Robust finite-time fuzzy H_∞ control for uncertain time-delay systems with stochastic jumps

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Abstract

This paper investigates the problem of robust finite-time H_∞ control for a class of uncertain discrete-time Markovian jump nonlinear systems with time-delays represented by Takagi–Sugeno (T–S) model. Initially, the concepts of stochastic finite-time boundedness and stochastic finite-time H_∞ stabilization are presented. Then, by using stochastic Lyapunov–Krasovskii functional approach, sufficient conditions are derived such that the resulting close-loop system is stochastically finite-time bounded and satisfies a prescribed H_∞ disturbance attenuation level in a given finite-time interval. Furthermore, sufficient criteria on stochastic finite-time H_∞ stabilization using a fuzzy state-feedback controller are provided, and the controller is designed by solving an optimization problem in terms of linear matrix inequalities. Finally, two numerical examples are exploited to show the validity of the proposed design techniques.

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1. Introduction

In the past few decades, the fuzzy logic control has been utilized as an alternative approach to conventional control for complex nonlinear systems. As one of most important form of fuzzy systems, Takagi–Sugeno (T–S) fuzzy model [1] has been recognized as a popular and powerful tool in approximating and describing complex nonlinear systems. The main reason of this attention to T–S fuzzy model is due to the fact that it can combine the merits of both fuzzy logic theory and linear systems, and then stability analysis and controller design of the overall fuzzy systems can be carried out in the Lyapunov function framework. Therefore, many problems have been tackled and some appealing results for T–S fuzzy systems have been reported in the literature. For example, the problems of stability analysis of T–S fuzzy systems have been discussed in [2]. The robust H_∞ control has been addressed in [3]. The H_∞ filtering problem has been considered in [4–8] and the output-feedback control has been studied in [9,10]. Some other important results like the guaranteed cost control, variable structure control and reliable control have also been presented for a class of T–S fuzzy systems in [11–13]. More results on fuzzy systems can be found in [14–16] and the references therein.

On the other hand, considerable attention has been paid to Markovian jump systems (MJSs) in the control community since the family of stochastic systems have been extensively applied to modeling various practical processes that can experience abrupt changes in their structures and parameters, possibly caused by the phenomena such as component failures, sudden environmental disturbances, and changing subsystem interconnections, and so on. Therefore, many attracting results and a large variety of control problems have been reported in the literature. For example, the authors considered the stability analysis and state-feedback stabilization problems for MJSs in [17]. The references [18,19] studied the robust H_∞ control problem, and the H_∞ filtering problem has been presented in [20–22]. The sliding mode control and passivity analysis for a class of stochastic systems have also been investigated in [23–25]. More detailed results on the topic could be found in [26] and the references therein. Recently, fuzzy MJSs, as a special form of MJSs, have also received many researches. Some results on fuzzy MJSs have been investigated and studied, such as the stability analysis and state-feedback stabilization problem [27], the H_∞ control [28] and the output-feedback stabilization [29].

In many practical applications, however, many concerned problems are that the described system state does not exceed a certain bound in a given finite-time interval. Compared with classical Lyapunov asymptotical stability, finite-time stability or short-time stability were investigated to deal with the transient performances of system trajectories in a specified finite-time interval. Finite-time stability or short-time stability were first introduced in the 1960s [30], and then the definition of finite-time stability was extended to finite-time boundedness in [31]. Further, applying the linear matrix inequality (LMI) technique and Lyapunov approach, many results on finite-time stability and stabilization have been investigated for linear systems, nonlinear systems, stochastic systems, switching systems, fuzzy systems and singular systems. For instance, the authors in [32] studied the state-feedback finite-time stabilization for discrete-time linear systems. The problem of finite-time stability and stabilization was tackled for nonlinear stochastic hybrid systems in [33]. The results on robust finite-time stabilization were provided for uncertain continuous-time fuzzy MJSs in [34]. For more details of the literature related to finite-time stability, finite-time stabilization and finite-time H_∞ control, the reader is referred to [35–39].

Motivated by aforementioned discussions, in this paper, we deal with the problem of robust finite-time H_∞ control for uncertain discrete-time Markovian jump T–S fuzzy systems with time-delays. The concepts of stochastic finite-time boundedness and stochastic finite-time H_∞ stabilization for a class of stochastic systems are first presented. Then, sufficient conditions of stochastic finite-time boundedness or stochastic finite-time H_∞ stabilization via a fuzzy state-feedback controller are obtained for the class of fuzzy stochastic systems. Sufficient criteria on stochastic finite-time boundedness or stochastic finite-time H_∞ stabilization can be tackled by a feasibility problem in terms of LMIs with a fixed parameter. Finally, two numerical examples are provided to illustrate the validity of the proposed methods. The contributions of this paper lie in the following three aspects: (i) by using stochastic Lyapunov function method and LMI technique, the stochastic finite-time H_∞ stabilization analysis is provided for discrete-time time-delay T–S fuzzy systems with Markovian jumps; (ii) when parameter uncertainties appear in discrete-time T–S fuzzy systems with Markovian jumps and time-delays, the robust stochastic finite-time H_∞ stabilization criteria are presented in terms of LMIs by applying the matrix decomposition techniques; and (iii) as an affiliated result, we also derive the sufficient conditions of stochastic finite-time boundedness for the class of discrete-time T–S fuzzy nonlinear systems with Markovian jumps and time-delays. Therefore, the main purpose of this paper is to make the first attempt to tackle the aforementioned contributions.

Notations: The notations in this paper are quite standard. $\mathbf{R}^n$, $\mathbf{R}^{n\times m}$ and $\mathbf{Z}_{k\geq 0}$ are used to denote the sets of n component real vectors, $n\times m$ real matrices, and the set of nonnegative integers, respectively. $\sigma_{\min}(A)$ and $\sigma_{\max}(A)$ denote the smallest and the largest eigenvalue of matrix A, respectively. M^T and M^{-1} stand for the transpose and the inverse of matrix M, respectively. The symbol $*$ represents a matrix which can be inferred by symmetry and diag$\{\cdots\}$ denotes a block-diagonal matrix. $(\Omega,\mathcal{F},\mathcal{P})$ is probability space, Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. In addition, $\mathbf{E}\{.\}$ stands for the mathematical expectation with some probability measure.

2. Problem statement and preliminaries

Consider the following discrete-time Markovian jump system (DMJS) with time-delays which could be represented by a T–S fuzzy model over the probability space $(\Omega,\mathcal{F},\mathcal{P})$:

Plant rules i: IF θ_1 is μ_{i1}, θ_2 is μ_{i2}, …, θ_g is μ_{ig}, THEN

$$x(k+1)=A_i(r_k,k)x(k)+A_{di}(r_k,k)x(k-d)+B_i(r_k,k)u(k)+G_i(r_k,k)w(k), \tag{1a}$$

$$z(k)=C_i(r_k)x(k)+C_{di}(r_k)x(k-d)+D_{1i}(r_k)u(k)+D_{2i}(r_k)w(k), \tag{1b}$$

$$x(k)=\phi(k), k\in\{-d,\ldots,0\}, \tag{1c}$$

where $x(k)\in\mathbf{R}^{n_1}$, $u(k)\in\mathbf{R}^{m_1}$ and $z(k)\in\mathbf{R}^{p_1}$ are the system state, the control input, and the control output, respectively, d is a positive integer denoting the constant time-delay of the system state, $\phi(k), k\in\{-d,\ldots,0\}$ is a vector-valued initial discrete sequence. The stochastic jump process $\{r_k, k\geq 0\}$ is a discrete-time, discrete-state Markov chain taking values in a finite set $\mathcal{S}=\{1,2,\ldots,s\}$ with transition probabilities π_{lm}, where $\pi_{lm}>0$ and $\sum_{m=1}^{s}\pi_{lm}=1$ for all $l\in\mathcal{S}$. $A_i(r_k,k), A_{di}(r_k,k), B_i(r_k,k)$ and $G_i(r_k,k)$ are appropriately dimensioned system matrices with time-varying parametric uncertainties, which are assumed to be of the form:

$$\begin{aligned}&\begin{bmatrix}A_i(r_k,k) & A_{di}(r_k,k) & B_i(r_k,k) & G_i(r_k,k)\end{bmatrix}\\ &\quad=\begin{bmatrix}A_i(r_k) & A_{di}(r_k) & B_i(r_k) & G_i(r_k)\end{bmatrix}\\ &\qquad+F_i(r_k)\Delta(r_k,k)\begin{bmatrix}E_{1i}(r_k) & E_{2i}(r_k) & E_{3i}(r_k) & E_{4i}(r_k)\end{bmatrix},\end{aligned} \tag{2}$$

where $\Delta(r_k, k)$ is an unknown, time-varying matrix function and satisfies $\Delta^T(r_k, k)\Delta(r_k, k) \leq I$ for all $r_k \in \mathcal{S}$ and $k \in \mathbf{Z}_{k \geq 0}$. $A_i(r_k), A_{di}(r_k), B_i(r_k), G_i(r_k), C_i(r_k), F_i(r_k), E_{1i}(r_k), E_{2i}(r_k), E_{3i}(r_k)$, $E_{4i}(r_k)$, $C_{di}(r_k), D_{1i}(r_k)$ and $D_{2i}(r_k)$ are known constant matrices of appropriate dimensions. θ_j and μ_{ij} $(i = 1, \ldots, f, j = 1, \ldots, g)$ are respectively the premise variables and the fuzzy sets, f is the number of IF–THEN rules. The fuzzy basis functions are given by

$$h_i(\theta(k)) = \frac{\prod_{j=1}^{g} \mu_{ij}(\theta_j(k))}{\sum_{i=1}^{f} \prod_{j=1}^{g} \mu_{ij}(\theta_j(k))}, \tag{3}$$

in which $\mu_{ij}(\theta_j(k))$ represents the grade of membership of $\theta_j(k)$ in μ_{ij}. It is obvious that $\sum_{i=1}^{f} h_i(\theta(k)) = 1$ with $h_i(\theta(k)) > 0$. Moreover, the noise signal $w(k) \in \mathbf{R}^{p_2}$ satisfies

$$\mathbf{E}\left\{ \sum_{k=0}^{\infty} w^T(k)w(k) \right\} \leq \varpi^2, \quad \varpi \geq 0. \tag{4}$$

To simplify the notation, in the sequel, for each possible $r_k = l, l \in \mathcal{S}$, matrix $M_i(r_k)$ will be denoted by $M_{i,l}$; for instance, $A_i(r_k)$ will be denoted by $A_{i,l}$, $A_{di}(r_k)$ by $A_{di,l}$, $A_i(r_k, k)$ by $A_{i,l}(k)$, and so on. In addition, $h_i(k), \Lambda$ and $\check{P}_l$ denote $h_i(\theta(k)), \{1, \ldots, f\}$ and $\sum_{m=1}^{s} \pi_{lm} P_l$, respectively.

By using the fuzzy blending method, the overall fuzzy DMJS could be inferred as follows:

$$x(k+1) = \sum_{i=1}^{f} h_i(k)[A_{i,l}(k)x(k) + A_{di,l}(k)x(k-d) + B_{i,l}(k)u(k) + G_{i,l}(k)w(k)], \tag{5a}$$

$$z(k) = \sum_{i=1}^{f} h_i(k)[C_{i,l}x(k) + C_{di,l}x(k-d) + D_{1i,l}u(k) + D_{2i,l}w(k)], \tag{5b}$$

$$x(k) = \phi(k), k \in \{-d, \ldots, 0\}. \tag{5c}$$

The design of controllers in this paper is performed through the parallel distributed compensation and the overall controller is thus inferred as

$$u(k) = \sum_{i=1}^{f} h_i(k)K_{i,l}x(k), \tag{6}$$

where $K_{i,l}$ is the state-feedback gain to be designed. Then, the resulting closed-loop fuzzy DMJS can be written in the following form:

$$x(k+1) = A_l(h,k)x(k) + A_{dl}(h,k)x(k-d) + G_l(h,k)w(k), \tag{7a}$$

$$z(k) = C_l(h)x(k) + A_{dl}(h)x(k-d) + D_l(h)w(k), \tag{7b}$$

where

$$A_l(h,k) = \sum_{i=1}^{f} \sum_{j=1}^{f} h_i(k)h_j(k)[A_{i,l}(k) + B_{i,l}(k)K_{j,l}],$$

$$A_{dl}(h,k) = \sum_{i=1}^{f} h_i(k)A_{di,l}(k), G_l(h,k) = \sum_{i=1}^{f} h_i(k)G_{i,l}(k),$$

$$C_l(h) = \sum_{i=1}^{f} \sum_{j=1}^{f} h_i(k)h_j(k)[C_{i,l} + D_{1i,l}K_{j,l}],$$

$$C_{dl}(h) = \sum_{i=1}^{f} h_i(k)C_{di,l}, D_l(h) = \sum_{i=1}^{f} h_i(k)D_{2i,l}.$$

Definition 2.1. (see [38], stochastically finite-time stable (SFTS)). The time-delay fuzzy DMJS (7a) with $w(k) = 0$ and $u(k) = 0$ is said to be SFTS with respect to $(\delta_x, \epsilon, R_l, N)$, where $0 < \delta_x < \epsilon$, $R_l > 0$ and $N \in \mathbf{Z}_{k \geq 0}$, if

$$\mathbf{E}\{x^T(k_1)R_lx(k_1)\} \leq \delta_x^2 \Rightarrow \mathbf{E}\{x^T(k_2)R_lx(k_2)\} < \epsilon^2, k_1 \in \{-d, \ldots, 0\}, k_2 \in \{1, 2, \ldots, N\}. \quad (8)$$

Definition 2.2. (see [38], stochastically finite-time bounded (SFTB)). The time-delay fuzzy DMJS (7a) is said to be SFTB with respect to $(\delta_x, \epsilon, R_l, N, \varpi)$, where $0 < \delta_x < \epsilon$, $R_l > 0$ and $N \in \mathbf{Z}_{k \geq 0}$, if the constraint relation (8) holds.

Definition 2.3. (see [38], stochastically finite-time H_∞ stabilizable). The time-delay fuzzy DMJS (7a) and (7b) is said to be stochastic finite-time H_∞ stabilizable with respect to $(\delta_x, \epsilon, R_l, N, \gamma, \varpi)$, where $0 < \delta_x < \epsilon$, $R_l > 0$, $\gamma > 0$ and $N \in \mathbf{Z}_{k \geq 0}$, if the time-delay fuzzy DMJS (7a) and (7b) is SFTB with respect to $(\delta_x, \epsilon, R_l, N, \varpi)$ and under the zero-initial condition the output $z(k)$ satisfies

$$\mathbf{E}\left\{ \sum_{k=0}^{N} z^T(k)z(k) \right\} < \gamma^2 \mathbf{E}\left\{ \sum_{k=0}^{N} w^T(k)w(k) \right\} \quad (9)$$

for any nonzero $w(k)$ which satisfies (4), where γ is a prescribed positive scalar. Moreover, the control law (6) is called as finite-time H_∞ controller of the time-delay fuzzy DMJS (1a)–(1c).

Lemma 2.1 (*Zhang et al. [35,38]*). *If matrices $U, F(k)$ and V are of appropriate dimensions, and time-varying matrix function $F(k)$ satisfying $F^T(k)F(k) \leq I$ for all $k \in \mathbf{Z}_{k \geq 0}$, then, for an arbitrary positive constant θ, the following inequality holds*

$$UF(k)V + [UF(k)V]^T \leq \theta UU^T + \theta^{-1}V^TV. \quad (10)$$

The main aim of this paper is to design a fuzzy state-feedback controller of the form (6) which can ensure stochastic finite-time H_∞ stabilization of the time-delay fuzzy DMJS (7a) and (7b).

3. Main results

In this section, we consider stochastic finite-time H_∞ stabilization analysis of the time-delay fuzzy DMJS described by Eqs. (1a)–(1c). LMI conditions will be established to show that the nominal or uncertain fuzzy DMJS (7a) and (7b) is SFTB and the output $z(k)$ and disturbance $w(k)$ satisfies the constraint relation (9).

Theorem 3.1. *The time-delay fuzzy DMJS (7a) is SFTB with respect to $(\delta_x, \epsilon, R_l, N, \varpi)$, if there exist scalars $\lambda \geq 1$ and $\epsilon > 0$, a symmetric positive-definite matrix Q, sets of symmetric positive-definite matrices $\{P_l, l \in \mathcal{S}\}$ and $\{Q_l, l \in \mathcal{S}\}$, for all $l \in \mathcal{S}$ and $i, j \in \Lambda$, such that*

$$\begin{bmatrix} A_l^T(h,k)\check{P}_lA_l(h,k) - \lambda P_l + Q & * & * \\ A_{dl}^T(h,k)\check{P}_lA_l(h,k) & -Q + A_{dl}^T(h,k)\check{P}_lA_{dl}(h,k) & * \\ G_l^T(h,k)\check{P}_lA_l(h,k) & G_l^T(h,k)\check{P}_lA_{dl}(h,k) & -Q_l + G_l^T(h,k)\check{P}_lG_l(h,k) \end{bmatrix} < 0, \quad (11a)$$

$$[\overline{\sigma}_P + \sigma_{\overline{Q}}d]\delta_x^2 + \sigma_Q\varpi^2 < \underline{\sigma}_P\lambda^{-N}\epsilon^2, \quad (11b)$$

$\sigma_Q = \max_{l \in \mathcal{S}}\{\sigma_{\max}(Q_l)\}$, $\overline{P}_l = R_l^{-1/2} P_l R_l^{-1/2}$ *and* $\overline{Q}_l = R_l^{-1/2} Q R_l^{-1/2}$.

Proof. We first choose the following stochastic Lyapunov–Krasovskii functional:

$$V(k) = x^T(k)P_l x(k) + \sum_{n=k-d}^{k-1} x^T(n)Qx(n). \tag{12}$$

Then, a simple computation yields

$$\begin{aligned}\mathbf{E}\{V(k+1)\} - V(k) &= \mathbf{E}\left\{\sum_{m=1}^{s} \Pr\{r_{k+1} = m | r_k = l\} \times [x^T(k+1)P_l x(k+1) + \sum_{n=k-d+1}^{k} x^T(n)Qx(n)]\right\} - V(k) \\ &= \sum_{m=1}^{s} \pi_{lm} x^T(k+1)P_l x(k+1) - x^T(k)P_l x(k) + x^T(k)Qx(k) - x^T(k-d)Qx(k-d) \\ &= \xi^T(k)\begin{bmatrix} A_l^T(h,k)P_l A_l(h,k) - P_l + Q & * & * \\ A_{dl}^T(h,k)P_l A_l(h,k) & -Q + A_{dl}^T(h,k)P_l A_{dl}(h,k) & * \\ G_l^T(h,k)P_l A_l(h,k) & G_l^T(h,k)P_l A_{dl}(h,k) & G_l^T(h,k)P_l G_l(h,k)\end{bmatrix}\xi(k).\end{aligned} \tag{13}$$

where $\xi^T(k) = [x^T(k)\ x^T(k-d)\ w^T(k)]$. From conditions (11a) and (13), we can establish

$$\begin{aligned}\mathbf{E}\{V(k+1)\} - V(k) &< (\lambda - 1)x^T(k)P_l x(k) + w^T(k)Q_l w(k) \\ &\leq (\lambda - 1)V(k) + \sigma_Q w^T(k)w(k).\end{aligned} \tag{14}$$

Therefore, we have

$$\mathbf{E}\{V(k+1)\} < \lambda \mathbf{E}\{V(k)\} + \sigma_Q \mathbf{E}\{w^T(k)w(k)\}. \tag{15}$$

Notice that $\lambda \geq 1$, it follows from Eq. (15) that

$$\mathbf{E}\{V(k)\} < \lambda^k \mathbf{E}\{V(0)\} + \sigma_Q \mathbf{E}\left\{\sum_{n=0}^{k-1} \lambda^{k-n-1} w^T(n)w(n)\right\} \leq \lambda^k \mathbf{E}\{V(0)\} + \lambda^k \sigma_Q \varpi^2. \tag{16}$$

Let $\overline{P}_l = R_l^{-1/2} P_l R_l^{-1/2}$, $\overline{Q}_l = R_l^{-1/2} Q R_l^{-1/2}$ and notice that $\mathbf{E}\{x^T(n)R_l x(n)\} \leq \delta_x^2$ for all $n \in \{-d, \ldots, 0\}$, we have

$$\mathbf{E}\{V(0)\} \leq \overline{\sigma}_{\overline{P}} \mathbf{E}\{x^T(0)R_l x(0)\} + \sigma_{\overline{Q}} \mathbf{E}\left\{\sum_{n=-d}^{-1} x^T(n)R_l x(n)\right\} \leq [\overline{\sigma}_{\overline{P}} + \sigma_{\overline{Q}} d]\delta_x^2. \tag{17}$$

On the other hand, for all $l \in \mathcal{S}$, we can obtain

$$\mathbf{E}\{V(k)\} \geq \mathbf{E}\{x^T(k)P_l x(k)\} = \mathbf{E}\{x^T(k)R_l^{1/2}\overline{P}_l R_l^{1/2} x(k)\} \geq \underline{\sigma}_{\overline{P}} \mathbf{E}\{x^T(k)R_l x(k)\}. \tag{18}$$

From (16) to (18), we can deduce

$$\mathbf{E}\left\{x^T(k)R_l x(k)\right\} < \frac{[\overline{\sigma}_{\overline{P}} + \sigma_{\overline{Q}} d]\lambda^k \delta_x^2 + \sigma_Q \lambda^k \varpi^2}{\underline{\sigma}_{\overline{P}}}. \tag{19}$$

Therefore, it follows from (11a) that $\mathbf{E}\{x^T(k)R_l x(k)\} < \epsilon^2$ for all $k \in \{1, 2, \ldots, N\}$. The proof of this theorem is completed. □

Theorem 3.2. *The time-delay fuzzy DMJS (7a) and (7b) is stochastically finite-time H_∞ stabilizable with respect to $(\delta_x, \epsilon, R_l, N, \gamma, \varpi)$, if there exist scalars $\lambda \geq 1, \epsilon > 0, \gamma > 0$, a symmetric positive-definite matrix Q, a set of symmetric positive-definite matrices $\{P_l, l \in \mathcal{S}\}$, for all $l \in \mathcal{S}$*

and $i, j \in \Lambda$, *such that*

$$\begin{bmatrix} -\lambda P_l + Q & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -\lambda^{-N}\gamma^2 I & * & * \\ L_{1\,l}(h,k) & L_{2\,l}(h,k) & L_{3\,l}(h,k) & -P^{-1} & * \\ C_l(h) & C_{dl}(h) & D_l(h) & 0 & -I \end{bmatrix} < 0, \tag{20a}$$

$$[\overline{\sigma}_{\overline{P}} + \sigma_{\overline{Q}} d]\delta_x^2 + \lambda^{-N}\gamma^2\varpi^2 < \underline{\sigma}_{\overline{P}}\lambda^{-N}\epsilon^2, \tag{20b}$$

where

$$L_{1\,l}^T(h,k) = \left[\sqrt{\pi_{l1}}A_l^T(h,k) \quad \sqrt{\pi_{l2}}A_l^T(h,k) \quad \cdots \quad \sqrt{\pi_{ls}}A_l^T(h,k) \right],$$

$$L_{2\,l}^T(h,k) = \left[\sqrt{\pi_{l1}}A_{dl}^T(h,k) \quad \sqrt{\pi_{l2}}A_{dl}^T(h,k) \quad \cdots \quad \sqrt{\pi_{ls}}A_{dl}^T(h,k) \right],$$

$$L_{3\,l}^T(h,k) = \left[\sqrt{\pi_{l1}}G_l^T(h,k) \quad \sqrt{\pi_{l2}}G_l^T(h,k) \quad \cdots \quad \sqrt{\pi_{ls}}G_l^T(h,k) \right],$$

$$P = \mathrm{diag}\{P_1, P_2, \ldots, P_s\}.$$

Proof. By the Schur complements, it follows that condition (20a) implies

$$\begin{bmatrix} -\lambda P_l + Q & * & * & * \\ 0 & -Q & * & * \\ 0 & 0 & -\lambda^{-N}\gamma^2 I & * \\ L_{1\,l}(h,k) & L_{2\,l}(h,k) & L_{3\,l}(h,k) & -P^{-1} \end{bmatrix} < 0. \tag{21}$$

Denote $Q_l = \lambda^{-N}\gamma^2 I$ and notice the form of $P_l, L_{1\,l}(h,k), L_{2\,l}(h,k), L_{3\,l}(h,k)$ and P, it is obvious that Eq. (21) is equivalent to condition (11a). Thus, conditions (21) and (20b) can guarantee that the time-delay DMJS (7a) is SFTB with respect to $(\delta_x, \epsilon, R_l, N, \varpi)$ according to Theorem 3.1. On the other hand, consider the similar fuzzy Lyapunov–Krasovskii functional as Theorem 3.1. Taking into account condition (20a) and $Q_l = \lambda^{-N}\gamma^2 I$, one can derive from Eq. (20a) that the following inequality:

$$\mathbf{E}\{V(k+1)\} < \lambda V(k) - z^T(k)z(k) + \gamma^2\lambda^{-N}w^T(k)w(k) \tag{22}$$

holds for all $l \in \mathcal{S}$. According to Eq. (22), we can derive

$$\mathbf{E}\{V(k)\} < \lambda^k \mathbf{E}\{V(0)\} - \sum_{j=0}^{k-1} \lambda^{k-j-1}\mathbf{E}\{z^T(j)z(j)\} + \gamma^2\lambda^{-N}\mathbf{E}\left\{ \sum_{j=0}^{k-1} \lambda^{k-j-1}w^T(j)w(j) \right\}. \tag{23}$$

Under the zero-value initial condition and noting that $V(k) \geq 0$ for all $k \in \mathbf{Z}_{k\geq 0}$, we have

$$\sum_{j=0}^{k-1} \lambda^{k-j-1}\mathbf{E}\{z^T(j)z(j)\} < \gamma^2\lambda^{-N}\mathbf{E}\left\{ \sum_{j=0}^{k-1} \lambda^{k-j-1}w^T(j)w(j) \right\}. \tag{24}$$

Notice that $\lambda \geq 1$, we have

$$\mathbf{E}\left\{ \sum_{k=0}^{N} z^T(k)z(k) \right\} = \sum_{k=0}^{N} \mathbf{E}\{z^T(k)z(k)\} \leq \sum_{k=0}^{N} \mathbf{E}\{\lambda^{N-k}z^T(k)z(k)\}$$

$$< \gamma^2\lambda^{-N}\mathbf{E}\left\{\sum_{k=0}^{N}\lambda^{N-k}w^T(k)w(k)\right\} \leq \gamma^2\mathbf{E}\left\{\sum_{k=0}^{N}w^T(k)w(k)\right\}. \tag{25}$$

Thus, condition (9) holds. This completes the proof. □

To solve Theorem 3.2, the following theorem gives LMI conditions to ensure stochastic finite-time H_∞ stabilization via fuzzy state-feedback control design for the nominal fuzzy DMJS (1a)–(1c).

Theorem 3.3. *Consider the time-delay nominal fuzzy DMJS (7a) and (7b), there exists a state feedback control gain $K_{i,l} = Y_{i,l}X_l^{-1}$ such that the DMJS (7a) and (7b) is stochastically finite-time H_∞ stabilizable with respect to $(\delta_x, \epsilon, R_l, N, \gamma, \varpi)$, if there exist scalars $\lambda \geq 1, \sigma > 0, \epsilon > 0, \gamma > 0, \eta_1 > 0$ and $\eta_2 > 0$, a symmetric positive-definite matrix M, a set of mode-dependent symmetric positive-definite matrices $\{X_l, l \in \mathcal{S}\}$, a set of matrices $\{Y_{i,l}, i \in \Lambda, l \in \mathcal{S}\}$, for all $l \in \mathcal{S}$ and $i, j \in \Lambda$, such that*

$$\Xi_{ii,l} < 0, \tag{26a}$$

$$\Xi_{ij,l} + \Xi_{ji,l} < 0, \quad i < j, \tag{26b}$$

$$\sigma R_l^{-1} < X_l < R_l^{-1}, \tag{26c}$$

$$\eta_1 R_l^{-1} < M < \eta_2 R_l^{-1}, \tag{26d}$$

$$\begin{bmatrix} (\varpi^2\gamma^2 - \epsilon^2)\lambda^{-N} & * & * \\ \delta_x & -\sigma & * \\ \delta_x\sqrt{d} & 0 & -\eta_1 \end{bmatrix} < 0, \tag{26e}$$

where

$$\Xi_{ij,l} = \begin{bmatrix} -\lambda X_l & * & * & * & * \\ 0 & -\lambda^{-N}\gamma^2 I & * & * & * \\ L_{1ij,l} & L_{3i,l} & -X + L_{2i,l}ML_{2j,l}^T & * & * \\ L_{4ij,l} & D_{2i,l} & C_{di,l}ML_{2j,l}^T & -I + C_{di,l}MC_{dj,l}^T & * \\ X_l & 0 & 0 & 0 & -M \end{bmatrix},$$

$$L_{1ij,l}^T = \left[\sqrt{\pi_{l1}}A_{ij,l}^T \quad \sqrt{\pi_{l2}}A_{ij,l}^T \quad \cdots \quad \sqrt{\pi_{ls}}A_{ij,l}^T\right],$$
$$L_{2i,l}^T = \left[\sqrt{\pi_{l1}}A_{di,l}^T \quad \sqrt{\pi_{l2}}A_{di,l}^T \quad \cdots \quad \sqrt{\pi_{ls}}A_{di,l}^T\right],$$
$$L_{3i,l}^T = \left[\sqrt{\pi_{l1}}G_{i,l}^T \quad \sqrt{\pi_{l2}}G_{i,l}^T \quad \cdots \quad \sqrt{\pi_{ls}}G_{i,l}^T\right],$$
$$L_{4ij,l} = C_{i,l}X_l + D_{1i,l}Y_{j,l}, A_{ij,l} = A_{i,l}X_l + B_{i,l}Y_{j,l},$$
$$X = \text{diag}\{X_1, X_2, \ldots, X_s\}.$$

Proof. Denote

$$A_l(h) = \sum_{i=1}^{f}\sum_{j=1}^{f}h_i(k)h_j(k)[A_{i,l} + B_{i,l}K_{j,l}], A_{dl}(h) = \sum_{i=1}^{f}h_i(k)A_{di,l}, G_l(h) = \sum_{i=1}^{f}h_i(k)G_{i,l}.$$

Then, condition (20a) is converted into the following inequality:

$$\begin{bmatrix} -\lambda P_l + Q & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -\lambda^{-N}\gamma^2 I & * & * \\ L_{1\ l}(h) & L_{2\ l}(h) & L_{3\ l}(h) & -P^{-1} & * \\ C_l(h) & C_{dl}(h) & D_l(h) & 0 & -I \end{bmatrix} < 0, \tag{27}$$

where

$$L_{1\ l}^T(h) = [\sqrt{\pi_{l1}}A_l^T(h)\sqrt{\pi_{l2}}A_l^T(h)\ldots\sqrt{\pi_{ls}}A_l^T(h)],$$

$$L_{2\ l}^T(h) = [\sqrt{\pi_{l1}}A_{dl}^T(h)\ \sqrt{\pi_{l2}}A_{dl}^T(h)\ldots\sqrt{\pi_{ls}}A_{dl}^T(h)],$$

$$L_{3\ l}^T(h) = [\sqrt{\pi_{l1}}G_l^T(h)\sqrt{\pi_{l2}}G_l^T(h)\ldots\sqrt{\pi_{ls}}G_l^T(h)].$$

Applying the Schur complements, we can verify that Eq. (27) is equivalent to

$$\begin{bmatrix} -\lambda P_l + Q & * & * & * \\ 0 & -\lambda^{-N}\gamma^2 I & * & * \\ L_{1\ l}(h) & L_{3\ l}(h) & -P^{-1} + L_{2\ l}(h)Q^{-1}L_{2\ l}^T(h) & * \\ C_l(h) & D_l(h) & C_{dl}(h)Q^{-1}L_{2\ l}^T(h) & -I + C_{dl}(h)Q^{-1}C_{dl}^T(h) \end{bmatrix} < 0. \tag{28}$$

Pre- and -post-multiplying (28) by the block-diagonal matrix diag$\{P_l^{-1}, I, I, I\}$ and using Schur complement lemma, this results in the following inequality holds

$$\sum_{i=1}^{f}\sum_{j=1}^{f} h_i(k)h_j(k)\Theta_{ij,l} < 0, \tag{29}$$

where

$$\Theta_{ij,l} = \begin{bmatrix} -\lambda P_l^{-1} & * & * & * & * \\ 0 & -\lambda^{-N}\gamma^2 I & * & * & * \\ \overline{L}_{1ij,l}P_l^{-1} & L_{3i,l} & -P^{-1} + L_{2i,l}Q^{-1}L_{2j,l}^T & * & * \\ C_{ij,l}P_l^{-1} & D_{2i,l} & C_{di,l}Q^{-1}L_{2j,l}^T & -I + C_{di,l}Q^{-1}C_{dj,l}^T & * \\ P_l^{-1} & 0 & 0 & 0 & -Q^{-1} \end{bmatrix},$$

$$\overline{L}_{1ij,l}^T = [\sqrt{\pi_{l1}}\overline{A}_{ij,l}^T\ \sqrt{\pi_{l2}}\overline{A}_{ij,l}^T\ldots\sqrt{\pi_{ls}}\overline{A}_{ij,l}^T],$$
$$L_{2i,l}^T = [\sqrt{\pi_{l1}}A_{di,l}^T\sqrt{\pi_{l2}}A_{di,l}^T\ldots\sqrt{\pi_{ls}}A_{di,l}^T],$$
$$L_{3i,l}^T = [\sqrt{\pi_{l1}}G_{i,l}^T\sqrt{\pi_{l2}}G_{i,l}^T\ldots\sqrt{\pi_{ls}}G_{i,l}^T],$$
$$C_{ij,l} = C_{i,l} + D_{1i,l}K_{j,l}, \overline{A}_{ij,l} = A_{i,l} + B_{i,l}K_{j,l}.$$

Denote

$$L_{1ij,l}^T = (\overline{L}_{1ij,l}P_l^{-1})^T = [\sqrt{\pi_{l1}}A_{ij,l}^T\ \sqrt{\pi_{l2}}A_{ij,l}^T\ldots\sqrt{\pi_{ls}}A_{ij,l}^T],$$
$$L_{4ij,l} = C_{ij,l}P_l^{-1} = C_{i,l}P_l^{-1} + D_{1i,l}K_{j,l}P_l^{-1},$$

where $A_{ij,l}=\overline{A}_{ij,l}P_l^{-1}=A_{i,l}P_l^{-1}+B_{i,l}K_{j,l}P_l^{-1}$. By setting $X_l=P_l^{-1}, Y_{j,l}=K_{j,l}X_l,\ M=Q^{-1}, X=P^{-1}$, it follows that

$$\begin{aligned}&X=P^{-1}=\operatorname{diag}\{X_1,X_2,\ldots,X_s\},\\&A_{ij,l}=A_{i,l}X_l+B_{i,l}K_{j,l}X_l=A_{i,l}X_l+B_{i,l}Y_{j,l},\\&L_{4ij,l}=C_{i,l}X_l+D_{1i,l}K_{j,l}X_l=C_{i,l}X_l+D_{1i,l}Y_{j,l}.\end{aligned}$$

Thus, we can deduce that condition (29) is equivalent to the following inequality:

$$\sum_{i=1}^{f}\sum_{j>i}^{f}h_i(k)h_j(k)[\Xi_{ij,l}+\Xi_{ji,l}]+\sum_{i=1}^{f}h_i^2(k)\Xi_{ii,l}<0. \tag{30}$$

Therefore, conditions (26a) and (26b) can guarantee that condition (20a) holds.

Taking into account that $P_l=X_l^{-1}, \overline{P}_l=R_l^{-1/2}P_lR_l^{-1/2}$ and $\overline{Q}_l=R_l^{-1/2}QR_l^{-1/2}$, it follows from (23c) and (23d) that $1<\underline{\sigma}_{\overline{P}}, \overline{\sigma}_{\overline{P}}<\sigma^{-1}, \sigma_{\overline{Q}}<\eta_1^{-1}$. Then, a sufficient condition to guarantee (20b) is that

$$(\sigma^{-1}+\eta_1^{-1}d)\delta_x^2+\lambda^{-N}\gamma^2\varpi^2<\lambda^{-N}\epsilon^2. \tag{31}$$

By Schur complement property, condition (31) is equivalent to Eq. (26e). Further, conditions (23c)–(23e) can guarantee that condition (20b) holds. Thus, the proof of the theorem is completed. □

The following corollary is an easy consequence of Theorem 3.3.

Corollary 3.1. *Consider the time-delay nominal fuzzy DMJS (7a), there exists a state-feedback control gain $K_{i,l}=Y_{i,l}X_l^{-1}$ such that the fuzzy DMJS (7a) is SFTB with respect to $(\delta_x,\epsilon,R_l,N,\varpi)$, if there exist scalars $\lambda\geq 1,\sigma>0,\epsilon>0,\delta_1>0,\delta_2>0,\eta_1>0$ and $\eta_2>0$, a symmetric positive-definite matrix M, sets of mode-dependent symmetric positive-definite matrices $\{X_l, l\in\mathcal{S}\}$ and $\{Q_l, l\in\mathcal{S}\}$, a set of matrices $\{Y_{i,l}, i\in\Lambda, l\in\mathcal{S}\}$, for all $l\in\mathcal{S}$ and $i,j\in\Lambda$, such that* (26c) and (26d) *and the following conditions hold*

$$\overline{\Xi}_{ii,l}<0, \tag{32a}$$

$$\overline{\Xi}_{ij,l}+\overline{\Xi}_{ji,l}<0,\quad i<j, \tag{32b}$$

$$\delta_1 I<Q_l<\delta_2 I, \tag{32c}$$

$$\begin{bmatrix}\varpi^2\delta_2-\epsilon^2\lambda^{-N} & * & * \\ \delta_x & -\sigma & * \\ \delta_x\sqrt{d} & 0 & -\eta_1\end{bmatrix}<0, \tag{32d}$$

where

$$\overline{\Xi}_{ij,l}=\begin{bmatrix}-\lambda X_i & * & * & \\ 0 & -Q_l & * & * \\ L_{1ij,l} & L_{3i,l} & -X+L_{2i,l}ML_{2j,l}^T & * \\ X_l & 0 & 0 & -M\end{bmatrix}.$$

Consider the robust finite-time H_∞ stabilization problem of the time-delay uncertain fuzzy DMJS (1a)–(1c), we have the following theorem stated:

Theorem 3.4. *Consider the time-delay uncertain fuzzy DMJS (7a) and (7b), there exists a state-feedback control gain $K_{i,l} = Y_{i,l}X_l^{-1}$ such that the DMJS (7a) and (7b) is stochastically finite-time H_∞ stabilizable with respect to $(\delta_x, \epsilon, R_l, N, \gamma, \varpi)$, if there exist scalars $\lambda \geq 1, \sigma > 0, \epsilon > 0, \gamma > 0, \eta_1 > 0$ and $\eta_2 > 0$, a set of positive scalars $\{\theta_{ij,l}, i, j \in \Lambda, l \in \mathcal{S}\}$, a mode-dependent symmetric positive-definite matrix M, a set of mode-dependent symmetric positive-definite matrices $\{X_l, l \in \mathcal{S}\}$, a set of matrices $\{Y_{i,l}, i \in \Lambda, l \in \mathcal{S}\}$, for all $l \in \mathcal{S}$ and $i, j \in \Lambda$, such that Eqs. (26c)–(26e) and the following inequalities hold*

$$\Gamma_{ii,l} = \begin{bmatrix} \Gamma_{1ii,l} & * \\ \Gamma_{2ii,l} & \Gamma_{3ii,l} \end{bmatrix} < 0, \tag{33a}$$

$$\Lambda_{ij,l} = \begin{bmatrix} \Lambda_{1ij,l} & * \\ \Lambda_{2ij,l} & \Lambda_{3ij,l} \end{bmatrix} < 0, \quad i < j, \tag{33b}$$

where

$$\Gamma_{1ii,l} = \begin{bmatrix} -\lambda X_l & * & * \\ 0 & -\lambda^{-N}\gamma^2 I & * \\ L_{1ii,l} & L_{3i,l} & -X + L_{2i,l}ML_{2i,l}^T + \overline{F}_{ii,l} \end{bmatrix},$$

$$\Gamma_{2ii,l} = \begin{bmatrix} L_{4ii,l} & D_{2i,l} & C_{di,l}ML_{2i,l}^T \\ L_{5ii,l} & E_{4i,l} & E_{2i,l}ML_{2i,l}^T \\ X_l & 0 & 0 \end{bmatrix},$$

$$\Gamma_{3ii,l} = \begin{bmatrix} -I + C_{di,l}MC_{di,l}^T & * & * \\ E_{2i,l}MC_{di,l}^T & -\theta_{ii,l}I + E_{2i,l}ME_{2i,l}^T & * \\ 0 & 0 & -M \end{bmatrix},$$

$$\Lambda_{1ij,l} = \begin{bmatrix} -2\lambda X_l & * & * \\ 0 & -2\lambda^{-N}\gamma^2 I & * \\ L_{1ij,l} + L_{1ji,l} & L_{3i,l} + L_{3j,l} & -2X + \frac{1}{2}L_{2ij,l}ML_{2ij,l}^T + \overline{F}_{ij,l} + \overline{F}_{ji,l} \end{bmatrix},$$

$$\Lambda_{2ij,l} = \begin{bmatrix} L_{4ij,l} + L_{4ji,l} & D_{2i,l} + D_{2j,l} & \frac{1}{2}C_{dij,l}MC_{dij,l}^T \\ L_{5ij,l} & E_{4i,l} & \frac{1}{2}E_{2i,l}ML_{2ij,l}^T \\ L_{5ji,l} & E_{4j,l} & \frac{1}{2}E_{2j,l}ML_{2ij,l}^T \\ X_l & 0 & 0 \end{bmatrix},$$

$$\Lambda_{3ij,l} = \begin{bmatrix} -2I + \frac{1}{2}C_{dij,l}MC_{dij,l}^T & * & * & * \\ \frac{1}{2}E_{2i,l}MC_{dij,l}^T & -\theta_{ij,l}I + \frac{1}{2}E_{2i,l}ME_{2i,l}^T & * & * \\ \frac{1}{2}E_{2j,l}MC_{dij,l}^T & \frac{1}{2}E_{2j,l}ME_{2i,l}^T & -\theta_{ji,l}I + \frac{1}{2}E_{2j,l}ME_{2j,l}^T & * \\ 0 & 0 & 0 & -\frac{1}{2}M \end{bmatrix},$$

$\overline{F}_{ij,l} = \theta_{ij,l} \times [\sqrt{\pi_{l1}}F_{i,l}^T \cdots \sqrt{\pi_{ls}}F_{i,l}^T]^T[\sqrt{\pi_{l1}}F_{i,l}^T \cdots \sqrt{\pi_{ls}}F_{i,l}^T]$,
$L_{2ij,l} = L_{2i,l} + L_{2j,l}, C_{dij,l} = C_{di,l} + C_{dj,l}, L_{5ij,l} = E_{1i,l}X_l + E_{3i,l}Y_{j,l}$.

Proof. By Theorem 3.3, we only need to prove that Eqs. (33a) and (33b) can guarantee that Eq. (20a) holds. Pre- and -post-multiplying Eq. (20a) by the block-diagonal matrix $\mathrm{diag}\{P_l^{-1}, I, I, I, I\}$, we can observe that Eq. (20a) is equivalent to the following inequality:

$$\sum_{i=1}^{f}\sum_{j=1}^{f} h_i(k)h_j(k)[\Psi_{0ij,l} + \Psi_{1ij,l}(k)] < 0, \tag{34}$$

where

$$\Psi_{0ij,l} = \begin{bmatrix} -\lambda P_l^{-1} + P_l^{-1}QP_l^{-1} & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -\lambda^{-N}\gamma^2 I & * & * \\ \overline{L}_{1ij,l}P_l^{-1} & L_{2i,l} & L_{3i,l} & -P^{-1} & * \\ C_{ij,l}P_l^{-1} & C_{di,l} & D_{2i,l} & 0 & -I \end{bmatrix},$$

$$\Psi_{1ij,l}(k) = U_{i,l}\Delta_l(k)V_{ij,l} + (U_{i,l}\Delta_l(k)V_{ij,l})^T,$$

with

$$U_{i,l}^T = [0\ 0\ 0\ \sqrt{\pi_{l1}}F_{i,l}^T \ldots \sqrt{\pi_{ls}}F_{i,l}^T\ 0],$$
$$V_{ij,l} = [E_{1i,l}P_l^{-1} + E_{3i,l}K_{j,l}P_l^{-1}\ E_{2i,l}\ E_{4i,l}\ 0\ \ldots\ 0\ 0].$$

According to Lemma 2.1, for arbitrary $\theta_{ij,l} > 0, i, j \in \Lambda, l \in \mathcal{S}$, we have

$$U_{i,l}\Delta_l(k)V_{ij,l} + (U_{i,l}\Delta_l(k)V_{ij,l})^T \le \theta_{ij,l}U_{i,l}U_{i,l}^T + \theta_{ij,l}^{-1}V_{ij,l}^TV_{ij,l}. \tag{35}$$

Notice that $\Psi_{0ij,l}$ is a symmetric matrix. Thus, a sufficient condition to guarantee Eq. (34) is that

$$\sum_{i=1}^{f}\sum_{j=1}^{f} h_i(k)h_j(k)\Psi_{ij,l} < 0, \tag{36}$$

where $\Psi_{ij,l} = (\Psi_{0ij,l} + \theta_{ij,l}U_{i,l}U_{i,l}^T) + \theta_{ij,l}^{-1}V_{ij,l}^TV_{ij,l}$. Thus, we can deduce that condition (36) is equivalent to the following inequality:

$$\sum_{i=1}^{f}\sum_{j>i}^{f} h_i(k)h_j(k)[\Psi_{ij,l} + \Psi_{ji,l}] + \sum_{i=1}^{f} h_i^2(k)\Psi_{ii,l} < 0. \tag{37}$$

Therefore, it follows that a sufficient condition to guarantee Eq. (37) is that

$$\Psi_{ii,l} < 0, \tag{38a}$$

$$\Psi_{ij,l} + \Psi_{ji,l} < 0. \tag{38b}$$

Applying similar approach to Theorem 3.3, we can prove that Eqs. (33a) and (33b) are equivalent to Eqs. (38a) and (38b), respectively. Thus, Eqs. (33a) and (33b) can guarantee that Eq. (20a) holds. □

By Theorem 3.4, the following corollary can be easily obtained.

Corollary 3.2. *Consider the time-delay uncertain fuzzy DMJS (7a), there exists a state-feedback control gain $K_{i,l} = Y_{i,l}X_l^{-1}$ such that the DMJS (7a) is SFTB with respect to $(\delta_x, \epsilon, R_l, N, \varpi)$, if there exist scalars $\lambda \ge 1, \sigma > 0, \epsilon > 0, \delta_1 > 0, \delta_2 > 0, \eta_1 > 0$ and $\eta_2 > 0$, a set of positive scalars $\{\theta_{ij,l}, i, j \in \Lambda, l \in \mathcal{S}\}$, a mode-dependent symmetric positive-definite matrix M, sets of* *mode-dependent symmetric positive-definite matrices* $\{X_l, l \in \mathcal{S}\}$ *and* $\{Q_l, l \in \mathcal{S}\}$, *a set of matrices* $\{Y_{i,l}, i \in \Lambda, l \in \mathcal{S}\}$, *for all* $l \in \mathcal{S}$ *and* $i, j \in \Lambda$, *such that* Eqs. (26c), (26d), (32c), (32d) *and the following conditions hold*

$$\overline{\Gamma}_{ii,l} = \begin{bmatrix} \overline{\Gamma}_{1ii,l} & * \\ \overline{\Gamma}_{2ii,l} & \overline{\Gamma}_{3ii,l} \end{bmatrix} < 0, \tag{39a}$$

$$\overline{\Lambda}_{ij,l} = \begin{bmatrix} \overline{\Lambda}_{1ij,l} & * \\ \overline{\Lambda}_{2ij,l} & \overline{\Lambda}_{3ij,l} \end{bmatrix} < 0, \quad i < j, \tag{39b}$$

where

$$\overline{\Gamma}_{1ii,l} = \begin{bmatrix} -\lambda X_l & * & * \\ 0 & -Q_l & * \\ L_{1ii,l} & L_{3i,l} & -X + L_{2i,l} M L_{2i,l}^T + \overline{F}_{ii,l} \end{bmatrix},$$

$$\overline{\Gamma}_{2ii,l} = \begin{bmatrix} L_{5ii,l} & E_{4i,l} & E_{2i,l} M L_{2i,l}^T \\ X_l & 0 & 0 \end{bmatrix},$$

$$\overline{\Gamma}_{3ii,l} = \begin{bmatrix} -\theta_{ii,l} I + E_{2i,l} M E_{2i,l}^T & * \\ 0 & -M \end{bmatrix},$$

$$\overline{\Lambda}_{1ij,l} = \begin{bmatrix} -2\lambda X_l & * & * \\ 0 & -2Q_l & * \\ L_{1ij,l} + L_{1ji,l} & L_{3i,l} + L_{3j,l} & -2X + \frac{1}{2} L_{2ij,l} M L_{2ij,l}^T + \overline{F}_{ij,l} + \overline{F}_{ji,l} \end{bmatrix}$$

$$\overline{\Lambda}_{2ij,l} = \begin{bmatrix} L_{5ij,l} & E_{4i,l} & \frac{1}{2} E_{2i,l} M L_{2ij,l}^T \\ L_{5ji,l} & E_{4j,l} & \frac{1}{2} E_{2j,l} M L_{2ij,l}^T \\ X_l & 0 & 0 \end{bmatrix},$$

$$\overline{\Lambda}_{3ij,l} = \begin{bmatrix} -\theta_{ij,l} I + \frac{1}{2} E_{2i,l} M E_{2i,l}^T & * & * \\ \frac{1}{2} E_{2j,l} M E_{2i,l}^T & -\theta_{ji,l} I + \frac{1}{2} E_{2j,l} M E_{2j,l}^T & * \\ 0 & 0 & -\frac{1}{2} M \end{bmatrix}.$$

Remark 3.1. It is significant to observe that conditions (26a), (26b), (26e), (33a) and (33b) are not strict LMIs, however, once we fix the parameter λ, the conditions can be turned into LMIs-based feasibility problem. Thus, one can obtain that the feasibility of conditions stated in Theorems 3.3 and 3.4 can be turned into the following LMIs-based feasibility problem with a fixed parameter λ, respectively:

$$\begin{aligned} &\min_{X_l, Y_{i,l}, M, \eta_1, \eta_2, \sigma} \ (\epsilon^2 + \gamma^2) \\ &\text{s.t.} \quad \text{LMIs(26a)–(26e)}. \end{aligned} \tag{40}$$

$$\begin{aligned} &\min_{X_l, Y_{i,l}, M, \eta_1, \eta_2, \theta_{ij,l}, \sigma} \ (\epsilon^2 + \gamma^2) \\ &\text{s.t.} \quad \text{LMIs(26c)–(26e)}, \text{(33a) and (33b)}. \end{aligned} \tag{41}$$

Remark 3.2. In order to get the minimum optimal value of $\epsilon^2 + \gamma^2$, by Theorems 3.3 and 3.4, one can first find the range of the feasible solution of the parameter λ, and then the locally convergent solution can be obtained by applying the program *fminsearch* in the unconstrained nonlinear optimization toolbox of Matlab.

Remark 3.3. Noted that Theorems 3.3 and 3.4 have presented the sufficient conditions to guarantee stochastic finite-time H_∞ stabilization of the nominal or uncertain discrete-time fuzzy MJSs with time-delays, respectively. From the optimization problem (40) or (41), one can see that $\epsilon^2+\gamma^2$ is taken as the optimal value and optimized over value $\epsilon^2+\gamma^2$. Similarly, we can also choose γ^2 as the optimal value and optimize over value γ^2. However, the former optimization approach should be suitable in order to let the system trajectories take the smaller bound as much as possible in a specified finite-time interval.

Remark 3.4. For conservatism reduction purpose, fuzzy or piecewise Lyapunov functional approaches can be applied to addressing the problems of H_∞ filtering and output-feedback control for fuzzy nonlinear systems with Markovian jumps and time-delays, see the Refs. [4,10,16]. Based on the novel Lyapunov functionals, we may also deal with the finite-time H_∞ control problem for continuous- or discrete-time fuzzy MJSs with time-varying delays and obtain the corresponding results in the future work.

4. Numerical examples

In this section, we provide two simulation examples to illustrate the proposed results.

Example 4.1. Consider the following time-delay nominal fuzzy DMJS involving two modes with the following parameters:

$$A_{1,1}=\begin{bmatrix}1.2 & 0.5\\ 0.8 & 1\end{bmatrix},\quad A_{d1,1}=\begin{bmatrix}0.6 & 0\\ 0 & -0.5\end{bmatrix},\quad A_{1,2}=\begin{bmatrix}-0.6 & 0.4\\ 1 & 1.5\end{bmatrix},$$

$$A_{d1,2}=\begin{bmatrix}-0.6 & 0.1\\ 0 & -0.4\end{bmatrix},\quad A_{2,1}=\begin{bmatrix}-0.5 & 0.2\\ 1 & 1\end{bmatrix},\quad A_{d2,1}=\begin{bmatrix}0.5 & -0.41\\ 0 & -0.6\end{bmatrix},$$

$$A_{2,2}=\begin{bmatrix}-0.6 & 0.8\\ 0.1 & 1.2\end{bmatrix},\quad A_{d2,2}=\begin{bmatrix}0.6 & -0.4\\ 0 & -0.5\end{bmatrix},\quad B_{1,1}=B_{2,1}=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix},$$

$$B_{1,2}=B_{2,2}=\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix},\quad G_{1,1}=G_{2,1}=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix},\quad G_{1,2}=G_{2,2}=\begin{bmatrix}0 & 1\\ 0 & 0.5\end{bmatrix},$$

$$C_{1,1}=C_{2,1}=C_{1,2}=C_{2,2}=\begin{bmatrix}0.5 & 0\end{bmatrix},\quad C_{d1,1}=C_{d2,1}=C_{d1,2}=C_{d2,2}=\begin{bmatrix}0.2 & 0\end{bmatrix},$$

$$D_{11,1}=D_{12,1}=D_{11,2}=D_{12,2}=\begin{bmatrix}0.5 & 0.2\end{bmatrix},\quad D_{21,1}=D_{22,1}=D_{21,2}=D_{22,2}=\begin{bmatrix}1 & 1\end{bmatrix}.$$

We assume that the transition rate matrix is given by

$$\Pi=\begin{bmatrix}0.6 & 0.4\\ 0.3 & 0.7\end{bmatrix}.$$

For simplicity, $h_1(k)$ and $h_2(k)$ are fuzzy basis functions defined, respectively, as $h_1(k)=h(x_1(k))$ and $h_2(k)=1-h(x_1(k))$ with

$$h(x_1(k)) = \begin{cases} \frac{1}{6}[3 - x_1(k)], & |x_1(k)| < 3, \\ 1, & |x_1(k)| \geq 3. \end{cases}$$

Let $R_1 = R_2 = I_2, \delta_x = 1, N = 5, \varpi = 2$ and $d = 1$, by Theorem 3.3, the optimal bound with minimum value of $\epsilon^2 + \gamma^2$ relies on the parameter λ. One can find a feasible solution when $1.90 \leq \lambda \leq 36.50$. Fig. 1 shows the optimal value with different values of λ. Then, by using the program *fminsearch* in the optimization toolbox of Matlab starting at $\lambda = 2$, we can also find the locally convergent solution $\lambda = 2.1626$, $\gamma = 15.7421$, $\epsilon = 33.9467$, and the state-feedback control gains can be derived as

$$K_{1,1} = \begin{bmatrix} 0.2804 & -0.4109 \\ -1.2848 & -0.4353 \end{bmatrix}, \quad K_{2,1} = \begin{bmatrix} -1.4482 & -0.5412 \\ 0.4945 & -0.2787 \end{bmatrix},$$

$$K_{1,2} = \begin{bmatrix} -1.3937 & -0.0070 \\ 0.0348 & -0.8134 \end{bmatrix}, \quad K_{2,2} = \begin{bmatrix} -0.7539 & 0.5136 \\ -0.4498 & -0.9513 \end{bmatrix}.$$

Now, we set the initial conditions as $x(-1) = x(0) = [0.6 \;\; -0.8]^T$ and the initial mode $r_0 = 1$. We further assume that $w(k) = [0.9e^{-k} \;\; 0.8e^{-k} \;\; \sin\, k]^T$, then, the simulation of the jumping

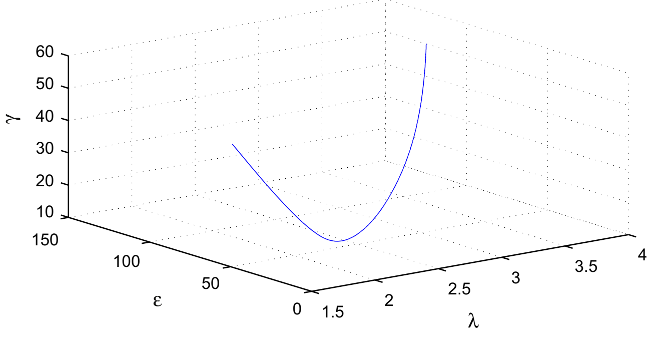

Fig. 1. The local optimal bound of ϵ and γ.

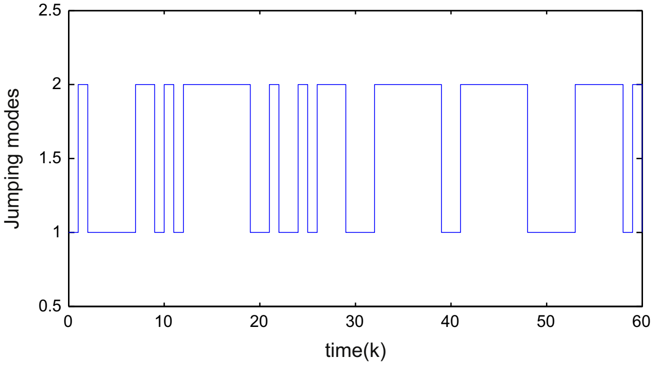

Fig. 2. The switching signal of the system with the initial mode 1.

modes and the state response of the closed-loop time-delay fuzzy DMJS are depicted in Figs. 2 and 3, which show the effectiveness of the proposed methods.

Example 4.2. To show stochastic finite-time boundedness of the time-delay uncertain fuzzy DMJS with disturbance attenuation γ, let

$$F_{1,1} = F_{2,1} = F_{1,2} = F_{2,2} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T,$$

$$E_{11,1} = F_{12,1} = F_{11,2} = F_{12,2} = \begin{bmatrix} 0.01 & 0.1 \end{bmatrix},$$

$$E_{21,1} = F_{22,1} = F_{21,2} = F_{22,2} = \begin{bmatrix} 0 & 0.01 \end{bmatrix},$$

$$E_{31,1} = F_{32,1} = F_{31,2} = F_{32,2} = \begin{bmatrix} 0 & 0.1 \end{bmatrix},$$

$$E_{41,1} = F_{42,1} = F_{41,2} = F_{42,2} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},$$

and $\Delta_l(k) = [\varsigma_l(k)]$, where $\varsigma_l^T(k)\varsigma_l(k) \leq 1$ for all $l \in \{1, 2\}$. Moreover, other matrix variables and the transition rate matrix are defined in Example 4.1.

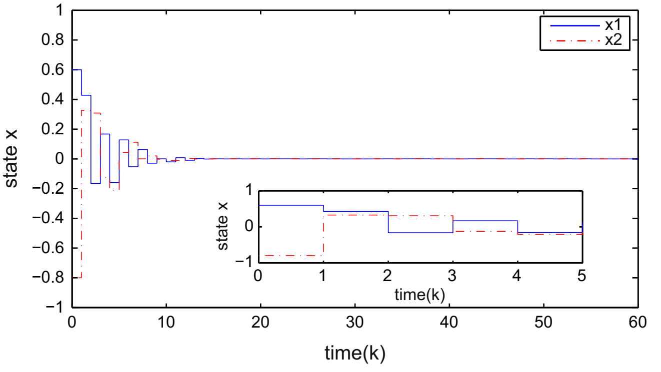

Fig. 3. The response of the system state $x(k)$ in the different time interval.

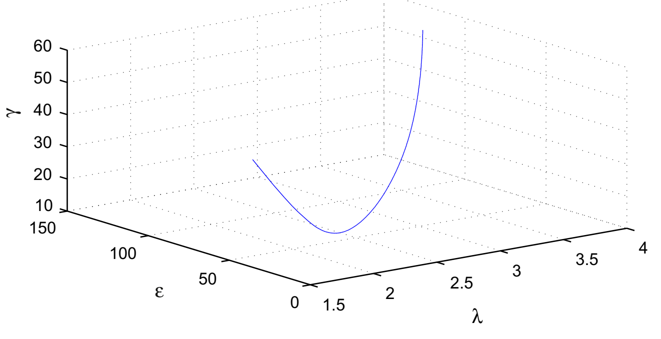

Fig. 4. The local optimal bound of ϵ and γ.

Let $R_1 = R_2 = I_2, \delta_x = 1, N = 5, \varpi = 2$ and $d = 1$, by Theorem 3.4, the optimal bound with minimum value of $\epsilon^2 + \gamma^2$ relies on the parameter λ. One can find a feasible solution when $1.95 \leq \lambda \leq 36.34$. Fig. 4 shows the optimal value with different values of λ. Then, by using the program *fminsearch* in the optimization toolbox of Matlab starting at $\lambda = 2$, we can also find the locally convergent solution $\lambda = 2.1644$, $\gamma = 16.1363, \epsilon = 34.6719$ and the state-feedback control gains can be derived as

$$K_{1,1} = \begin{bmatrix} 0.2380 & -0.3724 \\ -1.2400 & -0.4708 \end{bmatrix}, \quad K_{2,1} = \begin{bmatrix} -1.3966 & -0.4939 \\ 0.4585 & -0.3122 \end{bmatrix},$$

$$K_{1,2} = \begin{bmatrix} -1.4625 & 0.0439 \\ 0.0245 & -0.7991 \end{bmatrix}, \quad K_{2,2} = \begin{bmatrix} -0.4601 & 0.7330 \\ -0.4588 & -0.9601 \end{bmatrix}.$$

Figs. 5 and 6 give the jumping modes and the state response of the closed-loop time-delay uncertain fuzzy DMJS under the same initial conditions, parameters $w(k)$ and $\varsigma_l(k)$ as Example 4.1.

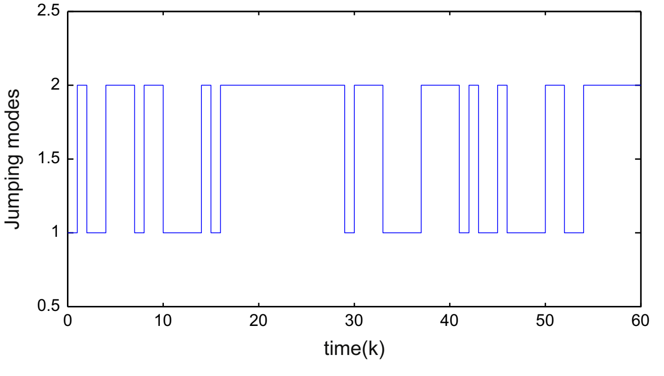

Fig. 5. The switching signal of the system with the initial mode 1.

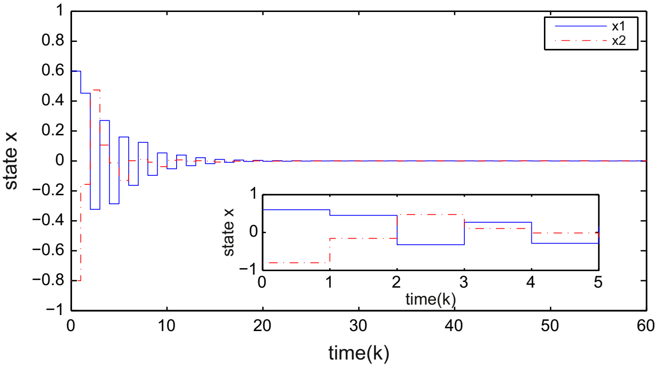

Fig. 6. The response of the system state $x(k)$ in the different time interval.

5. Conclusions

This paper studies the problem of robust finite-time H_∞ control for a class of uncertain discrete-time Markovian jump T–S fuzzy systems with time-delays. By applying the stochastic Lyapunov–Krasovskii function method, a finite-time H_∞ controller is designed such that the resulting closed-loop system is stochastically finite-time bounded and satisfies a prescribed H_∞ performance level in the given finite-time interval. Sufficient criteria are provided for the solvability of the problem, which can be tackled by a feasibility problem in terms of LMIs. Two examples are presented to show the validity of the proposed design approaches. It should be also pointed out that stochastic finite-time H_∞ filtering problems of time-delay fuzzy MJSs including continuous-time case and discrete-time one will be investigated in our future work.

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