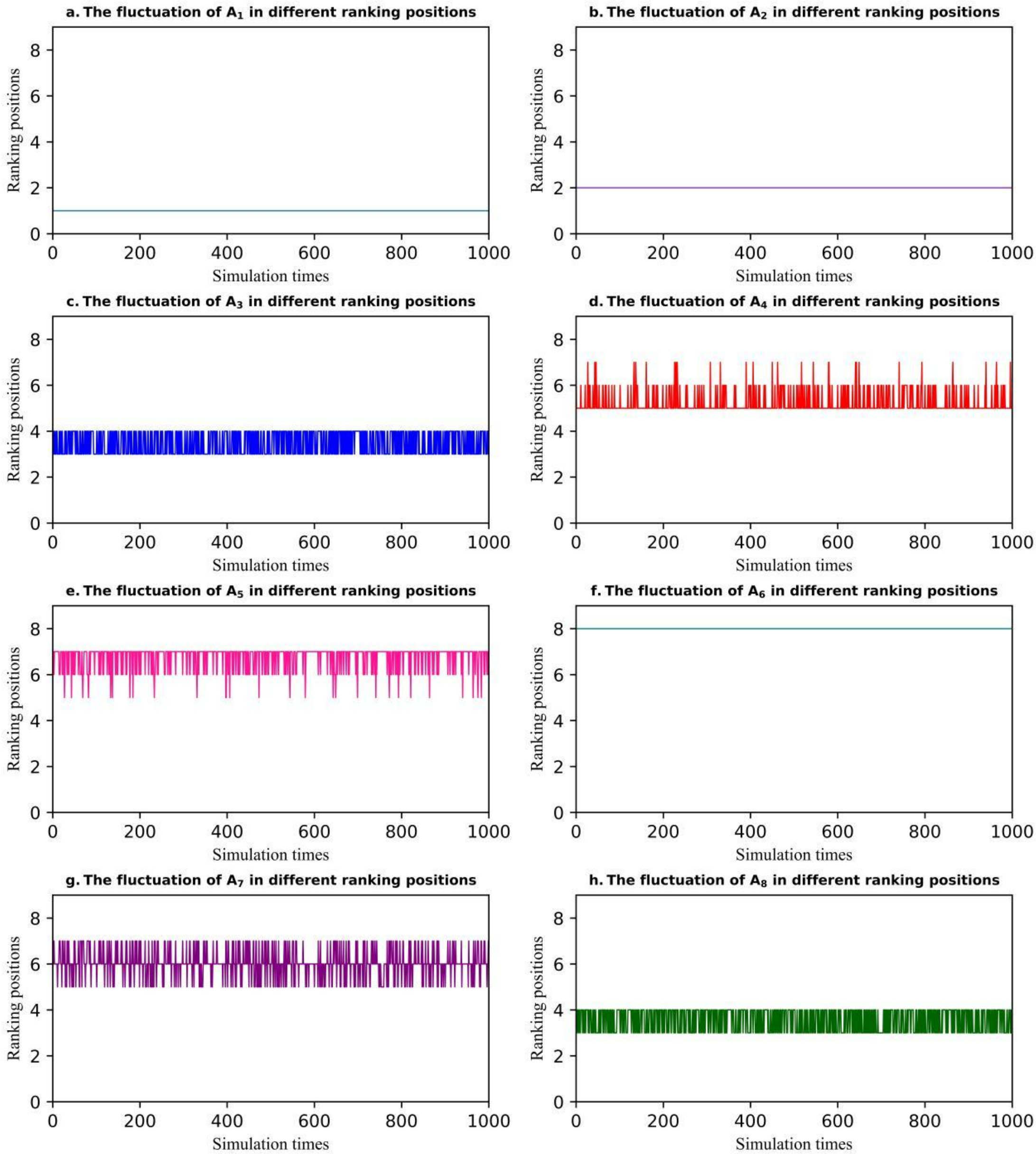

Fig. 10. The fluctuation of each alternative in different ranking positions by q-ROFCI (Liang et al., 2019)

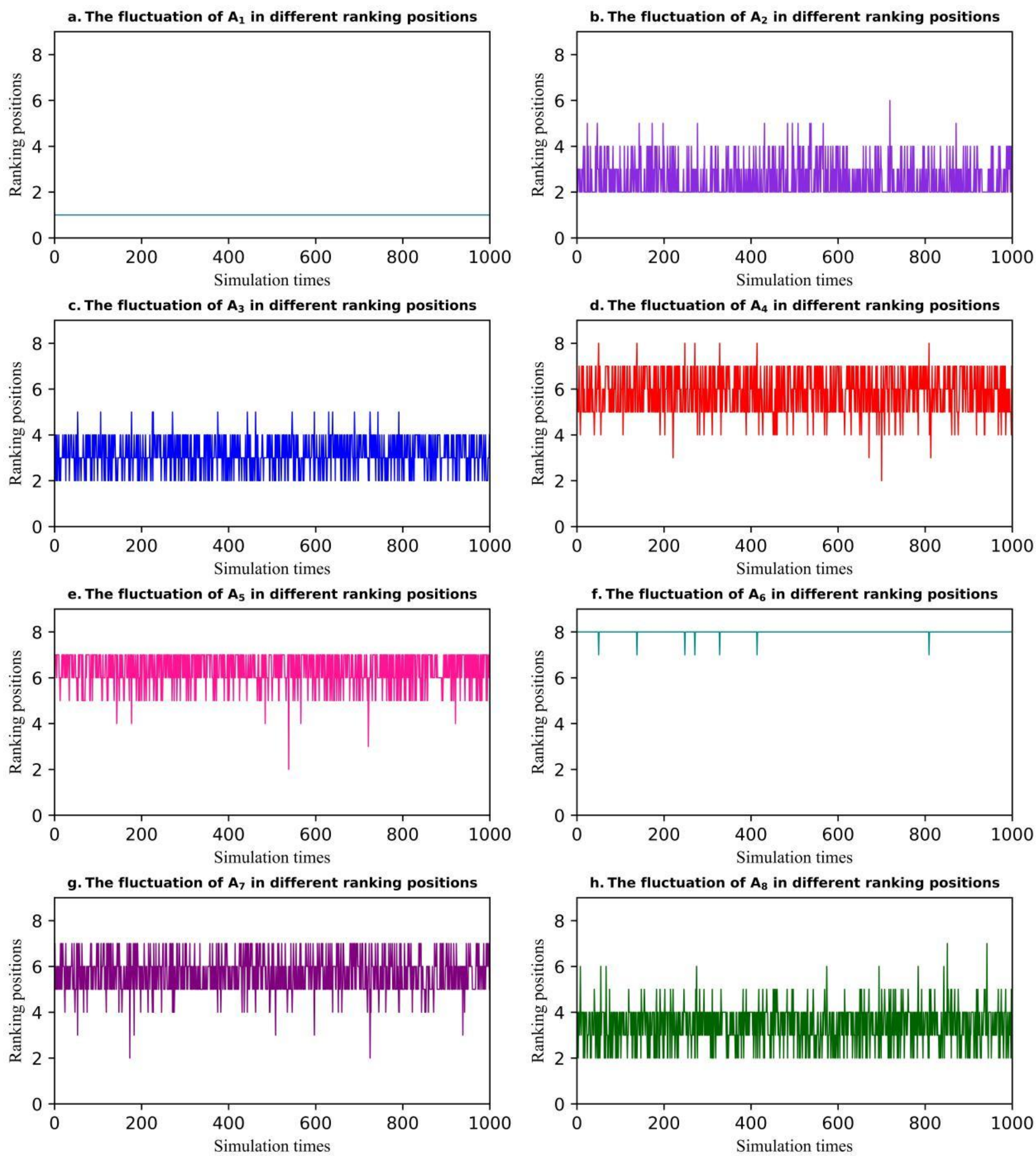

Fig. 11. The fluctuation of each alternative in different ranking positions by the proposed q-ROFZI

For computational complexity, Mu & Zeng (2019)'s IFZA, Mu et al. (2018)'s IVIFZA and our proposed q-ROFZI are all based on the Zhenyuan integral, so they have similar computational complexity. They all need $2^n - 2$ parameters to obtain fuzzy measures. In addition, when the attribute are independent, the Zhenyuan integral and Choquet integral become a weighted average operator. In such case, Darko & Liang (2020)'s Wq-ROFHA operator, Liang et al (2019)'s q-ROFCI and our proposed q-ROFZI are all based on the weighted average operator, so they have similar computational complexity. Their time complexity is $O(mn)$. When the attribute are interactive, Liang et al (2019)'s q-ROFCI only requires n parameters to derive fuzzy measures, whereas our proposed q-ROFZI need $2^n - 2$ parameters. Moreover, Liu & Wang (2019)'s q-ROFWABM operator, Liu & Wang (2020)'s q-ROFWGMSM operator and Yang & Pang (2019)'s q-ROFWPBM operator are based on the BM, MSM and PBM, respectively, so their time complexity are $O(mn^2)$, $O(mC_n^k)$ and $O(mn^2)$, where C_n^k represents the binomial coefficient. Therefore, when the criteria are interactive, the proposed q-ROFZI have similar computational complexity with Mu & Zeng (2019)'s IFZA and Mu et al. (2018)'s IVIFZA;

when the criteria are independent, the proposed q-ROFZI have similar computational complexity with Darko & Liang (2020)'s Wq-ROFHA operator and Liang et al (2019)'s q-ROFCI.

5.3 Summary about the advantages of the proposed method

The detailed comparative analysis revealed that the proposed method has the following advantages over existing methods.

- Compared with existing fuzzy AHP models, our method is simpler and more reliable. There are $2n-3$ comparisons in the proposed method, which are far less than comparisons in AHP ($n(n-1)/2$). In addition, our method derives more reliable outcomes as the comparisons are more consistent than existing fuzzy AHP models owing to the elimination of redundant comparisons.
- Compared with the existing q-ROF BWM (Mi et al., 2019), our method is more generic and effective. The existing q-ROF BWM assumes that the attributes are independent from each other. Hence, that method can only deal with MADM problems with independent attributes. In contrast, our method is based on fuzzy measures, and it can reflect the interactive characteristics among attributes. Hence, the proposed method can handle MADM problems involving both independent and dependent attributes. Its application scope is wilder than the existing q-ROF BWM.
- Compared with other fuzzy BWMs, the proposed method is more superior and applicable to real-world problems. Exiting fuzzy BWMs assume the independence of the attributes and neglect the influence of interaction between the attributes on the respective importance. Such setting is unrealistic in many real-world applications and can lead to poor decision-making performance. In contrast, our method is based on the Shapley value, which can capture the interaction between attributes and revise the weights of criteria. Therefore, the proposed method can work well in the problems with interactive attributes.
- Compared with the existing fuzzy Zhenyuan integrals (Mu & Zeng, 2019; Mu et al., 2018), the proposed method is more flexible and practical. The existing fuzzy Zhenyuan integrals are based on IFNs and IVIFNs, whose abilities to represent fuzzy information are limited. However, the proposed method can address this issue by using the q-ROFNs. We can represent any fuzzy number by adjusting the parameter q. This function is flexible and very useful in real-world applications.
- Compared with the existing q-ROF aggregation operators, our proposed is more rational and robust. Existing q-ROF aggregation operators consider interrelationship of attributes, but they cannot reflect the overall heterogeneous interaction among attributes. This may result in less rational outcomes. On the other hand, our method is based on the Zhenyuan integral, which can capture the total heterogeneous relationship among decision attributes in a more comprehensive and accurate manner. Therefore, in the real-world application scenarios, the proposed method is more applicable and reliable.

As such, we can conclude that the proposed method is better than the existing other methods for solving MADM problems with incomplete weights and interactive criteria.

6. Conclusion

In this study, to deal with the problems of mobile medical app evaluation, a novel q-ROF MADM was developed by integrating q-ROFNs, Zhenyuan integral, BWM and the Shapley value. First, in order to reflect the heterogeneous relationship of the attributes, a new nonlinear integral q-ROFZI over

q-ROFNs was proposed. This integral could be transformed into a q-ROF mathematical programming. As no approach exists for addressing such a type of q-ROF programming model, an approach was proposed to solving this mathematical programming problem. Meanwhile, its corresponding desirable features and special cases were discussed in detail. Then, two mathematical programming models based on the BWM and the Shapley value were established respectively for calculating optimal fuzzy measures in the scenarios where the attribute weight information is incompletely known or completely unknown. Next, a novel MADM method was developed for solving q-ROF MADM problems with incomplete weight information and interactive attributes. Furthermore, a case study regarding mobile medical app evaluation was provided to illustrate the specific application and the advantages of the developed approach with existing approaches in the literature. From the experimental results described in the case study, one can conclude that the proposed method can circumvent the issues associated to existing approaches to MADM problems within the context of q-ROFNs. The developed method offers a helpful-way for q-ROF MADM problems.

In practice, for some MADM problems, such as service quality evaluation of commercial banks (Tang et al., 2020a), credit risk assessment (Liu & Wang, 2020), and investment options (Liu & Wang, 2019; Tang et al., 2020), the decision information is mostly unknown, and many factors are affected by uncertainty. In such cases, q-ROFSs are suitable for depicting decision information because they can maximize the accuracy and integrity of uncertain information. Besides, for the above mentioned MADM problems, heterogeneous relationships often exist among the criteria. For example, when assessing the service quality of a commercial bank, the following attributes are considered: security, operational capacity, consultation service and service attitude. Here, operational capacity and service attitude can be considered as positively related attributes, security and consultation service can be considered as independent, while consultation service and service attitude can be considered as negatively related attributes. Complex heterogeneous relationships often can be observed among the attributes. As the proposed model can validly fuse q-ROFNs and globally reflect the heterogeneous relationship of the criteria, it is very suitable to handle such problems. As future research, we will analyze the application of the proposed method to above actual MADM problems. In addition, possible combinations between the Zhenyuan integral and other assessment expression models, such as the spherical fuzzy sets (Rayappan & Krishnaswamy, 2020), neutrosophic structured elements sets (Edalatpanah, 2020), linguistic q-ROFSs (Akram et al., 2021), complex q-rung picture fuzzy sets (Akram et al. 2021), linguistic intuitionistic cubic hesitant sets (Qiyas et al. 2021), intuitionistic fuzzy rough sets (Yahya et al., 2021), and probabilistic hesitant fuzzy sets (Naeem et al., 2021), to deal with various kinds of problems would be worth investigating.

Acknowledgement

This paper is supported by the National Natural Science Foundation of China (Nos.71771140, 71801142), the National Science Foundation for Post-doctoral Scientists of China (No.2021M691899), the Ministry of Education of Humanities and Social Science Project (No.21YJC630124), the Natural Science Foundation of Shandong Province (No.ZR2021QG013), the Postdoctoral Innovative Talent Support Plan of Shandong Province (SDBX2021009), the International Cooperation Research Project of Shandong University of Finance and Economics of China.

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