Available online at www.sciencedirect.com

SciVerse ScienceDirect

Journal of the Franklin Institute 350 (2013) 556–574

Journal of The Franklin Institute

www.elsevier.com/locate/jfranklin

Robust model predictive control of a Wiener-like system

J.L. Figueroa[a,c,1], S.I. Biagiola[a,c,*,1], M.P. Alvarez[b], L.R. Castro[b,c], O.E. Agamennoni[a,c,2]

[a]*Departamento de Ingeniería Eléctrica y de Computadoras, Universidad Nacional del Sur, Bahía Blanca, Argentina*
[b]*Departamento de Matemática, Universidad Nacional del Sur, Bahía Blanca, Argentina*
[c]*Instituto de Investigaciones en Ingeniería Eléctrica "Alfredo Desages", Argentina*

Received 23 April 2012; received in revised form 28 August 2012; accepted 18 December 2012
Available online 28 December 2012

Abstract

In this paper, a robust model predictive control for a Wiener-like system is presented. The proposed system consists of a lineal dynamic block represented by Laguerre or Kautz basis followed by a High Level Piecewise Linear function. The results are evaluated on the basis of a simulation of a distillation column.

1. Introduction

Although most of the contributions for controller design are based on a linear model of the process, typically industrial processes are inherently nonlinear. However, when the system is highly nonlinear and the operating point changes along a wide region, it is difficult to represent adequately a given system by means of a linear model. In such cases, there are very few controller design techniques that can be proven to stabilize processes in the presence of nonlinearities and constraints.

*Corresponding author at: Departamento de Ingeniería Eléctrica y de Computadoras, U.N.S., Avda. Alem 1253, (8000) Bahía Blanca, Argentina. Tel.: +54 291 4595101x3338; fax: +54 291 4595154.

E-mail addresses: figueroa@uns.edu.ar (J.L. Figueroa), biagiola@uns.edu.ar (S.I. Biagiola), palvarez@criba.edu.ar (M.P. Alvarez), lcastro@uns.edu.ar (L.R. Castro), oagamen@uns.edu.ar (O.E. Agamennoni).

[1]Also with CONICET.

[2]Also with CICpBA.

0016-0032/$32.00
http://dx.doi.org/10.1016/j.jfranklin.2012.12.016

For this reason, there has been much interest in nonlinear model-based control. A critical step in the application of these methods is the development of a suitable model of the process dynamics. One possible approach for modeling a dynamical system is to select a nonlinear black-box structure. Sjöberg et al. [1] described several black-box approaches for model development and they connected all of them in a common framework.

A special class of the so-called block oriented nonlinear models are those ‘‘cascade models’’ [2] in which a linear time invariant (LTI) dynamic block is preceded and/or followed by a static nonlinearity. The Wiener and Hammerstein models are special kinds of nonlinear systems where the nonlinear block is static and follows or is followed by a linear system, respectively. These models have applications in many engineering problems and therefore, identification of both Hammerstein and Wiener models has been an active research area for many years [3]. As regards applicability, these structures are present in a wide variety of fields. A detailed review on applications can be found in [3]. Among other practical uses of these cascade models, it can be mentioned many in the field of chemical engineering [4–12], communications [13,14], medicine [15] as well as biology [16–20].

Although several approaches can be found in the literature for nominal identification of these models there are not many developments to obtain uncertain models. It is a well-known fact that the high computational cost involved in the identification is inherent to the nature of both models [21]. Robust identification of both Wiener and Hammerstein structures is a subject under research, among the attempts for robust identification of Wiener models it can be mentioned [7,22–25], and for Hammerstein models [22,26–28]. A typical approach for robust identification is to define a set of possible models to represent all the process behaviors. One possible strategy concerning robust Wiener model identification is to concentrate the uncertainty in the nonlinear static block [23,29]. However, the validity of these results is limited to those cases in which the nonlinearity is invertible. In [30,25] no iterative algorithms for robust identification of Wiener and Hammerstein systems are presented. The adopted methodology allows to make a robust model in the sense that the overall data can be reproduced by the family of models obtained. In a recent work [31], an uncertain parametric Wiener-like model as the one described in Fig. 1 is treated. This is the modeling approach herein followed to represent the uncertain Wiener system. In this model, the static nonlinearity is represented by High Level Canonical Piecewise Linear (HLCPWL) functions [32–34], while the linear dynamic part consists of a finite number of Laguerre or Kautz filters [35,36]. In order to estimate the parameters of the HLCPWL functions, the uncertainty is described as a set of parameters,

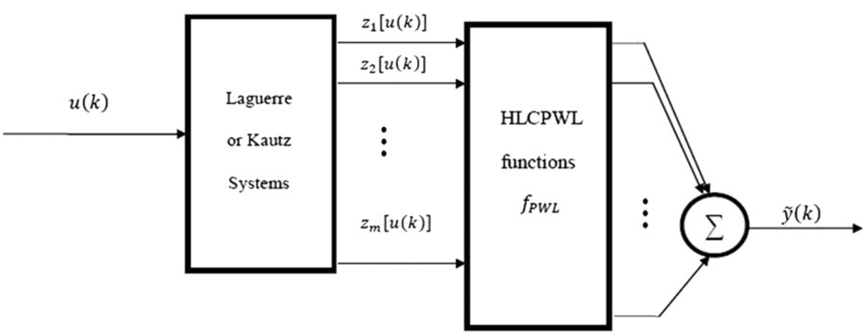

Fig. 1. Wiener-like model.

which are identified through the solution of an optimization problem. The approach herein followed can be connected with formulations made within the set membership theory framework (see, e.g. [37–40] for surveys on the topic).

The resulting methodology is robust since that the identified set of parameters is such that any of the collected data can be represented by at least one of the models in the set. This set of feasible parameters is obtained by solving a simple linear programming problem.

Once the block-structured model of the process is determined, the design of a suitable dedicated controller has to be dealt with. As regards the control of nonlinear systems, there are very few design techniques that can be proven to stabilize processes in the presence of nonlinearities and constraints. Model predictive control (MPC) is one of these techniques. MPC refers to a class of computer control algorithms that regulates the future behavior of a plant through the use of an explicit process model. At each control interval the MPC algorithm computes an open-loop sequence of manipulated variable adjustments in order to optimize future plant behavior. The first input in the optimal sequence is injected into the plant, and the entire optimization is repeated at subsequent control intervals [41].

With the introduction of a dynamic nonlinear model within the NMPC algorithm, the complexity of the predictive control problem increases significantly. This issue has been thoroughly dealt with in the review papers by Bequette [42] and Henson [43], where they presented the various approaches for handling nonlinear systems via MPC.

In particular, Wiener models have a special structure that facilitates their application to NMPC [6,44,29]. In these models, due to the static nature of the nonlinearities, they can be removed from the control problem, which allows solving the NMPC problem as a linear MPC one.

In a recent paper [25], a robust algorithm for MPC control of Wiener models was presented. In that work, the authors extended the ideas of [45] to Wiener models, by inverting the nonlinear gain. Nevertheless, this strategy is not applicable to the Wiener-like systems herein treated (Fig. 1) due to the dimension of the nonlinear PWL functions that makes impossible the inversion. To overcome this problem, we follow the ideas in [22], where the nonlinearities are transformed into polytopic descriptions. In such a way, the procedure enables the use of robust linear MPC techniques for controlling these Wiener-like structures, while convex optimization problem is retained.

The paper is organized as follows. In Section 2, the model structure is presented. The concepts and results about HLCPWL functions and Wiener modeling are briefly summarized and the approximation structures are described. The approach for identifying the HLCPWL mapping within the framework of set membership estimation theory is presented and the model structure is addressed. Section 3 presents the RMPC synthesis for such identified Wiener-like models. This is the main contribution of this paper: a robust controller design algorithm for MIMO Wiener-like model in the presence of uncertainty. In Section 4 a distillation column simulation model is presented to illustrate both the robust identification and control approaches. This process is an interesting benchmark due to its well-known nonlinear dynamics. This paper concludes with some final remarks in Section 5.

2. Wiener-like model

2.1. Model description

In this work, we focus on a particular and widely used type of block-oriented nonlinear models, the Wiener-like model, and we assume it has a parametric representation.

The structure of the model herein studied is shown in Fig. 1, where the linear block maps an input sequence $\{u(k)\} \in \Re^{N_u}$ to a sequence of intermediate signals $\{z(k)\} \in \Re^{N_z}$. The model output is $\hat{y}(k) = f_{pwl}(z(k)) \in \Re^{N_y}$. The linear part of model is represented by orthogonal bases [12,26]:

$$z_{i,j,p}(k) = B_{i,j,p}(q)u_j(k), \tag{1}$$

for $i = 1, \ldots, N_y$, $j = 1, \ldots, N_u$ and $p = 1, \ldots, N_z$; where q is the forward time operator, N_z is the number of terms in the orthonormal basis, u_j is the jth entry on the input vector and the $B_{i,j,p}(q)$ are the elements that relate the jth input to the ith output via the intermediate variables $z_{i,j,p}$. These bases are defined as

$$B_{i,j,0}(q) = \frac{(1-\xi_{i,j}^2)^{1/2}}{q-\xi_{i,j}} \tag{2}$$

and

$$B_{i,j,p}(q) = B_{i,j,p-1}(q)\left(\frac{1-\xi_{i,j}q}{q-\xi_{i,j}}\right), \quad p = 1, \ldots, m. \tag{3}$$

This basis allows to use the previous knowledge of the dominant modes of the systems, including them as parameters $\xi_{i,j}$.

To represent the nonlinear block, we use piecewise linear functions, these functions are defined on a rectangular compact domain S of the form

$$S \doteq \{x \in \Re^n : a_i \le x_i \le a_i + \delta ndiv; i = 1, 2, \ldots, n\}, \tag{4}$$

where $a_i \in \Re$, δ is the grid size and $ndiv \in \mathcal{Z}_+$ is the number of divisions associated with the x_i-axis. The domain S is partitioned by means of a simplicial boundary configuration H.

The space $PWL_H[S]$ of all continuous PWL mappings defined over the domain S partitioned with a simplicial boundary configuration H [46] is a linear vector space.

According to [34], any $f_{pwl} \in PWL_H[S]$ can be written as

$$f_{pwl}(x) = c^T \Lambda(x), \tag{5}$$

where c is the vector of parameters and Λ is the matrix of the basis functions defined on S.

Then, for the particular case we are dealing with, the dimension n stands for the number of filters (e.g. Laguerre filters) used in each input vector. In such a case, the nonlinear static block takes the form

$$\tilde{y}_i(k) = c_i^T \Lambda(z_i(k)), \tag{6}$$

for $i = 1, \ldots, N_y$, where $z_i(k)$ is the vector formed by the entries $z_{i,j,p}(k)$ (with $j = 1, \ldots, N_u$ and $p = 1, \ldots, N_z$) and c_i is the vector of unknown parameters of the nonlinear block which must be determined.

Note that different time constants for each model entry are considered. Moreover, the approach herein followed assumes the most general situation in which any input $u_i(k)$ can influence any linear block output $z_j(k)$. In the same way, it is assumed that any nonlinear block output can be influenced by any input $z_j(k)$ to this block.

Henceforth, it will be useful to formulate the model in a state-space form. For a particular input $u_j(k)$ and for any $i = 1, \ldots, N_y$, let us analyze the signal $z_{i,j}$. From the expression of the first base (2) we obtain

$$z_{i,j,0}(k+1)=\xi_{i,j}z_{i,j,0}(k)+(1-\xi_{i,j}^2)^{1/2}u_j(k). \tag{7}$$

Analogously, using Eq. (3) it results

$$\begin{aligned}z_{i,j,1}(k+1)&=\xi_{i,j}z_{i,j,1}(k)+z_{i,j,0}(k)-\xi_{i,j}z_{i,j,0}(k+1)\\&=\xi_{i,j}z_{i,j,1}(k)+z_{i,j,0}(k)-\xi_{i,j}(\xi_{i,j}z_{i,j,0}(k)+(1-\xi_{i,j}^2)^{1/2}u_j(k))\\&=\xi_{i,j}z_{i,j,1}(k)+(1-\xi_{i,j}^2)z_{i,j,0}(k)-\xi_{i,j}(1-\xi_{i,j}^2)^{1/2}u_j(k).\end{aligned} \tag{8}$$

In the same way,

$$\begin{aligned}z_{i,j,2}(k+1)&=\xi_{i,j}z_{i,j,2}(k)+z_{i,j,1}(k)-\xi_{i,j}z_{i,j,1}(k+1)\\&=\xi_{i,j}z_{i,j,2}(k)+(1-\xi_{i,j}^2)z_{i,j,1}(k)-\xi_{i,j}(1-\xi_{i,j}^2)z_{i,j,0}(k)+\xi_{i,j}^2(1-\xi_{i,j}^2)^{1/2}u_j(k).\end{aligned} \tag{9}$$

If we group these expression in a matricial form (see Appendix), we obtain

$$\mathbf{z}_{i,j}(k+1)=\mathbf{A}_{i,j}\mathbf{z}_{i,j}(k)+\mathbf{B}_{i,j}u_j(k). \tag{10}$$

Therefore, the following multivariable model is obtained:

$$\mathbf{x}(k+1)=\mathbf{A}\mathbf{x}(k)+\mathbf{B}u(k). \tag{11}$$

The vectors z_i can be obtained from the vector $\mathbf{x}$ by tacking the entries $z_{i,j,p}(k)$ for values $j=1,\dots,N_u$ and $p=1,\dots,N_z$, i.e.,

$$z_i(k)=C_i\mathbf{x}(k), \tag{12}$$

where the matrices C_i $(i=1,\dots,N_y)$ have ones in the appropriated places and zeros elsewhere.

2.2. Model identification

Provided an input–output data set from the plant, a nominal model of the process can be obtained; however, the aim is to perform a robust identification. An uncertain model can be described by defining a set of parameters $\mathcal{C}$ for the nonlinear static block, as follows:

$$\mathcal{C}=\{c : c^l\le c\le c^u\}, \tag{13}$$

where the inequalities apply entry by entry, i.e.,

$$c_{i,p}^l\le c_{i,p}\le c_{i,p}^u \tag{14}$$

for $i=1,\dots,N_y$ and $p=1,\dots,N_z$. In order to obtain an uncertain model, we must determine the parameter bounds c^l and c^u in such a way that every input data $u(k)$ can be mapped through the model to the corresponding $y(k)$.

Now, let us analyze this situation in order to compute the parameter bounds that satisfy this condition. This determination is based on the whole input/output data available.

Given the input datum $u(k)$, the linear block maps (at some specific time t) onto a signal $z_i(k)$ for $i=1,\dots,N_y$ by means of Eqs. (11)–(12). Subsequently, the PWL basis operates on $z_i(k)$ generating $\Lambda(z_i(k))$. Therefore, the estimated output $\tilde{y}_i(k)$ can be written in terms of the external input $u(k)$ and expressed as

$$\tilde{y}_i(k)=c_i^T\tilde{\Lambda}_i(u(k)). \tag{15}$$

Note that this output only depends on the input signal $u(k)$ and on some model parameters fixed a priori. In this way, and since the entries of $\lambda_i(u(k))$ are positive [34], it is possible to compute the bounds of the robust model as

$$\min_{c^l, c^u} \sum_p (c^u_{i,p} - c^l_{i,p}) \tag{16}$$

subject to

$$(c^l)^T \tilde{\Lambda}(u(k)) \leq y(k), \quad t = 1, \ldots, K, \tag{17}$$

$$(c^u)^T \tilde{\Lambda}(u(k)) \geq y(k), \quad t = 1, \ldots, K, \tag{18}$$

where K is the number of data measurement available.

Note that the proposed approach for the identification problem allows to transform it into a Linear Programming problem with convex feasible region. The number of optimization variables is twice the number of model parameters and the number of constrains is twice the number of the process data. Due to the suitable formulation of these problem, its solution is obtained in an efficient way.

3. Robust MPC for the Wiener-like model

The ‘‘multi-model’’ paradigm for RMPC was introduced in [45]. The underlying model is given as

$$x(k+1) = Ax(k) + Bu(k),\ y(k) = Cx(k),\ [A \ \ B] \in \Omega, \tag{19}$$

where $u(k) \in \Re^{n_u}$ is the control input, $x(k) \in \Re^{n_x}$ is the state of the plant, $y(k) \in \Re^{n_y}$ is the plant output, and Ω is a polytope that stands for the set of uncertainty

$$\Omega = Co\{[A_1 \ \ B_1], [A_2 \ \ B_2], \ldots, [A_L \ \ B_L]\} \tag{20}$$

where Co means convex hull. Note that if $[A \ \ B] \in \Omega$ then, for some $\lambda_i \geq 0;\ i = 1, \ldots, L$ with $\sum \lambda_i = 1$ it is possible to write

$$[A \ \ B] = \sum_{i=1}^{L} \lambda_i [A_i \ \ B_i]. \tag{21}$$

For this process description, the unconstrained RMPC performance objective can be possed as [45]

$$\min_{u(k+i|k), i=0,1,\ldots,m} \ \max_{[A \ \ B] \in \Omega} J_\infty(k), \tag{22}$$

where

$$J_\infty(k) = \sum_{i=0}^{\infty} [x(k+i|k)^T Q_1 x(k+i|k) + u(k+i|k)^T R u(k+i|k)], \tag{23}$$

where $x(k+i|k)$ and $u(k+i|k)$ are the state and control move, respectively, at time $k+i$, computed based on measurements at time k. In particular, $x(k|k)$ is the measured state at time k and $u(k|k)$ is the control move to be implemented at time k. It is assumed that there is no control action after time $k+C-1$ (i.e. $u(k+i|k) = 0$ for $i \geq C$) with C is the control horizon.

Eq. (22) is a "min–max" problem. The maximization is over the set of possible plants, and corresponds to choose a plant $[A\ B] \in \Omega$, which, if uses as model for predictions, would lead to the worst case value of $J_\infty(k)$ among all plants in Ω. This worst case value is then minimized over the present and future control moves $u(k+i|k)$, $i = 0, 1, \ldots, C$.

To solve this problem, in [45] there is developed an upper bound on $J_\infty(k)$ in the form of

$$J_\infty(k) \geq V(x(k|k)), \tag{24}$$

where $V(x) = x^T P x$ with $P > 0$. Thus

$$\max_{[A\ \ B] \in \Omega} J_\infty(k) \leq V(x(k|k)). \tag{25}$$

This expression gives an upper bound on the robust performance objective. Thus solution of the problem can be obtained on the basis of the following theorem [45].

Theorem 1. *Let $x(k) = x(k|k)$ be the state of the uncertain system (19). Then the state feedback matrix F in the control law $u(k+i|k) = Fx(k+i|k)$, $i \geq 0$ that minimizes the upper bound $V(x(k|k))$ on the robust performance objective function at sampling time k is given by*

$$F = YQ^{-1}, \tag{26}$$

where $Q > 0$ and Y are obtained from the solution (if it exists) of the following linear objective minimization problem with LMI constraints:

$$\min_{\gamma, Q, Y} \gamma \tag{27}$$

subject to

$$\begin{bmatrix} 1 & x(k|k)^T \\ x(k|k) & Q \end{bmatrix} \geq 0, \tag{28}$$

and

$$\begin{bmatrix} Q & QA_j^T + Y^T B_j^T & QQ_1^{1/2} & Y^T R^{1/2} \\ A_j Q + B_j Y & Q & 0 & 0 \\ QQ_1^{1/2} & 0 & \gamma I & 0 \\ R^{1/2} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \quad j = 1, 2, \ldots, L. \tag{29}$$

Proof. See [45].

This problem could be extended to consider constraints on the manipulated variables and on the process outputs. If constraints on the control variables are $\|u(k+i|k)\|_2 \leq u_{max}$ for $i \leq 0$ the following LMI should be included as constraint in problem (26):

$$\begin{bmatrix} u_{max}^2 I & Y \\ Y^T & Q \end{bmatrix} \geq 0. \tag{30}$$

In the case of output constraints of the form

$$\max_{[A\ \ B] \in \Omega} \|y(k+i|k)\|_2 \leq y_{max} \tag{31}$$

the LMI constraint to be include in Eq. (26) is

$$\begin{bmatrix} Q & (A_j Q + B_j Y)^T C^T \\ C(A_j Q + B_j Y) & y_{max}^2 I \end{bmatrix} \geq 0, \quad j = 1,2,\ldots,L. \tag{32}$$

In the context of systems modeled as Orthonormal Basis Functions, [47] extended this result to models in the form

$$\begin{aligned} \tilde{x}(k+1) &= \tilde{A}\tilde{x}(k) + \tilde{B}\tilde{u}(k), \\ \tilde{y}(k) &= \tilde{C}\tilde{x}(k), \end{aligned} \tag{33}$$

where the uncertainty is concentrated on matrix $\tilde{C}$ and matrices $\tilde{A}$ and $\tilde{B}$ are completely determined, as it is the case of the model identified in Section 2. In this model, the description of the uncertainty is in the form $\tilde{C} \in \Omega_C$, where

$$\Omega_C = Co\{C_1, C_2, \ldots, C_L\}. \tag{34}$$

In other words, if $\tilde{C} \in \Omega_C$ then, for some $\lambda_i \geq 0$; $i = 1, \ldots, L$ with $\sum \lambda_i = 1$ we have

$$\tilde{C} = \sum_{i=1}^{L} \lambda_i C_i. \tag{35}$$

By writing this model in function of deviation variables, and considering a constant setpoint signal on the horizon (w), the following model is obtained [47]:

$$\begin{aligned} \begin{bmatrix} \Delta\tilde{x}(k+1) \\ \tilde{y}(k+1) - w \end{bmatrix} &= \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C}\tilde{A} & I \end{bmatrix} \begin{bmatrix} \Delta\tilde{x}(k) \\ \tilde{y}(k) - w \end{bmatrix} + \begin{bmatrix} \tilde{B} \\ \tilde{C}\tilde{B} \end{bmatrix} \Delta\tilde{u}(k) \\ [y(k) - w] &= [0 \;\; I] \begin{bmatrix} \Delta\tilde{x}(k) \\ \tilde{y}(k) - w \end{bmatrix}, \end{aligned} \tag{36}$$

where $\Delta = 1 - q^{-1}$ and q^{-1} is the delay operator.

Now, defining

$$A = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C}\tilde{A} & I \end{bmatrix}, \quad B = \begin{bmatrix} \tilde{B} \\ \tilde{C}\tilde{B} \end{bmatrix}, \quad C = [0 \;\; I], \quad x = \begin{bmatrix} \Delta\tilde{x}(k) \\ \tilde{y}(k) - w \end{bmatrix}, \quad y(k) = \tilde{y}(k) - w,$$

$$u(k) = \Delta\tilde{u}(k), \quad A_i = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C}_i\tilde{A} & I \end{bmatrix} \quad \text{and} \quad B_i = \begin{bmatrix} \tilde{B} \\ \tilde{C}_i\tilde{B} \end{bmatrix} \quad \text{for } i = 1, \ldots, L,$$

it is possible to solve the RMPC problem with objective function

$$J_\infty(k) = \sum_{i=0}^{\infty} \{(\tilde{y}(k+i|k) - w)^T Q_1 (\tilde{y}(k+i|k) - w) + \Delta\tilde{u}(k+i|k)^T R \Delta\tilde{u}(k+i|k)\} \tag{37}$$

by means of the result of Theorem 1.

More recently, [25] extended this result to the control of Wiener models with uncertainties in linear and nonlinear blocks. An important drawback of such approach is it demands invertibility of the static nonlinearity, which is a rather restrictive condition. In the modeling approach herein followed it is considered that the static nonlinearity may be invertible or not, therefore, the controller design developed in [25] is not suitable and a different strategy must be developed for the controller synthesis. For this purpose, it is relevant the paper by Bloemen et al. [22] where the static nonlinearity is represented by a polytopic description which is linear in $z(k)$. Basically, conic sector bounds [48] of the nonlinear function are determined (which are valid in the operating region) and the following inequalities arise

$$C_{min}.z(k) \le y(k) \le C_{max}.z(k) \tag{38}$$

which involves the uncertain nonlinear functions in Eq. (15) pass through the origin [22]. The situation described in Eq. (38) is illustrated in Fig. 2, which illustrate the conic sectors. In other words,

$$y(k) = C(z(k)).z(k) \tag{39}$$

with

$$C(z(k)) \in \Omega_G = Co\{G_{min}, G_{max}\}. \tag{40}$$

In this way, it is possible to use the algorithm presented in [45], to design a robust controller for the Wiener-like model proposed in Section 2.

The resultant robust control design method can be applied to a general model structure in which the nonlinearities are transformed into polytopic descriptions. Therefore, this procedure enables the use of robust linear MPC techniques for controlling these Wiener-like structures. An important advantage of this approach is it retains the convexity of the optimization problem associated to the control movement calculation.

Note that the control design problem, which is stated in the framework of LMI theory, is able to deal with model uncertainty as well as input and output constraints.

Another advantage of the presented approach is it uses polytopic descriptions instead of removing the nonlinearities from the control problem. In such a way, the effect of the nonlinearities on the input–output behavior of the plant is still considered. As the nonlinearity is present in the original design criterion, a better control performance can be attained.

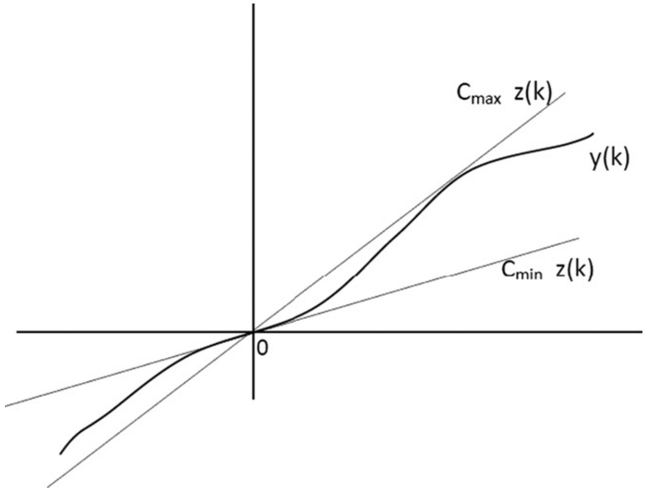

Fig. 2. Linear bounds for $y(k)$.

4. Simulation example: distillation column

The goal of this section is to illustrate the application of the proposed methodology regarding both identification and control. For this purpose, a distillation column has been selected. This is an appealing application as it is one of the most common unit operations in the chemical industry. Its relevance as well as its complex nature have been the main reasons for being a favorite subject in process systems engineering field. Moreover, in the areas of modeling and control, distillation columns have captured the attraction of many researchers. Such is the case of Skogestad et al. [49,50], whose Column A has been widely diffused. This simulation example is herein selected to illustrate the proposed identification methodology for both Wiener and Hammerstein models.

In this case the LV control structure is used. This Liquid–Vapor (LV) strategy is a common control pairing in binary distillation column system. In order to maintain mole fraction of distillate and bottom product, this structure will manipulate the reflux flow rate and steam flow rate. Then, the input $u(k) = [V_B \; L_T]^T$ is a vector formed by the boilup and the reflux flows, respectively. On the other hand, the output $y(k) = [x_B \; x_D]^T$ is a vector formed by the liquid bottom and the distillate product compositions, respectively. Therefore, a two input–two output process is considered for the identification.

4.1. Model identification

Simulation of Column A was accomplished in order to collect the required input–output data of this nonlinear process. For such purpose, random signals with uniform distribution around 2% of the nominal steady-state operating point were considered for the inputs (i.e. manipulated variables). A sample time of 50 s was assumed.

The dominant poles in the Laguerre basis were chosen taking into account a preliminary linear identification. In this case, a Laguerre expansion of order 1 was selected with poles $\xi_{i,1} = 0.5451$ and $\xi_{i,2} = 0.6805$ for $i = 1,2$. As regards the PWL approximation, the domain was defined as $z_{i,1,1} \in [-0.02, 0.02]$ and $z_{i,2,1} \in [-0.002, 0.002]$ for $i = 1,2$, and each dimension was divided into 10 partitions.

A nominal model was computed by minimizing a quadratic criterion [12] to show the suitability of the proposed Wiener-like structure for modeling the distillation process herein considered. Figs. 3 and 4 illustrate the approximation achieved for both outputs (x_B and x_D) with the Wiener-like structure and the linear model. Note that the improvement due to the inclusion of a nonlinear block justifies the use of the Wiener-like model.

The results of the robust identification are depicted in Figs. 5 and 6. From these plots it is clear that the measurement data are completely represented by the uncertain model obtained.

4.2. Robust control

The algorithm developed in Section 3 is used to achieve a robust MPC control for the Wiener-like model. The following design parameters are considered: $Q_1 = diag[1/0.01 \; 6/0.99]$ and $R = diag[1 \; 1]$.

To calculate the sequence of control inputs, the RMPC algorithm requires the uncertain model which represents the observed behavior of the process to be covered with a conic sector. These sectors are $G_{max} = [1.7799 - 0.0532]$ and $G_{min} = [0.3341 - 0.8744]$ for x_B and $G_{max} = [1.5709 - 0.1227]$ and $G_{min} = [0.2865 - 1.0329]$ for x_D.

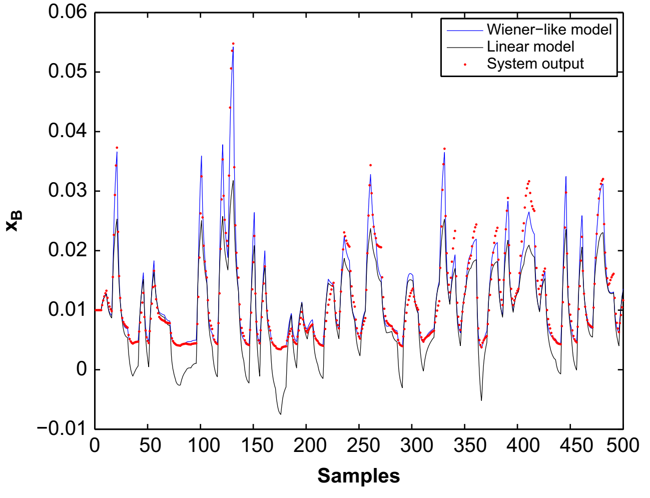

Fig. 3. System output and predictions for x_B.

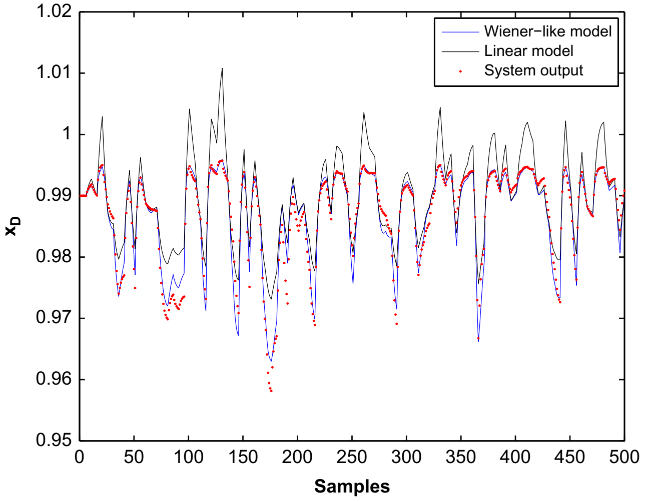

Fig. 4. System output and predictions for x_D.

A sequence of wide range changes is applied to the setpoints. Simulation results are shown in Figs. 7 and 8 for both controlled variables. In these plots are included the performances achieved with both the robust and the nominal nonlinear model predictive controller. The nominal scheme is implemented by direct optimization of a quadratic objective function under the constraint of the nominal Wiener-like model. It should be remarked that the robust controller outperforms the nominal one because the latter sometimes fails in tracking the setpoint, while the robust one succeeds in tracking every new setpoint value. Figs. 9 and 10 show the corresponding manipulated variables movements for both controllers (i.e. the robust and the nominal ones).

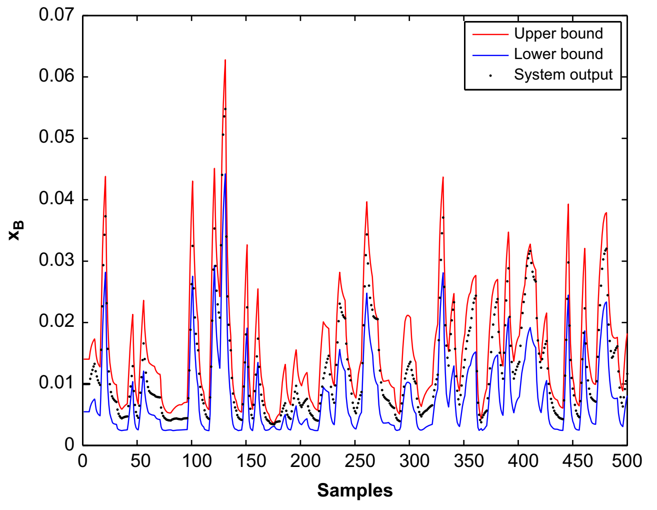

Fig. 5. Lower and upper bounds on the output x_B.

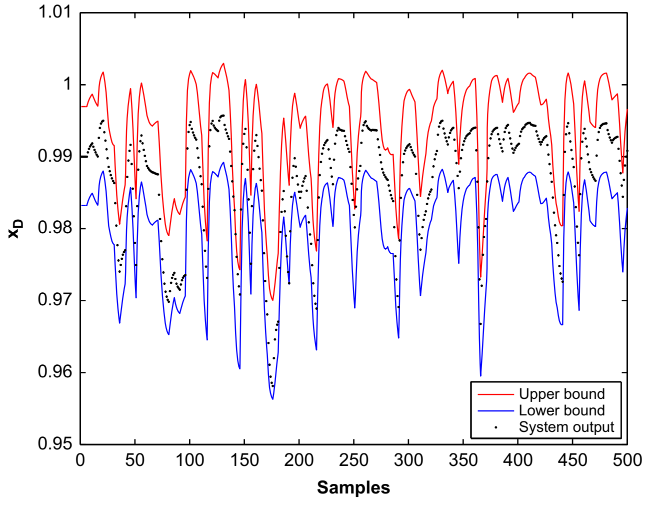

Fig. 6. Lower and upper bounds on the output x_D.

As the optimization problem should be solved at each sample time, it is interesting to consider the necessary time to obtain the solution. The implementation performed in this work made use of MATLAB LMI Toolbox [51]. In the complete simulation, which was accomplished with an Intel Processor Core I3 330 M (2.13 GHz), the largest time demand to perform the optimization was 0.4 s, which is even lower than the sample time (50 s). On the other hand, the nominal MPC algorithm execution involved a computational time which exceeded a second.

Additional simulation tests were accomplished to investigate the influence of the design parameters Q_1 and R (see Eq. (23)) on the controllers performance. For this purpose,

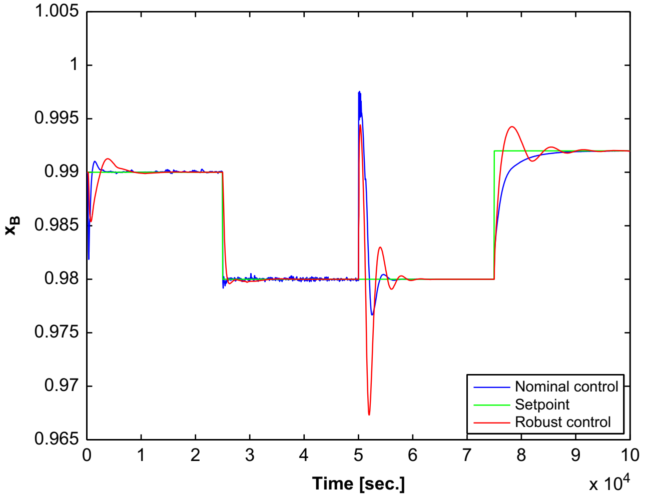

Fig. 7. Nominal and robust MPC for output x_B.

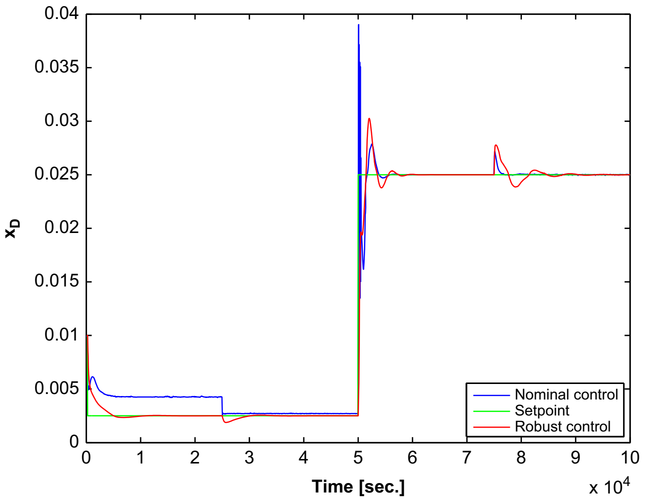

Fig. 8. Nominal and robust MPC for output x_D.

different values were given to the weight R in order to modify the influence of the control movements on the controller design. Simulation results are shown in Figs. 11 and 12 for both controlled variables. Note that the obtained outputs variables clearly agree with Eq. (23), i.e., the lower the weight R, the faster the achievable output response. Therefore, this physical meaning of the design parameter R is verified.

5. Conclusions

In the present work, both identification and control of a Wiener-like system are dealt with. The dynamic linear part is represented by a finite set of discrete Laguerre or Kautz

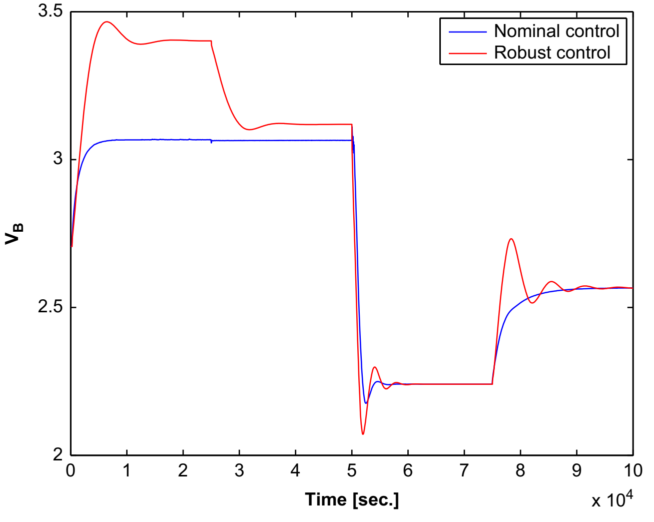

Fig. 9. Required control movements in V_B for nominal and robust MPC.

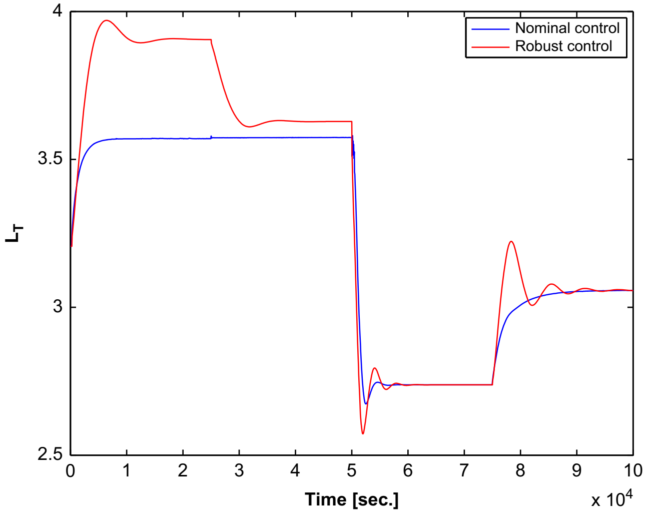

Fig. 10. Required control movements in L_T for nominal and robust MPC.

transfer functions, while the nonlinear static part is realized by high level canonical piecewise linear basis functions (HLCPWL). It must be pointed out that though this modeling structure can lead to a larger number of potentially necessary parameters than other Wiener-like systems such as those in [28], better approximation quality can be achieved with the models herein considered. It is well-known that this structure allows to uniformly approximate any causal, time-invariant, nonlinear discrete dynamic system with fading memory [24,52]. The modeling problem is tackled using a parametric identification approach which is stated and worked out as a linear programming problem.

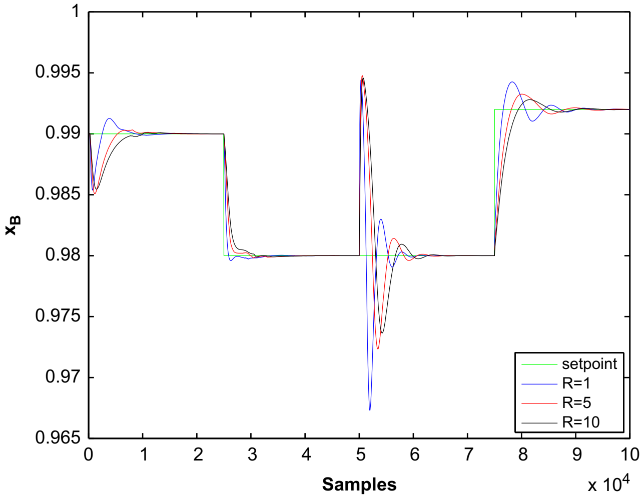

Fig. 11. Effect of the design parameter R on the controlled variable X_B.

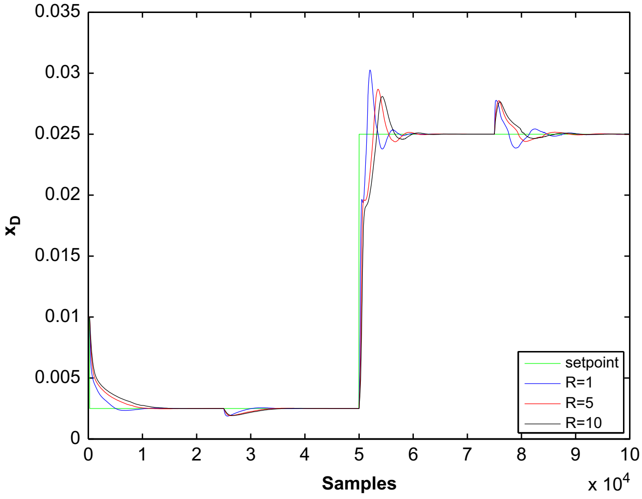

Fig. 12. Effect of the design parameter R on the controlled variable X_D.

The main contribution of the paper consists in a method for robust control of a Wiener-like model structure where the nonlinearities are transformed into polytopic descriptions. In such a way, the procedure enables the use of robust linear MPC techniques for controlling these Wiener-like structures, while convex optimization problem is retained.

The whole control problem (which deals with model uncertainty as well as input and output constraints) is formulated in the framework of LMI theory. For this purpose, Lyapunov functions are used that are a widely diffused tool for stability assessment. Therefore, an LMI optimization problem is stated and solved.

The main advantage of using the polytopic descriptions instead of removing the nonlinearities from the control problem is that the effect of the nonlinearities on the input–output behavior of the plant is still taken into account. Because the nonlinearity is present in the original design criterion, a better control performance is obtained. This fact has been illustrated by a simulation example which involves composition control of a distillation column.

Acknowledgments

This work was financially supported by the Agencia Nacional de Promoción Científica y Tecnológica, CONICET, CICpBA and Universidad Nacional del Sur.

Appendix

$$z_{i,j}(k)=\begin{bmatrix} z_{i,j,0}(k+1) \\ z_{i,j,1}(k+1) \\ z_{i,j,2}(k+1) \\ \vdots \\ z_{i,j,N_z}(k+1) \end{bmatrix}, \tag{41}$$

$$\mathbf{A}_{i,j}=\begin{bmatrix} \xi_{i,j} & 0 & 0 & \cdots & 0 \\ 1-\xi_{i,j}^2 & \xi_{i,j} & 0 & \cdots & 0 \\ -\xi_{i,j}(1-\xi_{i,j}^2) & 1-\xi_{i,j}^2 & \xi_{i,j} & \cdots & 0 \\ & & \vdots & & \\ (-\xi_{i,j})^{N_z-1}(1-\xi_{i,j}^2) & (-\xi_{i,j})^{N_z-2}(1-\xi_{i,j}^2) & (-\xi_{i,j})^{N_z-3}(1-\xi_{i,j}^2) & \cdots & \xi_{i,j} \end{bmatrix}, \tag{42}$$

$$\mathbf{B}_{i,j}=\begin{bmatrix} (1-\xi_{i,j}^2)^{1/2} \\ -\xi_{i,j}(1-\xi_{i,j}^2)^{1/2} \\ \xi_{i,j}^2(1-\xi_{i,j}^2)^{1/2} \\ \vdots \\ (-\xi_{i,j})^{N_z}(1-\xi_{i,j}^2)^{1/2}) \end{bmatrix}, \tag{43}$$

$$\mathbf{x}(k)=\begin{bmatrix} z_{1,1}(k+1) \\ z_{1,2}(k+1) \\ \vdots \\ z_{i,j}(k+1) \\ \vdots \\ z_{N_y,N_u}(k+1) \end{bmatrix}, \tag{44}$$

$$\mathbf{A}=[diagblocks\{\mathbf{A}_{1,1}\mathbf{A}_{1,2}\cdots\mathbf{A}_{i,j}\cdots\mathbf{A}_{N_y,N_u}\}], \tag{45}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{1,1} & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{B}_{1,2} & 0 & \cdots & 0 \\ & & \vdots & & \\ 0 & \cdots & \mathbf{B}_{i,j} & \cdots & 0 \\ & & \vdots & & \\ \mathbf{B}_{2,1} & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{B}_{2,2} & 0 & \cdots & 0 \\ & & \vdots & & \\ 0 & \cdots & 0 & \cdots & \mathbf{B}_{N_y,N_u} \end{bmatrix}. \tag{46}$$

References

[1] J. Sjöberg, O. Zhang, L. Ljung, A. Benveniste, B. Delyon, P.Y. Glorennec, H. Hjalmarsson, A. Juditsky, Nonlinear black-box modeling in system identification: a unified overview, Automatica 12 (12) (1995) 1691–1724.

[2] R. Haber, H. Unbehauen, Structure identification of nonlinear dynamic system a survey on input/output approaches, Automatica 26 (4) (1990) 651–677.

[3] A. Janczak, Identification of Nonlinear Systems using Neural Networks and Polynomial Models, vol. 310, Springer, Berlin, 2005.

[4] A. Kalafatis, N. Arifin, L. Wang, W.R. Cluett, A new approach to the identification of pH processes based on Wiener model, Chemical Engineering Science 50 (23) (1995) 3693–3701.

[5] K.P. Fruzzetti, A. Palazoglu, K.A. McDonald, Nonlinear model predictive control using Hammerstein models, Journal of Process Control 7 (1997) 31–41.

[6] S.J. Norquay, A. Palazoglu, J.A. Romagnoli, Model predictive control based on Wiener models, Chemical Engineering Science 53 (1) (1998) 75–84.

[7] A. Visala, H. Pitkänen, J. Paanajärvi, Wiener-NN models and robust identification, in: International Joint Conference on Neural Networks (IJCNN'99), vol. 3, 10–16 July, Washington, DC, USA, 1999, pp. 2188–2193.

[8] Y. Zhu, Distillation column identification for control using Wiener model, in: Proceedings of American Control Conference, vol. 5, 2–4 June, San Diego, CA, USA, 1999, pp. 3462–3466.

[9] E. Eskinat, S. Johnson, W. Luyben, Use of Hammerstein models in identification of nonlinear systems, AIChE Journal 37 (1991) 255–268.

[10] G. Sentoni, O. Agamennoni, A. Desages, J. Romagnoli, Approximate models for nonlinear process control, AIChE Journal 42 (1996) 2240–2250.

[11] R.K. Pearson, M. Pottmann, Gray-box identification of block-oriented nonlinear models, Journal of Process Control 10 (2000) 301–315.

[12] J.C. Gómez, E. Baeyens, Identification of block-oriented nonlinear systems using orthonormal bases, Journal of Process Control 14 (2004) 685–697.

[13] S. Hadjiloucas, A. Izhac, R.K.H. Galvao, J.W. Bowen, V.M. Becerra, State-space model identification of a wireless THz network, in: Conference Digest of the 2004 Joint 29th International Conference on Infrared and Millimeter Waves, and 12th International Conference on Terahertz Electronics, 27 September–1 October, 2004, pp. 375–376.

[14] A. Huang, J.M.A. Tanskanen, I.O. Hartimo, Design of optimum power estimator based on Wiener model applied to mobile transmitter power control, in: Proceedings of the 1998 IEEE International Symposium on Circuits and Systems (ISCAS 98), Monterey, CA, USA, May, 1998, pp. 249–252.

[15] P. Celka, P. Colditz, Nonlinear nonstationary Wiener model of infant EEG seizures, IEEE Transactions on Biomedical Engineering 49 (6) (2002) 556–564.

[16] I.W. Hunter, M.J. Korenberg, The identification of nonlinear biological systems: Wiener and Hammerstein cascade models, Biological Cybernetics 55 (1986) 135–144.

[17] W. Durfee, K. Maclean, Methods for estimating the isometric recruitment curve of electrically stimulated muscle, IEEE Transactions on Biomedical Engineering 36 (1989) 654–667.

[18] D. Westwick, R. Kearney, Separable least squares identification of nonlinear Hammerstein models: application to stretch reflex dynamics, Annals of Biomedical Engineering 29 (8) (2001) 707–718.
[19] E. Dempsey, D. Westwick, Identification of Hammerstein models with cubic spline nonlinearities, IEEE Transactions on Biomedical Engineering 51 (2) (2004) 237–245.
[20] F. Le, I. Markovsky, C. Freeman, E. Rogers, Identification of electrically stimulated muscle models of stroke patients, Control Engineering Practice 18 (2010) 396–407.
[21] M. Sznaier, Computational complexity analysis of set membership identification of Hammestein and Wiener systems, Automatica 45 (2009) 701–705.
[22] H.H. Bloemen, T.J.J. Van Den Boom, H.B. Verbruggen, Model-based predictive control for Hammerstein–Wiener systems, International Journal of Control 74 (2001) 482–495.
[23] I. Skrjanc, S. Blazic, O.E. Agamennoni, Interval fuzzy modeling applied to PWL Wiener models with uncertainties, IEEE Transactions on Systems, Man, and Cybernetics, Part B 35 (5) (2005) 1092–1095.
[24] M.P. Alvarez, L.R. Castro, O.E. Agamennoni, Set membership estimation theory for Wiener modelling using HLCPWL functions, International Journal of Modelling, Identification and Control 14 (1/2) (2011) 13–26.
[25] S.I. Biagiola, J.L. Figueroa, Robust model predictive control of Wiener systems, International Journal of Control 84 (3) (2011) 432–444.
[26] P. Falugi, L. Giarré, G. Zappa, Approximation of the feasible parameter set in worst-case identification of Hammerstein models, Automatica 41 (6) (2005) 1017–1024.
[27] P.M. Mäkilä, On robustness in control and LTI identification: near-linearity and non-conic uncertainty, Automatica 42 (2006) 601–612.
[28] S.I. Biagiola, J.L. Figueroa, Identification of uncertain MIMO Wiener and Hammerstein models, Computers and Chemical Engineering 35 (12) (2011) 2867–2875.
[29] A. Lussón-Cervantes, O.E. Agamennoni, J.L. Figueroa, A nonlinear model predictive control scheme based on Wiener piecewise linear models, Journal of Process Control 13 (2003) 655–666.
[30] S.I. Biagiola, J.L. Figueroa, Wiener and Hammerstein uncertain models identification, Mathematics and Computers in Simulation 79 (11) (2009) 3296–3313.
[31] M.P. Alvarez, L.R. Castro, J.L. Figueroa, O.E. Agamennoni, Set membership estimation theory for coupled MIMO Wiener-like models, in: Proceedings of the XIV Reunión en Procesamiento, Información y Control, 16–18 November 2011, Oro Verde, Entre Ríos, Argentina.
[32] S.M. Kang, L.O. Chua, A global representation of multidimensional piecewise-linear functions with linear partitions, IEEE Transactions on Circuits and Systems 25 (1978) 938–940.
[33] J.N. Lin, H. Xu, R. Unbehahuen, A generalization of canonical piecewise-linear functions, IEEE Transactions on Circuits and Systems 41 (1994) 345–347.
[34] P. Julián, A.C. Desages, O.E. Agamennoni, High level canonical piecewise linear representation using a simplicial partition, IEEE Transactions on Circuits and Systems CAS-46 (1999) 463–480.
[35] B. Wahlberg, System identification using Kautz models, IEEE Transactions on Automatic Control 39 (6) (1994) 1276–1282.
[36] B. Wahlberg, P.M. Makila, On approximation of stable linear dynamic systems using Laguerre and Kautz functions, Automatica 32 (5) (1996) 693–708.
[37] M. Milanese, Estimation theory and prediction in the presence of unknown but bounded uncertainty: a survey, in: M. Milanese, R. Tempo, A. Vicino (Eds.), Robustness in Identification and Control, Plenum Press, New York, 1989.
[38] M. Milanese, C. Novara, Model quality in identification of nonlinear systems, IEEE Transactions on Automatic Control 50 (10) (2005) 1606–1611.
[39] M. Milanese, R. Tempo, Optimal algorithms theory for robust estimation and prediction, IEEE Transactions on Automatic Control 30 (8) (1985) 730–738.
[40] M. Milanese, R. Tempo, Optimal algorithms theory for dynamic systems with set membership uncertainty: an overview, Automatica 27 (1991) 997–1009.
[41] S.J. Qin, T.A. Badwell, An overview of industrial model predictive control technology, in: Carlos E. Garcia, Jeffrey C. Kantor, Brice Carnahan (Eds.), Proceedings of the Fifth International Conference on Chemical Process Control, AIChE Symposium Series 316, vol. 93, 1997, pp. 232–256.
[42] B.W. Bequette, Nonlinear control of chemical processes: a review, Industrial and Engineering Chemistry Research 30 (1991) 1391–1413.
[43] M.A. Henson, Nonlinear model predictive control: current status and future directions, Computers and Chemical Engineering 23 (1998) 187–202.

[44] S. Gerkšič, D. Juričic, S. Strmčnic, D. Matko, Wiener model based nonlinear predictive control, International Journal of Systems Science 31 (2000) 189–202.
[45] M.V. Kothare, V. Balakrishnan, M. Morari, Robust constrained model predictive control using linear matrix inequalities, Automatica 32 (1996) 1361–1379.
[46] L.O. Chua, L.P. Ying, Solving nonlinear resistive networks using piecewise-linear analysis and simplicial subdivision, IEEE Transactions on Circuits and Systems CAS-24 (1977) 305–317.
[47] H.X. Araújo, G.H.C. Oliveira, An LMI approach for output feedback robust predictive control using orthonormal basis functions, in: Proceedings of the Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference, December 16–18, Shangai, PR China, 2009.
[48] S.I. Biagiola, O.E. Agamennoni, J.L. Figueroa, H_∞ control of a Wiener type system, International Journal of Control 77 (6) (2004) 572–583.
[49] S. Skogestad, M. Morari, Understanding the dynamic behaviour of distillation columns, Industrial and Engineering Chemistry Research 27 (10) (1988) 1848–1862.
[50] S. Skogestad, I. Postlethwaite, Multivariable Feedback Control. Analysis and Design, second ed., John Wiley & Sons, 2007.
[51] P. Gahinet, A. Nemirovski, A.J. Laub, M. Chilali, LMI Control Toolbox, The MathWorks, MA, USA, 1995.
[52] S. Boyd, L.O. Chua, Fading memory and the problem of approximating nonlinear operators with Volterra series, IEEE Transactions on Circuits and Systems CAS-32 (11) (1985) 1150–1161.