Finite-time synchronization of multi-layer nonlinear coupled complex networks via intermittent feedback control

Daoyuan Zhang, Yanjun Shen

College of Electrical Engineering & New Energy,

China Three Gorges University, Yichang, Hubei, 443002, China

Jun Mei

Department of Electrical, Electronic and Computer Engineering,

University of Pretoria, Pretoria 0002, South Africa

Abstract

This paper addresses the problem of finite-time synchronization for a class of multi-layer nonlinear coupled complex networks via intermittent feedback control. Firstly, based on finite-time stability theory, some novel criteria are given to guarantee that the error system of drive-response systems is still finite-time stable under an inherently discontinuous controller. Then, by proposing two kinds of intermittent feedback control laws, sufficient conditions of finite-time synchronization of two kinds of multi-layer complex networks are derived, respectively. The time delay between different layers is also taken into consideration. Finally, a numerical example is provided to verify the effectiveness of the proposed methods.

Index Terms

Complex networks, finite-time synchronization, multi-layer, intermittent feedback control.

I. Introduction

In the past few decades, the synchronization problem of complex networks has attracted more and more attention in practical applications [1], [2], [3], [4], [5], [6]. A basic complex network consists of some nodes and links between the nodes, where each node is a dynamic system. Since the problem of synchronization of chaotic systems has been studied in [1], synchronization as a potential engineering application has been applied into secure communication, neural network, biology and information processing [7], [8], [9], [10], [11]. Up till now, there are lots of different types of synchronization, for instance, complete synchronization [12], anti-synchronization [13], projective synchronization [14] and cluster synchronization [15], [16].

It should be noted that information of different nodes is transmitted based on a shared band-limited digital communication network. Thus, it is interesting to study synchronization of complex networks with delayed coupling. For example, global synchronization of a general linear coupled network has been studied with a time-varying coupling delay in [17]. Then, a developed generalized mixed outer synchronization are also studied with a time-varying coupling delay [18]. In [19], local and global synchronization of complex networks have been studied with a fixed delay. In [20], global exponential synchronization of nonlinear coupled dynamical networks are also considered with a delayed coupling. However, the aforementioned results are based on one or two layers network. Multi-layer networks which have more than two layers can be seen as some sub-networks distributed in different layers. For example, there exists a three-layers network about information transmission in a simple telephone network. Moreover, different transmission delays between different layers should also be taken into account. Therefore, synchronization of multi-layer networks with delayed coupling are more significant.

Different from continuous control methods, intermittent controller is implemented intermittently during a control period. Because of easier implementation and smaller control cost, the problem of synchronization under intermittent control has attracted lots of attention [21], [22], [23], [24], [25], [26], since the intermittent control is firstly proposed in [27]. Synchronization with finite time convergence has advantages to enhance the robustness and to overcome the disturbance in practical control and applications [28]. The existing results about finite-time stability and finite-time synchronization have been considered in [29], [30], [31], [32], [33], [34], [35], [36]. Therefore, it is very interesting to investigate finite-time synchronization of complex networks via intermittent feedback control. Some related results have been studied in our previous works [37], [38], [39], [40], however, the linear coupling is adopted in these works.

In this paper, finite-time synchronization of multi-layer nonlinear coupled complex networks is studied via intermittent feedback control. Firstly, based on finite-time stability theory, some novel criteria are given to guarantee that the nonlinear system is still finite-time stable. Then, by proposing two kinds of intermittent feedback controllers, sufficient conditions of finite-time synchronization of two complex networks are derived. The main contributions of this paper include: i) some novel criteria are given to guarantee finite-time synchronization of the error system of the drive-response systems under an intermittent controller; ii) then, based on these presented criteria, finite-time synchronization of two kinds of multi-layer nonlinear coupled networks is studied via periodically intermittent feedback control and aperiodically intermittent feedback control, respectively. The time delay between different layers is also taken into consideration. The corresponding sufficient conditions are also given to guarantee that the error system is finite-time stable.

This paper is organized as follows. In Section II, some definitions of finite-time stability and some novel finite-time criteria are given. In Section III, by proposing two kinds of intermittent feedback controllers, sufficient conditions of finite-time synchronization of delayed complex networks are derived respectively. Section IV provides an example to illustrate the validity of the proposed design methods. Finally, this paper is concluded in Section V.

II. Preliminaris

Let $\mathbb{R}^n$ denote n-dimension real space and $\mathbb{R}^+$ denote 1-dimension positive real space. For any $x \in \mathbb{R}^n$, let $\|x\| = (x^T x)^{1/2}$. For a matrix $P \in \mathbb{R}^{n\times n}$, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the largest and the smallest eigenvalues of the symmetric matrix P, respectively. Consider the following master system (drive system):

$$\dot{x}(t) = \phi(x(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $x(0) = x_0$, $\phi(\cdot) : \mathscr{D} \to \mathbb{R}^n$ is continuous on an open neighborhood $\mathscr{D}$ of the origin $x(t) = 0$ with $\phi(0) = 0$.

Definition 1: [41] The zero solution of (1) is finite-time convergent if there is an open neighborhood $\mathcal{U} \subset \mathscr{D}$ of the origin and a function $\mathcal{T} : \mathcal{U} \setminus \{0\} \to (0, \infty)$, such that $\forall x_0 \in \mathcal{U}$, the solution $\psi(t, x_0)$ of system (1) is defined and $\psi(t, x_0) \in \mathcal{U}$ $\{0\}$ for

$t \in [0, \mathcal{T}(x_0))$, and $\lim_{t \to \mathcal{T}(x_0)} \psi(t, x_0) = 0$. Then, $\psi(t, x_0)$ is called the settling time. If the zero solution of system (1) is finite-time convergent, the set of point x_0 such that $\psi(t, x_0) \to 0$ is called the domain of attraction of the solution.

Definition 2: [41] The zero solution of (1) is finite-time stable if it is Lyapunov stable and finite-time convergent. When, $\mathcal{U} = \mathscr{D} = \mathbb{R}^n$, the zero solution is said to be globally finite-time stable.

Consider the following slave system (response system):

$$\dot{y}(t) = \varphi(y(t), u(t)), \tag{2}$$

where $y(t) \in \mathbb{R}^n$, $y(0) = y_0$, $u(t) \in \mathbb{R}^q$ is the controller, $u(0) = u_0$, $\varphi(\cdot) : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^n$ is continuous. Denote the solutions of (1) and (2) as $x(t, x_0)$ and $y(t, y_0, u_0)$, respectively. For the notational simplicity, we denote $x(t, x_0)$ simply by $x(t)$, and $y(t, y_0, u_0)$ by $y(t)$. Next, we give the definition of finite-time synchronization of systems (1) and (2).

Definition 3: Systems (1) and (2) are said to be synchronization in finite time if there exists an open neighborhood $\mathcal{U} \subset \mathbb{R}^n$ of the origin such that $e_0 = y_0 - x_0 \in \mathcal{U}$ and a function $\mathcal{T}_1 : \mathcal{U} \setminus \{0\} \to (0, +\infty)$ and

$$\lim_{t \to \mathcal{T}_1(e_0)} \|e(t)\| \to 0,$$
$$\|e(t)\| = 0, \forall t > \mathcal{T}_1(e_0),$$

where $e(t) = y(t) - x(t)$ denotes the synchronization error of systems (1) and (2).

A continuous controller is designed in the form of $u(t) = \mathcal{F}(e(t)), \forall t \in [t_0, +\infty)$. If there exists a Lyapunov function $V(e(t))$ defined on a neighborhood $\mathcal{U} \subset \mathbb{R}^n$ of the origin such that $\dot{V}(e(t)) \leq -\alpha V^{\eta}(e(t))$, where $\alpha > 0$, $0 < \eta < 1$, from [42] and Definition 1, the error system (2)-(1) is synchronized in finite time. Based on our previous work [37], a new controller is proposed as follows:

$$\begin{cases} u(t) = 0, & t_0 + kT \leq t < t_0 + (k + h_1)T, \\ u(t) = \mathcal{F}(e(t)), & t_0 + (k + h_1)T \leq t < t_0 + (k + h_2)T, \\ u(t) = 0, & t_0 + (k + h_2)T \leq t < t_0 + (k + 1)T, \end{cases} \tag{3}$$

where $0 \leq h_1 < h_2 \leq 1$, $T > 0$ is the control period, $h_2 - h_1$ is the control rate and $k \geq 0$ is a nonnegative integer. Now, sufficient conditions are given to guarantee that the error systems (1)-(2) is synchronized in finite time via the controller (3).

Theorem 1: Consider systems (2) and (1) with controller (3), if there is a Lyapunov function $V(e(t))$ defined on a neighborhood $\mathcal{U} \subset \mathbb{R}^n$ of the origin such that

$$\begin{cases} \dot{V}(e(t)) \leq 0, & t_0 + kT \leq t < t_0 + (k + h_1)T, \\ \dot{V}(e(t)) \leq -\alpha V^{\eta}(e(t)), & t_0 + (k + h_1)T \leq t < t_0 + (k + h_2)T, \\ \dot{V}(e(t)) \leq 0, & t_0 + (k + h_2)T \leq t < t_0 + (k + 1)T, \end{cases} \tag{4}$$

hold, where $0 \leq h_1 < h_2 \leq 1$, $\alpha > 0$, $0 < \eta < 1$, then, the error system (2)-(1) is synchronized in finite-time. In addition, for any given t_0, the following inequality holds:

$$V^{1-\eta}(e(t)) \leq V^{1-\eta}(e_0) - \alpha(1 - \eta)(h_2 - h_1)(t - t_0 - h_1 T), \quad t_0 \leq t \leq T', \tag{5}$$

and $V(e(t)) \equiv 0, \forall t > T'$, where $T' = \frac{V^{1-\eta}(e_0)}{\alpha(1-\eta)(h_2-h_1)} + t_0 + h_1 T$ denotes the settling time.

Proof: The proof is based on a recursive approach and the following auxiliary function

$$H(t) = V^{1-\eta}(t) - M + \alpha(1 - \eta)(h_2 - h_1)t, \tag{6}$$

where $M = V^{1-\eta}(e_0) + \alpha(1-\eta)(h_2 - h_1)(t_0 + h_1 T)$. It is also easy to obtain that $H(t_0) < 0$. For simplicity, we denote $V(e(t))$ as $V(t)$.

Step 1: For any $t \in [t_0, t_0 + h_1 T)$, we have

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0).$$

Then, we can obtain

$$H(t) \leq V^{1-\eta}(t_0) - M + \alpha(1-\eta)(h_2 - h_1)t < 0.$$

For any $t \in [t_0 + h_1 T, t_0 + h_2 T)$, we have

$$\begin{aligned} V^{1-\eta}(t) &\leq V^{1-\eta}(t_0 + h_1 T) - \alpha(1-\eta)(t - t_0 - h_1 T) \\ &\leq V^{1-\eta}(t_0) - \alpha(1-\eta)t + \alpha(1-\eta)(t_0 + h_1 T). \end{aligned}$$

Then,

$$\begin{aligned} H(t) &\leq V^{1-\eta}(t_0) - \alpha(1-\eta)t + \alpha(1-\eta)(t_0 + h_1 T) - M + \alpha(1-\eta)(h_2 - h_1)t \\ &\leq -\alpha(1-\eta)t + \alpha(1-\eta)(t_0 + h_1 T) - \alpha(1-\eta)(h_2 - h_1)(t_0 + h_1 T) \\ &\quad + \alpha(1-\eta)(h_2 - h_1)t \\ &\leq \alpha(1-\eta)(1 - (h_2 - h_1))(t_0 + h_1 T - t) \leq 0. \end{aligned} \tag{7}$$

For any $t \in [t_0 + h_2 T, t_0 + T)$, we have

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0 + h_2 T) \leq V^{1-\eta}(t_0) - \alpha(1-\eta)(t_0 + h_2 T) + \alpha(1-\eta)(t_0 + h_1 T).$$

Then,

$$\begin{aligned} H(t) &\leq V^{1-\eta}(t_0) - \alpha(1-\eta)(t_0 + h_2 T) + \alpha(1-\eta)(t_0 + h_1 T) - M + \alpha(1-\eta)(h_2 - h_1)t \\ &\leq \alpha(1-\eta)(h_2 - h_1)(t - t_0 - h_1 T - T) < 0. \end{aligned} \tag{8}$$

Step 2: For any $t \in [t_0 + T, t_0 + (1 + h_1)T)$, we have

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0 + T) \leq V^{1-\eta}(t_0) - \alpha(1-\eta)(t_0 + h_2 T) + \alpha(1-\eta)(t_0 + h_1 T).$$

Then,

$$\begin{aligned} H(t) &\leq V^{1-\eta}(t_0) - \alpha(1-\eta)(t_0 + h_2 T) + \alpha(1-\eta)(t_0 + h_1 T) - M + \alpha(1-\eta)(h_2 - h_1)t \\ &\leq \alpha(1-\eta)(h_2 - h_1)(t - t_0 - h_1 T - T) < 0. \end{aligned} \tag{9}$$

For any $t \in [t_0 + (1 + h_1)T, t_0 + (1 + h_2)T)$, we have

$$\begin{aligned} V^{1-\eta}(t) &\leq V^{1-\eta}(t_0 + T + h_1 T) - \alpha(1-\eta)(t - t_0 - h_1 T - T) \\ &\leq V^{1-\eta}(t_0) - \alpha(1-\eta)t + \alpha(1-\eta)[t_0 + (h_1 + 1)T - (h_2 - h_1)T]. \end{aligned}$$

Then,

$$\begin{aligned} H(t) &\leq V^{1-\eta}(t_0) - \alpha(1-\eta)t + \alpha(1-\eta)[t_0 + (h_1 + 1)T - (h_2 - h_1)T] - M + \alpha(1-\eta)(h_2 - h_1)t \\ &\leq -\alpha(1-\eta)t + \alpha(1-\eta)[t_0 + (h_1 + 1)T - (h_2 - h_1)T] - \alpha(1-\eta)(h_2 - h_1)(t_0 + h_1 T) \\ &\quad + \alpha(1-\eta)(h_2 - h_1)t \\ &\leq \alpha(1-\eta)(1 - (h_2 - h_1))(t_0 + h_1 T + T - t) \leq 0. \end{aligned} \tag{10}$$

Inductive Step : For any $T' > t_0$, there exists $k \geq 0$ such that $T' \in [t_0 + kT, t_0 + (k+1)T)$. Thus, if $T' \leq t_0 + (k + h_1)T$, for any $t \in [t_0 + kT, T']$, we have

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0 + kT) \leq V^{1-\eta}(t_0) - \alpha(1-\eta)(h_2 - h_1)kT,$$

then,

$$H(t) \leq -\alpha(1-\eta)(h_2 - h_1)(t_0 + h_1 T + kT - t) < 0.$$

For any $t \in (T', +\infty)$, we have

$$V^{1-\eta}(t) \leq V^{1-\eta}(T') \leq V^{1-\eta}(t_0) - \alpha(1-\eta)(h_2 - h_1)(T' - t_0) \leq 0.$$

Note that $V(t) \geq 0$, thus, $V(t) = 0$.

If $t_0 + (k + h_1)T < T' \leq t_0 + (k + h_2)T$, then, for any $t \in [t_0 + (k + h_1)T, T']$, we have

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - \alpha(1-\eta)t + \alpha(1-\eta)[t_0 + (h_1 + k)T - (h_2 - h_1)kT].$$

Then,

$$H(t) \leq -\alpha(1-\eta)(1 - (h_2 - h_1))t + \alpha(1-\eta)(1 - (h_2 - h_1))(t_0 + h_1 T + kT) \leq 0.$$

For any $t \in (T', +\infty)$, it is easy to find that

$$V^{1-\eta}(t) \leq V^{1-\eta}(T') \leq 0.$$

Note that $V(t) \geq 0$, thus, $V(t) = 0$.

If $T' > t_0 + (k + h_2)T$, for any $t \in [t_0 + (k + h_1)T, T']$, we have

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - \alpha(1-\eta)t + \alpha(1-\eta)[t_0 + (h_1 + k)T - (h_2 - h_1)kT].$$

Then,

$$H(t) \leq -\alpha(1-\eta)(1 - (h_2 - h_1))t + \alpha(1-\eta)(1 - (h_2 - h_1))(t_0 + h_1 T + kT) \leq 0.$$

For any $t \in (T', +\infty)$, we have

$$V^{1-\eta}(t) \leq V^{1-\eta}(T') \leq 0.$$

Therefore,

$$H(t) \leq -\alpha(1-\eta)(h_2 - h_1)[(k+1)T + t_0 + h_1 T] + \alpha(1-\eta)(h_2 - h_1)t < 0. \tag{11}$$

Overall, when $t \in [t_0, T']$, we have $H(t) < 0$ and when $t > T'$, $V(t) \equiv 0$. The proof is completed.

Remark 1: If $h_1 = 0$ and $h_2 = 1$, Theorem 1 is reduced to that considered in [42], that is so-called full-control or continuous control; If $h_1 = 0$ and $0 < h_2 < 1$, Theorem 1 is reduced to that considered in [37], that is so-called front-control which belongs to a type of intermittent feedback control. The authors in [37] studied the finite-time synchronization of a class of complex networks. Especially, only controlling the front part of one control period is considered, which is not practical in nature. In most cases, the control part is stochastic in one period. If $0 < h_1 < 1$ and $h_2 = 1$, the results could be seen as another type of intermittent feedback control.

It is very important to notice that constants h_1 and h_2 are adopted in (4). In what follows, the different values of h_k^1 and h_k^2 are considered, which have a more general form.

Corollary 1: Suppose there is a Lyapunov function $V(e(t))$ defined on a neighborhood $\mathcal{U} \subset \mathbb{R}^n$ of the origin, such that the following conditions

$$\begin{cases} \dot{V}(e(t)) \le 0, & t_0 + kT \le t < t_0 + (k + h_k^1)T, \\ \dot{V}(e(t)) \le -\alpha V^{\eta}(e(t)), & t_0 + (k + h_k^1)T \le t < t_0 + (k + h_k^2)T, \\ \dot{V}(e(t)) \le 0, & t_0 + (k + h_k^2)T \le t < t_0 + (k + 1)T, \end{cases} \tag{12}$$

hold, where $0 \le h_k^1 < 1 - \theta$, $h_k^2 = h_k^1 + \theta$, $\theta > 0$, $\alpha > 0$, $0 < \eta < 1$, $T > 0$ is the control period and $k \ge 0$ is a nonnegative integer. Then, the origin of error system (2)-(1) is finite-time stable. In addition, for any given t_0, the following inequality holds:

$$V^{1-\eta}(e(t)) \le V^{1-\eta}(e_0) - \alpha\theta(1 - \eta)(t - t_0 - \theta T), \quad t_0 \le t \le T', \tag{13}$$

and

$$V(e(t)) \equiv 0, \quad \forall t > T',$$

where $T' = t_0 + \theta T + \frac{V^{1-\eta}(e_0)}{\alpha\theta(1-\eta)}$ denotes the settling time.

Proof: Construct an auxiliary function $H_1(t) = V^{1-\eta}(t) - M_1 + \alpha\theta(1 - \eta)t$, where $M_1 = V^{1-\eta}(e_0) + \alpha\theta(1 - \eta)(t_0 + \theta T)$. By using a similar recursive approach to Theorem 1, then, the results could be obtained.

Remark 2: Obviously, when $h_k^1 = h_1$ and $h_k^2 = h_2$, the result of Corollary 1 is reduced to that of Theorem 1. In Corollary 1, the control parts in different periods are different. This case can be used widely in practice. It should be noticed that the length of the control parts in different control periods are constant. A time-varying length of the control part will be considered in future. In addition, in one control period, we can divide it into many small intervals. It is very important to show that when the length of the control part is constant, such a case is reduced to Corollary 1 in essence.

III. Finite-time synchronization of complex networks via intermittent feedback control

Consider the following m layers complex dynamical networks consisting of N nonlinearly identical nodes:

$$\dot{x}_i(t) = f_i(x_i(t)) + \sum_{j=1}^{N} a_{ij}^0 g_{0j}(x_j(t)) + \sum_{r=1}^{m-1} \sum_{j=1}^{N} a_{ij}^r g_{rj}(x_j(t - \tau_r)), i = 1, 2, \cdots, N, \tag{14}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t)]^T \in \mathbb{R}^n$ denotes the state vector of the ith node. Functions $f_i(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ and $g_{rj}(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ are continuous. The abbreviation $x(t) = [x_1(t)^T, \cdots, x_N(t)^T]^T$, $f(x) := [f_1(x_1), \cdots, f_N(x_N)]^T$, $g(x) = [g_0(x)^T, \cdots, g_{m-1}(x)^T]^T$ and $g_r(x) := [g_{r1}, \cdots, g_{rN}]^T$ can be used to simply the notations. $\tau_r \ge 0$ $(r = 1, \cdots, m - 1)$ denote the different coupling delays. It should be noticed that $\{\tau_r\}$ is a strictly increasing sequence. $A_r = (a_{ij}^r) \in \mathbb{R}^{N\times N}$ $(r = 0, 1, \cdots, m-1)$ denote the weight configuration matrices. If there is a connection between nodes i and j $(i \ne j)$, then $a_{ij}^r = a_{ji}^r > 0$; otherwise, $a_{ij}^r = a_{ji}^r = 0$. The diagonal elements of matrices A_r are defined as

$$a_{ii}^r = -\sum_{j=1, j\ne i}^{N} a_{ij}^r, \quad r = 0, 1, \cdots, m - 1. \tag{15}$$

For simplicity, the drive system (14) can be written in following form:

$$\dot{x}(t) = f(x(t)) + A_0 g_0(x(t)) + \sum_{r=1}^{m-1} A_r g_r(x(t - \tau_r)). \tag{16}$$

Remark 3: When $m = 2$ and $g(x) = \Gamma x$, (Γ denotes the inner-coupling matrix), the finite-time synchronization control for system (14) has been studied in [37]. Then, based on the same control strategy of [37], the finite-time lag synchronization for system (14) with $m = 2$ is studied in [43]. More similar systems to (14) have been studied in [29], [30], [38], [39], [40]. Compared with the aforementioned systems, the system (14) has a more general form.

Assumption 1: [44] The function $f(\cdot) \in QUAD\ (P, \Delta, \xi)$, if there exists a positive definite diagonal matrix $P = diag\{P_1, P_2, \cdots, P_N\} \in \mathbb{R}^{nN\times nN}$, a diagonal matrix $\Delta = diag\{\Delta_1, \Delta_2, \cdots, \Delta_N\} \in \mathbb{R}^{nN\times nN}$ and a scalar $\xi > 0$, such that

$$(y(t) - x(t))^T P[f(y(t)) - f(x(t)) - \Delta(y(t) - x(t))] \leq -\xi(y(t) - x(t))^T (y(t) - x(t)),$$

holds for any $x, y \in \mathbb{R}^{nN}$, where P_i, $\Delta_i \in \mathbb{R}^{n\times n}$ are diagonal matrices.

Remark 4: Assumption 1 has been considered in [37], [40]. In fact, it can be applied to all the well-known systems, such as Lorenz system, Chen system, Chua's system and so on.

Assumption 2: There exists a positive constant l such that $g(\cdot)$ satisfies the following Lipschitz condition:

$$\|g(x(t)) - g(y(t))\| \leq l\|(x(t) - y(t))\|, \tag{17}$$

for all $x, y \in \mathbb{R}^{nN}$ and $t \geq 0$.

The following lemmas are useful for our main results.

Lemma 1: [45] Given any real matrices X, Y and K of appropriate dimensions and a scalar $\varepsilon > 0$ such that $K = K^T > 0$. Then, the following inequality holds:

$$X^T Y + Y^T X \leq \varepsilon X^T K X + \varepsilon^{-1} Y^T K^{-1} Y.$$

Lemma 2: [37] For any real vectors $x_1, x_2, \cdots, x_n \in \mathbb{R}^n$, there exists $0 < q < 2$ such that

$$(\|x_1\|^2 + \|x_2\|^2 + \cdots + \|x_n\|^2)^{q/2} \leq \|x_1\|^q + \|x_2\|^q + \cdots + \|x_n\|^q.$$

Note that the drive system is in form of (16), thus, the response system can be written as follows:

$$\dot{y}(t) = f(y(t)) + A_0 g_0(y(t)) + \sum_{r=1}^{m-1} A_r g_r(y(t - \tau_r)) + u(t), \tag{18}$$

where $y(t) = [y_1(t)^T, \cdots, y_N(t)^T]^T$, $y_i(t) = [y_{i1}(t), y_{i2}(t), \cdots, y_{in}(t)]^T \in \mathbb{R}^n$ denotes the response state vector of $x_i(t)$ of the ith node. $u(t) = [u_1(t)^T, \cdots, u_N(t)^T]^T$, $u_i(t)$ denotes the control input of the ith node.

In view of (18)- (16), the synchronization errors system can be obtained

$$\begin{aligned} \dot{e}(t) = {} & f(y(t)) + A_0 g_0(y(t)) + \sum_{r=1}^{m-1} A_r g_r(y(t - \tau_r)) + u(t) \\ & - f(x(t)) - A_0 g_0(x(t)) - \sum_{r=1}^{m-1} A_r g_r(x(t - \tau_r)), \end{aligned} \tag{19}$$

where $e(t) = [e_1(t)^T, e_2(t)^T, \cdots, e_N(t)^T]^T$, $e_i(t) = y_i(t) - x_i(t)$.

In order to achieve finite-time synchronization of (16) and (18), the periodically intermittent controller $u_i(t)$ ($i = 1, 2, \cdots, N$) are designed as follows:

$$u_i(t) = \begin{cases} 0, & \|e_i(t)\| = 0 \ \texttt{or} \ t \in [kT, (k+h_1)T), \\ -\lambda_i e_i(t) - \gamma \frac{\lambda_{\max}(P)^\eta}{\lambda_{\min}(P)} sign(e_i(t))|e_i(t)|^{2\eta-1} - (\gamma \sum\limits_{r=1}^{m-1} \int_{t-\tau_r}^{t} e_i(s)^T P_i e_i(s) ds)^\eta \frac{e_i}{\|e_i\|^2} \frac{1}{\lambda_{\min}(P)}, \\ \qquad t \in [(k+h_1)T, (k+h_2)T), \\ 0, & t \in [(k+h_2)T, (k+1)T), \end{cases} \tag{20}$$

where $\lambda_i > 0$ denotes the control gain, $\gamma > 0$ is a tunable constant and $\frac{1}{2} < \eta < 1$, T denotes the control period, $h_2 - h_1$ is the control rate.

Now, we give the main results.

Theorem 2: Suppose that Assumption 1 and Assumption 2 hold. If there exist two positive definite diagonal matrices P and Δ and three positive constants ξ, ξ_1, ξ_2 such that

$$\begin{aligned} & -\tfrac{\xi I^{nN}}{\lambda_{\max}(P)} + \Delta + \xi_1 A_0^T A_0 + \tfrac{l^2 I^{nN}}{\xi_1} + \xi_2 \sum_{r=1}^{m-1} A_r^T A_r + (m-1) I^{nN} \le 0, \\ & l^2 - \xi_2 \le 0, \end{aligned} \tag{21}$$

where $I^{nN} \in \mathbb{R}^{nN}$ denotes identity matrix, then the error system (19) is synchronized under the periodically intermittent controller (20) in a finite time:

$$T_2 = \frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)(h_2 - h_1)} + t_0 + h_1 T,$$

where $V(t_0) = \frac{1}{2} e(t_0)^T P e(t_0) + \sum\limits_{r=1}^{m-1} \int_{t_0-\tau_r}^{t_0} e(s)^T P e(s) ds$.

Proof: Consider the following Lyapunov-Krasovskii functional:

$$V(t) = V_1(t) + V_2(t), \tag{22}$$

where $V_1(t) = \frac{1}{2} e(t)^T P e(t)$, $V_2(t) = \sum\limits_{r=1}^{m-1} \int_{t-\tau_r}^{t} e(s)^T P e(s) ds$.

The derivative of $V_1(t)$ along the error system (19) is given by

$$\begin{aligned} \dot{V}_1(t) = e(t)^T P \dot{e}(t) = e(t)^T P[f(y) - f(x(t))] \\ + e(t)^T P A_0 [g_0(y) - g_0(x(t))] + e(t)^T P u(t) \\ + \sum_{r=1}^{m-1} e(t)^T P A_r [g_r(t-\tau_r) - g_r(t-\tau_r)]. \end{aligned} \tag{23}$$

By Assumption 1, Assumption 2 and Lemma 1, we have

$$e(t)^T P[f(y) - f(x(t) - \Delta e(t)] \le -\xi e(t)^T e(t)$$

and

$$e(t)^T P A_0 [g_0(y) - g_0(x(t))] \le \xi_1 e(t)^T A_0^T P A_0 e(t) + \tfrac{l^2}{\xi_1} e(t)^T P e(t),$$

and

$$e(t)^T P A_r [g_r(t-\tau_r) - g_r(t-\tau_r)] \le \xi_2 e(t)^T A_r^T P A_r e(t) + \tfrac{l^2}{\xi_2} e(t-\tau_r)^T P e(t-\tau_r),$$

where $\xi_1, \xi_2 > 0$ are positive constants.

Then, it follows from (23) that

$$\begin{aligned}\dot{V}_1(t) \leq& -\xi e(t)^T e(t) + e(t)^T \Delta e(t) + \xi_1 e(t)^T A_0^T P A_0 e(t)\\&+\tfrac{l^2}{\xi_1} e(t)^T P e(t) + \sum_{r=1}^{m-1} \xi_2 e(t)^T A_r^T P A_r e(t)\\&+\sum_{r=1}^{m-1} \tfrac{l^2}{\xi_2} e(t-\tau_r)^T P e(t-\tau_r) + e(t)^T P u(t).\end{aligned} \tag{24}$$

When $t \in [(k+h_1)T, (k+h_2)T)$, $u_i(t) = -\lambda_i e_i(t)$ $-\gamma \frac{\lambda_{\max}(P)^\eta}{\lambda_{\min}(P)}$ $\times sign(e_i(t))|e_i(t)|^{2\eta-1} - (\gamma \sum_{r=1}^{m-1} \int_{t-\tau_r}^{t} e_i(s)^T P_i e_i(s) ds)^\eta \frac{e_i}{\|e_i\|^2}$ $\times \frac{1}{\lambda_{\min}(P)}$. By Lemma 2, we have

$$\begin{aligned}e(t)^T P u(t) =& -\sum_{i=1}^{N} [e_i(t)^T P_i \lambda_i e_i(t) + \gamma \tfrac{\lambda_{\max}(P)^\eta}{\lambda_{\min}(P)} e_i(t)^T P_i sign(e_i(t))|e_i(t)|^{2\eta-1} + e_i(t)^T P_i\\&\times(\gamma \sum_{r=1}^{m-1} \int_{t-\tau_r}^{t} e_i(s)^T P_i e_i(s) ds)^\eta \tfrac{e_i}{\|e_i\|^2} \tfrac{1}{\lambda_{\min}(P)}]\\\leq& -e(t)^T P \lambda e(t) - \gamma \lambda_{\max}(P)^\eta \sum_{i=1}^{N} e_i(t)^T sign(e_i(t))|e_i(t)|^{2\eta-1} - \sum_{i=1}^{N} (\gamma \sum_{r=1}^{m-1} \int_{t-\tau_r}^{t} e_i(s)^T P_i e_i(s) ds)^\eta\\\leq& -e(t)^T P \lambda e(t) - \gamma \lambda_{\max}(P)^\eta |e(t)^T e(t)|^\eta - (\gamma \sum_{r=1}^{m-1} \int_{t-\tau_r}^{t} e(s)^T P e(s) ds)^\eta,\end{aligned} \tag{25}$$

where $\lambda = [\lambda_1, \lambda_2, \cdots, \lambda_n]^T$. Then, we can obtain

$$\begin{aligned}\dot{V}_1(t) \leq& -\xi e(t)^T e(t) + e(t)^T \Delta e(t) + \xi_1 e(t)^T A_0^T P A_0 e(t) + \tfrac{l^2}{\xi_1} e(t)^T P e(t) + \sum_{r=1}^{m-1} \xi_2 e(t)^T A_r^T P A_r e(t)\\&+\sum_{r=1}^{m-1} \tfrac{l^2}{\xi_2} e(t-\tau_r)^T P e(t-\tau_r) - e(t)^T P \lambda e(t) - \gamma \lambda_{\max}(P)^\eta |e(t)^T e(t)|^\eta - (\gamma \sum_{r=1}^{m-1} \int_{t-\tau_r}^{t} e(s)^T P e(s) ds)^\eta\\\leq& e(t)^T P[-\tfrac{\xi I^{nN}}{\lambda_{\max}(P)} + \Delta + \xi_1 A_0^T A_0 + \tfrac{l^2 I^{nN}}{\xi_1} + \xi_2 \sum_{r=1}^{m-1} A_r^T A_r - \lambda I^{nN}] e(t) + \sum_{r=1}^{m-1} \tfrac{l^2}{\xi_2} e(t-\tau_r)^T P e(t-\tau_r)\\&-\gamma \lambda_{\max}(P)^\eta |e(t)^T e(t)|^\eta - (\gamma \sum_{r=1}^{m-1} \int_{t-\tau_r}^{t} e(s)^T P e(s) ds)^\eta.\end{aligned} \tag{26}$$

The derivative of $V_2(t)$ satisfies

$$\dot{V}_2(t) = \sum_{r=1}^{m-1} [e(t)^T P e(t) - e(t-\tau_r)^T P e(t-\tau_r)]. \tag{27}$$

From (26) and (27), we have

$$\begin{aligned}\dot{V}(t) \leq& e(t)^T P[-\tfrac{\xi I^{nN}}{\lambda_{\max}(P)} + \Delta + \xi_1 A_0^T A_0 + \tfrac{l^2 I^{nN}}{\xi_1} + \xi_2 \sum_{r=1}^{m-1} A_r^T A_r - \lambda I^{nN} + (m-1) I^{nN}] e(t)\\&+(\tfrac{l^2}{\xi_2} - 1) \sum_{r=1}^{m-1} e(t-\tau_r)^T P e(t-\tau_r) - \gamma \lambda_{\max}(P)^\eta |e(t)^T e(t)|^\eta - (\gamma \sum_{r=0}^{m-1} \int_{t-\tau_r}^{t} e(s)^T P e(s) ds)^\eta\\\leq& -\gamma V^\eta(t).\end{aligned} \tag{28}$$

When $t \in [kT, (k+h_1)T)$ or $t \in [(k+h_2)T, (k+1)T)$, $u_i(t) = 0$,

$$\begin{aligned}\dot{V}(t) \leq& e(t)^T P[-\tfrac{\xi I^{nN}}{\lambda_{\max}(P)} + \Delta + \xi_1 A_0^T A_0 + \tfrac{l^2 I^{nN}}{\xi_1} + \xi_2 \sum_{r=1}^{m-1} A_r^T A_r + (m-1) I^{nN}] e(t)\\&+(\tfrac{l^2}{\xi_2} - 1) \sum_{r=1}^{m-1} P e(t-\tau_r)^T e(t-\tau_r) \leq 0.\end{aligned} \tag{29}$$

Therefore, from (28) and (29), we have

$$\dot{V}(t) \leq \begin{cases} 0, & t \in [kT, (k+h_1)T), \\ -\gamma V^\eta(t), & t \in [(k+h_1)T, (k+h_2)T), \\ 0, & t \in [(k+h_2)T, (k+1)T). \end{cases} \tag{30}$$

From the proof of Theorem 1, we can obtain that when $t \to T'$

$$\lambda_{\min}^{1-\eta}(P)\|e(t)\|^{2(1-\eta)} \le V^{1-\eta}(e(t)) \le V^{1-\eta}(e_0) - \alpha(1-\eta)(h_2-h_1)(t-t_0-h_1T) \to 0,$$

and

$$\lambda_{\min}^{1-\eta}(P)\|e(t)\|^{2(1-\eta)} \le V(e(t)) = 0, \quad \forall t > T'.$$

Note that P is a positive definite matrix and $\eta \in (\frac{1}{2}, 1)$, thus from Definition 3, the finite-time synchronization of systems (14) and (18) is achieved under the periodically intermittent controller (20). The proof is completed.

It should be noticed that the intermittent controller (20) is periodic. However, sometimes the controller may be implemented aperiodically in different control periods. Thus, the following aperiodically intermittent controller can be constructed:

$$u_i(t) = \begin{cases} 0, & \|e_i(t)\| = 0 \ \texttt{or} \ t \in [kT, (k+h_k^1)T), \\ -\lambda_i e_i(t) - \gamma \frac{\lambda_{\max}(P)^\eta}{\lambda_{\min}(P)} sign(e_i(t))|e_i(t)|^{2\eta-1} - (\gamma \sum\limits_{r=1}^{m-1} \int_{t-\tau_r}^{t} e_i(s)^T P_i e_i(s) ds)^\eta \frac{e_i}{\|e_i\|^2} \frac{1}{\lambda_{\min}(P)}, & t \in [(k+h_k^1)T, (k+h_k^2)T), \\ 0, & t \in [(k+h_k^2)T, (k+1)T), \end{cases} \tag{31}$$

where $\frac{1}{2} < \eta < 1$, $0 \le h_k^1 < 1-\theta$, $h_k^2 = h_k^1 + \theta$ and $\theta > 0$ denotes the control rate, $\lambda_i > 0$ denotes the control gain, $\gamma > 0$ is a tunable parameter.

Then, we can obtain the following results.

Theorem 3: Suppose that Assumption 1 and Assumption 2 hold. If there exist two positive definite diagonal matrices P and Δ and three positive constants ξ, ξ_1, ξ_2 such that

$$\begin{aligned} &-\frac{\xi I^{nN}}{\lambda_{\max}(P)} + \Delta + \xi_1 A_0^T A_0 + \frac{l^2 I^{nN}}{\xi_1} + \xi_2 \sum_{r=1}^{m-1} A_r^T A_r + (m-1) I^{nN} \le 0, \\ &l^2 - \xi_2 \le 0, \end{aligned} \tag{32}$$

where $I^{nN} \in \mathbb{R}^{nN}$ denotes identity matrix, then the error system (19) is synchronized under the aperiodically intermittent controller (31) in a finite time:

$$T_3 = \frac{V^{1-\eta}(t_0)}{\alpha\theta(1-\eta)} + t_0 + \theta T,$$

where $V(t_0) = \frac{1}{2} e(t_0)^T P e(t_0) + \sum\limits_{r=1}^{m-1} \int_{t_0-\tau_r}^{t_0} e(s)^T P e(s) ds$.

Proof: By using Corollary 1, the similar proof to Theorem 2 can be achieved. Thus, it is omitted here.

Remark 5: The aperiodically intermittent controller (31) could be seen as an extension of the periodically intermittent controller (20). When $h_k^1 = h_1$ and $h_k^2 = h_2$, the controller (31) is reduced to the controller (20). Though the aperiodically intermittent controller (31) is more complex than (20), it is more pragmatic and worktable in practical appications.

IV. Numerical simulation

In this section, we give a simple example to verify the effectiveness of our proposed methods.

Example 1: Consider the following multi-layer complex network with 10 identical nodes:

$$\dot{x}(t) = f(x(t)) + A_0 g_0(x(t)) + \sum_{r=1}^{2} A_r g_r(x(t-\tau_r)), \tag{33}$$

where

$$f_i(x_i(t)) = \begin{pmatrix} -a & a & 0 \\ b & -1 & 0 \\ 0 & 0 & -c \end{pmatrix} \begin{pmatrix} x_{i1}(t) \\ x_{i2}(t) \\ x_{i3}(t) \end{pmatrix} + \begin{pmatrix} 0 \\ -x_{i1}(t)x_{i3}(t) \\ x_{i1}(t)x_{i2}(t) \end{pmatrix},$$

$$\begin{aligned} g_0(x(t)) &= sin(x(t)), \\ g_1(x(t-\tau_1)) &= sin(x(t-\tau_1)), \\ g_2(x(t-\tau_2)) &= 0.3x(t)cos(x(t-\tau_2)), \end{aligned}$$

$$A_0 = \begin{pmatrix} -4 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & -4 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -5 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & -6 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & -7 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -3 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & -6 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & -5 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -3 \end{pmatrix},$$

$$A_1 = \begin{pmatrix} -7 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & -5 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & -5 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & -6 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & -5 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & -4 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & -4 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -3 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & -4 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -6 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & -4 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & -6 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -3 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -4 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & -7 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & -5 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & -5 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -4 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & -4 \end{pmatrix},$$

where the parameters are selected as $a = 10$, $b = 30$, $c = 8/3$, $A_r \in \mathbb{R}^{10\times 10}$ $(r = 0, 1, 2)$ are symmetrically coupling matrices and the time delays $\tau_1 = 0.05$s and $\tau_2 = 0.1$s. It is valuable to note that A_i $(i = 0, 1, 2)$ are randomly generated such that (15) holds. The chaotic attractor of the Lorenz system $\dot{x}(t) = f(x(t))$ is shown in Figure 1. Moreover, the initial values are given as: $x(0) = (3 + i, 5 + 2i, 7 + 2i)^T$, $y(0) = (-2 + 7i, -5 + 6i, -7 + 8i)^T$ $(i = 1, \cdots, 10)$. The other parameters of the controller are $\lambda_i = 1$, $\eta = 5/8$ and $\gamma = 8$. Take $P_i = diag\{0.5, 0.4, 0.2\}$ and $\Delta_i = diag\{50, 50, 50\}$ as [43], it is easy to verify that Assumption 1 holds with $\xi = 43.48$. Given $T = 0.5$s, $h_1 = 0.1$ and $h_2 = 0.7$, the trajectories of errors are illustrated with a periodically intermittent controller in Figures 2-4. When $h_1 = 0$ and $h_2 = 1$, that is, a full controller is designed, the trajectories of errors are illustrated with a continuous controller in Figures 5-7. From Theorem 2, the settling time of the intermittent controller is larger than that of a full controller. By making a comparison of these figures, it is reasonable to show that the convergent time under the intermittent controller is larger than that of the full controller. In addition, the trajectories of errors are illustrated with a mixed intermittent controller in Figures 8-10 with $h^1_{2k} = 0.1$, $h^2_{2k} = 0.7$ and $h^1_{2k+1} = 0.3$, $h^2_{2k+1} = 0.9$ $(k \geq 0)$, respectively. We can also find that the convergent time under a mixed intermittent controller is larger than that of a full controller. Figures 11-13 show that the trajectories of errors are not convergent with $h_1 = h_2$, that is, the controller is always 0. The results show that the system itself is not convergent and the proposed controller can work well.

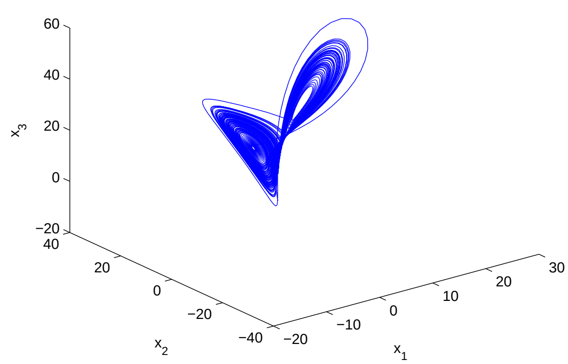

Fig. 1 Trajectories of the Lorenz system.

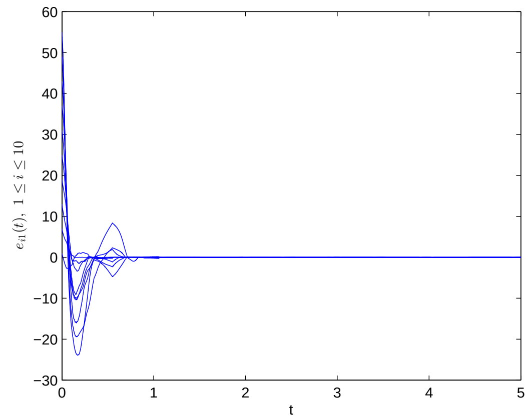

Fig. 2 Trajectories of the synchronization errors e_{i1} with control parameters $\gamma = 8$, $T = 0.5$s, $h_1 = 0.1$ and $h_2 = 0.7$.

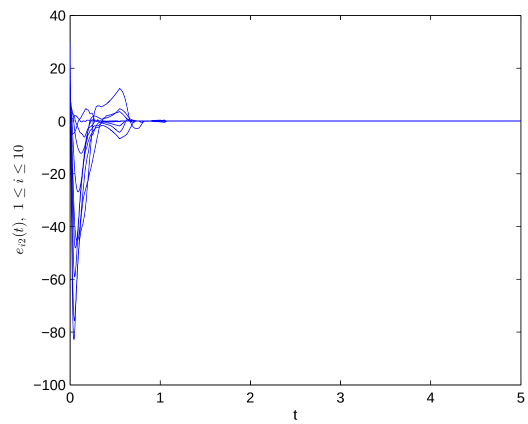

Fig. 3 Trajectories of the synchronization errors e_{i2} with control parameters $\gamma = 8$, $T = 0.5$s, $h_1 = 0.1$ and $h_2 = 0.7$.

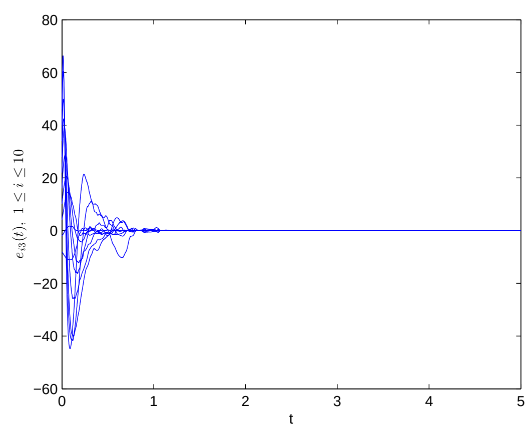

Fig. 4 Trajectories of the synchronization errors e_{i3} with control parameters $\gamma = 8$, $T = 0.5$s, $h_1 = 0.1$ and $h_2 = 0.7$.

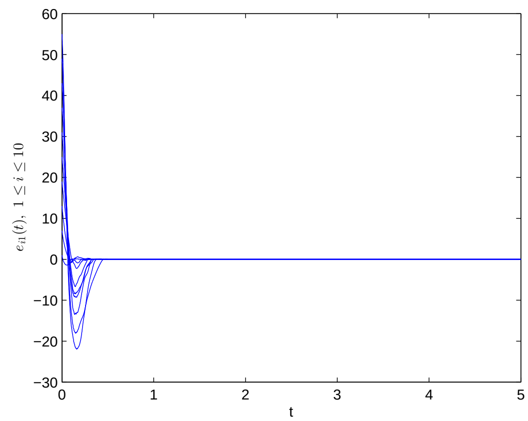

Fig. 5 Trajectories of the synchronization errors e_{i1} with control parameters $\gamma = 8$, $T = 0.5$s, $h_1 = 0$ and $h_2 = 1$.

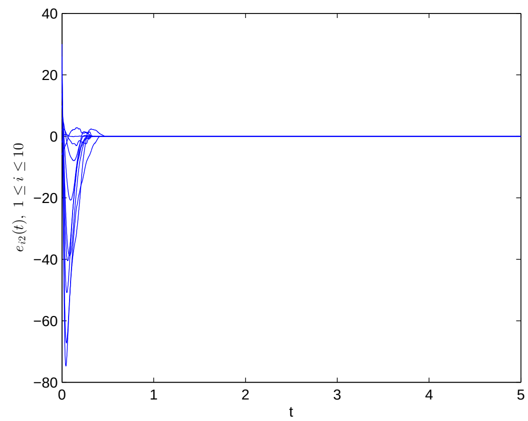

Fig. 6 Trajectories of the synchronization errors e_{i2} with control parameters $\gamma = 8$, $T = 0.5$s, $h_1 = 0$ and $h_2 = 1$.

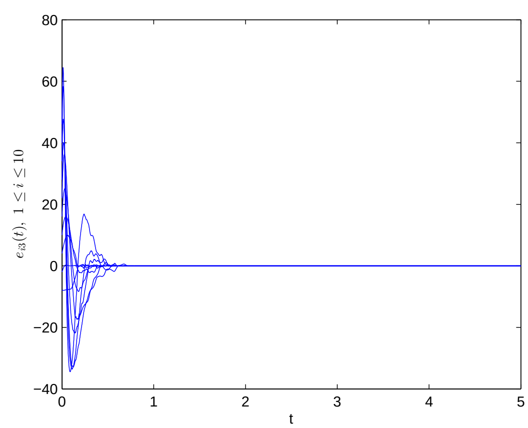

Fig. 7 Trajectories of the synchronization errors e_{i3} with control parameters $\gamma = 8$, $T = 0.5$s, $h_1 = 0$ and $h_2 = 1$.

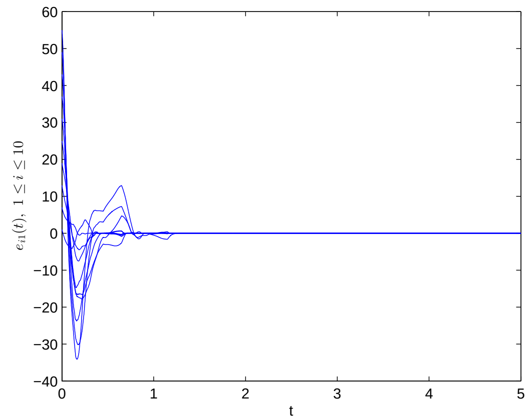

Fig. 8 Trajectories of the synchronization errors e_{i1} with control parameters $\gamma = 8$, $T = 0.5$s, $h^1_{2k} = 0.1$, $h^2_{2k} = 0.7$ and $h^1_{2k+1} = 0.3$, $h^2_{2k+1} = 0.9$ ($k \geq 0$), respectively.

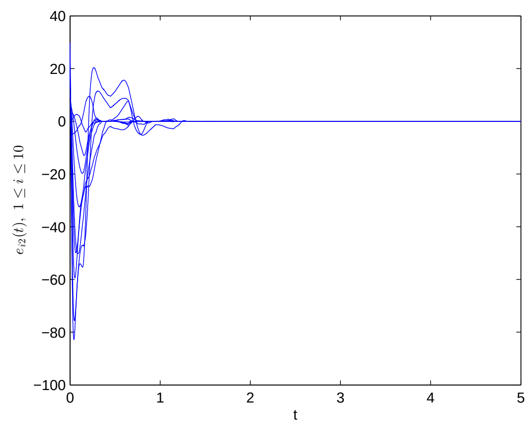

Fig. 9 Trajectories of the synchronization errors e_{i2} with control parameters $\gamma = 8$, $T = 0.5$s, $h^1_{2k} = 0.1$, $h^2_{2k} = 0.7$ and $h^1_{2k+1} = 0.3$, $h^2_{2k+1} = 0.9$ ($k \geq 0$), respectively.

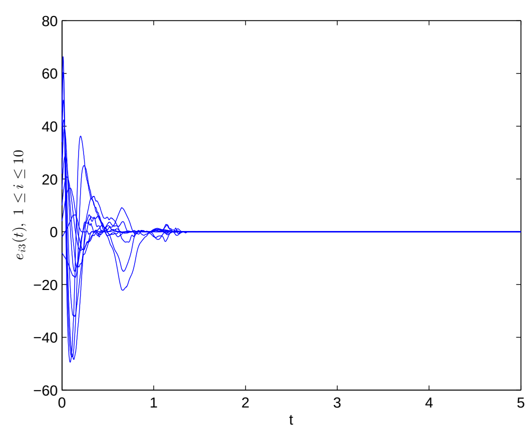

Fig. 10 Trajectories of the synchronization errors e_{i3} with control parameters $\gamma = 8$, $T = 0.5$s, $h^1_{2k} = 0.1$, $h^2_{2k} = 0.7$ and $h^1_{2k+1} = 0.3$, $h^2_{2k+1} = 0.9$ ($k \geq 0$), respectively.

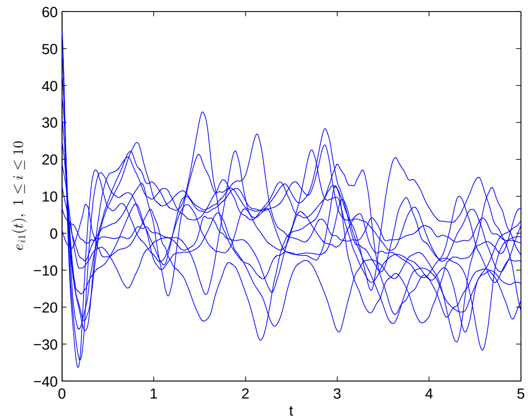

Fig. 11 Trajectories of the synchronization errors e_{i1} with control parameters $\gamma = 8$, $T = 0.5$s, $h_1 = h_2$.

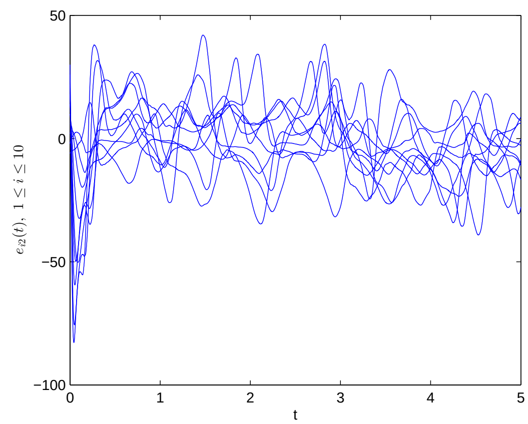

Fig. 12 Trajectories of the synchronization errors e_{i2} with control parameters $\gamma = 8$, $T = 0.5$s, $h_1 = h_2$.

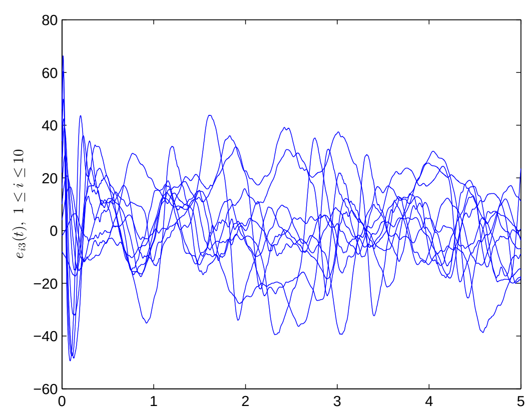

Fig. 13 Trajectories of the synchronization errors e_{i3} with control parameters $\gamma = 8$, $T = 0.5$s, $h_1 = h_2$.

V. Conclusion

This paper has addressed the problem of finite-time synchronization for a class of multi-layer nonlinear coupled complex networks via intermittent feedback control. Based on finite-time stability, some novel criteria were proposed to guarantee that the nonlinear system was still finite-time stable. Then, by proposing two kinds of different intermittent feedback controllers, sufficient conditions of finite-time synchronization of two complex networks were derived, respectively. At last, a numerical example was provided to verify the effectiveness of the proposed methods. It should be noticed that the same control rate is considered in this paper. In the future works, a mixed intermittent controller with different control rates may be studied. To the author's knowledge, the intermittent controller with different control rates can be applied more widely with a more natural property.

Acknowledgments

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