Class-Imbalanced Complementary-Label Learning via Weighted Loss

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Abstract

Complementary-label learning (CLL) is widely used in weakly supervised classification, but it faces a significant challenge in real-world datasets when confronted with class-imbalanced training samples. In such scenarios, the number of samples in one class is considerably lower than in other classes, which consequently leads to a decline in the accuracy of predictions. Unfortunately, existing CLL approaches have not investigate this problem. To alleviate this challenge, we propose a novel problem setting that enables learning from class-imbalanced complementary labels for multi-class classification. To tackle this problem, we propose a novel CLL approach called Weighted Complementary-Label Learning (WCLL). The proposed method models a weighted empirical risk minimization loss by utilizing the class-imbalanced complementary labels, which is also applicable to multi-class imbalanced training samples. Furthermore, we derive an estimation error bound to provide theoretical assurance. To evaluate our approach, we conduct extensive experiments on several widely-used benchmark datasets and a real-world dataset, and compare our method with existing state-of-the-art methods. The proposed approach shows significant improvement in these datasets, even in the case of multiple class-imbalanced scenarios. Notably, the proposed method not only utilizes complementary labels to train a classifier but also solves the problem of class imbalance.

Keywords: weakly supervised learning, complementary labels, class imbalanced, multi-class classification

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1. Introduction

Ordinary supervised classification requires precise ground-truth label of each training instance, making it time-consuming for large-scale datasets. To tackle this problem, weakly supervised learning (WSL) has been increasingly explored in the last decades, which allows training a classifier from less costly data. Previous works in WSL include semi-supervised learning (Chapelle et al., 2006; Tarvainen and Valpola, 2017; Miyato et al., 2018; Izmailov et al., 2020), noisy-label learning (Menon et al., 2015; Ghosh et al., 2017; Ma et al., 2018; Wang et al., 2019; Han et al., 2020), partial-label learning (Tang and Zhang, 2017; Xie and Huang, 2018; Lv et al., 2020; Zhang et al., 2021; Gong et al., 2022), unlabeled-unlabeled learning (Du Plessis et al., 2013; Golovnev et al., 2018; Chapel et al., 2020; Hu et al., 2021; Su et al., 2021; Gerych et al., 2022).

In this paper, we focus on complementary-label learning (CLL) (Ishida et al., 2017; Yu et al., 2018; Ishida et al., 2019; Chou et al., 2020; Feng et al., 2020; Xu et al., 2020a; Gao and Zhang, 2021), another scenario of WSL, which specifies the class label that the training instance does not belong to. Complementary labels are more accessible than ground-truth labels in some domains (Ishida et al., 2017). For instance, compared to labeling an instance with a precise ground-truth label from all candidate labels, a complementary label enables crowdsourced workers to choose one label from a set of incorrect classes, thereby simplifying the labeling process. In *K*-class classification when K > 2, the complementary label is sampled randomly and uniformly from K - 1 class. When an instance is labeled with complementary label *i*, it implies that any one of the ordinary labels $\{1, \ldots, i - 1, i + 1, \ldots, K\}$ could potentially be the ground-truth label.

However, a significant challenge arises due to class imbalance in practical complementary-label learning. For instance, in the medical disease detection (Johnson and Khoshgoftaar, 2019), there are typically more samples from healthy people than from patients, particularly in privacy-sensitive diseases (Kaneko et al., 2019). As illustrated in Figure 1, the DDSM ¹ (Digital Database for Screening Mammography) dataset inherently holds privacy attributes and requires professional knowledge to correctly identify cancer images. Considering the protection of privacy and reducing the difficulty of labeling work, collecting complementary labels may be a more viable alternative to obtaining accurate ground-truth labels. Furthermore, there is an inherent class imbalance in DDSM dataset, wherein a

¹DDSM: http://www.eng.usf.edu/cvprg/Mammography/Database.html



Figure 1: The comparative analysis of complementary label and ground-truth label on DDSM datasets. To protect the privacy of patients, collecting complementary labels may be a more viable alternative to obtaining accurate ground-truth labels. In addition, the DDSM dataset exhibits an imbalanced distribution with normal images constituting 61%, benign images 19%, and cancer images 20%. Here, The term "TL" refers to ground-truth label, whereas "CL" denotes complementary laebl.

significant portion of the dataset is composed of normal images, accounting for approximately 61%. In contrast, only 19% and 20% of the dataset are comprised of benign and cancer images, respectively. This inherent class imbalance presents a challenge for the model to effectively learn from complementary labels. Unfortunately, existing CLL methods have not investigated the issue of learning from class-imbalanced complementary labels, which leads to reduced prediction accuracy, particularly in imbalanced classes. Previous CLL approaches assume a balanced distribution of labeled samples in each class, which is difficult to maintain in real-world datasets(Dong et al., 2018; Liu et al., 2019). As a result, we propose a novel CLL approach to learn from class-imbalanced complementary labels.

Class imbalance is considered to be one of the main reasons for the performance decline (Yang et al., 2009; Kim and Sohn, 2020; Chen et al., 2021). Models trained on class-imbalanced datasets tend to favor the majority class and ignore the minority class. To mitigate this problem, supervised classification rebalance data by undersampling or oversampling it, but this results in a huge lack of information or simply duplication of existing data (Chen et al., 2021; Richhariya and Tanveer, 2020). Alternatively, cost-sensitive learning assigns higher weights to minority classes to minimize overall losses (Yang et al., 2009; Kim and Sohn, 2020; Ganaie et al., 2022b). However, these methods are not suitable for CLL since they rely on ground-truth labels. Therefore, this inspires us to propose a novel CLL approach to learn from class-imbalanced complementary labels.

In this study, we investigate the issue of class imbalance in complementary labels for the first time. Specifically, we propose a novel problem setting characterized by a significant disparity in the number of training samples among certain classes. This phenomenon results in an imbalance of complementary labels, which poses a challenge for accurate classification and prediction of existing CLL. To alleviate this issue, we propose a novel CLL approach, called Weighted Complementary-Label Learning (WCLL), which introduces a weighted loss function into the empirical risk to reduce the overall loss. It is theoretically proved that the proposed method can converge to the optimal solution. Additionally, our proposed method can also be applied to multi-class imbalanced setting. To demonstrate the effectiveness of our approach, we conduct comprehensive experiments on benchmark datasets and compared it with state-of-the-art CLL methods. The following is a summary of the major contributions:

- We propose a novel approach for learning from class-imbalanced complementary labels by introducing a weighted loss term to the empirical risk to minimize the total loss value. This approach not only utilizes complementary labels to train a classifier but also effectively alleviate the issue of class imbalance.
- It is theoretically proved that the classifier learned from the class-imbalanced complementary labels can converge to $O(1/\sqrt{n})$ with the training samples increasing.
- Our experiments show that better performance can be achieved on classimbalanced complementary labels. Furthermore, we believe this study can provide useful guidance for researchers interested in bringing imbalance problems into CLL.

The rest of this paper is structured as follows. Section 2 reports on related work. Section 3 gives formal definitions about ordinary multi-class classification and complementary lables learning. Section 4 presents the proposed CLL approach with theoretical analyses and algorithmic specifics. Section 5 reports the results of the comparative experiment. Finally, Section 6 concludes this paper.

2. Literature Review

In this section, we provide an overview of the relevant references, which are mainly categorized into two themes: class imbalance and complementary labels.

2.1. Class Imbalance

Real-world data typically exhibits a class-imbalanced label distribution, which hinders the generalizability of machine learning models (Yang et al., 2009; Dong et al., 2018; Liu et al., 2019). To alleviate this issue, various algorithms have been proposed. The most commonly used strategy is to re-balance the training samples (Buda et al., 2018). Existing approaches can be roughly divided into three categories.

The first directly balances the sampling distribution at the data level by oversampling the minority class or under-sampling the majority class, or both (He and Garcia, 2009; Byrd and Lipton, 2019; Guo and Li, 2022). Richhariya and Tanveer (2020) and Ganaie et al. (2022a) utilized universum data to re-balance the classimbalanced data, but this method is not effective for highly imbalanced data due to the loss of substantial information.

The second alleviates the imbalance problem from the model side. Chen et al. (2021) embedded ensemble learning into deep convolutional neural networks by attaching multiple auxiliary classifiers to different layers of the CNN model. Ganaie et al. (2022b) proposed a large-scale fuzzy least squares twin support vector machines to solve the overfitting problem and avoid the operation of matrix inverse, but this method is only applicable to binary classification.

The third modifies the loss function by assigning higher costs to examples from minority classes (Yang et al., 2009; Kim and Sohn, 2020). Yang et al. (2009) utilized an inverse proportional regularization penalty to reweight unbalanced classes. Kim and Sohn (2020) proposed a hybrid neural network with a cost-sensitive support vector machine to tackle class imbalance, but this method is only suitable for multimodal data. Fernando and Tsokos (2021); van der Meer et al. (2022) proposed a rebalancing strategy that utilizes a dynamic weighted loss function, which assigns weights based on class frequency and prediction probability. Zhang et al. (2019) proposed a recommended method for deep neural network learning based on item embedding and weighted loss function. de La Torre et al. (2018) discussed the application of the weighted Kappa loss function in multiclass classification with ordinal data.

These three distinct approaches provide solutions to alleviate class imbalance from varying perspectives, thereby offering valuable guidance for this issue in CLL. However, these methods rely on ground-truth labels, which is unsuitable for direct application to CLL scenarios.

2.2. Complementary labels

Recently, complementary labels learning (CLL) has been widely used in our daily lives (Kaneko et al., 2019; Xu et al., 2020b; Rezaei et al., 2020), particularly in data privacy. For certain private questions (Ishida et al., 2017; Yu et al., 2018), eliminating false responses can protect the correct one. However, the utilization of complementary labels presents a challenge, as complementary labels are less informative than ordinary labels. Previous research has explored various methods to alleviate this issue, which can be categorized into two branches.

The first branch aims to model the relationship between the complementary label \bar{y} and the ground-truth label y. Ishida et al. (2017) proposed an unbiased risk estimator (URE) with the assumption that the relationship is unbiased and provided theoretical analysis. However, this approach is only effective for specific loss functions. To alleviate this problem, Ishida et al. (2019) proposed a general URE framework for arbitrary loss functions. In contrast, Yu et al. (2018) used a multi-class Cross-Entropy loss function to solve CLL tasks with the assumption that the relationship is biased, which additionally requires a set of anchor instances for transition probability estimation.

Methods	Complementarys label used	Loss assump. free	Model assump. free	Class imbanlance
Ishida et al. (2017)	\checkmark	×	\checkmark	×
Yu et al. (2018)	\checkmark	×	×	×
Ishida et al. (2019)	\checkmark	\checkmark	\checkmark	×
Gao and Zhang (2021)	\checkmark	\checkmark	\checkmark	×
Ishiguro et al. (2022)	\checkmark	\checkmark	\checkmark	×
Gao et al. (2023)	\checkmark	\checkmark	\checkmark	×
Proposed (WCLL)	✓	\checkmark	✓	✓

Table 1: Comparison of the proposed methods with previous works

The second branch directly models from the output of classifiers. Gao and Zhang (2021) proposed a discriminative model that directly models from the output of trained classifiers. The goal is to make the predictive probability of the complementary label approach zero. In addition, there are some interesting setting

within the domain of complementary label learning. For instance, Ishiguro et al. (2022) explored the introduction of noise in the process of complementary label learning and proposed robust loss functions to learn from noisy complementary labels. Gao et al. (2023) conducted a comprehensive study on learning a classifier specifically designed for multiple labeled complementary labeling instances.

Unfortunately, existing CLL approaches do not take into account the problem of class imbalance. Therefore, there is an urgent need to investigate a novel approaches to alleviate this problem. In this regard, we propose a novel CLL approach, called Weighted Complementary-Label Learning, which not only utilizes complementary labels but also effectively alleviate the issue of class imbalance.

3. Formulation

In this section, we give notations and review the formulations of ordinary multi-class classification and complementary-label learning.

3.1. Ordinary Multi-Class Classification

Let $\mathcal{X} \subset \mathbb{R}^d$ denotes the feature space, and $\mathcal{Y} = \{1, 2, \ldots, K\}$ denotes the label space. Consider a random sample $(\mathbf{x}, y) \sim P(\mathbf{x}, y)$ drawn from a joint probability distribution over \mathcal{X} and \mathcal{Y} . Let $\mathcal{D}_L = \{(\mathbf{x}_i, y_i)\}_{i=1}^{N_l}$ denotes the set of training samples, which are sampled independently and identically from $P(\mathbf{x}, y)$. In ordinary multi-class classification, each instance is associated with a groundtruth label. The goal is to learn a classifier that predicts the probability of the class label, i.e., $P(y|\mathbf{x})$, and establishes a decision function $f: \mathbf{x} \to \{1, 2, \ldots, K\}$. Here, f denotes an arbitrary function, and $f_i(\mathbf{x})$ denotes the *i*-th element of $f(\mathbf{x})$. The classification risk for the decision function f is defined with respect to a loss function ℓ as:

$$R(f;\ell) = \mathbb{E}_{(\mathbf{x},y)\sim P(\mathbf{x},y)}[\ell(f(\mathbf{x}),y)],\tag{1}$$

where \mathbb{E} refers to the expectation. The approximating empirical risk is defined as:

$$\hat{R}(f;\ell) = \frac{1}{N_l} \sum_{i=1}^{N_l} \ell(f(\mathbf{x}_i), y_i),$$
(2)

where N_l refers to the number of training samples.

3.2. Learning from Complementary Labels

In contrast to ordinary multi-class classification, complementary labels learning (CLL) assigns a complementary label to each instance, which specifies the label that the instance does not belong to. Let $\bar{\mathcal{Y}} = \{1, 2, \dots, K\}$ denotes the complementary label space. Considering a joint probability distribution $\bar{P}(\mathbf{x}, \bar{y}) \neq$ $P(\mathbf{x}, y)$, where (\mathbf{x}, \bar{y}) denotes a training instance with its corresponding complementary label \bar{y} . It is important to note that Ishida et al. (2017) proposed a URE framework for CLL, assuming an unbiased relationship between \bar{y} and y. The classification risk can be defined as follows:

$$R(f;\ell) = \mathbb{E}_{(\boldsymbol{x},\bar{y})\sim\bar{P}(\boldsymbol{x},\bar{y})}[\bar{\ell}(f(\boldsymbol{x}),\bar{y})].$$
(3)

In addition, Gao and Zhang (2021) modelled directly from the classifier's output by using $\bar{f}(\mathbf{x}) = 1 - f(\mathbf{x})$. The loss function $\bar{\ell}$ can be written as:

$$\bar{\ell}(f(\boldsymbol{x}), \bar{y}) = \ell(1 - f(\boldsymbol{x}), y).$$
(4)

Therefore, the risk of CLL classification can be expressed as:

$$R(f;\ell) = \mathbb{E}_{(\mathbf{x},\bar{y})\sim\bar{P}(\mathbf{x},\bar{y})}[\ell(1-f(\mathbf{x}),y)].$$
(5)

4. Method

This section introduces the problem setting and proposes the weighted model. Additionally, we also derive the estimation error bound of the proposed method to provide a theoretical guarantee.

4.1. Problem Setting

4.1.1. Notation

Let $\mathcal{X} \subset \mathbb{R}^d$ denotes the feature space, and $\overline{\mathcal{Y}} = \{1, 2, \dots, K\}$ denotes the complementary label space. Let $\overline{\mathcal{D}}_L = \{(\mathbf{x}_i, \overline{y}_i)\}_{i=1}^{N_l}$ be sampled from $\overline{P}(\mathbf{x}, \overline{y})$ at an imbalanced scale α , where N_l denotes the number of training samples. The dataset $\overline{\mathcal{D}}_L$ is composed of two subsets: $\overline{\mathcal{D}}_{min}$ and $\overline{\mathcal{D}}_{maj}$, where $\overline{\mathcal{D}}_{min} = \{(\mathbf{x}, \overline{y}) \mid \overline{y} \in \mathcal{T}_{maj}\}$. Note that $\overline{\mathcal{D}}_{min} \cup \overline{\mathcal{D}}_{maj} = \overline{\mathcal{D}}_L, \overline{\mathcal{D}}_{min} \cap \overline{\mathcal{D}}_{maj} = \emptyset$ and $N_{min} \ll N_{maj}$. Moreover, let $\mathcal{T}_{min} = \{\overline{y} \mid (\mathbf{x}, \overline{y}) \in \overline{\mathcal{D}}_{min}\}$ and the number of each class from \mathcal{T}_{min} is equal, i.e., $N_i = N_j, \forall i \neq j, i, j \in \mathcal{T}_{min}$. Similarly, let $\mathcal{T}_{maj} = \{\overline{y} \mid (\mathbf{x}, \overline{y}) \in \overline{\mathcal{D}}_{maj}\}$ and $N_i = N_j, \forall i \neq j, i, j \in \mathcal{T}_{maj}$. Let p be the class-imbalanced proportion, which denotes the radio between the number of samples in the classes from \mathcal{T}_{maj} and \mathcal{T}_{min} , i.e., $p = \frac{N_i}{N_i}, i \in \mathcal{T}_{maj}, j \in \mathcal{T}_{min}$.

4.1.2. Class-Imbalanced Setup

During the training phase, the dataset \overline{D}_L is sampled from $\overline{P}(\mathbf{x}, \overline{y})$ at an imbalanced scale denoted by the vector $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_i\}$, where α_i denotes the number of samples belonging to the *i*-th class. In this study, the value of α_i is defined as follows:

$$\alpha_{i} = \begin{cases} N_{l} * \frac{1}{crad(\mathcal{T}_{min}) + p * crad(\mathcal{T}_{maj})}, & i \in \mathcal{T}_{min} \\ N_{l} * \frac{p}{crad(\mathcal{T}_{min}) + p * crad(\mathcal{T}_{maj})}, & i \in \mathcal{T}_{maj} \end{cases},$$
(6)

where $crad(\mathcal{T}_{min})$ denotes the size of \mathcal{T}_{min} and $crad(\mathcal{T}_{maj})$ denotes the size of \mathcal{T}_{maj} . As mentioned above, a complementary label specifies the class label that the instance does not belong to. In our setting, the number of complementary labels from \mathcal{T}_{min} is more than other labels from \mathcal{T}_{maj} . We assume that the number of complementary labels for each class is inversely proportional to the number of samples in that class, i.e., the proportion of complementary label for the *j*-th class, denoted as π_j , can be defined as follows:

$$\pi_j = \frac{1/\alpha_j}{\sum_{i=1}^K 1/\alpha_i}.$$
(7)

During the training phase, we utilize class-imbalanced complementarily labeled data. While in the testing phase, we utilize class-balanced data, which also employed by previous CLL approaches. Our goal is to derive an optimal decision function f to minimize the empirical risk, i.e., $f^* = argmin\hat{R}(f, \ell)$.

4.2. The Weighted Model

Given this novel setting, the data at hand is imbalanced, which increases the difficulty of CLL multi-class classification tasks. As previously mentioned, the classification risk of the proposed CLL setting can be described as:

$$R(f;\ell) = \mathbb{E}_{(\mathbf{x},\bar{y})\sim\bar{P}(\mathbf{x},\bar{y})}[\ell(f(\mathbf{x}),\bar{y})],\tag{8}$$

where ℓ denotes the complementary loss. The imbalanced sampling distribution creates an imbalance in the generated complementary labels, thereby making it challenging to minimize the total loss due to the increasing class loss with an increasing number of class samples. Additionally, data from minor classes may be characterized as noisy labels, which makes the classifier training process become



Figure 2: The training process of the proposed method. The proposed method involves computing weights for the class-imbalanced dataset and incorporating them into the complementary labeled losses to achieve balanced class losses. During training, our method only use complementary label to train the neural network.

more challenging. In order to boost the performance of the trained classifier, we propose an adjustment to the loss function to rebalance the sampling distribution. Specifically, we introduce a weighted loss term for each class sample to minimize the total loss value to complementary loss, as defined below:

$$\hat{\ell}(f(\boldsymbol{x}), \bar{y}) = \omega \bar{\ell}(f(\boldsymbol{x}), \bar{y}), \tag{9}$$

where ω denotes the K-dimensional loss weight vector for K-class, which is inversely proportional to the percentage of complementary labels. In accordance with the proposed problem setting, the complementary loss $\hat{\ell}$ can be expressed as:

$$\hat{\ell}(f(\boldsymbol{x}), j) = -(K-1)\omega_j \ell(f(\boldsymbol{x}), j) + \sum_{i=1}^{K} \omega_i \ell(f(\boldsymbol{x}), i),$$
(10)

where $j \in \{1, ..., K\}$, and ω_j denotes the weight assigned to the *j*-th class.

By using (10), the classification risk can be described as:

$$R(f;\ell) = \sum_{j=1}^{K} \pi_j \mathbb{E}_{(\mathbf{x},\bar{y})\sim\bar{P}(\mathbf{x},\bar{y})} [-(K-1)\omega_j \ell(f(\mathbf{x}),j) + \sum_{i=1}^{K} \omega_i \ell(f(\mathbf{x}),i)].$$
(11)

Algorithm 1 Weighted Complementary-Label Learning

Input:

The set of complementarily labeled training instances, $\{X_i\}, i$ \in $\{1, 2, \ldots, K\}$, where X_i denotes the samples labeled as class *i*, which means the class label that the instance does not belong to;

The number of epochs, T;

An external stochastic optimization algorithm, A;

Output:

model parameter θ for $f(\mathbf{x}, \theta)$;

- 1: **for** t = 1 to T **do**
- Shuffle $\{X_i\}$ into B mini-batches; 2:
- Calculate ω_i accoding to equation (13); 3:
- for b = 1 to B do 4:

5: Denotes
$$r_i^b(\theta) = -(K-1)\omega_i\pi_i\ell(f(\mathbf{x}_b),i) + \sum_{j=1}^{K}\omega_j\pi_j\ell(f(\mathbf{x}_b),j);$$

- Denotes $L^{b}(\theta) = \sum_{i=1}^{K} r_{i}^{b}(\theta)$ Set gradient $\bigtriangledown_{\theta} L^{b}(\theta)$ 6:
- 7:
- Update θ by \mathcal{A} 8:
- 9: end for

10: end for

where π_j denotes the base rate of the *j*-th class. Since the training dataset $\overline{\mathcal{D}}_L$ is sampled from $P(\mathbf{x}, \bar{y})$, the expression (11) can be approximated by

$$\hat{R}(f;\ell) = \frac{1}{N_l} \{ \sum_{z=1}^K \pi_z \sum_{i=i}^{N_z} [-(K-1)\omega_z \ell(f(\boldsymbol{x}_i), z) + \sum_{j=1}^K \omega_j \ell(f(\boldsymbol{x}_i), j)] \}.$$
 (12)

Here, we set $\omega = [\omega_1, \omega_2, \dots, \omega_j]$. ω_j denotes the *j*-th element of ω and can be defined as follows:

$$\omega_j = \frac{1/\pi_j}{\sum_{i=1}^K 1/\pi_i},$$
(13)

It is important to note that the weights in the proposed model satisfy the condition $\sum_{j=1}^{K} \omega_j = 1$ and $\omega_j \ge 0$. The incorporation of these weighted losses allows the model to effectively learn from training data with limited samples. Consequently, this approach is expected to enhance both the classification accuracy of imbalanced classes and the overall performance of the model. To provide a comprehensive understanding of the proposed method, Algorithm 1 illustrates the overall algorithmic procedure, while Figure 2 illustrates the training process.

4.3. Estimation Error Bound

Here, we analyze the generalization estimation error bound for the proposed method. Let f be the classification vector function in the hypothesis set \mathcal{F} . Let $\ell[f(\mathbf{x}), i] \leq L_f, \pi_i \leq \pi_K, \omega_j \leq \omega_K, i, j \in \{1, 2, \dots, K\}$. Using L_{ϕ} to denote the Lipschitz constant (Mohri et al., 2018) of ϕ , we can establish the following lemma.

Lemma 1. For any $\delta > 0$, with the probability at least $1 - \delta/2$, we have

$$\sup_{f \in \mathcal{F}} |\hat{R}(f) - R(f)| \leq 2\omega_K L_{\phi}[\pi_K(K-1) + K] \Re_{N_l}(\mathcal{F}) + \sqrt{\frac{I^2 \log \frac{4}{\delta}}{2N_l}}, \quad (14)$$

where $R(f) = \mathbb{E}_{\bar{P}}$, $\hat{R}(f)$ denotes the empirical risk estimator of R(f), and $\mathfrak{R}_{N_l}(\mathcal{F})$ are the Rademacher complexities (Mohri et al., 2018) of \mathcal{F} for the sampling of size N_l from $\bar{P}(\mathbf{x}, \bar{y})$, $I = 2\omega_K L_f[\pi_K(K-1) + K]$.

The proof is given in Appendix. Based on Lemma 1, the estimation error bound can be expressed as follows.

Theorem 1. For any $\delta > 0$, with the probability at least $1 - \delta/2$, we have

$$R(\hat{f}) - R(f^*) \leqslant 4\omega_K L_{\phi}[\pi_K(K-1) + K] \Re_{N_l}(\mathcal{F}) + \sqrt{\frac{2I^2 \log\frac{4}{\delta}}{N_l}}, \qquad (15)$$

where \hat{f} denotes the trained classifier, $R(f^*) = \min_{f \in \mathcal{F}} R(f)$.

The proof is provided in the Appendix. Lemma 1 and Theorem 1 show that our method exists an estimation error bound. With the deep network hypothesis set \mathcal{F} fixed, we can establish that $\Re_{N_l}(\mathcal{F}) = \mathcal{O}(1/\sqrt{N_l})$. Consequently, as $N_l \longrightarrow \infty$, the equation $R(\hat{f}) = R(f^*)$ holds, which prove that our method could converge to the optimal solution.

5. Experiments

This section conducts a comparative study to assess the performance of the proposed method in comparison to state-of-the-art approaches. The proposed method is short for WCLL, which incorporates a class-imbalanced complementary loss function defined in Eq.(12). The mini-batch size and number of epochs were set to 256 and 100, respectively. During the training phase, class-imbalanced complementarily labeled samples generated in the previous section were employed, while class-balanced complementarily labeled data was used for testing phase. All experiments are implemented using PyTorch.



Figure 3: Experiments results of test classification accuracy on the MNIST and CIFAR-10 datasets for 5 trials. The dark colors show the mean accuracy of the imbalanced class and the light colors show the standard deviation of the imbalanced class. Here, p denotes the denotes the number of radio between the class from \mathcal{T}_{maj} and \mathcal{T}_{min} . Im-class refers to the imbalance class. The proposed method has the highest accuracy on imbalanced class.

5.1. Experimental Setting

Datasets: Consistent with previous studies on CLL (Ishida et al., 2017, 2019; Feng et al., 2020; Gao and Zhang, 2021), we evaluate our proposed method on widely-used benchmark datasets, including MNIST (LeCun et al., 1998), CIFAR-10 (Torralba et al., 2008), and Tiny-Imagenet. A summary of the key statistics for each dataset is presented below:

- The MNIST dataset is a handwritten digits dataset, which is composed of 10 classes. Each sample is a 28 × 28 grayscale image. The MNIST dataset has 60k training examples and 10k test examples.
- The CIFAR-10 dataset has 10 classes of various objects: airplane, automobile, bird, cat, etc. This dataset has 50k training samples and 10k test samples and each sample is a colored image in 32 × 32 × 3 RGB formats.
- The Tiny-Imagenet dataset has 200 classes. Each class has 500 training images, 50 validation images, and 50 test images.

Compared Methods: We consider seven current state-of-the-art approaches, which include the following:

- PC(Ishida et al., 2017): For the first time, *Ishida et al.* proposed the concept of complementary labels and an unbiased estimator for learning from them. However, this method is only effective for specific loss functions.
- NN(Ishida et al., 2019): A non-negative risk estimator to overcome overfitting issue in CLL.
- FREE(Ishida et al., 2019): An unbiased estimator for learning from complementary labels without limitations of models and loss functions.
- LOG(Feng et al., 2020): *Feng et al.* proposed a multiple complementarylabel learning method. *Feng et al.* treated multiple complementary labels as a whole, and LOG is an upper-bound surrogate loss function of MAE.
- EXP (Feng et al., 2020): EXP is also a multiple complementary label learningmethod proposed by *Feng et al.* and EXP is the another upper-bound surrogate loss function of MAE.
- LW(Gao and Zhang, 2021): *Gao et al.* employed the highly confident predictions in the early stage of learning to boost the performance of succeeding updating of the model. They introduced a weighted loss term to minimize the loss value in CLL.
- L-UW(Gao and Zhang, 2021): Gao et al. proposed the discriminative model that directly estimates $P(\bar{y} \mid x)$ from the output of classifier. They define the prediction probability of complementary label as $\bar{f}(x) = 1 f(x)$.

- NCLL(Ishiguro et al., 2022): *Ishiguro et al.* analyzed the problem setting where complementary labels may be affected by label noise. They derived conditions for the loss function, ensuring that the learning algorithm remains robust against noise in complementary labels.
- ML_CLL(Gao et al., 2023): *Gao et al.* proposed an unbiased risk estimator with an estimation error bound to learn a multi-labeled classifier from complementary labeled data.

5.2. Comparison of Single Class Imbalance

Setup: For MNIST, we conduct experiments to validate the superiority of our method by introducing imbalanced treatment on class 0, class 1, and class 2. Specifically, we considered three scenarios: 1) $\mathcal{T}_{min} = \{0\}, \mathcal{T}_{maj} = \{1, 2, ..., K\};$ 2) $\mathcal{T}_{min} = \{1\}, \mathcal{T}_{maj} = \{0, 2, ..., K\};$ and 3) $\mathcal{T}_{min} = \{2\}, \mathcal{T}_{maj} = \{0, 1, 3, ..., K\}.$ A linear model is trained on this dataset with imbalanced proportion p set to 2 and 2.5. We fix weight decay to 1e - 4 and select the learning rate from $\{5e - 4, 1e - 4, 5e - 5, 5e - 6\}$ for Adam optimizer.

For CIFAR-10 and Tiny-Imagenet, we set the value of K to 5 and utilized only the data from the first five classes of the original training set, as well as the test set. We conduct imbalanced treatment on class 0, class 1, and class 2 individually. Th e proposed method employed a simple CNN network comprising a 9-layer convolutional network architecture. Each convolutional layer was followed by a Batch Normalization layer and a ReLU layer, with max pooling and drop-out layers applied after every three convolutions. The network concluded with a fully connected layer. The weight decay was set to 1e - 4, and the learning rate was chosen from the set $\{5e - 5, 5e - 6\}$.

Results: In Figure 3, we present the results of the imbalanced class for PC, LW, and the proposed method. As depicted in Figure 3, our method demonstrates a significantly higher average test classification accuracy compared to the PC and LW methods. Additionally, our method exhibits a smaller standard deviation in test classification accuracy, suggesting its stability compared to other methods. Notably, the proposed method maintains consistent performance when the imbalance proportion changes from 2 to 2.5 on the MNIST dataset, while the accuracy of the NN and PC methods slightly decreases on the imbalanced class.

Table 2 presents the classification accuracy for all methods across all classes, based on five trials conducted on various datasets. In view of the suboptimal performance exhibited by NCLL and ML_CLL algorithms when applied to the MNIST dataset, this study exclusively presents their outcomes exclusively on the

Dataset	Р	Class	Method	Imbalanced class = 1	Imbalanced $class = 2$	Imbalanced class = 3
MAUCT	2	1~10	PC (Ishida et al., 2017) NN (Ishida et al., 2019) FREE (Ishida et al., 2019) EXP (Feng et al., 2020) LW (Gao and Zhang, 2021) L-UW (Gao and Zhang, 2021) WCLL (our)		$\begin{array}{c} 69.73 \pm 3.77 \\ 34.15 \pm 1.04 \\ 59.91 \pm 1.41 \\ 30.04 \pm 2.98 \\ 70.14 \pm 3.02 \\ 66.91 \pm 4.25 \\ \textbf{70.55} \pm \textbf{1.44} \end{array}$	$\begin{array}{c} 67.54 \pm 1.64 \\ 35.78 \pm 0.81 \\ 59.88 \pm 0.76 \\ 30.65 \pm 7.32 \\ 69.71 \pm 2.05 \\ 67.04 \pm 6.25 \\ \textbf{71.07} \pm \textbf{1.72} \end{array}$
MINIST	2.5	1~10	PC (Ishida et al., 2017) NN (Ishida et al., 2019) FREE (Ishida et al., 2019) EXP (Feng et al., 2020) LW (Gao and Zhang, 2021) L-UW (Gao and Zhang, 2021) WCLL (our)	$\begin{array}{c} 65.60 \pm 2.18 \\ 30.50 \pm 1.13 \\ 58.35 \pm 1.64 \\ 30.41 \pm 6.65 \\ 63.10 \pm 3.81 \\ 66.91 \pm 4.56 \\ \textbf{71.40} \pm \textbf{1.62} \end{array}$	71.34 ± 0.83 30.68 ± 1.24 58.53 ± 1.68 28.09 ± 4.80 64.49 ± 5.64 69.19 ± 5.26 72.13 ± 1.33	$\begin{array}{c} 66.68 \pm 1.78 \\ 31.76 \pm 1.39 \\ 59.16 \pm 1.64 \\ 29.45 \pm 3.71 \\ 57.50 \pm 5.20 \\ 68.50 \pm 3.05 \\ \textbf{73.01} \pm \textbf{1.05} \end{array}$
CIFAR-10	5	1~5	PC (Ishida et al., 2017) NN (Ishida et al., 2019) FREE (Ishida et al., 2019) LOG (Feng et al., 2020) EXP (Feng et al., 2020) LW (Gao and Zhang, 2021) L-UW (Gao and Zhang, 2021) NCLL (Ishiguro et al., 2022) ML_CLL (Gao et al., 2023) WCLL (our)	$\begin{array}{c} 54.11 \pm 0.61 \\ 47.67 \pm 0.63 \\ 48.15 \pm 2.70 \\ 57.50 \pm 0.95 \\ 57.70 \pm 0.99 \\ 53.40 \pm 0.96 \\ 56.59 \pm 0.82 \\ 49.16 \pm 1.67 \\ 55.43 \pm 0.39 \\ \textbf{58.02 \pm 0.70} \end{array}$	$\begin{array}{c} 54.97 \pm 1.01 \\ 44.84 \pm 1.12 \\ 49.43 \pm 1.80 \\ 57.16 \pm 0.54 \\ 57.06 \pm 0.56 \\ 53.61 \pm 1.15 \\ 56.42 \pm 0.89 \\ 43.32 \pm 0.18 \\ \textbf{59.44} \pm \textbf{0.42} \\ 56.54 \pm 0.74 \end{array}$	$\begin{array}{c} 59.08 \pm 0.33 \\ 55.07 \pm 0.88 \\ 54.64 \pm 1.32 \\ 62.74 \pm 0.27 \\ \textbf{62.85} \pm \textbf{0.17} \\ 59.60 \pm 0.26 \\ 61.42 \pm 0.17 \\ 57.34 \pm 0.94 \\ 61.84 \pm 0.10 \\ 58.88 \pm 0.41 \end{array}$
Tiny- Imagenet	10	$1 \sim 5$	PC (Ishida et al., 2017) NN (Ishida et al., 2019) FREE (Ishida et al., 2019) LOG (Feng et al., 2020) EXP (Feng et al., 2020) LW (Gao and Zhang, 2021) L-UW (Gao and Zhang, 2021) NCLL (Ishiguro et al., 2022) ML_CLL (Gao et al., 2023) WCLL (our)	$\begin{array}{c} 51.22 \pm 1.02 \\ 43.69 \pm 2.30 \\ 49.40 \pm 2.12 \\ 49.43 \pm 3.08 \\ 48.78 \pm 3.16 \\ 51.19 \pm 2.47 \\ 51.45 \pm 1.27 \\ 43.29 \pm 0.20 \\ 43.99 \pm 0.30 \\ \textbf{52.55 \pm 1.33} \end{array}$	$\begin{array}{c} 50.66 \pm 2.17 \\ 46.11 \pm 0.87 \\ 48.44 \pm 2.71 \\ 52.84 \pm 1.55 \\ 52.63 \pm 2.27 \\ 51.84 \pm 1.93 \\ 52.86 \pm 1.64 \\ 43.60 \pm 0.71 \\ 48.40 \pm 0.70 \\ \textbf{53.15} \pm \textbf{1.63} \end{array}$	$\begin{array}{c} 55.41 \pm 1.86 \\ 48.42 \pm 1.85 \\ 51.54 \pm 3.88 \\ 59.76 \pm 1.17 \\ \textbf{60.20} \pm \textbf{0.91} \\ 56.63 \pm 1.34 \\ 58.19 \pm 1.33 \\ 48.91 \pm 0.80 \\ 48.80 \pm 0.30 \\ 57.31 \pm 1.30 \end{array}$

Table 2: Test classification accuracy of total class for 5 trials with mean and standard deviation (mean \pm std %) for MNSIT, CIFAR-10 and Tiny-Imagenet dataset. Here, the training data at hand is single class imbalanced. The best performance is shown in bold.

Dataset	Р	Class	Method	Imbalanced $class = 1, 2$	Imbalanced $class = 1, 3$	Imbalanced $class = 2, 3$
MNIST	2.5	1~10	PC (Ishida et al., 2017) NN (Ishida et al., 2019) FREE (Ishida et al., 2019) EXP (Feng et al., 2020) LW (Gao and Zhang, 2021) L-UW (Gao and Zhang, 2021) WCLL (our)		$\begin{array}{c} 57.51 \pm 1.62 \\ 24.58 \pm 1.27 \\ 48.50 \pm 1.49 \\ 29.23 \pm 5.72 \\ 62.83 \pm 1.34 \\ 61.25 \pm 4.06 \\ \textbf{68.76} \pm \textbf{1.82} \end{array}$	
CIFAR-10	5	$1 \sim 5$	PC (Ishida et al., 2017) NN (Ishida et al., 2019) FREE (Ishida et al., 2019) LOG (Feng et al., 2020) EXP (Feng et al., 2020) LW (Gao and Zhang, 2021) L-UW (Gao and Zhang, 2021) NCLL (Ishiguro et al., 2022) ML_CLL (Gao et al., 2023) WCLL (our)	$\begin{array}{c} 43.53 \pm 0.87 \\ 32.19 \pm 0.43 \\ 42.02 \pm 2.06 \\ 47.07 \pm 1.35 \\ 46.77 \pm 1.97 \\ 42.70 \pm 0.99 \\ 43.86 \pm 1.95 \\ 29.90 \pm 1.70 \\ 43.98 \pm 0.96 \\ \textbf{50.99} \pm \textbf{1.00} \end{array}$	$\begin{array}{c} 49.82 \pm 0.86 \\ 43.73 \pm 0.72 \\ 45.71 \pm 1.42 \\ 52.48 \pm 1.05 \\ 52.16 \pm 1.35 \\ 48.45 \pm 1.05 \\ 50.53 \pm 0.99 \\ 44.66 \pm 0.12 \\ 51.66 \pm 0.10 \\ \textbf{54.82} \pm \textbf{0.70} \end{array}$	$\begin{array}{c} 49.85 \pm 0.93 \\ 41.62 \pm 1.03 \\ 44.06 \pm 1.12 \\ 53.60 \pm 0.52 \\ 52.68 \pm 1.65 \\ 47.81 \pm 1.32 \\ 50.18 \pm 1.20 \\ 41.83 \pm 0.05 \\ 51.94 \pm 0.34 \\ \textbf{53.80} \pm \textbf{0.80} \end{array}$

Table 3: Test classification accuracy (mean \pm std %) of total class for 10 trials. Here, the training data at hand is two class imbalanced. The best performance is shown in bold.

CIFAR10 and Tiny-imagenet datasets. The proposed method demonstrates consistently high accuracy across all classes in the three datasets, which progressively increase in complexity. However, a slight decrease in accuracy is observed in case 2 and 3 of the CIFAR-10 dataset. We hypothesize that this decrease may be attributed to the presence of a negative term inherent in our method, which leads to overfitting. To alleviate this issue, we plan to conduct further investigations in the future to enhance and refine our model.

5.3. Comparison of Multi-Class Imbalance

Setup: In this experiment, we assess the performance of the proposed method on multi-class imbalanced data. Specifically, for the MNIST dataset, we conducted imbalanced treatment on class 0 and 1, class 0 and 2, and class 1 and 2, denoted as: 1) $\mathcal{T}_{min} = \{0, 1\}, \mathcal{T}_{maj} = \{2, \ldots, K\}; 2\} \mathcal{T}_{min} = \{0, 2\}, \mathcal{T}_{maj} = \{1, 3, \ldots, K\}; 3\} \mathcal{T}_{min} = \{1, 2\}, \mathcal{T}_{maj} = \{0, 3, \ldots, K\}$. In each setting, we conducted 10 trials with an imbalanced proportion of 2.5. We employed a linear

	MN	VIST	CIFAR-10				
Mathad	p=5	p=5	p=5	p=10			
Method	class=1~10	$class=1\sim10$	class= $1 \sim 5$	class=1~5			
	im-class=1,3,5,7,9	im-class=2,4,6,8,10	im-class=1,2,3,4	im-class=1,2,3,4			
PC (Ishida et al., 2017)	36.80 ± 1.18	36.20 ± 0.98	44.03 ± 0.58	36.27 ± 0.80			
NN (Ishida et al., 2019)	14.24 ± 1.52	14.14 ± 1.16	40.89 ± 2.17	33.47 ± 3.22			
FREE (Ishida et al., 2019)	20.07 ± 2.28	19.44 ± 1.38	46.50 ± 1.92	40.02 ± 1.66			
LOG (Feng et al., 2020)	9.80 ± 0.00	9.80 ± 0.00	47.58 ± 1.01	39.57 ± 0.89			
EXP (Feng et al., 2020)	16.40 ± 4.56	15.82 ± 4.49	47.77 ± 1.05	40.11 ± 0.69			
LW (Gao and Zhang, 2021)	34.87 ± 5.64	36.20 ± 5.04	47.43 ± 0.90	38.77 ± 0.88			
L-UW (Gao and Zhang, 2021)	31.72 ± 6.88	36.18 ± 5.64	48.16 ± 0.85	39.67 ± 1.11			
NCLL (Ishiguro et al., 2022)	_	_	40.83 ± 0.35	34.89 ± 0.81			
ML_CLL (Gao et al., 2023)	_	_	41.58 ± 0.60	33.40 ± 0.62			
WCLL (our)	55.22 ± 2.26	$\textbf{53.61} \pm \textbf{2.37}$	$\textbf{48.34} \pm \textbf{0.61}$	$\textbf{40.38} \pm \textbf{0.85}$			

Table 4: Test classification accuracy (mean \pm std %) of total class for 10 trials with more imbalance classes. The best performance is shown in bold.

model with weight-decay set to 1e - 4 and learning rate selected from $\{5e - 5, 5e - 6\}$.

For CIFAR-10, we utilize only the first five classes of the training set and the test set. Imbalanced treatment was applied to class 0 and 1, class 0 and 2, and class 1 and 2, with imbalanced proportion set to 5 and 10. The same model utilized in the previous section was employed in this experiment.

Results: Table 3 reports the test classification accuracy for 10 trials on two multi-class imbalanced datasets. Since compared methods exhibit poor performance on imbalanced classes, we focus our experiment on the test classification accuracy across all classes. The proposed method demonstrates a significant improvement in accuracy when multiple classes are imbalanced compared to other methods.

5.4. Additional Experiments

Extreme Case: To further demonstrate the effectiveness of our method on imbalanced data, we test our method's performance on data with more imbalanced classes. For MNIST, we tested two cases: 1) $K = 10, T_{min} = \{1, 3, 5, 7, 9\}$, and 2) $K = 10, T_{min} = \{2, 4, 6, 8, 10\}$. For CIFAR-10, we tested two cases: 1) $K = 5, p = 5, T_{min} = \{1, 2, 3, 4\}$, and 2) $K = 5, p = 10, T_{min} = \{1, 2, 3, 4\}$. The results are shown in Table 4, where we report the test classification accuracy

Random	Bias=0.05	Bias=0.1	Bias=0.15	Bias=0.2	Bias=0.3	Bias=0.5	WCLL (our)
$\begin{array}{r} 14.86 \\ \pm 3.55 \end{array}$	$54.66 \\ \pm 0.92$	$54.94 \\ \pm 1.05$	$54.45 \\ \pm 1.12$	$54.04 \\ \pm 1.09$	53.47 ± 1.17	$52.78 \\ \pm 1.20$	$72.30 \\ \pm 1.23$

Table 5: Test classification accuracy (mean \pm std %) of total class for 10 trials on MNIST datatset. The best performance is shown in bold.

Table 6: Test classification accuracy (mean \pm std %) of total class for 10 trials on MNIST datatset. The best performance is shown in bold.

P Under-	-sampling Ov	ver-sampling	WCLL (our)
2 49.56	$5 \pm 1.75 40$	6.08 ± 2.56	$\textbf{72.30} \pm \textbf{1.23}$
2.5 48.66	$6 \pm 1.26 40$	6.02 ± 2.42	$\textbf{71.40} \pm \textbf{1.62}$
5 44.38	8 ± 2.78 43	5.87 ± 2.57	$\textbf{68.71} \pm \textbf{1.61}$

of the total class. As demonstrated by the results, our method maintains high accuracy even in more extreme cases of imbalanced data, which further proves the superiority of our model.

Issue of Assumption: To verify our assumption on class imbalance of complementary labels, we conduct an additional experiment where we compare the results when π_j is not satisfied. We develop two distinct weighting strategies to alleviate this scenario. The first approach involves randomly generating a set of weights to the loss function. The second approach involves adding a bias to the proposed method, specifically, $\pi'_j = \pi_j + bias$, where the bias value was set to 0.05, 0.1, 0.15, 0.2, 0.3, and 0.5. The results of the test accuracy for 10 trials using the two different approaches are presented in Table 5. The performance is noticeably lower when π_j was not met, confirming the significance of our assumption on class imbalance of complementary labels.

Issue of Samping: We conducted additional experiments to compare the effectiveness of reweighting and resampling techniques on the same datasets. For undersampling, we removed the excess majority class samples and balanced them with the number of minority class samples. Conversely, oversampling was implemented by increasing the samples of the minority class to match the majority class. Table 6 presents the comparative results of these two techniques on the MNIST dataset. As depicted in the table 6, the proposed method outperforms the under-sampling and over-sampling techniques by a significant margin.

Method	total accuracy	benign accuracy	cancer accuracy
PC (Ishida et al., 2017)	34.62	4.23	25.38
NN (Ishida et al., 2019)	36.54	0.39	12.31
FREE (Ishida et al., 2019)	45.00	33.46	3.46
LOG (Feng et al., 2020)	40.26	3.85	25.77
EXP (Feng et al., 2020)	38.85	_	28.08
LW (Gao and Zhang, 2021)	38.85	_	28.85
L-UW (Gao and Zhang, 2021)	39.87	0.38	33.46
NCLL (Ishiguro et al., 2022)	36.41	_	10.77
ML_CLL (Gao et al., 2023)	37.32	_	15.77
WCLL (our)	57.56	76.54	34.23

Table 7: Test classification accuracy of DDSM dataset. The best performance is shown in bold.

Real-world Scenario: The performance results on the DDSM dataset are presented in Table 7, showcasing the outcome on a real-world dataset. The proposed method exhibits superior performance on the DDSM dataset, surpassing all baseline methods across all classes with a remarkable accuracy improvement of 17.56%. Notably, our method also demonstrates superior performance on the two unbalanced classes, namely benign and cancer accuracy. Specifically, our proposed method achieves a 43.08% increase in benign accuracy and a 0.77% increase in cancer accuracy compared to the best baseline method. This is a significant improvement, as all baseline methods failed to improve accuracy on both classes. Our findings clearly highlight the superiority of our proposed method.

6. Conclusion

Class imbalance is a common problem in real-world datasets, yet existing CLL approaches have not sufficiently alleviated this issue. Therefore, we propose a novel CLL problem setting specifically for class-imbalanced data. Our proposed approach is a weighted CLL method that learns from class-imbalanced data and has been theoretically proven to converge to the optimal solution. Our method not only leverages complementary labels to train a classifier, but also effectively alleviates the issue of class imbalance. The experimental results demonstrate that our method also outperforms other state-of-the-art methods when dealing with multi-class imbalanced samples.

It is noteworthy that the proposed method suffers from overfitting issue due to the negative problem. In the future, we consider designing a regularization penalty to alleviate this issue and improve the model's performance. Furtherore, the complementary labels were chosen at random, which also appears in other weakly supervised learning approaches. Taking the initiative to select some complementary labels that can enhance the model's performance would be interesting. In addition, dynamically weighted loss has been shown to be effective in dealing with class imbalance (Fernando and Tsokos, 2021; van der Meer et al., 2022), and it would be valuable to investigate how to create a dynamically weighted strategy for data with class-imbalanced complementary labels.

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Appendix A. The Proof of Lemma 1

Lemma 2. For any $\delta > 0$, with the probability at least $1 - \delta/2$, we have

$$\sup_{f\in\mathcal{F}}|\hat{R}(f)-R(f)| \leq 2\omega_K L_{\phi}[\pi_K(K-1)+K]\mathfrak{R}_{N_l}(\mathcal{F}) + \sqrt{\frac{I^2 log\frac{4}{\delta}}{2N_l}}.$$

Proof. Let According to McDiarmid inequality, for $\forall \varepsilon > 0$ we have

$$P(|R(f) - \hat{R}(f) - \mathbb{E}_{\bar{\mathcal{D}}_L}[R(f) - \hat{R}(f)]| \ge \varepsilon) \le 2exp(\frac{-2\varepsilon^2}{\sum_{N_l} C_i^2})$$

where $\overline{\mathcal{D}}_L = \{(x_i, \overline{y}_i)\}_{i=1}^{N_l}$. If x_i in $\overline{\mathcal{D}}_L$ is replaced with x'_i , we have $\overline{\mathcal{D}}'_L$ and

$$\begin{aligned} C_i &= \sup_{f \in \mathcal{F}} |[\hat{R}(f) - R(f)] - [\hat{R}'(f) - R(f)]| \\ &= \sup_{f \in \mathcal{F}} |\hat{R}(f) - \hat{R}'(f)| \\ &= \sup_{f \in \mathcal{F}} \frac{1}{N_l} \{ [\pi_z(K-1)\omega_z[f(x_i, z)] + \sum_{j=1}^K \omega_j[f(x_i, j)]] - [\pi_z(K-1)\omega_z[f(x'_i, z)] + \sum_{j=1}^K \omega_j[f(x'_i, j)]] \} \\ &\leqslant \frac{1}{N_l} \{ 2\omega_K L_f[\pi_K(K-1) + K] \} = \frac{I}{N_l} \end{aligned}$$

where $\pi_z \leqslant \pi_K, \omega_z \leqslant \omega_K, L[f(x_i, z)] \leqslant L_f, I = 2\omega_K L_f[\pi_K(K-1) + K]$, and $\hat{R}'(f) = \mathbb{E}_{\bar{\mathcal{D}}'_L}.$ Let $1 - P(|R(f) - \hat{R}(f) - \mathbb{E}_{D_l}[R(f) - \hat{R}(f)]| \leqslant \varepsilon) \leqslant \frac{\delta}{2}$, we have $2exp(\frac{-2\varepsilon^2}{\sum_{N_l} C_i^2}) = \frac{\delta}{2} \Rightarrow \varepsilon = \sqrt{\frac{I^2 log \frac{4}{\delta}}{2N_l}}$

Then, we can obtain the

$$|\hat{R}(f) - R(f) - \mathbb{E}_{\bar{\mathcal{D}}_L}[\hat{R}(f) - R(f)]| \leq \sqrt{\frac{I^2 log_{\bar{\delta}}^4}{2N_l}}$$

with the probability at least $1 - \frac{\delta}{2}$. Therefore,

$$\begin{aligned} \sup_{f \in \mathcal{F}} |\hat{R}(f) - R(f)| &\leq \mathbb{E}_{\bar{\mathcal{D}}_L}[\hat{R}(f) - R(f)] + \sqrt{\frac{I^2 log_{\delta}^4}{2N_l}} \\ &\leq 2\Re_{N_l}(\hat{\ell}o\mathcal{F}) + \sqrt{\frac{I^2 log_{\delta}^4}{2N_l}} \\ &\leq 2\omega_K L_{\phi}[\pi_K(K-1) + K]\Re_{N_l}(\mathcal{F}) + \sqrt{\frac{I^2 log_{\delta}^4}{2N_l}}, \end{aligned}$$

which proves the Lemma 1.

Appendix B. The Proof of Theorem 1

Theorem 2. For any $\delta > 0$, with the probability at least $1 - \delta/2$, we have

$$R(\hat{f}) - R(f^*) \leqslant 4\omega_K L_{\phi}[\pi_K(K-1) + K] \Re_{N_l}(\mathcal{F}) + \sqrt{\frac{2I^2 log_{\overline{\delta}}^4}{N_l}}.$$

Proof.

$$R(\hat{f}) - R(f^*) = (\hat{R}(\hat{f}) - \hat{R}(f^*)) + (R(\hat{f}) - \hat{R}(\hat{f})) + (\hat{R}(f^*) - R(f^*))$$

with $N_l \to \infty$, we have $\hat{R}(\hat{f}) - \hat{R}(f^*) = 0$. By using $R(\hat{f}) - \hat{R}(\hat{f}) \leq |R(\hat{f}) - \hat{R}(\hat{f})|$, $\hat{R}(f^*) - R(f^*) \leq |\hat{R}(f^*) - R(f^*)|$, and Lemma 1, we have

$$R(\hat{f}) - R(f^*) \leq 0 + 2 \sup_{f \in \mathcal{F}} |\hat{R}(f) - R(f)|$$
$$\leq 4\omega_K L_{\phi} [\pi_K(K-1) + K] \Re_{N_l}(\mathcal{F}) + \sqrt{\frac{2I^2 \log \frac{4}{\delta}}{N_l}}$$

with the probability at least $1 - \frac{\delta}{2}$, which finishes the proof.