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# Dual-Arm Cooperative Manipulation under Joint Limit Constraints

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## Abstract

Cooperative manipulation of a rigid object is challenging and represents an interesting and active research area, especially when these robots are subject to joint and task prioritization constraints. In cooperative manipulation, a primary task is to maintain the coordination of motions, to avoid severe damage caused by the violation of kinematic constraints imposed by the closed chain mechanism. This paper proposes a kinematic controller for dual-arm cooperative manipulation that ensures safety by providing relative coordinated motion as highest priority task and joint limit avoidance and world-space trajectory following at a lower priority. The coordination of motions is based on modular relative Jacobian formulation. The approach is applicable to systems composed of redundant or non-redundant manipulators. Experiments in simulation demonstrate the behavior of the approach under different redundancy configurations. Experiments on two robots with different number of redundant motions show the applicability of the proposed approach to cooperative manipulation under joint limit constraints.

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## 1. Introduction

The use of multiple cooperative manipulators is advantageous in handling long, large and heavy objects as well as for performing cooperative tasks [1, 2]. Potential applications for cooperative manipulation range from manufacturing  
5 industry [3] to hazardous environments such as nuclear sites [4], underwater [5] and space [6, 7]. Cooperation can lead to decreased costs since a wide range of tasks can be accomplished through the use of multiple simpler and less expensive robots. At times the cost efficiency leads to the use of a heterogeneous group of robots [8]. However, the control of a dual arms system can be difficult when  
10 it is composed by two manipulators having a different number of redundant motions (system with mixed redundancy). Indeed, when the number of critical joints (the number of joints close to their operational limits) of a cooperating manipulator is higher than its redundant motions, then some joints limits are not avoided and the cooperation quality is drastically degraded, even if the other  
15 cooperating manipulator has available redundant motions.

A pair of cooperative manipulators that manipulates a rigid body forms a closed kinematic chain [9]. For this reason, the manipulators need to maintain precise kinematic coordination and movement synchronisation to avoid damage. To maintain the coordination, we adopt the relative Jacobian method which was  
20 introduced by Lewis [10] and recently expressed in a more compact formulation by Jamisola *et al.* [11]. The relative Jacobian approach allows to develop a single kinematic control for two cooperative manipulators as a single redundant manipulator, which possesses the end-effector motion equals to the relative end-effector motion, and the number of joints equals to the total number of joints  
25 possessed by the cooperating manipulators.

Although the method enables cooperative tasks, its application under constraints, such as joint limits, is not straightforward. The handling of joint

motion constraints, that is to say limits on joint position or velocity, has been addressed extensively in the single manipulator case [12, 13, 14]. In manipulators with redundancy, joint position limits can be usually avoided without  
 30 sacrificing the end-effector position tracking accuracy while in non-redundant systems joint limit avoidance can cause trajectory tracking errors. Joint limit avoidance of cooperative manipulators has been considered by only few authors [15, 16] even though the additional coordinated motion constraint is a potential  
 35 cause of complications.

In this paper, we propose a kinematic controller for cooperative manipulation under joint limit constraints by using of the relative Jacobian method. In particular, the proposed controller assigns a priority execution level to each sub-task of the cooperative manipulation, in the following decreasing order of  
 40 priority:

1. relative end-effector motion;
2. world-space end-effector motion;
3. joint limit avoidance by using redundant motions.

The combination of a redundant and a non-redundant manipulator is redundant with respect to relative motions. However, our study considers the case  
 45 where there is also a world-frame task with a lower priority. This world-frame task constrains all degrees of freedom of the non-redundant manipulator. Therefore, the world frame task cannot necessarily be satisfied if the non-redundant manipulator is at a joint limit. On the other hand, the redundant manipulator is able to satisfy the world frame task even at a joint limit. This is true even  
 50 though the number of total degrees of freedom is the same in both cases.

Therefore, when the redundancy is not able to avoid all joint limits, the proposed controller temporarily assigns the joint limit avoidance task a higher priority equal to the world-space end-effector motion. In this way, the performance on trajectory tracking is temporarily reduced, while the relative end-  
 55 effector motion (with highest priority) is not affected.

In order to evaluate the efficiency of the proposed controller, this paper con-

siders both redundant and non-redundant manipulators as well as a combination of the two, that is, one redundant and one non-redundant manipulator; all these cases within the same framework.

The main contributions of this work are:

- a kinematic controller based on a smooth activation function that provides gradual activation of joint limit avoidance for both redundant and non-redundant manipulators;
- simulation results that illustrate the behavior of the approach for two 3-DoF planar manipulators, whose number of redundant motions depends on the assigned Cartesian task;
- experiments on a heterogeneous dual-arm system, which is composed of a redundant 7-DoF manipulator and a 6-DoF non-redundant manipulator.

The paper is organized as follows. Section 2 presents the related works, while the theoretical background on the relative Jacobian is summarized in Section 3. Section 4 describes the proposed approach, which is applied in the simulation studies shown in Section 5. Experimental results about the cooperation of KUKA LWR 4+ and Kinova Jaco robots are reported in Section 6. The results are discussed in Section 7, conclusion and future works complete the paper in Section 8.

## 2. Related Work

Several studies have been performed on single redundant manipulators affected by joint limits, many of which are based on the Gradient Projection Method (GPM) [17]. This method aims to minimize the distance of the joints to their middle range position by projecting the gradient of a quadratic cost function. The main disadvantage of this method is that it uses all available redundant motions for keeping all joint positions in the middle of their range. In order to save some redundant motions, other approaches make use of the potential fields [18], operating only on those joints whose positions are close to

their limits. A recent work [13] projects a joint limit avoidance function based on Prescribed Performance Control methodology (PPC) into the Jacobian null space of the desired task. However, this method treats joint limit avoidance as a low priority task, hence, avoidance is not guaranteed. A method that ensures joint limit avoidance is described in [19]. It concerns a supervisory controller that permits to switch from the classical projection operator to a new large projection operator based on the directional redundancy method proposed in [20]. The method proposed in [21] consists of a hierarchy of controllers, in order to divide the main task into several subtasks. In particular, individual subtasks are progressively interrupted by a supervisory controller to ensure sufficient degrees of motion for joint limit avoidance. Finally, the Saturation in the Null Space (SNS) algorithm [22] considers the projection of the exceeded end-effector velocity into the null space of the main task Jacobian matrix, namely partial Jacobian matrix, composed by the not-saturated joints velocity. This method also includes a main task velocity scaling procedure, when the partial Jacobian null space becomes rank deficient.

Recently, the control of two cooperative manipulators under joint constraints has been proposed in [15] and [16]. Both approaches employ the relative Jacobian to coordinate relative motions and a prioritized task hierarchy to coordinate between possibly conflicting tasks. Both methods exhibit binary switching behavior in joint limit avoidance. As identified in [16], this may cause strong velocity transients that would require extremely high accelerations and could be a possible source of system failure.

In order to avoid such transients, we propose to use a smooth activation function for the joint limit avoidance task, similar to [14, 18, 23] for individual manipulators. In particular, we propose to use the hyperbolic tangent as the activation function [18]. The function provides a smooth ( $C^\infty$ ) but thin transition and its advantage compared to a binary activation matrix was experimentally demonstrated in [24]. As a further difference to [15, 16], this work considers a heterogeneous setting, which allowed us to make interesting observations and findings about control of heterogeneous dual-arm systems, as presented in Sec-

tion 7.

### 3. The relative Jacobian method

In order to define the relative Jacobian method for possibly redundant manipulators, we begin by introducing the definitions of redundancy. A generic open chain manipulator is defined as *kinematically redundant* when its degree of motion  $n$  (number of joints) is higher than the number of variables  $r$  that are necessary to describe a given task (dimension of the task space),  $n > r$  [25]. In particular, when the manipulator also has a degree of motion higher than the dimension of the space  $m$  in which the manipulator operates,  $n > m$  with  $m \geq r$ , it is defined as *intrinsically redundant*. Otherwise, if  $n = m$  with  $m > r$  (and thus  $n > r$ ), it is defined as *functionally redundant*. The last definition does depend on the number of the task variables  $r$ , so that the same manipulator can be functionally redundant with respect to a specific task  $T_1$  with  $r_1 < m$ , and non-redundant with respect to another task  $T_2$  with  $r_2 = m$ . This property will be used in the Section 5 in order to study the cooperation of two planar manipulators having equal kinematic structures but different degree of redundancy.

#### 3.1. Background on relative Jacobian method

Let us consider two cooperative manipulators  $A$  and  $B$  as shown in Figure 1, which possess a number of joints equal to  $n_a$  and  $n_b$ , respectively. Manipulator  $A$  is considered to have the role of master. The frame of the end-effector  $B$  (denoted as  $B_e$ ) is expressed with respect to the frame of end-effector  $A$  (denoted as  $A_e$ ), by using vectors  $\vec{P}_R$  and  $\vec{\phi}_R$ . In detail, the vector  $\vec{P}_R$  defines the end-effector's relative position by Cartesian coordinates, while the vector  $\vec{\phi}_R$  describes the end-effector's relative orientation by using a minimum orientation representation (see, e.g., [25]).

Differentiating these two vectors with respect to time, it is possible to define a unique vector  $\dot{\vec{x}}_{R_k}$ , which contains the end-effector's relative velocity compo-

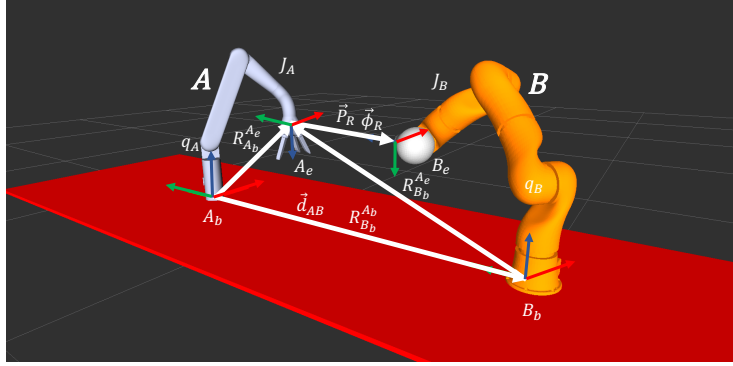


Figure 1: Heterogeneous two-robot system: coordinate frame transformation for the relative Jacobian formulation.

145 nents at time instant  $k$

$$\dot{\vec{x}}_{R_k} = \begin{bmatrix} \dot{\vec{P}}_R \\ \dot{\vec{\phi}}_R \end{bmatrix} \quad (1)$$

where  $\dot{\vec{x}}_{R_k}$  is a  $r_{ab}$  dimensional column vector; note that  $r_{ab} \leq 6$  in a three dimensional space.

The kinematic relations between the two manipulators permit to obtain a single equivalent manipulator having a number of joints equal to  $n_{ab} = n_a +$   
 150  $n_b$  and relative end-effector velocity components defined by the vector  $\dot{\vec{x}}_{R_k}$ . Therefore, it is possible to calculate the Jacobian matrix associated with this equivalent manipulator, namely relative Jacobian  $\mathbf{J}_R$ , which is a  $(r_{ab} \times n_{ab})$  dimensional matrix, which can be expressed in the following compact form [11]

$$\mathbf{J}_R(\vec{q}_{a_k}, \vec{q}_{b_k}) = \begin{bmatrix} -\psi_{B_e}^{A_e} \Omega_{A_b}^{A_e} \mathbf{J}_A & \Omega_{B_b}^{A_e} \mathbf{J}_B \end{bmatrix} \quad (2)$$

where  $\mathbf{J}_R$  matrix depends on joint position vectors of both manipulators,  $n_a$ -  
 155 dimensional column vector  $\vec{q}_{a_k}$  and  $n_b$ -dimensional column vector  $\vec{q}_{b_k}$ , respectively. The  $\mathbf{J}_R$  matrix, expressed in the compact form, shows explicitly the analytic Jacobians of the standalone manipulators, namely  $\mathbf{J}_A$  of dimension  $(r_{ab} \times n_a)$ , and  $\mathbf{J}_B$  of dimension  $(r_{ab} \times n_b)$ . Since the relative end-effector motions are expressed in frame  $A_e$ , the two block diagonal matrices  $\Omega_{A_b}^{A_e}$  and  $\Omega_{B_b}^{A_e}$   
 160 are used in (2) to transform the Jacobians from each manipulator base frame,



$A_b$  and  $B_b$ , to frame  $A_e$ , as shown in Figure 1. These matrices can be expressed in terms of rotation matrices as

$$\Omega_{A_b}^{A_e} = \begin{bmatrix} \mathbf{R}_{A_b}^{A_e} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{A_b}^{A_e} \end{bmatrix} \quad \Omega_{B_b}^{A_e} = \begin{bmatrix} \mathbf{R}_{B_b}^{A_e} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{B_b}^{A_e} \end{bmatrix} \quad (3)$$

Finally,  $\psi_{B_e}^{A_e}$  represents the wrench transformation matrix that compensates the translation component of the relative velocity, originating from the cross product of end-effector  $A$  orientation velocity  $\dot{\vec{\phi}}_A$  with the end-effector position vector  $\vec{P}_R$ ,

$$\psi_{B_e}^{A_e} = \begin{bmatrix} \mathbf{I} & -\mathbf{S}(\vec{P}_R) \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \quad (4)$$

where  $\mathbf{S}(\vec{P}_R)$  is the skew symmetric matrix generated by vector  $\vec{P}_R$ .

The relative Jacobian matrix  $\mathbf{J}_R$  expressed in (2) presents several advantages. First of all, the relative Jacobian is simple to obtain from the Jacobians of the individual manipulators. Moreover, configuration changes are easy to handle since it is sufficient to change the respective Jacobian matrix with that of the new manipulator.

The relation between the relative end-effector motion vector  $\dot{\vec{x}}_{R_k}$  and the joint velocity vectors of both manipulators,  $\dot{\vec{q}}_{a_k}$  and  $\dot{\vec{q}}_{b_k}$ , can be written using  $\mathbf{J}_R$  as

$$\dot{\vec{x}}_{R_k} = \mathbf{J}_R \dot{\vec{q}}_{ab_k} \quad (5)$$

where  $\dot{\vec{q}}_{ab_k} = [\dot{\vec{q}}_{a_k}, \dot{\vec{q}}_{b_k}]^T$  is a  $n_{ab}$ -dimensional vector. Since  $r_{ab} < n_{ab}$ , the inversion of (5) admits infinite solutions, therefore a criterion has to be adopted to choose the suitable solution. A possible criterion is to minimize the norm of  $\dot{\vec{q}}_{ab_k}$  using the Moore-Penrose pseudo-inverse resulting in [25]

$$\mathbf{J}_R^\dagger = \mathbf{J}_R^T (\mathbf{J}_R \mathbf{J}_R^T)^{-1} \quad (6)$$

Finally, in order to reduce the relative end-effector position and orientation error  $\vec{e}_{R_k}$ , defined as the difference between desired relative end-effector pose  $\vec{x}_{R_{d_k}}$  and the measured pose  $\vec{x}_{R_k}$ , namely

$$\vec{e}_{R_k} = \vec{x}_{R_{d_k}} - \vec{x}_{R_k} = \begin{bmatrix} \vec{P}_{R_d} & \vec{\phi}_{R_d} \end{bmatrix}^T - \begin{bmatrix} \vec{P}_R & \vec{\phi}_R \end{bmatrix}^T \quad (7)$$

it is possible to add a feedback correction term to inversion of (5). Therefore, the following first-order kinematic algorithm, based on Closed-Loop Inverse Kinematics (CLIK) [17], can be adopted

$$\dot{\vec{q}}_{ab_k} = \mathbf{J}_R^\dagger (\dot{\vec{x}}_{R_{dk}} + \mathbf{K}_R \vec{e}_{R_k}) \quad (8)$$

where  $\dot{\vec{x}}_{R_{dk}}$  is the desired relative end-effector motion vector, while  $\mathbf{K}_R$  is a constant positive-defined gain matrix that defines the convergence rate of  $\vec{e}_{R_k}$  [25].

## 4. Methodology

### 4.1. Hierarchic prioritized task architecture

Redundancy can be used to perform one or more secondary tasks, which possess lower execution priority with respect to the main task. The maximum number of tasks  $l$ , that can be simultaneously handled, depends on the number of degrees of motion equal to the number of joints  $n_{ab}$  and the rank  $s$  of the Jacobian associated with each task [26]. Therefore, when choosing tasks in a non-conflicting way, it is possible to add tasks until

$$\sum_{i=1}^l s_i = n_{ab} \quad (9)$$

In order to avoid task conflicts, Chiaverini *et al.* [27] introduced a hierarchic prioritized task architecture, in which lower priority tasks are projected on the null space of the higher priority ones. In this way, the lower priority tasks do not affect performance of the higher priority tasks and the performance of the highest priority task is always guaranteed.

Therefore, given a generic single manipulator and three prioritized tasks  $\dot{\mathbf{x}}_1$ ,  $\dot{\mathbf{x}}_2$  and  $\dot{\mathbf{x}}_3$  (where the subscript having lowest value indicates the highest priority task), it is possible to obtain the joint velocity vector  $\dot{\vec{q}}$  according to the hierarchic prioritized task architecture

$$\dot{\vec{q}} = \mathbf{J}_1^\dagger \dot{\mathbf{x}}_1 + \mathbf{P}_1 \left( \mathbf{J}_2^\dagger \dot{\mathbf{x}}_2 + \mathbf{P}_2 \mathbf{J}_3^\dagger \dot{\mathbf{x}}_3 \right) \quad (10)$$

where  $\mathbf{J}_i^\dagger$  ( $i = 1, 2, 3$ ) is the pseudo inverse of the Jacobian of each task, while  $\mathbf{P}_j$  ( $j = 2, 3$ ) indicates the orthogonal projector on the null space  $\mathbf{N}_j$ , that is obtained by  $\mathbf{I} - \mathbf{J}_j^\dagger \mathbf{J}_j$  (where  $\mathbf{I}$  indicates the identity matrix).

As proposed in [11, 28], in the dual arm system based on the Jacobian null space projection, we set the highest priority task to retain motion coordination represented by  $\dot{\mathbf{x}}_{R_{dk}}$ , while the secondary task is given by the desired  $A_e$  motion  $\dot{\mathbf{x}}_{A_{dk}}$ , which is expressed with respect to its base frame  $A_b$ . Moreover, if the system possesses redundant motions, in accordance with (9), it is possible to add a third task in order to satisfy further constraints (e.g., increasing the system manipulability [29] or avoiding joint position limits [28]). Since the study in this paper is focused on the joint limits avoidance, it can be expressed as a repulsive joint velocity vector  $\dot{\mathbf{q}}_k^+$ , which pushes the joint positions away from their limits. By expressing the three tasks described in (10) according to the relative Jacobian of (8), a kinematic controller for the dual arm system can be written as

$$\dot{\mathbf{q}}_{abk} = \mathbf{J}_R^\dagger (\dot{\mathbf{x}}_{R_{dk}} + \mathbf{K}_R \mathbf{e}_{Rk}) + \mathbf{P}_R \left( [\mathbf{J}_A \ \mathbf{0}]^\dagger (\dot{\mathbf{x}}_{A_d} + \mathbf{K} \mathbf{e}_k) + \mathbf{P}_{AB} \dot{\mathbf{q}}_k^+ \right) \quad (11)$$

where  $\mathbf{K}$  is the feedback gain related to the end-effector  $A$  position error  $\mathbf{e}_k = \mathbf{x}_{A_{dk}} - \mathbf{x}_{A_k}$ . Finally, the  $(n_{AB} \times n_{AB})$  dimensional matrix,  $\mathbf{P}_{AB}$ , can be defined as

$$\mathbf{P}_{AB} = \begin{bmatrix} \mathbf{P}_A & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_B \end{bmatrix} \quad (12)$$

which contains the orthogonal projectors into the Jacobian null spaces of the both manipulators  $A$  and  $B$ , respectively  $\mathbf{P}_A = (\mathbf{I} - \mathbf{J}_A^\dagger \mathbf{J}_A)$  and  $\mathbf{P}_B = (\mathbf{I} - \mathbf{J}_B^\dagger \mathbf{J}_B)$ , while  $\mathbf{P}_R = (\mathbf{I} - \mathbf{J}_R^\dagger \mathbf{J}_R)$  is the orthogonal projector matrix into the relative Jacobian null space  $\mathbf{N}_R$ .

**Remark.** Note that the subscript  $k$  related to time will be ignored in the following to improve readability.

#### 4.2. Proposed joint limit avoidance strategy

A classical approach to avoid joint limits is to define the gradient of a cost function as the lowest priority task [25]. This approach guides each joint towards

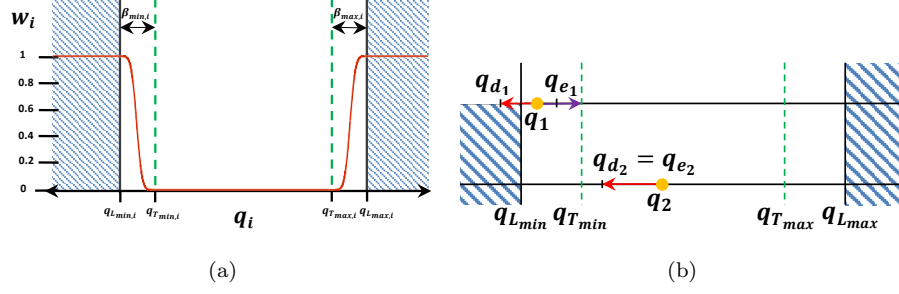


Figure 2: (a) Activation function for one component  $w_i$  of  $W$  matrix, (b) Operating principle of the joint limits avoidance strategy for a generic not redundant manipulator

the middle of its range, regardless of the joint position's closeness to the limit. In order to optimize the number of redundant motions that are employed in the joint limit avoidance task, we use a repulsive joint velocity that moves only the critical joints away from their limits [14]. In particular, we define for the  $i$ -th joint of position  $q_i$ , the following sets:

- joint position limits,  $q_{L_i} = [q_{Lmin,i}, q_{Lmax,i}]$ ;
- the activation threshold for the repulsive motion,  $q_{T_i} = [q_{Tmin,i}, q_{Tmax,i}]$ , where  $q_{Tmin,i} \geq q_{Lmin,i}$  and  $q_{Lmax,i} \geq q_{Tmax,i}$ ;
- the size of the interval where the activation function is smooth,  $\beta_i = [\beta_{min,i}, \beta_{max,i}] = [q_{Tmin,i} - q_{Lmin,i}, q_{Lmax,i} - q_{Tmax,i}]$ , as shown in Figure 2(a);
- the distances from its limits,  $\alpha_i = [\alpha_{min,i}, \alpha_{max,i}] = [q_i - q_{Lmin,i}, q_{Lmax,i} - q_i]$ .

Then the  $i$ -th joint is said to be a critical joint if  $\alpha_{min,i} < \beta_{min,i} \vee \alpha_{max,i} < \beta_{max,i}$ , i.e., if a generic  $q_i$  crosses the activation threshold  $q_{Tmin,i}$  or  $q_{Tmax,i}$ , as shown in Figure 2(a). In detail, the symbols  $\beta_{min,i}$  and  $\beta_{max,i}$  are parameters that can be chosen by the designer: higher absolute values imply a smoother (slower but safer) repulsion motion, while smaller values a stiffer (faster but less safe) repulsion.  $\alpha_{min,i}$  and  $\alpha_{max,i}$  indicate instead the current distance between

the joint from its lower and upper limits. When the  $i$ -th joint of position  $q_i$  gets closer to those limits, then the repulsive velocity has the objective to push it away from the closest limit. Following the form of the smooth activation function provided in (15), then the repulsive velocity applies as soon as the  $i$ -th joint crosses the actuation threshold, before reaching the respective limit. In this way, the joint can actively avoid its limit, before getting too close to it.

For the dual arm system composed of two manipulators, it is possible to obtain a repulsive joint velocity vector for each manipulator,  $\dot{\vec{q}}_A^+$  and  $\dot{\vec{q}}_B^+$ , respectively as

$$\dot{\vec{q}}_A^+ = \mathbf{H}_A \mathbf{W}_A (\vec{q}_{T_A} - \vec{q}_A) \quad (13)$$

$$\dot{\vec{q}}_B^+ = \mathbf{H}_B \mathbf{W}_B (\vec{q}_{T_B} - \vec{q}_B) \quad (14)$$

where  $\vec{q}_{T_A}$  and  $\vec{q}_{T_B}$  are two  $n_a$  and  $n_b$  dimensional column vectors, which contain the joint threshold positions closest to the current joint position (i.e.,  $q_{T_{A_i}} = q_{T_{A_{i_{min}}}}$  when  $\alpha_{A_{i_{min}}} < \alpha_{A_{i_{max}}}$ ), while  $\vec{q}_A$  and  $\vec{q}_B$  are the current joint positions. Regarding to  $\mathbf{H}_A$  and  $\mathbf{H}_B$ , they are  $(n_A \times n_A)$  and  $(n_B \times n_B)$  dimensional diagonal matrices representing the gains of the control law of this task. However, these gains are weighted by the two smooth activation diagonal matrices  $\mathbf{W}_A$  and  $\mathbf{W}_B$ , whose components  $w_i(q_i)$  values belong to  $[0, 1]$ , as shown in Figure 2(a). By expressing  $w_i$  functions with respect to  $\alpha_i$ , it is possible to define formally the Figure 2(a), as reported in (15).

$$w_i(\alpha_i) = \begin{cases} 1 & \alpha_{min,i} < 0 \vee \alpha_{max,i} < 0 \\ \frac{1}{2} \left[ 1 - \tanh \left( \frac{1}{1 - \frac{\alpha_{min,i}}{\beta_{min,i}}} - \frac{\beta_{min,i}}{\alpha_{min,i}} \right) \right], & \alpha_{min,i} \in [0, \beta_{min,i}] \\ \frac{1}{2} \left[ 1 - \tanh \left( \frac{1}{1 - \frac{\alpha_{max,i}}{\beta_{max,i}}} - \frac{\beta_{max,i}}{\alpha_{max,i}} \right) \right], & \alpha_{max,i} \in [0, \beta_{max,i}] \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

The smooth transition allows to reduce the discontinuities in the joint velocity signals compared to a binary activation matrix [18].

Finally, setting  $\dot{\vec{q}}^+ = [\dot{\vec{q}}_A^+, \dot{\vec{q}}_B^+]^T$  in (11), it is possible to obtain the following

final compact matrix equation

$$\begin{aligned} \dot{\vec{q}}_{ab} = & \mathbf{J}_R^\dagger (\dot{\vec{x}}_{R_d} + \mathbf{K}\vec{e}_R) + \mathbf{P}_R \left( [\mathbf{J}_A \ \mathbf{0}]^\dagger (\dot{\vec{x}}_{A_d} + \mathbf{K}\vec{e}) + \right. \\ & \left. \mathbf{P}_{AB} \mathbf{H}_{AB} \mathbf{W}_{AB} (\vec{q}_{TAB} - \vec{q}_{AB}) \right) \end{aligned} \quad (16)$$

where  $\vec{q}_{TAB} = [\vec{q}_{TA}, \vec{q}_{TB}]^T$ ,  $\vec{q}_{AB} = [\vec{q}_A, \vec{q}_B]^T$ , while  $\mathbf{H}_{AB}$  and  $\mathbf{W}_{AB}$  are defined as

$$\mathbf{H}_{AB} = \begin{bmatrix} \mathbf{H}_A & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_B \end{bmatrix} \quad \mathbf{W}_{AB} = \begin{bmatrix} \mathbf{W}_A & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_B \end{bmatrix}. \quad (17)$$

Note that the joint limit avoidance task has lower priority than trajectory following. This can appear counterintuitive. However, if the system is redundant, the joint limit avoidance task avoids losing trajectory tracking performance while still using the redundancy to avoid joint limits. In case of a dual-arms system is composed by at least one non-redundant manipulator, the submatrices  $\mathbf{P}_A$  (or/and  $\mathbf{P}_B$ ) of the  $\mathbf{P}_{AB}$  matrix projects the joint limit avoidance task into the zero dimension null-spaces, so that it results to be irrelevant. Hence, it is necessary to assign the same priority execution of the trajectory tracking task to the joint limit avoidance task, by opportunely editing the  $\mathbf{P}_{AB}$  matrix.

In detail, to ensure that the joint limits are satisfied, we let the trajectory performance related to  $\dot{\vec{x}}_{A_d}$  degrade gradually and temporarily by having same priority for trajectory tracking and joint limit avoidance. To keep this specific to the non-redundant manipulator, Eq. (16) is adapted by replacing the null-space projection matrix of the non-redundant manipulator with an identity matrix  $\mathbf{I}$  having the same dimension. In other words,  $\mathbf{P}_{AB}$  in (16) can be replaced with one of the orthogonal projectors

$$\mathbf{P}_{IB} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_B \end{bmatrix} \quad \mathbf{P}_{IA} = \begin{bmatrix} \mathbf{P}_A & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \quad (18)$$

where  $\mathbf{P}_{IX}$  is the projector matrix when the manipulator  $X$  is non-redundant. Finally, if both manipulators are non-redundant,  $\mathbf{P}_{AB}$  matrix is replaced by an identity matrix  $\mathbf{I}_{AB}$ .

Figure 2(b) shows an example of the proposed joint limit avoidance for a standalone cooperating non-redundant manipulator having the same joint limit values. If the manipulator has a critical joint  $q_1$ , this joint will converge to an equilibrium joint position  $q_{e_1}$ , where two opposite velocity components cancel out each other. In detail, the first component, defined as  $\dot{q}_{d_1} = k_1(q_{d_1} - q_1)$ , pushes  $q_1$  towards its desired position  $q_{d_1}$  according to the desired end-effector trajectory, while the second one, defined as  $\dot{q}_1^+ = h_1 w_1 (q_{T_1} - q_1)$  (with  $w_1 \neq 0$ ), pushes  $q_1$  towards the minimum threshold position  $q_{T_{min}}$ . On the other hand, a non-critical joint  $q_2$  is not affected by the second velocity signal ( $w_2 = 0$ ), therefore its equilibrium joint position  $q_{e_2}$  will converge to its desired position  $q_{d_2}$ . Therefore, it is possible to formalize this example according to (16):

$$\begin{aligned} \begin{bmatrix} q_{e_1} \\ q_{e_2} \end{bmatrix} &= \left( \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} q_{d_1} - q_1 \\ q_{d_2} - q_2 \end{bmatrix} + \begin{bmatrix} h_1 & 0 \\ 0 & h_2 \end{bmatrix} \begin{bmatrix} w_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_{T_{min_1}} - q_1 \\ q_{T_{min_2}} - q_2 \end{bmatrix} \right) \Delta t \\ &\quad + \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \end{aligned} \quad (19)$$

where  $\Delta t$  indicates the sample time. However, the equilibrium joint positions,  $q_{e_1}$  and  $q_{e_2}$ , are not the final joint positions, because of the added effect due to the relative end-effectors motions, as expressed by the first term of (19). Therefore, this example shows that an out-of-limit joint position is managed by the proposed method by allowing world-space trajectory deviations, but not losing cooperation performance.

## 5. Simulation results

In this section, we show how to use the proposed methodology to ensure that joint limits are preserved (i.e., there is no violation of joint position constraints) in a dual-arm system performing a desired task. We consider two identical planar manipulators with 3-DoF in three different cases:

**Case I** both manipulators are non-redundant;

**Case II** both manipulators have one redundant motion;

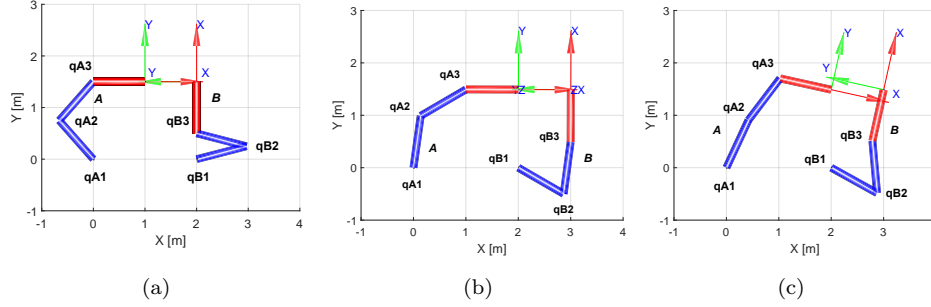


Figure 3: Manipulator configurations: (a) initial, (b) Case I and III final configuration, (c) Case II final configuration.

320 **Case III** manipulator *A* has no redundant motions, while manipulator *B* has one redundant motion.

The manipulators considered in the simulation study are functionally redundant, as described in Section 3. Therefore, the degrees of redundancy of each manipulator,  $dr_A$  and  $dr_B$ , can be calculated by applying the rank-nullity theorem [25] as  $n - r = dr$ . Since both manipulators have the same number of joints (325  $n_A = n_B = 3$ ), the degree of redundancy of each manipulator can be changed by assigning different number of task variables to each end-effector. Hence, the tasks proposed in **Case I** are defined by three variables  $r_A = r_B = 3$ , so that the degree of redundancy is equal to zero for both manipulator  $dr_A = dr_B = 0$ .

330 **Case II** presents two tasks described by only two motion variables  $r_A = r_B = 2$  (translation motions along  $x$  and  $y$  axes), so that the degree of redundancy is equal to one,  $dr_A = dr_B = 1$ . Finally, **Case III** presents the cooperation of manipulators having different degree of redundancy. The task of **Case I** is assigned to manipulator *A*, while the task of **Case II** is assigned to manipulator 335 *B*. Therefore, the degrees of redundancy are  $dr_A = 0$  and  $dr_B = 1$ .

### 5.1. Task description

The proposed task consists of translating the dual arm system from the initial *A* end-effector position  $P_1 = [1, 1.5]$  m (see Figure 3(a)) to final position



$P_2 = [2, 1.5]$  m, while keeping the relative pose constant. Moreover, two joint limits  $q_{A2}$  and  $q_{B1}$  are enforced using (13) and (14) with the values reported in Table 1 (with  $\beta = \beta_{min} = \beta_{max}$ ).

Table 1: Joint limits.

Critical Joint	$q_{L[min,max]}$	$q_{T[min,max]}$	$\beta$	$k_i$	$h_i$
$q_{A2}$	$[-1.7, 3.14]$	$[-1.5, 2.94]$	0.2	10	20
$q_{B1}$	$[-0.7, 3.14]$	$[-0.5, 2.94]$	0.2	10	20

The proposed task is decomposed into prioritized subtasks according to (16). Since the execution of the joint limit avoidance depends on  $dr_A$  and  $dr_B$ , their execution priority changes for each cooperation case, as reported in Table 2.

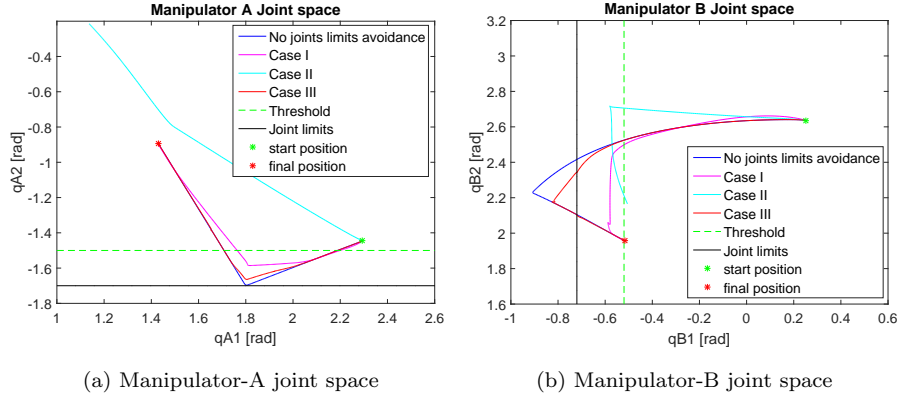


Figure 4: A (B) manipulator joint position space relative to  $q_{A1}$ - $q_{A2}$  (a) ( $q_{B1}$ - $q_{B2}$  (b)).

## 5.2. Results and discussion

### 5.2.1. Case I

In this case both manipulators have no degree of redundancy available for the execution of the joint limit avoidance,  $dr_A = dr_B = 0$ . Therefore, it is possible to assign to this task the priority of the higher priority task  $\dot{\vec{x}}_{A_d}$ , by

Table 2: Decomposition of the task into prioritized subtasks.

Case	Priority	Sub-task description	dim
I	3	$\dot{\vec{x}}_{R_d} = [0 \text{ m/s}, 0 \text{ m/s}, 0 \text{ rad/s}]^T$	3
	2	$\dot{\vec{x}}_{A_d} = [0.01 \text{ m/s}, 0.01 \text{ m/s}, 0 \text{ rad/s}]^T$	3
	2	$\dot{\vec{x}}_{B_d} = [0.01 \text{ m/s}, 0.01 \text{ m/s}, 0 \text{ rad/s}]^T$	3
	2	$\dot{q}_{A2}^+ = 20w_{A2}(-1.5 \text{ rad} - q_{A2})$	1
	2	$\dot{q}_{B1}^+ = 20w_{B1}(-0.5 \text{ rad} - q_{B1})$	1
II	3	$\dot{\vec{x}}_{R_d} = [0 \text{ m/s}, 0 \text{ m/s}, 0 \text{ rad/s}]^T$	3
	2	$\dot{\vec{x}}_{A_d} = [0.01 \text{ m/s}, 0.01 \text{ m/s}]^T$	2
	2	$\dot{\vec{x}}_{B_d} = [0.01 \text{ m/s}, 0.01 \text{ m/s}]^T$	2
	1	$\dot{q}_{A2}^+ = 20w_{A2}(-1.5 \text{ rad} - q_{A2})$	1
	1	$\dot{q}_{B1}^+ = 20w_{B1}(-0.5 \text{ rad} - q_{B1})$	1
III	3	$\dot{\vec{x}}_{R_d} = [0 \text{ m/s}, 0 \text{ m/s}, 0 \text{ rad/s}]^T$	3
	2	$\dot{\vec{x}}_{A_d} = [0.01 \text{ m/s}, 0.01 \text{ m/s}, 0 \text{ rad/s}]^T$	3
	2	$\dot{\vec{x}}_{B_d} = [0.01 \text{ m/s}, 0.01 \text{ m/s}]^T$	2
	2	$\dot{q}_{A2}^+ = 20w_{A2}(-1.5 \text{ rad} - q_{A2})$	1
	1	$\dot{q}_{B1}^+ = 20w_{B1}(-0.5 \text{ rad} - q_{B1})$	1

replacing the  $\mathbf{P}_{AB}$  in (16) with an identity matrix  $\mathbf{I}_{AB}$  as described in Section 3. Figure 4 shows the joint position space relative to the first two joints of each manipulator. In particular, the difference between the joint trajectory without the proposed joint limit avoidance strategy (blue line) and with it (violet line). The figure shows that the joint limit avoidance works correctly, pushing both joints away from their limits when  $q_{A2}$  and  $q_{B1}$  exceed their threshold limits (green dotted line). Due to the joint limit avoidance, the trajectory tracking accuracy is degraded temporarily for both end-effectors, as shown in Figure 5.

Moreover, Figure 3(b) shows that the final end-effector orientations assumed by both manipulators are equal to the initial ones (see Figure 3(a)), enforced by the relative motion task  $\dot{\vec{x}}_{R_d}$ .

### 5.2.2. Case II

In this case each manipulator possesses one degree of redundancy, because both end-effector orientation velocities are not specified. Therefore, (16) is applied directly. Since a redundant manipulator admits an infinite number of solutions for the inverse kinematic problem, it is possible to note in Figure 4 that the joint trajectories obtained (sky blue) are completely different from the trajectories tracked without joint limit avoidance (blue). However, the repulsive joint velocities generate self-motions in each manipulator not affecting their relative pose or their translation task. Therefore, the final configurations of the manipulators obtained (Figure 3(c)) are different from the starting ones (Figure 3(a)), and the path following error is negligible (green line in Figure 5).

### 5.2.3. Case III

This last case is a combination of the previous ones. Since manipulator  $A$  is non-redundant ( $\dot{\phi}_A$  is assigned), it is necessary to replace the  $\mathbf{P}_{AB}$  in (16) with the matrix  $\mathbf{P}_{IB}$  defined in (18). In this way,  $\dot{q}_{A2}$  has the execution priority of  $\dot{\vec{x}}_{A_d}$ , while  $\dot{q}_{B1}$  has lower priority (see Table 2). It is interesting to note that while the joint limit of  $q_{A2}$  is satisfied (see red line in Figure 3(a)), the joint

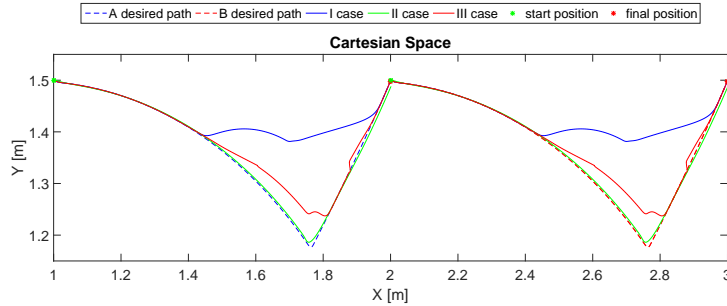
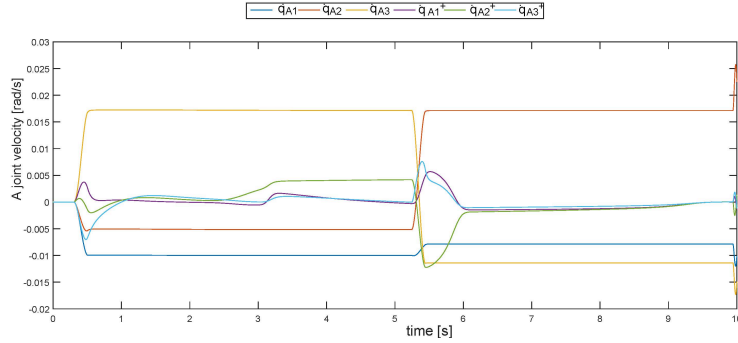
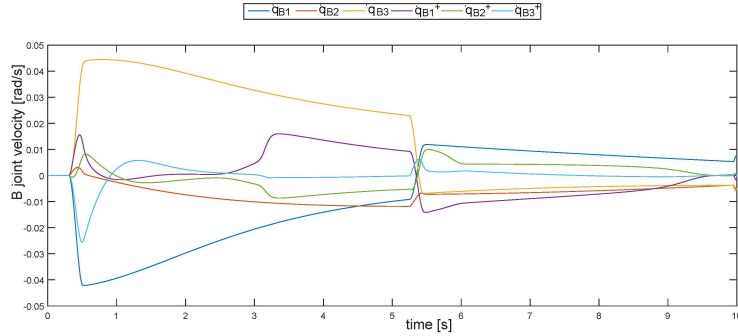


Figure 5:  $A$  and  $B$  manipulators Cartesian path for the three considered simulation cases.

limit of  $q_{B1}$  is violated (see red line in Figure 3(b)). In fact, although the  $\dot{\phi}_B$  is not assigned,  $\phi_B$  must be kept constant respect to  $\phi_A$  as specified from highest priority task  $\dot{\mathbf{x}}_{R_d}$ . Therefore,  $\dot{\phi}_B$  depends on  $\dot{\phi}_A$ , and consequently the  $B$  manipulator loses its degree of redundancy, so that the  $\dot{\mathbf{q}}_B^+$  can not be performed. In order to satisfy the joint limit of  $q_{B1}$ , it is necessary to assign to  $q_{B1}^+$  the same priority execution level of the higher priority task  $\dot{\mathbf{x}}_{B_d}$  as demonstrated in **Case I**. Therefore, the trajectory tracking performances of both manipulators are temporary degraded as described in **Case I**, and the final end-effector orientations are equal to those shown in Figure 3(b).



(a) Manipulator A



(b) Manipulator B

Figure 6: Desired and repulsive joint velocity signals obtained in case I for manipulator A (a) and manipulator B (b)

Joint velocities during motion are illustrated in Figure 6. This shows that

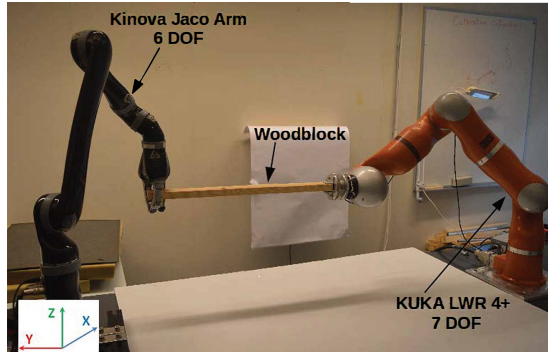


Figure 7: Two robot manipulators holding a woodblock to perform tightly coordinated cooperative operations.

unlike the existing method proposed in [16], the proposed controller strategy and the smooth activation function eliminate discontinuities in joint velocity commands.

## 6. Experimental results

The experimental setup is composed of two dissimilar robots holding a woodblock using their end-effector's (Figure 7). The coordinate framed for the set-up are depicted in Figure 1. Using the set-up, we investigate four cases (Cases A–  
 395 D) of cooperative manipulation, with joint position constraints shown in Table 3 (— denoting no joint limits). The goal of the experiment is to analyse how the proposed controller handles the task and joint constraints in the practical setting with different sources of error and uncertainty.

The manipulators adopted in the experimental set-up are a 6-DOF Kinova  
 400 Jaco (non-redundant) [30] and a 7-DOF KUKA LWR4+ (intrinsically redundant) [31] placed opposite and parallel to each other with a distance of 1.48 m. Their task is to cooperatively manipulate a woodblock of 0.9kg. The task priorities are similar to earlier:

1. Task 1 (highest priority): maintaining relative position and orientation of  
 405 the end-effector's such that  $\vec{P}_R = (0, 0.5, 0)$  m.

Table 3: Experimental cases.

Case	Kinova Jaco (non-redundant)	Kuka LWR (redundant)	Controller
Case A	—	—	Predefined path
Case B	<i>Joint Limit</i>	—	Online path change
Case C	—	<i>Joint Limit</i>	Similar to Case A
Case D	<i>Joint Limit</i>	<i>Joint Limit</i>	Similar to Case B

2. Task 2: moving the Jaco end-effector  $A_e$  along the predefined path.
3. Task 3: joint limit avoidance for cooperative manipulators (see Table 3).

The target trajectory is a circle with radius 0.04 m, as illustrated in Figure 8(a). Jaco starts from initial position  $A_e^o = [0.262, -0.258, 0.388]^T$  m. Joint limits to be enforced are shown in Table 4 (with  $\beta = \beta_{min} = \beta_{max}$ ).

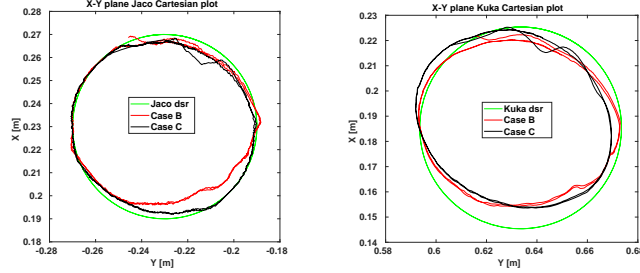
Table 4: Joint limit parameters.

Critical Joint	$q_{L_{[min,max]}}$	$q_{T_{[min,max]}}$	$\beta$	$k_i$	$h_i$
$q_{A3}$	[0.1, 0.59]	[0.15, 0.54,]	0.05	10	20
$q_{B1}$	[0.1, 0.88]	[0.15, 0.83]	0.05	10	20

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Figure 9 illustrates the Cartesian tracking errors and the relative pose error for Cases A-C. Case D is not shown as its behavior is almost identical to Case B. Case A (without joint limits) serves as a reference for the achievable performance due to limitations of the hardware, in particular Kinova Jaco. The maximum  
415 relative pose error during steady state is 5.1 mm.

In Case B with joint limit in the non-redundant robot (Jaco), the joint limit is avoided as shown in Figure 10(a). During joint limit avoidance, path following accuracy is temporarily sacrificed to up to 15 mm position error (shown by the red line in Figure 9(a)–(b)). The coordination of motion (Task 1) is kept  
420 enforced as the relative motion error is not increased over baseline (Figure 9(c)),



(a) Trajectory tracking by Jaco (b) Cooperative tracking by Kuka

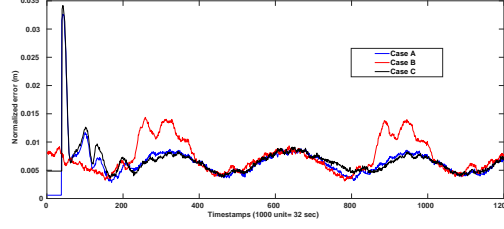
Figure 8: Coordinated manipulation results of tightly-coupled manipulators under joint constraints (Case B and C).

with maximum error 5.9 mm. The same behavior is illustrated in the Cartesian space in Figures 8(a)–(b).

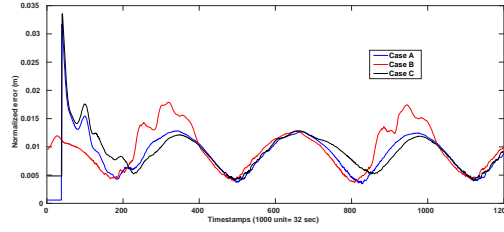
In Case C with joint limit in the redundant robot (KUKA LWR), the proposed controller is able to avoid the joint limit as illustrated in Figure 10b. Due to the redundancy, the accuracy of Cartesian trajectory or relative position are not deteriorated as shown in Figures 8 and 9 where the maximum relative pose error is 4.9 mm.

Even though KUKA can use its redundancy to avoid conflicts in joint and task space, this extra degree of freedom does not solve the problem of joint limit occurring on Jaco. Therefore the cooperative behaviour in Case D (joint limits for both) is similar to Case B (Jaco joint limit).

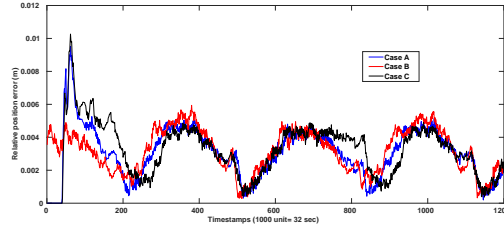
Looking more closely at the relative position errors in Figure 9(c), the errors remain small in all cases. In Case B the cooperative manipulators need to change Cartesian path to avoid joint limits, increasing the path following error temporarily but the limited relative motion error is maintained. This demonstrates the ability of the proposed approach to handle cooperative manipulation in a safe manner. Finally, cooperation between intrinsically redundant and non-redundant coupled robots demonstrates the proposed controller ability to take advantage of redundancy, as well as to temporarily sacrifice the Cartesian trajectory accuracy to comply with joint limits.



(a) (Cases A-C): Jaco path following error



(b) (Cases A-C): Kuka path following error



(c) (Cases A-C): Relative position error (RPE)

Figure 9: Cooperative performance of tightly-coupled manipulators for the Cases (Case A-C), refer to Table 3.

## 7. Discussion

The experimental results indicate one interesting observation for heterogeneous systems: The lower performance robot sets the performance limit for the entire system. This has a significant consequence for the system design in that the lower performance robot should be used as the master, whose trajectory tracking error is used as feedback. That is, in the context of this paper the lower performance robot should be manipulator *A* whose trajectory error,  $\vec{e}$  in (11), should be used as the trajectory feedback. This ensures that the perfor-



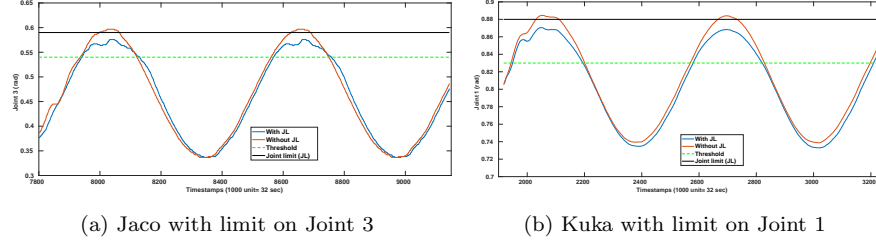


Figure 10: Controller treats joint limits enforced in manipulators.

mance of the highest priority coordinated motion task will not depend on the  
 450 performance of the lower performance robot.

Experimental implementation of a controller for heterogeneous robots is chal-  
 lenging because the access to hardware is usually not uniform. To implement  
 the controller, we developed a hardware plugin compatible with ROS-control  
 framework to abstract the two systems as a single robot and developed the con-  
 455 troller presented in this paper for that abstraction. The implemented controller  
 communicated with robot-specific native low-level controllers at 100 Hz. The  
 low-level controller of Kinova Jaco was fixed and did not allow tuning. The  
 performance of the low-level controllers differed significantly. To address this,  
 the gains of the proposed controller were tuned manually. Nevertheless, the  
 460 performance of the low-level controller for Jaco was limited which can be seen  
 in Figure 9(a)–(b) as tracking errors even for the baseline Case A without joint  
 limits. Despite this limitation, the experiments indicated no increase in tracking  
 error unless forced by the joint limit avoidance.

The relative position errors were at most 5 mm in all configurations. Thus  
 465 the proposed approach is applicable in practice to collaborative manipulation.  
 The remaining position errors were due to limitations in the performance of na-  
 tive low-level controllers as shown with the simulation results where the relative  
 pose errors remained much below measurement accuracy in all cases. Appli-  
 cations requiring higher performance would likely need an integrated custom  
 470 low-level controller controlling both manipulators.

## 8. Conclusion and future works

This paper presented a kinematic controller for coordinated cooperative manipulation based on the relative Jacobian method. Using a hierarchy of tasks and smooth activation, the controller ensures coordinated motion and joint limit avoidance. The same formulation allows joint limit avoidance for both redundant and non-redundant manipulators. Results from simulations showed that the approach avoids discontinuities in velocity commands unlike existing methods for the same problem. Experimental results from a two-robot system of mixed redundancy show that the relative position errors remained small, indicating that the proposed approach is applicable in practice to collaborative manipulation.

This paper studied collaborative manipulation with two robots. The relative Jacobian method expresses a constraint between two coordinate frames. An extension of the method to systems with more than two robots requires to study how the coordinated motion constraints between all robots are expressed in terms of pairwise constraints. This is an appealing topic for further research.

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