Multi-Unit Demand Auctions with Synergies:
Behavior in Sealed-Bid versus Ascending-Bid Uniform-price Auctions*

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Extended Abstract

In multi-unit demand, uniform-price auctions with synergies there are two opposing economic forces at work: Demand reduction resulting from the monopsony power that bidders with multiple-unit demands have and synergies which promote more aggressive bidding. To capture these synergies a bidder often needs to bid above the standalone value of units. This generates losses when the desired packages do not materialize. This *exposure problem*, and the possibility that acquiring the desired package will cost more than anticipated, may introduce an additional efficiency distorting force as bidders drop out without obtaining the desired packages. All three forces are present in a highly simplified economic setting that we explore experimentally, comparing sealed-bid with ascending-price (clock) private-value auctions. The behavioral space is very rich and demanding, with demand reduction forces winning out at lower valuations, the competing forces being at peak tension at intermediate valuations, and the synergy effect dominating at higher valuations. The clock auctions provide the opportunity for more refined responses, at intermediate valuations, than the sealed-bid auctions as bidders can make use of rivals drop-out prices to calculate whether or not it pays to try and capture the synergy bonus. If bidders make proper use of this information, and are not affected by the exposure problem, this can produce both higher revenue and economic efficiency than the sealed-bid auctions.

Bidding outcomes are closer to equilibrium in a clock compared to sealed-bid auctions. However, there are substantial deviations from equilibrium play in both cases, with patterns of out-of-equilibrium play differing between the two auction institutions. The most interesting and dramatic differences occur at the intermediate valuations as the exposure effect is much more pronounced in a clock compared to sealed-bid auctions. The net effect of these differences in out-of-equilibrium play is that the efficiency is the same, or somewhat higher, in the sealed-bid auctions (contrary to equilibrium predictions) and revenue is consistently, and significantly higher in the sealed-bid auctions.

Introduction

The recent FCC spectrum auctions have reinvigorated theoretical and empirical research on auctions in efforts to better understand the effects of different auction institutions when individual bidders demand multiple units of a given commodity. One line of research has focused on the performance of auctions with uniform-price rules, where all winning bids pay the same highest rejected bid.[1] It is well known by now that in such auctions, when the marginal value of additional units is non-increasing, bidders have an incentive to reduce demand to exploit the monopsony power they have when demanding multiple units. This strategy may result in winning fewer units, but when it does, it also reduces the price on units earned. (See, for example, Ausubel and Cramton, 1996 and Englebrecht-Wiggans and Kahn, 1995.) Demand reduction reduces economic efficiency and revenue relative to a full demand revelation. Experimental, and quasi-experimental research confirms that the demand reduction incentives are reasonably transparent and practiced even by relatively naive bidders (Kagel and Levin, in press; List and Lucking-Reily, 2000). Further, experiments comparing sealed-bid auctions with English-clock auctions reveal that although both auctions have the same normal form game representation, bidding is significantly closer to equilibrium in the clock auction, suggesting there are behavioral elements not fully captured in the theory (Kagel and Levin, in press).

Auctions which involve synergies, or increasing marginal values for additional units earned, provide additional incentives and generate radically different bidding strategies under uniform-price auction rules. Although the demand reduction incentive still exists, synergies

[1]Particular attention has been given to the effects of uniform-price auction rules because they are relatively easy to characterize and to implement, and are reasonably close in format to the one employed in the spectrum auctions (see Cramton, 1995).

create a powerful opposing force for aggressive bidding in order to acquire desired packages with their super additive value. In sealed-bid uniform-price auctions, that do not permit package bids, strong synergies may dictate submitting bids above the standalone values for individual units in order to increase the probability of winning a multi-unit package. However, this strategy is risky since if a bidder fails to acquire the whole package and wins only parts instead, she is likely to earn negative profits. Thus, in addition to the competing equilibrium incentives, an important "behavioral" force may affect bidding as well: Depending on the size of the potential loss, and risk preferences, bidders may refrain from such aggressive bidding in order to avoid exposure to such losses, despite the benefits of doing so (Bykowsky et al., 1995; Ausubel et al., 1997; Rothkopf et al., 1998). This avoidance has been referred to as the "exposure problem," a serious concern in some quarters at least, in designing auctions that do not permit package bids.[2]

The present paper reports the results of an experiment in a highly simplified auction environment designed to maintain the essential richness of the economic and behavioral forces present in multi-unit demand auctions with synergies. The simplicity of our design allows for reasonable interpretations of the experimental data, and serves as a benchmark against which to evaluate more complex studies. We compare outcomes of sealed-bid and ascending-bid (English-clock) uniform-price auctions. Under our design, the net effect of the demand reduction force and the synergy force is that in equilibrium: (1) at lower valuations, the demand reduction force dominates so that bidders shave their bids on marginal units, (2) at the highest valuations the

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[2]Bykowsky et al. discuss two types of exposure problems that may exist in complex environments with synergies. In our simple environment only the first of these potential problems exists namely exposure to bidding above the stand-alone value and not obtaining the desired package or obtaining it but at higher prices than anticipated. Bidder responsiveness to the exposure problem in this case is akin to *loss aversion* (Kahneman and Tversky, 1979). Package bidding has its own problems. These include the free rider/threshold problem and the computational complexity problem (Charles River Associates and Market Design Inc., 1998).

synergy force dominates so that bidders "go for it," bidding high enough to insure winning the items, and (3) at middle valuations the two forces are at peak tension and are counterbalancing each other, with bidding above value (but short of "going for it") in the sealed-bid auctions and "going for it," conditional on rivals' observed drop-out prices, in the clock auctions. The exposure problem works against the synergy effect, being most prominent at middle valuations when bidding above standalone values but short of going for it, and when valuations are such that going for it does not insure earning a positive profit.

Under our experimental design a human subject demanding two units of a commodity competes against different numbers of rivals demanding a single unit of the commodity in a uniform-price auction. The role of single-unit buyers is played by computers whose bids are equal to their private values (a dominant strategy for single-unit buyers in these auctions).[3] The standalone values for both items are the same for the human bidder, v_h, but earning both units generates *three* times the standalone value ($3v_h$). With independent private values drawn from a uniform distribution and with supply of two units, the equilibrium predictions for the "large" bidder correspond to the three regions characterized above. Thus, the experimental design is simple enough to yield equilibrium predictions while still maintaining the tension between the demand reduction and synergy forces. The design also abstracts away from the strategic uncertainties inherent in interactions between human bidders (e.g., problems of learning best responses given rivals' out-of-equilibrium bids). Finally, the experimental design allows us to employ a limited number of values for the human bidders without distorting the equilibrium

[3]That is, we announce the fact that subjects are playing against computers and that the computers will always bid their value, but do not discuss the basis for the computers' bidding strategy.

predictions. We exploit this by limiting the number of standalone values in each experimental session to three, with a number of replications at each value, thereby providing bidders with more systematic, and easier to process, feedback in this relatively complicated bidding environment. The standalone values employed span the strategy space and induce maximum differences in strategic behavior between the sealed-bid and clock auctions, while providing a number of replications at each value against which to evaluate behavior.

We do not expect bidders to be able to calculate and respond precisely to the fine cut off points associated with such a complex auction environment. Thus, we focus attention on the following questions: Are bidders sensitive to the tradeoffs inherent in uniform-price auctions with synergies? Do they behave differently in cases where the demand reduction force dominates compared to cases where the synergy bonus is strong enough to dominate? What role, if any, does the exposure problem play in bidding? What are the nature of deviations from optimal bidding strategies, and are there systematic differences in the patterns of deviations between the two types of auctions studied? Are there public policy implications resulting from any systematic deviations from optimal bidding?

There has been some recent experimental work on multi-unit demand auctions with synergies. The work falls roughly into two categories: First, "test bed" experiments designed to explore the effects of different auction rules on outcomes in environments with strong synergies (Banks, Ledyard, and Porter, 1989; Ledyard, Porter, and Rangle, 1997; Plott, 1997). Most of these experiments take place in environments that are substantially more complicated than ours, for which a *simple* (i.e., non-combinatorial) auction can implement global efficiency (maximizing the value of an assignment) with competitive equilibrium prices. (For details, see the references cited above.) Second, are experiments designed to explore the feasibility of Vickrey style auctions with package bids (Isaac and James, 2000; Brenner and Morgan, 1997). We do not address the question of Vickrey style auctions here. Finally, Ausubel et al. (1997) and Morten and Spiller (1996) examine license interdependencies in some of the early FCC spectrum auctions.

The auction model underlying our experiment is similar to one developed in Krishna and Rosenthal (1996) to explore simultaneous sealed-bid auctions with synergies. In both cases there is a single bidder demanding two units competing against a number of rivals demanding a single unit. The primary difference between the two models is that in Krishna and Rosenthal the bidder demanding multiple units competes in two separate second-price auctions against *n* single-unit demand bidders in each market.[4] In other words, in our model the two goods are perfect substitutes and sold together in a single uniform-price auction. In Krishna and Rosenthal the two goods are imperfect substitutes and sold in two separate second price auctions. The net effect is that there is no demand reduction force present in their model as there is in ours. However, in regions of our experimental design where the synergy force dominates the demand reduction force, the two models make remarkably similar predictions: Single-unit bidders always bid their value. When bidding above value, the bidder with increasing returns always bids the same on both units with bids increasing in the valuation drawn.[5] Once the valuation is high enough, there is a

[4]Krishna and Rosenthal extend their analysis to auctions in which there is more than one bidder with increasing returns. The Krishna-Rosenthal model is (arguably) closer, in some respects, to the U. S. spectrum auctions than our experimental design. Their model does not, however, deal with possible synergies *within* a given market as ours does. It is these underlying economic and behavioral forces that our experiment is designed to investigate rather than any effort to faithfully replicate any particular spectrum sale design.

[5]This result comes about for different reasons in the two models: In our case, bidding the same on both items follows from a dominance argument (see Appendix A). In Krishna and Rosenthal it follows from the assumption of equal numbers of single-unit bidders in each market in conjunction with their focusing on symmetric Nash bidding strategies.

discontinuous jump in the bid function, so that the bidder with increasing demand "goes for it." Finally, bids of the multi-unit demand bidder are weakly decreasing with more competition. Krishna and Rosenthal do not extend their analysis to ascending-bid clock auctions as we do here.

Our experiment yields a number of basic insights: Bidders are always closer to optimal bidding strategies in a clock compared to sealed-bid auctions. Bidders are responsive to the underlying economic forces present in the auction even though there is considerable out-of-equilibrium play. Further, out-of-equilibrium play differs substantially between the two institutions. In the sealed-bid auctions there is a clear tendency for bidders to overbid at low values and underbid at high values. In contrast, in the clock auctions, absent secure, positive expected profits, there is a general reluctance to bid above value when optimality requires it, consistent with the "exposure problem." At least part of this differential sensitivity to the exposure problem in clock compared to sealed-bid auctions results from the obvious ability to stop the bidding and assure a positive profit in the clock auctions. This is indicative of a clear presentation format effect, an outcome observed in other auction settings as well (Kagel, Harstad, and Levin, 1987; Levin, Kagel, and Richard, 1996; Kagel and Levin, in press). As a result, the clock auction *fails* to improve efficiency relative to the sealed-bid auctions where the theory predicts it should, and the sealed-bid auctions generate uniformly higher revenue.

The plan of the paper is as follows: Section I develops the theoretical predictions for both ascending-bid and sealed-bid auctions. The experimental design is outlined in Section II. Results of the experiment are reported in Section III. We close with a brief summary and discussion of our major results.

I. Theoretical Predictions:

We investigate bidding in IPV auctions with (n+1) bidders and *m indivisible*, identical objects for sale, where $n \$ m$. Each bidder i (i = 1, ... , n) demands a single unit of the good, placing a value v_i on the good. These bidders are indexed by their values so that $v_1 \$ v_2 \$, ..., \$ v_n$. Bidder *h,* the (n+1)th bidder, demands two units of the good, with the value of each unit *by itself* equal to v_h. However, earning two units generates synergies so that the value of winning both units = $2v_h + \alpha v_h$. Bidders' values were independent and identical draws (*iid)* from a *uniform* distribution on the interval [0,V]. In what follows we work with $m = 2$, $\alpha = 1$, as these are the values employed in the experiment, and analyze behavior within the unit interval (V=1).[6]

For both sealed-bid and clock auctions there are three bidding regions, with distinctly different bidding strategies for the human bidder in each region. These are summarized in Figure 1. Note, that for regions 1 and 3 outcomes are essentially the same between sealed-bid and clock auctions: demand reduction in region 1 and "go for it" in region 3. However, for the clock auctions, there is more flexibility in executing the optimal strategy in region 1, and the interval for region 3 is smaller. In contrast, outcomes are distinctly different between the two auction formats in region 2. In the sealed-bid auction, optimal bidding requires submitting two equal bids set *higher* than the value of a single unit, which may result in winning one unit and earning negative profits. In the clock auctions, the multi-unit demand bidder conditions her drop-out price on whether the second highest rival's value, v_2, is lower or higher than a predetermined cut off value, $\mathbf{P}^*$. If $\mathbf{P}^* > v_2$ she should "go for it" in an all out effort to win both units. Otherwise, she should

[6]Setting $\alpha = 1$ and $m = 2$ greatly simplifies bidders' calculations while still yielding the rich behavioral space characterized in Figure 1 below.

drop out on both units at **P***. As a result, and in contrast to the sealed-bid auctions, bidders should never earn a single unit. Detailed analysis of the auction procedures and outcomes follow.

Sealed-bid auctions: In the sealed-bid auction, each bidder simultaneously submits sealed bids for each of the units demanded. These are ranked from highest to lowest, with the m highest bids each winning an item and paying a price equal to the m+1 highest bid.

For bidders $i = 1, \ldots, n$, demanding a single unit of the commodity, there is a dominant strategy (the same as in the familiar single-unit Vickrey auction) to bid their value, v_i . Bidder h demands two units of the commodity and submits two bids, b_1 and b_2 for unit one and two respectively. Without loss of generality assume that $b_1 \$ b_2$. The optimal bidding strategy varies dramatically with v_h, since this directly affects the tradeoffs between incentives promoting demand reduction and those promoting bidding above v_h in order to benefit from the synergy bonus. There are three regions of interest:

1. For $v_h < ½$, $b_1 = v_h$ and $b_2 = 0$. (For $v_h = ½$, $b_1 = b_2 = ½$.)[7].

2. For $½ < v_h < v(n)$, $v_h < b_1 = b_2 < 1$. That is, the optimal bidding strategy calls for submitting two equal bids above $v_{h,}$ but not high enough to assure winning both units. Both the size of this interval and how much to bid above v_h varies with n, with a wider interval and more aggressive bidding the smaller n is (see, for example, Table 1 below). As our proof in the appendix uses only dominance arguments to yield $b_1 = b_2$ for this region, these criteria hold for risk aversion as well, although the precise level of the bids would be affected by risk preferences.

3. For $v_h \$ v(n)$, $b_1 = b_2 \$ 1$. That is, the optimal bidding strategy is to "go for it," bidding high enough to insure wining both units.

[7] Appendix A provides derivations of the equilibrium bidding strategies for the sealed-bid auctions.

Ascending-bid (Clock) Auctions: The ascending-bid version of the uniform-price auction (also referred to as a clock auction or an English-clock auction) starts with the price being zero and increasing continuously thereafter. Bidders start out actively bidding on all units demanded, choosing the price to drop out of the bidding. Dropping out is irrevocable so that a bidder can no longer bid on a unit she has dropped out on.[8] The drop-out price which equates the number of *remaining* active bids to the number of items for sale sets the market price. Winning bidders earn a profit equal to the value of their winning unit less the market price. All other units earn zero profit. Posted on each bidder's screen at all times is the current price of the item, the number of items for sale, and the number of units actively bid on, so that *h* can tell at exactly what price a rival has dropped out. Further, there is a brief pause in the progress of the price clock following a drop out during which *h* can drop out as well. Dropouts during the pause are recorded as having dropped out at the same price, but are indexed as having dropped later than the dropout that initiated the pause.[9]

Bidders i = 1, ..., *n* demanding a single unit have a dominant strategy to remain active until the clock price reaches their value, v_i. As in the sealed-bid version of the auction there are three regions of interest:

1. For $v_h < ½$, the optimal bidding strategy for *h* is comparable to the sealed-bid auction strategy in the sense that *h* earns greater expected profit by winning a single unit and reducing the price paid

[8]Given that the computers follow the dominant strategy, *h* has no incentive to drop out and re-enter the auction. However, we plan to conduct additional experiments where the irrevocable exit rule may become relevant.

[9]The auction is formally modeled as a continuous-time game. However, we want to take into account the possibility that bidder *i*'s strategy may be to reduce her quantity at a given time, while bidder *j*'s strategy may be to reduce his quantity at the soonest possible instant after bidder *i* does. This requires allowing "moves that occur consecutively at the same moment in time" (Simon and Stinchcombe, 1989; also see Ausubel, 1997).

by not winning a second unit. However, there is considerably more flexibility in carrying out the optimal policy:

If $v_2 \leq v_h$, $d_1 = v_h$ and $0 \leq d_2 \leq v_2$, and

If $v_2 > v_h$, then $d_1 \in [v_h, \max(v_h, v_3)]$ and $d_2 \in [0, \max(v_h, v_3)]$

where d_i is *h*'s dropout price on unit *i*.

2. For $v \in [½, 2/3)$ the optimal bidding strategy uses the information revealed by rivals' drop-out prices. In particular there is a cutoff point, $\mathbf{P^*} = [3v_h - 1]$ such that:

If $v_2 \leq \mathbf{P^*}$, $d_1 = d_2 \geq 1$ and

If $v_2 > \mathbf{P^*}$, $d_1 = d_2 \in [\mathbf{P^*}, \max\{\mathbf{P^*}, v_3\}]$.

This yields distinctly different outcomes than the sealed-bid auctions in this interval. Note that, in contrast to the sealed-bid auction, the bidding rule in this region does not depend on the number of rivals as the information revealed in the second highest computer's dropout price is sufficient to determine the optimal bidding strategy.

3. For $v_h \geq 2/3$, $d_2 \geq 1$, so that *h* has to "go for it," winning both for sure. Note that "going for it" in this interval assures positive profits, as *h* earns at least 2 and pays at most 2, so that the exposure problem is not an issue here. This strategy yields higher expected profits for any realization of v_2 compared to stopping the auction and earning a single unit. Further, the size of this interval is smaller than the interval in which bidders "go for it" in the sealed-bid auctions.

II. Experimental Design:

Valuations were i.i.d. from a uniform distribution with support [0, \$7.50].[10] Bidders with

[10]The underlying support for valuations, along with the number of computer rivals, were chosen with an eye towards comparing behavior here with earlier multi-unit demand auctions with flat demands (no synergies) which call for demand reduction at all values (Kagel and Levin, in press).

single-unit demands were represented by computers programmed to submit bids equal to their value in the sealed-bid auctions or to stay active until the price reached their value in the clock auctions. Bidder *h* was played by subjects drawn from a wide cross-section of undergraduate and graduate students at the University of Pittsburgh and Carnegie-Mellon University.[11] Each *h* operated in her own market with her own set of computer rivals, knew they were bidding against computer rivals and the number of computer rivals, as well as the computers' bidding strategy.

The use of computer rivals has a number of advantages in a first foray into this area: *h*s face all of the essential strategic tradeoffs involved in auctions of this sort but in a very "clean" environment. The latter include no strategic uncertainty regarding other bidders' behavior and no issues of whether or not "common knowledge" assumptions are satisfied.

Since single-unit bidders have a dominant strategy independent of *h*'s valuation, which permits us to employ a limited number of values for *h* without distorting the equilibrium predictions. Given the complexity of the environment, we limited ourselves to four values designed to fully represent/span the strategy space, and to induce maximum differences in strategic behavior between the sealed-bid and clock auctions. By repeating the use of the same valuations within an experimental session we provide bidders with considerable experience at each value, which might be expected to ease decision making in such a complex environment, while providing us with multiple observations against which to evaluate behavior.

Sessions employed either three or five computer rivals (n = 3 or 5), with the number of computer rivals remaining constant within each session. In each session v_h varied randomly over

[11]Students were recruited through fliers posted throughout both campuses, advertisements in student newspapers, electronic bulletin board postings, and classroom visits.

three of the four values employed using a block random design.[12] Values employed with each set of computer rivals, along with equilibrium predictions for *h*'s bids are reported in Table 1. The lowest v_h, $3.00, calls for complete demand reduction in both auctions, and is employed exclusively with $n = 3$. (The expected cost of deviating from the optimal strategy with v_h = $3.00 and $n = 5$ is quite small, and involves a rather large opportunity cost in terms of foregone observations at more salient values.) The highest $v_{h,}$ $5.10, requires "going for it," and insures a *secure* (minimum) profit of 30¢ per auction. It is employed exclusively with n =5 out of cost considerations and the fact that there is little 'bang for the buck' at this value in auctions with n = 3.[13]

The middle values make different predictions between sealed-bid and clock auctions and were employed with both $n = 3$ and 5.

v_h = $4.00: In the sealed-bid auctions $b_1 = b_2$ = $4.34 with $n = 3$ and $b_1 = b_2$ = $4.16 with n = 5. The clock auction also requires bidding above value on both units: If v_2 # **P*** = $4.50, *h* goes for it as the expected value of winning two units is positive and greater than the value of stopping the auction at p = v_2 and winning a single unit. If v_2 > **P***, *h* drops on both units at the cutoff point **P*.**

v_h = $4.40: It pays to "go for it" in the sealed-bid auctions ($b_1 = b_2$ $$7.50), regardless of whether $n = 3$ or 5. The clock auction also calls for bidding above value, with the cutoff point **P***

[12] All three values occurred in each consecutive series of three auctions, but in random sequence.

[13] In the clock auctions with $n = 3$, v_1 is often below $5.10, so there is very little information to be gained about whether or not *h*'s recognize that "going for it" is the optimal strategy in these cases. This problem is reduced substantially with $n = 5$. Further, rather substantial deviations from the optimal bidding strategy of $b_1 = b_2$ $$7.50 (for example, bidding above $5.10 but below $7.50) frequently incur no penalty in sealed-bid auctions with $n = 3$, but such deviations are much more likely to be punished with $n = 5$.

= \$5.70.

Finally, for both intermediate values, if h mistakenly stays beyond max $\{\mathbf{P}^*, v_3\}$ when $v_2 > \mathbf{P}^*$ the certain loss associated with stopping the auction and winning one unit is greater than the expected loss associated with remaining active and winning both units. Thus in equilibrium or outside equilibrium, h should never win only one unit in the clock auction.

All clock auctions employ a "digital" clock with price increments of \$0.01 per second. Following each computer drop out there was a brief pause of 3 seconds. h's dropping out during a pause counted as dropping at the same price, but later than, the computer's dropout. h could drop out on a single unit by hitting any key other than the number 2 key. The first stroke of the key pad dropped out unit 2. Hitting the number 2 key, or a second key during the pause, permitted h to drop out on both units at the same price.

In the sealed-bid auctions the sequencing required subjects to submit bids on unit 1 followed by unit 2. Any nonnegative bid was accepted for unit 1, with the bid for unit 2 required to be the same or lower than the bid on unit 1.[14] There was no opportunity to submit a single bid for both units combined or a bid contingent on winning only one or winning both units.

In all sessions, instructions were read out loud, with copies for subjects to read as well. The instructions included examples of how the auctions worked as well as indicating some of the basic strategic considerations inherent in the auctions. For example, the instructions pointed out that the higher h's value, the more valuable the synergy bonus was, hence the greater value of earning two rather than one unit, and that when bidding above v_h winning a single unit would

[14]Earlier multi-unit demand auctions without synergies demonstrate that this restriction has no effect on bidding strategies (Kagel and Levin, in press).

necessarily involve losses (see the instructions for full details). Finally, it was emphasized to subjects that "...in thinking about bidding, earning an item is of no intrinsic value. Your sole objective should be to maximize your earnings."

Subjects were told that in each auction period the computers would (and did) receive fresh values. At the conclusion of each auction, bids were ranked from highest to lowest and posted along with the corresponding values. Winning bids were identified, prices were posted, profits were calculated, and cash balances were updated. Bidders only observed the outcomes for their own market. Sessions began with three dry runs to familiarize subjects with the auction procedures, followed by thirty-three auctions played for cash.

Bidders were given starting capital balances of $5. Positive profits were added to this and negative profits subtracted from it. End-of-experiment balances were paid in cash. Expected profits were sufficiently high that we did not provide any participation fee.[15] Sessions lasted between 1.5 and 2 hours.

III. Experimental Results:

This is a relatively complicated experiment both for the subjects and for providing a concise, yet comprehensive, analysis of the results. To help guide the reader, the result's section consists of a series of conclusions along with support for those conclusions. Our focus will be on bidding behavior, with some attention to revenue and efficiency as well. Throughout the analysis we will concentrate on bidding in the last 6 auctions under each v_h, when bidders would have become reasonably familiar with the environment (recall, that there were 11 auctions played for

[15]In those few cases where end-of-experiment earnings were below $2.00, a token $2.00 payment was provided.

cash at each $v_{h,}$ and 1 dry run).[16]

A. Bid Patterns

1. Frequency of Optimal Play in Sealed-bid Versus Clock Auctions: Table 2 compares the frequency of optimal play between the two auction institutions. This table employs rather strict definitions for optimal play.[17] Conclusion 1 is based on these results.

Conclusion 1: Bids are consistently closer to optimal outcomes in the clock auction, with differences between auction institutions most pronounced at the two extreme values - $3.00 and $5.10.

Bids being closer to equilibrium/optimal play in clock versus sealed-bid auctions are consistent with results from a large number of auction experiments: single-unit private value auctions (Kagel, Harstad, and Levin, 1987), single-unit common value auctions (Levin, Kagel, and Richard, 1996), and multi-unit demand, uniform-price auctions without synergies (Kagel and Levin, in press).[18] Further, it is clear from the experimental manipulations reported in Kagel and Levin (in press) that it is the clock auction format, in conjunction with the information revealed to *h* by dropout prices, that is responsible for its superior performance.[19]

What's missing from Table 2, and will provide the focus of the detailed analysis that

[16]The choice of the last 6 auctions is somewhat arbitrary, but it does distinguish more from less experienced play, and the results are robust to adding or dropping an auction or two to either side of 6. We will occasionally make reference to obvious learning/adjustment patterns in bids, but forgo any kind of detailed analysis as the paper is already quite long.

[17]We provide 5¢ "allowances" for "trembles" throughout. For example, with v_h = $3.00, in the clock auctions we count as equilibrium $d_2 \# v_2 + 0.05$ when $v_2 \# v_h$. Our results are robust to either eliminating these allowances or increasing them a bit.

[18]For reviews of these results see Kagel (1995) and Kagel and Levin (2000).

[19]Kagel and Levin (in press) show that in a multi-unit demand, uniform-price auction, with flat demand, a clock auction *with no feedback* on rivals' drop-out prices looks no different from the sealed-bid version of the auction. Further, a sealed-bid auction with the crucial dropout information employed in the clock auction *provided by the experimenter*, improves performance, but still comes up short compared to a clock auction with rivals' drop-out prices announced.

follows, is the pattern of deviations from equilibrium, which is quite different between the two auction institutions.

2. Bidding in Sealed-bid auctions:

> *Conclusion 2: In the sealed-bid auctions the data show a clear pattern of increasing bids at higher values as the theory predicts. However, bidders are imperfectly calibrated, as bids are substantially higher than they should be at lower values, and are lower than they should be at higher values, with the possible exception of v_h = $5.10, where bids are close to optimal.*

Support for this conclusion is reported in Table 3, where we have fit random effect Tobits to the bid data. We employ Tobits as there is a mass point at $7.50, and we truncate all bids greater than $7.50.[20] An error components specification is employed with the error term $\varepsilon_{it} = \delta_i + \zeta_{it}$, where δ_i is a subject-specific error term assumed to be constant between auctions within an experimental session, and ζ_{it} is an auction period error term. Standard assumptions regarding the error terms are employed.[21] With n = 3 we use v_h = $3.00 as the intercept of the bid function and create a dummy variable (DV4) for v_h = $4.00 (DV4 = 1 if v_h $ $4.00, 0 otherwise), and a second dummy, (DV440) for v_h = $4.40 (DV440 = 1 if v_h = $4.40, 0 otherwise). For n = 5 we use v_h = $4.00 for the intercept of the bid function and create separate dummy variables v_h = $4.40 (DV440 = 1 if v_h $ $4.40, 0 otherwise) and for v_h = $5.10 (DV510 = 1 if v_h = $5.10, 0 otherwise). Also reported, for the reader's convenience, are the 95% confidence intervals for bids predicted by the regression equations.

For the n = 3 both dummy variables are significant at the 5% level or better, indicating that

[20] In our experimental design, bids above $7.50 have essentially the same impact of bids of $7.50 and are treated as such.

[21] δ_i - (0, σ_δ) and ζ_{it} - (0, σ_ζ) where δ_i and ζ_{it} are independent among each other and among themselves. Note, in our experimental design, the use of computer rivals, clearly supports the assumption that the δ_i are independent among each other.

bids were increasing in v_h, as they should be, for both b_1 and b_2. Further, the lower bound of the 95% confidence interval for unit 1 bids is well above the optimal bid when v_h = \$3.00 and \$4.00 (optimal bids of \$3.00 and \$4.34, respectively), with unit 2 bids showing a similar pattern. While the upper bound of the 95% confidence interval for unit 1 bids is \$7.50 for v_h = \$4.40, the upper bound for unit 2 bids falls well short of the optimal bid of \$7.50. Finally, for v_h = \$4.00 the upper bound for unit 2 bids falls short of the lower bound for unit 1 bids. This is indicative of a general failure to follow the requirement that $b_1 = b_2$ when bidding above value.[22]

Similar results are reported for the $n = 5$ case. Both dummy variables are significant at the 5% level or better, indicating that bids were increasing in v_h, as they should be, for both b_1 and b_2. The lower bound of the 95% confidence interval for unit 1 bids is well above the optimal bid of \$4.16 with v_h = \$4.00, as is the lower bound for unit 2 bids. For v_h = \$4.40, the upper bound of the 95% confidence interval for unit 1 bids includes \$7.50, but is short of the optimal bid of \$7.50 for unit 2 bids. What is different from the $n = 3$ case is that there is overlap between unit 1 and unit 2 bids in all cases. This is clear from the raw data as well, which shows that conditional on $b_1 \$ v_h$, the relative frequency of $b_1 > b_2$ has been cut by 50% compared to $n = 3$.[23] No doubt these differences result from the fact that bidding less on unit 2 compared to unit 1 (conditional on $b_1 > v_h$) is more costly with $n = 5$ than $n = 3$.[24] Finally, the regression shows that for v_h = \$5.10, the

[22]The raw data on this score is as follows: Conditional on $b_1 > v_h + 0.05$, the frequency of $b_1 - b_2 > .25$ or $b_2 \# v_h$ is 51.0% with v_h = \$3.00, 35.6% with v_h = \$4.00 and 31.9% with v_h = \$4.40. (Note, our calculations provide small allowances for deviations from optimal play to leave some room for "trembles.")

[23]For $n = 5$, conditional on $b_1 > v_h + 0.05$, the frequency of $b_1 - b_2 > .25$ or $b_2 \# v_h$ is 17.2% with v_h = \$4.00, 17.7% with v_h = \$4.40 and 12.0% with v_h = \$5.10.

[24]For example, with v_h = \$4.00, numerical evaluation of outcomes for the rule of thumb, $b_1 = 1.5v_h$ and $b_2 = v_h$, yields positive profits, but profits that are lower by approximately 33¢ per auction, than the optimal strategy with $n = 3$. With n = 5, this rule yields negative profits which are lower by approximately 53¢ per auction than profits generated by the optimal strategy.

lower bound of the 95% confidence interval for unit 1 bids is $7.50, and the 95% confidence interval for unit 2 bids includes the optimal bid of $7.50 as well. As such, even though the equilibrium requirement of $b_1 \$ b_2 \$\$7.50$ is only satisfied 40.6% of the time, bidders win both units 71.9% of the time. The net result is that deviations from optimality in payoff space are substantially smaller than deviations from optimality in the choice space.

3. Bidding in Clock Auctions:

Conclusion 3: There is directional consistency in the clock auctions in the sense that two units are won more often at higher valuations (when they should be won), and at v_h = $5.10 bidders "go for it" most of the time, as they should. However, at the intermediate valuations of region 2, there are large deviations from optimal bid patterns that are best explained by the exposure problem.

Support for Conclusion 3 can be found in Table 4. First note that for v_2 # v_h the frequency of winning two units is increasing in v_h. Further, consistent with optimal bidding, two units are won close to 100% of the time (in 83.3% of all auctions) with v_h = $5.10. However, substantial deviations from optimal bid patterns are reported in region 2, at the intermediate values of $4.00 and $4.40, where bidders face an exposure problem. This exposure problem expresses itself in three distinct ways for these valuations:

(1) For v_2 # v_h the primary deviation from equilibrium bidding involves demand reduction (winning 1 unit with positive profits).[25] Further, in most cases this involves complete demand reduction; i.e., dropping out at the same time (or prior to) v_2, thereby not affecting the market price (87.5% of all cases with v_h = $4.00, 74.7% of all cases with v_h = $4.40) and never having to bid above v_h.

[25] Other deviations from optimality include winning 0 units and winning 1 unit with negative profits ("going for it" but then reconsidering) with deviations slightly favoring the latter case.

(2) In cases where $v_h < v_2 \# \mathbf{P^*}$, where optimality requires "going for it," and necessarily involves bidding above v_h, there is a high frequency of winning zero units. The combined frequency of winning zero units and winning 1 unit (which involves "going for it" but then reconsidering when bidding above v_h) is consistently higher than the frequency of winning two units. Further, bidders win two units consistently less often when $v_2 \# v_h$.[26] And the exposure problem is necessarily more extreme when $v_h < v_2 \# \mathbf{P^*}$ than when $v_2 \# v_h$. In contrast, the theory predicts no difference in the frequency of winning two units between these two cases.

(3) In cases where $v_2 > \max(\mathbf{P^*}, v_h)$ equilibrium play calls for bidding up to $\mathbf{P^*}$ and dropping on both units at this point. This necessarily involves bidding above v_h, but rarely happens. Rather the modal response in three out of four of these cases is to drop out *prior to* $\mathbf{P^*}$, typically dropping out very close to v_h.[27]

In contrast to bidding at these intermediate valuations ($4.00 and $4.40), bidders have little problem bidding above their valuation for v_h = $5.10, when they are assured of a minimum profit of 30¢ by going for it. This response to the exposure problem in the clock auction is in marked contrast to the sealed-bid auction where bidders show no reluctance to bid above v_h on both units. This suggests that it is both the fear of losses, in conjunction with the auction format, that is responsible for the greater response to the exposure problem in the clock auction. What is it about the clock auction that accounts for this heightened exposure effect? The clock auction format (with feedback on bidders' drop-out prices) makes it much more transparent to bidders,

[26]Support for the statistical significance of these differences is provided buy random effect probits, with subject as the random component.

[27]And dropping too soon is within a whisker of the modal response in the fourth case ($n = 3$ and v_h = $4.40).

compared to sealed-bid auctions, that they are liable to lose money as a consequence of bidding above value (Kagel, Harstad and Levin, 1986; Kagel, 1995; Kagel and Levin, in press). In single-unit private value auctions, and in multi-unit demand auctions without synergies, this heightened awareness of the perils of bidding above one's valuation helps to improve bidder profits and to move play closer to the equilibrium outcome. Here it holds bidders back from achieving maximum profit and generates deviations from the equilibrium outcome.[28]

One final thing to note in Table 4 is the sharp (and consistent) difference between the frequency of winning two units for n = 3 versus n = 5 when v_2 # v_n, and when v_2 # **P*** for the region 2 valuations of $4.00 and $4.40.[29] We suspect that this has little to do with the different number of rivals in the two treatments, but is a hysteresis effect brought on by the different behavior patterns rewarded under the remaining valuation in each case: the $3.00 value which calls for complete demand reduction with n = 3 and the $5.10 value which calls for "going for it" with n = 5. This result is summarized in Conclusion 4.

Conclusion 4: There appears to be a strong hysteresis effect in the data, as with resale values of $4.00 and $4.40, the exposure problem is much more severe with n = 3 than with n = 5.

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[28]We should add that in cases where v_2 # v_h, the saliency of the demand reduction option ("stop the clock and win 1 unit assuring positive profits") is much more pronounced in clock compared to sealed-bid auctions when demand reduction is consistent with optimal bidding (Kagel and Levin, in press).

[29]These differences are statistically significant at conventional levels. Tests for statistical significance consisted of the following: Take all cases where v_2 # v_h. Run a random effects probit (with subject as the random component) and dependent variable = 1 when a bidder wins 1 unit with positive profits), 0 otherwise. Let v_h = $3.00 serve as the baseline and define dummy variables DV4 = 1 if v_h $ $4.00, 0 otherwise; DV440 = 1 if v_h $ $4.40, 0 otherwise; and DV510 = 1 if v_h = $5.10, 0 otherwise; and DN5 = 1 if n = 5, 0 otherwise. This yields:

$$\underset{(0.292)^{+}}{\text{Win1} = 0.502} - \underset{(0.222)^{**}}{0.601\text{DV4}} - \underset{(0.159)}{0.187\text{DV440}} - \underset{(0.258)^{**}}{0.630\text{DV510}} - \underset{(0.358)^{**}}{0.932\text{DN5}}$$

with standard errors in parentheses and + and ** indicating statistical significance at the 10% and 1% levels respectively. The DN5 dummy is negative and significant at the 1% level.

4. Learning and Adjustments Over Time: The next conclusion takes a closer look at bidding with v_h = \$5.10. In almost all respects this should be (and is) the valuation for which play is closest to optimal in both auction formats as it only takes a little arithmetic to realize that "going for it" yield a secure, minimum profit of 30¢ per auction. As a result, with repeated exposure to the problem one would expect more subjects to "get it." And they do as the data show a clear learning effect, converging toward optimal play.

Conclusion 5: With v_h = \$5.10, we observe clear adjustments over time toward optimal play in both sealed-bid and clock auctions. However, winning two units is still more pronounced in the clock auctions.

Support is provided by the random effect probits reported in Table 5, where we have pooled the data for both clock and sealed-bid auctions for v_h = \$5.10. The dependent variable takes on a value of 1 in cases where two units were won (as optimal bidding requires) and 0 otherwise.[30] We use the data for all auctions, excluding the dry runs. Model 1 includes a single dummy variable, Dclock = 1 if a clock auction, 0 if sealed bid. The coefficient for the dummy is positive and significant at the 10% level, indicating that play is closer to the optimal outcome in the clock auctions. Model 2 introduces a second dummy variable, Dearly = 1 for the first 5 auctions, 0 for the last 6, in an effort to identify possible learning/adjustment effects. These are clearly present as indicated by the relatively large, statistically significant negative coefficient value for the Dearly dummy. Further, the introduction of the Dearly dummy has virtually no effect on the magnitude of the Dclock dummy or on its standard error.[31]

[30]Focusing on outcomes (win 2 units) in comparing between auction formats controls for the censoring problem associated with bids in the clock auctions in comparing between auction formats.

[31] A third specification introducing an interaction effect between the Dearly and Dclock dummies fails to reduce the log likelihood function by a significant amount.

This is the one case where we observe clear learning/adjustments toward optimal play in the data. Specifications searching for learning/adjustment effects for other valuations reveal considerably more noise in early play (higher variances and less stable coefficient values), as opposed to any clear adjustments toward optimal play.

B. Profits, Efficiency and Revenue

This section examines profits, revenue and efficiency. Bidders' profits provide a measure of success in terms of payoffs, and provide a convenient way of characterizing performance in terms of a single outcome measure. Efficiency is defined as the sum of the values of two units sold in each auction period (including the synergy bonus, if relevant) as a percentage of the *highest* total value that would have been obtained in a full information Nash equilibrium. Revenue is what the seller would have earned in each period. Note, in computing revenue and efficiency we are keenly aware that the same results might not emerge in auctions where all bidders are human. As noted, computer rivals were employed to minimize possible complications associated with learning against human rivals who may be playing out-of-equilibrium strategies. Out-of-equilibrium play may affect different institutions differently. On the other hand, there is very limited experimental data on efficiency and revenue, measures of central importance to economists, in environments such as this, and we believe that the present data are at least suggestive of what will be observed in interactive settings. Finally, in reporting revenue and efficiency we provide a benchmark against which future experiments with all human bidders can compare results on these important issues.

1. *Profits:*

Conclusion 6: Profits are consistently and significantly less than would have been achieved with optimal bidding in all but one case (v_h = \$4.00, n = 3, sealed-bid auction). Profits *are consistently higher in clock compared to sealed-bid auctions, but fail to be significantly higher in several cases where the theory predicts that profits will be higher.*

Table 6 reports profits -- actual, predicted and the difference between the two -- for both auctions, along with the difference between actual profits in the two auction formats (sealed-bid less clock). At v_h = \$3.00, bidders earned negative profits averaging -60¢ and -15¢ in the sealed-bid and clock auctions respectively. Profits in the sealed-bid auctions were significantly below zero ($t = 2.80$, $p < .01$, 2-tailed test), and significantly less than those realized in the clock auctions ($t = -1.93$, $p < .10$, 2-tailed test), reflective of the large differences in equilibrium play between the two auctions.[32] Recall that under optimal bidding profits are predicted to be the same in both auctions for v_h = \$3.00.

For $n = 3$, with v_h = \$4.00 and \$4.40 optimal bidding predicts 20–24% higher profits in clock compared to sealed-bid auctions. These higher predicted profits result from the greater flexibility afforded by the information revelation in the clock auctions. These higher profits do not materialize, however, as (i) the exposure problem serves to promote the strong demand reduction found in the clock auctions, which wipes out most of the advantages resulting from information revelation in this case, and (ii) the deviations from equilibrium in the sealed-bid auctions are not so severe as to have disastrous effects on earnings. Even larger percentage differences between profits are predicted with optimal bidding at v_h = \$4.00 and \$4.40 with $n = 5$. In contrast to the $n = 3$ case, these differences are largely realized (at least directionally) as the clock auctions raise an average of 43¢ and 21¢ more per auction at \$4.00 and \$4.40 respectively ($t = -2.80$, $p < .01$, two-tailed test with v_h = \$4.00; $t = -0.89$ with v_h = \$4.40). The higher profits

[32]Average profits per subject are the unit of observation in these calculations.

realized in this case result from (i) the weaker demand reduction effect in the clock auctions with $n = 5$ (recall Conclusion 4) and (ii) the harsher effects of deviating from optimal bidding in the sealed-bid auctions with $n = 5$. Finally, the relatively modest differences in bidding with v_h = \$5.10 do *not* translate into significant differences in actual earnings between the two auction formats (t = -1.29).[33]

2. *Efficiency:*

Conclusion 7: Optimal bidding predicts either the same or higher efficiency in clock compared to sealed-bid auctions. In contrast to these predictions, actual efficiency differences are quite mixed, with the only significant difference recorded in favor of the sealed-bid auction.

Table 7 reports efficiency outcomes. At the \$3.00 value actual efficiency is very close to predicted levels in both sealed-bid and clock auctions. This is not surprising in the clock auctions where bidding is relatively close to equilibrium, but is somewhat unexpected in the sealed-bid auctions with its large deviations from equilibrium outcomes. These deviations apparently have minimal impact on efficiency since the overbidding occasionally produces large efficiency gains (relative to the predicted outcome) as a result of the synergy bonus. At the \$4.00 value, where the tension between the synergy bonus and the demand reduction effect is strongest, the clock auction is predicted to yield large efficiency gains compared to the sealed-bid auction. However, these gains go almost entirely unrealized as (i) the exposure problem serves to promote demand reduction in the clock auctions, which wipes out most of the efficiency gains predicted, and (ii) deviations from equilibrium bidding in the sealed-bid auctions have essentially no effect relative to predicted efficiency. No doubt this last result is due to the fact that with the synergy bonus

[33] The differences we do find are aided by the higher predicted profits in the clock auctions. The latter results from sampling variability in terms of the computer values drawn, as optimal bidding yields the same equilibrium outcome in both cases.

overbidding occasionally produces large efficiency gains relative to the predicted outcome.[34] Finally, actual efficiency is well below predicted efficiency at higher values in both auction institutions as bidders do not "go for it" often enough to take full advantage of the synergy bonus.

3. *Revenue:*

Conclusion 8: Revenue is consistently higher in the sealed-bid auctions, and is significantly higher in four out of six cases. These higher revenues in the sealed-bid auctions do not come at the expense of any significant efficiency losses relative to the clock auctions.

Revenues are reported in Table 8. Note that revenues are predicted to be substantially higher in the clock auctions with v_h = 4.00 as the information revelation in the clock auction at this valuation promotes more aggressive bidding compared to a sealed-bid auction. In contrast, at the \$4.40 value the sealed-bid auction is predicted to raise more revenue as bidders should "go for it" all the time, but still condition their actions on rivals' dropout prices in the clock auction. The actual data, however, show uniformly higher revenues in the sealed-bid auction: 4.0% to 18.8% higher revenue with n = 3, 3.0% to 12.7% higher revenue with n = 5 (calculated as a percentage of realized revenue in the clock auctions). Note that these increased revenues do not come at the expense of reduced efficiencies, as Table 7 reports no significant decreases in efficiency in sealed-bid compared to clock auctions, and one case with significantly higher efficiency in the sealed-bid auction.

[34]Actual efficiency is greater than predicted efficiency in the sealed-bid auctions in these two cases. There is no inconsistency here since predicted efficiency is less than 100%; i.e., predicted efficiency is the value of the two highest units sold (including the synergy bonus) under the Nash equilibrium with incomplete information divided by the value of the two units sold under the full information Nash equilibrium.

C. Experienced Bidders

This section briefly discusses results from two additional sealed-bid auction sessions using bidders who had participated in one of the sealed-bid or clock auctions reported above. Both of the experienced subject sessions employed $n = 3$, with valuations of $3.00, $4.00 and $4.40.

Conclusion 9: Experienced subjects in sealed-bid auctions do not perform materially better than inexperienced ones. There are, however, significant differences depending on bidders' past experience: Those with clock experience bid less aggressively at lower valuations compared to those with sealed-bid experience. These differences constitute a direct carryover of the differences observed between auction institutions for inexperienced bidders.

Support for this conclusion is provided in Table 9, where separate bid functions have been estimated as a function of subjects prior experience - sealed-bid or clock. Random effect Tobits, similar to the ones reported in Table 3 have been estimated in both cases. As in Table 3, v_h = $3.00 serves as the intercept of the bid function, with dummy variables DV4 = 1 if v_h $ $4.00 (0 otherwise) and DV440 = 1 if v_h = $4.40 (0 otherwise).

Results for those with sealed-bid auction experience are quite similar to those reported for inexperienced bidders in Table 3. Qualitatively, bids are well above what they should be to maximize profits for v_h = $3.00 and $4.00, and bids are too conservative (not "going for it" often enough) with v_h = $4.40. Quantitatively, the 95% confidence intervals for unit 1 and unit 2 bids overlap for all three values with those reported for inexperienced bidders in Table 3. (However, there is some tendency for experienced bidders to bid more aggressively at v_h = $4.00 and $4.40.) In contrast, those with clock experience bid far less aggressively, with the 95% confidence intervals for unit 1 and unit 2 bids (i) uniformly lower, with no overlap, compared to subjects with sealed-bid experience (Table 9), and (ii) uniformly lower, but with some overlap, compared to inexperienced bidders (recall Table 3).

These bid differences result in large differences in profits at the lowest and highest v_h values: Substantially higher profits were earned for those with clock experience at v_h = \$3.00 (0.01¢ per auction) versus those with sealed-bid experience (-0.65¢ per auction).[35] Earnings were essentially the same at v_h = \$4.00 (0.83¢ per auction for those with clock experience versus 0.74¢ per auction for those with sealed-bid experience). Finally, for v_h = \$4.40 profits were 39¢ higher per auction for those with sealed-bid experience (\$1.72 per auction versus \$1.33; $t = 4.26$, $p < .01$, 2-tailed test).[36]

C. Comparison to Multi-Unit Demand Auctions Without Synergies

We have conducted an experiment similar to this one with one major difference -- the synergy bonus was set to zero (Kagel and Levin, in press). In this case demand reduction dominates at all valuations, as it does in region 1 here. This section briefly compares the outcomes observed in this earlier experiment with the outcomes in region 1 here (v_h = \$3.00).

Conclusion 10: It is clearly much harder for bidders to exercise demand reduction, when it is called for, in auctions with synergies than in auctions without synergies. This holds true for both sealed-bid and clock auctions.

For the clock auctions, with v_2 # v_h demand reduction (partial and complete) is practiced in 85.7% of all auctions without synergies compared to 63.2% of auctions with synergies and v_h =

[35] $t = -3.96$, $p < .01$, 2-tailed test. Average profit per subject serves as the unit of observation. All profit calculations here are based on simulations using actual bids for human bidders in each auction and 500 simulated draws for the computers for each auction. This helps to smooth out the variability inherent in using actual draws for the computer from the auctions, and are employed here because of the relatively small number of subjects involved.

[36] In these two sessions we also varied the order of v_h values to see if this had any impact on behavior. In one session valuations were rotated in a block random design, as in the inexperienced subject sessions, for 24 auctions, with the session terminating with 8 auctions with v_h = \$3.00. In the other session valuations were presented in blocks of 8 auctions: 8 auctions with v_h = \$3.00, followed by 8 auctions with v_h = \$4.40, followed by 8 auctions with v_h = \$4.00, and ending with 8 auctions with v_h = \$3.00. There were no statistically significant, or economically significant, differences between the block random design and the session with values fixed for 8 auctions, indicating the absence of any kind of hysteresis effect on this dimension. It will be interesting to see if this result holds for inexperienced bidders.

$3.00. Further, there is a substantially higher frequency of winning two units in the auctions with synergies (21.3%) versus those without (5.6%). Recall, that at most, a single unit should be won in both cases.

For sealed-bid auctions there is a much higher frequency of bidding above value on unit 1 in auctions with synergies (77.8%) than in those without (37.9%).[37] Similar differences in the frequency of bidding above value are found with respect to unit 2: 54.5% of all auctions with synergies versus 24.7% without. And there is substantially less demand reduction (defined as bidding more than 5¢ below value) in auctions with synergies (30.1%) versus those without (58.6%).

Of course, neither of these differences is terribly surprising. The introduction of relatively large synergies creates a potent force acting in opposition to the demand reduction forces inherent in uniform-price auctions. As such it is not terribly surprising to find increased deviations from optimal bidding even when the demand reduction forces should dominate.

IV. Summary and Conclusions:

We report results from an experiment comparing sealed-bid and ascending-bid uniform-price auctions where individual bidders demand multiple units and there are synergies between units. We use a simple demand structure: Several single-unit demand bidders and one bidder demanding up to two units. We further simplify the structure by having computers play the dominant strategy that single-unit bidders have of always bidding their value. In spite of its simplicity, the key economic incentives present in uniform-price auctions with synergies are all captured: the synergy effect, which promotes bidding above standalone values; the exposure

[37]These measures include a 5¢ allowance for trembles in both cases: $b_1 > v_h + 0.05$.

problem which may deter bidders from pursing this aggressive biding strategy, thereby reducing economic efficiency; and the monopsony power that multi-unit demand bidders can exploit to reduce prices in a uniform-price auction.

The experiment shows that bidding is closer to optimal play in the clock auctions, consistent with evidence from a number of other auction environments (Kagel, Harstad, and Levin, 1987; Levin, Kagel, and Richard, 1996; Kagel and Levin, in press). Further, in most cases bidders behave sensibly, though not optimally: Bidding under the highest valuation, where the optimal play is quite transparent, generates by far the highest levels of optimal play, levels that are comparable to the highest levels reported for subjects in any experimental auction environment. Demand functions estimated for the sealed-bid auctions are monotonically increasing in bidders' valuations. In the clock auctions, there is a higher frequency of "going for it" at higher valuations, when multi-unit demand bidders should strive to obtain both units.

Nevertheless, there is much out-of-equilibrium play under both institutions, with the most interesting and dramatic differences occurring at intermediate valuations where the theory requires balancing demand reduction incentives against the synergy bonus, while exposing bidders to possible losses. At these values the exposure problem promotes relatively strong demand reduction in the clock auctions in contrast to optimal bidding which requires that bidders "go for it." In contrast, the exposure problem is barely present in the sealed-bid auctions, with bidders consistently bidding above value on both units. This suggests that it is both the fear of losses, in conjunction with the auction format, that is responsible for the greater response to the exposure problem in the clock auction. The clock auction format (with feedback on bidders drop-out prices) makes it much more transparent to bidders that they are liable to lose money as a consequence of bidding above value (Kagel, Harstad and Levin, 1986; Kagel, 1995; Kagel and Levin, in press). This heightened awareness of the perils of bidding above ones' valuation helps to improve bidder profits and to move play closer to the equilibrium outcome in single-unit private value auctions and in multi-unit demand auctions without synergies. With synergies it holds bidders back from achieving maximum profit and generates deviations from the equilibrium outcome.

How relevant are these findings of out-of-equilibrium play to "real world" auctions? We argue that for low stakes field settings, or those that for one reason or another, bidders do not employ high-powered consultants (as occurred in at least one U. S. spectrum auction), our observations are directly relevant.[38]

But what about high-stakes settings with high-powered consultants on all sides, as is typically the case in auctions involving many millions or even billions of dollars? Extrapolating laboratory results in such cases is clearly more problematic and one must be much more careful in assessing the applicability of the results. However, we do know that the exposure problem is a major concern in auctions with synergies.[39] Our experiment demonstrates that it is more prevalent in an ascending price format where the consequences of stopping the auction and winning a limited number of items with positive gains are accentuated by the auction format. It is also clear from "war stories" of colleagues and consultants involved in advising companies that not all advice is good advice, and not all good advice is acted on. Further, "real" auctions are

[38]See, for example Naik (1996) who discusses problems small firms had in PCS auctions and Mills (1997) who discusses problems a number of bidders had in the Interactive Video and Data Services (IVDS) auction.

[39] The recent Wye River conference sponsored by the FCC/NSF/Stanford was called, and devoted to, "Combinatorial Bidding." Combinatorial, or package, bidding is one possible solution to the exposure problem as bidders can bid aggressively to acquire a package with supper additive value, without the risk of getting just parts of the package at cost that exceeds the value. This demonstrates that the exposure avoidance problem is presumed "alive and well" among more sophisticated bidders.

substantially more complicated than the one implemented here, so much so, that it is usually impossible to identify optimal bidding strategies. Thus, the answer to questions of applicability of laboratory results to field settings remains the same here as it does elsewhere: Laboratory economic systems are real economic systems. Thus, behavioral processes identified in the lab are presumed to hold beyond the lab. This, shifts the burden of proof to those who argue that such findings will not generalize to provide a factual basis for their argument in field data.

There are a number of obvious and interesting extensions to the experimental results reported here. One would be to conduct these auctions with all human bidders to see what differences possible out-of-equilibrium play by single-unit bidders would have on multi-unit demand bidders. Another would be to permit the use of package bidding, either under the present set-up or with all human bidders, to see how well this serves to overcome the exposure problem and to identify what, if any, “complexity problems”issues this might pose for bidders. Finally, a natural next step would be to extend the analysis to environments with several multi-unit demand bidders and/or to environments in which items are imperfect substitutes and sold in separate auction markets.

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Figure 1

Bid Tactics By Regions and Auction Institution

Sealed Bid Auction	Region 1 $0 \le v_h < \frac{1}{2}$	Region 2 $\frac{1}{2} < v_h < v(n)$	Region 3 $v_h \ge v(n)$
	$b_1 = v_h$ $b_2 = 0$	$v_h < b_1 = b_2 < 1$	$b_1 = b_2 \ge 1$
Ascending Clock Auction	Region 1 $v \in [0, \frac{1}{2})$	Region 2 $v \in [\frac{1}{2}, \frac{2}{3})$	Region 3 $v_h \ge \frac{2}{3}$
	If $v_2 \le v_h,\ d_1 = v_h$ *and* $0 \le d_2 \le v_2$ *If* $v_2 > v_h,\ d_1 \in [v_h,\ \max(v_h, v_3))$ *and* $d_2 \in [0,\ \max(v_h, v_3)]$	*If* $v_2 \le \mathbf{P^*},\ d_1 = d_2 \ge 1$ *If* $v_2 > \mathbf{P^*},\ d_1 = d_2 \in [\mathbf{P^*},\ \max\{\mathbf{P^*}, v_3\}]$	$d_1 = d_2 \ge 1$
Comments	Allocations and prices are the same in both auctions. However, there is considerably more flexibility in carrying out the optional policy in clock auctions	Prices and allocations differ between auctions. Multi-unit demand bidder (h) never wins one unit in clock auctions, but does so at times in sealed bid.	"Go for it" in both auctions, but size of Region 3 is smaller in clock auctions

$v(3) \approx 0.545; v(5) \approx 0.573$

$\mathbf{P^*} = [3v_h - 1]$

TABLE 1

Bidders Values and Equilibrium Predictions

Bidder value ( )	Sealed Bid Auctions	Clock Auctions
$\$3.00$ (with 3 computer rivals only)		If , $d_1 = \$3.00$ and $d_2 \le v_{2.}$ If , $d_1 \in [\$3.00, \max(\$3.00, v_3)]$ and $d_2 \in [0, \max(\$3.00, v_3)]$.
$\$4.00$ (with 3 and 5 computer rivals)	with 3 computers: $b_1 = b_2 = \$4.34$ with 5 computers: $b_1 = b_2 = \$4.16$	If $= \$4.50$, $d_1 = d_2 = \$7.50$. If $= \$4.50$, $d_1 = d_2 \in [\$4.50, \max(\$4.50, v_3)]$.
$\$4.40$ (with 3 and 5 computer rivals)	$b_1 = b_2 \ge \$7.50$ (earn two units)	If $\$5.70$, $d_1 = d_2 = \$7.50$. If $> \$5.70$, $d_1 = d_2 \in [\$5.70, \max(\$5.70, v_3)]$
$\$5.10$ (with 5 computer rivals only)	$b_1 = b_2 \ge \$7.50$ (earn two units)	$d_1 = d_2 = \$7.50$.

Table 2

Comparing Frequency of Equilibrium Play Under Different Auction Institutions
(raw data in parentheses)

v_h	No. Computers	Clock	Sealed Bid
\$3.00	3	46.3% (111/240)	2.6% (5/189)
\$4.00	3	23.7% (57/240)	1.6% (3/188)
	5	22.3% (54/240)	3.1% (6/192)
\$4.40	3	38.8% (93/240)	27.7% (52/188)
	5	35.8% (86/240)	27.1% (52/192)
\$5.10	5	79.2% (190/240)	40.6% (78/192)

a. One subject left before her session ended resulting in fewer than 6 auctions at each v_h value for that subject.

Table 3
Sealed Bid Auctions: Random Effect Tobit Estimates of Bid Function

No. of Computers		95% confidence interval for bids v_h=\$3.00	v_h=\$4.00	v_h=\$4.40	V_h=\$5.10	No. Subjects	No. Observations
3	b_1= 6.19 V3 + 2.08 DV4 + 1.62 DV440 (0.782)** (0.948)* (0.989)	4.65-7.50	6.68-7.50	7.50	NA	32	565
	b_2 = 4.19V3 + 1.33 DV4 + 0.67 DV440 (0.491)** (0.513)** (0.524)	3.23-5.16	4.54-6.50	5.20-7.18	NA		
5	b_1 = 6.99 V4 + 1.04 DV440 + 1.69 DV510 (0.515)** (0.452)* (0.482)**	NA	5.98-7.50	6.99-7.50	7.50	32	576
	b_2 = 6.09 V4 + 0.70 DV440 + 0.88 DV510 (0.155)** (0.154)** (0.160)**	NA	5.78-6.39	6.48-7.10	7.34-7.50		

* significantly different from 0 at the .05 level, 2-tailed test
** significantly different from 0 at the .01 level, 2-tailed test

TABLE 4

Bid Patterns in Clock Auctions (raw data in parentheses)													
$\nu_2 \leq \nu_h$					$P^* \geq \nu_2 > \nu_h$			$\nu_2 > \text{Max}\{\nu_h, P^+\}$					
No. Computers	Value	Win 2 Units	Demand Reduction	Other	Win 2 Units	Win 0 Units	Win 1 Unit	Equilibrium	Drop Too Soon & Win 0	Drop Too Late & Win 0	Win 1	Win 2
3	\$3.00	23.7% (18/76)	63.2% (48/76)	13.2% (10/76)	NA	NA	NA	44.5% (73/164)	13.4% (22/164)	14.6% (24/164)	20.1% (33/164)	7.3% (12/164)
	\$4.00	33.8% (44/130)	49.2% (64/130)	16.9% (22/130)	25.0% (6/24)	50.0% (12/24)	25.0% (6/24)	8.1% (7/86)	53.5% (46/86)	8.1% (7/86)	17.4% (15/86)	12.2% (11/86)
	\$4.40	47.0% (70/149)	43.0% (64/149)	10.1% (15/149)	38.0% (19/50)	38.0% (19/50)	24.0% (12/50)	9.8% (4/41)	39.0% (16/41)	4.9% (2/41)	2.4% (1/41)	43.9% (18/41)
5	\$4.00	45.2% (28/62)	25.8% (16/62)	29.0% (18/62)	39.1% (9/23)	39.1% (9/23)	21.7% (5/23)	11.0% (17/155)	47.7% (74/155)	14.8% (23/155)	12.9% (20/155)	13.5% (21/155)
	\$4.40	60.8% (45/74)	25.7% (19/74)	13.5% (10/74)	47.2% (34/72)	31.9% (23/72)	20.8% (15/72)	7.4% (7/94)	38.3% (36/94)	12.8% (12/94)	19.1% (18/94)	22.3% (21/94)
	\$5.10	83.3% (95/114)	11.4% (13/114)	5.3% (6/114)	75.4% (95/126)	14.3% (18/126)	10.3% (13/126)	NA	NA	NA	NA	NA

NA: Not applicable

Table 5
Probits Comparing Winning 2 Units in Clock vs Sealed Bid Auctions: v_h = $5.10

Variable	Model 1	Model 2
Constant	0.706 (0.211)**	0.981 (0.222)**
DClock	0.439 (0.252)+	0.458 (0.255)+
DEarly	-----	-0.540 (0.122)**
Log Likelihood	-362.9	-352.8
No. Observations	792	798
No. Subjects	72	72

+ Significantly different from 0 at the 10% level, 2-tailed test

** Significantly different from 0 at the 1% level, 2-tailed test

Table 6
Profits (in dollars)
(standard errors of mean in parentheses)

No. Computers	v_h value	Sealed Bid Auctions Actual	Sealed Bid Auctions Predicted	Sealed Bid Auctions Difference	Clock Auctions Actual	Clock Auctions Predicted	Clock Auctions Difference	Difference (actual): Sealed Bid less Clock (t-statistics)
3	\$3.00	-0.60 (0.214)	0.35 (0.056)	-0.959** (0.187)	-0.15 (0.127)	0.36 (0.032)	-0.495** (0.124)	-0.45 (-1.93)+
	\$4.00	0.72 (0.176)	0.88 (0.176)	-0.168 (0.111)	0.73 (0.120)	1.09 (0.144)	-0.365** (0.104)	-0.01 (-0.02)
	\$4.40	1.24 (0.207)	1.74 (0.240)	-0.497** (1.33)	1.29 (0.150)	2.10 (0.183)	-0.805** (0.146)	-0.05 (-0.20)
5	\$4.00	-0.25 (0.123)	0.21 (0.073)	-0.459** (0.092)	0.18 (0.093)	0.44 (0.104)	-0.260** (0.074)	-0.43 (-2.80)**
	\$4.40	0.25 (0.144)	0.54 (0.260)	-0.286** (0.077)	0.46 (0.145)	0.84 (0.140)	-0.385** (0.074)	-0.21 (-0.89)
	\$5.10	2.03 (0.263)	2.69 (0.149)	-0.661** (0162)	2.45 (0.202)	2.93 (0.138)	-0.490** (0.129)	-0.42 (-1.29)

+ Significantly different from 0 at the 10% level, 2 tailed t-test
** Significantly different from 0 at the 1% level, 2 tailed t-test

TABLE 7

EFFICIENCY
(standard errors in parentheses)

No. Computers	v_h Value	Sealed Bid Auctions			Clock Auctions			Difference (actual): Sealed Bid less Clock (t-statistic)
		Actual	Predicted	Difference	Actual	Predicted	Difference	
3	$3.00	91.9% (0.83)	92.7% (0.96)	-0.8% (1.18)	92.9% (0.72)	93.5% (0.56)	-0.6% (0.81)	-1.00 (-0.93)
	$4.00	91.9% (1.04)	90.8% (0.78)	1.2% (1.17)	89.0% (1.05)	98.9% (0.19)	-9.8%** (1.08)	2.90 (1.93)+
	$4.40	90.6% (1.29)	99.8% (0.08)	-9.2%** (1.29)	88.4% (1.34)	99.7% (0.07)	-11.4%** (1.35)	2.20 (1.17)
5	$4.00	93.8% (0.63)	92.6% (0.70)	1.2% (1.00)	94.0% (0.62)	97.9% (0.28)	-3.9%** (0.70)	-0.20 (-0.15)
	$4.40	94.0% (0.78)	99.0% (0.17)	-5.0%** (0.84)	93.6% (0.80)	99.2% (0.15)	-5.7%** (0.86)	0.40 (0.39)
	$5.10	94.4% (1.39)	100% (0.00)	-5.6%** (1.39)	95.4% (1.20)	100% (0.00)	-6.1%** (1.20)	-1.00 (-0.52)

** Significantly different from 0 at the 1% level, 2 tailed t-test

TABLE 8

Revenue (in dollars)
(standard error in parentheses)

No. Computers	v_h Value	Sealed Bid Auctions			Clock Auctions			Difference (actual): Sealed Bid less Clock (t-statistic)
		Actual	Predicted	Difference	Actual	Predicted	Difference	
3	$3.00	8.48 (0.33)	5.80 (0.12)	2.68** (0.29)	7.14 (0.24)	5.84 (0.12)	1.30** (0.24)	1.34 (3.38)**
	$4.00	9.33 (0.24)	8.42 (0.09)	0.91** (0.23)	8.31 (0.23)	9.93 (0.13)	-1.62** (0.24)	1.02 (3.01)**
	$4.40	9.60 (0.26)	11.45 (0.24)	-1.85** (0.30)	9.23 (0.31)	10.81 (0.18)	-1.58** (0.26)	0.37 (0.88)
5	$4.00	11.10 (0.17)	9.01 (0.12)	2.08** (0.19)	9.85 (0.16)	10.16 (0.13)	-0.31 (0.16)	1.25 (5.24)**
	$4.40	11.66 (0.17)	12.64 (0.14)	-1.00** (0.16)	10.88 (0.21)	11.72 (0.13)	-0.84** (0.19)	0.78 (2.78)**
	$5.10	12.13 (0.16)	12.60 (0.15)	-0.47** (0.15)	11.78 (1.27)	12.37 (0.14)	-0.59 (0.16)	0.35 (1.43)

** Significantly different from 0 at the 1% level, 2 tailed t-test.

Table 9

Experienced Bidders: Estimated Bid Functions for Sealed Bid Auctions

Prior Experience		95% confidence interval for bids $v_h = \$3.00$	 $v_h = \$4.00$	 $v_h = \$4.40$	No. Subjects	No. Observations
Sealed Bid	b_1= 5.45 V3 + 2.72 DV4 + 1.10 DV440 (0.561)** (0.696)** (0.740)	4.35-6.55	7.00-7.50	7.50	15	270
	b_2 = 4.59 V3 + 1.83 DV4 + 0.72 DV440 (0.374)** (0.291)** (0.304)*	3.85-5.32	5.67-7.16	6.38-7.50		
Clock	b_1 = 4.20 V3 + 1.12 DV4 + 0.3.2 DV440 (0.071)** (0.097)** (0.097)**	4.05-4.33	5.17-5.45	5.49-5.77	18	324
	b_2 = 3.19 V3 + 1.33 DV4 + 0.33 DV440 (0.092)** (0.127)** (0.127)**	3.01-3.37	4.34-4.70	4.67-5.03		

\+ significantly different from 0 at the .05 level, 2-tailed test

* significantly different from 0 at the .05 level, 2-tailed test

** significantly different from 0 at the .01 level, 2-tailed test