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Adaptive Entropy-based Learning with Dynamic Artificial Neural Network

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Abstract: Entropy models the added information associated to data uncertainty, proving that stochasticity is not purely random. This paper explores the potential improvement of machine learning methodologies through the incorporation of entropy analysis in the learning process. A multi-layer perceptron is applied to identify patterns in previous forecasting errors achieved by a machine learning methodology. The proposed learning approach is adaptive to the training data through a re-training process that includes only the most recent and relevant data, thus excluding misleading information from the training process. The learnt error patterns are then combined with the original forecasting results in order to improve forecasting accuracy, using the Rényi entropy to determine the amount in which the original forecasted value should be adapted considering the learnt error patterns. The proposed approach is combined with eleven different machine learning methodologies, and applied to the forecasting of electricity market prices using real data from the Iberian electricity market operator – OMIE. Results show that through the identification of patterns in the forecasting error, the proposed methodology is able to improve the learning algorithms' forecasting accuracy and reduce the variability of their forecasting errors.

Keywords: Artificial Neural Networks, Electricity Market Prices, Entropy, Forecasting, Information Theory, Machine Learning

1. Introduction

Machine learning (and time series forecasting, in particular) is an attractive domain in the power and energy systems field, as it is essential to enable an adequate energy resources management [1]. With the increase of renewable generation, with an intermittent nature, and the consequent need for the increase in consumers' flexibility; forecasting energy generation, consumption and energy market prices, becomes crucial [2]. Regression approaches and Artificial Intelligence (AI) based methods, such as Artificial Neural Networks (ANN) are the most common approaches.

Several relevant advances have been accomplished in forecasting energy market prices. A novel learning algorithm based on improvement of conventional extended Kalman filter is proposed in [3], with the purpose of surpassing the limitations of feedforward ANN when training data contains a certain number of outliers. In [4] authors propose a model that estimates the speed of energy price adjustment to its target level by extracting energy price bubbles and showing that bubbles help explaining price adjustment. The work presented in [5] includes a study on the impact of intra-day price relationships and market fundamentals for short and mid-term forecasting of electricity prices. Market price forecasting models for India are proposed in [6], using Model Confidence Set (MCS) approach to test the utility of these models and picking up the "best" models. Other domain-specific market price forecasting approach is proposed in [7], with focus on the identification of market price spikes in the Ontario electricity market. The work presented in [8], on the other hand, studies the influence of specificities of the market and weather conditions on the electricity price. [9] addresses the perspective of market integration in the scope of day-ahead market price forecasting.

47 A review that reports the current state on the subject of extreme learning machine (ELM) is
48 presented in [10]. ELM is used for classification, regression, clustering, feature selection,
49 representational learning and many other learning tasks. Hardware implementation and parallel
50 computation techniques have substantially sped up the training of ELM, making it feasible for big
51 data processing and real-time reasoning. ELM are also used in [11] to improve wavelet neural
52 networks in probabilistic forecasting of hourly electricity prices (please refer to [12] for a complete
53 review on probabilistic forecasting of electricity market prices). ELM are also being used as core
54 components of hybrid methods, such as [13], which combines ELM with Seasonal and Trend
55 decomposition using Loess (STL). In fact, the trend for methods' hybridization is gaining significant
56 strength in recent years; see e.g. [14], which proposes a hybrid multi-step ahead forecasting method
57 for the energy system; [15], which proposes a novel method combining interval Holt's exponential
58 smoothing method (HoltI) and multi-output support vector regression (MSVR) for interval-valued
59 time series; [16], in which a hybrid forecasting approach is used for multi-objective optimization; or
60 [17], which proposes a hybrid combination between wavelet-particle swarm optimization and
61 adaptive neuro-fuzzy inference systems for short-term electricity prices forecasting. This type of
62 hybridization generally shows good performance in dealing with multiple input variables. Also, with
63 the objective of facilitating the combination of multiple variables, in [18], authors propose a clustering
64 based methodology to identify similar days and time periods concerning solar intensity. Using this
65 identification, the training process of forecasting methodologies is adapted in order to achieve better
66 forecasting results.

67 In fact, identifying relevant data values and how these influence the variability of the data series,
68 leads to the concept of entropy [19]. Entropy usually refers to stochasticity or uncertainty, and aims
69 to measure the amount of information brought by each data point. The concept of information
70 entropy was introduced by C. Shannon in [19]. Entropy has been often explored as a means of
71 improving machine learning methodologies. The work proposed in [20] uses an entropy based
72 method to process initial data of photovoltaic power generation and unify these data according to
73 their expected information gain. The refined data is used to train an ELM for short-term forecasting.
74 In [21] fast ensemble empirical mode decomposition and sample entropy are used to pre-process
75 data. The prediction model is then built to forecast the sub-series, whose inputs and outputs are
76 obtained in accordance to phase space reconstruction. This work uses real wind speed data for short-
77 term wind speed forecasting. Also for wind speed forecasting, [22] proposes a novel hybrid model
78 that uses wavelet decomposition based on entropy to create a set of data subseries and estimate their
79 uncertainty. AdaBoost and wavelet filters are then used to forecast the wind speed values. Another
80 hybrid methodology, this time for load forecasting, combining entropy based feature selection, fuzzy
81 inductive reasoning, random forest and ANN is proposed in [23].

82 Entropy proves that stochasticity is not random; and the improvement of forecasting results
83 when incorporating entropy analysis shows that uncertainty is associated to a degree of added
84 information. Learning error is therefore also not random, and thereby it should be possible to identify
85 error patterns and extract further relevant information to improve the learning process. However, as
86 identified in this related work review, the existing forecasting methods focus on maximizing the
87 forecasting performance by minimizing the prediction error according to the input data. There is no
88 explicit work on analysing the tendencies of forecasting errors throughout the time, in order to
89 identify patterns in these errors and incorporate these patterns as a way to reduce the original
90 forecasting results. On the other hand, entropy is traditionally used in the feature extraction process,
91 but this concept is not extended or applied to analysing the divergence/convergence between the
92 forecasting errors and the actual forecasting values.

93 This work contributes to surpassing the identified gap by proposing an adaptive entropy-based
94 learning methodology based on the analysis of the forecasting error, instead of using entropy to
95 model and select training data, as performed by current entropy based forecasting models. A
96 dynamic feedforward ANN with backpropagation is modelled to analyse and learn patterns in the
97 historical set of forecasting errors achieved by a learning methodology. The proposed learning
98 approach is adaptive to the training data through a re-training process that includes only the most
99 recent and relevant data, thus excluding misleading information from the training process. Three

100 dynamic training strategies are proposed to model training data according to different perspectives
101 when analysing the historical data patterns. The learnt error patterns are then combined with the
102 original forecasting results in order to improve forecasting accuracy, using the Rényi entropy as
103 measure to determine the amount in which the original forecast should be adapted taking into
104 account the perceived error values. The proposed approach is combined with eleven different
105 machine learning methodologies, and applied to the forecasting of electricity market prices using real
106 data from the Iberian electricity market operator – OMIE [24]. Results show that through the
107 identification of patterns in the forecasting error, the proposed methodology is able to improve the
108 learning algorithms' forecasting accuracy.

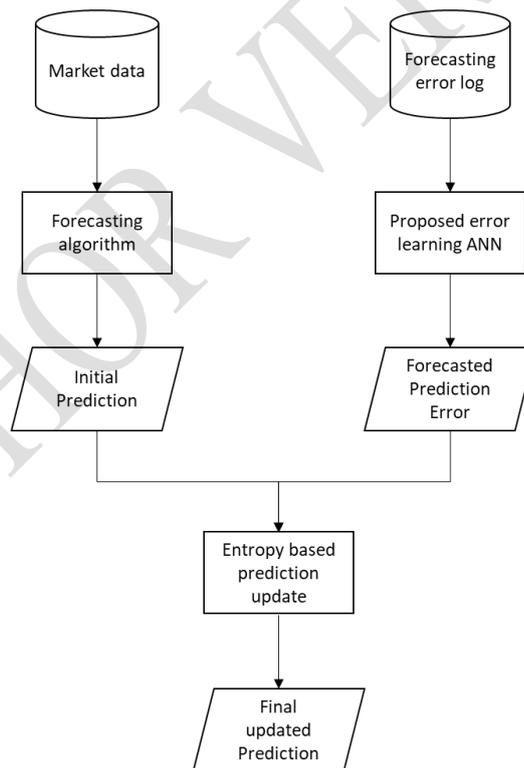
109 After this introductory section, section 2 describes the proposed dynamic learning approach,
110 including the proposed learning model using the considered entropy measures, and the dynamic
111 training process. Section 3 presents the case study based on real electricity markets data, and section
112 4 presents the most relevant conclusions from this work.

113 2. Proposed methodology

114 1. Overview

115 The proposed learning model aims at identifying patterns in the history log of forecasting errors
116 measured through the application of a given machine learning methodology. The objective is,
117 therefore, to enable reaching predictions on the expected error of future forecasts provided by the
118 supporting learning method, as depicted in Fig. 1.

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120

121

Figure 1. Flowchart of the proposed methodology

122 Fig. 1 shows that an initial prediction of the target output is provided by a supporting forecasting
123 algorithm (which may be any forecasting method). In order to reach this prediction, the forecasting
124 method analyses the historical data (in the case of the particular case study depicted in this paper,
125 the historical electricity market data).

126 On the other hand, a historical log of the forecasting errors achieved by the same forecasting
127 method is maintained, and updated every time the accuracy of a new forecast may be evaluated. The
128 ANN described in section 2.2 is then applied in order to find patterns in the historical forecasting

129 errors, and trained according to the dynamic training process explained in section 2.3. This ANN
130 provides as output the predicted/expected error associated to the most recent forecast that has been
131 achieved.

132 Considering the expected error of the initial prediction, the initial forecasted value is updated
133 according to the entropy based process explained in section 2.2, thus resulting in the final, updated,
134 prediction that is the final output of the proposed learning approach.

135 2. Learning model

136 The forecasting errors may be positive if the predicted value is higher than the actual real value;
137 or negative, if the predicted value is lower than the actual target value. The predicted/expected
138 forecasting error y_t provided by the proposed learning model is used to adapt the initial forecasted
139 value F_t , in order to make it converge towards the actual target value, at each time t . In order to
140 determine the amount in which y_t should influence F_t , the Rényi entropy $H_\alpha(X)$ is used [25]. Rényi
141 entropy is applied because it enables representing the amount of gained information by reflecting the
142 difference between the reality and the forecast results. Among the several entropy and divergence
143 methods available in the literature [26-31], Rényi entropy is the best solution for this problem, as it
144 generalizes several of the most widely used entropy measures, namely the Hartley entropy [32], the
145 Shannon entropy [19], the collision entropy and the min entropy. The Rényi entropy of order α is
146 applied as in (1).

$$H_\alpha(y_t) = \frac{1}{1-\alpha} \log \sum p_{y_t}^\alpha \quad (1)$$

147 where $\alpha \geq 0$ and $\alpha \neq 1$. p_{y_t} is the probability associated to the predicted/expected forecasting error y_t in
148 time t .

149 By using entropy as a measure for calculating the amount in which a new event should affect an
150 already accomplished forecasted value, the proposed methodology enables adapting the current
151 forecast and reaching a new forecasted value that is closer to the real value, as in (2).

$$F'_t = F_t - y_t H_\alpha(y_t) \quad (2)$$

152 The adapted forecasting value F'_t is therefore achieved by subtracting the expected error from
153 the given original forecast, in a measure given the calculation of the Rényi entropy.

154 The learning process and prediction of the expected error y_t at each time is performed using a
155 feedforward ANN, receiving as inputs the historic of forecasting errors achieved by the supporting
156 forecasting methodology. The ANN considers two nodes in the intermediate layer (as result of
157 extensive preliminary sensitivity analysis), and one output – the predicted forecasting error.

158 Backpropagation using the gradient descent method [33] has been used as training algorithm
159 for the ANN. This requires calculating the derivative of the squared error function with respect to
160 the weights of the network. The squared error function E for the single output neuron is defined as
161 in (3).

$$E = \frac{1}{2}(t - y)^2 \quad (3)$$

162 where t is the target output for a training sample, and y is the actual output of the output neuron.

163 For each neuron j , its output o_j is defined by feedforward calculation, as in (4).

$$o_j = f \left(\sum_{k=1}^n w_{kj} x_k \right) \quad (4)$$

164 where n is the number of input units to neuron j , and w_{kj} is the weight between neurons k and j . Hence,
165 the input for the activation function f of a neuron is the weighted sum of outputs o_k of the previous
166 neurons. The used activation function f is the logistic function, a log-sigmoid function, which can be

167 defined as in (5).

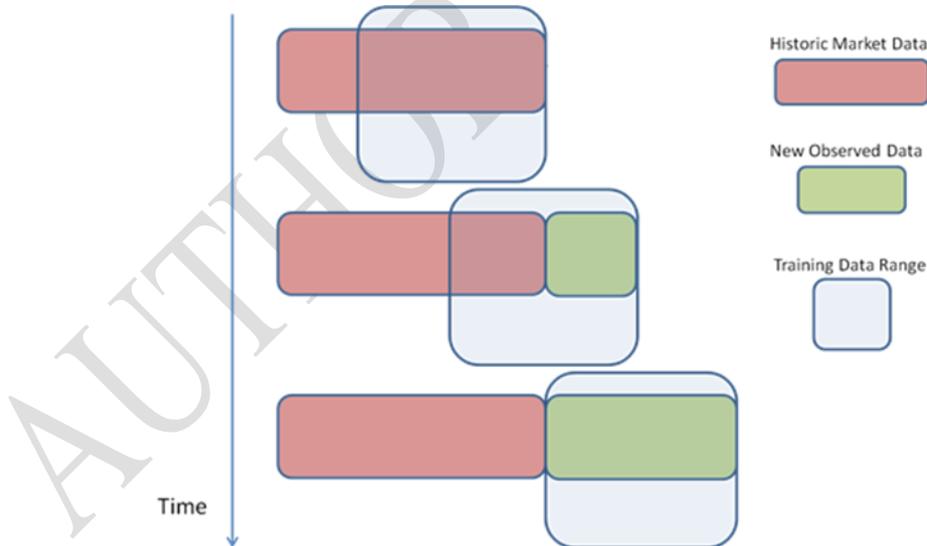
$$f(z) = \frac{1}{1 + e^{-z}} \quad (5)$$

168 The backpropagation algorithm is used as the training method of the designed artificial neural
169 network. The backpropagation algorithm includes the following steps [33]:

- 170 1. Initialize weights as small random numbers;
- 171 2. Introduce training data to the ANN and calculate the output by propagating the input
172 forward through the network using (4);
- 173 3. Calculate the error using (3);
- 174 4. Propagate the sensitivities backward through the network by simply taking the derivative
175 of the activation function (5) with respect to the network parameters;
- 176 5. Calculate w_{kj} updates;
- 177 6. Update the values of w_{kj} ;
- 178 7. Repeat steps 2 to 6 until all examples are classified correctly.

179 3. Dynamic training

180 In order to guarantee that the learning process is always updated throughout each simulation,
181 the methodology is re-trained every time that new data is observed. The data used for training the
182 ANN in the first iterations is the historic data from the electricity market. As the simulation results
183 start appearing, these are used a continuation of the previous historic, *i.e.*, they will be used to
184 complement the previous information, and will, as the time progresses, be the main subject of
185 analysis for the training of the ANN. Fig. 2 provides a visualization of this process.



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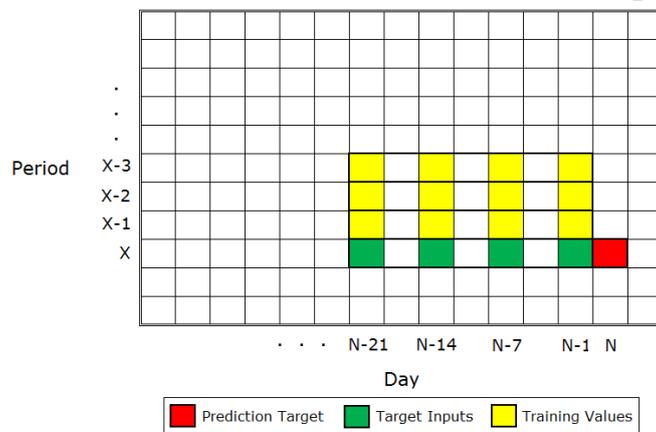
187 **Figure 2.** Data range for the training process of the proposed learning methodology

188 In addition to the re-training process that guarantees the constant update of the learning
189 procedure, the training data is also selected carefully. Since the objective of the proposed approach is
190 to find patterns in the forecasting error, the way historical error data should be approached is a crucial
191 aspect. This process must be ad-hoc and dependent on the original training data itself. In the case
192 addressed by this paper, the used training data refers to the historic electricity market prices. Since
193 electricity market prices depend directly on generation of consumption in each moment, they have a
194 strong seasonal variation (due to the variation in the use of heating and cooling devices). Also, there
195 is a variation depending on the week day due to consumption patterns (especially from business days

196 to weekends). Also, there are differences from hour to hour due to the variability of generation
 197 (especially solar, which is only positive during the day) and consumption patterns. Due to this
 198 entropy, the training data must be selected carefully in order to improve the performance of the
 199 methodology. Three different training strategies are used, according to the entropy identified above.

200 These three approaches use the ANN with one value in the output layer - the value of the
 201 expected error, two intermediate nodes, and an input layer of four units. The input layer considers
 202 different values in the different approaches. These values depend on how the history of the error is
 203 considered:

- 204 • Strategy A - This strategy makes a prediction along the 24 hourly periods per day, using
 205 for the training of each period the error of the same hour for:
 - 206 ○ the previous day;
 - 207 ○ the previous 7 days;
 - 208 ○ the previous 14 days;
 - 209 ○ the previous 21 days.



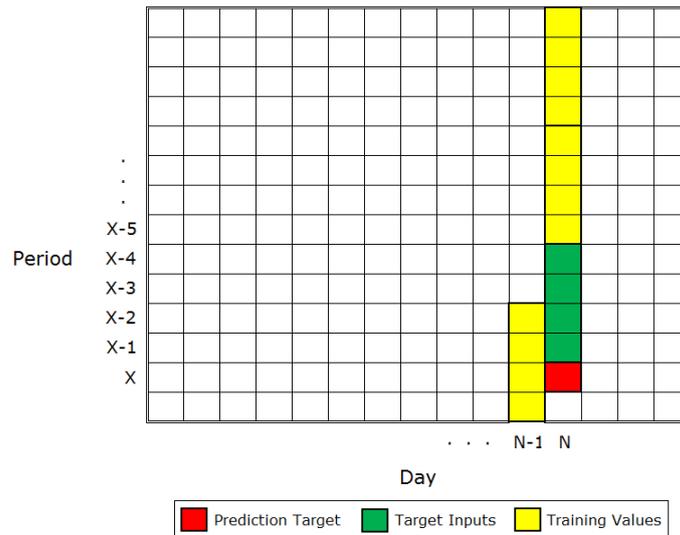
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211

Figure 3. Strategy A training data structure

212 Having Fig. 3 as support, if an error prediction is required for day N and period X , the input
 213 values of the ANN are $N-1$, $N-7$, $N-14$ and $N-21$, all for hour X . The data of the previous hours is used
 214 to train the ANN, considering the same days.

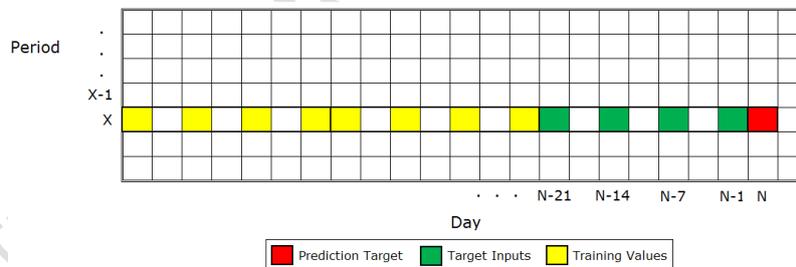
- 215 • Strategy B - This strategy makes a prediction along the days, using the error of the
 216 following hourly periods:
 - 217 ○ previous period;
 - 218 ○ 2 previous periods;
 - 219 ○ 3 previous periods;
 - 220 ○ 4 previous periods.



221
222 **Figure 4.** Strategy B training data structure

223 If an error prediction is required for day N and period X , the input values of the ANN are the
224 previous four hourly periods, considering the same day. The previous periods are considered to train
225 the ANN. When all the periods of the current day have contributed to the training, the periods of the
226 prior day start being considered, as shown in Fig. 4.

- 227
- 228 • Strategy C - this strategy makes a prediction considering always the errors of the same
229 period (the hourly period in question), using the error for:
 - 230 ○ the previous day;
 - 231 ○ the previous 7 days;
 - 232 ○ the previous 14 days;
 - the previous 21 days.



233
234 **Figure 5.** Strategy C training data structure

235 This approach always considers the same period. It ignores the data concerning all periods other
236 than the required one, and uses the previous days to train the ANN, as depicted by Fig. 5.

237 **3. Case study**

238 *1. Specifications*

239 This section presents the case study used to assess the performance of the proposed
240 methodology. The proposed approach is applied to the day-ahead forecasting of electricity market
241 prices, using real data from the Iberian market operator – OMIE. Real data from 3 years is used,
242 namely from 1, November 2014 to 31 October 2017. These data can be consulted and downloaded in
243 [34]. From these data, the first two years are used as training data, and the last year as test and
244 validation data. The used data has the characteristics presented in Table 1.
245

Table 1. *Experimental data characteristics overview*

Time interval	Max	Min	STD	Median	Mean
3 years (1, November 2014 to 31 October 2017)	12.32	0.00	1.31	6.84	6.32

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Eleven different forecasting methods are used as basis approaches to provide the primary electricity market price forecasts. The proposed methodology is then applied to the analysis and learning of the forecasting errors originated by each of these approaches in order to reach the updated prediction values. The results from the application of the proposed methodology to the different methods is compared to the individual performance of the methods themselves. These methods have been chosen as they are the most widely applied methods to the electricity market forecasting problem in the literature. The considered basis forecasting methods are:

- AutoRegressive Integrated Moving Average (ARIMA) model;
- Feedforward ANN trained with backpropagation, as presented in [35];
- Support Vector Machine (SVM) for regression, as presented in [36];
- Wang and Mendel's (WM) Fuzzy Rule-Based System (FRBS) model [37];
- Hybrid neural Fuzzy Inference System (HyFIS) [38];
- Genetic fuzzy systems for fuzzy rule learning based on the MOGUL methodology (GFS.FR.MOGUL) [39];
- Genetic lateral tuning and rule selection of linguistic fuzzy systems (GFS.LT.RS) [40];
- The simplified TSK fuzzy rule generation method using heuristics and gradient descent method (FS.HGD) [41];
- Extreme Learning Machine (ELM) [42];
- Echo State Network (ESN) [43];
- Convolutional neural network [44].

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In order to enable a fair comparison between these models, similar input and output structures have been used. The output of all methods is the predicted market price. The inputs are: (i) for ARIMA and SVM, the market price in the same hour of the previous four days, and the market price of the same hour, referring to one week, two weeks, three weeks and four weeks before the target day, (ii) for all the other methods, besides the inputs used for the ARIMA and SVM, inputs also include the volume of traded power in the same hour. This difference in training is due to the bigger difficulty of ARIMA and SVM in achieving quality regression results when associating different variables. The inclusion of extra data in this case only worsens the forecasting results, and it is, therefore, unnecessary, and thereby excluded.

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2. Results

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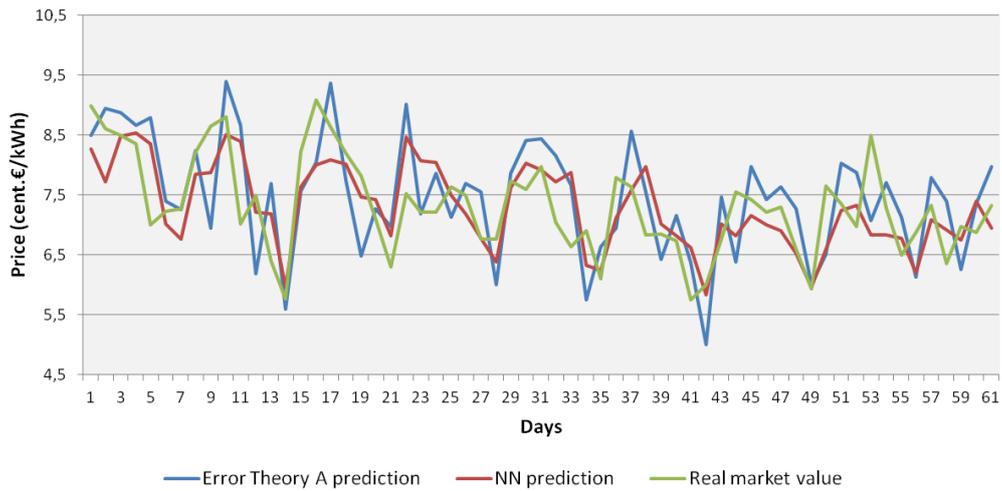
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Fig. 6, 7 and 8 show a graphical comparison of the real market price, the price forecasted by the ANN and the adapted prediction price resulting from the proposed methodology, throughout the last 2 months of test data, using the three strategies considered for dynamic training, as explained in section 3.3.

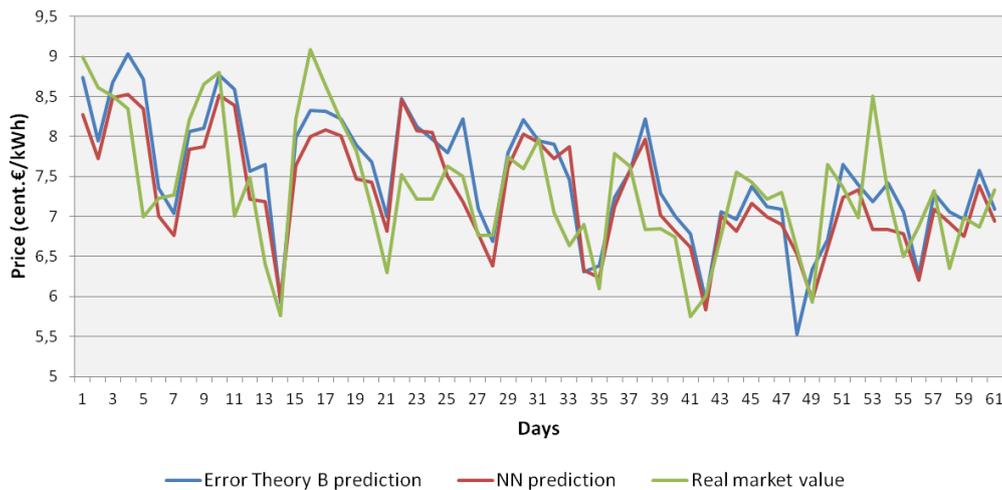
Fig. 6 shows the results when using training strategy A, comparing its prediction with the value received by the auxiliary strategy (ANN), to which the expected error is applied. In this comparison there is presented the actual market price, so one can check the quality of the prediction strategy.



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Figure 6. Results using training strategy A

287 As can be seen from Fig. 6, the calculation and application of the expected error, using strategy
 288 A, is very close to the actual market value. From Fig. 6 it is visible that the proposed methodology is
 289 able to improve the ANN forecasted values, as the green line is, in most cases, closer to the real market
 290 price values than the ANN forecast. The proposed approach is able to identify patterns on the ANN
 291 forecasting errors, learn from these errors history log, and use this learning process to adapt the
 292 prediction in a way that the predicted values become closer to the real market price values. In Fig. 7,
 293 the same information is presented, for the case in which strategy B is applied.



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Figure 7. Results using training strategy B

296 From Fig. 7 it is visible that strategy B has had worst results when compared with strategy A,
 297 for the considered time period. Finally, Fig. 8 shows the results from the application of strategy C.

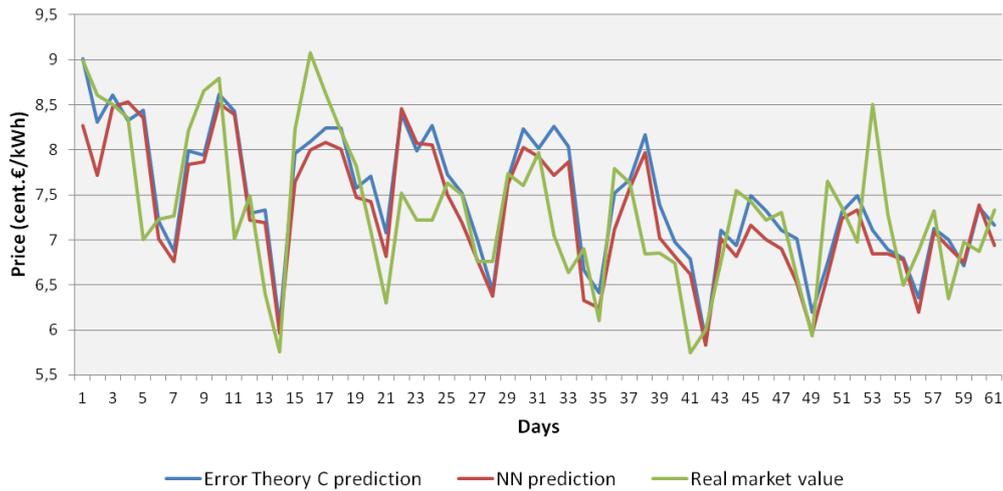


Figure 8. Results using training strategy C

Analysing the results when using strategy C, it can be seen that strategy C achieves better results than strategy B for the considered time period, regarding the level of proximity of the forecast with the actual values of the market, but still worse than strategy A.

From these results it can be concluded that the best strategy to be applied is training strategy A, which is the one applied in the experimental tests shown henceforward.

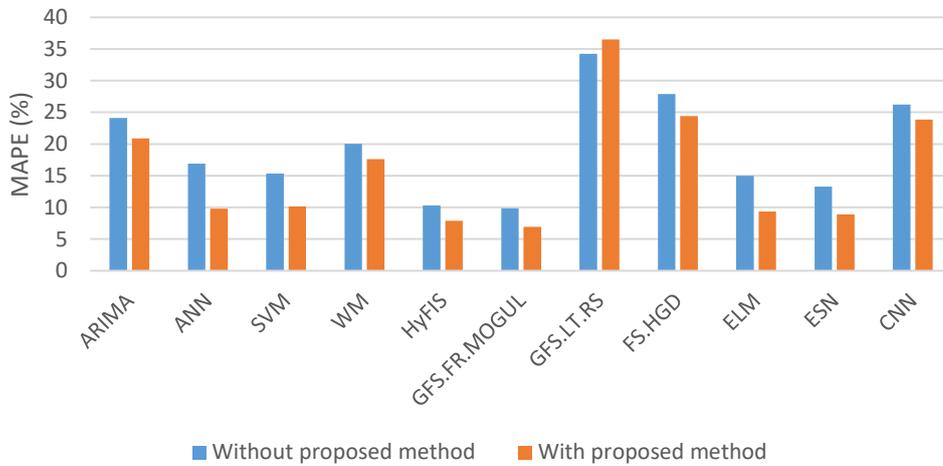
Table 2 presents the forecasting error achieved by the eleven forecasting methods, and the error achieved by the proposed methodology (using training strategy A) applied to each of these approaches. These results refer to the average results over the entire year of test data. The error is measured through the Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE). The Standard Deviation (STD) of the forecast errors is also presented, as well as the value of Pearson Correlation (PC) [-1, 1] between the real data and the forecasted data [45].

Table 2. Results of the forecasting methods and of the proposed methodology

Method	Without the proposed methodology				With the proposed methodology			
	RMSE	MAPE (%)	STD	PC	RMSE	MAPE (%)	STD	PC
ARIMA	16.4	24.12	5.3	0.74	13.3	20.87	4.9	0.79
ANN	10.09	16.90	4.6	0.81	7.87	9.83	3.2	0.88
SVM	9.95	15.36	5.1	0.82	8.06	10.16	4.2	0.89
WM	12.31	20.06	6.3	0.78	11.45	17.63	4.8	0.81
HyFIS	7.25	10.32	3.6	0.88	5.36	7.87	2.9	0.93
GFS.FR.MOGUL	6.83	9.86	3.1	0.89	4.36	6.93	2.6	0.94
GFS.LT.RS	22.31	34.23	13.3	0.63	24.36	36.49	11.1	0.61
FS.HGD	18.14	27.88	8.4	0.68	14.80	24.42	7.2	0.70
ELM	9.45	14.98	4.9	0.83	7.21	9.37	3.1	0.90
ESN	8.89	13.26	4.2	0.86	6.92	8.89	3.0	0.91
CNN	17.23	26.21	8.6	0.69	13.12	23.87	8.2	0.71

From Table 2 it is visible that the proposed methodology has been able to reduce the forecasting error of nearly all the applied forecasting methodologies. The only exception is the GFS.LT.RS method, whose performance is bad from the start. There is no tendency found in the results, and thus the adaptation of the original value is not done properly. There is no found pattern in the errors achieved by this method. Nevertheless, the proposed approach is still able to decrease the errors STD, smoothing the variation of the errors outliers. In all the other methods the results are improved, regarding both the reduction of the forecasting error and of the STD. The method that achieves the best results is the GFS.FR.MOGUL, followed by the HyFIS, with both approaches being able to reach

321 a MAPE error below 8% when using the proposed methodology, which is a relevant result. Fig. 9
 322 shows the MAPE errors of the different methodologies, to allow a visual comparison of their
 323 performance.
 324



325
 326 **Figure 9.** MAPE forecasting errors of the different methods and results when applying the proposed
 327 methodology

328 From Fig. 9 it is visible that the proposed method enables a reduction of the forecasting error of
 329 all methods but GFS.LT.RS, by capturing patterns in their forecasting errors, and adjusting the
 330 predicted value accordingly, in order to reduce the final prediction error. Table 3 shows the results
 331 of the forecasting error prediction, when analysing the error achieved by each of the forecasting
 332 methodologies.

333 **Table 3.** Results of the predictions' forecasting error prediction

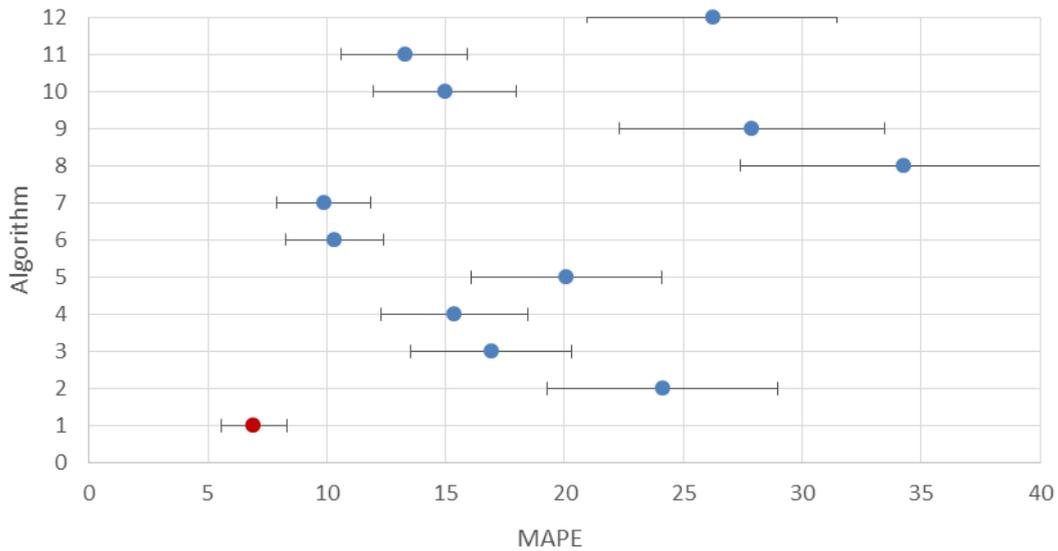
Method	RMSE	MAPE (%)	STD
ARIMA	6.23	23.16	3.2
ANN	2.45	8.34	1.3
SVM	3.16	9.12	1.8
WM	6.32	24.17	2.3
HyFIS	4.26	10.34	2.2
GFS.FR.MOGUL	3.99	11.06	2.3
GFS.LT.RS	10.3	48.12	5.2
FS.HGD	6.06	22.34	3.6
ELM	2.95	9.01	1.7
ESN	2.89	8.86	1.6
CNN	5.98	21.06	3.8

334
 335 Table 3 shows that the proposed model is able to predict the forecasting error of the different
 336 methodologies with an acceptable accuracy. By matching Table 3 with Table 2 it can be seen that the
 337 methods that have the better improvement when using the proposed methodology are those for
 338 which the proposed method is able to achieve a smaller error in predicting their forecasting errors.
 339 This is especially relevant in the case of the ANN (and also clearly visible for the SVM and
 340 GFS.FR.MOGUL), which is the method with the smaller forecasting error prediction, and the method
 341 with the biggest improvement when using the proposed approach. The contrary is visible in the case
 342 of the GFS.LT.RS method; the prediction of its forecasting error is not successful (as seen by the large
 343 values in Table 3), and this leads to no improvement when applying the proposed methodology
 344 (Table 2).

345 In order to validate the significance of the proposed method against the eleven considered
 346 benchmark methods, a statistical analysis is performed [46]. The Kruskal-Wallis test is the
 347 nonparametric test used to compare three or more independent samples. It indicates if there is a
 348 difference between at least two of them. This is used to test the null hypothesis that all populations
 349 have equal distribution functions against the alternative hypothesis that at least two of the
 350 populations have different distribution functions. In this way it is assumed that equality of averages
 351 when equality of equal distributions exists [47].

352 By the test Kruskal-Wallis it is possible to obtain the value of $p = 0$ that gives us indication of
 353 rejection of the null hypothesis that all data samples come from the same distribution at a 1%
 354 significance level. Given the result of the test that gives the indication of the null hypothesis, the
 355 comparison between the pairs of groups is made in order to verify which of the samples differ from
 356 each other.

357 The Bonferroni procedure is performed in order to make the comparison in pairs. Fig. 10
 358 represents the 95% confidence interval for all sample groups (12 methods, in which group 1 is the
 359 proposed method using the best supporting forecasting approach: GFS.FR.MOGUL; and the other 11
 360 groups are the 11 benchmark methods considered in this study). In this way, it is possible to see
 361 which groups differ in the value of the average, using the Bonferroni procedure.



362
 363 **Figure 10.** Bonferroni confidence interval by 95%

364 By analyzing the graph of Fig. 10, it is possible to observe that all methods have significantly
 365 different mean values. Table 4 shows the results of this analysis.

366

Table 4. Bonferroni procedure

Group pairs		p-value
1	1	1
1	2	1
1	3	1
1	4	1
1	5	1
1	6	1
1	7	1
1	8	1
1	9	1
1	10	1
1	11	1
1	12	1

367 Since the p-value is equal to 1 in all these group tests, the null hypothesis where the groups are
368 considered to have similar means with an error of 5% is accepted.

369 Taking into account this analysis, it is concluded that the applied benchmark methods achieve
370 significantly different results, thus supporting the relevance of the proposed approach.

371 4. Conclusions

372 The study of how the uncertainty associated to different types of data can be modelled and how
373 it influences the prediction of future events, is a relevant domain for several decades. Entropy has
374 arisen as a recognized means to deal with this uncertainty and to extract the information that is given
375 by the different data.

376 This paper incorporates learning of past data uncertainty into the forecasting process. This is
377 accomplished by applying a multi-layer perceptron to analyse the historic of forecasting errors
378 originated by a forecasting methodology, and reach predictions on the expected forecasting error.
379 The expected error is then used to adapt the original forecasted value in order to reach a more
380 accurate prediction, using the Rényi entropy as measure of the amount in which the adaption of the
381 original forecasted value should be applied.

382 The proposed methodology has been applied and compared against eleven different state of the
383 art forecasting methodologies, including ANN, SVM, ARIMA, ELM, ESN, and hybrid fuzzy based
384 inference systems. The forecasting process has been experimented on a log of real electricity market
385 data from the last three years. Results show that the proposed methodology is able to successfully
386 find patterns in the history of forecasting errors originated by the different learning methodologies,
387 and use these error predictions to improve the quality of the forecasts. The forecasting error has been
388 decreased, as well as the standard deviation of the forecasting errors.

389 As a future work, different approaches will be experimented to learn the patters from past error
390 data, namely probabilistic approaches and hybrid forecasting methods. Also, the combination
391 between the expected error and the original values will be enhanced by exploring different means of
392 reaching intelligent and adaptive combinations of these values.

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