An Algorithm for Clock Synchronization with the Gradient Property in Sensor Networks

Rodolfo M. Pussente Valmir C. Barbosa^{*}

Universidade Federal do Rio de Janeiro Programa de Engenharia de Sistemas e Computação, COPPE Caixa Postal 68511 21941-972 Rio de Janeiro - RJ, Brazil

Abstract

We introduce a distributed algorithm for clock synchronization in sensor networks. Our algorithm assumes that nodes in the network only know their immediate neighborhoods and an upper bound on the network's diameter. Clock-synchronization messages are only sent as part of the communication, assumed reasonably frequent, that already takes place among nodes. The algorithm has the gradient property of [2], achieving an O(1) worst-case skew between the logical clocks of neighbors. As in the case of [3, 8], the algorithm's actions are such that no constant lower bound exists on the rate at which logical clocks progress in time, and for this reason the lower bound of [2, 5] that forbids constant skew between neighbors does not apply.

Keywords: Distributed computing, Sensor networks, Clock synchronization, Gradient property in clock synchronization.

1 Introduction

We consider a network of sensors and assume it may be represented by a connected undirected graph G = (N, E) whose nodes stand for sensors and undirected edges for bidirectional communication channels. We also assume that channels are fully reliable and deliver messages with delays bounded by a constant. We let n = |N|, use $N_i \subset N$ to denote the set of node *i*'s neighbors, and $t \geq 0$ to denote real time.

No node has access to the value of t but rather relies on a hardware clock to estimate it. For node i, the hardware clock at time t is denoted by $H_i(t) \ge 0$.

^{*}Corresponding author (valmir@cos.ufrj.br).

Ideally, $H_i(t)$ should evolve in "lockstep" with t, but we assume instead that its progress occurs at a positive rate that may drift as t elapses. We assume an additive drift, which at time t is denoted by $\rho_i(t) \in [-\hat{\rho}, \hat{\rho}]$ for some constant $\hat{\rho} \in [0, 1)$. The rate at which $H_i(t)$ progresses is then $1 + \rho(t)$ at time t, and it follows that

$$H_i(t) = \int_{r=0}^t [1 + \rho(r)] \, dr. \tag{1}$$

Because the instantaneous drifts may be different throughout G for any given t, nodes may only acquire a common estimate of real time by resorting to clock synchronization. At node i, this amounts to maintaining a logical clock $L_i(t) \geq 0$ that normally progresses at a rate proportional to that of the node's hardware clock but can be updated as i learns about the logical clocks of other nodes in G.

We assume that $L_i(t)$ is never allowed to run backwards (i.e., $L_i(t') \ge L_i(t)$ for all t' > t), and note that this is sometimes made more stringent by requiring a constant lower bound $b \in (0, 1]$ on the rate of progress of every node's logical clock.¹ When the latter is the case, enforcing the requirement is easy if $\hat{\rho}$ is known to the nodes: it suffices to set $dL_i(t)/dH_i(t) \ge b/(1-\hat{\rho})$, since

$$\frac{dL_i(t)}{dt} = \frac{dL_i(t)}{dH_i(t)} \frac{dH_i(t)}{dt} \ge \frac{b}{1-\hat{\rho}} (1-\hat{\rho}) = b.$$
(2)

The goal of a distributed algorithm for clock synchronization is to minimize the skew $|L_i(t) - L_j(t)|$ for all pairs i, j of distinct nodes and all t. While significant progress was achieved in the past (cf., e.g., [3, 7, 8, 1]), with a single exception to be discussed shortly it seems that all algorithms to date admit a worst-case skew of O(D), where D is the diameter of G, even between neighbors in the graph. The problem with this in the context of sensor networks is that, for tasks as fundamental as that of data fusion [6], for example, nearby nodes must synchronize their clocks much more strictly than this, while for distant nodes the larger skew is not a problem.

This observation has motivated the introduction in [2] of a new property of clock skews, the so-called gradient property. For f a positive, nondecreasing real function of distances in G, and d_{ij} the distance between nodes i and j, the gradient property requires

$$|L_i(t) - L_j(t)| \le f(d_{ij}) \tag{3}$$

for all pairs i, j of distinct nodes and all t. To our knowledge, the only algorithm to date that guarantees clock skews for which the gradient property holds is the one of [4]. In this algorithm, we have that

$$f(d_{ij}) \text{ is } \begin{cases} O(d_{ij}\sqrt{D}), & \text{if } d_{ij} \le \sqrt{D+1}; \\ O(D), & \text{otherwise,} \end{cases}$$
(4)

so in the worst case the clock skew between neighbors in G is $O(\sqrt{D})$.

¹As in [7, 1], but not in [3, 8], for example.

Achieving this, however, requires a relatively strong assumption on what is known to the nodes and also that nodes communicate frequently with their neighbors. The assumption is that both D, the graph's diameter, and $\hat{\rho}$, the maximum drift of hardware-clock rates, are known to all nodes. As for communicating with neighbors, a node is required to do so whenever its logical clock reaches a new integer value or is updated in the wake of the reception of a message.

While for some sensor networks the assumption may be regarded as reasonable, since it may be possible to bound both D and $\hat{\rho}$ from above in the environment in question, we find the need for frequent communication with neighbors to be generally incompatible with the power-consumption constraints normally associated with sensor networks. So we maintain the assumption, in part, but strive to reduce communication requirements as much as possible.

2 A new algorithm

Unlike the algorithm of [4], the algorithm we introduce in this paper targets sensor networks directly. For this reason, we adopt the same two assumptions as [5] regarding the communication among sensors:

- (i) Messages sent between neighbors in G are delivered instantaneously;
- (ii) If t and t' are instants at which two neighbors communicate in one of the two directions without any intervening communication in the same direction between them in the meantime, then $|t t'| \le d$ for some d > 0.

We aim at synchronizing clocks without any messages sent exclusively for this purpose, that is, by attaching clock-synchronization messages to whatever communication is already guaranteed to take place by assumption (ii).

We assume that nodes know their local neighborhoods (i.e., the neighbor set N_i for node i) and, like [4], that the diameter D (or an upper bound on it) is also known to them. We assume further that no node has access to the value of $\hat{\rho}$ or d, and that clock synchronization is started concurrently at any number of nodes, from which it propagates. If i is one of these nodes, then we assume $L_i(t) = 0$ for t the time at which clock synchronization is started at node i; if not, then we assume $L_i(t) = 0$ for t the time at which clock synchronization is first reached by a clock-synchronization message.

For $j \in N_i$, node *i* maintains a variable L_i^j to store its current view of the logical clock of *j*. If *t* is the instant at which $L_j(t)$ is communicated by *j* to *i*, and if L_i^j results from this communication, then assumption (i) implies that $L_i^j = L_j(t)$. For all $i \in N$ and all $j \in N_i$, we assume $L_i^j = 0$ before the reception at *i* of the first clock-synchronization message from *j*.

Now let α_i be the number by which the current rate of progress of $L_i(t)$ is proportional to that of $H_i(t)$; that is, let $\alpha_i = dL_i(t)/dH_i(t)$. Our algorithm uses α_i as the minimum of multiple α_i^j 's, one for each of node *i*'s neighbors, that is,

$$\alpha_i = \min_{j \in N_i} \alpha_i^j. \tag{5}$$

We assume that, initially, $\alpha_i^j = 1$ for all $i \in N$ and all $j \in N_i$. Our algorithm is based on lowering α_i as needed whenever node *i* detects, upon receiving a clocksynchronization message, that its logical clock is ahead of that of the message's sender by a certain amount *c* or more.

Other than this manipulation of α_i , our algorithm strives at node *i* to advance $L_i(t)$, if appropriate, toward the greatest L_i^j , so long as this does not leave the least L_i^j behind by the same *c* as above or more. We now describe our algorithm in terms of how node *i* responds to the reception of $\langle L \rangle$ in a clock-synchronization message from node $j \in N_i$ at time *t*. Notice that, by assumption (i), $L = L_j(t)$. Node *i*'s response to the message from *j* comprises the following two steps, whose processing is also assumed instantaneous.

Step 1. $L_i^j := L$.

Step 2. With $L^- = \min_{j \in N_i} L_i^j$ and $L^+ = \max_{j \in N_i} L_i^j$:

- (a) If $L_i(t) \ge L_i^j + c$, then $\alpha_i^j := 1/D$, otherwise $\alpha_i^j := 1$.
- (b) $L_i(t) := \max\{L_i(t), \min\{L^- + c, L^+\}\}.$

Step 1 is devoted simply to updating node *i*'s view of node *j*'s logical clock. Step 2 attempts to reduce α_i^j to 1/D, in case $L_i(t) \ge L_i^j + c$ (and thus $L_i(t) \ge L^- + c$); or to restore α_i^j to 1, in case $L_i(t) < L_i^j + c$; or yet to advance $L_i(t)$, in case both $L_i(t) < L^- + c$ and $L_i(t) < L^+$.

The value of $L_i(t)$ that results from Steps 1 and 2 continues to evolve before it gets sent in a clock-synchronization message to some of *i*'s neighbors. If such a message is sent at some time *t'* before *i* receives the next clock-synchronization message, then $\langle L_i(t') \rangle$ gets sent along with it such that $L_i(t') \leq L_i(t) + \alpha_i(1+\hat{\rho})d$.

3 Worst-case clock skews

Step 2(b), with $c = (1 + \hat{\rho})\sqrt{D + 1}$, is the essence of the algorithm in [4]. The reason why that algorithm guarantees a maximum skew of $O(\sqrt{D})$ between the logical clocks of neighbors in G is intimately related to this particular choice for c and to how this choice relates to the worst-case skew between any two nodes, which is always less than $(1 + \hat{\rho})D + 1$. There are other factors involved, but this one is crucial and a closer examination of [4] reveals that choosing c to be O(1), for example, disrupts the clocks' gradient property.

Considered within the assumptions of our model, the problem with letting c be O(1) in Step 2(b) is that a length-O(D) wait chain may occur in G in which each node finds out that its logical clock is ahead of the next node's by at least c. In this chain, the node whose logical clock is ahead of all others' may dart still farther ahead unchecked for an O(D) amount of time, which will then

be the worst-case skew between neighbors. So, in order to accommodate the possibility of a constant value for c along with the gradient property for some f, a mechanism is needed to slow down the progress of logical clocks that are ahead of others by c or more. This is what Step 2(a) does, provided $c \leq (1 + \hat{\rho})d$, as we assume henceforth. As we demonstrate shortly, an f is achieved that implies constant skew between the logical clocks of neighbors.

Let us now examine the worst-case skews that logical clocks may have under Steps 1 and 2. We start with the skew between any two nodes, in which case it suffices that we consider a chain of D + 1 nodes and the algorithm's start-up process. After the algorithm is initiated by one of the nodes (this gives us the worst case as far as the number of initiators is concerned), it may take as long as Dd time units for all others to have started their logical clocks, during which time the initiator may advance its logical clock from 0 to at most $(1 + \hat{\rho})Dd$. This is then the largest skew between any two logical clocks.

We now turn to the worst-case skew between the logical clocks of neighbors in G. As we indicated above, Step 2(a) has a crucial role to play in ensuring that this skew remains bounded within the desired limits of O(1). In order to see that this is really the case, first recall that, in the absence of Step 2(a), Step 2 would be ineffectual at time t if we had $L_i(t) \ge L_j(t) + c$. The following, then, is fundamentally dependent on Step 2(a).

Let us consider the same (D + 1)-node chain as above and look at the situation in which $L_i(t) = L_j(t) + c$, $L_j(t) = L_k(t) + c$ for some $k \neq i$, and so on through the chain. Clearly, this scenario can only involve so many edges of the chain. If we let ℓ be this number of edges, then our previous result on the maximum skew between any two nodes, together with the fact that $c \leq (1 + \hat{\rho})d$, implies that

$$\ell = \min\left\{D, \frac{(1+\hat{\rho})Dd}{c}\right\} = D.$$
(6)

By time t + d, each of the first ℓ nodes in the chain (i, j, and so on) has found out that it is waiting for its neighbor down the chain to catch up with it, and consequently has reduced its rate to 1/D. The $\ell + 1$ st node has caught up with its predecessor, but j will not be able to catch up with i for another $(\ell - 1)d$ time units, every d of which sees a new node ready to raise its rate back to 1 and catch up with its own predecessor.

During the first d time units past time t, node i's logical clock may increase by as much as $\alpha_i(1+\hat{\rho})d \leq (1+\hat{\rho})d$, node j's by as little as $\alpha_j(1-\hat{\rho})d \geq (1-\hat{\rho})d/D$, thus causing the logical clocks of i and j to undergo a further separation of at most

$$(1+\hat{\rho})d - \frac{(1-\hat{\rho})d}{D} \le (1+\hat{\rho})d.$$
 (7)

During the remaining $(\ell - 1)d$ time units, the logical clock of node *i* may increase by as much as

$$\alpha_i (1+\hat{\rho})(\ell-1)d = \frac{(1+\hat{\rho})(\ell-1)d}{D}.$$
(8)

The logical clock of node j, in turn, may during this time increase by as little as

$$\alpha_j (1-\hat{\rho})(\ell-1)d = \frac{(1-\hat{\rho})(\ell-1)d}{D}.$$
(9)

At time $t + \ell d$, then, the greatest possible skew between the logical clocks of the neighboring nodes i and j is

$$c + (1+\hat{\rho})d + \frac{2\hat{\rho}(\ell-1)d}{D} \le c + (1+3\hat{\rho})d,\tag{10}$$

since $\ell = D$. Our algorithm is then seen to achieve the gradient property in such a way that

$$f(d_{ij}) \text{ is } O(d_{ij}) \tag{11}$$

for all $d_{ij} \in [1, D]$, so the worst-case clock skew between neighbors is O(1).

4 Discussion

In [2], and also in [5] for the specific case of assumptions (i) and (ii), it is proven that $f(d_{ij})$ is $\Omega(d_{ij} + \log D/\log \log D)$. This is proven for all clocksynchronization algorithms that have the gradient property and for which the constant lower bound *b* mentioned earlier on the rate of progress of all logical clocks exists. Such a property would be seriously at odds with our claim of an O(1) worst-case skew between neighbors, so in this section we discuss its relation to our algorithm. Specifically, we demonstrate that our approach admits no constant lower bound on $dL_i(t)/dt$ that holds for all *i* and all *t*, so the lower bound on $f(d_{ij})$ does not hold.

We first discuss the definability of $dL_i(t)/dt$. For fixed t, let $t_1 < t$ and $t_2 > t$ be such that the value of α_i does not change in the time interval $[t_1, t)$ or in the interval $(t, t_2]$. Then $dL_i(t)/dt$ is in principle definable indistinctly as

$$\lim_{t_1 \to t} \frac{L_i(t) - L_i(t_1)}{t - t_1} = \alpha_i \lim_{t_1 \to t} \frac{H_i(t) - H_i(t_1)}{t - t_1} = \alpha_i \frac{dH_i(t)}{dt}$$
(12)

or

$$\lim_{t_2 \to t} \frac{L_i(t_2) - L_i(t)}{t_2 - t} = \alpha_i \lim_{t_2 \to t} \frac{H_i(t_2) - H_i(t)}{t_2 - t} = \alpha_i \frac{dH_i(t)}{dt}.$$
 (13)

However, if t is precisely the time at which node i changes the value of α_i through Step 2(a) (by reducing some α_i^j from 1 to 1/D while all others remain equal to 1, or by raising the single α_i^j whose value is 1/D back to 1), then the two limits above are inconsistent with each other and $dL_i(t)/dt$ remains undefined.

But the values of t for which $dL_i(t)/dt$ is undefined are only finitely many, so one naturally wonders about the other, infinitely many instants at which the derivatives are defined. For these other instants, notice that Step 2(a) never causes α_i to be reduced below 1/D, so one might still consider, for all $i \in N$, the existence of the lower bound b on $dL_i(t)/dt$, provided

$$b \le \frac{(1-\hat{\rho})}{D}.\tag{14}$$

Such a bound, however, would not be a constant, as it would depend on G.

5 Concluding remarks

Our algorithm's Step 2 embodies two competing trends in its two parts (a) and (b). The aim of part (a) is to slow down nodes whose logical clocks are ahead of any of their neighbors' by c or more. Part (b), on the other hand, forces a node's logical clock to move ahead toward its neighbors' whenever possible. Both trends are fundamental to the algorithm's proper operation. Without Step 2(a), the O(1) worst-case skew between neighbors would be unachievable; without Step 2(b), the presence of a single slow-moving hardware clock would slow down all nodes' logical clocks, turning them into poor approximations of real time.

One relevant open question at this point is how the two trends balance each other, both in theory and in practice. Our algorithm relies strongly on the possibility of altering, in Step 2(a), the rates at which nodes' logical clocks follow their hardware clocks. Even though there is a clear provision for such rates to return to their original value of 1 whenever safe, further investigation is needed to clarify their most important properties. One of these concerns the duration of the periods during which the rates get reduced. Another is related to how rate reduction affects the logical clocks' main purpose, which is to track the progress of real time in as synchronized a way as possible.

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References

- J. Elson, L. Girod, and D. Estrin. Fine-grained network time synchronization using reference broadcasts. *Operating Systems Review*, 36:147–163, 2002.
- [2] R. Fan and N. Lynch. Gradient clock synchronization. Distributed Computing, 18:255–266, 2006.
- [3] L. Lamport and P. M. Melliar-Smith. Synchonizing clocks in the presence of faults. *Journal of the ACM*, 32:52–78, 1985.
- [4] T. Locher and R. Wattenhofer. Oblivious gradient clock synchronization. In Proceedings of the Twentieth International Symposium on Distributed Computing, volume 4167 of Lecture Notes in Computer Science, pages 520–533, Berlin, Germany, 2006. Springer-Verlag.
- [5] L. Meier and L. Thiele. Brief announcement: gradient clock synchronization in sensor networks. In Proceedings of the Twenty-Fourth Annual ACM Symposium on Principles of Distributed Computing, page 238, 2005.

- [6] H. Qi, X. Wang, S. S. Iyengar, and K. Chakrabarty. Multisensor data fusion in distributed sensor networks using mobile agents. In *Proceedings of the International Conference on Information Fusion*, pages 11–16, 2001.
- [7] T. K. Srikanth and S. Toueg. Optimal clock synchronization. Journal of the ACM, 34:626–645, 1987.
- [8] J. L. Welch and N. Lynch. A new fault-tolerant algorithm for clock synchronization. *Information and Computation*, 77:1–36, 1988.