Approximate Robotic Mapping from sonar data by modeling Perceptions with Antonyms[☆]

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Abstract

This work, inspired by the idea of "Computing with Words and Perceptions" proposed by Zadeh in [57, 59], focuses on how to transform measurements into perceptions [22] for the problem of map building by Autonomous Mobile Robots. We propose to model the perceptions obtained from sonar-sensors as two grid maps: one for obstacles and another for empty spaces. The rules used to build and integrate these maps are expressed by linguistic descriptions and modeled by fuzzy rules. The main difference of this approach from other studies reported in the literature is that the method presented here is based on the hypothesis that the concepts "occupied" and "empty" are antonyms rather than complementary (as it happens in probabilistic approaches), or independent (as it happens in the previous fuzzy models).

Controlled experimentation with a real robot in three representative indoor environments has been performed and the results presented. We offer a qualitative and quantitative comparison of the estimated maps obtained by the probabilistic approach, the previous fuzzy method and the new antonyms-based fuzzy approach. It is shown that the maps obtained with the antonyms-based approach are better defined, capture better the shape of the walls and of the empty-spaces, and contain less errors due to rebounds and short-echoes. Furthermore, in spite of noise and low resolution inherent to the sonar-sensors used, the maps obtained are accurate and tolerant to imprecision.

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1. Introduction

A commonly required capability of Autonomous Mobile Robots (AMR) is map building. Without a given map, the robot has to navigate, perceive the environment (exteroception), integrate each actual perception with the previous ones, and maintain a coherent and sufficiently accurate representation of the environment. On the one hand, maps can be used as references for navigation, in particular for robot localization and path planning. On the other hand, they can be used as themselves for mapping purposes.

Mapping is an active research area, and there is no final taxonomy of map types. There are several ways to represent a map, for instance, a classical differentiation is between metric and topological maps; metric maps are based on Cartesian reference systems; topological maps emphasize the relations between environmental elements, typically rooms and corridors. This work focuses on metric maps, in particular occupation grid maps, in which the information is represented by bidimensional grids. Habitually, each grid's cell corresponds to a squared space region parallel to the floor and at the height of the robot sensors, and it contains the available knowledge about the cell.

Habitual sensors used in indoor AMRs in order to build maps include: odometric sensors (that measure the relative position of the robot with respect to previous ones), and range sensors (that measure the distance to obstacles). Main range sensors are ultrasonic, or sonar, and radial laser sensors. Although laser range sensors offer a greater angular resolution, sonar sensors have reduced cost, are present on almost every robot platform, and require the computation of a smaller raw data volume.

This work focuses only on sonar exteroception because it suffers from a higher imprecision that other kinds of exteroceptions [6], and therefore an accurate mapping is more difficult to achieve, and because the results could be later extend to other kinds of exteroceptions. An objective of this work is to study how fuzzy logic can help us manage this imprecision [58, 12].

Odometric sensors suffer from some problems [5]; they have a limited resolution and offer incorrect readings when the wheel slips. Consequently, imprecision of odometric estimation grows with distance and number of maneuvers.

Sonar sensing also suffers from several problems [29]; the measure of "time of flight" (TOF) has imprecision inherent in the measuring instrument, it has a poor angular resolution due to the transducers aperture, and the signal emitted forms an open solid angle which does not permit us to know exactly in which part of the wavefront the obstacle is located [35]. Additionally, if the angle of incidence of the beam in respect to the surface is greater than half of the sensor aperture, the echo may not return, or may return after being reflected on other surfaces. This effect is more likely when the surface is planar and smooth. These kinds of surfaces, such as glass, marble, polished wood, plastic, etc. are

commonly found in indoor environments. In general, it can be said that a model of the AMR position based on the distances obtained from ultrasonic sensors is not continuous and not linear.

The specific problem we focus on is how to build an accurate and robust map of the environment by integrating the actual perception with past perceptions [14, 3, 9]. This problem includes the sub-problem of how to handle sensor noise and the contradictions that arise during the process.

> *"A fundamental difference between measurements and perceptions is that, in general, measurements are crisp numbers whereas perceptions are fuzzy numbers or, more generally, fuzzy granules, that is, clumps of objects in which the transition from membership to nonmembership is gradual rather than abrupt".* L. Zadeh in [57]

The approach taken in this paper is based on the assumption that a perception can be represented by means of linguistic descriptions [59, 22], which express imprecise constraints and therefore are gradually satisfied [60, 23], and on the hypothesis that occupied and empty are antonyms instead of complementary. In consequence, we propose to build a fuzzy model of obstacles and another of empty-spaces that verify the properties of antonyms.

The main differences of this work with previous ones are:

- It is not assumed that obstacles and empty-spaces are complementary, and that one is the complement of other, as happens in probabilistic models.
- It is not assumed that obstacles and empty-spaces are independent as happens in previous fuzzy models.
- We assume that obstacles and empty-spaces form a pair of antonyms and should be modeled as such.
- It is not assumed that observations are independent as happens in probabilistic models, in fact some observations are used to correct others.
- It is not assumed that the exact position of the robot is know, so there is imprecision about it. So the model should tolerate that imprecision and still produce robust maps.
- We dealt with rebounds, short echoes, and other noises by defining a set of fuzzy rules that capture our knowledge about the problem.
- The way of building the aggregated maps by means of linguistic quantifiers differs greatly from previous approaches, and it allowed us to handle the partial contribution of each sonar reading to the aggregated map.

The main contributions of this work are:

- A robust model based on antonyms that can properly handle the imprecision and contradictions that arise in the process of building navigation maps.

- The antonyms-based model allows to discard rebounds and short-echoes and reduce greatly the contradictions and errors. Also, by dealing explicitly with contradictions, this model is able to distinguish between two kinds of unknown cells, the ones that are unknown due to contradictions and the ones that are unknown because are unexplored. This allowed to the robot to recognize which zones needed to be navigated with care and which ones needed to be explored later on.

The maps obtained by the antonyms-based method are better defined, capture better the shape of the walls and of the empty-spaces, and contain less errors due to rebounds and short-echoes. The use of approximate maps allowed us to synthesize the accumulated information from samples in a way that kept the data structure constant while the accuracy of the representation increased with the number of samples. The proposed method obtains better maps with higher confidences, and therefore is more robust to noise and to imprecision of sonar-sensors. Based on the qualitative and quantitative comparison performed, we can conclude that the antonyms-based method performs better than the probabilistic method and the previous fuzzy method, obtaining a better recall of obstacles and empty-spaces, a good balance between precision and recall, a higher TCR and a smaller MAE in the three experiments performed.

The rest of the paper is organized as follows: Section 2 presents related works and highlight their differences with the current work. Section 3 explains the theoretical foundations that support the contribution. Section 4 details the main contribution, antonyms-based fuzzy maps. Section 5 presents the experiments and results. Finally, conclusions are covered in Section 6. Additionally, an appendix containing all the maps for an easy comparison has been included.

2. Related Work

In the last decade some approximations of the AMR mapping and localization problems have been published [13, 5, 26, 8], but definitive solutions have not been found. Depending on the kind of sensors used, different approaches are taken, for example: in the case of sonar sensors [18, 53], in the case of laser rangefinder sensors [37, 36], in the case of video sensors [44, 17], or combinations of them as in [33, 7].

The three main approaches to robot map building are: topological [28, 45], feature-based [31] and grid-based [16]. Recently other approaches have appeared using sparse matrices [54] or graph-based maps [21].

In the case of grid-based maps, a typical way of representing the robot observations is by means of an occupancy grid, in which each cell can have two states, empty or occupied, and the grid contains the probability of the cell being occupied, as it was proposed by Elfes in [15, 16], and recently used in [47, 29]. To have a feasible algorithm to build this occupancy grid using a probabilistic approach it is necessary to assume that the current observation is independent from the previous ones, and that the probability of being occupied for each cell is independent from the others, but in most practical cases these assumptions

fail, and the maps based on them contain severe errors, (see [41, 62] for a comparison). A difference between the work presented here and probabilistic approaches is that our model does not need to assume that observations are independent, in fact, the opposite is assumed when some observations are used to correct others (see Section 4.2 for details).

Several sensor models have been developed by different authors [47]. These models provide different description levels of factors that produce uncertainty. Probabilistic models usually use stochastic techniques [15, 19, 26] (see in [47] a classification of probabilistic sensor models) in order to build an occupancy grid (where cells contain the probability of being occupied), but in [43, 41] it was proved that fuzzy models are more robust (have a higher recall of obstacles) and better suited for managing this imprecision and for building approximate maps (where cells contain the degree of being an obstacle, or of being an empty space).

The model presented here is a fuzzy model, because the approximate maps built contain the degree of being an obstacle, or of being an empty space. However, the main difference of this model with previous fuzzy models [42, 43, 41, 39, 20, 1, 27, 53] is that in our model obstacles and empty-space form a pair of antonyms. And that impose certain constraints (see Section 3.3) over the obstacle and empty-spaces maps that allows us to handle explicitly contradictions that arouse in the process, and to warranty that the integrated map will contain much less contradictions.

Recently some works have focused on the problem of dealing with the contradictions that arise during the mapping and navigation process, for example Lee and Chung following a probabilistic approach proposed in [29] to use a conflict evaluated maximum approximated likelihood (CEMAL) approach to deal with that contradictions, although they assumed that observations are independent and that the exact position of the robot is known in advance. As it has been said before in our model it is not assumed the independency of observations nor that the exact robot position is know in advance (see Section 5 for details).

3. Fundamentals

Occupancy grid maps are built using a grid of cells with a certain size. Each cell is associated with a state and a confidence degree, that in the case of probability maps [15] is the probability mass function, or in the case of fuzzy maps [39] is the possibility degree. Before introducing the foundations of antonyms, probabilistic maps and fuzzy maps are presented.

3.1. Probabilistic Maps

In the probabilistic approach, to be able to tackle grids of a size big enough (e.g 100x100) it is assumed that cells states are independent and complete. That is, each cell has a state s, occupied or empty $s(C_{ij}) \in \{E, O\}$, with a certain probability defined by the function $P : \{E, O\} \rightarrow [0, 1]$, that can be represented

by a pair of matrices of size $(n \times m)$, with the probability associated with each state:

$$\begin{aligned} Map_{Occup} &= \begin{bmatrix} P[s(C_{11}) = O] & ... & P[s(C_{1m}) = O] \\ \vdots & P[s(C_{ij}) = O] & \vdots \\ P[s(C_{n1}) = O] & ... & P[s(C_{nm}) = O] \end{bmatrix} \\ Map_{Empty} &= \begin{bmatrix} P[s(C_{11}) = E] & ... & P[s(C_{1m}) = E] \\ \vdots & P[s(C_{ij}) = E] & \vdots \\ P[s(C_{n1}) = E] & ... & P[s(C_{nm}) = E] \end{bmatrix} \end{aligned} \tag{1}$$

In this case, and since both Maps are complementary

$$P[s(C_{ij} = E)] + P[s(C_{ij}) = O] = 1, \tag{2}$$

then maintaining one map is enough.

Each cell is associated with a probability mass function, which is estimated from a sensor model that measures confidence, and an integration process done by the Bayes-Rule. For example, in [41] Ribo and Pinz proposed a sensor model detailed below[1], although other models of the sensor measurements $p[r|s(C_{ij}) = X]$ have been defined (see [46] for details).

$$p[r|s(C_{ij}) = O] = p_1[r|s(\theta, \rho) = O] + p_2[\theta, \rho|r] \tag{3}$$

$$p_1[r|s(\theta,\rho)] = \begin{cases} (1-\lambda)\,0.5 + \lambda p_E & 0 \le \rho < r - 2\delta r, \\ (0.5 - p_E)\left(1 - \lambda\left(\frac{r-\rho-\delta r}{\delta r}\right)^2\right) & r - 2\delta r \le \rho < r - \delta r, \\ \lambda\,(p_O - 0.5)\left(1 - \left(\frac{r-\rho}{\delta r}\right)^2\right) & r - \delta r \le \rho < r + \delta r, \\ 0.5 & \rho \ge r + \delta r \end{cases} \tag{4}$$

$$p_2[\theta, \rho|r] = \begin{cases} p_E, & 0 \le \quad \rho < r - \delta r, \\ 0.5, & r - \delta r \le \quad \rho \end{cases} \tag{5}$$

Where r is a given range reading, ρ is the distance from the sensor to C_{ij}, θ is the angular distance between the beam axis and C_{ij}, $2\delta r$ is the width considered proximal to the reading, or the error in measure, and $\lambda = \Gamma(\rho) \cdot \Delta(\theta)$ is the confidence in the reading expressed by the confidence in the distance (expressed in meters) and in the angle (expressed in radians), as follows:

$$\Gamma(\rho) = 1 + \frac{1 + tanh(2(\rho - \rho_v))}{2} \tag{6}$$

$$\Delta(\theta) = \begin{cases} 1 - \left(\frac{\theta}{0.2182}\right)^2, & 0 \le |\theta| \le 0.2182 \\ 0, & \text{other} \end{cases} \tag{7}$$

[1]Small mistakes presented in the original formula have been corrected

These values come from the analysis of the response of the sensors (see Figure 15), and from the increase of the width of the cone with the distance (see [38]). Then using the sensor measurements model and the upcoming sensors readings, the updating process of the probabilistic occupancy grid is done using the Bayes rule:

$$P[s(C_{ij}) = O|r] = \frac{p[r|s(C_{ij}) = O] \cdot P[s(C_{ij}) = O]}{\sum_{X \in \{E,O\}} p[r|s(C_{ij}) = X] \cdot P[s(C_{ij}) = X]} \tag{8}$$

3.2. Fuzzy Maps

In this case the empty and occupied maps are defined by two fuzzy sets μ_E and μ_O [39, 41]; and two different sensor models are used, one for building the occupied map $f_O(\rho, r)$, and another for building the empty map $f_E(\rho, r)$.

$$f_O(\rho, r) = \begin{cases} 0, & 0 \leq \rho < r - \delta_r, \\ k_O \left(1 - (\frac{r-\rho}{\delta_r})^2\right), & r - \delta_r \leq \rho < r + \delta_r, \\ 0, & \rho \geq r + \delta_r, \end{cases} \tag{9}$$

$$f_E(\rho, r) = \begin{cases} k_E, & 0 \leq \rho < r - \delta_r, \\ k_E(\frac{r-\rho}{\delta_r})^2, & r - \delta_r \leq \rho < r + \delta_r, \\ 0, & \rho \geq r, \end{cases} \tag{10}$$

k_O and k_E are defined such that $k_O \leq 1$ and $k_E \leq 1$

Then for each reading r the partial maps are built using the sensor model and the confidence in the reading $\lambda = \Gamma(\rho) \cdot \Delta(\theta)$.

$$\mu_O^r(C_{ij}) = \Gamma(\rho) \cdot \triangle(\theta) \cdot f_O(\rho, r), \tag{11}$$

$$\mu_E^r(C_{ij}) = \Gamma(\rho) \cdot \triangle(\theta) \cdot f_E(\rho, r). \tag{12}$$

To aggregate several readings in a global map, in [41] the authors propose to use a t-conorm S, for example $S(a, b) = a + b - a \cdot b$.

$$\mu_O = \bigcup_{i=1}^{n} \mu_O^{r_i} \qquad\qquad \mu_E = \bigcup_{i=1}^{n} \mu_E^{r_i}$$

In fact, fuzzy measures need less axioms than probabilities, so a wider choice of operators are available for modeling the imprecision, and for aggregating information from different sources [40].

3.3. Antonyms in Fuzzy Logic

Nature shows a lot of geometrical symmetries; the mathematical concept of symmetry is of paramount importance for nature's scientific study. Underlying the concept of symmetry is the concept of opposite. Humans tend to perceive and categorize the world by means of opposite concepts and to find different

kind of symmetries. Linguistic expressions that describe these perceptions and concepts have, therefore, incorporated antonyms to express opposite meanings.

Antonymy is a phenomenon of language based on pairs of opposite words (P, Q) called pairs of antonyms by grammarians. Antonyms (or opposites) play a key role in perceptions and knowledge organization [10]. Antonyms have not received too much attention form the point of view of classical logic, perhaps because of the barely syntactical character of that phenomenon in language. However, fuzzy logic deals more with semantical aspects of language than classical logic, and the concept of antonym was early considered in fuzzy logic by Zadeh in [55].

The importance of antonyms in linguistics has been studied by Lyons [34], Lehrer [30] and many others, and is evidenced by the fact that there are many dictionaries of antonyms and synonyms. In this context an antonym of a word (or term) P is defined as an opposite word (or term) aP, and it can be related "opposite of meaning" with "symmetry of use" between P and aP. In fact, to fully understand the meaning of a predicate P, one usually need to understand the meaning of one of its antonyms aP; that is, we need to know both, the use of P and the use of aP, because their uses are entwined. In some cases the antonym aP coincides with the negation of P, but this is a limited case and only occurs when there is not middle term, such as in *(dead, alive)* or *(even, odd)*.

In fuzzy logic, one of the most important values of pairs of antonyms is that they allow us to build linguistic variables [55]. A linguistic variable is a set of linguistic labels usually built from a pair of antonyms, the medium term (when it exists), a set of linguistic modifiers (i.e. not, very, moderately, quite, ...) and combinations of them by conjunctions (i.e. and, or). For example, the linguistic variable "Height" may be defined by the set of linguistic labels {tall, short, medium, very tall, moderately tall, quite tall, not very short, medium or tall, ...}. For more details about modifiers and about linguistic variables, see [55], and for more details about conjunctions and disjunctions, see [2, 24]. Different models of antonyms have been studied in the context of fuzzy logic in [11, 49], which later were compiled and extended in [50].

Language is too complex to admit strictly formal definitions [51] as it happens for antonyms. Nevertheless, there are some properties that different authors attribute to a pair of antonyms (see [30]). These properties were summarized in [50], but we will use here the following two:

1. Involution: In considering a pair (P, Q) of antonyms, Q is an antonym of P, and P is an antonym of Q.

$$P = a(aP), \text{ and } Q = a(aQ)$$

2. Coherence with the negation: Pairs of antonyms (P, Q) are N-contradictory [48, 25], for certain strong negation; it is expressed by:

$$P \leq not\, Q, \text{ and } Q \leq not\, P$$

(see for example the pair of antonyms $(Far, Near)$ in Figure 1 or $(None, Some)$ in Figure 2).
Therefore they should verify that $P \wedge Q \leq not\, P \wedge not\, Q$, for some conjunction $\wedge$ usually a t-norm.

For a more complete study of fuzzy models of antonyms, further examples and proofs see [50]. In the next section we will try to take advantage of these properties to obtain a proper representation of the pair of antonyms $(Occupied, Empty)$.

4. Antonyms-based Fuzzy Maps

Let us make a reflection about the nature of the concepts occupied and empty in the context of occupancy grid maps for robot navigation. We consider that occupied and empty are antonyms and not complementary. First, because if some space is not being occupied, we cannot infer that it is empty, it could be unknown or ambiguous, and second because there is a middle term which represents the cells that are not occupied and not empty. A cell may be unknown if the robot perception has not reached it yet, and may be ambiguous if it is partially occupied and partially empty, so it cannot be assigned to any of them without introducing errors.

In consequence, we propose to build a pair of fuzzy models, one of obstacles and another one of empty-spaces, that verify the properties of antonyms. If we want that occupied and empty maps form a pair of antonyms, they must fulfill certain conditions: involution and N-Contradiction; they were explained in Section 3.3. In the following sections we present how these maps can be built from observations, and how they can be refined by properly handling the contradictions that appear from the data during the process.

First, we defined a set of linguistic rules that allowed us to build obstacles and empty maps in a coherent way. And second, we defined another set of linguistic rules to build a contradictions map, that allows the robot to handle the contradictions that arise in the process, and an integrated map, that allows the robot to integrate properly the obstacles and empty space maps by removing the contradictions.

4.1. Building the obstacles and empty-space maps

Let us now introduce a linguistic description of how the robot should model its perceptions of obstacles and empty spaces. By means of this antonyms-based model we will be able to model properly the imprecision inherent in the robot perception process. The ways obstacles and empty-spaces are modeled are different since the information provided by sonar sensors must be interpreted differently for obstacles than for empty-spaces.

Let us first introduce a linguistic description of the obstacles perception model from the reading of one ultrasonic sensor. Basically, the model must assign some degree to the grid cells affected by the sensor reading, that is, the cells inside the circular sector projected by the sensor cone over the grid.

- Confidence rule:
  - If the measure is small (an obstacle is near), then assign a high confidence to the measure (μ_{Near}).
- Local perception rule:
  - If the measure is r at angle θ, then put an obstacle at this position (μ_{Occup}).
- Aggregation rule:
  - If an obstacle is perceived some times, then increase the confidence on its presence (μ_{Some}).

These rules translate into the following formulas:

$$\mu_{Near}(r) = \frac{1 + tanh(\frac{200-x}{30})}{2} \tag{13}$$

$$\mu^{k}_{Occup}(C_{ij}|(r,\theta)) = \mu_{Approx}(d,r) \cdot \mu_{Approx}(\alpha,\theta) \tag{14}$$

$$\mu_{Some} = \begin{cases} 0 & 0 \leq x \leq 1 \\ \frac{x-1}{2} & 1 < x \leq 3 \\ 1 & 3 < x \leq 10 \end{cases} \tag{15}$$

These fuzzy sets (see Figures 1 and 2) try to capture the fact that $not-near$ readings tend to be confusing for the obstacle map, and that the robot need to see an obstacle more than once (*some* times) to know that it is there. After analyzing the data coming from the sensors in different situations, we tuned the parameters of these fuzzy sets.

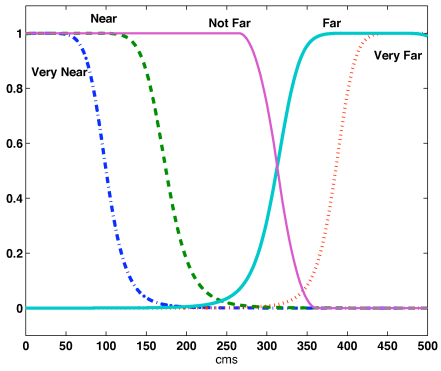

Figure 1: Distance labels

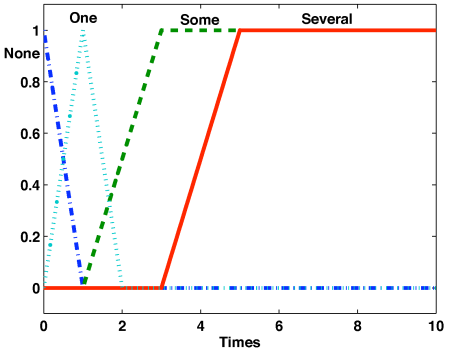

Figure 2: Times labels

Analogously, we established the following linguistic descriptions of how to model the empty space perception from each ultrasonic sensor:

- Confidence rule:
  - If the measure is not big (an obstacle is not far), then assign a high confidence to the measure ($\mu_{Not-Far}$).
- Local perception rule:
  - If the measure is r at angle θ, then assign empty space inside the circular sector with radius r and aperture δ_α (μ_{Empty}).
- Aggregation rule:
  - If an empty space is perceived several times, then increase the confidence on its emptiness ($\mu_{Several}$).

These rules translate into the following formulas:

$$\mu_{Not-Far}(r) = 1 - \frac{1 + tanh(\frac{x-300}{30})}{2} \tag{16}$$

$$\mu^k_{Empty}(C_{ij}|(r,\theta)) = \mu_{Smaller}(d,r) \cdot \mu_{Approx}(\alpha,\theta) \tag{17}$$

$$\mu_{Several} = \begin{cases} 0 & 0 \leq x \leq 3 \\ \frac{x-3}{2} & 3 < x \leq 5 \\ 1 & 5 < x \leq 10 \end{cases} \tag{18}$$

These fuzzy sets (see Figures 1 and 2) try to capture the fact that far away readings tend to be wrong, and that the robot needs to see an empty-space *several* times before knowing that it is empty. After analyzing the data coming from the sensors in different situations, we tuned the parameters of these fuzzy sets.

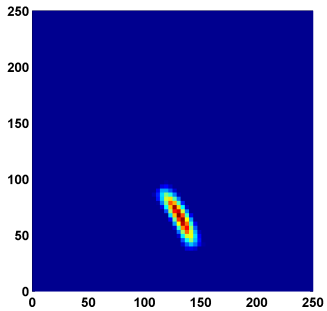

Figure 3: Obstacle with $(r,\theta) = (150, 30^o)$

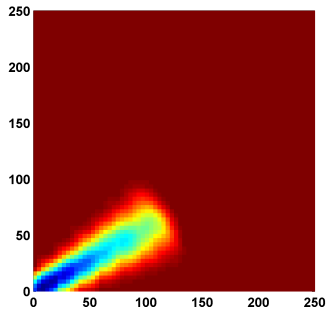

Figure 4: Empty-space with $(r,\theta) = (150, 30^o)$

The linguistic labels used in these rules can be seen in Figures 1 and 2. Examples of the contribution to the obstacles map and empty-spaces maps produced by a single sonar reading $\mu^k_{Occup}(C_{ij}|(r,\theta))$ and $\mu^k_{Empty}(C_{ij}|(r,\theta))$ are presented in Figures 3 and 4, respectively.

These rules try to take into account the problems underlying sonar readings and allow us to build two approximate maps in such a way that every cell contains a number between 0 and 1, which represents the degree of the cell being an obstacle, or the degree of being an empty-space.

In order to define the fuzzy sets μ_{Approx} an $\mu_{Smaller}$ and to allow a fair comparison with other approaches, we have used the values of the parameters ($\delta_r = 15\,cm, \delta_\alpha = radians(30^o) = 0.2618\,rad$) that were used in [41]:

$$\mu_{Approx}(d, r) = 1 - \frac{(d-r)^2}{\delta_r^2} \tag{19}$$

$$\mu_{Approx}(\alpha, \theta) = 1 - \frac{|\alpha-\theta|^2}{\delta_\alpha^2} \tag{20}$$

$$\mu_{Smaller}(d, r) = 1 - \frac{1 + tanh(\frac{(d-r)}{50})}{2} \tag{21}$$

r is a given range reading, d is the distance from the sensor to C_{ij}, θ is the sensor bearing and α is the angle between the beam axis and C_{ij}.

Every cell may receive contributions from several sensor readings. The approximate maps are built by the aggregation of a sequence of readings $R = \{(r_1, \theta_1), ..., (r_k, \theta_k), ..., (r_n, \theta_n)\}$ as follows (see Figures 5 and 6 for obstacles and empty-space maps of the Office):

$$\mu_{Occup}(C_{ij}|R) = \mu_{Some}\left(\sum_{k=1}^{n}(\mu_{Near}(r_k) \cdot \mu^k_{Occup}(C_{ij}|(r_k, \theta_k)))\right) \tag{22}$$

$$\mu_{Empty}(C_{ij}|R) = \mu_{Several}\left(\sum_{k=1}^{n}(\mu_{Not-Far}(r_k) \cdot \mu^k_{Empty}(C_{ij}|(r_k, \theta_k)))\right) \tag{23}$$

This way of building the aggregated maps by means of linguistic quantifiers (see [12]) differs greatly from other approaches [15, 39, 32], and allows us to easily handle the partial contribution of each sonar reading to the aggregated map.

Figures 5 and 6 show an example of an obstacles map (obstacles in red) and an empty-space map (empty space in blue), respectively. In these figures, the limits of the place may easily be recognized although due to the presence of rebounds and short echoes some outliers also appear.

4.2. Building the contradiction map and the integrated map

If one cell is considered simultaneously empty and occupied, then we have a contradiction in that cell. This situation violates the principle of coherence with the negation (the value of the cell in one map must be smaller than the negation of the value in the other map, see Section 3.3).

In order to analyze the contradictions we built a contradiction map by the conjunction of the obstacles and empty space maps. Since all t-norms are smaller

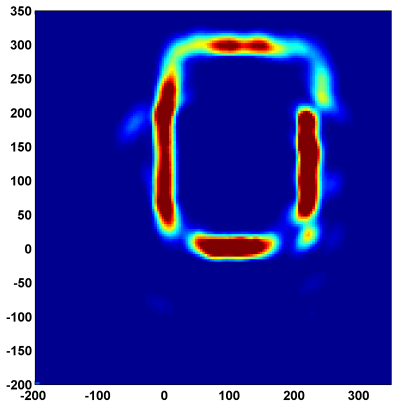

Figure 5: Obstacles map

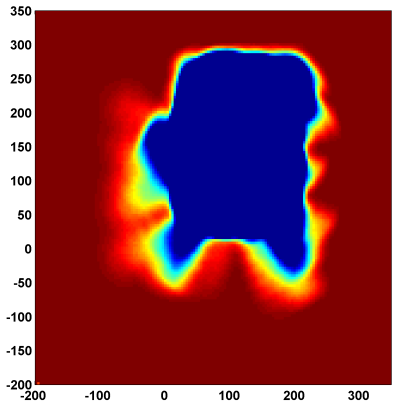

Figure 6: Empty spaces map

than minimum (see [40]), we used minimum to obtain the largest degree of contradiction and to work in the worst case.

$$\mu_{Contra} = min(\mu_{Occup}, \mu_{Empty}) \tag{24}$$

The integrated map is built by combining the occupied and empty maps by the following procedure. In this case, to work in a very restrictive case, we used the Łukasiewicz t-norm $W(x, y) = max(0, x + y - 1)$ (see [40]).

1. Calculate the occupied cells that are not empty:

$$\mu_{Occup^*} = W(\mu_{Occup}, 1 - \mu_{Empty}) = max(0, \mu_{Occup} - \mu_{Empty}) \tag{25}$$

2. Calculate the empty cells that are not occupied:

$$\mu_{Empty^*} = W(\mu_{Empty}, 1 - \mu_{Occup}) = max(0, \mu_{Empty} - \mu_{Occup}) \tag{26}$$

3. Aggregate both maps in an integrated map $\mu_{Integ} : [cells] \rightarrow [-1, 1]$:

$$\mu_{Integ}(C_{ij}) = \begin{cases} \mu_{Occup^*}(C_{ij}) & \text{if } \mu_{Occup}(C_{ij}) > \mu_{Empty}(C_{ij}) \\ -\mu_{Empty^*}(C_{ij}) & \text{if } \mu_{Occup}(C_{ij}) \leq \mu_{Empty}(C_{ij}) \end{cases} \tag{27}$$

The integrated map (see Figure 8) is defined from -1 to 1, with -1 meaning that the cell is completely empty (represented as blue in the figures), 1 meaning that the cell is completely occupied (represented as red in the figures) and 0 meaning that the cell is unknown (represented as green in the figures). Cells can be unknown due to two reasons: first because the cell's state is ambiguous (or contradictory) since the cell is occupied and empty simultaneously, and second because the robot has never explored that zone and the cell's state is neither occupied nor empty.

It can be seen in the contradictions map, Figure 7, that there are many contradictions between obstacles and empty-space maps and, therefore, that the integrated map plotted in Figure 8 contains several errors. For instance, there is a hole in the left wall, and there are obstacles around corners.

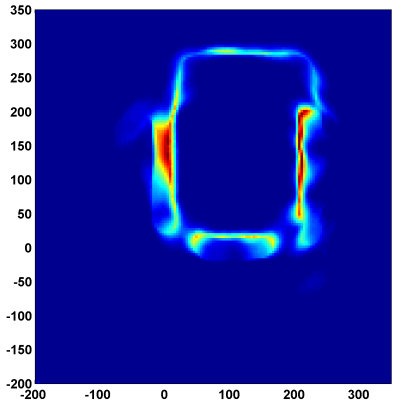

Figure 7: Contradictions map

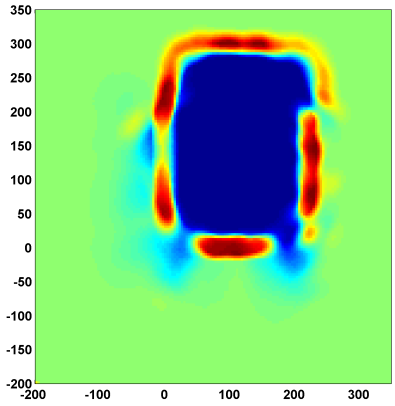

Figure 8: Integrated map

Analyzing the contradictions map we can see that it reflects the existence of errors and imprecision. If we could have perfect knowledge and there were no errors, then there would be no contradictions. These contradictions are usually solved, in a conservative approach, by marking them as unsafe to avoid collisions (see [20] or [41]). One problem of this conservative approach is that when a cell is marked unsafe, it eventually will remain that way forever.

There are two known situations that cause cell contradictions: rebounds and short echoes. We can model our knowledge about them in order to correct as many contradictions as possible.

A rebound is a range larger than the real distance to the obstacle. It generates some false empty cells inside its circular segment and false occupied cells in its arc. We have empirically observed that it is very infrequent to obtain a rebound in a short range reading (bellow more or less 150 centimeters), this explains the formula choused for the confidence on readings $\Gamma(\rho)$.

A short echo is a range shorter than the real distance to the obstacle. Short echoes are mainly produced by the significant aperture of ultrasound sensors. Although an echo is usually produced only at one point of the wavefront, several grid cells (those of the arc) are assigned as occupied. This effect is more dramatic when the echo is produced at one of the sides of the wavefront. In general, this phenomenon generates false occupied cells along of the arc. Short echoes are perceived as false obstacles that habitually disappear if the robot gets closer to them (bellow more or less 150 centimeters). A typical example are corners, perceived from far as round shapes.

False occupied cells and false empty cells can be detected if they have been

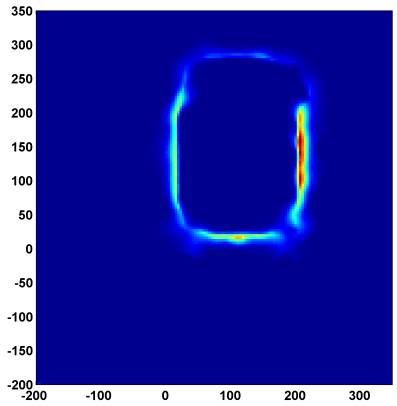

Figure 9: Errors due to Short-Echoes

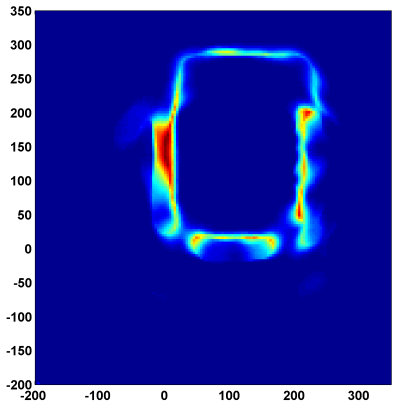

Figure 10: Errors due to Rebounds

correctly captured by another range. To ensure this correction, we additionally require that the other range is obtained near the cell.

We model this knowledge about rebounds and short echoes by means of the following fuzzy rules:

- If one cell is occupied and empty, but from near looks empty, then it is a short-echo (a false obstacle) (see Figure 9).
- If one cell is occupied and empty, but from near looks occupied, then it is a rebound (a false empty-space) (see Figure 10).

Using these rules we can remove false obstacles from the obstacles map and false empty cells from the empty-space map (see Figures 11 and 12). After removing these false obstacles and false empty cells we obtained a new contradiction map and integrated map shown in Figures 13 and 14, which show less contradictions and contains less errors, respectively.

5. Experiments and results

5.1. System Description

The proposed model have been validated by means of experimentation with real data. In order to obtain data-sets from real environments, the robot Sancho-2[2], an indoor mobile robot for research purposes, has been used. It is a 50 cm x 50 cm x 50 cm platform with motor and sensory capabilities, over which a

[2]Sancho-2 has been designed and built by Gracian Triviño at the Universidad Politécnica de Madrid [52]

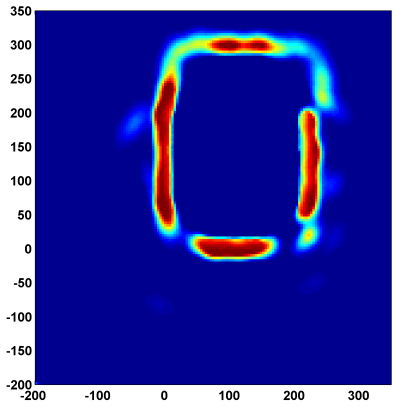

Figure 11: Corrected obstacles map

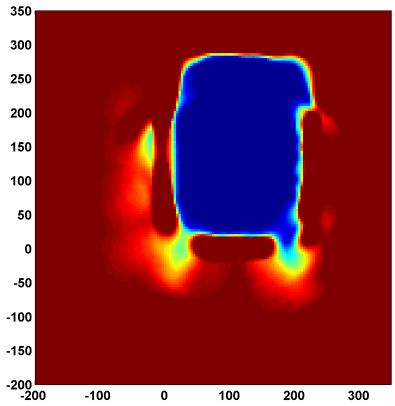

Figure 12: Corrected empty-spaces map

laptop PC is placed and connected through a serial connection, for controlling the robot. The wheels are disposed in tricycle configuration with two driven wheels and one passive castor wheel. Each driven wheel is controlled in closed-loop by means of a coupled encoder disk. The resolution of these odometers, projected over the floor, is 1.2 cm. The sensory capability is disposed by a ring of twelve ultrasonic, or sonar, Polaroid 6500 sensors distributed at angles of 30^o around it. A detailed description of the Polaroid ultrasonic sensor may be found in [6]. The range resolution produced by our sensor control hardware is 4 cm, and the aperture of the transducer's main lobe (cone) is 30^o at 3dB although in some cases the second and even the third lobe can be reflected and received (see Figure 15). This causes considerable imprecision about the location of the object that generates the returned echo. This imprecision grows with the measured range.

5.2. Experiments setup

Several controlled experiments have been performed to validate the proposed mapping approach. Three real indoor places: an office (see Figure 16), a medium sized hall (see Figure 17) and a large corridor (see Figure 18), were selected. Sonar and position data were collected from real navigation of Sancho-2 around these environments. Periodically, the robot was stopped to manually measure its pose (position and orientation). These recorded data were post-processed to obtain reduced data sets with controlled position ground truth. All these experimentation steps are detailed as follows.

The first selected place (see Figure 16) was a 25 square meters office, reduced to a 2 by 2.5 meters navigation area due to the furniture. The obstacles, excepting the upper line, were made of polished surfaces susceptible to generate rebounds. The second place was a hall (see Figure 17) of about 6 by 8 meters in its central zone. There was no furniture, walls were covered by smooth plastic

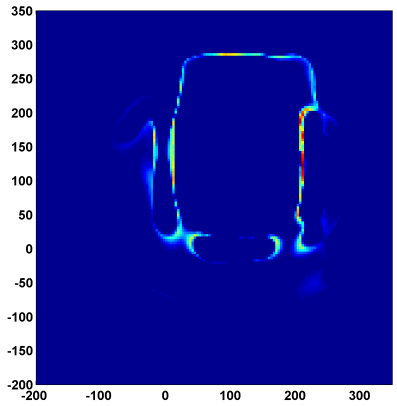

Figure 13: Contradictions map after corrections

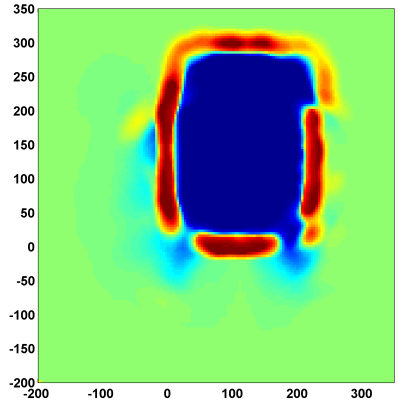

Figure 14: Integrated map after corrections

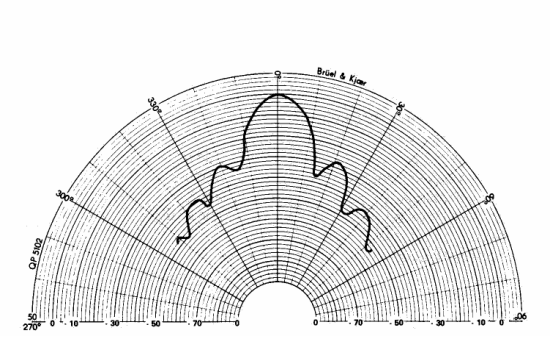

Figure 15: Polaroid 6500 sonar beam (extracted from [6])

panels, and people passed and sometimes stopped to see the robot movements. The third place was a large corridor which can be seen in Figure 18. It had an approximate size of 10 by 30 meters, the walls were covered by smooth plastic panels, it included several doors (which remained closed during the experiment) and several benches and columns on the right side. These three places showed many of the problems that we wanted to deal with, and are representative of the places that a mobile robot may find in indoor environments.

The robot trajectories were programmed to visit the main zones of each place in order to bring the robot near walls and corners, for perceiving them from close and from far. In the office the task was to visit sequentially a list of predefined positions. In the hall a first trajectory visited the four extremes of an imaginary rectangle through its diagonals, describing an 8-like figure over the floor. In the second group of trajectories, the task was to visit a random position and return to the starting point. In this case the movement of people was not

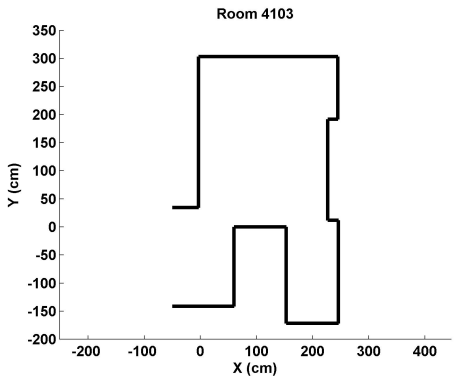

Figure 16: Office

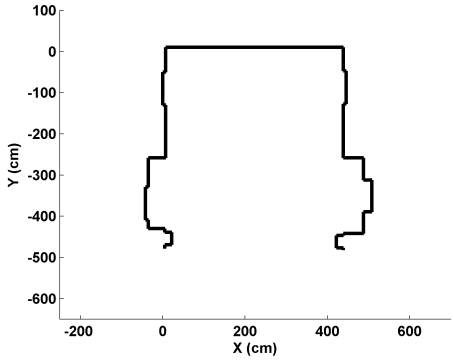

Figure 17: Hall

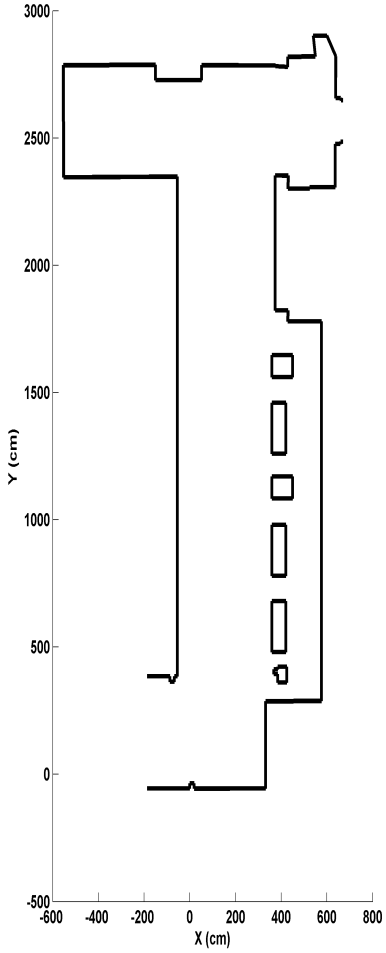

Figure 18: Corridor

restricted, so in the perception data unexpected obstacles appeared sometimes. During navigation, the robot estimated its pose using only odometry.

In every experiment, the robot stopped periodically to allow the human operator to manually measure the robot pose. The recorded position by the robot was an odometric estimation, periodically corrected by the introduction of the manual measurement. Every advanced 50 cm, sonar sensors were fired, and its readings logged, so each recorded position (x, y, α) was associated with twelve sonar readings $(d1, d2, ..., d12)$. The resulting data set, called raw trace, is a chronologically ordered set of vectors:

$$(x, y, \alpha, d1, d2, ..., d12)$$

This trace contained some positions with an estimation error too large for mapping or localization purposes. These positions were those recorded long after a manual measure was introduced, and its accumulated odometric error was not tolerable. Therefore, the raw trace was post-processed to obtain a trace with controlled position error, called controlled trace. When a manual measurement was introduced the position estimation error of the last odometric position was calculated. The post-process assumed a linear increase of the odometric error, starting at 1 cm, 1^o in every manual measurement, and ending at the calculated estimation error in the next manual measurement. So an estimation of error was computed for each position of the raw trace, and positions with error greater than 25 cm, 15^o were rejected.

5.3. Mapping Results

Results for the first place, the office, have been explained in Section 4, so here we present the results for the other places: the hall and the corridor.

The initial integrated map is shown in Figure 19, where the false obstacles at the upper side (the small green areas) correspond to people. A clear improvement can be seen in the final integrated map in Figure 20, when contradictions were detected and corrected. Note that there are fewer outliers, more empty space is detected, and it is nearer to the walls. When the rebounds were discarded, the locations of the walls could be appreciated.

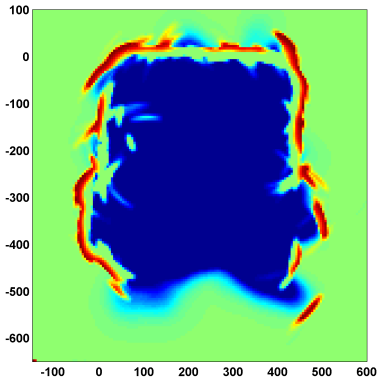

Figure 19: Initial integrated map of the hall

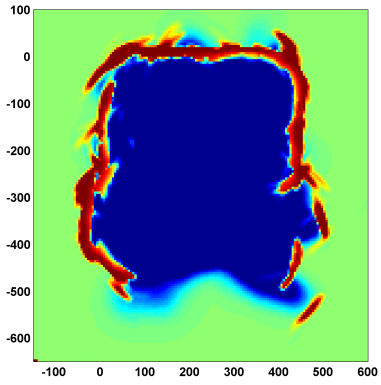

Figure 20: Final integrated map of the hall

A similar effect can be seen in Figures 21 and 22, where initially most of the obstacles were considered unknown but later were recovered and shown in the final integrated map.

Looking at Figure 20 a good map of the hall can be seen, containing few rebounds and short-echoes. However, looking at Figure 22 it is harder to see some of the walls of the corridor although the empty-space represents the place very well.

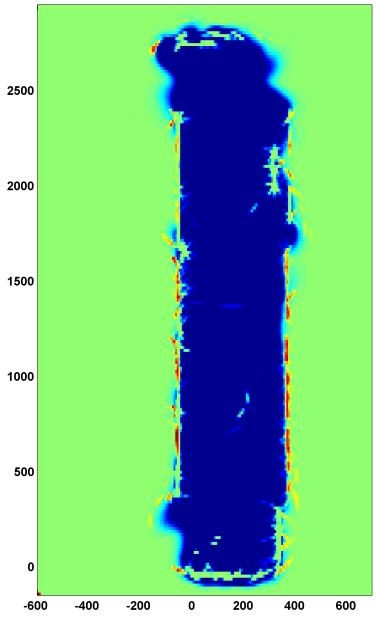

Figure 21: Initial Integrated Map of the Corridor

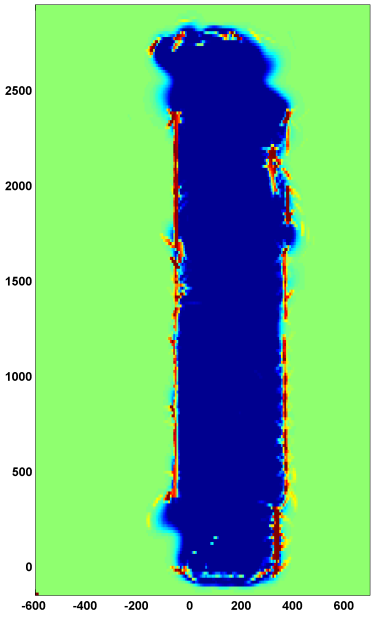

Figure 22: Final Integrated Map of the Corridor

5.4. Comparison with other approaches

In order to compare this approach with previous approaches, we ran the algorithms compared in [41]: a probabilistic approach based on [15] and a fuzzy approach based on [39]. We used the parameters' values used in the corresponding papers: $\rho_v = 1.2m$, $\delta_r = 0.15m$, $p_O = 0.6$ for the probability approach, and $k_E = 0.45$ and $k_O = 0.65$ for the fuzzy approach.

In Appendix A, the complete set of maps obtained by the three approaches are presented (see Figures A.29 to A.37). It can be seen that, due to rebounds and short-echoes, many obstacles (i.e. walls) are missing in the probabilistic and previous fuzzy methods.

If one magnifies into the up side of Figure A.32, it can be seen that although empty spaces are more or less well captured, most obstacles are missed by the probabilistic approach (see details in Figure 23). Something similar occurs to the previous fuzzy method which can be seen in figure 24. Nevertheless, we can see by magnifying Figure A.34 that, although obstacles are in some zones imprecise, the map built by the antonyms-based method follows the wall very well (see details in Figure 25).

In the case of the fuzzy approach, we can also compare obstacles maps and empty space maps directly, and compare the integrated map obtained using

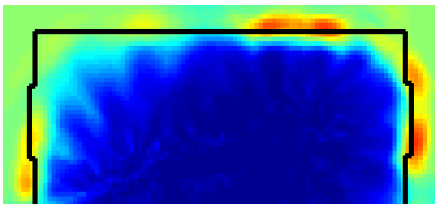

Figure 23: Zoom into the Probabilistic map

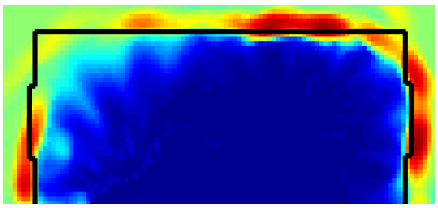

Figure 24: Zoom into the Fuzzy Map

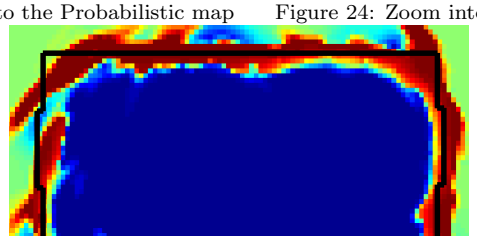

Figure 25: Zoom into the Antonyms-Based Map

the antonyms-based approach with the safe map obtained using the fuzzy approach proposed in [41], a map that directly excludes contradictory cells and indeterminate cells.

It can be seen in this comparison with the other approaches that the antonyms-based method has created better maps with more defined walls and clear empty-spaces, in spite of imprecision inherent to the sensors. If necessary, we could introduce thresholds to decide more precisely the limits of obstacles and of empty spaces, but in that case we would be forcing the model by introducing more constraints. However, antonyms-based maps recall most of the obstacles and empty spaces (see quantitative comparison bellow for details), since with this approach, rebounds and short echoes can be detected, differentiated, and removed from the integrated map.

5.5. Quantitative comparison

In order to perform a quantitative comparison, we needed to define a set of measures that allowed us to compare the results of the different solutions. First, we defined a reference occupancy map obtained from the manual measurement of the walls' positions. In the reference map obstacles, empty-spaces and unknown cells were represented by 1, -1 and 0 values respectively (as a ternary classification problem). Then, we quantified the accuracy of the maps, estimated by comparing the resulting map of the three approaches with the reference map, by means of the precision, recall, f-measure and mean absolute error (MAE) measures.

To make the comparison and discretization easier we re-scaled all the obtained maps to [-1,1] with -1 meaning completely empty, 1 meaning completely occupied and 0 meaning unknown.

We discretized the integrated maps obtained by assigning to obstacles those cells that had a value between α and 1, assigning to unknowns those cells that

had a value between $-\alpha$ and $+\alpha$ and assigning to empty-spaces those cells that had a value between -1 and $-\alpha$. That means that a cell needed to have a degree greater than α to be considered an obstacle (this resembles an α-cut of the fuzzy set of obstacles), smaller than $-\alpha$ to be considered an empty-space (this resembles an α-cut of the fuzzy set of empty-spaces), and considered unknown in other cases. From that we built the following confusion matrix:.

		Actual		
		Obstacles	Empty	Unknown
Predicted	Obstacles	oto	efo	ufo
	Empty	ofe	ete	ufe
	Unknow	ofu	efu	utu

where *oto* stands for Obstacle-True-Obstacle, which means that the method predicted an obstacle when it was obstacle, efo stands for Empty-False-Obstacle, which means that the method predicted an empty-space when it was an obstacle, ofe stands for Obstacle-False-Empty, which means that the method predicted an obstacle when it was an empty-space, and so on.

From the confusion matrix, Precision (P) and Recall (R) for obstacles and empty-spaces are defined as:

$$\mathrm{P}_O = \frac{oto}{oto + efo + ufo} \qquad \mathrm{R}_O = \frac{oto}{oto + ofe + ofu} \tag{28}$$

$$\mathrm{P}_E = \frac{ete}{ete + ofe + ufe} \qquad \mathrm{R}_E = \frac{ete}{efo + ete + efu} \tag{29}$$

We didn't calculate the precision and recall of unknown space since it was complementary to the other two and didn't add further information.

From precision and recall the f-measure is defined using the weighted harmonic mean

$$F_\beta = \frac{(1+\beta)}{\frac{1}{\text{Precision}} + \frac{\beta}{\text{Recall}}} \tag{30}$$

In this case, since recall is more important than precision (to avoid collisions with obstacles), we used the F_2 measure, whose weights recall twice as much as precision.

To aggregate and obtain a Total Combined Rate (TCR) from the results of obstacles and empty-spaces, we used the arithmetic mean of f-measures from the obstacles F_O and from the empty-spaces F_E (the results can be see in tables 1, 2 and 3).

$$TCR = \frac{F_O + F_E}{2} \tag{31}$$

Different elections of the confidence threshold α returned different solutions (see Figures 26 to 28), but to show a detailed comparison, a reasonable choice was to split the range $[-1, 1]$ in three parts $([-1, -\alpha], [-\alpha, \alpha], [\alpha, 1])$ and take $\alpha = \frac{1}{3}$. The numerical results of this choice can be seen in the tables 1, 2 and 3.

To perform an exhaustive comparison, we calculated the Total Combined Rate (TCR) for 30 different $\alpha \in (0, 1)$, for each method and for each place. The results of these comparisions can be seen in Figures 26, 27, and 28.

Table 1: Precision, Recall and F_2 measures for the obstacles and empty-spaces maps of the office, and the Total Combined Rate (TCR) for each method

Method	P_O	R_O	F_O	P_E	R_E	F_E	TCR
Probabilistic	**40**%	25%	29%	99%	55%	64%	47%
Fuzzy	30%	50%	**41**%	**99**%	56%	65%	**53**%
Antonyms	22%	**54**%	36%	91%	**62**%	**69**%	**53**%

Table 2: Precision, Recall and F_2 measures for the obstacles and empty-spaces maps of the hall, and the Total Combined Rate (TCR) for each method

Method	P_O	R_O	F_O	P_E	R_E	F_E	TCR
Probabilistic	**52**%	14%	18%	**99**%	77%	83%	51%
Fuzzy	39%	39%	39%	**99**%	78%	84%	61%
Antonyms	33%	**75**%	**53**%	97%	**80**%	**85**%	**69**%

Table 3: Precision, Recall and F_2 measures for the obstacles and empty-spaces maps of the corridor, and the Total Combined Rate (TCR) for each method

Method	P_O	R_O	F_O	P_E	R_E	F_E	TCR
Probabilistic	**78**%	2%	3%	**99**%	75%	81%	42%
Fuzzy	43%	8%	10%	**98**%	74%	81%	46%
Antonyms	40%	**39**%	**40**%	94%	**91**%	**92**%	**66**%

To obtain a more global evaluation of the obtained maps Obt we also calculated the Mean Absolute Error (MAE) with respect to the reference map Ref (the results can be seen in table 4).

$$MAE = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} |Ref(C_{ij}) - Obt(C_{ij})|}{n \cdot m} \tag{32}$$

5.6. Discussion

As it can be seen in Appendix A the maps obtained by the antonyms-based method are better defined, capture better the shape of the walls and of the empty-spaces, and contain less errors due to rebounds and short-echoes. While in the probabilistic and in the previous fuzzy method many obstacles (i.e. walls)

Table 4: Mean Absolute Error (MAE) for each method

	MAE		
Method	Office	Hall	Corridor
Probabilistic	0.2115	0.1985	0.1512
Fuzzy	0.2184	0.1848	0.1400
Antonyms	**0.1688**	**0.1395**	**0.0767**

are missing, and many empty-spaces not clear enough. The quantitative measures of performance (see Tables 1 to 4) of the three different methods in the three places come to confirm these qualitative evaluation.

For instance, in Tables 1, 2 and 3 it can be seen that although the probabilistic method has the highest precision it has the lowest recall; while the antonyms-based method has the highest recall, and the best balance between precision and recall. This means that antonyms-based method detects more obstacles and more empty spaces, and have a sound compromise between precision and recall.

In addition, the antonyms-based method has the highest f-measure F_2 and the highest Total Combined Rate TCR, which means that more obstacles and empty-spaces are captured, while less empty-spaces are considered as obstacles and less obstacles are considered as empty-spaces.

Looking at Figures 26, 27 and 28, it can be seen that antonyms-based method obtains higher and more stable TCR for different thresholds α, while the others degrade faster when confidence threshold α grows. This means that antonyms-based method obtains better maps with higher confidences, and therefore is more robust to noise and to imprecision of sonar-sensors.

When the environment becomes bigger, precision of obstacles and empty-spaces increases while recall of obstacles decreases and recall of empty-spaces increases. This is due to the fact that there are many more empty-spaces than obstacles and also that empty-spaces are easier to identify than obstacles. One question that goes beyond the scope of this work is how these methods could be applied to a dynamic environments where many obstacles are moved, removed or added (see [4, 12] for solutions to this problem).

For a more global comparison we also calculated the Mean Absolute Error (MAE), obtaining as result that antonyms-based method has the smallest MAE in the three environments, as can be seen in table 4, and therefore it can be considered more accurate than the other methods, since the map obtained is closer to the reference map.

6. Conclusions

In this paper we have proposed new ways of representing the perceptions that an Autonomous Mobile Robot can obtain of its environment, emphasizing that environment and sensor characteristics must be considered as a whole. The

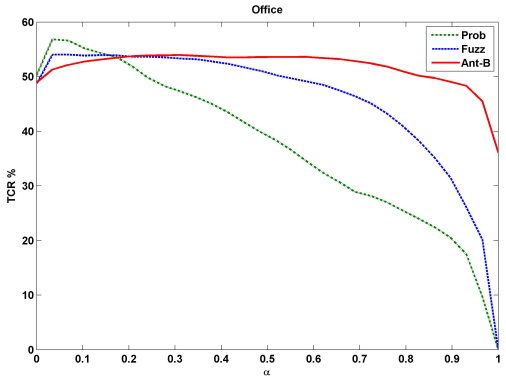

Figure 26: Comparison of TCR for different α for the office

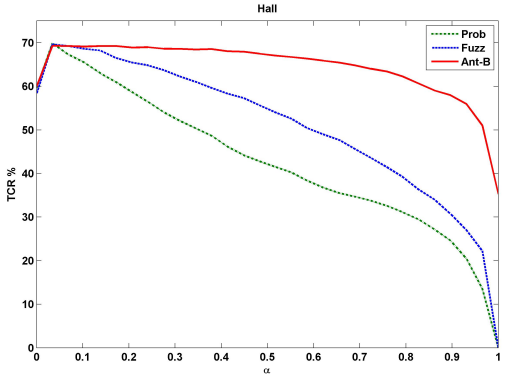

Figure 27: Comparison of TCR for different α for the hall

aim has been to model these perceptions properly by taking into account the imprecision inherent in the environment and the robot sensors. For that purpose we have built a robust model based on antonyms that can properly handle the imprecision and contradictions that arise in the process of building these maps, which, in addition, represents a new way of building antonyms from data.

Fuzzy logic theory has proved to be adequate and useful for representing the inherent imprecision in this robotic mapping problem. This theory has allowed us to successfully integrate the information obtained from different observations, and to build accurate maps for exploration tasks. In particular, antonyms abstraction has made possible to model and handle the concepts "occupied space" and "empty space" when reasoning about the integration of past and actual perceptions during the navigation of a mobile robot. From that we have built a set of fuzzy grid maps that represent the robot's perception of "obstacles", of "empty spaces", of "short-echoes", and of "rebounds", respectively. These

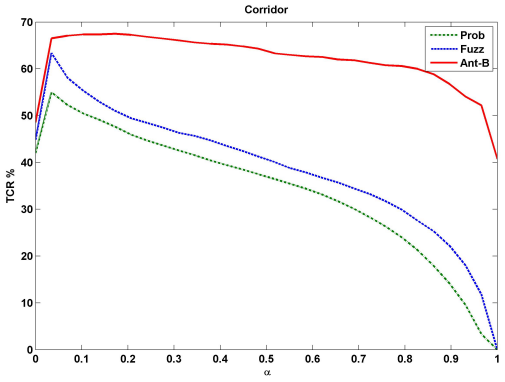

Figure 28: Comparison of TCR for different α for the corridor

maps were later fused to obtain an integrated map.

A controlled experimentation with a real robot equipped with sonar exteroception was performed in three representative real indoor places. Obtained maps show high fidelity with respect to the real architectural walls map, demonstrating the suitability and robustness of this approach to robotic mapping. Furthermore, it should be noted that these results were obtained using low cost sensors of limited accuracy.

Based on the qualitative and quantitative comparison performed, we can conclude that the antonyms-based method performs better than the probabilistic method and the previous fuzzy method, obtaining a better recall of obstacles and empty-spaces, a good balance between precision and recall, a higher TCR and a smaller MAE in the three experiments performed.

The inclusion of antonyms also allowed us to discard rebounds and short-echoes and reduce greatly the contradictions and errors. We dealt with rebounds, short echoes, and other noises by defining a set of fuzzy rules that capture our knowledge about the problem. Also, the way of building the aggregated maps by means of linguistic quantifiers differs greatly from previous approaches, and it allowed us to handle the partial contribution of each sonar reading to the aggregated map.

Thanks to maintaining obstacles and empty maps as a pair of antonyms, we were able to deal with contradictions, and build an accurate and robust integrated map. The use of approximate maps allowed us to synthesize the accumulated information from samples in a way that kept the data structure constant while the accuracy of the representation achieved increased with the number of samples.

An important point is that the obtained integrated map can be traced back to the contradictions map and from it to the obstacles and empty spaces maps, adding an explicative feature to the results. Also, by dealing explicitly with contradictions, we were able to distinguish between two kinds of unknown cells, the ones that are unknown due to contradictions and the ones that are unknown

because are unexplored. This allowed to the robot to recognize which zones needed to be navigated with care and which zones needed to be explored later on.

The main difference of this contribution from other published works is that our approach is based on the manipulation of perceptions by means of antonyms. In addition, the obtained results are more accurate, more robust and more understandable than the previous ones.

The approach to model perceptions by means of linguistic descriptions and antonyms, and its applicability to real problems can be seen as an initial step in the development of Computing with Words and Computational Theory of Perceptions proposed by Zadeh in [56, 59, 60, 61].

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Appendix A. Comparison of maps

Several maps have already appeared in the text of this paper to facilitate its explanations. For ease of comparison, we have repeated some of them here.

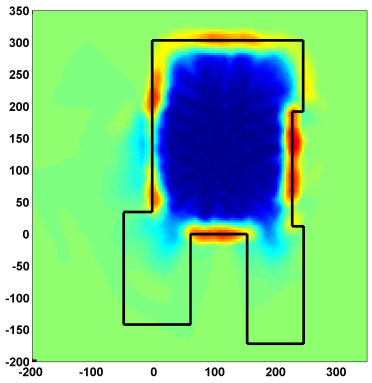

Figure A.29: Map of the office by the probabilistic method

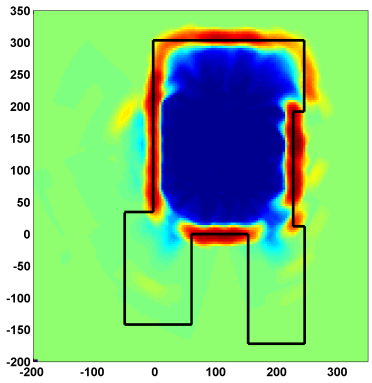

Figure A.30: Map of the office by the fuzzy method

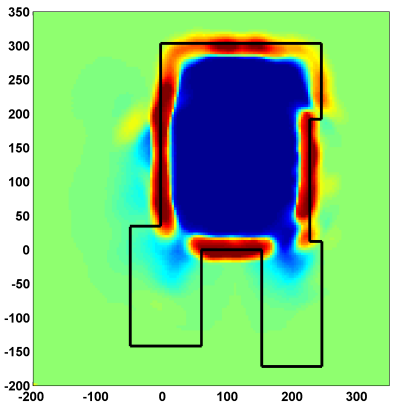

Figure A.31: Map of the office by the antonyms-Based method

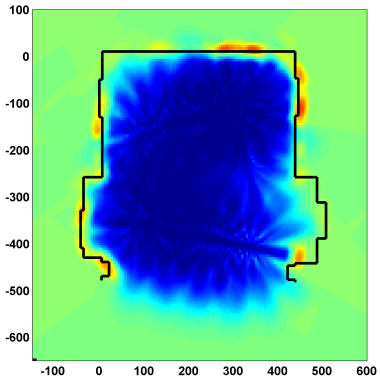

Figure A.32: Map of the hall by the probabilistic method

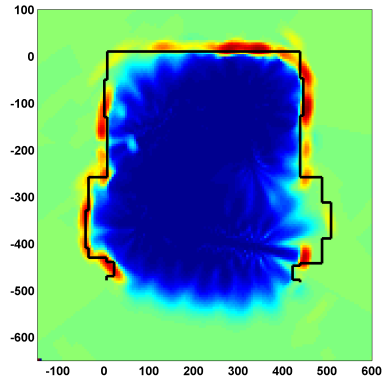

Figure A.33: Map of the hall by the fuzzy method

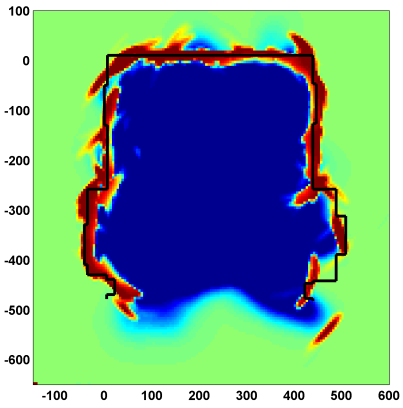

Figure A.34: Map of the hall by the antonyms-based method

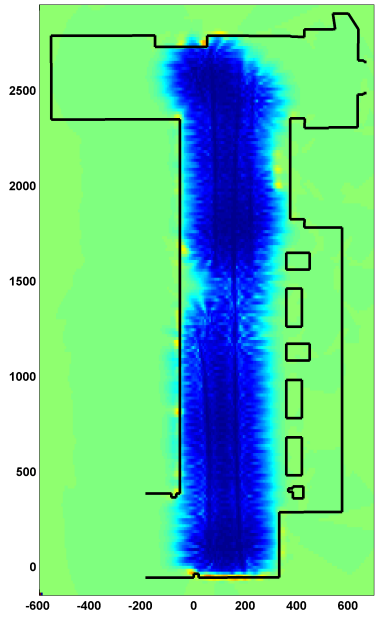

Figure A.35: Map of the corridor by the probabilistic method

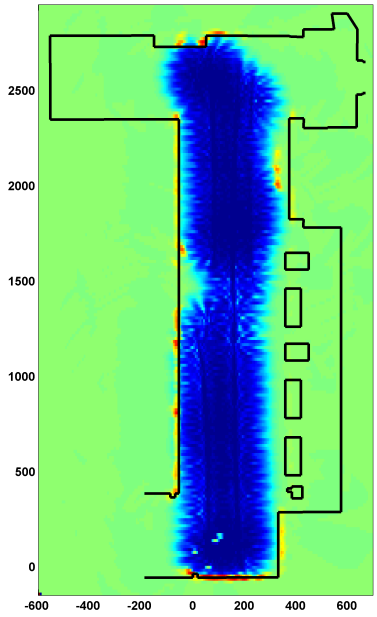

Figure A.36: Map of the corridor by the fuzzy method

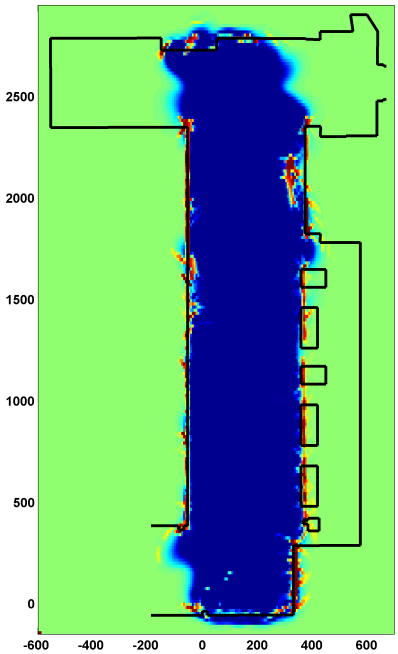

Figure A.37: Map of the corridor by the antonyms-based method