Fitting missing data by means of adaptive meshes of Bézier curves

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Abstract

In this paper we present a method to fit missing data -i.e., to fit a dataset containing a region in which no data are provided- by means of a $\mathcal{C}^1$-quadratic patch. Such a patch is constructed to faithfully extend the shape and the geometric features of the dataset. To this end, a mesh of curves gathering the information about the *shape* of the dataset will be considered and extended to the interior of the hole. Next, a (unique) patch fitting such a mesh of curves will be computed. Several numerical and graphical examples showing the effectiveness of the proposed method are provided.

Keywords: Powell-Sabin finite element; missing data; surface reconstruction; energy functional; shape-preserving; Bézier curves.

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1. Introduction.

The problem of handling sets of scattered data points in which there is some hidden region, some lack of information -usually due to imperfections of the surface or solid to be scanned, to difficulties linked to the scan process, or to accessibility limitations, occlusions, reflecting spaces or surfaces parallel to camera- is rather common. This situation arises in all sorts of fields in which image reconstruction is involved: engineering problems, 3D human body scanning, dental reconstruction, archaeology, CAGD, Earth Sciences, computer vision in robotics, image reconstruction from satellite and radar information, physics, etc. Several papers in the literature address the question of fitting under these geometrical

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difficulties or, more in general, under obstacles due to the dataset itself or to geometrical features or other additional constraints to be achieved.

Regarding the problem of fitting with additional drawbacks, we also find the one of fitting *missing data*, i.e., handling of datasets containing regions where no information -or information with not enough quality- is provided, or the more general of having a surface -understood as the graphic of a bivariate function- suffering from such a lack of information -a *hole*-. Most of the fitting missing data methods considered up to now are developed for arbitrary dataset points, i.e., they do not specifically consider the particular geometric feature of the dataset to be fitted. Applying these common fitting methods very often give rise to surfaces that tend to be 'flat' insofar as a reasonable way to define a patch is by minimizing some kind of measure, like the stretching or the bending energy. As a consequence, most of the existing methods work well for certain functions (as long as they are 'flat'), but they do not lead to proper results in other cases. To illustrate this fact we can consider, for example, the case of filling a hole of information in the top of a semisphere: Depending on what the role of the fitting patch will be, we may want to fill the hole with a 'minimal' criteria -i.e., minimizing some linked measure-, or we may want to fill the hole by a bending patch with *semispherical* shape. It is desirable then to develop methods to fit missing or unstructured data, or to fill holes, providing fitting or filling patches restoring characteristics of the models ([1]), sharp features ([2]) or fulfilling some specific geometrical constraints of industrial or design type, in such a way that the fitting patch will be no longer flat but faithful to the shape of known dataset. In short, we may be more interested in obtaining a global fitting function faithful to the dataset that in obtaining a minimal fitting patch.

With the aim of obtaining fitting patches somehow *inheriting* the shape of the known information, several approaches have been considered: e.g., in [3] the authors propose a method consisting of minimizing energy functionals to extend the shape of the dataset towards the interior of the hole by means of a patch fitting not only the data, but also some of its 'estimated' partial derivatives. Nevertheless, most approaches do not specifically adapt to the particular shape of the data to be fitted. In this work we propose a fitting method over missing data which, for each specific dataset, 'calibrates' its shape by means of several representative curves. These curves will be extended over the missing data preserving, as much as possible, the shape they have over the known data, becoming somehow the cornerstones of the global fitting surface. Such *wireframe* of curves will therefore play an essential role as long as, not only will identify the behavior -near the boundary of the missing data- of the surface to be fitted, but it will also determine the geometry of the fitting patch, that is required to preserve the shape of the known surface.

All over the fitting process, we will be interested in obtaining $\mathcal{C}^1$-splines with the minimum possible degree in order to simplify computational aspects. That's why we choose the $\mathcal{C}^1$-quadratic Powell-Sabin finite element and, insofar we want to work with this finite element, the first step we will have to carry out will be to fit the scattered data points by means of a $\mathcal{C}^1$-quadratic fitting surface. Next, the fitting method we propose will consist mainly of three steps:

i) To the define the wireframe of curves inside the known surface gathering the information about its shape;

ii) To extend such a wireframe of curves towards the interior of the hole;

iii) To construct a fitting patch over the *filled* wireframe of curves in *ii*).

The single-variate fitting process referred to in *ii*) will be carried out by means of Bézier curves, which give a more natural and wide frame to work with geometric features as tangent lines, osculating planes or curvature and torsion ([4] is a basic handbook for Bézier techniques). Regarding *iii*), it is to mention that a surface fitting method based on one-dimensional fittings was for the first time considered in [5]. Nevertheless, the method therein proposed suffers from two drawbacks: on the one hand, it sits on a cartesian wireframe of curves, restricting then to 'measure' the shape of the surface along straight lines parallel to the axis and not allowing, thus, to consider other paths better gathering the shape of the surface. On the other hand, the fitting method in [5] considers just interpolation of function and derivatives values at boundary points of the curves to be fitted, both drawbacks leading to a more restrictive fitting frame as long as derivatives values on their own do not, in general, give a complete insight of the geometry of a surface.

The fitting method we propose in this work leads, especially in the case of more irregularly shaped functions, to more accurate fitting patches. This fact is a direct consequence of improving two of the main aspects of previous fitting methods:

i) The wireframe over which the fitting patch lies is obtained by means of Bézier techniques, which allows to more faithfully extend inside the hole the shape of the surface to be fitted;

ii) The fitting patch fits a wireframe which is general enough -no longer cartesian- to more faithfully gather the information of the surface to be fitted.

Moreover, this method constitutes, to our knowledge, the first one in which the fitting patches are obtained by means of Bézier curves, that more properly deal with geometric features.

The paper is organized as follows: in Section 2 we briefly present a review of some papers in the literature regarding the general problem of fitting data and the more specific one of fitting data under additional drawbacks. In Section 3 we briefly recall the basic concepts on Powell-Sabin triangulations and Bézier curves that will be used throughout the work. In Section 4 we deal with the problem that we want to solve: we fix the notation to be used, we formulate the problem and we show the existence and uniqueness of the solution. Section 5 is devoted to present several graphical and numerical examples which show the effectiveness of the method proposed. We end up with a conclusions section.

2. A brief review of fitting data methods.

Among the techniques commonly applied in data fitting problems we find, for example, *B-splines*, *radial basis functions (RBF)*, *algebraic fitting* or *discrete energy minimization*.

RBF-fitting methods (see e.g. [6] or [7]) have the advantages that are mesh-free and that they become a powerful tool when handling multidimensional fitting data, while B-spline fitting methods become computationally harder when fine meshes are considered. On the contrary, using B-splines has the advantage that insofar as basis functions have small local supports, they lead to sparse matrix that, moreover, are symmetric and definite positive under some conditions on the basis functions. Besides, B-splines are easier to implement than RBF since they are piecewise polynomial functions, usually with low degree.

Algebraic surface fitting, consisting in fitting a dataset by means of a polynomial implicit surface $f(x, y, z) = 0$ where the coefficients of f are usually chosen to minimize the mean square distance from the dataset to the implicit surface, is a natural approach to the fitting problem. Apart from the fact that manipulating polynomials is computationally more efficient than doing it with arbitrary analytic functions, algebraic surfaces provide enough generality to accurately model almost all complicated rigid objects. On the contrary, this kind of fitting often suffers from instability and numerical problems (see e.g. [8] or [9] and references therein).

Discrete energy minimization is an extended model in computer vision. One key advantage of this method is that it allows to handle a great variety of problems related to this researching field, like image denoising, segmentation, motion estimation object, recognition and image editing. Nevertheless, modern vision problems involve complex models and larger datasets that give rise to hard energy minimization problems (see e.g. [10] or [11] and references therein).

Among the existing techniques developed to handle fitting data with additional constraints we find the *biharmonic optimization*, used to overcome the problem of the flatness of the surfaces or regions that some methods based on minimization of stretching or bending energy provide (see e.g. [12]); *transfinite interpolation*, used e.g. in [13] to construct a Hermite interpolant matching values and normal derivatives of a given function on the boundary of a simply connected planar domain; or other advanced techniques, like the one developed in [14], where algorithms to handle weakly defined control points by means of B-spline surfaces are provided.

Two interesting papers in the surface modeling field are [15] and [16]. In [15] a freeform modeling framework for unstructured triangle meshes based on constraint shape optimization is presented. As in this paper, in [15] the authors also consider the minimization of quadratic energy functionals that is carried out by means of the corresponding Euler-Lagrange equations leading to surfaces with minimal area, minimal surface bending, or minimal variation of linearized curvature. On the other hand, in [16] the authors explore discretizations of Laplacian and Laplacian gradient energies PDE's on meshes by using mixed finite elements, and they demonstrate applications in several geometric modeling problems, such as hole filling.

References

[1] L.-C. Wang, Y.-C. Hung, Hole filling of triangular mesh segments using systematic grey prediction, Computer-Aided Design 44 (12) (2012) 1182–1189. doi:10.1016/j.cad.2012.07.007.

[2] C. Wang, P. Hu, A hole-filling algorithm for triangular meshes in engineering, International Journal for Computational Methods in Engineering Science and Mechanics 14 (5) (2013) 465–471. doi:10.1080/15502287.2013.784385.

[3] M. A. Fortes, P. González, A. Palomares, M. Pasadas, Filling holes with shape preserving conditions, Math. and Comp. in Simul. 118 (2015) 198–212. doi:10.1016/j.matcom.2014.12.008.

[4] G. Farin, Curves and Surfaces in CAGD. A Practical Guide, 5th Edition, Morgan Kaufmann, 2002. doi:10.1016/B978-1-55860-737-8.X5000-5.

[5] M. A. Fortes, P. González, A. Palomares, M. L. Rodríguez, Filling holes using a mesh of filled curves, Math. and Comp. in Simulation 164 (2019) 78–93.

[6] M. Buhmann, Radial basis functions: theory and implementations, No. 12 in Cambridge Monographs on Applied and Computational Mathematics, Cambridge University Press, Cambridge, 2003.

[7] G. Fasshauer, Meshfree Approximation Methods with Matlab, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2007.

[8] G. Taubin, An improved algorithm for algebraic curve and surface fitting, Proceedings of the 4^{th} International Conference on Computer Vision (1993) 658–665. doi:10.1109/ICCV.1993.378149.

[9] B. Jüttler, A. Felis, Least-squares fitting of algebraic spline surfaces, Advances in Computational Mathematics 17 (2002) 135–152. doi:10.1023/A:1015200504295.

[10] J. H. Kappes, et al., A comparative study of modern inference techniques for discrete energy minimization problems, Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (2013) 1328–1335. doi:10.1109/CVPR.2013.175.

[11] R. Szeliski, et al., A comparative study of energy minimization methods for markov random fields with smoothness-based priors, Proceedings of the EEE Transactions on Pattern Analysis and Machine Intelligence 30 (6) (2008) 1068–1080. doi:10.1109/TPAMI.2007.70844.

[12] A. V. Smurygin, I. V. Zhurbin, Biharmonic optimization of piecewise planar surfaces, Optoelectronics, Instrumentation and Data Processing 51 (2) (2015) 170–174. doi:10.3103/S8756699015020107.

[13] C. Dyken, M. S. Floater, Transfinite mean value interpolation, Computer Aided Geometric Design 26 (2009) 117–134. doi:10.1016/j.cagd.2007.12.003.

[14] V. Weiss, L. Andor, G. Renner, T. Várady, Advanced surface fitting techniques, Computer Aided Geometric Design 19 (2002) 19–42. doi:10.1016/S0167-8396(01)00086-3.

[15] M. Botsch, L. Kobbelt, An intuitive framework for real-time freeform modeling, ACM Transactions on Graphics 23 (3) (2004) 630–634. doi:10.1145/1015706.1015772.

[16] A. Jacobson, E. Tosun, O. Sorkine, D. Zorin, Mixed finite elements for variational surface modeling, Computer Graphics Forum 29 (5) (2010) 1565–1574. doi:10.1111/j.1467-8659.2010.01765.x.