A Tensor Based Hyper-heuristic for Nurse Rostering

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Abstract

Nurse rostering is a well-known highly constrained scheduling problem requiring assignment of shifts to nurses satisfying a variety of constraints. Exact algorithms may fail to produce high quality solutions, hence (meta)heuristics are commonly preferred as solution methods which are often designed and tuned for specific (group of) problem instances. Hyper-heuristics have emerged as general search methodologies that mix and manage a predefined set of low level heuristics while solving computationally hard problems. In this study, we describe an online learning hyper-heuristic employing a data science technique which is capable of self-improvement via tensor analysis for nurse rostering. The proposed approach is evaluated on a well-known nurse rostering benchmark consisting of a diverse collection of instances obtained from different hospitals across the world. The empirical results indicate the success of the tensor-based hyper-heuristic, improving upon the best-known solutions for four of the instances.

Keywords: Nurse Rostering, Personnel Scheduling, Data Science, Tensor Factorization, Hyper-heuristics.

1. Introduction

Hyper-heuristics are high level improvement search methodologies exploring space of heuristics (Burke et al., 2013). According to (Burke et al., 2010c), hyper-heuristics can be categorized in many ways. A hyper-heuristic either selects from a set of available low level heuristics or generates new heuristics from components of existing low level heuristics to solve a problem,

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leading to a distinction between *selection* and *generation* hyper-heuristics, respectively. Also, depending on the availability of feedback from the search process, hyper-heuristics can be categorized as *learning* and *no-learning*. Learning hyper-heuristics can further be categorized into online and offline methodologies depending on the nature of the feedback. Online hyper-heuristics learn *while* solving a problem whereas offline hyper-heuristics process collected data gathered from training instances prior to solving the problem.

Nurse rostering is a highly constrained scheduling problem which was proven to be NP-hard (Karp, 1972) in its simplified form. Solving a nurse rostering problem requires assignment of shifts to a set of nurses so that 1) the minimum staff requirements are fulfilled and 2) the nurses' contracts are respected (Burke et al., 2004). The general problem can be represented as a constraint optimisation problem using 5-tuples consisting of set of nurses, days (periods) including the relevant information from the previous and upcoming schedule, shift types, skill types and constraints.

In this study, a novel selection hyper-heuristic approach is employed to tackle the nurse rostering problem. The proposed framework (which is an extension to the framework in (Asta and Özcan, 2015)) is a single point based search algorithm which fits best in the online learning selection hyper-heuristic category, even if it is slightly different to the other online learning selection hyper-heuristics. A selection hyper-heuristic has two main components: heuristic selection and move acceptance method. While the task of the heuristic selection is to select low level heuristics based on a strategy, the acceptance method decides whether or not the solution produced by the selected heuristic shall be accepted. Over the years many heuristic selection and move acceptance methods have been proposed. Examples of heuristics selection strategies are Simple Random (SR) and Random Gradient (RG)(Cowling et al., 2001), Choice Function (CF) (Cowling et al., 2001), Reinforcement Learning (RL) (Nareyek, 2004) and Tabu Search (TS) (Burke et al., 2003). Examples of early (primitive) acceptance mechanisms are Improvement Only (IO) (Cowling et al., 2001), Improving and Equal (IE) (Bilgin et al., 2007), and Naive Acceptance (NA) (Burke et al., 2010a). The IO acceptance criteria only accepts improving solutions (compared to the current solution) and solutions equal or worsening to the quality of the current solution are rejected. The IE acceptance criteria accepts improving as well as equal solutions. This is while the NA acceptance method improving solutions are always accepted and equal or worsening solutions are accepted according to a fixed probability (usually 0.5). More sophisticated acceptance algorithms such as Simulated Annealing(SA), Late Acceptance (LA) and Great Deluge (GD) can be found in the scientific literature (Burke et al., 2013).

Our proposed approach consists of running the simple random heuristic selection strategy in four stages. In the first two stages, the acceptance mechanism is NA, while in the second stage, we use IE as acceptance mechanism. The trace of the hyper-heuristic in each stage is represented as a 3-rd order tensor. After each stage commences, the respective tensor is factorized which results in a score value associated to each heuristic. The space of heuristics is partitioned into two distinct sets, each representing a different acceptance mechanism (NA and IE respectively) and lower level heuristics associated to it. Subsequently, a hyper-heuristic is created which uses different acceptance methods in an interleaving manner, switching between acceptance methods periodically. In the third stage, the parameter values for heuristics is extracted by running the hybrid hyper-heuristic and collecting tensorial data similar to the first two stages. Subsequently, the hybrid hyper-heuristic equipped with heuristic parameter values is run for a specific time. The above mentioned procedure continues until the maximum allowed time is reached.

Compared to the method proposed in (Asta and Özcan, 2015), the framework here has few modifications. First, the framework in (Asta and Özcan, 2015) has been extended to accommodate for an arbitrary number of acceptance criteria to be involved in the framework. That is, in contrast to the work in (Asta and Özcan, 2015) where tensor data was collected for one acceptance criteria and the space of heuristics was partitioned into two disjoint sets, in this study, data collection and tensor analysis is performed for each hyper-heuristic separately. Moreover, low level heuristics are partitioned dynamically, rather than only once which was the case in (Asta and Özcan, 2015) where ten (nominal) minute runs were considered. Mining search data periodically allows us to investigate whether the framework is capable of extracting new knowledge as the search makes progress. This could be useful in a variety of applications (i.e. life-long learning as in (Silver et al., 2013), (Hart and Sim, 2014) and (Sim and Hart, 2014) or apprenticeship learning as in (Asta et al., 2013) and (Asta and Ozcan, 2014)). Finally, the framework here is different than the one proposed in (Asta and Özcan, 2015) when parameter control for each low level heuristic is considered. While no parameter control was done in (Asta and Özcan, 2015), in this study, parameters of each heuristic is tuned using tensor analysis. The good results achieved in this study shows that tensor analysis can also play a parameter control role.

The paper is organised as follows. Section 2 overviews the nurse rostering problem covering the problem definition, benchmarks in the area and related work. An introduction to tensor analysis is given in Section 3 where tensor representation of data, its advantages along with techniques widely employed to analyse tensorial data are explained. A detailed account of the proposed approach is provided in Section 4. The settings used in our experiments and the results of these experiments are laid out in Sections 5.1 and 5 respectively. Finally, concluding remarks and plans towards future work are discussed in Section 6.

2. Nurse Rostering

In this section, we define the nurse rostering problem dealt with. Additionally, an overview of related work is provided.

2.1. Problem Definition

The constraints in the nurse rostering problem can be grouped into two categories: (i) those that link two or more nurses and (ii) those that only apply to a single nurse. Constraints that fall into the first category include the cover (sometimes called demand) constraints. These are the constraints that ensure a minimum or maximum number of nurses are assigned to each shift on each day. They are also specified per skill/qualification levels in some instances. Another example of a constraint that would fall into this category would be constraints that ensure certain employees do or do not work together. Although these constraints do not appear in most benchmark instances (including those used here), they do occasionally appear in practise to model requirements such as training/supervision, carpooling, spreading expertise etc. The second group of constraints model the requirements on each nurse's individual schedule. For example, the minimum and maximum number of hours worked, permissible shifts, shift rotation, vacation requests, permissible sequences of shifts, minimum rest time and so on.

In this study, our aim is to see whether any improvement is possible via the use of machine learning, particularly tensor analysis. We preferred using the benchmark provided at (Curtois, 2015) as discussed in the next section. These benchmark instances are collected from a variety of workplaces across the world and as such have different requirements and constraints, particularly the constraints on each nurse's individual schedule. This is because different organisations have different working regulations which have usually been defined by a combination of national laws, organisational and union requirements and worker preferences. To be able to model this variety, in (Burke and Curtois, 2014) a regular expression constraint was used. Using this domain specific regular expression constraint allowed all the nurse specific constraints found in these benchmarks instances to be modelled. The model is given below.

Sets

E = Employees to be scheduled, $e \in E$
T = Shift types to be assigned, $t \in T$
D = Days in the planning horizon, $d \in \{1, \cdots |D|\}$
R_e = Regular expressions for employee e, $r \in R_e$
W_e = Workload limits for employee e, $w \in W_e$

Parameters

r_{er}^{max} = Maximum number of matches of regular expression r in the work schedule of employee e.
r_{er}^{min} = Minimum number of matches of regular expression r in the work schedule of employee e.
a_{er} = Weight associated with regular expression r for employee e.
v_{ew}^{max} = Maximum number of hours to be assigned to employee e within the time period defined by workload limit w.
v_{ew}^{min} = Minimum number of hours to be assigned to employee e within the time period defined by workload limit w.
b_{ew} = Weight associated with workload limit w for employee e.
s_{td}^{max} = Maximum number of shifts of type t required on day d.
s_{td}^{min} = Minimum number of shifts of type t required on day d.
c_{td} = Weight associated with the cover requirements of shift type t on day d.

Variables

x_{etd} = 1 if employee e is assigned shift type t on day d, 0 otherwise.
n_{er} = The number of matches of regular expression r in the work schedule of employee e.
p_{ew} = The number of hours assigned to employee e within the time period defined by workload limit w.
q_{td} = The number of shifts of type t assigned on day d.

Constraints
Employees can be assigned only one shift per day.

$$\sum_{t \in T} x_{etd} \leq 1, \qquad \forall e \in E,\ d \in D \tag{1}$$

Objective Function

$$Min f(s) = \sum_{e \in E} \sum_{i=1}^{4} f_{e,i}(x) + \sum_{t \in T} \sum_{d \in D} \sum_{i=5}^{6} f_{t,d,i}(x) \tag{2a}$$

$$where$$

$$f_{e,1}(x) = \sum_{e \in R_e} max\{0, (n_{er} - r_{er}^{max}) a_{er}\} \tag{2b}$$

$$f_{e,2}(x) = \sum_{e \in R_e} max\{0, (r_{er}^{min} - n_{er}) a_{er}\} \tag{2c}$$

$$f_{e,3}(x) = \sum_{w \in W_e} max\{0, (p_{ew} - v_{ew}^{max}) b_{ew}\} \tag{2d}$$

$$f_{e,4}(x) = \sum_{w \in W_e} max\{0, (v_{ew}^{min} - p_{ew}) b_{ew}\} \tag{2e}$$

$$f_{e,5}(x) = max\{0, (s_{td}^{min} - q_{td}) c_{td}\} \tag{2f}$$

$$f_{e,6}(x) = max\{0, (q_{td} - s_{td}^{max}) c_{td}\} \tag{2g}$$

To facilitate comparing results and to remove the difficulties in comparing infeasible solutions, the benchmark instances were designed with only one hard constraint 1 which is always possible to satisfy. Every other constraint is modelled as a soft constraint, meaning that it becomes part of the objective function. If in practice, in one of the instances, a soft constraint should really be regarded as a hard constraint then it was given a very high weight (the Big M method). The objective function is thus given in equation 2a. It consists of minimising the sum of equations 2b to 2g. Equations 2b and 2c ensure that as many of the regular expression constraints are satisfied as possible. These constraints model requirements on an individual nurse's shift pattern. For example, constraints on the length of a sequence of consecutive working days, or constraints on the number of weekends worked, or the number of night shifts and so on. Equations 2d and 2e ensure that each nurse's workload constraints are satisfied. For example, depending on the instance, there may be a minimum and maximum number of hours worked per week, or per four weeks, or per month or however the staff for that organisation are contracted. Finally, equations 2f and 2g represent the demand (sometime called cover) constraints to ensure there are the required number of staff present during each shift. Again, depending upon the instance, there may be multiple demand curves for each shift to represent, for example, the minimum and maximum requirements as well as a preferred staffing level. The weights for the constraints are all instance specific because they represent the scheduling goals for different institutions.

2.2. Related Work

There are various benchmarks for nurse rostering. Curtois (2015) provides a comprehensive public benchmark including the latest best known results together with the approaches yielding them. The characteristics of the benchmark instances used in the experiments are summarized in Table 1.

There is a growing interest in challenges and the instances used during those challenges and resultant algorithms serve as a benchmark afterwards. The last nurse rostering competition was organised in 2010 (Haspeslagh et al., 2014) which consisted of three tracks where each track differed from others in maximum running time and size of instances. Many different algorithms have been proposed since then ((Valouxis et al., 2012), (Lü and Hao, 2012), (Pillay and Rae, 2012), (Anwar et al., 2014), (Rae and Pillay, 2014), and more). Since it has been observed that the previous challenge did not impose much difficulty for the competitors (Burke and Curtois, 2014), other than developing a solution method in limited amount of time, a second challenge has been organised which is ongoing [1]. In the second nurse rostering competition, the nurse rostering problem is reformulated as a multi-stage problem with fewer constraints where a solver is expected to deal with consecutive series of time periods (weeks) and consider longer planning horizon. The remaining part of this section covers the state-of-the-art solution methods applied to the benchmark instances from (Curtois, 2015) which are used during the experiments.

[1]The Second International Nurse Rostering Competition: mobiz.vives.be/inrc2/

Instance	No. of Staff	Shift Types	No. of Shifts	Best Known	Method (Ref.)
BCV-A.12.1	12	5	31	1294	(Xue et al., 2010)
BCV-3.46.1	46	3	26	3280	″
BCV-A.12.2	12	5	31	1875	hyper-heuristic (Chan et al., 2012)
CHILD-A2	41	5	42	1095	″
ERRVH-A	51	8	48	2135	″
ERRVH-B	51	8	48	3105	″
ERMGH-B	41	4	48	1355	″
MER-A	54	12	48	8814	″
Valouxis-1	16	3	28	20	variable neighborhood (Solos et al., 2013)
Ikegami-3Shift-DATA1.1	25	3	30	3	*
Ikegami-3Shift-DATA1.2	25	3	30	3	*
Ikegami-3Shift-DATA1	2	3	30	2	*
ORTEC01	16	4	31	270	branch&price (Burke and Curtois, 2014)
ORTEC02	16	4	31	270	″

Table 1: Characteristics of the nurse rostering benchmark instances (Curtois, 2015). The method and source from which the best known solution's objective value is obtained for each instance is listed in the last column. The entries indicated by * are taken from a private communication to Nobuo Inui, Kenta Maeda and Atsuko Ikegami.

In (Métivier et al., 2009), the nurse rostering problem was identified as an over-constrained one and it is modelled using soft global constraints. A variant of Variable Neighbourhood Search (VNS), namely VNS/LDS+CP (Loudni and Boizumault, 2008), was used as a meta-heuristic to solve the problem instances. The proposed approach was tested on nine different instances from (Curtois, 2015). The experimental results show that the method is relatively successful, though the authors have suggested to use specific new heuristic for instances such as `Ikegami` to improve the performance of the algorithm.

(Xue et al., 2010) ????? This approach is the best on the BCV-A.12.1 and BCV-3.46.1 instances.

Glass and Knight (2010) proposed a method based on mixed integer linear programming, solving four of the instances from (Curtois, 2015), namely, `ORTEC01`, `ORTEC02`, `GPost` and `GPost-B`. The method is able to solve those instances to optimality very quickly. The idea of implied penalties was introduced in the study. Employing implied penalties avoids accepting small improvements in the current rostering period at the expense of penalizing larger penalties on the next rostering period.

In (Burke et al., 2010d), a hybrid multi-objective model was presented to solve nurse rostering problems. The method is based on Integer Programming (IP) and Variable Neighbourhood Search (VNS). The IP method is used in the first phase of the algorithm to produce intermediary solutions considering all the hard constraints and a subset of soft constraints. The solution is further improved using the VNS method. The proposed approach is then applied to the `ORTEC` problem instances and compared to a commercial hybrid Genetic Algorithm (GA) and a hybrid VNS (Burke et al., 2008). The computational results show that the proposed approach outperforms both methods in terms of solution quality.

Chan et al. (2012) introduced a hyper-heuristic method inspired from pearl hunting and applied it to various nurse rostering instances. The proposed method is based on repeated intensification and diversification and can be described as a type of Iterated Local Search (ILS). Their experiments consisted of running the algorithm on various instances for several times, where each run is 24 CPU hours long. The algorithm discovered 6 new best-known results as illustrated in Table 1.

In (Solos et al., 2013), a generic two-phase variable neighbourhood approach was proposed for nurse rostering. After determining the value of the parameters which govern the performance of the algorithm, a random population of candidate solutions is generated. In the first phase of the algorithm, assigning nurses to working days is handled. Subsequent to this phase, in the second phase, assigning nurses to shift types is dealt with. Though, the proposed approach has been applied to few publicly available instances, the chosen instances are significantly different from one another. This is still the best approach producing the best known result for Valouxis-1 in the benchmark.

In (Burke and Curtois, 2014) a branch and price algorithm and an ejection chain method was employed for solving the nurse rostering problem instances which were collected from thirteen different countries by the authors and made publically available from (Curtois, 2015). Branch and price method is based on the branch and bound technique with the difference that each node of the tree is a linear programming relaxation and is solved through column generation. The column generation method consists of two parts: the restricted master problem and the pricing problem. The former is solved using a linear programming method while the latter is using a dynamic programming approach. Some of the latest results and best-known solutions regarding the instances is provided by this study. Also, a general problem modelling scheme has been proposed in (Burke and Curtois, 2014) which is also adopted here due to its generality over many problem instances. This approach is the best on the ORTEC01 and ORTAC02 instances.

Numerous other approaches have been proposed to solve the nurse rostering problem. In (Azaiez and Al Sharif, 2005) the nurse rostering problem was modelled using 0-1 Goal Programming Model. A shift sequence based approach was introduced in (Brucker et al., 2010). In (Burke et al., 2010b) a Scatter Search (SS) was presented to tackle the nurse rostering problem.

3. Tensor Analysis

Tensorial representation and tensor analytic approaches are widely used to investigate high dimensional data and extract latent patterns and correlations between various modes of data. The classical approach in data mining is to collect the data into datasets of matrix form without regard for the natural dimensionality of the data. Collapsing data in this fashion brings about the advantage of simplifying the problem of data mining which is much needed for many machine learning and statistical approaches which shape the core of data mining frameworks. However, collapsing data dimensions results in loss of information including latent relationships between various modes of data. Several recent studies dealt with this problem and proposed a solid remedy based on tensor analytic approaches ((Vasilescu and Terzopoulos, 2002),(Anandkumar et al., 2012) and (Acar et al., 2009)). These studies all show that tensorial representation and analytical approaches which come with it are capable of detecting patterns which where invisible to classical data mining approaches which have no regard for the natural dimensionality of the data.

Since their introduction, tensorial approaches have been employed and greatly contributed to a variety of research areas such as computer vision (Vasilescu and Terzopoulos, 2002), video processing (Krausz and Bauckhage, 2010), data compression (Wang et al., 2009), Signal Processing (Cichocki et al., 2014) and web mining (Acar et al., 2009), (Zou et al., 2015). Various problems produce data of high order of dimensionality in nature and tensors, as multidimensional arrays, are fully suited to represent such data. For instance, video streams constitute a data which is three dimensional (pixel coordinates and time) or higher (when information such as audio, text, change of environment and etc are also considered). Such data can surely be presented as a three dimensional array which is commonly referred to as a n^{th}-order tensor. The *order* of a tensor indicates its dimensionality where each dimension of a tensor is referred to as a *mode*. For example, a tensor representing a video is a 3^{rd}-order tensor.

The first step to extract latent patterns hidden within tensor data is to subject the tensor to factorization, say, decomposing the tensor into its basic factors. Several factorizations methods exist. Higher Order Singular Value Decomposition (HOSVD), Tucker decomposition, Parallel Factor decomposition and Non-negative Tensor Factorization (NTF) are among the most widely known factorization methods. HOSVD (Lathauwer et al., 2000) is a generalization of the Singular Value Decomposition (SVD) used in the Principal Component Analysis (PCA) method to higher dimensions. Tucker decomposition (Tucker, 1966c) decomposes a tensor into a set of matrices and a core tensor. Parallel Factor (a.k.a PARAFAC or CANDECOMP or CP) (Harshman, 1976) and Non-negative Tensor Factorization (NTF) (Shashua and Hazan, 2005) decompose a tensor into a sum of rank one tensors. Further information on tensors and applications and comparison between various factorization methods can be found in (Kolda and Bader, 2009).

Following (Kolda and Bader, 2009), the notations which are used in this paper are as the following. Tensors, matrices and vectors are denoted by boldface Euler script letters, boldface capital letters and boldface lower-case letters respectively (e.g., $\mathcal{T}$ is a tensor,$\mathbf{M}$ denotes a matrix and $\mathbf{v}$ is a vector). Scalar values such as tensor, matrix and vector entries are indexed by lower-case letters. For example, t_{pqr} is the (p, q, r) entry of a $3^{rd}-$order tensor $\mathcal{T}$.

3.1. CP Factorization

In this paper, following (Asta and Özcan, 2015), CP decomposition method is used for factorization. While Tucker decomposition is a generalization of the CP method, there are few advantages based on which the latter method is favoured over the former approach. Unlike the Tucker decomposition, the factors produced by the CP decomposition method are unique (under mild conditions)(Kolda and Bader, 2009). The tensor-based hyper-heuristic in this paper uses the scores achieved after factorization to rank heuristics. One should expect a reasonably high level of consistency between the heuristic rankings produced in various runs of the algorithm. Thus, uniqueness of the basic factors is a most crucial condition. Moreover, compared to Tucker decomposition, CP factorization produces *easy to interpret* basic factors. This is particularly desirable in the context of heuristics. Some applications (such as video, speech and text) produce easy-to-understand contents because there is a certain semantic concept naturally associated with the data. This is not the case in the data produced by hyper-heuristics. Therefore, it is very desirable to choose a decomposition method which offers some level of simplicity in the process of interpreting the basic factors.

CP decomposition uses the Alternating Least Square (ALS) algorithm ((Carroll and Chang, 1970))S to decompose a tensor into its basic factors. That is, the original tensor $\mathcal{T}$ of size $P \times Q \times R$ is approximated by another tensor $\hat{\mathcal{T}}$. The purpose of the ALS algorithm is to minimize the error difference between the original tensor and the estimated tensor, denoted as ε as follows:

$$\varepsilon = \frac{1}{2}||\mathcal{T} - \hat{\mathcal{T}}||_F^2 \tag{3}$$

where the subscript F refers to the Frobenious norm. That is:

$$||\mathcal{T} - \hat{\mathcal{T}}||_F^2 = \sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{r=1}^{R}\left(t_{pqr} - \hat{t}_{pqr}\right)^2 \tag{4}$$

The approximated tensor contains the factors of the original tensor and can be expressed as a sum of rank one[2] tensors as in Equation 5 (this is also illustrated in Figure 1).

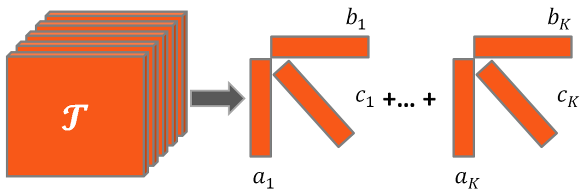


Figure 1: Factorizing a tensor to K components.

$$\hat{\mathcal{T}} = \sum_{k=1}^{K} \lambda_k \ \mathbf{a}_k \circ \mathbf{b}_k \circ \mathbf{c}_k \tag{5}$$

In Eq.5, $\lambda_k \in \mathbb{R}_+$, $\mathbf{a}_k \in \mathbb{R}^P$, $\mathbf{b}_k \in \mathbb{R}^Q$ and $\mathbf{c}_k \in \mathbb{R}^R$ for $k = 1 \cdots K$, where K is the number of desired components. Each component k is of the form $(\lambda_k \ \mathbf{a}_k \circ \mathbf{b}_k \circ \mathbf{c}_k)$ where λ_k is the weight of the component and the individual vectors (e.g. $\mathbf{a}_k$, $\mathbf{b}_k$ and $\mathbf{c}_k$) are called factors. The number of components is set depending on the application. In this paper, we set the number of components to be $K = 1$. Therefore, the Equation 5 is reduced to the following equation.

$$\hat{\mathcal{T}} = \mathbf{a} \circ \mathbf{b} \circ \mathbf{c} \tag{6}$$

The operator "$\circ$" indicates an outer product. The outer product of three vectors produces a 3^{rd}-order tensor. For instance, in Equation 6, $\hat{\mathcal{T}}$ is a 3^{rd}-order tensor which is obtained by the outer product of three vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. Subsequently, each tensor entry, denoted as $\hat{t}_{pqr}$ is computed through a simple multiplication like $a_p b_q c_r$.

Rather than using all the factors in the outer product which results in the approximated tensor $\hat{\mathcal{T}}$, using only two of the factors in the outer product results in a structure which we refer to as the *basic frame* (Equation 7). It

[2] a tensor is of rank one when it can be written as the outer product of N vectors. Many articles use the term *rank* to refer to tensor *order*. However, it should be noted that the meaning of the two words are entirely different.

was shown in several studies (e.g. (Krausz and Bauckhage, 2010)) that the basic frame exhibits very interesting phenomena.

$$\mathbf{B} = \mathbf{a} \circ \mathbf{b} \tag{7}$$

For instance the basic frame $\mathbf{B}$, which is actually a matrix, captures the hidden relationship between the first two modes of data. In other words, the matrix $\mathbf{B}$ contains score values which quantifies the relationship between the object pairs. For instance, in case of a video tensor and given that $\mathbf{a}$ and $\mathbf{b}$ represent x and y values of pixel coordinates, the basic frame $\mathbf{B}$ quantifies the "level of interaction" between various pixel regions within a video.

Tensor factorization represents the original tensor in a concise form as can be seen from Equation 5. When it comes to mining the approximated data, the estimated tensor $\hat{\mathcal{T}}$ is also more generalizable and also increases the resistance to disruptions caused by data anomalies such as missing values. Factorizing the tensor allow partitioning of the data into a set of more comprehensible sub-data which can be of specific use depending on the application according to various criteria. For example, in the field of computer vision, (Kim and Cipolla, 2009) and (Krausz and Bauckhage, 2010) separately show that factorization of a 3^{rd}-order tensor of a video sequence results in some interesting basic factors. These basic factors reveal the location and functionality of human body parts which move synchronously.

4. Proposed Approach

The proposed approach consists of the consecutive iteration of four stages as depicted in Algorithm 1 and Figure 2. In all stages, *simple* hyper-heuristic algorithms operating on top of a fixed set of low level heuristics (move operators) are used. Those low level heuristics are exactly the same low level heuristics implemented for the personnel scheduling problem domain (Curtois et al., 2009) under the Hyper-heuristic Flexible Framework (HyFlex) v1.0 (Ochoa et al., 2012). The low level heuristics in HyFlex are categorized into four groups: mutation (MU), ruin and re-create (RR), crossover (XO) and local search (LS). Consequently, one mutation operator is available for the nurse rostering problem domain which is denoted here by $MU0$. This operator randomly un-assigns a number of shifts while keeping the resulting solution feasible. Three ruin and re-create heuristics are available which are denoted by $RR0$, $RR1$ and $RR2$. These operators are based on the heuristics proposed in (Burke et al., 2008) and they operate by un-assigning all the shifts in one or more randomly chosen employees' schedule followed by a rebuilding procedure. These operators differ in the size of the perturbation they cause in the solution. Five local search heuristics, denoted by $LS0$, $LS1$, $LS2$, $LS3$ and $LS4$ are also used where the first three heuristics are hill climbers and the remaining two are based on variable depth search. Also, three different crossover heuristics are used which are denoted by $XO0$, $XO1$ and $XO2$. The crossover operators are binary operators and applied to the current solution in hand and the best solution found so far (which is initially the first generated solution). More information on these heuristics can be found in (Curtois et al., 2009).

During the first two stages (line 2 and 3), two different tensors are constructed. The tensor $\mathcal{T}_{NA}$ is constructed by means of an SR-NA algorithm and tensor $\mathcal{T}_{IE}$ is constructed from the data collected from running an SR-IE algorithm. Both stages use all the heuristics available to the problem domain which is denoted by $\mathbf{h}$. At the end of the second stage (line 4 and 5), each tensor is subjected to factorization to obtain basic frames ($\mathbf{B}_{NA}$ and $\mathbf{B}_{IE}$) and score values ($\mathbf{S}_{NA}$ and $\mathbf{S}_{IE}$) corresponding to each tensor. Using all the information we have on both tensors, the heuristic space is partitioned (line 6) to two distinct sets: $\mathbf{h}_{NA}$ and $\mathbf{h}_{IE}$. Subsequently, in the third stage, a hybrid algorithm is executed for a limited time (t_p) with random heuristic parameter values (depth of search and intensity of mutation). The hybrid algorithm consists of periodically switching between the two acceptance mechanisms NA and IE. Depending on the chosen acceptance method, the heuristics are chosen either from $\mathbf{h}_{NA}$ or $\mathbf{h}_{IE}$. In fact the hybrid algorithm is very similar to the algorithm in the final stage except that during the search process in this stage, the heuristic parameters are chosen randomly and a tensor using the heuristic parameter settings is constructed. Factorizing this tensor and obtaining the basic frame (similar to what is done in previous steps) results in good parameter value settings for heuristics. Hence this stage can be considered as a parameter control phase for the heuristics. The final (fourth) stage consists of running the previous stage for a longer time ($3 \times t_p$) and assigning values achieved in the previous stage to heuristic parameters. After the time specified for the fourth stage is consumed, the algorithm starts over from stage one. This whole process continues until the maximum time allowed (T_{max}) for a given instance is reached. Figure 2 illustrates this process.

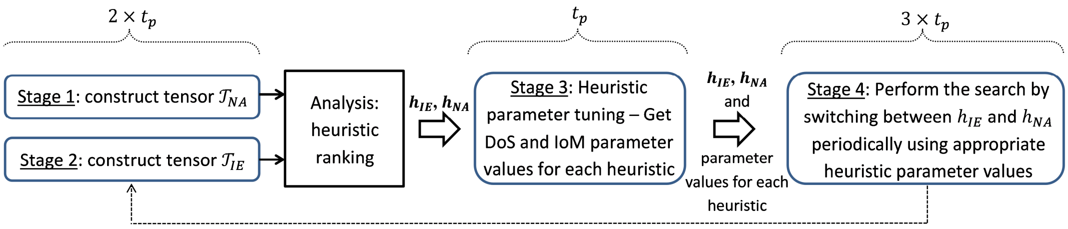


Figure 2: Overall approach with various stages.

Algorithm 1: Tensor-based hyper-heuristic

while $t < T_{max}$ **do**
- $(\mathcal{T}_{NA})$ = ConstructTensor($\mathbf{h}$, NA, t_p);
- $(\mathcal{T}_{IE})$ = ConstructTensor($\mathbf{h}$, IE, t_p);
- $(\mathbf{B}_{NA}, \mathbf{S}_{NA})$ = Factorization($\mathcal{T}_{NA}$,$\mathbf{h}$);
- $(\mathbf{B}_{IE}, \mathbf{S}_{IE})$ = Factorization($\mathcal{T}_{IE}$,$\mathbf{h}$);
- $(\mathbf{h}_{NA}, \mathbf{h}_{IE})$ = Partitioning($\mathbf{B}_{NA}, \mathbf{B}_{IE}, \mathbf{S}_{NA}, \mathbf{S}_{IE}$);
- $(\mathbf{P})$ = ParameterControl();
- Improvement($\mathbf{h}_{NA}, \mathbf{h}_{IE}, \mathbf{P}, \mathbf{S}_{NA}, \mathbf{S}_{IE}$);

end

4.1. Tensor Analysis for Dynamic Low Level Heuristic Partitioning

During first and second stages, given an acceptance criteria (such as NA or IE), a hyper-heuristic with a simple random heuristic selection and the given acceptance methodology is executed. During the run, data, in the form of a tensor is collected. Hence the collected tensor data ($\mathcal{T}_{NA}$ or $\mathcal{T}_{IE}$ depending on the acceptance criteria) is a 3^{rd}-order tensor of size $\mathbb{R}^{|\mathbf{h}|} \times \mathbb{R}^{|\mathbf{h}|} \times \mathbb{R}^{t}$, where $|\mathbf{h}|$ is the number of low level heuristics and t represents the number of tensor frames collected in a given amount of time. Each tensor is a collection of two dimensional matrices ($\mathbf{M}$) which are referred to as *tensor frames*. A tensor frame is a two dimensional matrix of heuristic indices. Column indices in a tensor frame represent the index of the current heuristic whereas row indices represent the index of the heuristic chosen and applied before the current heuristic. Algorithm 2 shows the tensor construction procedure.

The core of the algorithm is the SR selection hyper-heuristic (starting at the while loop in line 4) combined with the acceptance criteria which is given as input. Repeatedly, a new heuristic is selected at random (line 12) and is applied to the problem instance (line 14). The returned objective function value (f_{new}) is compared against the the old objective function value and the immediate change in objective function value is calculated as $\delta_f = f_{old} - f_{new}$. The method `Accept` (line 16) takes the δ_f value as input and returns a decision as to whether accept the new solution or reject it. In case the new solution is accepted , assuming that the indices of the current and previous heuristics are $h_{current}$ and $h_{previous}$ respectively, the tensor frame $\mathbf{M}$ is updated symmetrically: $m_{h_{previous},h_{current}} = 1$ and $m_{h_{current},h_{previous}} = 1$. The use of symmetry in the design of tensor frames ensures that the order in which heuristics are applied (which is random) does not influence the outcome of the factorization. The tensor frame $\mathbf{M}$ is only allowed to have $\lfloor |\mathbf{h}|/2 \rfloor$ elements with a value of 1 (line 5). Whenever this threshold is reached the tensor frame is added to the tensor and a new frame is initialized (lines 6 to 9). A high value for this threshold increases the likelihood of calling a heuristic more than once when filling a given frame. While the heuristic may be paired with different heuristics each time it is called, in such situations, the fact that the heuristics itself has been called multiple times will be lost to the tensor and it would look as if the heuristic has been called only once. Hence, such cases where loss of information is inevitable should be less likely. In other words, we are in favour of a sparse tensor. On the other hand, we also would like to ensure that adequate number of heuristics have been registered with each frame to make sure that each frame represents a considerable subset of heuristics. That is, we are not in favour of frames with only one active entry (which is the minimum number of entries a frame can have). The value of $\lfloor |\mathbf{h}|/2 \rfloor$ is determined experimentally to ensure that we have a sparse enough tensor which represents a wide enough range of available heuristics.

Assuming that the objective function value before updating the frame for the first time is f_{start} and assuming that the objective function value after the last update in the tensor frame is f_{end}, then the frame is labelled as $\Delta_f = f_{start} - f_{end}$ (line 7). In other word, the label of a frame (Δ_f) is the overall change in the objective function value caused during the construction of the frame. Δ_f is different from δ_f in the sense that the former measures the change in objective value inflicted by the collective application of active heuristic indexes inside a frame. The latter is the immediate change in objective value caused by applying a single heuristic. This whole process is repeated until a time limit (t_p) is reached (line 4). This procedure, creates a tensor of binary data for a given acceptance method. In order to prepare the tensor for factorization and increase the chances of gaining good patterns from the data, the frames of the constructed tensor are scanned for consecutive frames of positive labels($\Delta_f > 0$). Only these frames are kept in the tensor and all other frames are removed. This adds an extra emphasis on intra-frame correlations which is important in discovering good patterns. The resulting tensor for each acceptance criteria is then fed to the factorization procedure.

In the factorization stage, a tensor $\mathcal{T}$ is fed to the factorization procedure (Algorithm 3). This could be $\mathcal{T}_{NA}$ or $\mathcal{T}_{IE}$ depending on who calls the factorization procedure. Using the CP factorization, basic factors of the input tensor $\mathcal{T}$ are obtained (line 2). Using these basic factors the basic frame $\mathbf{B}$ is computed (line 3). To obtain the basic frame, Equation 6 is used where $K = 1$ and basic factors $\mathbf{a}$ and $\mathbf{b}$ represent previous and current heuristic indexes respectively (Figure 3). The values in the basic frame quantify the relationship between the elements along each dimension (basic factor). To make use of the basic frame, the maximum entry is pinpointed and the column corresponding to this entry is sorted. This results in a vector $\mathbf{S}$ which contains the score values achieved for heuristics.

The factorization stage is applied to both $\mathcal{T}_{NA}$ and $\mathcal{T}_{IE}$ tensors. That is, the procedures tensor construction (Algorithm 2) and factorization (Algorithm 3) are executed twice independently. Starting from an initial solution, first we execute the two procedure assuming NA as acceptance criteria. Consequently, we obtain a basic frame $\mathbf{B}_{NA}$ and a set of score values $\mathbf{S}_{NA}$. Fol-

Algorithm 2: The tensor construction phase

In: $\mathbf{h}$, acceptance criteria, $\mathbf{P}$, t_p;
Initialize tensor frame $\mathbf{M}$ to 0;
$counter = 0$;
while $t < t_p$ **do**
    if $counter = \lfloor |\mathbf{h}|/2 \rfloor$ **then**
        append $\mathbf{M}$ to $\mathcal{T}$;
        set frame label to Δ_f;
        Initialize tensor frame $\mathbf{M}$ to 0;
        $counter = 0$;
    end
    $h_{previous} = h_{current}$;
    $h_{current} = \text{selectHeuristic}(\mathbf{h})$;
    $f_{current} = f_{new}$;
    $f_{new} = \text{applyHeuristic}(h_{current})$;
    $\delta_f = f_{current} - f_{new}$;
    if *Accept(δ_f, acceptance criteria)* **then**
        $m_{h_{previous},h_{current}} = 1$;
        $m_{h_{current},h_{previous}} = 1$;
        $counter++$;
    end
end
Construct final tensor $\mathcal{T}$ from collected data;

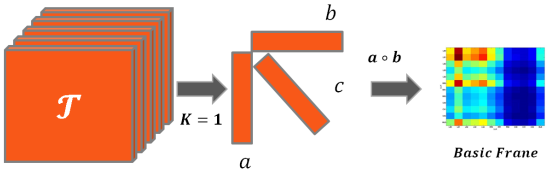


Figure 3: Extracting the basic frame for $K = 1$ in Eq.5.

lowing this, the two procedure (Algorithms 2 and 3) are executed assuming IE as acceptance criteria. This results in a basic frame $\mathbf{B}_{IE}$ and score values $\mathbf{S}_{IE}$. Consequently, for a given heuristic, there are two vectors of score values, one obtained from factorizing $\mathcal{T}_{NA}$ and the other obtained from factorizing $\mathcal{T}_{IE}$. These score vectors are sorted in line 5 and fed into the partitioning procedure (Algorithm 4). Sorting of scores is necessary during the ranking of heuristics and partitioning the heuristic space in Algorithm 4.

Algorithm 3: Factorization

1 In: $\mathcal{T}, \mathbf{h}$;
2 $\mathbf{a}, \mathbf{b}, \mathbf{c} = \mathrm{CP}(\mathcal{T}, K = 1)$;
3 $\mathbf{B} = \mathbf{a} \circ \mathbf{b}$;
4 $x, y = max(\mathbf{B})$;
5 $\mathbf{S} = sort(\mathbf{B}_{i=1:|\mathbf{h}|,y})$ //Scores;

Algorithm 4 is used to partition the space of heuristics. In lines 2 and 3 of the algorithm, the two score values for a given heuristic are compared to one another. The heuristic is assigned to the set $\mathbf{h}_{NA}$ if its score is higher (or equal) in the basic frame achieved from $\mathcal{T}_{NA}$ (that is, if $\mathbf{S}_{NA}(h) \geq \mathbf{S}_{IE}(h)$). Otherwise it is assigned to $\mathbf{h}_{IE}$. Note that, equal scores (say, $\mathbf{S}_{NA}(h) = \mathbf{S}_{IE}(h)$) rarely happens. At the end of this procedure, two distinct sets of heuristics, $\mathbf{h}_{NA}$ and $\mathbf{h}_{IE}$, are achieved where each group is associated to NA and IE acceptance methods respectively..

4.2. Parameter Control via Tensor Analysis

The next two stages of the framework (Algorithm 5 and 6 respectively) are very similar. The only detail which distinguishes the two is that, the first stage (Algorithm 5) is run for a shorter time with randomly chosen heuristic parameter values. These values are chosen from the range {0.1, 0.2, 0.3, 0.4,

Algorithm 4: Partitioning

In: $\mathbf{B}_{NA}, \mathbf{B}_{IE}, \mathbf{S}_{NA}, \mathbf{S}_{IE}$;
$\mathbf{h}_{NA} = \{h \in \mathbf{h} \mid \mathbf{S}_{NA}(h) \geq \mathbf{S}_{IE}(h)\}$;
$\mathbf{h}_{IE} = \{h \in \mathbf{h} \mid \mathbf{S}_{IE}(h) > \mathbf{S}_{NA}(h)\}$;

0.5, 0.6, 0.7, 0.8} denoted by $p \in \mathbf{P}$. The goal is to construct a tensor which contains selected heuristic parameter values per heuristic index. Factorizing this tensor would then help in associating each heuristic with a parameter value. This is while the final stage of the algorithm (Algorithm 6) runs for a longer time and uses these parameter values for each heuristic instead of choosing them randomly. Despite their similarity, each stage is described in detail here to provide the readers with a clearer picture of the logic

In Algorithm 5, the two sets of heuristics achieved in the previous stage together with their respective score values per heuristic are employed to run a hybrid acceptance hyper-heuristic. For the selected heuristic, a random parameter value is chosen and set (lines 11-12), the heuristic is applied and the relevant acceptance criteria is checked (lines 14-15). The heuristic selection is based on tournament selection. Depending on the tour size, few heuristics are chosen from the heuristic set corresponding to the acceptance mechanism and a heuristic with highest score (probability) is chosen and applied. In case of acceptance, the relevant frame entry is updated (line 16). Since this is a hybrid acceptance algorithm, each acceptance criteria has a budget which is expressed as the number of heuristic calls allocated to the acceptance method. If the acceptance criteria has used its budget, then a new random acceptance criteria is selected (lines 18-20). Obviously, in cases where there are only two acceptance methods available (like here) random selection can be replaced by simply toggling between the two acceptance methods. A random selection is however necessary for cases where there are more than two acceptance strategies available. After continuing this process for a time t_p, the final tensor ($\mathcal{T}_{Param}$) is constructed from collected frames and factorized (exactly in the same manner as in Algorithm 2), the basic frame is computed and the parameter values are extracted as suggested in line 25.

4.3. Improvement Stage

In the next phase of the algorithm, the two sets of heuristics achieved in previous stages, together with their score and parameter values are employed

Algorithm 5: Parameter Control

In: $\mathbf{h}, \mathbf{h}_{NA}, \mathbf{h}_{IE}$, t_p;
Initialize tensor frame $\mathbf{M}$ to 0;
$counter = 0$;
while $t < t_p$ **do**
    if $counter = \lfloor |\mathbf{h}|/2 \rfloor$ **then**
        append frame and initialize;
    if *acceptance criteria* $=$ *NA* **then**
        $h =$ SelectHeuristic($\mathbf{h}_{NA}$);
    else
        $h =$ SelectHeuristic($\mathbf{h}_{IE}$);
    $p_{current} = rand(\{0.1, 0.2, \cdots, 0.8\})$;
    setHeuristicParameter($p_{current}$);
    $f_{current} = f_{new}$;
    f_{new} =applyHeuristic($h_{current}$) , $\delta_f = f_{current} - f_{new}$;
    if *Acceptance(δ_f, acceptance criteria)* **then**
        $m_{h,p} = 1$, $counter++$;
    end
    if *callCounter* $> c$ **then**
        callCounter $= 0$;
        acceptance criteria = selectRandomAcceptance();
end
Construct final tensor $\mathcal{T}_{Param}$ from collected data;
$\mathbf{a}, \mathbf{b}, \mathbf{c} = \mathrm{CP}(\mathcal{T}_{Param}, K = 1)$;
$\mathbf{B} = \mathbf{a} \circ \mathbf{b}$;
$x, y = max(\mathbf{B})$;
$\mathbf{P} = sort(\mathbf{B}_{i=1:|\mathbf{h}|,y})$;

to run a hybrid acceptance hyper-heuristic (Algorithm 6). Each acceptance method is given a budget in terms of the maximum number of heuristic calls it is allowed to perform (`callCounter`). Whenever, the acceptance method uses its budget, the algorithm switches to a randomly chosen acceptance method, resetting the budget (line 10-12). Depending on the acceptance criteria in charge, a heuristic is selected (using the tournament selection method discussed above) from the corresponding set (lines 2-5). For instance, if NA is in charge a heuristic is selected from $\mathbf{h}_{NA}$. Later, depending on the nature of the heuristic (mutation, hill climbing or none) the parameter value of the heuristic is assigned (line 6) and the heuristic is applied (line 7). The achieved objective function value is then controlled for acceptance (line 9). This process continues until a time limit $(3 \times t_p)$ is reached.

Algorithm 6: Improvement

1 **while** $t < (3 \times t_p)$ **do**
2   **if** *acceptance criteria* $=$ *NA* **then**
3         $h =$ SelectHeuristic($\mathbf{h}_{NA}$);
4   **else**
5         $h =$ SelectHeuristic($\mathbf{h}_{IE}$);
6   setHeuristicParameter($\mathbf{P}(h)$);
7     $f_{new} =$ ApplyHeuristic(h);
8     $\delta_f = f_{current} - f_{new}$;
9   Acceptance(δ_f, acceptance criteria);
10   **if** *callCounter* $> c$ **then**
11   callCounter = 0;
12   acceptance criteria = selectRandomAcceptance();
13 **end**

5. Experimental Results

5.1. Experimental Design

The algorithm proposed here is a multi-stage algorithm where in each stage data samples are collected from the search process in form of tensors. Various approaches can be considered for data collection. While each stage can collect the data and ignore those collected in previous corresponding stages the data collected from various (corresponding) stages can be appended to one another. The former data collection approach has the advantage that collected data reflect the current search status independent from previous search stages allowing the algorithm to focus on the current state. However, ignoring previous data means discarding the knowledge that could have been extracted from experience. In order to assess the two data collection approaches, we employ both data collection approaches. That is, two methods are investigated here, both using the same algorithm (as in Algorithm 1). The only difference between them is that one algorithm (TeBHH 1) the data collection phase of the algorithm ignores previously collected data and over-writes the dataset. In the second algorithm (TeBHH 2) the data collected at each stage is appended to those collected in the same previous stage. Please note that, this does not mean that the data collected in the third stage is appended to those collected in the second stage. Each stage maintains its own dataset and e.g. stage 2 appends its data to the dataset designated for the same stage index.

Regardless of the data collection strategy they employ, both TeBHH 1 and TeBHH 2 have three configurable parameters, namely, the time allocated for data collection phase (t_p), the budget allocated to each acceptance criterion (`callCounter` in Algorithm 6) and the tournament size (employed in Algorithms 5 and 6). For each variant, a range of values are considered. Values considered for the variable t_p are $\{75, 125, 175\}$ seconds. For the call budget, values in $\{|\mathbf{h}|, 2 \times |\mathbf{h}|, 3 \times |\mathbf{h}|\}$ are considered. Also, three different tour sizes have been investigated: $\{2, \frac{|\mathbf{h}|}{2}, |\mathbf{h}|\}$. Each experiment performed with the proposed approach with each combination of those parameter values as the initial setting are indexed successively using the ordering as provided, starting from 1 denoting $(75, |\mathbf{h}|, 2)$ to 27 denoting $(175, 3 \times |\mathbf{h}|, |\mathbf{h}|)$.

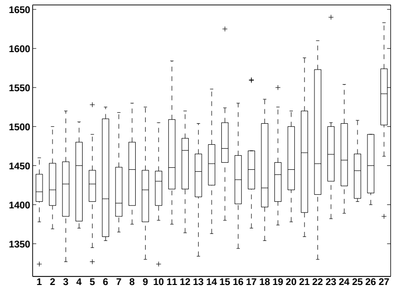
5.2. Selecting The Best Performing Parameter Setting

In order to determine the best performing parameter setting, each variant of the algorithm with a parameter value combination was run 10 times where each run terminates after two hours. Apart from detecting the best performing parameter configuration, we would like to know how sensitive the framework is with respect to the parameter settings. Seven instances were chosen for these experiments which would hopefully cover and represent a whole range of available instances. The chosen instances are: `BCV-A.12.1`, `BCV-A.12.2`, `CHILD-A2`, `ERRVH-A`, `ERRVH-B`, `Ikegami-3Shift-DATA1.2` and `MER-A`.

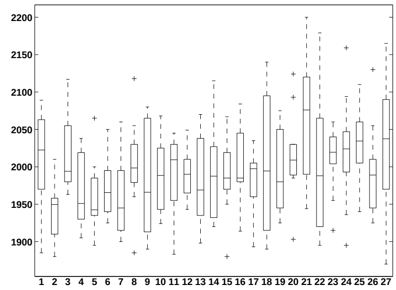
Figure 4 shows the results from these experiments for the TeBHH 1 variant. Although most of the configurations seem to achieve similar performances, there is no other parameter configuration which performs significantly better than the configuration with index 9 (for which $t_p = 175$ seconds, acceptance budget of $3 \times |\mathbf{h}|$ and tournament size of 2) on any of the cases, which is confirmed via a Wilcoxon signed rank test. Ranking all configuration based on the average results across the instances shows that this configuration performs slightly better than the others in the overall. Therefore, these values are chosen for the TeBHH 1 variant. A similar analysis shows that the parameter configuration with index 4 (for which $t_p = 75$ seconds, acceptance budget of $2 \times |\mathbf{h}|$ and tournament size of 2) is more suitable for the TeBHH 2 variant. It has been observed that tournament size of 2 is constantly a winner over the other values for tour size. The apparent conclusion is that both algorithms are very much sensitive to the value chosen for this parameter. Also, a shorter time for data collection in the TeBHH 2 variant makes sense, since it preserves the data collected in previous data collection sessions. The same is not true for TeBHH 1 which overwrites the old data in each stage. Thus, a longer data collection time in case of TeBHH 1 also makes sense. However, when the performance of variants with different data collection time values are compared, the emerging conclusion is that both algorithms are not very sensitive to the chosen value. A similar conclusion can be reached for the value of the acceptance budget.

5.3. Comparative Study

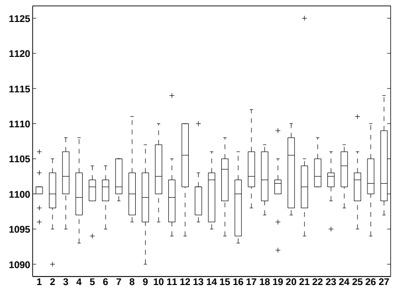
In the first round of experiments, an analysis is made as to compare the performance of the two proposed algorithms, namely, TeBHH 1 and TeBHH 2. The second column in Table 2 shows the result of this comparison. The



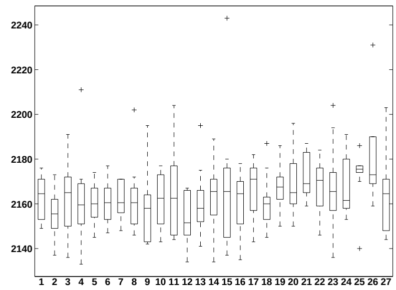
(a) BCV-A.12.1



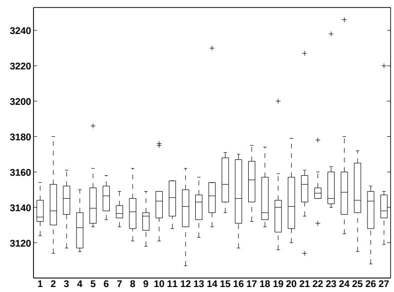
(b) BCV-A.12.2



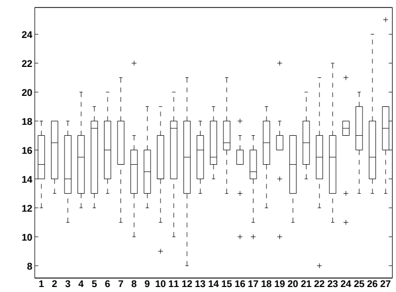
(c) CHILD-A2



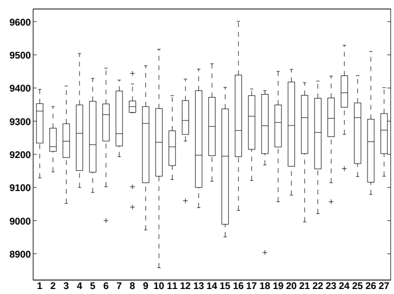
(d) ERRVH-A



(e) ERRVH-B



(f) Ikegami-3Shift-DATA1.2



(g) MER-A

Figure 4: Parameter configuration experiments using TeBHH 1. Each value on the X axis represent the index of the parameter setting of the approach as described at the end of Section 5.1. The Y axis represents the objective function values.

statistical test, based on Wilcoxon signed rank test, reveals that, the performance disparity between the two algorithms can vary from one instance to another. For instance, TeBHH 1 outperforms TeBHH 2 on 3 instances significantly. This is while on 2 other instances the situation is the opposite. Also, on 9 instances there is no statistically significance difference between the performance of the two algorithms. This makes sense since the only difference between the two algorithms is the way the dataset is treated throughout the time. While TeBHH 1 overwrites the data with newly collected dataset, TeBHH 2 appends the new data to the old dataset. Thus, it is natural that TeBHH 2 performs similarly to TeBHH 1 since much of their collected data can be similar. Also, the heuristics on all nurse rostering instances are quite slow and therefore there is a lack of data which is more the reason that the two algorithms perform similarly.

Overall, combining the entries in Table 2 and the minimum objective function value achieved by each algorithm (Table 3), it would be fair to say that TeBHH 1 performs slightly better than TeBHH 2. It is to say that it would be safer to refresh the dataset once in a while and handle the current search landscape independent from the experience achieved from other regions of the search landscape.

Subsequent to this conclusion another statistical experiment is conducted to compare the performance of the TeBHH 1 to its building block components, namely, SR-NA and SR-IE. The third and fourth columns in Table 2 shows that, given equal values as run time, TeBHH1 performs always better than the SR-IE hyper-heuristic. On only one instance, TeBHH 1 performs slightly (and not significantly) better. As for the comparison between TeBHH 1 and SR-NA, although TeBHH 1 still performs significantly better than SR-NA on the majority of instances, on `ORTEC` instances it performs very poorly.

The results of applying the two proposed algorithms on various nurse rostering instances is shown in Table 3. The two algorithms are also compared to various well-known algorithms. While some of these algorithms (like the one in (Chan et al., 2012)) are general-purpose search algorithms, some others are specifically designed to solve the given instance.

On the first seven instances in Table 3, both TeBHH 1 and TeBHH 2 outperform compared algorithms in terms of minimum objective function value. On the instance `Valouxis-1`, both algorithm can achieve the best known result (20), although much later than the state-of-the-art (Solos et al., 2013). Similarly, on `Ikegami` and `ORTEC` instances as well as `BCV-3.46.1`, the state-of-the-art performs better. The algorithms which solve aforementioned

Instance	**TeBHH 1 vs TeBHH 2**	**TeBHH 1 vs SRNA**	**TeBHH 1 vs SRIE**
BCV-A.12.1	>	>	>
BCV-3.46.1	<=	>=	>
BCV-A.12.2	<	>=	>
CHILD-A2	>	>	>=
ERRVH-A	>	>	>
ERRVH-B	>=	>	>
ERMGH-B	>=	>	>
MER-A	>=	>	>
Valouxis-1	>=	>	>
Ikegami-3Shift-DATA1.2	>=	>	>
Ikegami-3Shift-DATA1.1	<=	>	>
Ikegami-3Shift-DATA1	<=	>	>
ORTEC01	<	<	>
ORTEC02	>=	<=	>

Table 2: Statistical Comparison between TeBHH 1, TeBHH 2 and their building block components (SRIE and SRNA). Wilcoxon signed rank test is performed as a statistical test on the objective function values obtained over 20 runs from both algorithms. Comparing algorithm x versus y (x vs. y) $\geq$ ($>$) denotes that x (y) performs slightly (significantly) better than the compared algorithm (within a confidence interval of 95%), while $\leq$ ($<$) indicates vice versa.

Instance	**TeBHH 1**	**TeBHH 2**	**Time (sec.)**	**Best Known**	**Time$_{BestKnown}$ (sec.)**
BCV-A.12.1	**1270**	1280	41443	1294	13914
BCV-3.46.1	3282	3283	-	**3280**	20764
BCV-A.12.2	1858	**1844**	9080	1875	86400
CHILD-A2	**1087**	1089	8229	1095	86400
ERRVH-A	**2118**	2127	9175	2135	86400
ERRVH-B	**3090**	3095	10629	3105	86400
ERMGH-B	1217	**1214**	5	1355	86400
MER-A	8810	**8779**	22008	8814	86400
Valouxis-1	**20**	**20**	3184	**20**	17
Ikegami-3Shift-DATA1.1	6	8	-	**3**	2820
Ikegami-3Shift-DATA1.2	8	9	-	**3**	2820
Ikegami-3Shift-DATA1	6	3	-	**2**	21600
ORTEC01	285	280	-	**270**	69
ORTEC02	290	290	-	**270**	105

Table 3: Comparison between the two proposed algorithms and various well-known (hyper-/meta-)heuristics. The second and third columns contain the best objective function values achieved by TeBHH 1 and TeBHH 2 respectively. Fourth column gives the earliest CPU time (seconds) in which the reported result in bold has been achieved by the corresponding proposed algorithm. '-' denotes that the maximum CPU time has been used up without improving upon the best known result. Same quantities (minimum objective function values and earliest time it has been achieved) are also reported for compared algorithms in columns five and six.

instances are instance-specific and designed to solve a group of highly related instances, such as those in the `Ikegami` family. Overall, the two algorithms perform well on provided instances and produce new best known results for some of them (the first seven instances).

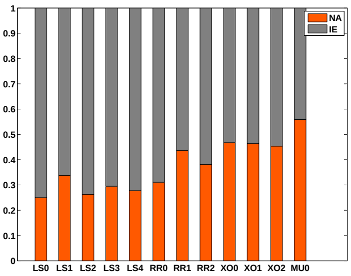
Figure 5 shows the distribution of heuristics to disjoint sets $\mathbf{h}_{NA}$ and $\mathbf{h}_{IE}$ throughout the 20 runs for some of the problem instances. Each run consists of up to 27 stages and in each stage, the set of heuristics is partitioned using tensor factorization. The histograms in Figure 5 is built by counting the number of times a heuristic is associated with the NA and IE move acceptance methods throughout all the runs for a given instance. The histograms vary from one instance to another. The difference between histograms are sometimes minor (as it is between histograms of `BCV-A.12.1` and `BCV-A.12.2`) and sometimes major (as is the case for the instance `MER-A` compared to the rest). However, the common pattern among most of these partitions is that the heuristic MU0 has been equally associated to both sets. Although the framework clearly shows the tendency to assign heuristics more to the $\mathbf{h}_{IE}$ set rather than $\mathbf{h}_{NA}$, Ruin Recreate and Crossover heuristics are likelier to be assigned to $\mathbf{h}_{NA}$ compared to local search heuristics. Since the heuristics in nurse rostering domain all deliver feasible solutions, it makes sense that the framework tries to increase the possibility of diversification by assigning diversifying heuristics to NA acceptance method.

During the improvement stage (Algorithm 6), the algorithm allocates a time budget to each acceptance method. Whenever this budget is consumed, the algorithm switches to a randomly chosen acceptance criteria. Since the tensor analysis is likelier to assign diversifying heuristics to $\mathbf{h}_{NA}$ (keeping the intensifying heuristics in $\mathbf{h}_{IE}$), it thus performs similar to a higher level Iterated Local Search (ILS) algorithm where each intensification step is followed by a diversification one. That in turn results in continuous improvement of the solution as is confirmed in Figure 6 for TeBHH 1 and TeBHH 2 respectively.

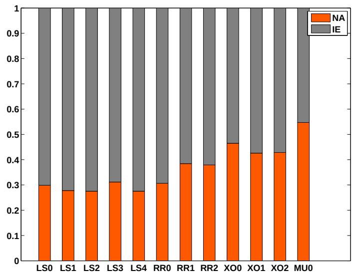
The progress plots corresponding to TeBHH 1 and TeBHH 2 (Figure 6) show that on many instances (particularly on `BCV-A.12.1`, `BCV-A.12.2` and `ERMGH-B`) both algorithm are rarely stuck in local optima. This is a good behaviour showing that given longer run times (similar to the experiments in (Chan et al., 2012)) there is a high likelihood that the algorithms proposed here provide better results with even lower objective function values.

6. Conclusions

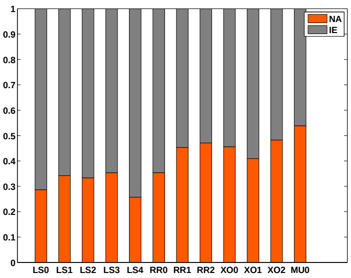
Nurse rostering is a real-world NP-hard combinatorial optimisation problem. A hyper-heuristic approach which benefits from an advanced data science technique, namely, tensor analysis is proposed in this study to tackle a nurse rostering problem. The proposed approach embedding a tensor based machine learning algorithm is tested on well-known benchmark problem instances collected from hospitals across the world. Two different remembering mechanisms, *memory length*s are used within the learning algorithm. One of them remembers all relevant changes from the start of the search process, while the other one refreshes its memory every stage. The results indicate



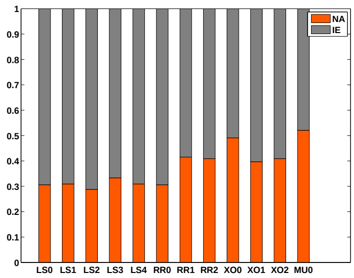
(a) BCV-A.12.1



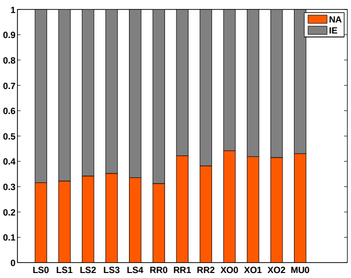
(b) BCV-A.12.2



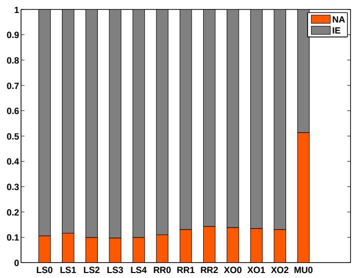
(c) ERRVH-A



(d) ERRVH-B

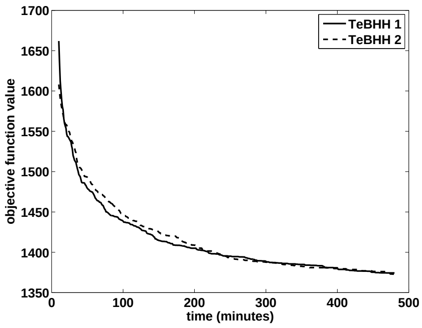


(e) MER-A

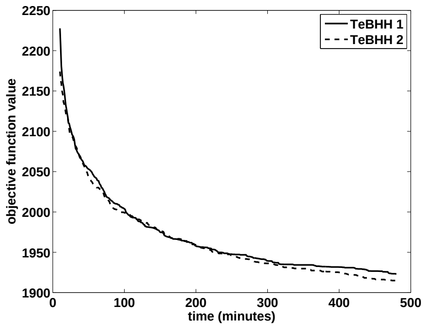


(f) ERMGH-B

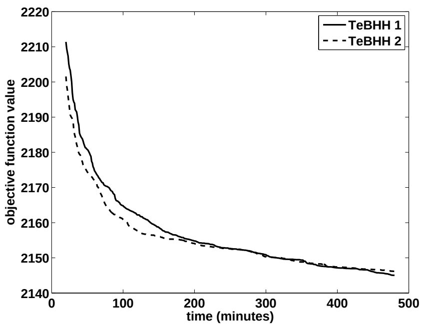
Figure 5: Distribution of heuristics in $\mathbf{h}_{NA}$ and $\mathbf{h}_{IE}$ partitions.



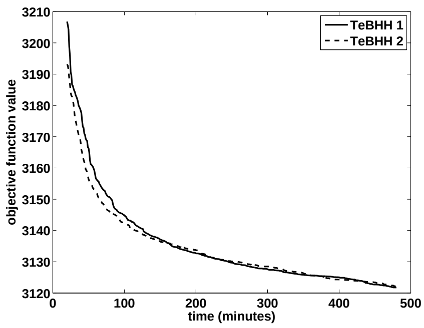
(a) BCV-A.12.1



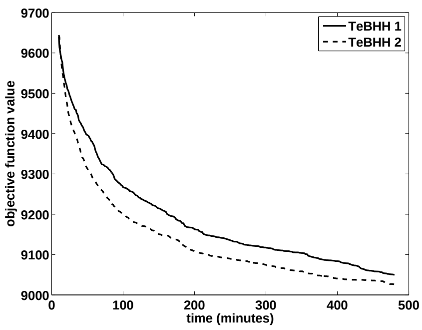
(b) BCV-A.12.2



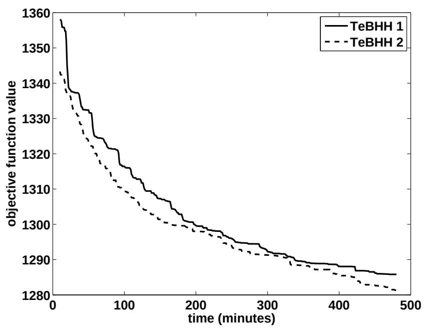
(c) ERRVH-A



(d) ERRVH-B



(e) MER-A



(f) ERMGH-B

Figure 6: The progress of the objective function value on average, obtained from 20 runs of TeBHH 1 and TeBHH 2.

that ‘forgetting’ is slightly more useful than remembering all. Hence, a strategy that decides on the memory length adaptively would be of interest as a future work. In this study, the tensor-based hyper-heuristic with memory refresh generated new best solutions for four benchmark instances and a tie on one of the benchmark instance.

The proposed approach cycles through four stages continuously and periodically, employing machine learning in the first three stages to configure the algorithm to be used in the final stage. The final stage approach itself is an iterated multi-stage algorithm, invoking a randomly chosen hyper-heuristic at each stage. Depending on the problem instance and even a trial, the nature of the low level heuristics allocated to each stage (hence the move acceptance) could change. However, experiments indicate that mutational heuristics often can get allocated to either of the hyper-heuristics. SR-NA allows worsening moves while SR-IE does not. Hence, the final stage component of the tensor-based hyper-heuristic acts as a high level Iterated Local Search algorithm (Lourenço et al., 2010), providing a neat balance between intensification and diversification using the appropriate low level heuristics which are determined automatically during the search process, resulting in continuous improvement in time. The overall approach is enabled to extract fresh knowledge periodically throughout the run time, which is an extremely desired behavior in life-long learning. Thus, the tensor-based hyper-heuristic proposed here can be considered in life-long learning applications.

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