Optimal Education and Pensions in an Endogenous Growth Model☆

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Abstract

In OLG economies with life-cycle saving and exogenous growth, competitive equilibria in general fail to achieve optimality because individuals accumulate amounts of physical capital that differ from the one that maximizes welfare along a balanced growth path (the Golden Rule). With human capital, a second potential source of departure from optimality arises, related to education decisions. We propose to recover the Golden Rule of physical and also human capital accumulation. We characterize the optimal policy to decentralize the Golden Rule balanced growth path when there are no constraints for individuals to finance their education investments, and show that it involves education taxes. Also, when the government subsidizes the repayment of education loans, optimal pensions are positive.

Key words: endogenous growth, human capital, intergenerational transfers, education policy
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1. Introduction

The original approach to endogenous growth theory (Romer (1986), Lucas (1988), Rebelo (1991)) is based on the idea that returns to some capital goods, such as the stock of human capital resulting from education, do not diminish as economies develop. Knowledge spillovers and education externalities are some of the mechanisms that help sustain the returns to the accumulation of capital. In turn, the presence of a positive externality implies that a Pigouvian subsidy is required to attain optimality. Within growth theory, the overlapping generations literature has specifically focused on the existence of intergenerational externalities (see, among others, Azariadis and Drazen (1990), Caballé (1995), Boldrin and Montes (2005), Docquier et al. (2007)). These are a consequence of children inheriting a portion of the human capital of their parents, or of the spillovers generated by the aggregate stock of human capital. Because individuals ignore the effect of their educational decision on future human capital, they underinvest in education. In overlapping generations economies (OLG hereafter), this source of departure from optimality adds to the potential discrepancy between laissez-faire and optimal physical capital accumulation well known since Diamond (1965). As a result, two policy instruments are required to attain optimality, typically education subsidies and intergenerational transfers.

The persistence in the result that education should be subsidized in a growth context masks the fact that it hinges on the optimality criterion adopted, to wit, the maximization of a discounted sum of utilities defined over consumption levels. In the case of infinitely lived individuals, they are assumed to discount future utility, so there is no difference between individual and social objectives. In an OLG economy with labour productivity growth that translates into increasing consumption levels, the planner cannot maximize individual utility along a balanced growth path simply because it grows without limit. The social planner is then assumed to maximize the sum of present and future utilities of consumption per unit of natural labour, discounting the latter at an arbitrary discount rate.

In this paper we assume, on the contrary, that the social planner purposively wants to treat all generations alike, while being respectful with individual preferences. One way to do this is to maximize a utility function whose arguments are consumption levels per unit of *efficient* labour. In a three overlapping generation economy where human capital accumulation is the engine of growth, we search for the balanced growth path that maximizes this notion of lifetime welfare subject to the constraint that the welfare of the representative individual of every other generation is fixed at the same level. This optimality criterion would appear as the counterpart in the current framework of the (two-part) Golden Rule (Diamond, 1965, Samuelson, 1968).

As will be made clear, embracing this criterion implies that it is no longer wrong for individuals to ignore the (positive) effect of their decisions on the human capital of others. What is now wrong from the point of view of the social planner is for individuals to ignore the effect of their decisions on the growth rate of the economy. This effect can generally be positive or negative, but it is definitely negative when the individual faces the Golden Rule wage and interest rates. Thus, an education *tax* is required to maintain the economy on its optimal balanced growth path. Furthermore, when the government subsidises the repayment of education loans, the optimal policy also involves *positive* pensions.

In our model, like in Boldrin and Montes (2005) and Docquier et al. (2007), agents are born with an endowment of knowledge equal to the human capital of their parents, and borrow when young in order to invest in education. Credit markets are perfect. Inherited and acquired human capital interact to produce human capital of the middle-aged. As middle-aged, agents work, pay back the loan, save and pay taxes or receive subsidies. Finally, as old-aged, they consume and pay taxes or receive subsidies. We use this model to identify the optimum, compare it to the laissez faire and identify the optimal policy. The objective of the social planner is to maximize a welfare index defined in terms of consumption per unit of efficient labour, and her decision variables are also defined in terms of output per unit of efficient labour.

The Golden Rule thus obtained consists of five conditions that need to be satisfied simultaneously. One of the conditions of the optimal allocation is that the marginal product of physical capital (per unit of efficient labour) equals the (endogenous) growth rate of the economy. This is like in models with exogenous productivity growth (Buiter, 1979). As far as human capital accumulation is concerned, a condition equating marginal benefits and marginal costs of investing in education is obtained. Unlike the case of exogenous growth models, where laissez-faire and socially optimal physical capital accumulation can coincide by chance, we show that, in our framework, the laissez faire with perfect credit markets cannot possibly attain the Golden Rule. Individuals will always over or under invest either in physical capital, human capital, or both. To be more precise, as illustrated in Table 1 below, when the laissez-faire accumulation of physical capital is smaller than the optimal one, the laissez-faire expenditure in education can either exceed or fall short of the optimal one. However, a laissez-faire accumulation of physical capital greater than or equal to the optimal one can only coexist with a too large expenditure in education. As a consequence, if the laissez-faire amount of physical capital coincides with the socially optimal level, there will be *over-accumulation* of human capital.

Therefore, as stated above, in order to attain the Golden Rule, education should be taxed. The reason is that individuals choose their human capital investments accounting only for the effects on their earnings and loan repayment costs, not on the growth rate of the economy. If faced with the optimal (i.e., Golden Rule) factor prices in the laissez faire, they would ignore the costs associated with maintaining these factor prices at their Golden Rule level as human capital increases due to their investments. Under these circumstances, they would over-invest in education. For this reason, a tax is required to maintain the economy on the optimal balanced growth path.

The rest of the paper is organized as follows. Sections 2 and 3 respectively present the model and the social optimum. Section 4 derives the decentralized market equilibrium in the presence of government. The tax instruments available are lump-sum taxes on both the working and the retired population and education subsidies. The laissez-faire can be easily characterized by setting the tax parameters equal to zero. Section 5 compares the laissez-faire balanced growth path with the Golden Rule. Finally, Section 6 characterizes the optimal policy and Section 7 concludes.

2. The Model

The basic framework of analysis is the overlapping generations model with both human and physical capital developed in Boldrin and Montes (2005) and Docquier et al. (2007). At period t, L_{t+1} individuals are born. They coexist with L_t middle-aged and L_{t-1} old-aged. Population grows at the exogenous rate n so that $L_t = (1+n)L_{t-1}$ with $n > -1$. Agents are born with an endowment of basic "knowledge". This knowledge is assumed to be equal to the level of human capital of their parents h_{t-1} and is measured in units of efficient labour per unit of natural labour. Human capital in period t is produced out of the amount of output invested in education d_{t-1} and basic knowledge h_{t-1} according to the production function $h_t = E(d_{t-1}, h_{t-1})$. Assuming constant returns to scale, the production of human capital can be written in intensive terms as $h_t/h_{t-1} = e(\tilde{d}_{t-1})$, where $e(.)$ satisfies the Inada conditions and $\tilde{d}_{t-1} = d_{t-1}/h_{t-1}$ is the amount of output devoted to education per unit of inherited human capital.

A single good Y_t is produced by means of physical capital K_t and human capital H_t, according to a constant returns to scale production function $Y_t = F(K_t, H_t)$. As explained below, only the middle-aged work and they inelastically supply one unit of natural labour, so that $H_t = h_t L_t$. Physical capital is assumed to fully depreciate each period. If we define $k_t = K_t/L_t$ as the physical capital per unit of natural labour ratio and $\tilde{k}_t = K_t/H_t = k_t/h_t$ as the physical capital per unit of efficient labour ratio, the technology can be described as $Y_t/H_t = f(\tilde{k}_t)$, where $f(.)$ also satisfies the Inada conditions.

The lifetime utility function of an individual born at period $t-1$ is

$$U_t = U(c_t^m, c_{t+1}^o), \tag{1}$$

where c_t^m and c_{t+1}^o denote her consumption levels as middle-aged and old-aged, respectively. This utility function is assumed to be strictly quasi-concave and homogeneous of degree $j > 0$. The reason why we only impose strict quasi-concavity instead of strict concavity of the individual utility function is that, as it will be made clearer shortly, we are only interested in *ordinal* preferences.

Total output produced in period t, $F(K_t, H_t)$, can be devoted to consumption, $c_t^m L_t + c_t^o L_{t-1}$, investment in education (or human capital), $d_t L_{t+1}$, and investment in physical capital, K_{t+1}. Thus, the aggregate feasibility constraint writes

$$F(K_t, H_t) = c_t^m L_t + c_t^o L_{t-1} + d_t L_{t+1} + K_{t+1} \tag{2}$$

or, expressed in units of natural labour:

$$h_t f(k_t/h_t) = c_t^m + \frac{c_t^o}{1+n} + (1+n)d_t + (1+n)k_{t+1} \tag{3}$$

Alternatively, we can divide (3) by h_t, which is given at time t, and obtain the aggregate feasibility constraint in period t measured in terms of output per unit of efficient labour:

$$f(\tilde{k}_t) = \tilde{c}_t^m + \frac{\tilde{c}_t^o}{e(\tilde{d}_{t-1})(1+n)} + (1+n)\tilde{d}_t + e(\tilde{d}_t)(1+n)\tilde{k}_{t+1} \tag{4}$$

where $\tilde{c}_t^m = c_t^m/h_t$ and $\tilde{c}_t^o = c_t^o/h_{t-1}$ denote respectively consumption when middle-aged and consumption when old-aged per unit of efficient labour.[1] Note that $h_{t+1}/h_t = e(\tilde{d}_t) = 1 + g_{t+1}$, where g_{t+1} is the growth rate of productivity from period t to period $t+1$.

Along a balanced growth path, all variables expressed in terms of output per unit of natural labour are growing at rate g. In consequence, all variables expressed in terms of output per unit of efficient labour remain constant: $\tilde{c}_t^m = \tilde{c}_{t+1}^m = \tilde{c}^m$, $\tilde{c}_t^o = \tilde{c}_{t+1}^o = \tilde{c}^o$, $\tilde{k}_t = \tilde{k}_{t+1} = \tilde{k}$ and $\tilde{d}_t = \tilde{d}_{t+1} = \tilde{d}$.

[1]Note that $c_t^m L_t$ and $c_t^o L_{t-1}$ are expressed in units of output. Since middle-aged individuals supply one unit of natural labour, c_t^m and c_t^o are expressed in units of output per unit of *natural* labour. The interpretation of $\tilde{c}_t^m$ and $\tilde{c}_t^o$ in terms of units of output per unit of *efficient* labour follows naturally.

3. The Social Optimum: the Golden Rule in the presence of Endogenous Growth

In the presence of productivity growth that translates into consumption growth (as is, of course, the case along a balanced growth path), consumption levels will grow without limit. It is for this reason that, in order to characterize the balanced growth path, consumptions (and all the other variables) have to be expressed in terms of output per unit of efficient labour. As argued in the introduction, an obvious consequence of c_t^m and c_{t+1}^o growing to infinity is that a social planner will be unable to choose the consumption levels that maximize $U_t = U(c_t^m, c_{t+1}^o)$ along a balanced growth path, because U_t tends to infinity. The standard way to sidestep this problem is to assume that the planner maximizes a discounted sum of utilities and to search for the sequence of consumptions leading to the optimal balanced growth path.

Assume, on the contrary, that the social planner wants to treat all generations equally while being respectful with individual preferences. One possibility is for her to consider a valuation function defined over $\tilde{c}_t^m$ and $\tilde{c}_{t+1}^o$. Since the utility function (1) is homogeneous, we can write:

$$\tilde{U}_t = U(\tilde{c}_t^m, \tilde{c}_{t+1}^o) = U\left(c_t^m/h_t, c_{t+1}^o/h_t\right) = (1/h_t^j)U(c_t^m, c_{t+1}^o) = (1/h_t^j)U_t \tag{5}$$

Thus, a "new" utility function is obtained by means of a monotonic transformation of the first one, this ensuring that *ordinal* preferences are respected. The arguments of this utility function are measured in terms of consumption per unit of efficient rather than natural labour. Notice that (5) and (1) have the same functional form, and are both homogeneous of degree j. Also, the slope, curvature and higher derivatives of indifference curves in $(\tilde{c}_t^m, \tilde{c}_{t+1}^o)$ space are the same as those of the corresponding indifference curves in (c_t^m, c_{t+1}^o) space.

We can now posit that the social planner's objective is to choose the balanced growth path that maximizes the welfare of a representative individual, as measured by (5), subject to the constraint that everyone attains the same utility level, $\tilde{U} = U(\tilde{c}^m, \tilde{c}^o)$. We adopt this approach, which is reminiscent of Diamond (1965)'s original treatment of the Golden Rule in an OLG framework with productive capital.[2] Note that, as shown by (5), although both U_t and h_t^j tend to infinity, the ratio $\tilde{U} = (1/h_t^j)U_t$ is well defined along a balanced growth path. Then, the social planner will choose $(\tilde{c}^m, \tilde{c}^o, \tilde{k}, \tilde{d})$ that maximize $U(\tilde{c}^m, \tilde{c}^o)$ subject to the balanced growth path version of (4) and the technological relationship $1 + g = e(\tilde{d})$. We obtain, from the first order conditions:

$$\frac{\partial U(\tilde{c}_*^m, \tilde{c}_*^o)/\partial \tilde{c}^m}{\partial U(\tilde{c}_*^m, \tilde{c}_*^o)/\partial \tilde{c}^o} = (1 + g_*)(1 + n) \tag{6}$$

$$f'(\tilde{k}_*) = (1 + g_*)(1 + n) \tag{7}$$

$$e'(\tilde{d}_*)\left(\frac{\tilde{c}_*^o}{\left[(1 + g_*)(1 + n)\right]^2} - \tilde{k}_*\right) = 1 \tag{8}$$

[2]Phelps' Golden Rule identifies the amount of physical capital that maximizes consumption per capita (Phelps, 1961). A wider view of the Golden Rule concept, suggested by Diamond (1965) and Samuelson (1968, 1975a and 1975b) in an OLG model without productivity growth, focuses on the resource allocation that maximizes the lifetime welfare of a representative individual subject to the constraint the everyone else's welfare is fixed at the same level. The resulting *two-part* Golden Rule encompasses both Phelps' Golden Rule and Samuelson(1958)'s biological interest rate.

$$\tilde{c}_*^m + \frac{\tilde{c}_*^o}{(1+g_*)(1+n)} = f(\tilde{k}_*) - (1+g_*)(1+n)\tilde{k}_* - (1+n)\tilde{d}_* \tag{9}$$

$$1 + g_* = e(\tilde{d}_*) \tag{10}$$

Definition 1. *The* Golden Rule *balanced growth path $(\tilde{c}_*^m, \tilde{c}_*^o, \tilde{k}_*, \tilde{d}_*)$ provides the maximum level of welfare $\tilde{U} = U(\tilde{c}^m, \tilde{c}^o)$ that can be achieved by a representative individual subject to the feasibility constraint and the additional constraint that everyone else attains the same level. It is characterized by expressions (6)-(10).*

The interpretation of these equations is simpler if we start from the exogenous productivity growth setting, that we obtain for a given g and (without loss of generality) $\tilde{d} = 0$. Then, the Golden Rule is characterized by (6), (7) and (9) with g given and $\tilde{d} = 0$ (see Buiter, 1979). Equation (6) is the equality of the marginal rate of substitution between second and third period consumptions and the counterpart in the current model of the so-called "biological" interest rate, i.e., the economy's growth rate. Equation (7) is the equality of the marginal product of physical capital (per unit of efficient labour) and the growth rate of labour measured in efficiency units (i.e., the sum of the rates at which the efficiency of labour and the natural units of labour respectively grow). Of course, if $g = 0$ we are back to Diamond's framework and we obtain the original two-part Golden Rule.

Turning now to the endogenous growth framework, (6) and (7) continue to hold with the same interpretation, with g_* being obtained from (10). Equation (8), however, requires a careful explanation. It points out that, along the optimal balanced growth path, the marginal benefit of an increase in the amount of output devoted to education (again per unit of efficient labour) must equal its marginal cost. This can be seen using the aggregate feasibility constraint (9). A rise in $\tilde{d}$ has a direct cost in terms of the third term in the right hand side of (9), as it reduces consumption possibilities by $(1 + n)$. It also has an indirect cost, given by $e'(.)(1 + n)\tilde{k}$ as a consequence of the effect of a rise in $\tilde{d}$ on the rate of growth g: indeed, the greater the productivity growth rate g, the greater the amount of output that must be devoted to investment in physical capital in order to keep $\tilde{k}$ constant. However, a rise in $\tilde{d}$ (and thus in g) also has a benefit, since a greater g implies a greater marginal rate of transformation between second and third period consumption in the LHS of (8). This amounts to an expansion of consumption possibilities that is captured by $e'(.)(1+n)\tilde{c}_*^o / \left[(1+g_*)(1+n)\right]^2$. At the optimum, the marginal benefit and the marginal costs must be equal:

$$(1+n)e'(\tilde{d}_*)\frac{\tilde{c}_*^o}{\left[(1+g_*)(1+n)\right]^2} = (1+n)e'(\tilde{d}_*)\tilde{k}_* + (1+n) \tag{11}$$

The terms involving $(1 + n)$ in (11) cancel out and (8) emerges.

For later use, (8) can be rewritten, using (7) and (9), as

$$e'(\tilde{d}_*)\left(f(\tilde{k}_*) - \tilde{k}_* f'(\tilde{k}_*) - \Lambda_*(\tilde{k}_*, \tilde{d}_*)\right) = f'(\tilde{k}_*) \tag{12}$$

with $\Lambda_*(\tilde{k}_*, \tilde{d}_*) = (1+g_*)(1+n)\tilde{k}_* + (1+n)\tilde{d}_* + \tilde{c}_*^m > 0$.

Notice that, in the exogenous growth framework, $\tilde{k}_*$ is univocally determined by (7) and the optimization problem can be solved sequentially. In contrast, with endogenous growth, (7) and (8) are not enough to determine $\tilde{k}_*$ and $\tilde{d}_*$ because (8) incorporates also $\tilde{c}^o_*$, which can only be obtained from (6) and (9). A sequential solution of the optimization problem is now impossible: all optimal variables are determined simultaneously.

To conclude this section, note that the definition of the social optimum that is posited in this paper differs in several ways from the standard one. First, the optimal allocation resulting from maximizing $\sum_{t=0}^{\infty} \gamma^t U(c^m_t, c^o_{t+1})$ depends upon the choice of a particular social discount factor γ. Second, the inter-temporal trajectory leading to the optimal balanced growth path depends on the initial conditions. Third, the optimum corresponding to the standard approach is contingent upon the particular degree of homogeneity of the utility function, and thus the precise cardinalization of preferences, (a point stressed in Del Rey and Lopez-Garcia, 2012). In contrast, when one adheres to the social planner's objective underlying Definition 1, the social optimum is independent of the choice of an arbitrary social discount rate and there is no need to choose a specific cardinalization of utility. Finally, the focus is on the choice of the best optimal balanced growth path, this implying that the role of the initial conditions is not an issue.

4. Decentralized Market Equilibrium with Government

In this section we analyse the behaviour of the economy in the presence of education subsidies and intergenerational transfers and characterize the equilibrium balanced growth path for given values of the policy parameters. The properties of the laissez-faire balanced growth path are discussed in section 5.

Since individuals not only decide about the allocation of their resources along their life cycle but also how much to invest in education, there are now two potential sources of divergence between socially optimal and individual choices. This is the reason why we posit that the government has two policy instruments at its disposal: education subsidies and intergenerational transfers from the middle-aged to the elderly. Among the different ways of tackling education subsidies, we choose to model them as subsidies to the repayment, in the second period of life, of the loans taken in the first one to pay for education. Let $z^m_t > 0$ [resp. < 0] be the lump-sum tax [transfer] the middle aged pay [receive], $z^o_t > 0$ [< 0] the lump-sum tax the old pay [the pension they receive] and let θ_t be the subsidy rate, all of them in period t. The laissez-faire equilibrium can then be retrieved by setting $z^m_t = z^o_t = \theta_t = 0$.[3]

Factor prices are determined under perfect competition by their marginal products, so that, if $1 + r_t$ and w_t are respectively the interest factor and the wage rate per unit of efficient labour,

$$\begin{aligned} 1 + r_t &= f'(\tilde{k}_t) && (13) \\ w_t &= f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t) && (14) \end{aligned}$$

Individuals choose, in their first period, the amount of education that maximizes their lifetime resources. They do so by borrowing any amount they wish in perfect credit markets. Concerning

[3]It is worth emphasizing that, in this paper, the government subsidises the repayment of education loans. Alternatively the government can directly subsidise expenditures in education in the first period. Main results concerning the optimal education subsidy are however robust to alternative designs of this policy. See below.

savings, they behave as pure life-cyclers, i.e., they save to transfer purchasing power from the second to the third period. Then, for an individual born at $t-1$, consumption when middle-aged and consumption when old can be written respectively

$$c_t^m = w_t h_t - (1+r_t)d_{t-1}(1-\theta_t) - z_t^m - s_t \tag{15}$$

$$c_{t+1}^o = (1+r_{t+1})s_t - z_{t+1}^o \tag{16}$$

where s_t are the savings of a middle-aged. Thus, the lifetime budget constraint of an individual born at period $t-1$ is:

$$c_t^m + \frac{c_{t+1}^o}{1+r_{t+1}} = w_t h_t - (1+r_t)d_{t-1}(1-\theta_t) - z_t^m - \frac{z_{t+1}^o}{1+r_{t+1}} \tag{17}$$

The first order conditions associated with the individual decision variables, d_{t-1}, c_t^m and c_{t+1}^o, are:

$$w_t e'(d_{t-1}/h_{t-1}) = (1+r_t)(1-\theta_t) \tag{18}$$

$$\frac{\partial U(c_t^m, c_{t+1}^o)/\partial c_t^m}{\partial U(c_t^m, c_{t+1}^o)/\partial c_{t+1}^o} = (1+r_{t+1}) \tag{19}$$

where use has been made of the homogeneity of degree one of the E function, i.e., $h_t = e(d_{t-1}/h_{t-1})h_{t-1}$. Equation (18) shows that the individual will invest in education up to the point where the marginal benefit in terms of second period earnings equals the marginal cost of investing in human capital allowing for subsidies. Rewriting (18) as $e'(\tilde{d}_{t-1}) = (1-\theta_t)\ (1+r(\tilde{k}_t))/w(\tilde{k}_t)$, this expression implicitly characterizes the optimal ratio $\tilde{d}_{t-1}$ as a function of $\tilde{k}_t$ and θ_t, i.e., $\tilde{d}_{t-1} = \phi(\tilde{k}_t, \theta_t)$. Since $e'' < 0$ it can readily be shown that the greater $\tilde{k}_t$ and θ_t are, the greater $\tilde{d}_{t-1}$ is.

The government finances education subsidies with revenues obtained from taxing the middle-aged and/or the old-aged:

$$z_t^m L_t + z_t^o L_{t-1} = \theta_t(1+r_t)d_{t-1}L_t \tag{20}$$

which plugged into (17) yields

$$c_t^m + \frac{c_{t+1}^o}{1+r_{t+1}} = \omega_t \tag{21}$$

where ω_t is the present value of the net lifetime income of an individual born at $t-1$:

$$\omega_t = w_t h_t - (1+r_t)d_{t-1}(1-\theta_t) - z_t^m - \frac{1+n}{1+r_{t+1}}[\theta_{t+1}(1+r_{t+1})d_t - z_{t+1}^m] \tag{22}$$

The homogeneity assumption on preferences implies that the c_{t+1}^o/c_t^m ratio is a function of r_{t+1} only. Substituted into the budget constraint (22) this allows us to write consumption in the second period as a fraction of lifetime income, $c_t^m = \pi(r_{t+1})\omega_t$. Equilibrium in the market for physical capital is achieved when the (physical) capital stock available in $t+1$, K_{t+1}, equals gross savings made by the middle-aged in t, $s_t L_t$, minus the amount of output devoted to human capital investment by the young in t, $(1+n)d_t L_t$, i.e., when

$$K_{t+1} = s_t L_t - (1+n)d_t L_t \tag{23}$$

Using (23), the budget constraints (15) and (16), the government budget constraint (20) and the equilibrium factor prices (13) and (14), one can obtain the feasibility constraint (3).

Since $s_t = w_t h_t - c_t^m - (1 + r_t)d_{t-1}(1 - \theta_t) - z_t^m$, equilibrium condition (23) can be written in terms of output per unit of efficient labour:

$$\tilde{k}_{t+1} = \frac{(1 - \pi(r_{t+1}))\,\tilde{\omega}_t}{e(\phi(\tilde{k}_{t+1}, \theta_{t+1}))(1 + n)} - \frac{\tilde{z}^m_{t+1}}{1 + r_{t+1}} - \frac{(1 - \theta_{t+1})\,\phi(\tilde{k}_{t+1}, \theta_{t+1})}{e(\phi(\tilde{k}_{t+1}, \theta_{t+1}))} \tag{24}$$

where $\tilde{\omega}_t = \omega_t / h_t$ and use has been made of the fact that $d_t = \phi(\tilde{k}_{t+1}, \theta_{t+1})h_t$.

This expression implicitly provides $\tilde{k}_{t+1}$ as a function $\Psi(\tilde{k}_t; \tilde{z}^m_t, \tilde{z}^m_{t+1}, \theta_t, \theta_{t+1})$. Along a balanced growth path, we can delete the time subscripts and write $\tilde{k} = \Psi(\tilde{k}; \tilde{z}^m, \theta)$. An equilibrium ratio of physical capital to labour in efficiency units along a balanced growth path in the presence of government intervention, $\tilde{k}_G$, will then be a fixed point of the Ψ function, i.e., $\tilde{k}_G = \Psi(\tilde{k}_G; \tilde{z}^m, \theta)$. Such an equilibrium will be locally stable provided that $0 < \partial\Psi(\tilde{k}_G; \tilde{z}^m, \theta)/\partial\tilde{k} < 1$. In what follows, we will focus on situations where the equilibrium is unique and stable so that the relationship between $\tilde{k}$ and the tax parameters can be written, with an obvious notation, as

$$\tilde{k} = \tilde{k}(\tilde{z}^m, \theta) \tag{25}$$

We can now turn to the determination of $\tilde{d}$ or, what is the same, the growth rate g. The amount of output devoted to education per unit of inherited human capital along a balanced growth path will be governed by the relationship arising from the education decision (18), i.e., $\tilde{d} = \phi(\tilde{k}, \theta)$. Using (25) we can write $\tilde{d} = \phi\left(\tilde{k}(\tilde{z}^m, \theta), \theta\right)$ or, for short,

$$\tilde{d} = \tilde{d}(\tilde{z}^m, \theta) \tag{26}$$

Letting g_G be the growth rate of any variable expressed in terms of output per unit of natural labour, since $1 + g = e(\tilde{d})$, we have $1 + g_G = e\left(\phi(\tilde{k}_G, \theta)\right)$. Finally, the growth rate of all variables expressed in absolute terms (physical capital, human capital and output) is $(1 + g_G)(1 + n)$. This follows from writing H [K] as hL [kL] and observing that h [k] grows at rate g_G while L grows at rate n.

Summarizing, the balanced growth path in the presence of government intervention is characterized by (25) and (26) satisfying

$$\frac{\partial U(\tilde{c}^m, \tilde{c}^o)/\partial\tilde{c}^m}{\partial U(\tilde{c}^m, \tilde{c}^o)/\partial\tilde{c}^o} = (1 + r) \tag{27}$$

$$we'(\tilde{d}) = (1 + r)(1 - \theta) \tag{28}$$

$$\tilde{c}^m + \frac{\tilde{c}^o}{1 + r} = \tilde{\omega} \tag{29}$$

where $\tilde{\omega} \equiv \omega/h$ is given by

$$\tilde{\omega} = w - \frac{(1 + r)\tilde{d}}{(1 + g)} + \left[\frac{(1 + r) - (1 + g)(1 + n)}{1 + g}\right]\theta\tilde{d} - \left[\frac{(1 + r) - (1 + g)(1 + n)}{1 + r}\right]\tilde{z}^m \tag{30}$$

and represents the present value of lifetime resources expressed in terms of output per unit of efficient labour.

	$\tilde{d}_{LF} > \tilde{d}_*$	$\tilde{d}_{LF} \leqslant \tilde{d}_*$
$\tilde{k}_{LF} \geqslant \tilde{k}_*$	feasible	not feasible
$\tilde{k}_{LF} < \tilde{k}_*$	feasible	feasible

Table 1: Different laissez-faire balanced growth paths

5. Laissez-faire and Golden Rule Optimum

We start with the characterization of the laissez-faire balanced growth path. The values of $\tilde{k}$ and $\tilde{d}$ along the laissez-faire balanced growth path, $\tilde{k}_{LF}$ and $\tilde{d}_{LF}$, can be readily obtained evaluating (25) and (26) at $\theta = \tilde{z}^m = 0$. It is well known that, in exogenous growth models like Diamond (1965), $\tilde{k}_{LF}$ may either be greater or less than $\tilde{k}_*$ (and both can, by chance, coincide). A natural conjecture is that this might be possible in the current framework as well, and, by symmetry, that $\tilde{d}_{LF}$ might also be higher/lower than (or equal to) $\tilde{d}_*$. This conjecture translates into the four possibilities associated with the four cells in Table 1. Note, however, that one of them is not feasible. To see this, consider first a balanced growth path with $\tilde{k}_{LF} \geqslant \tilde{k}_*$. This implies that $f'(\tilde{k}_{LF}) \leqslant f'(\tilde{k}_*)$ and $f(\tilde{k}_{LF}) - \tilde{k}_{LF} f'(\tilde{k}_{LF}) \geqslant f(\tilde{k}_*) - \tilde{k}_* f'(\tilde{k}_*)$. We can now compare, letting $\Sigma(\tilde{k}) = f'(\tilde{k})/(f(\tilde{k}) - \tilde{k} f'(\tilde{k}))$, equations (12) and (28) with $\theta = 0$ as follows:

$$e'(\tilde{d}_{LF}) = \Sigma(\tilde{k}_{LF}) \leqslant \Sigma(\tilde{k}_*) < \Sigma(\tilde{k}_*) + \frac{\Lambda_* e'(\tilde{d}_*)}{f(\tilde{k}_*) - \tilde{k}_* f'(\tilde{k}_*)} = e'(\tilde{d}_*) \tag{31}$$

and, therefore, $\tilde{d}_{LF} > \tilde{d}_*$ by the concavity of the production function of human capital. Hence, $\tilde{k}_{LF} \geqslant \tilde{k}_*$ is not compatible with $\tilde{d}_{LF} \leqslant \tilde{d}_*$, implying that cell a_{12} in Table 1 is not feasible. It also follows that, even if it happens that $\tilde{k}_{LF} = \tilde{k}_*$, it will be the case that $\tilde{d}_{LF} > \tilde{d}_*$. Consequently, even if the accumulation of physical capital is optimal, the accumulation of human capital will not.

If we now consider the case where $\tilde{k}_{LF} < \tilde{k}_*$, so that $\Sigma(\tilde{k}_{LF}) > \Sigma(\tilde{k}_*)$, following the same steps as before, we get

$$e'(\tilde{d}_{LF}) = \Sigma(\tilde{k}_{LF}) \lesseqgtr \Sigma(\tilde{k}_*) + \frac{\Lambda_* e'(\tilde{d}_*)}{f(\tilde{k}_*) - \tilde{k}_* f'(\tilde{k}_*)} = e'(\tilde{d}_*) \tag{32}$$

from which $\tilde{d}_{LF}$ can be greater, equal or lesser than $\tilde{d}_*$, as displayed in Table 1.[4] To show that the laissez faire will never be able to reach the optimal allocation, it suffices to show that, even if the accumulation of human capital is optimal, the accumulation of physical capital will not. Indeed, suppose that $\tilde{d}_{LF} = \tilde{d}_*$, then

$$\Sigma(\tilde{k}_{LF}) = e'(\tilde{d}_{LF}) = e'(\tilde{d}_*) = \Sigma(\tilde{k}_*) + \frac{\Lambda_* e'(\tilde{d}_*)}{f(\tilde{k}_*) - \tilde{k}_* f'(\tilde{k}_*)} > \Sigma(\tilde{k}_*) \tag{33}$$

and now it is the concavity of the production function for physical output in intensive terms that necessarily implies that $\tilde{k}_{LF} < \tilde{k}_*$. Thus we can state the following result.

[4]The feasibility of cells a_{11}, a_{21}, and a_{22} in Table 1 can be verified by means of the triple Cobb-Douglas example, i.e., using Cobb-Douglas production functions for output and human capital and logarithmic utility (see Del Rey and Lopez-Garcia (2009), Appendix C).

Proposition 1. *The laissez-faire equilibrium cannot possibly support the Golden Rule balanced growth path.*

The intuition for this negative result, fully developed in the next section, is naturally related to the divergence of objectives of the individual and the social planner. This divergence translates into different structures of costs and benefits. On the one hand, the marginal benefit for the social planner from an increase in $\tilde{d}$ is the enhanced *consumption* possibilities, captured by the term involving $\tilde{c}^o_*$ in (8), that stem from a rise in g. The marginal costs of raising $\tilde{d}$ include the direct cost (reduced consumption possibilities) and the indirect cost resulting from the rise in g (requirement to increase investment in physical capital in order to keep $\tilde{k}$ constant). These are captured by the terms not involving $\tilde{c}^o_*$ in (8). On the other hand, from (28) with $\theta = 0$, in the laissez faire the marginal benefit of a unit of output invested by the individual in education in the first period is the increase in second-period *earnings*, and the marginal cost is the interest factor to be paid. Clearly, there is no reason why these costs and benefits should coincide.

The previous analysis also shows that it is possible that $\tilde{d}_{LF} > \tilde{d}_*$ (see cells a_{11} and a_{21} in Table 1). As a consequence, the laissez faire productivity growth rate g_{LF} can be *larger* than the optimal growth rate g_*. This is certainly also a possibility in the overlapping generations framework with perfect credit markets considered by Docquier et al. (2007, p. 368), where the social planner maximizes a discounted sum of utilities. However, it differs from the result in standard models of endogenous growth with infinitely lived individuals, where laissez-faire growth rates are typically lower than optimal ones (Romer (1986), Lucas (1988), Barro (1990)). The fact that $\tilde{d}_{LF} > \tilde{d}_*$ is in particular the only possibility when $\tilde{k}_{LF} = \tilde{k}_*$ allows us to anticipate that the optimal public policy may involve a tax on education. We now characterize in full the public policy that complies with the social objective posited in Definition 1.

6. Optimal Public Policy

We can now discuss the optimal policy, i.e., the orthopaedics that allow to convert the laissez-faire equilibrium into the optimal Golden Rule balanced growth path. First, we deal with the optimal education subsidy and then investigate the direction of the optimal intergenerational transfer (i.e., from/to the middle-aged to/from the elderly). The optimality condition for the choice of $\tilde{d}$ can be stated as (12). It can now be rewritten in a way that can be directly compared to the condition associated with the individual's choice of education in the presence of government, (28):

$$e'(\tilde{d}_*)\left(f(\tilde{k}_*) - \tilde{k}_* f'(\tilde{k}_*)\right) = f'(\tilde{k}_*)\left(1 + \frac{e'(\tilde{d}_*)\Lambda_*(\tilde{k}_*, \tilde{d}_*)}{f'(\tilde{k}_*)}\right) \tag{34}$$

with $\Lambda_*(\tilde{k}_*, \tilde{d}_*) = (1 + g_*)(1 + n)\tilde{k}_* + (1 + n)\tilde{d}_* + \tilde{c}^m_* > 0$. Mere comparison of (28) and (34) shows that the optimal value of the tax parameter addressed to education, θ_*, is negative:

$$\theta_* = -\frac{e'(\tilde{d}_*)\Lambda_*(\tilde{k}_*, \tilde{d}_*)}{f'(\tilde{k}_*)} < 0 \tag{35}$$

This allows to state the following proposition.

Proposition 2. *Decentralizing the Golden Rule balanced growth path entails $\theta_* < 0$, i.e., an education tax.*

As we mentioned before, the intuition for this result can be easily grasped when one compares the structures of marginal benefits and marginal costs underlying the decisions of the social planner and the individual. Comparing (28) with $\theta = 0$ and (34) it is clear that, when choosing how much to invest in education, if the individual were confronted with the (optimal) wage and interest rates, $f(\tilde{k}_*) - \tilde{k}_* f'(\tilde{k}_*)$ and $f'(\tilde{k}_*)$, she would fail to take into account the terms collected by $\Lambda_*(\tilde{k}_*, \tilde{d}_*)$. In other words, she would neglect the investment in physical capital (per unit of efficient labour) required to keep $\tilde{k}_*$ constant $((1 + g_*)(1 + n)\tilde{k}_*)$ when g increases, its counterpart, referred to $\tilde{d}_*$ $((1 + n)\tilde{k}_*)$, as well as the total consumption of the middle aged (per unit of efficient labour) necessary to keep $\tilde{c}^m_*$ constant. In these circumstances, as the individual does not account for these costs, she over-invests in education and a tax is required to attain the Golden Rule.[5]

This is in contrast with the results from the standard approach to endogenous growth theory, where the social planner maximizes an infinite discounted sum of utilities (with consumptions measured in output per unit of natural labour). In that framework, comparing the individual and the planner's decisions shows that, when choosing the education level d_t, the individual neglects the effect of this decision on next generation's welfare through the inherited human capital. Due to this positive intergenerational externality, education subsidies are positive along the optimal balanced growth path. In our framework, human capital continues to be the engine of growth, but both the objective and the decision variables of the social planner are different. The objective is now to maximize a utility function whose arguments are consumptions per unit of efficient labour, and the welfare level is the same for all generations. The decision variables are measured in terms of output per unit of efficient labour. Individuals can now under or over-invest with respect to the optimal investment in education, i.e., $\tilde{d}_{LF}$ can be larger or less than $\tilde{d}_*$ (see Table 1). But, in any case, if confronted with the *optimal* wage and interest rates, they will fail to account for the aforementioned indirect costs, supported by all generations alike. This leads to over-investment in education and a corrective tax emerges as the natural way to deal with this excessive amount of education investment.

Note that Proposition 2 does not depend on the specific way education subsidies are designed, i.e., subsidies to the repayment of the loans. As we have seen, this result is directly driven from the optimality conditions and makes no use of the government budget constraint. Thus, as noted previously (see footnote 3) the fact that education should be taxed is independent of the way we design government intervention in education investments.[6]

Key to our results is, however, the assumption of perfect credit markets where individuals can borrow to finance their education. Boldrin and Montes (2005) consider an OLG economy

[5]Nevertheless, it is worth mentioning that, when the starting point is not the laissez-faire but an *arbitrary* balanced growth path, education subsidies can be welfare improving (see Del Rey and Lopez-Garcia (2009), section 5).

[6]This means that Proposition 2 holds when the government directly subsidizes education expenditures in the first period. The counterpart of the government budget constraint (20) is now: $z^m_t L_t + z^o_t L_{t-1} = \sigma_t d_t L_{t+1}$ where σ_t is the subsidy rate to the education expenditures of the L_{t+1} members of the younger generation. This has obvious consequences for the individual budget constraint and the equilibrium condition in the market for physical capital, but does not affect the result that, along the Golden Rule balanced growth path, the optimal education subsidy is negative, i.e., it is a tax.

essentially similar to the one set up in section 2 but where credit markets to finance education investments are missing. They show that public financing of education and public pensions can be designed to generate an intergenerational transfer scheme that decentralizes the complete market allocation. Such a government supported scheme would give rise to the same balanced growth path as the laissez-faire with credit markets characterized by expressions (27)-(29) with $\theta = \tilde{z}^m = 0$. Proposition 1 however states that this equilibrium balanced growth path does not provide the highest welfare in the Golden Rule sense of Definition 1.

Coming back to our framework with credit markets, we can now turn to the issue of the direction of the optimal intergenerational transfers, and, in particular, whether they adopt the form of positive or negative lump-sum taxes on the older generation.

The equilibrium condition in the market for physical capital, (24), can be rewritten along a balanced growth path as:

$$(1+g)(1+n)\tilde{k} = w - \frac{1+r}{1+g}\tilde{d}(1-\theta) - \tilde{z}^m - \tilde{c}^m - (1+n)\tilde{d} \tag{36}$$

On the other hand, from the individual budget constraint (29), and using (30) and (36):

$$\frac{\tilde{c}^o}{(1+r)^2} = \frac{1}{1+r}\left((1+g)(1+n)\tilde{k} + (1+n)(1-\theta)\tilde{d} + \tilde{z}^m \frac{(1+g)(1+n)}{(1+r)}\right) \tag{37}$$

Evaluating (37) at the Golden Rule, i.e. for the optimal tax parameters $\tilde{z}^m_*$ and θ_*, using (8), and making use of the fact that $1 + g_* = e(\tilde{d}_*)$, we obtain:

$$\frac{e'(\tilde{d}_*)}{e(\tilde{d}_*)/\tilde{d}_*} - 1 = e'(\tilde{d}_*)\left(\frac{\tilde{d}_*\theta_*}{1+g_*} - \frac{\tilde{z}^m_*}{(1+g_*)(1+n)}\right) < 0 \tag{38}$$

By the concavity of $e(.)$, $e'(\tilde{d}_*) < e(\tilde{d}_*)/\tilde{d}_*$ so that the left hand side of this expression is negative. It then follows that the expression in parenthesis on the right hand side of (38) is also negative. Writing the government budget constraint (20) in terms of output per unit of efficient labour yields $\tilde{z}^m + \tilde{z}^o/[(1+g)(1+n)] = \theta(1+r)\tilde{d}/(1+g)$. Then, at the Golden Rule:

$$\frac{\tilde{z}^o_*}{[(1+g_*)(1+n)]^2} = \frac{\tilde{d}_*\theta_*}{1+g_*} - \frac{\tilde{z}^m_*}{(1+g_*)(1+n)} \tag{39}$$

and, from (38), $\tilde{z}^o_*$ must be negative, i.e., pensions must be positive. We then have:

Proposition 3. *When the government subsidizes the repayment of education loans, decentralizing the Golden Rule balanced growth path involves $\tilde{z}^o_* < 0$, i.e., positive pensions to the elderly.*

In contrast, when the government directly subsidizes education expenditures in the first period, the sign of $\tilde{z}^o_*$ cannot be determined in general.

7. Concluding comments

This paper has focused on the relationship among education subsidies, a scheme of intergenerational transfers, and welfare in a life-cycle growth model with both physical and human capital. The welfare objective has been taken to be the Golden Rule, i.e., the balanced growth path that maximizes the lifetime welfare of a representative individual subject to the feasibility constraint and the requirement that everyone else's welfare is fixed at the same level. This social objective has been characterized in the only sensible way in an endogenous growth framework, i.e., by considering a utility function whose arguments are individual consumptions measured in terms of output per unit of efficient (instead of natural) labour. We have shown that the laissez-faire equilibrium cannot possibly support the Golden Rule balanced growth path and we have identified the optimal policy. This consists of education taxes and, when the government subsidizes the repayment of student loans, pensions for the old-aged.

The result that education should be taxed may seem shocking at first. The intuition is rooted in the fact that the planner does not choose the optimal education investment but the optimal growth rate. This is in contrast with individual behaviour, which does not take into account the effect of educational decision on the rate of growth. We have shown that this effect is negative when the individual faces the Golden Rule wage and interest rates, since individuals ignore the costs associated with maintaining these factor prices at their Golden Rule level as human capital increases due to their investments. As a result, a corrective tax is required to maintain the economy on its optimal balanced growth path.

To conclude, it is clear that some of the assumptions underlying the model, particularly that of perfect credit markets, are not realistic and must somehow be relaxed. Indeed, policy conclusions may be quite sensitive not only to whether or not individuals face constraints when trying to borrow to finance their education investments but also to the reasons for these constraints. It seems fair to say that more research on this subject is warranted.

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