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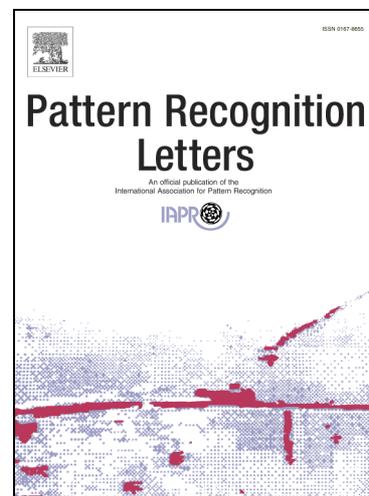
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1 Learning from Multiple Annotators: Distinguishing  
2 Good from Random Labelers

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14 **Abstract**

With the increasing popularity of online crowdsourcing platforms such as Amazon Mechanical Turk (AMT), building supervised learning models for datasets with multiple annotators is receiving an increasing attention from researchers. These platforms provide an inexpensive and accessible resource that can be used to obtain labeled data, and in many situations the quality of the labels competes directly with those of experts. For such reasons, much attention has recently been given to annotator-aware models. In this paper, we propose a new probabilistic model for supervised learning with multiple annotators where the reliability of the different annotators is treated as a latent variable. We empirically show that this model is able to achieve state of the art performance, while reducing the number of model parameters, thus avoiding a potential overfitting. Furthermore, the proposed model is easier to implement and extend to other classes of learning problems such as sequence

labeling tasks.

15 *Keywords:* Multiple Annotators, Crowdsourcing, Latent Variable Models,  
16 Expectation-Maximization, Logistic Regression

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## 17 **1. Introduction**

18 Crowdsourcing (Howe, 2008) is rapidly changing the way datasets are  
19 built. With the development of crowdsourcing platforms such as Amazon  
20 Mechanical Turk (AMT)<sup>1</sup>, it is becoming increasingly easier to obtain la-  
21 beled data for a wide range of tasks covering different areas such as Com-  
22 puter Vision, Natural Language Processing, Speech Recognition, etc. The  
23 attractiveness of these platforms comes not only from their low cost and ac-  
24 cessibility, but also from the surprisingly good quality of the labels obtained,  
25 which in many cases competes directly with those of “experts” (Snow et al.,  
26 2008). Furthermore, by distributing the workload among multiple annota-  
27 tors, labeling tasks can be completed much faster.

28 The current trend of social web, where citizens’ participation is growing  
29 in many forms, has come to stay, and information is being produced at  
30 a massive rate. This information can take many forms: document tags,  
31 opinions, product ratings, user clicks, contents, etc. These new sources of  
32 data also motivate the development of new machine learning approaches for  
33 learning from multiple sources.

34 On another perspective, there are tasks for which ground truth labels  
35 simply cannot be obtained due to their highly subjective nature. Consider

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<sup>1</sup><http://www.mturk.com>

36 for instance the tasks of sentiment analysis, movie rating or keyphrase ex-  
37 traction. These tasks are subjective in nature and hence no absolute gold  
38 standard can be defined. In such cases the only attainable goal is to build a  
39 model that captures the *wisdom of the crowds* (Surowiecki, 2004) as well as  
40 possible. For such tasks crowdsourcing platforms like AMT become a natural  
41 solution. However, the large amount of labeled data needed to compensate  
42 for the heterogeneity of annotators' expertise can rapidly rise its actual cost  
43 beyond acceptable values. Since different annotators have different levels of  
44 expertise, it is important to consider how *reliable* the annotators are when  
45 learning from their answers, and a parsimonious solution needs to be de-  
46 signed that is able to deal with such real world constraints (e.g. annotation  
47 cost) and heterogeneity.

48 Even in situations where a ground truth can be obtained, it may be too  
49 costly. For example, in Medical Diagnosis, determining whether a patient  
50 has cancer may require a biopsy, which is an invasive procedure, and thus  
51 should only be used as a last resource. On the other hand, it is rather easy  
52 for a diagnostician to consult its colleagues for their opinions before making a  
53 decision. Therefore, although there is no crowdsourcing involved here, there  
54 are still multiple experts, with different levels of expertise, providing their  
55 own (possibly wrong) opinions, from which we have to be able to learn from.

56 Many approaches have recently been proposed that deal with this increas-  
57 ingly important problem of supervised learning from multiple annotators in  
58 different paradigms: classification (Raykar et al., 2009; Yan et al., 2011),  
59 regression (Groot et al., 2011), ranking (Wu et al., 2011), etc. However,  
60 most of the work developed so far is centered on the *unknown* true labels of

61 the data, for which noisy versions are provided by the various annotators.  
62 Therefore, there has been a tendency to include these *unobserved* true labels  
63 as latent variables in a probabilistic framework, which, as we demonstrate,  
64 is not necessarily the best option. Furthermore, this choice of latent vari-  
65 ables hinders a natural extension of these approaches to structured prediction  
66 problems such as sequence labeling tasks due to combinatorial explosion of  
67 possible outcomes of the latent variables. Contrarily to these approaches, we  
68 argue that the focus should be on the annotators, and that including the also  
69 *unknown* reliabilities of the annotators as latent variables can be preferable,  
70 since it not only leads to simpler models that are less prone to overfitting,  
71 but also bypasses the problem of the high number of possible labelings to  
72 marginalize over.

73 In this paper, we propose a new probabilistic model that explores these  
74 ideas, and explicitly handles the annotators' reliabilities as latent variables.  
75 We empirically show, using both simulated annotators and human annota-  
76 tors from AMT, that for many tasks the new model can be competitive with  
77 the state of the art methods, and can even significantly outperform previ-  
78 ous approaches under certain conditions. Although we focus on multi-class  
79 Logistic Regression as the base classifier, the proposed model is simple and  
80 generic enough to be implemented with other classifiers. Furthermore the  
81 extension to structured prediction problems such as sequence labeling tasks  
82 can be much easier than with latent ground truth models (e.g. Raykar et al.  
83 (2010); Yan et al. (2011)).

84 The remainder of this paper is organized as follows: Section 2 provides  
85 the reader with an overview of state of the art; Section 3 clarifies the problem

86 with latent ground truth models; Section 4 presents the proposed model, and  
87 Section 5 compares the results obtained by this model with two majority  
88 voting baselines and a state of the art approach; the article will end with a  
89 short discussion and conclusions (Section 6).

## 90 2. State of the art

91 There is considerable work on estimating ground truth labels from the  
92 responses of multiple annotators. Most of the early important works were in  
93 the fields of Biostatistics and Epidemiology. In 1979, Dawid and Skene (1979)  
94 proposed an approach for estimating the error rates of multiple patients  
95 (annotators) given their responses (labels) to multiple medical questions.  
96 However, like most of the early works with multiple annotators, this work  
97 only focused on estimating the unobserved ground truth labels. Only later,  
98 researchers started paying more attention to the specific problem of learning  
99 a classifier from the multiple annotator's data. In 1995, Smyth et al. (1995)  
100 proposed a similar approach to the one from Dawid and Skene (1979) to  
101 estimate the ground truth from the labels of multiple experts, which was  
102 then used to train a classifier. As with previous works, the authors employed  
103 a model where the unknown true labels were treated as latent variables.

104 More recently, with the increasing popularity of AMT and other crowd-  
105 sourcing and work-recruiting platforms, researchers started recognizing the  
106 importance of the problem of learning from the labels of multiple non-expert  
107 annotators. The researchers' interest grew even further with works such as  
108 (Snow et al., 2008) and (Novotney and Callison-Burch, 2010), which show  
109 that, for many tasks, learning from multiple non-experts can be as good as

110 learning from an expert.

111 With the rising interest in crowdsourcing as a source of labeled data,  
112 more challenging approaches for learning from multiple annotators started  
113 to appear. In 2009, Raykar et al. (2009) proposed an innovative probabilis-  
114 tic approach where the unknown ground truth labels and the classifier are  
115 learnt jointly. By handling the unobserved ground truth labels as latent vari-  
116 ables, the authors are able to find the maximum likelihood parameters for  
117 their model by iteratively estimating the posterior distribution of the ground  
118 truth labels and then using this estimate to determine the qualities of the  
119 annotators and the parameters of a Logistic Regression model. Unlike most  
120 of the previous works, this approach also has the advantage of relaxing the  
121 requirement of repeated labeling, i.e. the same instance being annotated by  
122 multiple annotators. Later works then relaxed other assumptions made by  
123 the authors. For example, Yan et al. (2010) relaxed the assumption that  
124 the quality of the labels provided by the annotators does not depend on the  
125 instance they are labeling.

126 This main line of work also inspired many variations and extensions in the  
127 past couple of years. Groot et al. (2011) proposed an extension of Gaussian  
128 processes to do regression in a multiple annotator setting. In the field of  
129 ranking, Wu et al. (2011) presented an approach to learn how to rank from  
130 the opinions of multiple annotators. In an active learning setting, Yan et al.  
131 (2011) proposed an approach for multiple annotators by providing answers to  
132 the following questions: what instance should be selected to be labeled next  
133 and which annotators should label it? On a different perspective, in (Donmez  
134 et al., 2010) the authors propose the use of a particle filter to model the

135 time-varying accuracies of the different annotators. Despite the plausibility  
 136 of their assumptions, i.e. it is legitimate to assume that the quality of the  
 137 labels provided by an annotator will vary with time, the results obtained  
 138 showed only a small improvement on the performance of their model through  
 139 the inclusion of this time dependence.

140 The approaches above mentioned typically treat the *unknown* ground  
 141 truth labels as latent variables and build a model on that basis. We argue  
 142 that explicitly handling the reliabilities of the annotators as latent variables,  
 143 as opposed to the true labels, in a fashion that slightly resembles a mixture of  
 144 experts (Jacobs et al., 1991; Bishop, 2006), brings many attractive advantages  
 145 and can, under certain conditions, outperform latent ground truth models.

### 146 3. The problem with latent ground truth models

147 In order to help motivate the proposed model, we now introduce a typ-  
 148 ical class of approaches for learning from multiple annotators, in which the  
 149 *unknown* true labels are treated as latent variables (e.g. Raykar et al. (2009,  
 150 2010); Yan et al. (2010)).

151 Let  $y_i^r$  be the label assigned to instance  $\mathbf{x}_i$  by the  $r^{th}$  annotator, and let  
 152  $y_i$  be the true (unobserved) label for that instance. Contrarily to a typical  
 153 classification problem with a single annotator, in a setting with  $R$  annotators,  
 154 a dataset  $\mathcal{D}$  with size  $N$  consists of a set of labels  $\{y_i^1, y_i^2, \dots, y_i^R\}$  for each of  
 155 the  $N$  instances  $\mathbf{x}_i$ .

156 In general, the class of models we refer to as “latent ground truth mod-  
 157 els” tend to assume the following generative process: for each instance  $\mathbf{x}_i$   
 158 there is an *unobserved* true label  $y_i$ , and each of the different annotators in-

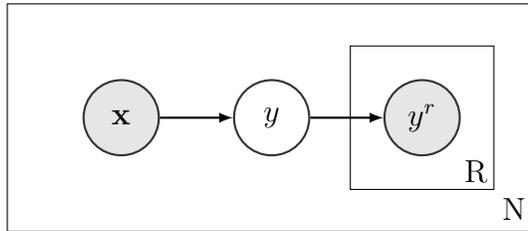


Figure 1: Plate representation of general latent ground truth model.

159 dependently provides its own version ( $y_i^r$ ) of this true label, which in practice  
 160 corresponds to an approximation to the real label  $y_i$ . Figure 1 depicts such  
 161 a model in plate notation. Shaded nodes represent observed variables, and  
 162 non-shaded nodes represent unobserved (latent) variables.

If besides the dataset  $\mathcal{D} = \{y_i^1, \dots, y_i^R, \mathbf{x}_i\}_{i=1}^N$  we were given the true labels  
 $\mathcal{Y} = \{y_i\}_{i=1}^N$  as well, the likelihood for this model would take the form

$$p(\mathcal{D}, \mathcal{Y}) = \prod_{i=1}^N \left( p(y_i | \mathbf{x}_i) \prod_{r=1}^R p(y_i^r | y_i) \right). \quad (1)$$

163 Since we do not actually observe the true labels  $y_i$  we must treat them as  
 164 latent variables and marginalize them out of the likelihood, and this leads  
 165 us to the first problem with this approach: although this marginalization is  
 166 not difficult for classification problems where the number of classes ( $K$ ) is  
 167 small, for other types of problems like sequence labeling tasks (or any task  
 168 with structured outputs), marginalizing over the output space is intractable  
 169 in general (Sutton, 2012). If we consider, for example, the tasks of part-  
 170 of-speech (POS) tagging or Named Entity Recognition (NER), which are  
 171 usually handled as a sequence labelling problems, it is easy to see that the  
 172 number of possible label sequences grows exponentially with the length of  
 173 the sentence, deeming the marginalization over the output space intractable.

174 The second problem with this class of models is related with the prob-  
 175 ability  $p(y_i^r|y_i)$ , which for a classification problem with  $K$  classes requires a  
 176  $K \times K$  table of parameters for each annotator. Even though this approach  
 177 allows to capture certain biases in the annotators answers, like for example  
 178 the tendency to confuse two classes, in practice, on a crowdsourcing platform  
 179 like AMT, each annotator only labels a rather small set of instances. There-  
 180 fore, under such conditions, having a model with so many parameters for  
 181 the reliability of the annotators can easily lead to overfitting. Consider, for  
 182 example, a classification problem with 10 classes. Such a problem requires a  
 183 total of 100 parameters (a  $10 \times 10$  probability table) to model the expertise  
 184 of a single annotator. To effectively learn such a number of parameters, each  
 185 annotator would be required to label a large number of instances, at least in  
 186 the order of the thousands, something that is both unrealistic and hard to  
 187 control in a crowdsourcing platform.

188 Taking these issues into consideration, we developed a new probabilis-  
 189 tic model for learning from multiple annotators, which we present in the  
 190 following section.

## 191 4. Proposed model

### 192 4.1. Maximum likelihood estimator

Given a dataset  $\mathcal{D} = \{y_i^1, \dots, y_i^R, \mathbf{x}_i\}_{i=1}^N$  with  $N$  instances and  $R$  different  
 annotators, and assuming that the instances are independent and identically  
 distributed (i.i.d.), the likelihood is given by

$$p(\mathcal{D}|\theta) = \prod_{i=1}^N p(y_i^1, \dots, y_i^R | \mathbf{x}_i, \theta) \quad (2)$$

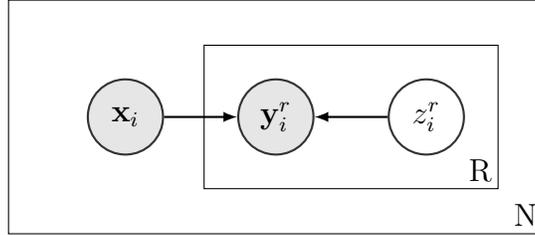


Figure 2: Plate representation of the proposed model.

193 where  $\theta$  denotes the model parameters.

Let us now assume the following generative process of the annotators' labels: when the annotators are asked to provide a label to a given instance  $\mathbf{x}_i$ , they flip a biased coin, and based on the outcome of those coin flips, they decide whether or not to provide the correct label. This intuition amounts to introducing a binary random variable  $z_i^r$ , whose value indicates whether the  $r^{\text{th}}$  annotator labeled the  $i^{\text{th}}$  instance correctly or not. Hence,  $z_i^r \sim \text{Bernoulli}(\pi_r)$ , where  $\pi_r$  is the accuracy of the  $r^{\text{th}}$  annotator, and

$$p(z_i^r | \pi_r) = (\pi_r)^{z_i^r} (1 - \pi_r)^{1 - z_i^r}. \quad (3)$$

194 The expectation of this Bernoulli random variable  $\mathbb{E}[z_i^r] = p(z_i^r = 1)$  can be  
 195 interpreted as the probability of an annotator providing a correct label or, in  
 196 other words, as an indicator of how reliable an annotator is. For the sake of  
 197 simplicity, we assume that an unreliable annotator provides labels according  
 198 to some random model  $p_{\text{Rand}}(y_i^r = k | \mathbf{x}_i)$ .

199 Figure 2 shows a plate representation of this generative model. Notice  
 200 that the variables  $z_i^r$  are not observed in this model, hence their nodes are  
 201 not shaded in the figure.

If we were told the true values for  $\mathcal{Z} = \{z_i^1, \dots, z_i^R\}_{i=1}^N$ , and assuming

the annotators make their decisions independently of the each other, the complete-data likelihood could then be factored as

$$p(\mathcal{D}, \mathcal{Z}|\theta) = \prod_{i=1}^N \prod_{r=1}^R p(z_i^r|\pi_r) p(y_i^r|\mathbf{x}_i, z_i^r, \mathbf{w}) \quad (4)$$

202 where  $\theta = \{\boldsymbol{\pi}, \mathbf{w}\}$  are the model parameters. The values of  $\boldsymbol{\pi} = \{\pi_r\}_{r=1}^R$   
 203 correspond to the parameters of the  $R$  Bernoulli distributions (one for each  
 204 annotator). In turn,  $\mathbf{w}$  are the weights of a Logistic Regression model.

Following the generative process described above, we can now define  $p(y_i^r|\mathbf{x}_i, z_i^r, \mathbf{w})$  as

$$p(y_i^r|\mathbf{x}_i, z_i^r, \mathbf{w}) = \left( p_{\text{LogReg}}(y_i^r|\mathbf{x}_i, \mathbf{w}) \right)^{z_i^r} \left( p_{\text{Rand}}(y_i^r|\mathbf{x}_i) \right)^{1-z_i^r} \quad (5)$$

where  $p_{\text{LogReg}}(y_i^r|\mathbf{x}_i, \mathbf{w})$  denotes the likelihood of the label provided by the  $r^{\text{th}}$  annotator for the instance  $\mathbf{x}_i$  according to a multi-class Logistic Regression model with parameters  $\mathbf{w}$ , which for a classification task with  $K$  classes is given by

$$p_{\text{LogReg}}(y_i^r = k|\mathbf{x}_i, \mathbf{w}) = \frac{\exp(\mathbf{w}_k^T \mathbf{x}_i)}{\sum_{k'=1}^K \exp(\mathbf{w}_{k'}^T \mathbf{x}_i)}. \quad (6)$$

Similarly,  $p_{\text{Rand}}(y_i^r|\mathbf{x}_i)$  denotes the likelihood of the label  $y_i^r$  according to a random model, which we assume to be uniformly distributed. Hence,

$$p_{\text{Rand}}(y_i^r = k|\mathbf{x}_i) = \frac{1}{K}. \quad (7)$$

205 To summarize, this is akin to saying that if  $z_i^r = 1$  then the label provided  
 206 by the  $r^{\text{th}}$  annotator ( $y_i^r$ ) fits a Logistic Regression model, which is assumed  
 207 to capture the correct (true) labeling process. Conversely, if  $z_i^r = 0$  then  
 208  $y_i^r$  is assumed to be drawn from a random model where all the classes are  
 209 equiprobable.

Since we do not actually observe the set  $\mathcal{Z}$  we must treat the variables  $z_i^r$  as latent and marginalize them out of the likelihood by summing over all its possible outcomes. The (observed) data likelihood then becomes

$$p(\mathcal{D}|\theta) = \prod_{i=1}^N \prod_{r=1}^R \sum_{z_i^r \in \{0,1\}} p(z_i^r|\pi_r) p(y_i^r|\mathbf{x}_i, z_i^r, \mathbf{w}). \quad (8)$$

Making use of equations 3 and 5, this expression can be further simplified, giving

$$p(\mathcal{D}|\theta) = \prod_{i=1}^N \prod_{r=1}^R \left( \pi_r p_{\text{LogReg}}(y_i^r|\mathbf{x}_i, \mathbf{w}) + (1 - \pi_r) p_{\text{Rand}}(y_i^r|\mathbf{x}_i) \right). \quad (9)$$

210 Our goal is then to estimate the maximum likelihood parameters  $\theta_{\text{ML}}$ ,  
 211 which are found by determining  $\theta_{\text{ML}} = \arg \max_{\theta} \ln p(\mathcal{D}|\theta)$ .

212 At this point, it is important to note that extending this approach to  
 213 sequence labeling problems, or any kind of structured prediction problems  
 214 in general, could be as simple as replacing in equation 5 the probabilities  
 215  $p_{\text{LogReg}}(y_i^r|\mathbf{x}_i, \mathbf{w})$  and  $p_{\text{Rand}}(y_i^r|\mathbf{x}_i)$  with their sequence labeling counterparts,  
 216 which for  $p_{\text{LogReg}}(\cdot)$  could be an Hidden Markov Model (HMM) or a Condi-  
 217 tional Random Field (CRF), and updating the remaining equations accord-  
 218 ingly.

#### 219 4.2. Expectation-Maximization

220 As with other latent variable models, we rely on the Expectation-  
 221 Maximization (EM) algorithm (Dempster et al., 1977) to optimize this oth-  
 222 erwise intractable maximization problem. The EM algorithm is an iterative  
 223 method for finding maximum likelihood solutions for probabilistic models  
 224 with latent variables, and consist of two steps: the E-step and M-step. In

225 the E-step the posterior distribution of the latent variables is computed based  
 226 on the current model parameters. This posterior distribution is then used  
 227 to estimate the new model parameters (M-step). These two steps are then  
 228 iterated until convergence.

229 If we observed the complete dataset  $\{\mathcal{D}, \mathcal{Z}\}$  then the loglikelihood func-  
 230 tion would simply take the form  $\ln p(\mathcal{D}, \mathcal{Z}|\theta)$ . Since we only have access to  
 231 the “incomplete” dataset  $\mathcal{D}$ , our state of the knowledge about the values  
 232 of  $\mathcal{Z}$  (the reliabilities of the annotators) can be given by the posterior dis-  
 233 tribution  $p(\mathcal{Z}|\mathcal{D}, \theta)$ . Therefore, instead of the complete data loglikelihood,  
 234 we consider its expected value under the posterior distribution of the latent  
 235 variable  $p(\mathcal{Z}|\mathcal{D}, \theta)$ , which corresponds to the E-step of the EM algorithm.  
 236 Hence, in the E-step we use the current parameter values  $\theta^{old}$  to find the  
 237 posterior distribution of the latent variables in  $\mathcal{Z}$ . We then use this poste-  
 238 rior distribution to find the expectation of the complete-data loglikelihood  
 239 evaluated for some general parameter values  $\theta$ . This expectation is given by

$$\begin{aligned} & \mathbb{E}_{p(\mathcal{Z}|\mathcal{D}, \theta_{old})} \left[ \ln p(\mathcal{D}, \mathcal{Z}|\theta) \right] \\ &= \sum_{\mathcal{Z}} p(\mathcal{Z}|\mathcal{D}, \theta_{old}) \ln p(\mathcal{D}, \mathcal{Z}|\theta) \\ &= \sum_{i=1}^N \sum_{r=1}^R \sum_{z_i^r \in \{0,1\}} p(z_i^r | y_i^r, \mathbf{x}_i, \theta_{old}) \ln \left( p(z_i^r | \pi_r) p(y_i^r | \mathbf{x}_i, z_i^r, \mathbf{w}) \right). \end{aligned} \quad (10)$$

The posterior distribution of the latent variables  $z_i^r$  (denoted by  $\gamma(z_i^r)$ )

can be estimated using the Bayes theorem giving

$$\begin{aligned}
\gamma(z_i^r) &= p(z_i^r = 1 | y_i^r, \mathbf{x}_i, \theta^{old}) \\
&= \frac{p(z_i^r = 1 | \pi_r^{old}) p(y_i^r | \mathbf{x}_i, z_i^r = 1, \mathbf{w}^{old})}{p(z_i^r = 1 | \pi_r^{old}) p(y_i^r | \mathbf{x}_i, z_i^r = 1, \mathbf{w}^{old}) + p(z_i^r = 0 | \pi_r^{old}) p(y_i^r | \mathbf{x}_i, z_i^r = 0, \mathbf{w})} \\
&= \frac{\pi_r^{old} p_{\text{LogReg}}(y_i^r | \mathbf{x}_i, \mathbf{w}^{old})}{\pi_r^{old} p_{\text{LogReg}}(y_i^r | \mathbf{x}_i, \mathbf{w}^{old}) + (1 - \pi_r^{old}) p_{\text{Rand}}(y_i^r | \mathbf{x}_i)} \quad (11)
\end{aligned}$$

240 where we also made use of equations 3 and 5.

The expected value of the complete data loglikelihood then becomes

$$\begin{aligned}
\mathbb{E}_{p(\mathcal{Z} | \mathcal{D}, \theta_{old})} \left[ \ln p(\mathcal{D}, \mathcal{Z} | \theta) \right] &= \sum_{i=1}^N \sum_{r=1}^R \gamma(z_i^r) \ln \left( \pi_r p_{\text{LogReg}}(y_i^r | \mathbf{x}_i, \mathbf{w}) \right) \\
&\quad + (1 - \gamma(z_i^r)) \ln \left( (1 - \pi_r) p_{\text{Rand}}(y_i^r | \mathbf{x}_i) \right). \quad (12)
\end{aligned}$$

In the M-step of the EM algorithm we maximize this expectation with respect to the model parameters  $\theta$ , obtaining new parameter values  $\theta^{new}$  given by

$$\theta^{new} = \arg \max_{\theta} \mathbb{E}_{p(\mathcal{Z} | \mathcal{D}, \theta_{old})} \left[ \ln p(\mathcal{D}, \mathcal{Z} | \theta) \right]. \quad (13)$$

241 The EM algorithm can then be summarized as follows:

242 **E-step** Compute the posterior distribution of the latent variables  $z_i^r$  by mak-  
243 ing use of equation 11.

**M-step** Estimate the new model parameters  $\theta^{new} = \{\boldsymbol{\pi}^{new}, \mathbf{w}^{new}\}$  given by

$$\mathbf{w}^{new} = \arg \max_{\mathbf{w}} \sum_{i=1}^N \sum_{r=1}^R \gamma(z_i^r) \ln p_{\text{LogReg}}(y_i^r | \mathbf{x}_i, \mathbf{w}) \quad (14)$$

$$\hat{\mathcal{Y}}^{new} = \arg \max_{\hat{\mathcal{Y}}} p_{\text{LogReg}}(\hat{\mathcal{Y}} | \mathcal{X}, \mathbf{w}^{new}) \quad (15)$$

$$\pi_r^{new} = \text{accuracy}_r = \frac{\#\{i : y_i^r = \hat{y}_i\}}{N_r} \quad (16)$$

where  $N_r$  denotes the number of instances labeled by annotator  $r$ . In order to optimize equation 14 we use limited-memory BFGS (Liu and Nocedal, 1989). The first order derivate is given by

$$\nabla_{\mathbf{w}} = \sum_{i=1}^N \sum_{r=1}^R \left( \gamma(z_i^r) \sum_{k=1}^K \left( t_{ik}^r - p_{\text{LogReg}}(y_i = k | \mathbf{x}_i, \mathbf{w}) \right) \mathbf{x}_i \mathbf{x}_i^T \right) \quad (17)$$

244 where  $\mathbf{t}_i^r$  is a vector representation of  $y_i^r$  in a 1-of- $K$  coding scheme, thus  $t_{ik}^r$   
 245 would be 1 when  $k$  corresponds to the label provided by the  $r^{\text{th}}$  annotator  
 246 and 0 otherwise.

247 Notice that this is very similar to the typical training of a multi-class  
 248 Logistic Regression model. However, in this case, the contributions of the  
 249 labels provided by each annotator to the loglikelihood are being weighted by  
 250 her reliability, or in other words, by how likely it is for her to be correct.  
 251 This makes our proposed approach quite easy to implement in practice.

## 252 5. Experiments

253 The proposed Multiple-Annotator Logistic Regression (MA-LR)<sup>2</sup> model  
 254 was evaluated using both multiple-annotator data with simulated annotators  
 255 and data manually labelled using AMT. The model was compared with the  
 256 multi-class extension of the model proposed by Raykar et al. (2009, 2010),  
 257 which is a latent ground truth model, and with two majority voting baselines:

- 258 • Soft Majority Voting (MVsoft): this corresponds to a multi-class Logistic  
 259 Regression model trained with the *soft* probabilistic labels resultant  
 260 from the voting process.

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<sup>2</sup>Source code is available at: <http://amilab.dei.uc.pt/fmpr/malr.tar.gz>

- 261 • Hard Majority Voting (MVhard): this corresponds to a multi-class  
262 Logistic Regression model trained with the most voted labels resultant  
263 from the voting process (i.e. the most voted class for a given instance  
264 gets “1” and the others get “0”).

265 In all experiments the EM algorithm was initialized with majority voting.

### 266 5.1. Simulated annotators

267 With the purpose of comparing the presented approaches in different  
268 classification tasks we used six popular benchmark datasets from the UCI  
269 repository<sup>3</sup> - a collection of databases, domain theories, and data generators  
270 that are used by the machine learning community for the empirical analysis  
271 of machine learning algorithms. Since these datasets do not have labels from  
272 multiple annotators, the latter were simulated from the ground truth using  
273 two different methods. The first method, denoted “label flips”, consists in  
274 randomly flipping the label of an instance with a given uniform probability  
275  $p(\text{flip})$  in order to simulate an annotator with an average reliability of  $(1 -$   
276  $p(\text{flip}))$ . The second method, referred to as “model noise”, seeks simulating  
277 annotators that are more consistent in their opinions, and can be summarized  
278 as follows. First, a multi-class Logistic Regression model is trained on the  
279 original training set. Then, the resulting weights  $\mathbf{w}$  are perturbed, such that  
280 the classifier consistently “fails” in a coherent fashion throughout the test set.  
281 To do so, the values of  $\mathbf{w}$  are standardized, and then random “noise” is drawn  
282 from a Gaussian distribution with zero mean and  $\sigma^2$  variance and added  
283 to the weights  $\mathbf{w}$ . These weights are then “unstandardized” (by reversing

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<sup>3</sup><http://archive.ics.uci.edu/ml/index.html>

Table 1: Details of the UCI datasets

Dataset	Num. Instances	Num. Features	Num. Classes
Annealing	798	38	6
Image Segmentation	2310	19	7
Ionosphere	351	34	2
Iris	150	4	3
Parkinson's	197	23	2
Wine	178	13	3

284 the standardization process previously used), and the modified multi-class  
 285 Logistic Regression model is re-applied to the training set in order to simulate  
 286 an annotator. The quality of this annotator will vary depending on the value  
 287 of  $\sigma^2$  used.

288 Since in practice each annotator only labels a small subset of all the in-  
 289 stances in the dataset, we introduce another parameter in this annotator  
 290 simulation process: the probability  $p(\text{label})$  of an annotator labeling an in-  
 291 stance.

292 Table 1 describes the UCI datasets used in these experiments. Special care  
 293 was taken in choosing datasets that correspond to real data and that were  
 294 among the most popular ones in the repository and, consequently, among  
 295 the Machine Learning community. Datasets that were overly unbalanced,  
 296 i.e. with too many instances of some classes and very few instances of oth-  
 297 ers, were avoided. Despite that, the selection process was random, which  
 298 resulted in a rather heterogeneous collection of datasets: with different sizes,

299 dimensionalities and number of classes.

300 Figures 3 and 4 show the results obtained using 5 simulated annotators  
301 with different reliabilities using distinct simulation methods: “label flips”  
302 and “model noise” respectively. Although not all the results (i.e. using both  
303 simulation methods on all the six datasets) are presented here, we note that  
304 the omitted results are similar to those shown. Hence, to avoid redundancy  
305 and preserve brevity, only a random subset of these are presented. All the  
306 experiments use 10-fold cross-validation. Due to the stochastic nature of the  
307 simulation process of the annotators, each experiment was repeated 30 times  
308 and the average results were collected. The plots on the left show the root  
309 mean squared error (RMSE) between the estimated annotators accuracies  
310 and their actual accuracies evaluated against the ground truth. The plots  
311 on the center and on the right show, respectively, the trainset and testset  
312 accuracies. Note that here, unlike in “typical” supervised learning tasks,  
313 trainset accuracy is quite important since it indicates how well the models  
314 are estimating the *unobserved* ground truth labels from the opinions of the  
315 multiple annotators.

316 From a general perspective on the results of figures 3 and 4 we can con-  
317 clude that both methods for learning from multiple annotators (MA-LR and  
318 Raykar) tend to outperform the majority voting baselines under most condi-  
319 tions. Not surprisingly, as the value of  $p(\text{label})$ , and consequently the average  
320 number of instances labeled by each annotator, decreases, both the trainset  
321 and testset accuracies of all the approaches decrease or stay roughly the same.  
322 As expected, a higher trainset accuracy usually translates in a higher testset  
323 accuracy and a better approximation of the annotators accuracies (i.e. lower

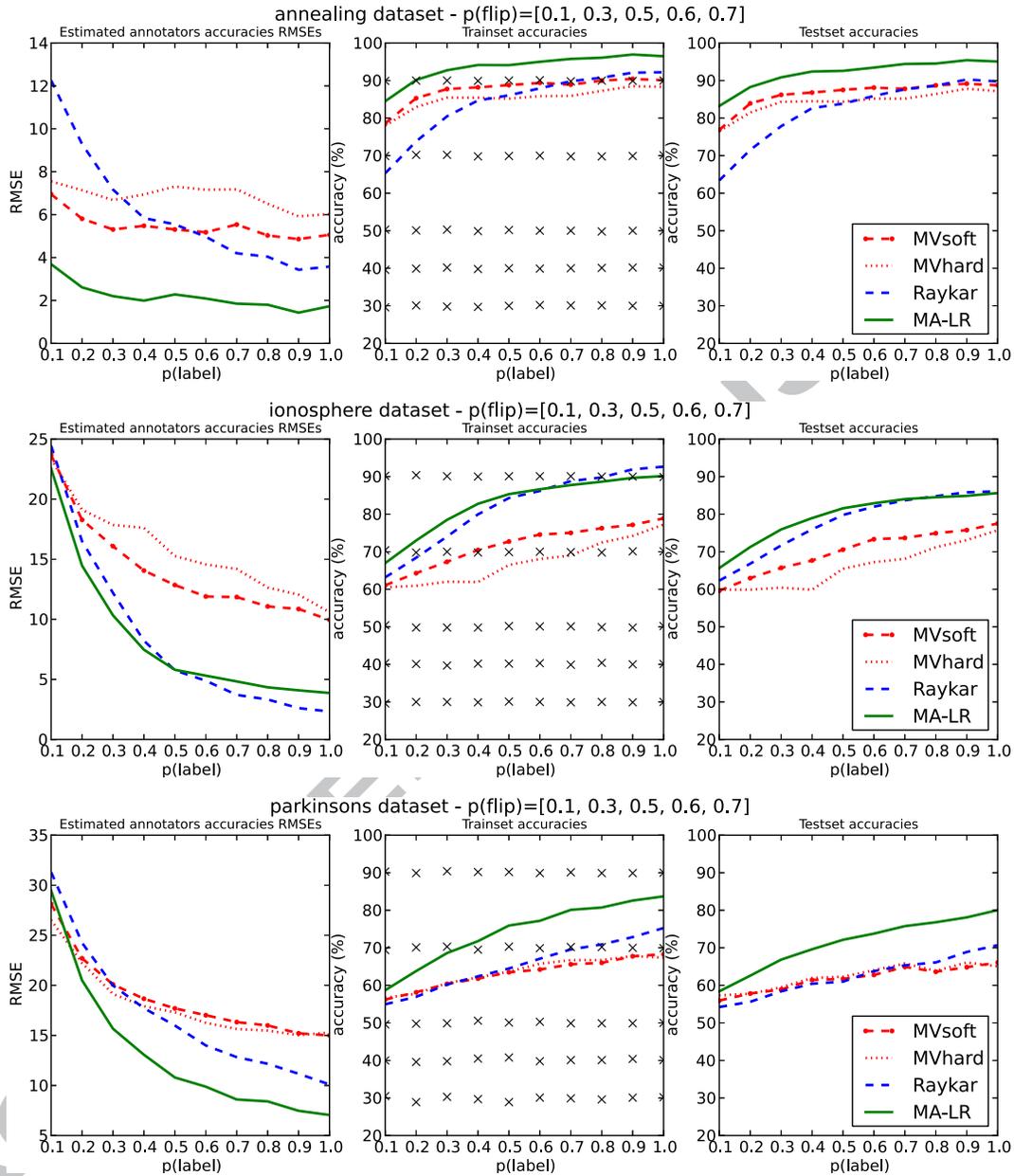


Figure 3: Results for the Annealing, Ionosphere and Parkinsons datasets using the “label flips” method for simulating annotators. The “x” marks indicate the average true accuracies of the simulated annotators.

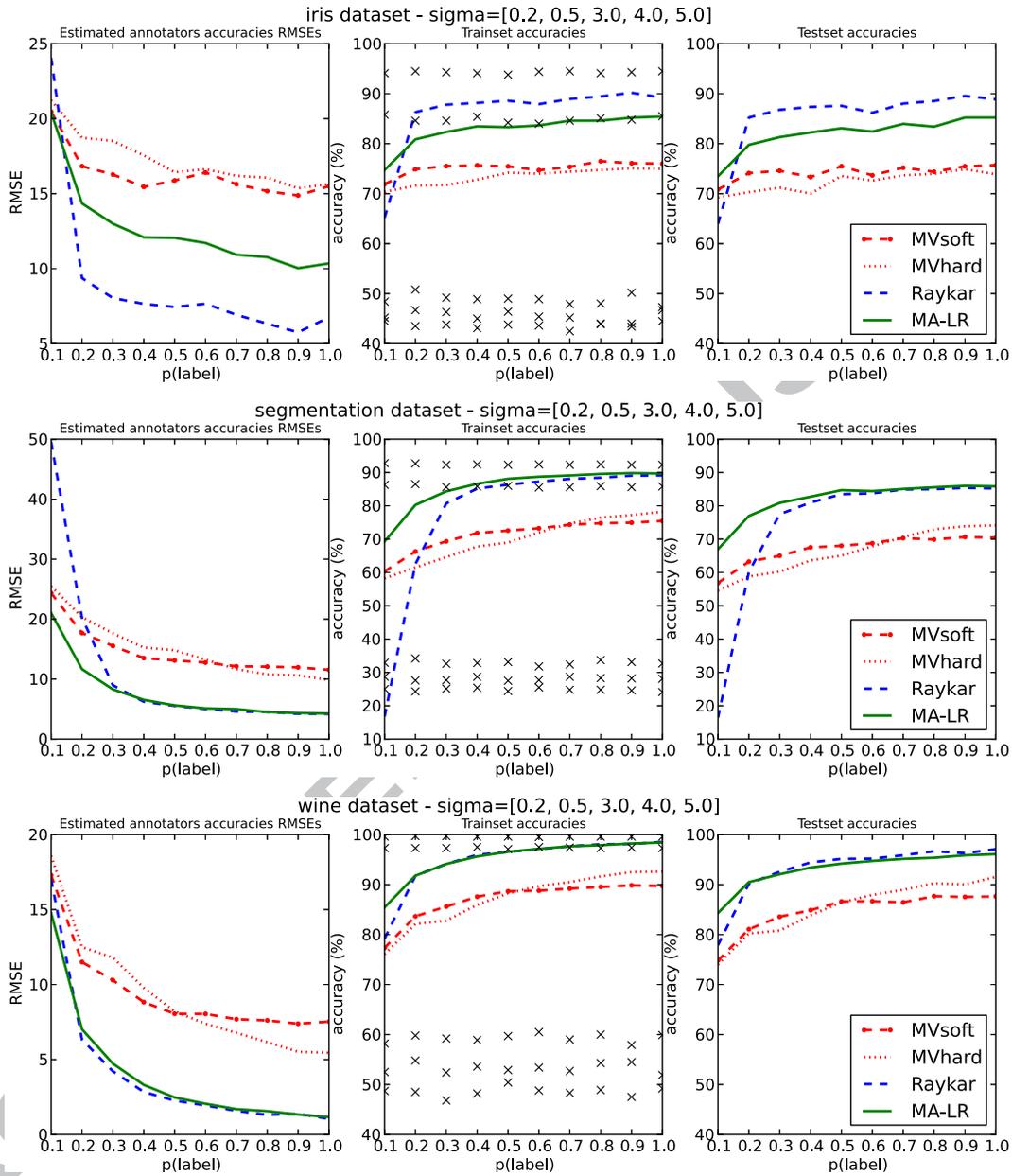


Figure 4: Results for the Iris, Segmentation and Wine datasets using the “model noise” method for simulating annotators. The “x” marks indicate the average true accuracies of the simulated annotators.

324 RMSE), since the approximation of the ground truth is also better.

325 A more careful analysis of the results reveals that, contrarily to the model  
326 by Raykar et al. (2009, 2010), the proposed model (MA-LR) is less prone  
327 to overfitting when the number of instances labeled by each annotator de-  
328 creases. This is a direct consequence of the number of parameters used to  
329 model the annotators expertise. While the model by Raykar et al. (2009,  
330 2010) uses a  $K \times K$  confusion matrix for each annotator, making a total of  
331  $RK^2$  parameters, the proposed model only has  $R$  parameters. However, it is  
332 important to note that there is a tradeoff here, since the model by Raykar et  
333 al. can capture certain biases in the annotators answers by keeping a  $K \times K$   
334 confusion matrix for each annotator, which is not possible with the MA-LR  
335 model. Notwithstanding, in practice, on crowdsourcing platforms like AMT,  
336 the number of instances labeled by each annotator is usually low. Hence, we  
337 believe that the proposed model is preferable in most situations. Further-  
338 more, our experimental results show that even when the number of instances  
339 labeled by each annotator is high, the MA-LR model can achieve similar or  
340 even better results than the model by Raykar et al. (2009, 2010).

### 341 5.2. Amazon Mechanical Turk

342 In order to assess the performance of the proposed model in learning from  
343 the labels of multiple non-expert human annotators and compare it with the  
344 other approaches, two experiments were conducted using AMT: sentiment  
345 polarity and music genre classification<sup>4</sup>.

346 The sentiment polarity experiment was based on the sentiment analysis

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<sup>4</sup>Datasets are available at: <http://amilab.dei.uc.pt/fmpr/mturk-datasets.tar.gz>

347 dataset introduced by Pang and Lee (2005), which corresponds to a collection  
348 of more than ten thousand sentences extracted from the movie review website  
349 RottenTomatoes<sup>5</sup>. These are labeled as positive or negative depending on  
350 whether they were marked as “fresh” or “rotten” respectively. From this  
351 collection, a random subset of 5000 sentences were selected and published on  
352 Amazon Mechanical Turk for annotation. Given the sentences, the workers  
353 were asked to provide the sentiment polarity (positive or negative). The  
354 remaining 5428 sentences were kept for evaluation.

355 For the music genre classification experiment, the audio dataset intro-  
356 duced by Tzanetakis and Cook (2002) was used. This dataset consists of  
357 a thousand samples of songs with 30 seconds of length and divided among  
358 10 different music genres: classical, country, disco, hiphop, jazz, rock, blues,  
359 reggae, pop and metal. Each of the genres has 100 representative samples.  
360 A random 70/30 train/test split was performed on the dataset, and the 700  
361 training samples were published on AMT for classification. In this case, the  
362 workers were required to listen to a 30-second audio excerpt and classify it  
363 as one of the 10 genres enumerated above.

364 On both experiments, the AMT workers were required to have an *HIT*  
365 *approval rate* - an AMT quality indicator that reflects the percentage of  
366 accepted answers of a worker - of 95%, which ensures some reliability on the  
367 quality of the answers.

368 Table 2 shows some statistics about the answers of the AMT workers for  
369 both datasets. Figure 5 further explore the distributions of the number of

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<sup>5</sup><http://www.rottentomatoes.com/>

Table 2: Statistics of the answers of the AMT workers for the two experiments performed. Note that the worker accuracies correspond to trainset accuracies.

	Sentiment polarity	Music genre
Number of answers collected	27747	2946
Number of workers	203	44
Avg. answers per worker ( $\pm$ std)	$136.68 \pm 345.37$	$66.93 \pm 104.41$
Min. answers per worker	5	2
Max. answers per worker	3993	368
Avg. worker accuracy ( $\pm$ std)	$77.12 \pm 17.10\%$	$73.28 \pm 24.16\%$
Min. worker accuracy	20%	6.8%
Max. worker accuracy	100%	100%

370 answers provided by each annotator and their accuracies for the sentiment  
 371 polarity and music genre datasets. The figure reveals a highly skewed dis-  
 372 tribution of number of answers per worker, which support our intuition that  
 373 on this kind of crowdsourcing platforms each worker tends to only provide  
 374 a small number of answers, with only a couple of workers performing high  
 375 quantities of labelings.

376 Standard preprocessing and features extraction techniques were performed  
 377 on both experiments. In the case of the sentiment polarity dataset, the stop-  
 378 words were removed and the remaining words were reduced to their root by  
 379 applying a stemmer. This resulted in a vocabulary with size 8919, which still  
 380 makes a bag-of-words representation computationally expensive. Hence, La-  
 381 tent Semantic Analysis (LSA) was used to further reduce the dimensionally

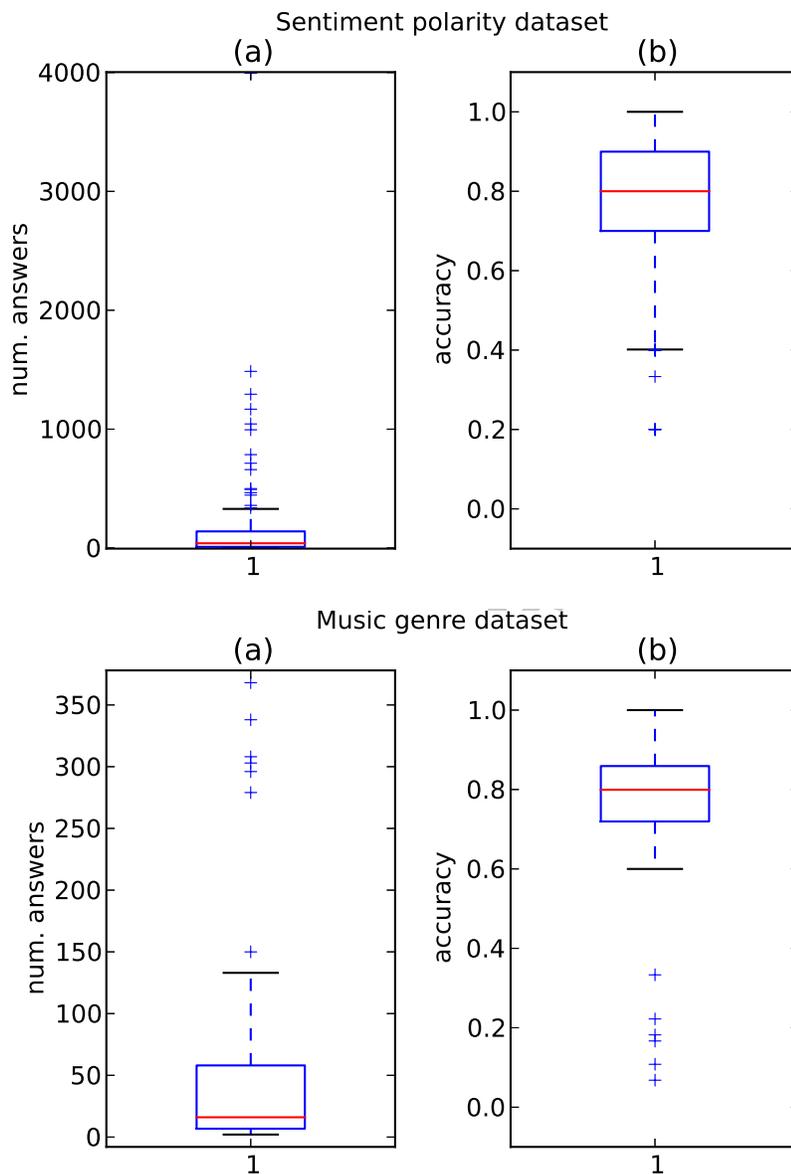


Figure 5: Boxplots for the number of answers (a) and for the accuracies (b) of the AMT workers for the sentiment polarity (top) and music genre (bottom) datasets.

Table 3: Trainset and testset accuracies for the different approaches on the datasets obtained from AMT.

Method	Sentiment polarity		Music genre	
	Train acc.	Test acc.	Train acc.	Test acc.
MVsoft	80.70%	71.65%	67.43%	60.33%
MVhard	79.68%	70.27%	67.71%	59.00%
Raykar	49.91%	48.67%	9.14%	12.00%
Raykar (w/prior)	84.92%	70.78%	71.86%	63.00%
MA-LR	85.40%	72.40%	72.00%	64.00%

382 of the dataset to 1200 features.

383 Regarding the music genre dataset, we used Marsyas<sup>6</sup>, a standard music  
 384 information retrieval tool, to extract a collection of commonly used features  
 385 in this kind of tasks (Tzanetakis and Cook, 2002). These include means and  
 386 variances of timbral features, time-domain Zero-Crossings, Spectral Centroid,  
 387 Rolloff, Flux and Mel-Frequency Cepstral Coefficients (MFCC) over a texture  
 388 window of 1 second. A total of 124 features were extracted. The details on  
 389 these features fall out of the scope of this article. The interested reader is  
 390 redirected to the appropriate literature (e.g. Aucouturier and Pachet (2003);  
 391 Tzanetakis and Cook (2002)).

392 Table 3 presents the results obtained by the different methods on the sen-  
 393 timent polarity and music genre datasets. As expected, the results indicate  
 394 that both annotator-aware methods are clearly superior when compared to

<sup>6</sup><http://marsyasweb.appspot.com>

395 the majority voting baselines. Also, notice that due to the fact some anno-  
396 tators only label a very small portion of instances, the “standard” model by  
397 Raykar et al. (2009, 2010) performs very poorly (as bad as a random classi-  
398 fier) due to overfitting. In order to overcome this, a prior had to be imposed  
399 on the probability distribution that controls the quality of the annotators.  
400 In the case of the sentiment polarity task, a  $Beta(1, 1)$  prior was used, and  
401 for the music genre task we applied a symmetric Dirichlet with parameter  
402  $\alpha = 1$ . Despite the use of a prior, the model by Raykar et al. (2009, 2010)  
403 still performs worse than the proposed MA-LR model, which takes advan-  
404 tage of its single quality parameter per annotator to produce better estimates  
405 of the annotators’ reliabilities. These results are coherent with our findings  
406 with the simulated annotators, which highlights the quality of the proposed  
407 model.

## 408 6. Conclusions and Future Work

409 In this paper we presented a new probabilistic model for supervised multi-  
410 class classification from multiple annotator data. Unlike previous approaches,  
411 in this model the reliabilities of the annotators are treated as latent variables.  
412 This design choice results in a model with various attractive characteristics,  
413 such as: its easy implementation and extension to other classifiers, the nat-  
414 ural extension to structured prediction problems (as opposed to the com-  
415 monly used latent ground truth models), and the ability to overcome the  
416 overfitting to which more complex models of the annotators expertise are  
417 susceptible as the number of instances labeled by each annotator decreases.

418 We empirically showed, using both simulated annotators and human-

419 labeled data from Amazon Mechanical Turk, that under most conditions,  
420 the proposed approach achieves comparable or even better results when com-  
421 pared to a state of the art model (Raykar et al., 2009, 2010) despite its much  
422 smaller set of parameters to model the annotators expertise. In fact, it turned  
423 out that this reduced number of parameters plays a key role in making the  
424 model less prone to overfitting.

425 Future work will explore the behavior of the proposed model when we  
426 relax the assumption that the reliability of the annotators does not depend  
427 on the instances that they are labeling, similarly to what is done in Yan et al.  
428 (2010). Furthermore, the generalization to sequence labeling tasks will also  
429 be investigated.

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## Highlights:

We propose a new probabilistic model for learning with multiple annotators.

The reliability of the different annotators is treated as a latent variable.

Model is able to achieve state of the art performance (or superior).

Reduced number of model parameters is able to avoid overfitting.

Model is easier to implement and extend to other classes of learning problems.

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