# Journal Pre-proofs

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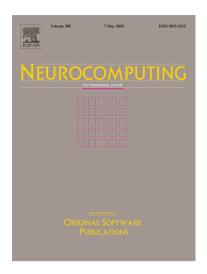
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Observer-Based Incremental Backstepping Sliding-Mode Fault-Tolerant

Control for Blended-wing-body Aircrafts

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**ABSTRACT** 

This paper presents an adaptive incremental nonlinear backstepping sliding-mode (INBSM) controller,

for fault tolerant tracking control of a blended wing body (BWB) aircraft with unknown disturbances

and actuator faults. The INBSM controller is based on a nonlinear dynamics model of the BWB aircraft.

In addition, a radial basis function neural network disturbance observer (RBF-NNDO) is proposed to

enhance the disturbance attenuation ability. A fault estimator is suggested to improve actuator fault

tolerant control level. The closed-loop control system of the BWB aircraft is proved to be globally

asymptotically stable using Lyapunov theory. Simulations of the combined NNDO-INBSM controller

are presented and compared with both the INBSM design and an adaptive fuzzy controller. The results

demonstrate an improved capability of the NNDO-INBSM control for the BWB aircraft to execute

realistic attitude tracking missions, even in the presence of center of gravity movement, unknown

disturbances, model uncertainties and actuator faults.

Keywords: Robust control of nonlinear system; neural network disturbance observer; incremental

nonlinear backstepping control; incremental nonlinear dynamic inversion; stability augmentation

control

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# 1. Introduction

The blended wing body aircraft is a creative airframe design which seamlessly combines a flying wing with a futuristic fuselage. It has potential advantages in lower carbon emissions, reduction of noise, higher speed, longer flight range, and broader internal volume [1, 2]. However, due to the flying wing configuration, the longitudinal stability of the BWB aircraft decreases as the angle of attack increases, even at small angles of attack. This is because the lift curve shows a nonlinear increase at small angles of attack because of flow separation [3,4], and the nonlinear change of pitching moment can easily make the BWB aircraft unstable due to the aerodynamic center moving forward. Furthermore, because the pitch arm of the BWB aircraft is shorter than that of classical configurations, the pitch trim ability of the BWB aircraft is insufficient when climbing at low speed and high angle of attack. These make the BWB aircraft design and flight control very challenging [5].

Qin and Vavalle initially studied the lift distribution of the BWB aircraft [6]. Castro studied the control stability augmentation and flying qualities of a BWB aircraft designed by Cranfield University [7]. Rahman & Whidborne proposed an integration strategy of propulsion and flight control for the BWB aircraft [8]. Yann and Joël studied multi-control surface optimization for the BWB aircraft under handling quality constraints [9]. However, most of these modelled the BWB aircraft as a linear time-invariant system and then used conventional linear control methods to design controllers.

To improve the control performance of an aircraft with nonlinear models, many nonlinear control methods have been studied, backstepping and feedback linearization are two common methods [10]. Liu and Sang proposed both backstepping and sliding mode backstepping control to realize trajectory tracking control of stratospheric airships [11-15]. Bacon and Ostroff expanded dynamic inversion into an incremental form by introducing an angular acceleration feedback [16]. Incremental nonlinear

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dynamic inversion (INDI), a method in which the dynamics are written in an incremental form, is a sensor-based control method that does not rely on accurate aircraft model, and it can realize aircraft's flight under certain uncertainties and structural faults [17]. Smeur, Chu et al. used an adaptive incremental nonlinear dynamic inversion to control the attitude of a miniature unmanned aerial vehicle (UAV) [18]. Recently Lu and Kampen proposed a fault tolerant trajectory tracking control based on INDI to deal with actuator faults [19], and Wang and Kampen proposed a combination of sliding mode control and incremental aerodynamic dynamic inversion, so that the gain of the sliding mode controller could be reduced while making up for the remaining perturbations [20]. Incremental backstepping control is another form of incremental nonlinear control which brings the implicitness of sensor-based architectures with Lyapunov-based control design [21]. Ali, Chu et al. proposed an adaptive INBS using immersion and invariance to control an F-16 aircraft [22]. Lu and Kampen studied the robustness problem of attitude control by using INBS [23]. Wang and Chen used a command-filtered incremental backstepping controller to realize a small UAV attitude control and improved the plant robustness [24]. Since aircraft are subjected to unknown disturbances such as wind and turbulence, a nonlinear disturbance observer-based control (DOBC) approach is introduced here to enhance the disturbance attenuation capability of the INBSM control. The disturbance observer-based technique has been applied to the nonlinear systems with unknown disturbances for three decades [25], where an observer is designed to estimate external disturbances and then compensate for them [26]. Recently a fuzzy disturbance observer and a neural network disturbance observer have been developed and applied to position systems and UAVs [27-28]. Meanwhile, neural networks, with strong approximation capacities, can be employed for robust control by combining with backstepping, thus model uncertainty and disturbances are all handled [29].

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Motivated by the weak static stability of blended wing body aircraft that make it sensitive to unknown wind and faults, and by the work of Castro [7], Acquatella and Chu [21], Chen [25] and Zheng [29], this paper mainly concerns the stability augmentation system and attitude tracking control for a BWB aircraft with unknown disturbances, model uncertainties and actuator faults. The main contributions are listed as follows. The stability augmentation system to improve dynamic stability characteristics for the BWB aircraft with nonlinear motions is studied. The influence of center of gravity (CG) position on the BWB aircraft stability is analyzed. The stability enhancement or large stability margins by using INBSM control and adaptive fuzzy control will be proved and numerically verified when the system has uncertainties or time delays. A novel RBF NNDO enhanced INBSM is proposed for the attitude tracking. An NNDO based approach is applied to enhance its disturbance attenuation capability and provide robustness against model uncertainties. The uncertainties of the dynamics and unobservable disturbances are compensated by using a robust adaptive RBF-NN. The INBSM method is used to realize robustness attitude tracking control. The NNDO observer is designed for aerodynamic coefficient uncertainty and unknown environmental disturbances, and the fault estimator is proposed for the actuator fault tolerant control.

This paper is organized as follows. Section 2 gives the nonlinear dynamics model of the BWB aircraft and presents the stability augmentation and attitude tracking problem. Section 3 proposes a nonlinear NNDO-INBSM control design, and stability is analyzed for the associated closed-loop tracking error system. Simulations and performances of three scenarios with the NNDO-INBSM control method are demonstrated in Section 4. Section 5 gives some conclusions.

# 2. Dynamics modeling and problem formulation

# 2.1 Dynamics modeling of the BWB aircraft

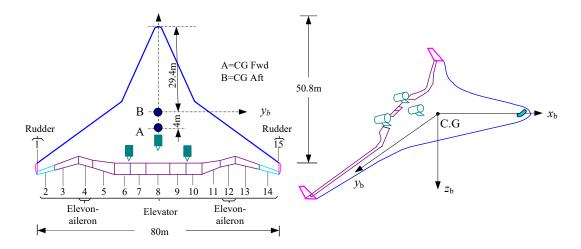


Fig.1 The BWB aircraft platform and geometry

The BWB aircraft is shown Fig.1. It is a flying-wing configuration with 50.8m length and 80m wing span, and has fifteen flaps. Flap 1 and Flap 15 are rudders, which are not in the wing plane. Flap 12 and Flap 4 are elevon-ailerons, while Flaps 2 –14 are the elevators. The aircraft is considered as a rigid body moving in air. The dynamics model is established as follows

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{pmatrix} -qw + rv \\ -ru + pw \\ -pv + qu \end{pmatrix} + g \begin{bmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} + \frac{1}{m} \begin{bmatrix} \overline{q}SC_{xa} \\ \overline{q}SC_{ya} \\ \overline{q}SC_{za} \end{bmatrix} + \frac{1}{m} \begin{bmatrix} \overline{q}SC_{x\delta} + T \\ \overline{q}SC_{y\delta} \\ \overline{q}SC_{z\delta} \end{bmatrix}$$
(1)

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = -\overline{I}^{-1} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{pmatrix} \overline{I} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \overline{I}^{-1} \begin{bmatrix} \overline{q}SbC_{la} \\ \overline{q}S\overline{c}C_{ma} \\ \overline{q}SbC_{na} \end{bmatrix} + \overline{I}^{-1} \begin{bmatrix} \overline{q}SbC_{l\delta} \\ \overline{q}S\overline{c}C_{m\delta} \\ \overline{q}SbC_{n\delta} \end{bmatrix}$$

$$(2)$$

where T denotes thrust,  $C_x$ ,  $C_y$ ,  $C_z$ ,  $C_l$ ,  $C_m$ ,  $C_n$  are aerodynamic force coefficients and moment coefficients as referred in [7, 8], subscripts a and  $\delta$  denote aerodynamic terms of surfaces without control and of control surfaces respectively.  $\textbf{\textit{V}}=[u\ v\ w]^T$  is the body-fixed velocity, the position in the flat earth axes is  $\boldsymbol{\xi}=[x\ y\ z]^T$ ,  $\boldsymbol{\omega}=[p\ q\ r]^T$  is the body-fixed rotational rare,  $\boldsymbol{\eta}=[\phi\ \theta\ \psi]^T$  is the aircraft attitude, m is the mass of the BWB aircraft, and  $\overline{\textbf{\textit{I}}}=\begin{bmatrix}I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz}\end{bmatrix}$  is the aircraft inertial moment

matrix.

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Now consider the attitude motion of the BWB aircraft. Denote  $\mathbf{x}^T = [\mathbf{x}_1 \ \mathbf{x}_2]$  as the state vector, where  $\mathbf{x}_1 = \boldsymbol{\eta}$ ,  $\mathbf{x}_2 = \boldsymbol{\omega}$ , the attitude motion model of the BWB aircraft can be rewritten as follows [17],

$$\dot{\mathbf{x}}_{1}(t) = \mathbf{J}(\boldsymbol{\eta})\mathbf{x}_{2}(t) \tag{3a}$$

$$\dot{\boldsymbol{x}}_{2}(t) = \boldsymbol{f}_{2} + G_{2}\boldsymbol{u} \tag{3b}$$

where  $f_2(\cdot)$  and  $G_2(\cdot)$  are continuous mapping functions as

$$f_{2} = -\overline{\boldsymbol{I}}^{-1} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \left( \overline{\boldsymbol{I}} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right) + \overline{\boldsymbol{I}}^{-1} \begin{bmatrix} \overline{q}SbC_{la} \\ \overline{q}S\overline{c}C_{ma} \\ \overline{q}SbC_{na} \end{bmatrix}, \tag{4}$$

$$G_{2} = \overline{\boldsymbol{I}}^{-1} \left( \boldsymbol{M}_{c} \right)_{\delta}, \boldsymbol{M}_{c} = \begin{bmatrix} \overline{q} Sb \left( C_{l_{\delta a}} \delta_{a} + C_{l_{\delta_{r}}} \delta_{r} \right) \\ \overline{q} S\overline{c} C_{m_{\delta e}} \delta_{e} \\ \overline{q} Sb \left( C_{n_{\delta_{r}}} \delta_{r} + C_{n_{\delta a}} \delta_{a} \right) \end{bmatrix}, \left( \boldsymbol{M}_{c} \right)_{\delta} = \frac{\partial \boldsymbol{M}_{c}}{\partial \delta} = \overline{q} S \begin{bmatrix} C_{l_{\delta a}} & 0 & C_{l_{\delta_{r}}} \\ 0 & C_{m_{\delta e}} & 0 \\ C_{n_{\delta a}} & 0 & C_{n_{\delta_{r}}} \end{bmatrix} \begin{pmatrix} b \\ \overline{c} \\ b \end{pmatrix}$$
 (5)

where  $\overline{q}$  denotes dynamic pressure, S is reference area of the wing, b and  $\overline{c}$  are wing span and mean aerodynamic chord length of the wing,  $C_{l\delta_a}$ ,  $C_{m\delta_e}$  and  $C_{n\delta_r}$  denote aerodynamic derivative coefficients of the aileron, elevator and rudder respectively. The transformation matrix J is

$$J(\eta) = \begin{pmatrix} 1 & s(\phi) \tan \theta & c(\phi) \tan \theta \\ 0 & c(\phi) & -s(\phi) \\ 0 & s(\phi) \sec \theta & c(\phi) \sec \theta \end{pmatrix}$$
(6)

where  $|\theta| < \pi/2$  is assumed to avoid the singularity of the matrix because  $\theta = \pm \pi/2$  is not likely to be encountered during practical operation of the aircraft,  $s(\cdot)$  and  $c(\cdot)$  denote the sine and cosine functions respectively.

Disturbances widely exist and act on the BWB aircraft, which adversely affect the performance and stability of the flight control system. The disturbance includes modelling errors caused by the uncertainty as well as the disturbances from the external environment. Since the disturbance is difficultly measured by sensors, then disturbance rejection becomes one challenge of the control system design. So the model uncertainty and external disturbance  $\mathbf{D} = \begin{bmatrix} \mathbf{D}_V^T & \mathbf{D}_\omega^T \end{bmatrix}^T$  are introduced

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meeting with  $|\mathbf{D}| < \mathbf{D}_{\text{U}}$ , where  $\mathbf{D}_{\text{U}}$  is a real value, and the system of (3) can be modified in an affine form,

$$\dot{\boldsymbol{x}}_{2}(t) = \boldsymbol{f}_{2} + \boldsymbol{G}_{2}\boldsymbol{u} + \boldsymbol{g}_{d} \boldsymbol{D}_{\omega} \tag{7}$$

where  $D_{\omega} \in \mathbb{R}^3$  denotes the attitude disturbance vector, assume  $D_{\omega}$  is bounded,  $g_{d_{\omega}}(x) = \overline{I}^{-1}$ .

# 2.2 The attitude tracking control problem

Consider the BWB aircraft models of (3) and (7). Let  $\eta_d(t):[0,\infty)\to\mathbb{R}^3$  be a given, sufficiently smooth, time-varying reference attitude with bounded time-derivatives of  $(\dot{\eta}_d(t), \ddot{\eta}_d(t))$ . The control task is to design a nonlinear controller such that the closed-loop system meets following requirements:

- 1) Stability of the closed-loop system is enhanced when the center of gravity (CG) of the BWB aircraft moves aftwards;
- 2) The attitude tracking error dynamics of the BWB aircraft are globally asymptotically stable under specified model uncertainty, external disturbances and actuator faults.

# 3 NNDO-INBSM augmentation control design

This section gives an overview of the NNDO-INBSM control for the BWB aircraft. The proposed controller structure is shown in Fig.2, which includes an augmentation control, a NN disturbance observer, an incremental backstepping sliding mode control, a fault estimator and the BWB aircraft dynamics model. The INBSM controller is there to realize the attitude tracking, and the NNDO to observe the observable disturbances; the unobservable disturbances can be approximated by the neural network. The stable filter and command filter are applied to suppress high frequency signal inputs. The controller design is presented in detail in the next section.

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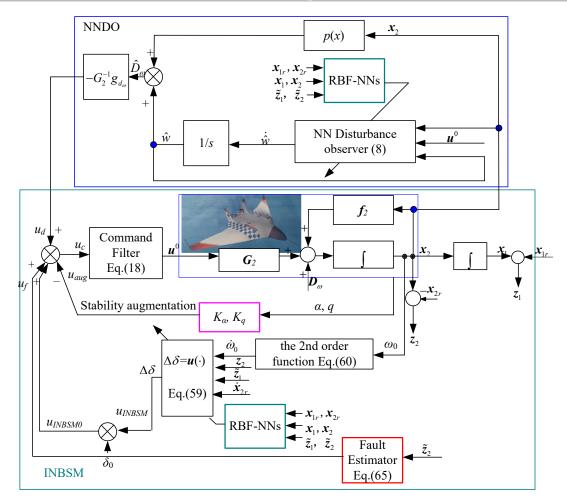


Fig.2 Block of the NNDO-INBSM controller for the BWB aircraft

# 3.1 Neural-Network Disturbance Observer Design

Since the tracking control system for (7) has the unknown bounded disturbance  $D_{\omega}$ , a neural-network disturbance observer is designed to improve the tracking precision [25, 30],

$$\begin{cases} \dot{\hat{w}}(t) = -l(x)g_{d_{\omega}}(x)\hat{w}(t) - l(x)\Big(g_{d_{\omega}}(x)p(x) + f_{2}(x) + G_{2}u + \hat{W}^{T}h(x)\Big) \\ \hat{D}_{\omega}(t) = \hat{w}(t) + p(x) \end{cases}$$
(8)

where  $\hat{w} \in \mathbb{R}^3$  is the internal state of the nonlinear observer,  $\hat{D}_{\omega}$  is the estimation of  $D_{\omega}$ ,  $\hat{W}$  is the estimation of the best weight vector of the RBF neural network, h(x) is the basis function as in (14), and p(x) is the nonlinear function to be designed.

The advantages of the RBF-NNDO are that not only unknown disturbances can be estimated, but also model uncertainties can be approximated by the radial basis function neural network, thus the

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disturbance attenuation ability is greatly enhanced. The NNDO gain l(x) is determined by

$$l(x) = \frac{\partial p(x)}{\partial x} \tag{9}$$

It has been shown in [25] that the NNDO asymptotically estimates the disturbance if the observer gain l(x) is chosen such that the following error plant (10) is asymptotically stable.

$$\dot{d}_{\omega} = \dot{D}_{\omega} - \dot{\hat{D}}_{\omega} = -\dot{\hat{D}}_{\omega} = -\dot{\hat{w}}(t) - \frac{dp(x)}{dt}$$

$$= l(x)g_{d_{\omega}}(x)\hat{w}(t) + l(x)\Big(g_{d_{\omega}}(x)p(x) + f_2(x) + G_2(x)u\Big) - \frac{dp(x)}{dt}$$

$$= l(x)\hat{D}_{\omega} - l(x)D_{\omega}$$

$$= -l(x)g_{d_{\omega}}d_{\omega}$$
(10)

where  $d_{\omega} = D_{\omega} - \hat{D}_{\omega}$  denotes the unobservable part of the overall disturbance  $D_{\omega}$ .

**Remark 1** The design of a disturbance observer (8) essentially is to choose an appropriate gain l(x) and associated p(x) such that the convergence of estimation error  $d_{\omega}$  is guaranteed. It is possible to choose l(x) as a constant matrix such that all the eigenvalues of matrix  $-l(x)g_{d_{\omega}}$  have negative real parts. Multiplying l(x) with the aircraft state x yields p(x) = l(x) x.

It has been shown that the estimation  $\hat{D}_{\omega}(t)$  of the NNDO approaches the disturbance  $D_{\omega}(t)$  exponentially, if the observer gain l(x) is chosen such that the error system (10) is global exponentially stable for all  $x \in \mathbb{R}^n$  [25]. Hence the disturbance compensator can be designed as

$$u_{d} = -G_{2}^{-1} \left( g_{d_{m}} \hat{D}_{\omega} \right) \tag{11}$$

By using the proposed nonlinear disturbance observer, the disturbance  $D_{\omega}$  has been reduced and the system (7) can be transformed as follows:

$$\dot{x}_{2}(t) = f_{2}(x) + G_{2}(x) (u_{INBS} + u_{d}) + g_{d_{\omega}} D_{\omega}$$

$$= f_{2}(x) + G_{2}(x) u_{INBS}(t) + g_{d_{\omega}} d_{\omega}(t)$$
(12)

Usually suppose that the unobservable disturbance  $d_{\omega}$  varies slowly relative to the observer

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dynamics, i.e.  $\dot{d}_{\omega} \approx 0$ , so this will produce conservative control, to break this assumption and reduce this conservativeness, in this paper a neural network is used to approximate this disturbance as follows

$$\dot{x}_2(t) = f_2(x) + G_2(x)u_{INBS}(t) + \left(W^{*T} \cdot h(x) + \varepsilon\right) \tag{13}$$

where  $W^*$  denotes the optimal weight matrix of the RBF-NN,  $\varepsilon$  is approximation error, which can be arbitrarily small variable by regulating the weight and nodes of the RBF-NN, so we assume that  $|\varepsilon| \le \overline{\varepsilon}$ ,  $\overline{\varepsilon} > 0$ . The output of the neural network is  $h(x) = [h_j(x)]^T$ , h(x) is the basis function which can be selected as following Gaussian function

$$h_j(x) = \exp\left(\frac{\|x - c_j\|^2}{2b_j^2}\right)$$
 (14)

where  $c_j$  is the centre value of neural net j,  $b_j$  is the width value of the Gaussian function of net j, j is the node number of the hidden layer. The disturbance term of  $\dot{d}_\omega t$  is approximated by  $y_{NN} = W^{*T} \cdot h(x) + \varepsilon \ .$ 

The faults of the aircraft actuators can be modeled as [31]

$$\mathbf{u}_{a}(t) = \mathbf{u}(t) + (\mathbf{I} - \overline{\boldsymbol{\rho}}_{a}(t))(\mathbf{f}_{a}(t) - \mathbf{u}_{a}(t))$$
(15)

that is,

$$\mathbf{u}_{c}(t) = \mathbf{u}_{a}(t) = \overline{\rho}_{a}\mathbf{u}(t) + (\mathbf{I} - \overline{\rho}_{a}(t))\mathbf{f}_{a}(t) \tag{16}$$

where  $\boldsymbol{u}_a = \left[u_{a,1}, u_{a,2}, \cdots, u_{a,m}\right] \in \mathbb{R}^m$  is the actual control output,  $\boldsymbol{u} \in \mathbb{R}^m$  is the synthesized control output, the diagonal matrix  $\overline{\boldsymbol{\rho}}_a = \operatorname{diag}\left\{\rho_{a,1}, \rho_{a,2}, \cdots, \rho_{a,m}\right\}$  denotes the operational effectiveness of the actuators,  $\boldsymbol{I}$  is  $m \times m$  identity matrix, the fault vector  $\boldsymbol{f}_a = \left[f_{a,1}, f_{a,2}, \cdots, f_{a,m}\right] \in \mathbb{R}^m$  denotes stuck, or floating value of the actuators. Since fault compensation mainly relies on the INBS control, so  $\boldsymbol{u}_{INBS}(t) \approx \boldsymbol{u}_a(t)$  and substituting Eq.(16) into Eq.(13) yields

$$\dot{\boldsymbol{x}}_{2}(t) = \boldsymbol{f}_{2}(t) + G_{2}\boldsymbol{\bar{\rho}}_{a}\boldsymbol{u}(t) + G_{2}(\boldsymbol{I} - \boldsymbol{\bar{\rho}}_{a}(t))\boldsymbol{f}_{a}(t) + \boldsymbol{W}^{*T} \cdot \boldsymbol{h}(x) + \boldsymbol{\varepsilon}$$

$$\tag{17}$$

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#### 3.2. Backstepping stability augmentation and attitude control

This section is to design the backstepping controller, and the objective is to make the closed-loop system stability augmentation and the attitude output  $\eta$  converge to the desired value vector  $\eta_d$ .

A command filter is used to meet magnitude and rate constraints of the input signal. The command filter can be obtained by using a second-order lower pass filter as follows

$$G_r(s) = \frac{X_r(s)}{X_r^0(s)} = \frac{\omega_n^2}{s^2 + 2\zeta_n \omega_n s + \omega_n^2}$$
(18)

where  $\zeta_n$  denotes the filter damping ratio,  $\omega_n$  denotes the filter nature frequency. To reduce the filter error  $e_r \triangleq x_r^0 - x_r$ , the filter frequency  $\omega_n$  is the bandwidth of  $X_r^0(s)$ , which is generally less than that of  $G_r(s)$ , and then  $\omega_n$  can be selected.

Due to mass and passenger changes, the position of CG of the BWB aircraft changes. When the CG position of the aircraft moves aftwards, the stability of the BWB aircraft decreases. To implement augmentation control, usually the angle of attack (AOA,  $\alpha$ ) and pith rate (q) are selected as the feedback signals to implement pitch-axis stability<sup>[32]</sup>, but the BWB aircraft is sensitive for AOA, the feedback signal of AOA is sometime replaced by normal load or unused, see Fig.2, where AOA and pitch rate can be obtained from AOA sensor and angular rate gyros. Similarly side-slip angle and yaw rate are feedback for the lateral dynamics stability augmentation. So stability of the closed-loop system is enhanced by the inner loop stability augmentation control.

$$\boldsymbol{u}_{aug}^{lon} = -K_{\alpha}\alpha - K_{q}q$$
, or  $\boldsymbol{u}_{aug}^{lon} = -K_{q}q$  (19)

Now the control objective is to track a reference signal  $x_{1r} = \eta_d$  with derivative  $\dot{x}_{1r}$  under unknown disturbances  $d_{\omega}$ . Thus the tracking error vectors of the attitudes are defined as

$$\begin{cases} z_1 \triangleq x_1 - x_{1r} \\ z_2 \triangleq x_2 - x_{2r} \end{cases}$$
 (20)

where  $x_{1r}$  is the reference or desired trajectory and  $x_{2r}$  is the output of a command filter.

Now consider the kinematics model (3) and the dynamics model (12). The attitude controller is derived in two steps.

**Step 1** (Backstepping for the variation of  $z_1$ ).

Consider the Lyapunov function  $V_1$ , which is required to be positive definite around the desired position as follows:

$$V_1 = \frac{1}{2} z_1^T z_1 \,. \tag{21}$$

To make the derivative function  $\partial V_1/\partial z_1$  negative definite, a virtual control is defined as

$$\boldsymbol{\alpha}_{1} = G_{1}^{-1} \left( \dot{\boldsymbol{x}}_{1r} - c_{1} \boldsymbol{z}_{1} \right) \tag{22}$$

where  $G_1 = J(\eta)$ , and then

$$\frac{\partial V_1}{\partial z_1} = \mathbf{z}_1^T \left( \dot{\mathbf{x}}_1 - \dot{\mathbf{x}}_{1r} \right) = \mathbf{z}_1^T \left( G_1 \boldsymbol{\alpha}_1 - \dot{\mathbf{x}}_{1r} \right) = -\mathbf{z}_1^T c_1 \mathbf{z}_1 < 0 \tag{23}$$

where  $c_1 > 0$  is often chosen as a diagonal matrix to simplify the design, i.e.,  $c_1 = \text{diag}(c_{11}, c_{12}, c_{13})$ , and  $c_{1i}$  (i = 1, 2, 3) are positive constant.

However, instead of directly applying this virtual control  $a_1$ , a new signal  $x_{2r}^0$  is defined as

$$\boldsymbol{x}_{2r}^{0} \triangleq \boldsymbol{\alpha}_{1} - \boldsymbol{\zeta}_{2} \tag{24}$$

where  $\zeta_2$  will be designed in step 2. The command signal  $\mathbf{x}_{2r}^0$  is filtered to produce the reference signal  $\mathbf{x}_{2r}$  and its derivative  $\dot{\mathbf{x}}_{2r}$ . It can be implemented to enforce magnitude and rate limits through this command filter. By design of the second order command filter (18), the signal  $\left(\mathbf{x}_{2,r} - \mathbf{x}_{2,r}^0\right)$  is bounded and small.

The effect of filtering on the tracking error  $z_1$  is estimated by the following stable linear filter

$$\dot{\zeta}_{1} = -c_{1}\zeta_{1} + G_{1}(x_{2,r} - x_{2,r}^{0})$$
(25)

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where  $c_1$  is as in (23). To remove the effect of filtering the stabilizing functions from the tracking error, the compensated tracking error is defined as

$$\tilde{z}_i \triangleq z_i - \zeta_i, (i=1,2) \tag{26}$$

Re-select the first Lyapunov function  $V_1$  as a quadratic function of the compensated tracking error,

$$V_1 = \frac{1}{2} \tilde{\mathbf{z}}_1^T \tilde{\mathbf{z}}_1 \tag{27}$$

whose derivative is

$$\dot{V}_{1} = \tilde{\mathbf{z}}_{1}^{T} \dot{\tilde{\mathbf{z}}}_{1}$$

$$= \tilde{\mathbf{z}}_{1}^{T} \left( \dot{\mathbf{x}}_{1} - \dot{\mathbf{x}}_{1r} - \dot{\boldsymbol{\zeta}}_{1} \right)$$

$$= \tilde{\mathbf{z}}_{1}^{T} \left( G_{1} \mathbf{x}_{2} - \dot{\mathbf{x}}_{1r} - \left( -c_{1} \boldsymbol{\zeta}_{1} + G_{1} (\mathbf{x}_{2r} - \mathbf{x}_{2r}^{0}) \right) \right)$$
(28)

Substituting Eqs.(22) and (24) into Eq.(28) yields

$$\dot{V}_{1} = \tilde{\boldsymbol{z}}_{1}^{T} \left( G_{1} \left( \boldsymbol{z}_{2} + \boldsymbol{\alpha}_{1} - \boldsymbol{\zeta}_{2} \right) \right) + \tilde{\boldsymbol{z}}_{1}^{T} \left( c_{1} \boldsymbol{\zeta}_{1} - \dot{\boldsymbol{x}}_{1r} \right)$$

$$= \tilde{\boldsymbol{z}}_{1}^{T} c_{1} \left( \boldsymbol{\zeta}_{1} - \dot{\boldsymbol{x}}_{1r} \right) + \tilde{\boldsymbol{z}}_{1}^{T} \left( G_{1} \tilde{\boldsymbol{z}}_{2} + \dot{\boldsymbol{x}}_{1r} - c_{1} \boldsymbol{z}_{1} \right)$$

$$= \tilde{\boldsymbol{z}}_{1}^{T} G_{1} \tilde{\boldsymbol{z}}_{2} - \tilde{\boldsymbol{z}}_{1}^{T} c_{1} \tilde{\boldsymbol{z}}_{1} \tag{29}$$

Now a SMC control is introduced to improve system adaptiveness, the sliding surface s is designed as

$$s = \lambda_1 \tilde{z}_1 + \tilde{z}_2 \quad , \tag{30}$$

where  $\lambda_1$  is the control gains to be determined.

The well-known characteristics of SMC are attraction and invariance, which means the condition for the state to reach the sliding mode surface s in finite time  $t_r$  and remain it, i.e.,

$$s\dot{s} < 0, \quad t < t_r \tag{31}$$

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$$s = \dot{s} = 0, \ t \ge t_{r} \tag{32}$$

For the sliding mode surface of Eq.(30), the reachability condition is designed as

$$\dot{s} = -\hbar s - \zeta \operatorname{sgn}(s) \,, \tag{33}$$

so their corresponding attraction and invariance for the SMC are as follows

$$s\dot{s} = -\hbar s^T s - \zeta |s| \tag{34}$$

where  $\hbar$ , and  $\varsigma$  are sliding mode surface parameters with  $\hbar > 0$ , and  $\varsigma > 0$ .

#### Step 2 (Calculate the INBSM control)

The effect of filtering the command signal on the tracking error  $\mathbf{z}_2$  is estimated by the following stable linear filter

$$\dot{\boldsymbol{\zeta}}_{2} = -c_{2}\boldsymbol{\zeta}_{2} + G_{2}\left(\boldsymbol{u}_{INBS} - \boldsymbol{u}_{DIBS}^{0}\right) \tag{35}$$

where  $G_2$  is as in (5),  $c_2$  is the filter gain to be determined,  $\boldsymbol{u}_{\scriptscriptstyle INBS}^0$  is the incremental backstepping sliding mode control to be designed, and  $\boldsymbol{u}_{\scriptscriptstyle INBS}$  is the filtered output of  $\boldsymbol{u}_{\scriptscriptstyle INBS}^0$ .

Now consider that the unobservable disturbance of  $d_{\omega}$ , it can be approximated by the neural network. Construct the second candidate Lyapunov function as

$$V_2 = V_1 + \frac{1}{2} s^T s + \frac{1}{2} (1/\gamma_W) \tilde{W}^T \tilde{W} , \qquad (36)$$

where  $\tilde{W}$  is estimation error of the weight function W meeting  $\tilde{W} = W^* - \hat{W}$ ,  $\gamma_w$  is a positive constant that determines the convergence speed of the NN approximation. Differentiating (36) yields

$$\dot{V}_{2} = -\tilde{\mathbf{z}}_{1}^{T} C_{1} \tilde{\mathbf{z}}_{1} + \tilde{\mathbf{z}}_{1}^{T} G_{1} \tilde{\mathbf{z}}_{2} + s^{T} \dot{s} + (1/\gamma_{W}) \tilde{W}^{T} \dot{\tilde{W}} \qquad . \tag{37}$$

Substituting (20), (23),(29), (35) into (37) yields

$$\dot{V}_{2} = -\tilde{\mathbf{z}}_{1}^{T} c_{1} \tilde{\mathbf{z}}_{1} + \tilde{\mathbf{z}}_{1}^{T} G_{1} \tilde{\mathbf{z}}_{2} + s^{T} \dot{s} - (1/\gamma_{w}) \tilde{W}^{T} \dot{\hat{W}}$$

$$= -\tilde{\boldsymbol{z}}_{1}^{T} \boldsymbol{c}_{1} \tilde{\boldsymbol{z}}_{1} + \tilde{\boldsymbol{z}}_{1}^{T} \boldsymbol{G}_{1} \tilde{\boldsymbol{z}}_{2} + \boldsymbol{s}^{T} \left( \lambda_{1} \dot{\tilde{\boldsymbol{z}}}_{1} + \dot{\tilde{\boldsymbol{z}}}_{2} \right) - (1/\gamma_{W}) \tilde{\boldsymbol{W}}^{T} \dot{\hat{\boldsymbol{W}}}^{T}$$

$$= -\tilde{\boldsymbol{z}}_{1}^{T} c_{1} \tilde{\boldsymbol{z}}_{1} + \tilde{\boldsymbol{z}}_{1}^{T} G_{1} \tilde{\boldsymbol{z}}_{2} + \boldsymbol{s}^{T} \left( \boldsymbol{\lambda}_{1} \left( c_{1} \tilde{\boldsymbol{z}}_{1} + G_{1}^{T} \tilde{\boldsymbol{z}}_{2} \right) + \left( \boldsymbol{f}_{2} + G_{2} \boldsymbol{u}_{INBS} + \boldsymbol{g}_{d_{\omega}} \boldsymbol{d}_{\omega} - \dot{\boldsymbol{x}}_{2r} + c_{2} \boldsymbol{\zeta}_{2} - G_{2} \left( \boldsymbol{u}_{INBS} - \boldsymbol{u}_{DBS}^{0} \right) \right) \right) - (1/\gamma_{W}) \tilde{W}^{T} \dot{\hat{W}}$$

$$= -\tilde{\boldsymbol{z}}_{1}^{T} c_{1} \tilde{\boldsymbol{z}}_{1} + \tilde{\boldsymbol{z}}_{1}^{T} G_{1} \tilde{\boldsymbol{z}}_{2} + \boldsymbol{s}^{T} \left( \boldsymbol{\lambda}_{1} \left( c_{1} \tilde{\boldsymbol{z}}_{1} + G_{1}^{T} \tilde{\boldsymbol{z}}_{2} \right) + \boldsymbol{f}_{2} + G_{2} \boldsymbol{u}_{DBS}^{0} + W^{*T} h + \varepsilon - \dot{\boldsymbol{x}}_{2r} + c_{2} (\boldsymbol{z}_{2} - \tilde{\boldsymbol{z}}_{2}) \right) - (1/\gamma_{W}) \tilde{W}^{T} \dot{\hat{W}}$$
(38)

By using the sliding surface (30) and reaching law (33), a desired control input (in non-incremental form) is designed as follows to make (38) semi-negative definite,

$$\boldsymbol{u}_{\text{\tiny DNRS}}^{0} = G_{2}^{-1} \left( -\lambda_{1} \left( c_{1} \tilde{z}_{1} + G_{1}^{T} \tilde{z}_{2} \right) - c_{2} (z_{2} - \tilde{z}_{2}) - G_{1}^{T} \tilde{z}_{1} - \boldsymbol{f}_{2} + \dot{\boldsymbol{x}}_{2r} - \hat{W}^{T} h - \left( \hbar \boldsymbol{s} + \varsigma \operatorname{sgn}(\boldsymbol{s}) \right) \right), \tag{39}$$

Since

$$s^T \tilde{d}_m = \tilde{d}_m^T s , \quad \tilde{z}_1^T G_1 \tilde{z}_2 = \tilde{z}_2^T G_1^T \tilde{z}_1$$

$$\tag{40}$$

Substituting (39) and (40) into (38) yields

$$\dot{V}_{2} = \left(-\tilde{\mathbf{z}}_{1}^{T} c_{1} \tilde{\mathbf{z}}_{1} + \tilde{\mathbf{z}}_{1}^{T} G_{1} \tilde{\mathbf{z}}_{2}\right) - \hbar \mathbf{s}^{T} \mathbf{s} - \varsigma \mid \mathbf{s} \mid + \mathbf{s}^{T} \left(\tilde{W}^{T} h + \varepsilon\right) - (1/\gamma_{w}) \tilde{W}^{T} \dot{\hat{W}}$$

$$\tag{41}$$

Choose the update law of the NN weight function as:

$$\dot{\hat{W}} = \gamma_W \mathbf{s}^T h(x) \tag{42}$$

and substitute (42) into (41) yields

$$\dot{V}_{2} = \left(-\tilde{\boldsymbol{z}}_{1}^{T} \boldsymbol{c}_{1} \tilde{\boldsymbol{z}}_{1} + \tilde{\boldsymbol{z}}_{1}^{T} \boldsymbol{G}_{1} \tilde{\boldsymbol{z}}_{2}\right) - \hbar \boldsymbol{s}^{T} \boldsymbol{s} - \boldsymbol{\varsigma} \mid \boldsymbol{s} \mid + \boldsymbol{s}^{T} \boldsymbol{\varepsilon}$$

$$\tag{43}$$

Define Q as a positive definite symmetric matrix

$$Q \triangleq \begin{bmatrix} c_1 + \hbar \lambda_1^2 & \lambda_1 \hbar - \frac{1}{2} G_1 \\ \lambda_1 \hbar - 0.5 G_1 & \hbar I_n \end{bmatrix}$$
(44)

where  $I_n$  denotes the  $n \times n$  unit matrix. Let  $Z_{12} \triangleq \begin{bmatrix} \tilde{z}_1^T & \tilde{z}_2^T \end{bmatrix}^T$ , this then yields

$$Z_{12}^{T}QZ_{12} = \tilde{\mathbf{z}}_{1}^{T}c_{1}\tilde{\mathbf{z}}_{1} + \hbar\boldsymbol{\lambda}_{1}^{2}\tilde{\mathbf{z}}_{1}^{T}\tilde{\mathbf{z}}_{1} + 2\boldsymbol{\lambda}_{1}\hbar\tilde{\mathbf{z}}_{1}^{T}\tilde{\mathbf{z}}_{2} - \tilde{\mathbf{z}}_{1}^{T}G_{1}\tilde{\mathbf{z}}_{2} + \hbar\tilde{\mathbf{z}}_{2}^{T}\tilde{\mathbf{z}}_{2}$$

$$= \tilde{\mathbf{z}}_{1}^{T}c_{1}\tilde{\mathbf{z}}_{1} - \tilde{\mathbf{z}}_{1}^{T}G_{1}\tilde{\mathbf{z}}_{2} + \hbar(\boldsymbol{\lambda}_{1}\tilde{\mathbf{z}}_{1} + \tilde{\mathbf{z}}_{2})^{T}(\boldsymbol{\lambda}_{1}\tilde{\mathbf{z}}_{1} + \tilde{\mathbf{z}}_{2})$$

$$= (\tilde{\mathbf{z}}_{1}^{T}c_{1}\tilde{\mathbf{z}}_{1} - \tilde{\mathbf{z}}_{1}^{T}G_{1}\tilde{\mathbf{z}}_{2}) + \hbar\mathbf{s}^{T}\mathbf{s} \quad .$$

$$(45)$$

Substituting (45) into (43) yields

$$\dot{V}_{2} = -Z_{12}^{T}QZ_{12} - \zeta \mid \mathbf{s} \mid +\mathbf{s}^{T} \varepsilon$$

(46)

Using (44), it is obtained that

$$|Q| = \begin{bmatrix} c_1 + \hbar \lambda_1^2 & \lambda_1 \hbar - 0.5G_1 \\ \lambda_1 \hbar - 0.5G_1 & \hbar I_n \end{bmatrix}$$

$$= \hbar (c_1 + \lambda_1 G_1) - G_1^T G_1 / 4. \tag{47}$$

If the following condition is satisfied,

$$\hbar(c_1 + \lambda_1 G_1) > G_1^T G_1 / 4, \qquad (48)$$

then Q is positive definite. Since  $|\varepsilon| \le \overline{\varepsilon}$  (from Eq.(14)), then  $\mathbf{s}^T \varepsilon \le \left|\mathbf{s}^T \mid \cdot |\varepsilon| < \overline{\varepsilon} \mid \mathbf{s}^T \mid$ , and if  $\zeta > \overline{\varepsilon} > 0$ , then we have

$$(-\zeta + \overline{\varepsilon}) |s| = -\zeta |s| + \overline{\varepsilon} |s^T| < 0,$$
(49)

and then  $-\zeta | \mathbf{s} | + \mathbf{s}^T \varepsilon < 0$ . So

$$\dot{V}_2 = -Z_{12}^T Q Z_{12} - \varsigma \mid \mathbf{s} \mid + \mathbf{s}^T \varepsilon < 0.$$
 (50)

Therefore, the closed-loop system is stable by Lyapunov theory.

**Remark 3** In order to effectively eliminate the chattering phenomenon of reachability condition (33) in the sliding mode control, a continuous function is used to define practical sliding surface dynamics using a "tanh" function,

$$\dot{s} = -\hbar s - \zeta \tanh(s/\phi), \qquad (51)$$

where  $\phi$  is sliding surface boundary layer parameters used to retain continuity of control as motion trajectories cross the sliding surface and prevent chattering. High frequency chattering of the control is prevented by using the boundary layer.

# 3.3 Incremental nonlinear backstepping attitude tracking control

It can clear that NDI control law depends on accurate knowledge of the aerodynamic model contained in both  $M_a$  and  $M_c$ , and depends on the model uncertainties and disturbances; if the

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disturbance information is unknown, it does not reject the disturbances. So an incremental backstepping control design is proposed to achieve a more flexible and augmented design. According to Fig.2, the composite controller input of the tracking system is

$$\boldsymbol{u}_{c} \triangleq \boldsymbol{u}_{INBSM0} + \boldsymbol{u}_{aug} + \boldsymbol{u}_{d} + \boldsymbol{u}_{f} \tag{52}$$

where  $\mathbf{u}_d$  are  $\mathbf{u}_f$  are compensation control inputs of disturbances and faults, respectively,  $\mathbf{u}_{INBSM\,0} = \delta_0 + \mathbf{u}_{INBSM} \ .$ 

To obtain an incremental Backstepping control  $u_{NBSM}$ , for simplicity, we start from Step 2 of the backstepping design procedure in Section 3.2, assuming that the outer subsystem's stabilizing control law are already obtained and stepped back up to the dynamic equation in consider.

For the BWB aircraft, the model (12) of the attitude dynamics can be represented as follows

$$\overline{I}\dot{\omega} = \left(-\omega \times (\overline{I}\omega) + M_a\right) + (M_c)_s \delta + g_{d_s} d_{\omega}$$
(53)

where  $\mathbf{M}_{a} = \overline{q}S[bC_{la} \quad \overline{c}C_{ma} \quad bC_{na}]^{T}$ ,  $\delta = \begin{bmatrix} \delta_{a} & \delta_{e} & \delta_{r} \end{bmatrix}^{T}$ ,  $(\mathbf{M}_{c})_{\delta}$  is as in (5).

Now we depart from the error dynamics equation by using Eqs (24) and (26),

$$\dot{\tilde{z}}_{2} = \dot{x}_{2} - \dot{x}_{2r} - \dot{\zeta}_{2} = \dot{x}_{2} - \dot{\overline{\alpha}}_{1}(\sigma, \omega) \tag{54}$$

that is,

$$\dot{\tilde{z}}_{2} = \dot{\omega} - \dot{\bar{\alpha}}_{1}(\sigma, \omega) = f_{2}(x) + G_{2}(x)\delta + g_{d_{\omega}}d_{\omega} - \dot{\bar{\alpha}}_{1}(\sigma, \omega)$$

$$(55)$$

where  $\sigma$  may represent a kinematics variable or a state stepped back from the outer subsystems,  $\dot{\bar{\alpha}}_1(\sigma,\omega)=\dot{\alpha}_1(\sigma,\omega)+G_1^{-1}\left(\ddot{\zeta}_1+c_1\dot{\zeta}_1\right)$ , according to the filter (18), the signal  $\left(x_{2,r}-x_{2,r}^0\right)$  is bounded and small, so  $\dot{\bar{\alpha}}_1$  can be approximated by  $\dot{\alpha}_1$ . As  $f_2(x)=\bar{I}^{-1}\left(M_a-\omega\times\left(\bar{I}\omega\right)\right)$ ,  $G_2=\bar{I}^{-1}\left(M_c\right)_{\delta}$ , for flight control law design, the goal is to stabilize the complete system described by the following augmented equation

$$\dot{\tilde{z}}_{2} = \overline{I}^{-1} \left( M_{a} - \omega \times (\overline{I}\omega) \right) + \overline{I}^{-1} \left( M_{c} \right)_{s} \delta + \overline{I}^{-1} g_{d} d_{\omega} - \dot{\overline{\alpha}}_{1}(\sigma, \omega)$$
(56)

Applying backstepping to Eq.(55), the actuator input of the BWB aircraft is

$$\delta = (\boldsymbol{M}_{c})_{\delta}^{-1} \overline{\boldsymbol{I}} \left[ -K_{\omega} \tilde{\boldsymbol{z}}_{2} - \overline{\boldsymbol{I}}^{-1} (\boldsymbol{M}_{a} - \omega \times (\overline{\boldsymbol{I}} \omega)) + \dot{\overline{\alpha}}_{1} (\sigma, \omega) + \Gamma_{d} \right]$$

$$(57)$$

where the control gain is  $K_{\omega} > 0$  and the damping term is  $\Gamma_d = -\overline{I}^{-1}\hat{W}^T h$ , which is used to reject the unobservable disturbances and model uncertainty. The robustness of such a backstepping design is improved by introducing its incremental counterpart, using the implicit approach with the recursive control law

$$\delta = \delta_0 + (\boldsymbol{M}_c)_{\delta}^{-1} \, \overline{\boldsymbol{I}} \left[ -K_{\omega} \tilde{\boldsymbol{z}}_2 - \dot{\boldsymbol{\omega}}_0 + \dot{\overline{\boldsymbol{\alpha}}}_1(\boldsymbol{\sigma}, \boldsymbol{\omega}) + \Gamma_d \right]$$
 (58)

According to the backstepping control law (39) and considering the approximate dynamics around the current reference state for the dynamics  $\dot{\omega} \approx \dot{\omega}_0 + G_2 \Delta \delta$ , the backstepping control (39) can be rewritten in incremental form,

$$\Delta \delta = G_2^{-1} \left( -\lambda_1 \left( c_1 \tilde{\mathbf{z}}_1 + G_1^T \tilde{\mathbf{z}}_2 \right) - c_2 (\mathbf{z}_2 - \tilde{\mathbf{z}}_2) - G_1^T \tilde{\mathbf{z}}_1 - \dot{\omega}_0 + \dot{\mathbf{x}}_{2r} - \hat{W}^T h - \left( \hbar \mathbf{s} + \varsigma \operatorname{sgn}(\mathbf{s}) \right) \right). \tag{59}$$

The output  $\dot{\omega}_0$  represents the current estimated angular acceleration, which is measured by differentiating the angular rates. The approximation function from the  $\omega_0$  to derivative  $\dot{\omega}_0$  is

$$H_{\omega}(s) = \frac{s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$
 (60)

Theorem 3.1 There exists a composite controller of  $u_{INBSM}$  and  $u_d$  with a disturbance observer of (8) that guarantees the closed-loop system of (7) meeting desired performances of (a) and (b), if the positive-definition control gain of  $K_{\omega}$  meets Eq.(57), the observer gain l(x) is appropriately determined such that all the eigenvalues of matrix  $-l(x)g_{d_{\omega}}$  have negative real part, and the positive-definition filter gain matrices of  $c_1$  and  $c_2$  meet Eqs (25) and (35), the sliding mode control (30) and (34) satisfy that Q and  $\lambda_1$  is positive definite, and  $\hbar > 0$ ,  $\varsigma > \overline{\varepsilon} > 0$ , and update law of the NN weight function of (42) meets  $\gamma_W > 0$ .

Proof.

If there exist a positive-definition control gains of  $K_{\omega}$  meets Eqs.(57) and (59), then the control input of the INBSM controller  $\boldsymbol{u}_{INBSM0}$  is obtained, so the INBSM controller of (57) and (59) can meet Eq.(39), which guarantees the candidate Lyapunov function  $V_2 > 0$ ,  $\dot{V}_2 < 0$ , yield the result.  $\blacksquare$  **Remark 4** It suffices to consider the new  $f_2 = \dot{x}_{2_0} = \dot{\omega}_0$ , noticing that we are replacing the accurate knowledge of  $f_2$  by a measurement (or an estimate) instead [21], and this trade-off results in a robustified backstepping control law which is not entirely dependent on the model. Comparing with Eq.(57), there does not exist an  $\dot{\alpha}_1$  term for (58), so it can be directly obtained.

For the BWB aircraft, the elevator, aileron, rudder and engine actuator dynamics can be simplified as 1st order models,

$$G_{\delta_i}(s) = \frac{T_{\delta_i}}{s + T_{\delta_i}} \tag{61}$$

where  $\delta_i = \delta_a, \delta_e, \delta_r, \delta_p$   $T_{\delta_i}$  are the actuator time constants.

Finally, consider fault of the system Eq.(17), two faults of actuator bias and loss of effectiveness are considered, and then the fault model of (16) can be rewritten as

$$\mathbf{u}_{a}(t) = \overline{\rho}_{a}\mathbf{u}(t) + \mathbf{u}_{f} \tag{62}$$

where  $\boldsymbol{u}_f = (\boldsymbol{I} - \overline{\boldsymbol{\rho}}_a) \boldsymbol{f}_a$ .

Theorem 3.2 There exists a composite controller of (52) with a disturbance observer (8) and a fault estimator (65) that guarantees the closed-loop system (7) meets desired performances of (a) and (b), if the positive-definition control gain of  $K_{\omega}$  meets Eq.(57), the observer gain l(x) is appropriately determined such that all the eigenvalues of matrix  $-l(x)g_{d_{\omega}}$  have negative real part, and the positive-definition filter gain matrices of  $c_1$  and  $c_2$  meet Eqs (25) and (35), the sliding mode control (30) and (34) satisfy that Q and  $\lambda_1$  is positive definite, and  $\hbar > 0$ ,  $\varsigma > \overline{\varepsilon} > 0$ , and update law of the NN weight function of (42) meets  $\gamma_W > 0$ , positive definite weight matrix  $P_f$  and  $\gamma_f$  make Eq.(65) be

convergent.

Proof

Based on **Theorem 3.1**, the second augmented closed-loop Lyapunov function of (29) is changed for backstepping as follows when actuator faults occur

$$V_{2} = V_{1} + \frac{1}{2} s^{T} s + \frac{1}{2} (1/\gamma_{W}) \tilde{W}^{T} \tilde{W} + \frac{1}{2\gamma_{d}} \tilde{\boldsymbol{u}}_{f}^{T} \boldsymbol{P}_{f}^{-1} \tilde{\boldsymbol{u}}_{f}$$
(63)

where  $\tilde{u}_f = \hat{u}_f - u_f$  is the fault estimation error, and  $P_f$  is a positive definite weight matrix.

Substituting (50) into the following derivative of  $V_2$  yields

$$\dot{V}_{2} = -Z_{12}^{T} Q Z_{12} - \varsigma \mid \mathbf{s} \mid + \mathbf{s}^{T} \varepsilon + \tilde{\boldsymbol{u}}_{f}^{T} \boldsymbol{P}_{f}^{-1} \dot{\tilde{\boldsymbol{u}}}_{f}$$

$$\tag{64}$$

The fault estimator is designed as:

$$\dot{\hat{\boldsymbol{u}}}_f = \boldsymbol{P}_f \boldsymbol{G}_2^T \tilde{\boldsymbol{z}}_2 - \boldsymbol{\gamma}_f \hat{\boldsymbol{u}}_f \tag{65}$$

which yields

$$\tilde{\boldsymbol{u}}_{f}^{T} \boldsymbol{P}_{f}^{-1} \dot{\tilde{\boldsymbol{u}}}_{f} = \tilde{\boldsymbol{u}}_{f}^{T} \boldsymbol{P}_{f}^{-1} \left( \dot{\hat{\boldsymbol{u}}}_{f} - \dot{\boldsymbol{u}}_{f} \right) = \tilde{\boldsymbol{u}}_{f}^{T} \boldsymbol{P}_{f}^{-1} \boldsymbol{P}_{f} \boldsymbol{G}_{2}^{T} \tilde{\boldsymbol{z}}_{2} - \tilde{\boldsymbol{u}}_{f}^{T} \boldsymbol{P}_{f}^{-1} \boldsymbol{\gamma}_{f} \hat{\boldsymbol{u}}_{f} - \tilde{\boldsymbol{u}}_{f}^{T} \boldsymbol{P}_{f}^{-1} \dot{\boldsymbol{u}}_{f}$$

$$= \tilde{\boldsymbol{u}}_{f}^{T} \boldsymbol{G}_{2}^{T} \tilde{\boldsymbol{z}}_{2} - \boldsymbol{P}_{f}^{-1} \boldsymbol{\gamma}_{f} \tilde{\boldsymbol{u}}_{f}^{T} \hat{\boldsymbol{u}}_{f} - \boldsymbol{P}_{f}^{-1} \tilde{\boldsymbol{u}}_{f}^{T} \dot{\boldsymbol{u}}_{f} \tag{66}$$

Since  $\| \boldsymbol{u}_f \| \le \| \overline{u}_{\text{max}} \| < \infty, \| \dot{\boldsymbol{u}}_f \| \le \| \overline{\dot{u}}_{\text{max}} \| < \infty$ ,  $\overline{u}_{\text{max}}$  and  $\overline{\dot{u}}_{\text{max}}$  denote upper bounds of the actuator position and rotating rate, then [33]

$$-\boldsymbol{P}_{f}^{-1}\boldsymbol{\gamma}_{f}\tilde{\boldsymbol{u}}_{f}^{T}\hat{\boldsymbol{u}}_{f}=-\boldsymbol{P}_{f}^{-1}\boldsymbol{\gamma}_{f}\tilde{\boldsymbol{u}}_{f}^{T}\left(\tilde{\boldsymbol{u}}_{f}+\boldsymbol{u}_{f}\right)$$

$$\leq \frac{1}{2} \left( -\boldsymbol{P}_{f}^{-1} \gamma_{f} \tilde{\boldsymbol{u}}_{f}^{T} \tilde{\boldsymbol{u}}_{f} + \boldsymbol{P}_{f}^{-1} \gamma_{f} \overline{\boldsymbol{u}}_{\text{max}}^{T} \overline{\boldsymbol{u}}_{\text{max}} \right)$$

$$(67)$$

$$-\boldsymbol{P}_{f}^{-1}\widetilde{\boldsymbol{u}}_{f}^{T}\dot{\boldsymbol{u}}_{f} \leq \frac{1}{2} \left(\boldsymbol{P}_{f}^{-1}\widetilde{\boldsymbol{u}}_{f}^{T}\widetilde{\boldsymbol{u}}_{f} + \boldsymbol{P}_{f}^{-1}\overline{\dot{\boldsymbol{u}}}^{T}\overline{\dot{\boldsymbol{u}}}\right) \leq \frac{1}{2} \left(\boldsymbol{P}_{f}^{-1}\widetilde{\boldsymbol{u}}_{f}^{T}\widetilde{\boldsymbol{u}}_{f} + \boldsymbol{P}_{f}^{-1}\overline{\dot{\boldsymbol{u}}}_{\max}^{T}\overline{\dot{\boldsymbol{u}}}_{\max}\right)$$
(68)

Substitute (67), (68) into (66) yields

$$\tilde{\boldsymbol{u}}_{f}^{T} \boldsymbol{P}_{f}^{-1} \dot{\tilde{\boldsymbol{u}}}_{f} \leq \tilde{\boldsymbol{u}}_{f}^{T} \boldsymbol{G}_{2}^{T} \tilde{\boldsymbol{z}}_{2} + \overline{\boldsymbol{c}} \tag{69}$$

where  $P_f = diag(1,1,1)$ ,  $\gamma_f = diag(1,1,1)$ 

$$\overline{c} = \frac{\gamma_f}{2\boldsymbol{P}_f} \left( -\tilde{\boldsymbol{u}}_f^T \tilde{\boldsymbol{u}}_f + \overline{u}_{\text{max}}^T \overline{u}_{\text{max}} \right) + \frac{1}{2\boldsymbol{P}_f} \left( \tilde{\boldsymbol{u}}_f^T \tilde{\boldsymbol{u}}_f + \overline{u}_{\text{max}}^T \overline{u}_{\text{max}} \right). \tag{70}$$

As fault estimation error  $\tilde{u}_f$  is a small variant for the fault estimator (65) [33], thus finally yields

$$\dot{V}_{2} = -Z_{12}^{T} Q Z_{12} - \varsigma \mid \mathbf{s} \mid + \mathbf{s}^{T} \varepsilon + \overline{c}$$

$$(71)$$

Let  $c_k = \min\{2|Q|, -\gamma_f+1\}$ , and if  $\varsigma > \overline{\varepsilon} > 0$ , then

$$\dot{V}_{2} \le -c_{k}V_{2} + \overline{c} \tag{72}$$

According to LaSalle-Yoshizawa Lemma [34] and reference [33], the closed-loop system tracking error will exponentially converge, and

$$V_2 \le \left(V_2(\mathbf{0}) - \overline{c} / c_{\scriptscriptstyle k}\right) e^{-c_k t} + \overline{c} / c_{\scriptscriptstyle k} \tag{73}$$

Hence

$$\frac{1}{2}Z_{12}^{T}Z_{12} \le V_{2}, \|Z_{12}\| \le \sqrt{\frac{2\overline{c}}{c_{k}}} \quad , \tag{74}$$

that is, the desired tracking error will exponentially converge to the set

$$\|\tilde{z}_1\| + \|\tilde{z}_2\| \le \sqrt{\frac{2\overline{c}}{c_{\scriptscriptstyle L}}},\tag{75}$$

and

$$\|\mathbf{x}_1\| = \|\tilde{\mathbf{z}}_1 + \mathbf{\zeta}_1\| \le \|\tilde{\mathbf{z}}_1\| + \|\mathbf{\zeta}_1\|, \|\mathbf{x}_2\| = \|\tilde{\mathbf{z}}_2 + \mathbf{\zeta}_2\| \le \|\tilde{\mathbf{z}}_2\| + \|\mathbf{\zeta}_2\|.$$
 (76)

According to Fig.2, the algorithm flow chart of the proposed NNDO-INBSM control is as follows,

Step 1 Calculate error signals

$$\frac{\boldsymbol{\eta}, \omega}{\boldsymbol{x}_{1r}, \boldsymbol{x}_{2r}} \underbrace{Eq.(20)}_{Eq.(20)} \underbrace{\begin{pmatrix} \boldsymbol{z}_{1} \\ \boldsymbol{z}_{2} \end{pmatrix}}_{\boldsymbol{z}_{1}}; \underbrace{\begin{array}{c} \boldsymbol{z}_{1}, \boldsymbol{z}_{2} \\ \boldsymbol{x}_{1r}, \boldsymbol{\eta} \end{array}}_{\boldsymbol{z}_{1}} \underbrace{Eq.(22)}_{Eq.(22)} \alpha_{1}; \underbrace{\begin{array}{c} \boldsymbol{\eta}, \boldsymbol{x}_{2r}^{0}, \boldsymbol{x}_{2r} \\ \boldsymbol{u}_{NBS}^{0}, \boldsymbol{u}_{INBS} \end{array}}_{Eq.(25), (35)} \underbrace{\begin{pmatrix} \boldsymbol{\zeta}_{1} \\ \boldsymbol{\zeta}_{2} \end{pmatrix}}_{\boldsymbol{z}_{2}}; \underbrace{\begin{array}{c} \boldsymbol{z}_{1} \\ \boldsymbol{z}_{1}, \boldsymbol{\zeta}_{2} \end{array}}_{\boldsymbol{\zeta}_{1}, \boldsymbol{\zeta}_{2}} \underbrace{\begin{array}{c} Eq.(26) \\ \boldsymbol{z}_{1} \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{2}}; \underbrace{\begin{array}{c} \boldsymbol{z}_{1} \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{2}} \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{2}} s; \underbrace{\begin{array}{c} \boldsymbol{z}_{1} \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{2}} \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{2}} s; \underbrace{\begin{array}{c} \boldsymbol{z}_{1} \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{2}} \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{2}} s; \underbrace{\begin{array}{c} \boldsymbol{z}_{1} \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{2}} \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{1} \end{array}}_{\boldsymbol{z}_{2}} s; \underbrace{\begin{array}{c} \boldsymbol{z}_{1} \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{1}} \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{1}} s; \underbrace{\begin{array}{c} \boldsymbol{z}_{1} \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{1}} \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{1}} s; \underbrace{\begin{array}{c} \boldsymbol{z}_{1} \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{1}} \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{1} \end{array}}_{\boldsymbol{z}_{2}} s; \underbrace{\begin{array}{c} \boldsymbol{z}_{1} \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{1}} \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{1} \end{array}}_{\boldsymbol{z}_{2}} s; \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{1} \end{array}}_{\boldsymbol{z}_{2}} \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{1} \end{array}}_{\boldsymbol{z}_{2}} s; \underbrace{\begin{array}{c} Eq.(30) \\ \boldsymbol{z}_{2} \end{array}}_{\boldsymbol{z}_{2}} s; \underbrace{\begin{array}{c} Eq.$$

Step 2 Calculate NNDO and disturbance estimation  $\hat{D}_{\omega}$  by Eqs (8), (11), and (14);

Step 3 Calculate the augmentation control  $\mathbf{u}_{\text{aug}}$  by Eq.(19)

Step 4 Calculate the incremental BSMC control  $u_{INBS}$  with NNs compensation by Eq.(59).

Step 5 Calculate the fault estimation  $\hat{\boldsymbol{u}}_f$  by Eq.(65)

Step 6 Calculate the total control input  $u_c$  by Eq.(52)

Step 7 repeat step1 to step6 until all tracking tasks are complemented.

# 4. Simulation and analysis

The considered model is a 50.8m length, 80m wing span BWB aircraft [7, 8], the aerodynamics coefficient parameters are as in Ref.[7], and the structure parameters of the BWB aircraft are listed in Table 1.

Table 1 Parameters for the studied BWB aircraft

Parameter	Value	Unit	Coefficient	Value	Unit
m	3.7×10 <sup>5</sup>	kg	$T_{\delta a}$	15	sec
S	841.7	$m^2$	$T_{\delta \mathrm{e}}$	15	sec
$\overline{c}$	12.31	m	$T_{ m \delta r}$	15	sec
b	80	m	$T_{\delta \mathrm{p}}$	1	sec
$x_G$ ,	30.4	m	$z_G$	0	m
$I_{\mathrm{xx}}$	$4.703 \times 10^7$	kg·m²	$I_{ m yy}$	$2.507 \times 10^{7}$	kg·m <sup>2</sup>
$I_{zz}$	$9.973 \times 10^{7}$	kg·m²	$I_{ m xz}$	0.0	kg·m <sup>2</sup>

The aircraft flight is at 20000ft (6096m) altitude and 375kts (191.9m/s) speed, the trim state  $x_e = \begin{bmatrix} u & w & q & \theta & h & x & v & p & r & \phi & \psi & y \end{bmatrix}_e^T$ ,

 $= [191.92\ 18.31\ 1.31\ 0.095\ 6096\ 1.18e-16\ 2.52e-20\ -1.40e-25\ -1.09e-25\ 5.93e-23\ 0\ 0]^T,\ unit: [u\ v\ w]\ (m/s), [p\ q\ r]\ (rad/s), [x\ y\ h]\ (m), [\phi,\theta,\psi](rad).$ 

$$u_e = \begin{bmatrix} \delta_e & \delta_p & \delta_a & \delta_r \end{bmatrix}_e^T = \begin{bmatrix} -0.11 & 0.33 & 5.59e-22 & 1.28e-21 \end{bmatrix}^T, \quad \text{unit: } [\delta_a \delta_e \delta_r] \text{ (rad)}.$$

The position range for the actuators are  $\delta_e \in [-0.35, 0.26]$  (rad),  $\delta_p \in [0, 1]$ ,  $\delta_a \in [-0.65, 0.65]$  (rad),  $\delta_r \in [-0.38, 0.38]$  (rad). The initial position is set to  $\xi_0 = [0, 0, -6096\text{m}]^T$ , with initial body velocity  $V_0 = [0, 0, 0]^T$ , initial attitude  $\eta_0 = [0, 5.5^\circ, 0]^T$ , and initial angular velocity  $\omega_0 = [0, 0, 0]^T$ . For the

command filter (18), the damping ratio is selected as  $\zeta_n = 0.9$ , and natural frequency is selected as  $\omega_n = 10$  rad/s to avoid large maneuver inputs.

Scenario I: Stability augmentation control with different center of gravity

According to stability requirement of the BWB aircraft [7, 8], center of gravity (CG) can move along the longitudinal axis in the range of 29.4m ~34.4m, see Fig.1. To illustrate the proposed method in stability augmentation, the controller parameters are designed as follows:

$$c_1 = \text{diag}([0.1, 0.3, 0.3]), \quad c_2 = \text{diag}([1 \ 1/3 \ 0.4]),$$
 (77)

and the augment stability control gains

$$K_{\alpha} = 0.01, \quad K_{q} = 0.3,$$

the RBF NN parameters are set as

$$\gamma_{\text{W}} = 10, c = [-1\ 0\ 1; -1\ 0\ 1; -1\ 0\ 1; -1\ 0\ 1; -1\ 0\ 1; -1\ 0\ 1]; b_{\text{i}} = 0.002; h(0) = [0.01, 0.01, 0.01]^{\text{T}};$$

the siding mode control parameters are designed as

$$\lambda_1 = 0.1* \text{diag}([1, 1, 1]), \hbar = 0.1* \text{diag}([1, 1, 1]), \varsigma = \text{diag}([1.0 \ 1.2 \ 1.0]), \phi = 0.4,$$

where  $c_1$  and  $c_2$  are in Eq.(25) and Eq.(35),  $K_{\alpha}$  and  $K_{\beta}$  are selected to satisfy the requirements in Section 3 after several design iterations. A doublet command is predefined as the desired attitude to verify the tracking performance of the NNDO-INBSM. An INDI proposed by Lu and Kampen [19] is used to compare the proposed controller performances, where control gains are designed as

$$K_{\eta} = \text{diag}([0.4 \ 0.4 \ 0.4]), \quad K_{\omega} = \text{diag}([22.5 \ 45 \ 1.125]),$$
 (78)

The simulation results are shown in Figs 3-6.

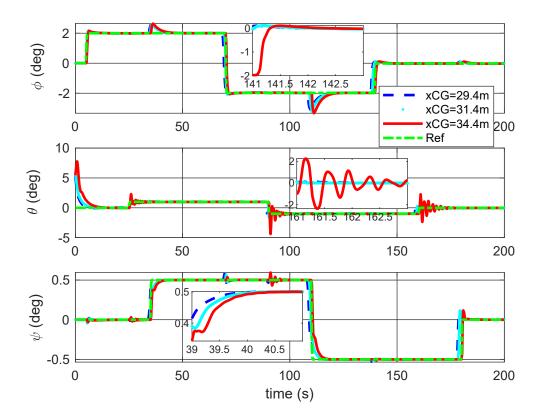


Fig.3. Stability augmentation responses by INBSM control under different CG positions

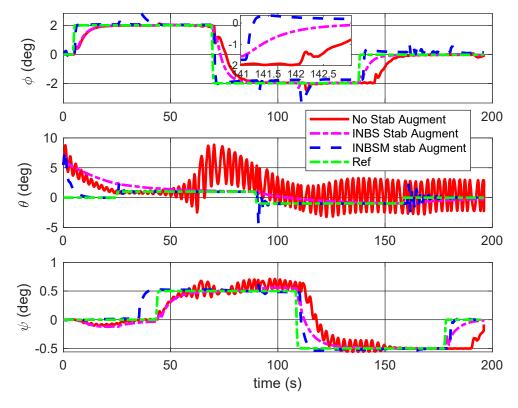


Fig.4. Stability augmentation control responses

From Figs 3 and 4, it can be seen that the responses of the forward CG position converge faster than those of the aftward CG position. When the CG position move backwards to  $x_{CG} = 34.4$ m, which is outside the range of the limitation, see Fig.1, the responses of pitch and yaw motion begin to oscillate and become unstable without augmentation control, see Fig.4. When the stability augmentation control is added, the tracking responses for the Euler angles are stable and converge to steady values, this is because the stability augmentation improves the system damping rate and frequency of pitch motion. Meanwhile, the tracking responses by using INBSM design are faster than those by using INBSM control, this shows the sliding mode control has better adaptive capability. The control inputs of the INBSM design are shown in Fig.5.

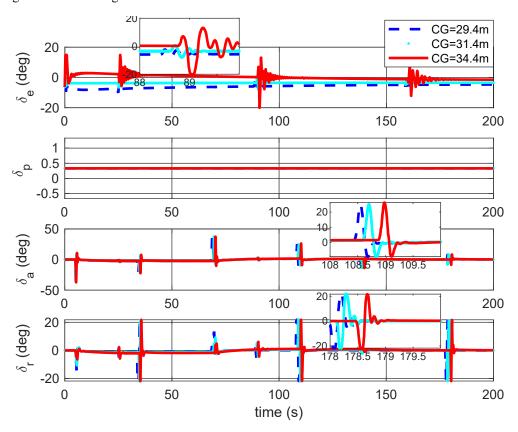


Fig.5. Control inputs of INBSM control under different CG positions

From Fig.5 it can be seen that the further back the CG position, the bigger the elevator inputs. This is because the more aftward the CG position, the more the aircraft is unstable, and it needs more elevator

input to balance pitch moment of aerodynamic forces, where the pitch moment arm of  $|x_{AC} - x_{CG}|$  is bigger (AC denotes aerodynamic center).

Scenario II: Attitude tracking control

Case 1: Unknown disturbances inputs.

In this scenario suppose that there is the unknown disturbance vector  $D_{\omega}$  acting on the BWB aircraft, given by

$$\mathbf{D}_{\omega}(t) = \ \overline{I} \times [0.025 - 0.025 \ 0.015]^{\mathrm{T}}. \tag{79}$$

The gains of the disturbance observer (8) are designed as

$$l(x) = diag([.25, .00022, .0105]).$$
 (80)

To illustrate the proposed method in attitude control, an INBS control proposed by Acquatella [21] and an adaptive fuzzy control referred as in Ref.[35]- Ref.[37] are used to compare, and the controller parameters of the INBS control are designed as Scenario I. For the adaptive fuzzy control, the fuzzy sets are defined as NL, NS, ZO, PS, and PL, which denote negative large, negative small, zero, positive small, positive large, and their center points are selected as  $-\frac{\pi}{6}$ ,  $-\frac{\pi}{12}$ , 0,  $\frac{\pi}{12}$ ,  $\frac{\pi}{6}$ , respectively. The fuzzy membership functions are chosen as

$$\mu_{F_{i,j}}(x) = \exp\left[-\left(\left(x_j + \frac{\pi}{6} - (i-1) \cdot \frac{\pi}{12}\right) / \frac{\pi}{24}\right)^2\right] (i = 1, 2, ..., 5, j = 1, 2, ..., n), \quad (81)$$

Assume function  $f_2(x)$ ,  $g_2(x)$  are unknown, The following fuzzy logic systems are used to approximate them,

$$\hat{f}_{2}(\hat{x} | \hat{\theta}_{f}) = \hat{\theta}_{f}^{T} \xi(\hat{x}), \quad \hat{g}_{2}(\hat{x} | \hat{\theta}_{g}) = \hat{\theta}_{g}^{T} \xi(\hat{x}),$$
 (82)

where  $\hat{x}$  is the estimation of the state x, that is,  $x^T = \begin{bmatrix} \eta^T & \omega^T \end{bmatrix}$ . The fuzzy basis functions

$$\xi_{i}(x) = \frac{\prod_{j}^{n} \mu_{i,j}(x_{j})}{\sum_{i=1}^{N} \left(\prod_{j}^{n} \mu_{i,j}(x_{j})\right)},$$
(83)

$$\hat{f}_2(x_d \mid \hat{\theta}_f) = \hat{\theta}_f^T \xi(x_d), \quad \hat{g}_2(x_d \mid \hat{\theta}_g) = \hat{\theta}_g^T \xi(x_d). \tag{84}$$

~

For each channel of roll, pitch and yaw, select the vector  $K = \begin{bmatrix} k_2 \\ k_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ , such that the following equation holds  $A^TP + PA = -Q$ , where  $A = \begin{bmatrix} 0 & -k_2 \\ 1 & -k_1 \end{bmatrix}$ ,  $Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ , P is a symmetric positive definite matrix. The adaptive laws are  $\dot{\theta}_f = -\gamma_1 e^T P b \xi(x)$ ,  $\dot{\theta}_g = -\gamma_2 e^T P b \zeta(x) u$ ,  $e = x_d - x$ ,  $\gamma_1 = 10$ ,  $\gamma_2 = 1$ ,  $b = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ , the fuzzy control output  $u_F = \frac{1}{\hat{g}_2} \left( -\hat{f}_2 + K^T e + \ddot{x}_{1r} \right)$  as the input of the inner loop of angular rate for nonlinear dynamic inversion control, so the adaptive fuzzy control for the outer loop of attitudes with a nonlinear dynamic inversion control for the inner loop of angular rate is implemented. The parameters of the nonlinear dynamic inversion control are the same as Eq.(78). The simulation results are shown in Figs 6-8.

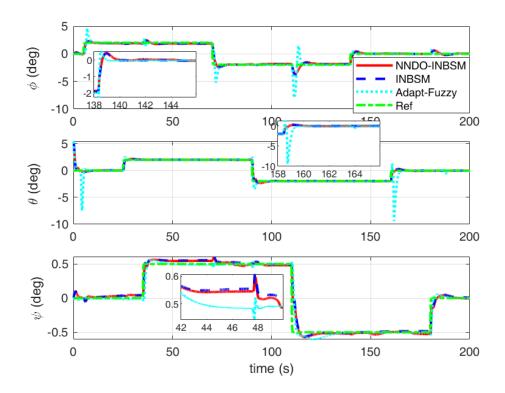


Fig.6. Attitude tracking responses under unknown disturbances

From Fig.6, it can be seen that the three methods can track the desired attitudes, but the adaptive fuzzy control responses have larger overshoots and errors than those of the INBS and NNDO-INBSM design, this is because adaptive fuzzy control requires more control inputs. Meanwhile, there are larger

overshoots of INBSM control than those of NNDO-INBSM control in pitch, roll and yaw motions, this shows NNDO-INBSM can compensate for the observable disturbances or input changes and make their responses smooth. The disturbances have been observed via disturbance observers as shown in Figs 7 and 8.

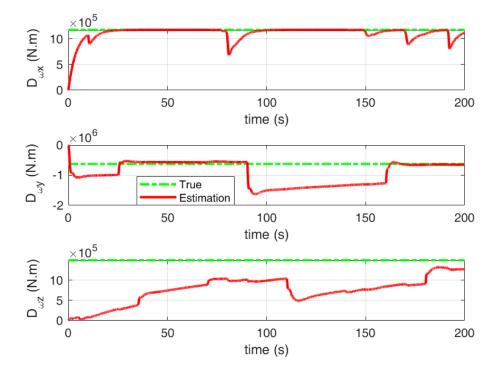


Fig.7. Observable disturbance estimation

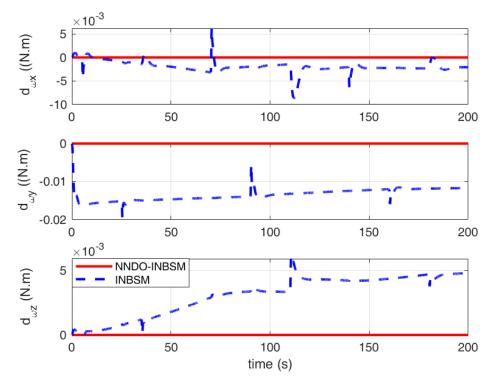


Fig.8. Unobservable disturbance approximation

From Fig.7 it can be seen that roll moment disturbance can be precisely estimated, but yaw and pitch moments have some errors during transition due to the disturbance estimation varying with the associated states, as p(x) = l(x) x in Remark 1, and their steady values converge to the true ones. Fig.8 shows that there are large steady errors of INBSM compared with those of NNDO-INBSM, which demonstrates that NNDO-INBSM can reduce the observable disturbances.

#### Case 2: Model parameter uncertainties with unknown disturbances

This scenario considers model parameter uncertainty due to aerodynamic derivatives varying accordingly with angle of attack (AOA) and side-slip angle. Hence the aerodynamic coefficients  $C_x$  are set with 20% uncertainties  $\Delta C$ =0.2 $C_x$ , where  $\triangle$  denotes the perturbation value. Here the parameter uncertainty and external disturbances are considered simultaneously. The nonlinear disturbance observer gain is selected as l(x) = diag([.13, .0032, .0331]). The simulation results are shown in Fig. 9.

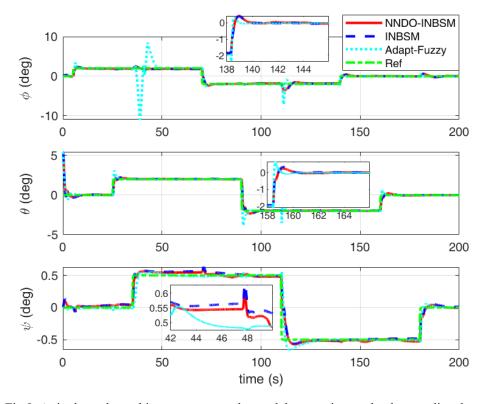


Fig.9. Attitude angle tracking responses under model uncertainty and unknown disturbances From Fig.9, it can be seen that the adaptive fuzzy control responses also have larger overshoots and errors than those of INBSM and NNDO-INBSM design due to the INBSM introducing the  $\dot{x}_{2r}$  feedback and more inputs required for the adaptive fuzzy control. Compared to INBSM, the NNDO-INBSM responses have lower overshoots, which shows that the NNDO-INBSM design has better capability under transient inputs.

#### Scenario III: actuator faults with model uncertainties and disturbances

This scenario considers actuator faults due to damages and loss of effectiveness. The actuator faults are set as follows

$$\delta_i = (1 - \rho_{f_i}) \times \delta_i^0 \quad (i = e, a, r), \tag{85}$$

where  $\rho_{f_i} = 0.1$  is the loss effectiveness factor of each actuator, the faults of the elevator, aileron and rudder occur at times of 50s, 80s, and 120s and remain until the end. Model parameter uncertainties and disturbances are also considered as for Scenario II. The nonlinear disturbance observer gain is

selected as l(x) = diag([.25, .005, .01]), the fault estimation estimator parameters are chosen as  $P_f = \text{diag}([1, 1, 60])$ ,  $\gamma_f = \text{diag}([0.2, 0.02, 0.2])$ . The simulation results are shown in Figs 10-14.

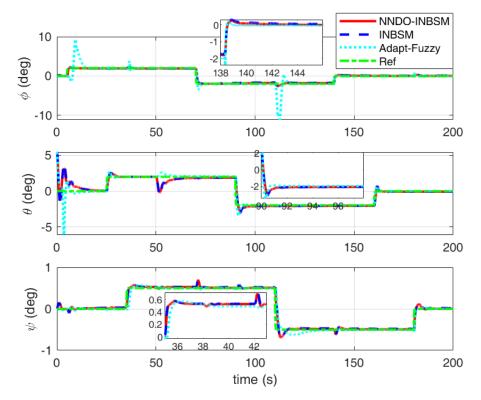


Fig.10. Attitude angle tracking responses under faults and model uncertainties

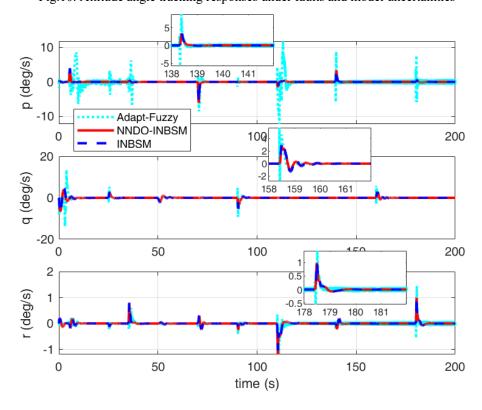


Fig.11. Angular rate responses under faults and model uncertainties

From Figs 10 and 11, it can be seen that the output tracking responses begin oscillating after the actuator faults at 50 and 80sec. The INBSM responses have quicker convergence in roll, pitch and yaw motions than the responses of the adaptive fuzzy control, this shows the INBS design has strong FTC capability. The steady tracking errors for the INBSM and NNDO-INBSM designs are smaller than those of adaptive fuzzy control. Compared with the INBSM control, the tracking responses of the NNDO-INBSM control have smaller overshoots and steady tracking errors, which show that the NNDO-INBSM design has better compensation capability for actuator faults and model uncertainties through the nonlinear disturbance observer. It is noticeable that there will be larger tracking error in both roll and yaw motions if the disturbances cannot be accurately estimated because the  $\dot{x}_{2r}$  feedback will amplify the disturbances or the fault inputs.

The disturbances have been observed via disturbance observers as seen in Figs 12 and 13 where it can be seen that observable roll disturbances can be tracked well while those of pitch and yaw motions have more estimation error in the transient process, this results from state varying and actuator faults, but all observable disturbance estimations converge to their true values even for actuator faults. Fig.13 shows that there are large estimation values of INBSM method compared with those of NNDO-INBSM control, which demonstrates that the NNDO-INBSM design can reduce the observable disturbances.

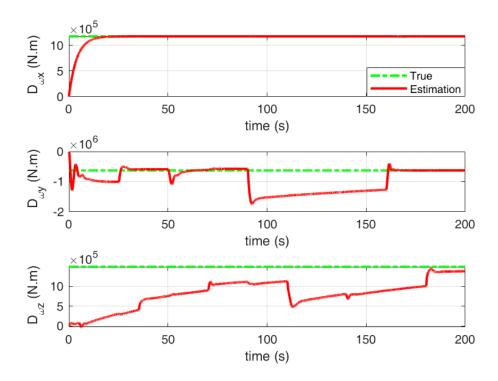


Fig.12. Observable disturbance estimation under faults and model uncertainties

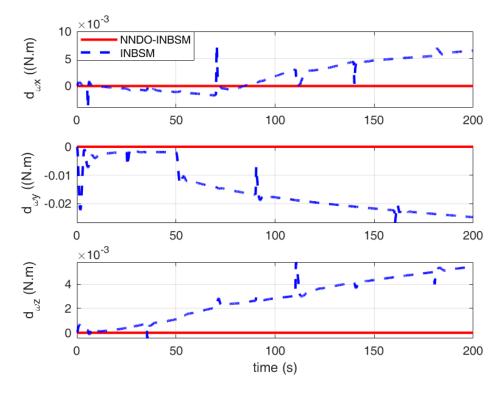


Fig.13. Unobservable disturbance approximation under faults and model uncertainties

The actuator fault estimation and actuator output responses are shown in Fig.14 and Fig.15, from which it can be seen that the actuator angles have been accurately estimated using the fault estimator of (65),

2.4

and the NNDO-INBSM has larger inputs than those of INBSM when commands change or there are input faults. This is because NNDO-INBSM adds a disturbance compensator  $u_{\rm d}$  of (11) and makes the output tracking responses smoother and quicker. Meanwhile, the adaptive fuzzy control inputs are bigger than the INBSM and NNDO-INBSM design, this shows that adaptive fuzzy control requires more control inputs, which makes the actuator saturate more easily when unknown disturbances and faults occur, see Fig.15.

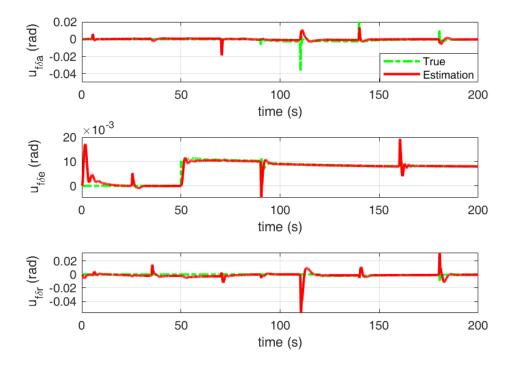


Fig.14. Actuator fault estimation

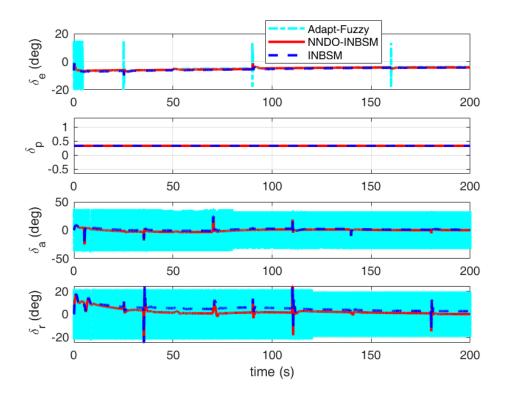


Fig.15. Actuator responses under faults and model uncertainties

# 5. Conclusion

In this paper we propose the NNDO-INBSM approach for the control of the BWB aircraft. Based on a 3-DOF nonlinear model, an attitude tracking controller is designed. The developed controller stabilizes the attitude and angular rate and enhances the stability of the BWB aircraft via the INBS and INDI methods with stability augmentation system. Furthermore, an NNDO is introduced to estimate the observable disturbances. Stability analysis shows that the closed-loop attitude tracking error dynamics is asymptotically stable. Three scenarios of the different CG position, unknown disturbances, model uncertainties and actuator faults have been simulated and show the proposed NNDO-INBSM control robustness. Compared with adaptive fuzzy control and INBSM controllers, the NNDO-INBSM achieves better attitude tracking performances even though the BWB aircraft is affected by a varying CG position, parametric uncertainties, external bounded disturbances and actuator faults. Therefore, the effectiveness and availability of the NNDO-INBSM design are

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demonstrated. Results from the nonlinear simulations confirm that the performance and robustness objectives are achieved. The methodology can be extended to trajectory control.

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# **CRediT** author statement

Roles of the authors,

Shiqian Liu: Methodology, control design and Simulation, Writing- Original draft

James F. Whidborne: Conceptualization, BWB aircraft model, Writing- Reviewing and Editing



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# Declaration of interest statement

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work, Journal Pre-proofs

there is no professional or other personal interest of any nature or kind in

any product, service and /or company that could be construed as

influencing the position presented in, or the review of, the manuscript

entitled.

Best Regards

Shiqian Liu, James F. Whidborne

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