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Recursive algorithm to directly obtain the sum of correlations in a CSS

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ABSTRACT

This paper presents a new recursive algorithm to directly obtain the sum of the autocorrelation functions of the M sequences belonging to a Complementary Set of Sequences (M-CSS). The proposed algorithm allows a digital filter with only one stage to be obtained, what significantly reduces the total number of memory requirements, multiplications and additions to be performed in comparison with the use of M correlators, even when these correlators could have an efficient implementation. The use of this algorithm can be of great relevancy since there is an increasing interest in the use of CSS (or derived sequences such as Loosely Synchronized—LS-sequences or Zero Correlation Zone—ZCZ-sequences) in the field of communications and sensor systems.

1. Introduction

Complementary Sets of Sequences [1] have extended their fields of applications from their first use in the infrared spectrometry area. Nowadays, they are extensively used in the area of communications, including CDMA (Code Division Multiple Access) in multiple access communications [2], new modulation schemes [3], infrared communications [4] and also in pulse compression for ultrasound systems [5,6].

The use of CSS allows ideal properties to be obtained in the additive autocorrelation and crosscorrelation functions (the sum of the correlations of the sequences belonging to a complementary set). On the other hand, in order to simultaneously emit the corresponding sequences of a CSS assigned to a certain transmitter, an M-ary modulation scheme has to be used (M-Phase Shift Keying, Frequency Shift Keying, etc.). Then, at the reception stage, after the M-ary demodulation, a set of M correlators is needed to detect the sequences of the set and, finally, the sum of all these correlations is computed. Each one of these M correlations can be implemented by using efficient filters available for CSS called as Efficient Set of Sequences Correlator (ESSC) [7]. Furthermore, the architecture proposed in Ref. [7] could be used to efficiently detect codes based on orthogonal CSS, such as Loosely Synchronized—LS-sequences [8] or Zero Correlation Zone—ZCZ-sequences [9]. These kinds of codes could

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be useful when the application requires large ZCZ around the origin of the correlation function to reduce the inter-symbol and multiple access interferences in communications applications or sensor systems applied to localization methods.

Thus, when methods based on Efficient Set of Sequences correlators are used, computations are reduced, allowing real-time processing, compared to straightforward correlators [5,7], but at the expense of an increase of the memory requirements. In this paper, a new recursive algorithm to directly obtain the sum of the autocorrelation functions of the M sequences belonging to a CSS is presented. It uses only one stage instead of the M stages commonly used, what significantly reduces the total number of operations to be performed without penalizing the memory requirements and thus improves the feasibility of these systems (even for very long sequences).

2. Recursive algorithm to obtain the ACFS in a CSS

A recursive algorithm for the generation and correlation of complementary sets of M sequences with length L has been presented in Ref. [7], where the number of sequences M is a power of two ($M=2^m$) and their length L is a power of M ($L=M^N=2^{mN}$ with m, N $0\in\mathbb{N}-\{0\}$). In order to obtain the correlation of a received signal $r[k]$, with each one of the sequences of a M-CSS, the recursive Eq. (1) has to be used, thus obtaining a vector $S_{M(N)}$ after $n=N$ iterations ($n\in\{1, 2, \ldots, N\}$), in which the ith row contains the correlation of the received signal with the ith sequence in the CSS

$$\mathbf{S}_{M(N)} = \mathbf{\Lambda}_{(N)}^{(m} \cdot \mathbf{D} \cdot \mathbf{S}_{M(N-1)} \quad (1)$$

where $\mathbf{D}$ is a set of delays; $\mathbf{\Lambda}_{(N)}^{(m}$ is a generative matrix for obtaining the set of sequences, as it is described below with $m\in\{1, 2, \ldots, \log_2(M)\}$; $M,N\in\mathbb{N}-\{0\}$, and

$$\mathbf{S}_{M(N)} = \begin{pmatrix} S_{1,M(N)}[k] \\ S_{2,M(N)}[k] \\ S_{3,M(N)}[k] \\ \vdots \\ S_{M,M(N)}[k] \end{pmatrix}_{M\times 1} ; \quad \mathbf{S}_{M(N-1)} = \begin{pmatrix} S_{1,M(N-1)}[k] \\ S_{2,M(N-1)}[k] \\ S_{3,M(N-1)}[k] \\ \vdots \\ S_{M,M(N-1)}[k] \end{pmatrix}_{M\times 1} \quad (2)$$

Finally, in the case $N=1$, $S_{M(0)}$ is defined as

$$\mathbf{S}_{M(0)} = \begin{pmatrix} r[k] \\ r[k] \\ \vdots \\ r[k] \end{pmatrix}_{M\times 1} \quad (3)$$

where $r[k]$ is the received input signal.

Furthermore, in Eq. (1), $\mathbf{\Lambda}_{(N)}^{(m}$ is a matrix of size $M\times M$ that contains the order and the way in which a set of binary coefficients ($w_{m,N}$) affects the correlation results obtained in the previous iteration ($N-1$), to compute the correlation results $\mathbf{S}_{M(N)}$ in the Nth iteration. To generate this matrix, the following recursive algorithm is defined in Ref. [7]:

$$\mathbf{\Lambda}_{(n)}^{(m} = \begin{pmatrix} \mathbf{\Lambda}_{(n)}^{(m-1} \otimes (w_{m,n}\cdot(-\mathbf{\Lambda}_{(n)}^{(m-1})) \\ \mathbf{\Lambda}_{(n)}^{(m-1} \otimes (w_{m,n}\cdot\mathbf{\Lambda}_{(n)}^{(m-1}) \end{pmatrix}; \quad \mathbf{\Lambda}_{(n)}^{(1} = \begin{pmatrix} 1 & w_{1,n} \\ 1 & -w_{1,n} \end{pmatrix} \quad (4)$$

where $m\in\{1, 2, \ldots, \log_2(M)\}$, $n\in\{1, 2, \ldots, N\}$; M, $N\in\rceil-\{0\}$.

The matrix $\mathbf{\Lambda}_{(n)}^{(m}$ is built from $\mathbf{\Lambda}_{(n)}^{(1}$ which has the coefficients to obtain binary sequences of 2-CSS, after m iterations of Eq. (4). The coefficients $w_{m,n}$ are those for a particular M-CSS in the nth iteration. The set of these coefficients forms the generative seed [7] of the set of sequences that is correlated with the input signal $r[k]$.

Finally, $\mathbf{D}$ is a diagonal matrix of size $M\times M$ containing a set of delays

$$\mathbf{D} = \begin{pmatrix} \delta[k-((M-1)\cdot D_n)] & 0 & 0 & \cdots & 0 \\ 0 & \delta[k-((M-2)\cdot D_n)] & 0 & \cdots & 0 \\ 0 & 0 & \delta[k-((M-3)\cdot D_n)] & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & \delta[k] \end{pmatrix} \quad (5)$$

where D_n is an arbitrary positive delay defined as $D_n=M^{Pn}$ and P_n is any permutation of the numbers $\{0,1, 2, \ldots, N-1\}$.

The autocorrelation function (*ACF*) of a sequence $S_{i,M}[k]$, belonging to a certain M-CSS, can be obtained with the iterative algorithm presented above, by using that sequence as the received signal in Eq. (2) and considering only the ith row in $\mathbf{S}_{M(N)}$. Obviously, to obtain the M *ACF*s of the corresponding sequences, for a certain M-*CSS*, the recursive algorithm must be applied M times. In every case, the sequence to be correlated is used as the received sequence: $r[k]=S_{i,M}[k]$ ($i\in\{1, 2, \ldots, M\}$). In the following, the vector $\mathbf{S}_{M(0)}$ presented in Eq. (3) will be renamed by adding an upper index referred to that sequence, $\mathbf{S}_{M(0)}^{Si}$, as follows:

$$\mathbf{S}_{M(0)}^{Si} = \begin{pmatrix} S_i[k] \\ S_i[k] \\ \vdots \\ S_i[k] \end{pmatrix}_{M\times 1} \quad (6)$$

The recursive algorithm to compute the *ACF*s of the corresponding sequences can be expressed as

$$\mathbf{S}_{M(n)}^{S_i} = \mathbf{\Lambda}_{(n)}^{(m} \cdot \mathbf{D} \cdot \mathbf{S}_{M(n-1)}^{S_i} \quad (7)$$

And thus, after $n=N$ iterations, the *ACF* of the ith sequence will be in the ith row of $\mathbf{S}_{M(N)}^{S_i}$.

In order to obtain the sum of all the *ACF*s (*SACF*), the results provided by applying Eq. (7) for every $Si[k]$ can be concatenated; thus obtaining a matrix $\mathbf{S}_{M(N)}^{Con}$ defined as

$$\mathbf{S}_{M(N)}^{Con} = \left(\mathbf{S}_{M(N)}^{S_1} \,\vdots\, \mathbf{S}_{M(N)}^{S_2} \,\vdots\, \cdots \,\vdots\, \mathbf{S}_{M(N)}^{S_M}\right) \quad (8)$$

In this matrix, the *ACF*s of the corresponding sequences of the CSS are in the main diagonal, and the *SACF* can be simply calculated by computing the trace

$$SACF = Tr(\mathbf{S}_{M(N)}^{Con}) \quad (9)$$

The recursive algorithm to obtain the *SACF* function for a M-CSS with length $L=M^N$ can be considered as a digital filter of N identical stages. For example, in the case of a 4-CSS with length $L=4$ the recursive equations to compute de *ACF*s are detailed in Eq. (10) [7]

$$S_{1,4(0)}^{Si}[k] = S_{2,4(0)}^{Si}[k] = S_{3,4(0)}^{Si}[k] = S_{4,4(0)}^{Si}[k] = Si[k]$$
$$S_{1,4(1)}^{Si}[k] = S_{1,4(0)}^{Si}[k-3\cdot D_1] - w_{2,1}\cdot S_{2,4(0)}^{Si}[k-2\cdot D_1] + w_{1,1}\cdot S_{3,4(0)}^{Si}[k-D_1] - w_{2,1}\cdot w_{1,1}\cdot S_{4,4(0)}^{Si}[k]$$

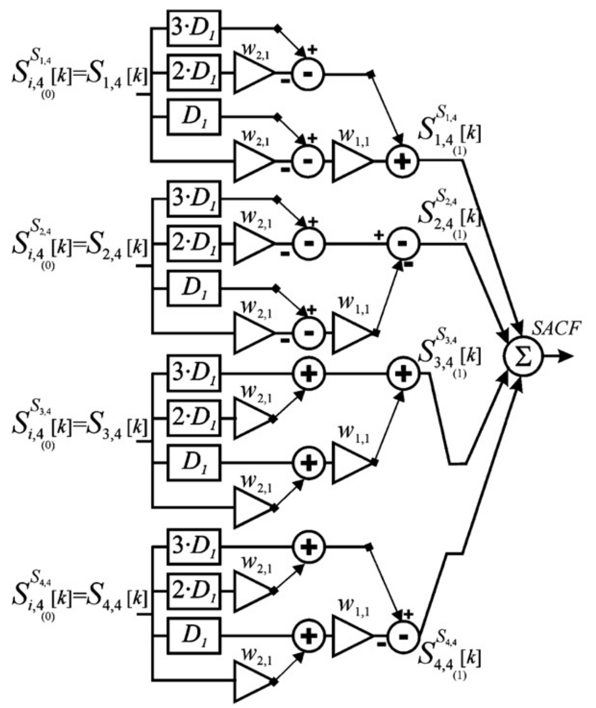

Fig. 1. Digital filter defined by Eq. (11) to compute the sum of autocorrelation functions (*SACF*) for the corresponding sequences from a 4-CSS of length $L=4$.

$$
\begin{aligned}
S^{Si}_{2,4(1)}[k] &= S^{Si}_{1,4(0)}[k-3\cdot D_1]-w_{2,1}\cdot S^{Si}_{2,4(0)}[k-2\cdot D_1]\\
&\quad -w_{1,1}\cdot S^{Si}_{3,4(0)}[k-D_1]+w_{2,1}\cdot w_{1,1}\cdot S^{Si}_{4,4(0)}[k]\\
S^{Si}_{3,4(1)}[k] &= S^{Si}_{1,4(0)}[k-3\cdot D_1]+w_{2,1}\cdot S^{Si}_{2,4(0)}[k-2\cdot D_1]\\
&\quad +w_{1,1}\cdot S^{Si}_{3,4(0)}[k-D_1]+w_{2,1}\cdot w_{1,1}\cdot S^{Si}_{4,4(0)}[k]\\
S^{Si}_{4,4(1)}[k] &= S^{Si}_{1,4(0)}[k-3\cdot D_1]+w_{2,1}\cdot S^{Si}_{2,4(0)}[k-2\cdot D_1]\\
&\quad -w_{1,1}\cdot S^{Si}_{3,4(0)}[k-D_1]-w_{2,1}\cdot w_{1,1}\cdot S^{Si}_{4,4(0)}[k]
\end{aligned}
\tag{10}
$$

In this case, the *SACF* can be obtained as shown in Eq. (11), after computing Eq. (10) for every Si ($i\in\{1, 2, \ldots, M\}$) and adding the corresponding equation; and the related digital filter is depicted in Fig. 1

$$
\begin{aligned}
SACF &= S_{1,4}[k-3\cdot D_1]-w_{2,1}\cdot S_{1,4}[k-2\cdot D_1]+w_{1,1}\cdot S_{1,4}[k-D_1]\\
&\quad -w_{2,1}\cdot w_{1,1}\cdot S_{1,4}[k]+\cdots+S_{2,4}[k-3\cdot D_1]\\
&\quad -w_{2,1}\cdot S_{2,4}[k-2\cdot D_1]-w_{1,1}\cdot S_{2,4}[k-D_1]\\
&\quad +w_{2,1}\cdot w_{1,1}\cdot S_{2,4}[k]+\cdots+S_{3,4}[k-3\cdot D_1]\\
&\quad +w_{2,1}\cdot S_{3,4}[k-2\cdot D_1]+w_{1,1}\cdot S_{3,4}[k-D_1]\\
&\quad +w_{2,1}\cdot w_{1,1}\cdot S_{3,4}[k]+\cdots+S_{4,4}[k-3\cdot D_1]\\
&\quad +w_{2,1}\cdot S_{4,4}[k-2\cdot D_1]-w_{1,1}\cdot S_{4,4}[k-D_1]\\
&\quad -w_{2,1}\cdot w_{1,1}\cdot S_{4,4}[k]
\end{aligned}
\tag{11}
$$

3. New recursive algorithm to directly obtain the *SACF* in a CSS

According to the previous section, the *SACF* function can be obtained by performing M times Eq. (7). In order to simplify and reduce the computational load of this method, a new recursive algorithm is proposed here. This algorithm is based on the transposition, in every iteration n, of the elements involved in Eq. (7) to correlate a set of sequences belonging to a CSS; and thus using all the sequences of the CSS as inputs in the iteration $n=0$. For the sake of clarity, in the case of 4-CSS ($M=4$) with length $L=4$, the new recursive equations of this algorithm are

$$
\begin{aligned}
&S'_{1,4(0)}[k]=S_{1,4}[k];\; S'_{2,4(0)}[k]=S_{2,4}[k];\; S'_{3,4(0)}[k]=S_{3,4}[k];\\
&S'_{4,4(0)}[k]=S_{4,4}[k]\\
&S'_{1,4(1)}[k]=S'_{3,4(0)}[k-3\cdot D_1]+S'_{4,4(0)}[k-3\cdot D_1]\\
&\qquad +S'_{2,4(0)}[k-3\cdot D_1]+S'_{1,4(0)}[k-3\cdot D_1]\\
&S'_{2,4(1)}[k]=w_{2,1}\cdot S'_{3,4(0)}[k-2\cdot D_1]-w_{2,1}\cdot S'_{4,4(0)}[k-2\cdot D_1]\\
&\qquad -w_{2,1}\cdot S'_{2,4(0)}[k-2\cdot D_1]+w_{2,1}\cdot S'_{1,4(0)}[k-2\cdot D_1]\\
&S'_{3,4(1)}[k]=w_{1,1}\cdot S'_{3,4(0)}[k-D_1]-w_{1,1}\cdot S'_{4,4(0)}[k-2\cdot D_1]\\
&\qquad -w_{1,1}\cdot S'_{2,4(0)}[k-D_1]+w_{1,1}\cdot S'_{1,4(0)}[k-D_1]\\
&S'_{4,4(1)}[k]=w_{2,1}\cdot w_{1,1}\cdot S'_{3,4(0)}[k]-w_{2,1}\cdot w_{1,1}\cdot S'_{4,4(0)}[k]\\
&\qquad +w_{2,1}\cdot w_{1,1}\cdot S'_{2,4(0)}[k]-w_{2,1}\cdot w_{1,1}\cdot S'_{1,4(0)}[k]
\end{aligned}
\tag{12}
$$

Then, the resulting set of recursive equations described in Eq. (12) is not the direct correlation of every sequence. Nevertheless, by computing the sum of the outputs as shown in Eq. (13), the *SACF* function described in Eq. (11) is directly obtained

$$
\begin{aligned}
SACF &= S_{1,4}[k-3\cdot D_1]+S_{2,4}[k-3\cdot D_1]+S_{3,4}[k-3\cdot D_1]\\
&\quad +S_{4,4}[k-3\cdot D_1]+\cdots-w_{2,1}\cdot S_{1,4}[k-2\cdot D_1]\\
&\quad -w_{2,1}\cdot S_{2,4}[k-2\cdot D_1]+w_{2,1}\cdot S_{3,4}[k-2\cdot D_1]\\
&\quad +w_{2,1}\cdot S_{4,4}[k-2\cdot D_1]+w_{1,1}\cdot S_{1,4}[k-D_1]\\
&\quad -w_{1,1}\cdot S_{2,4}[k-D_1]+w_{1,1}\cdot S_{3,4}[k-D_1]\\
&\quad -w_{1,1}\cdot S_{4,4}[k-D_1]+\cdots-w_{2,1}\cdot w_{1,1}\cdot S_{1,4}[k]\\
&\quad +w_{2,1}\cdot w_{1,1}\cdot S_{2,4}[k]+w_{2,1}\cdot w_{1,1}\cdot S_{3,4}[k]\\
&\quad -w_{2,1}\cdot w_{1,1}\cdot S_{4,4}[k]
\end{aligned}
\tag{13}
$$

The digital filter associated with the *SACF* function defined by this innovative recursive algorithm is shown in Fig. 2. Now, the new architecture called 4-ESSC^{T}, is the transposed version of the 4-ESSC described in Ref. [7], and by using only one adder to perform the sum of the outputs, the *SACF* function is obtained.

In the general case of an M-CSS with length $L=M^N$, the new recursive equations can be defined as follows:

$$
\mathbf{S}'_{M(0)}=\left[\begin{pmatrix} S_1[k] & S_2[k] & \cdots & S_M[k]\end{pmatrix}_{M\times 1}\right]^{T}
\tag{14}
$$

$$
\mathbf{S}'_{M(n)}=\mathbf{D}\cdot\left(\boldsymbol{\Lambda}^{(m}_{(N-n+1)}\right)^{\mathrm{T}}\cdot\mathbf{S}'_{M(n-1)}
\tag{15}
$$

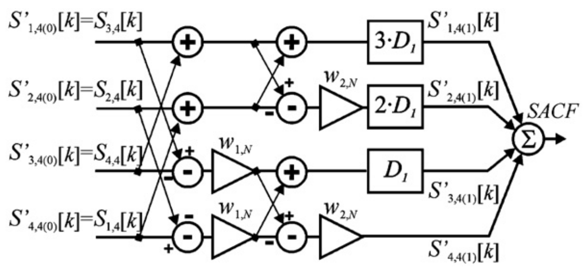

Fig. 2. Digital filter defined by Eq. (13) to compute the sum of autocorrelation (*SACF*) functions for the corresponding sequences from a 4-CSS of length $L=4$.

Table 1
Number of resources to obtain the *SACF* for an *M*-CSS with different methods.

Implementation	Multipliers	Adders	Memory elements
SACF using straightforward correlators	$2^m \cdot 2^{m \cdot N}$	$2^m \cdot 2^{m \cdot N}$	$2^m \cdot (2^{m \cdot N}-1)$
SACF using Eq. (8) and ESSC [5]	$2^m \cdot [(N-1) \cdot (m \cdot N \cdot 2^{m-1})+2^m-1]$	$2^m \cdot [(N-1) \cdot (m \cdot N \cdot 2^m)+2^m-1]$	$2^m \cdot (2^{m \cdot N}-1)$
SACF using ESSC^T	$m \cdot N \cdot 2^{m-1}$	$(2^m \cdot N \cdot m)+1$	$2^{m \cdot N}-1$

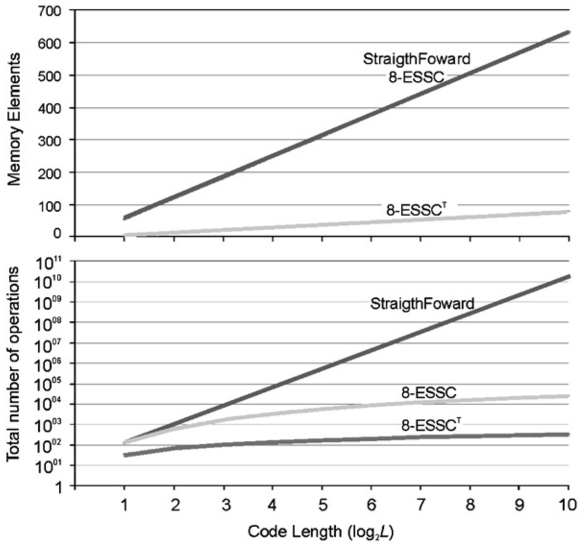

Fig. 3. Number of operations and memory resources required to obtain the SACF for 8-CSS (m=3) based on straightforward implementation, 8-ESSC and the 8-ESSC^T.

By performing the sum of rows in vector $\mathbf{S}'_{M(N)}$ (after n=N iterations), the *SACF* function is performed as follows:

$$SACF = Add(\mathbf{S}'_{M(N)}) \tag{16}$$

where *Add*(**A**) represents the addition of all the components of the *M*-dimensional column vector **A**.

As described in Ref. [7] for a generic *M*-CSS correlator, the property of regularity in the digital filter can be also maintained in the *SACF* architectures, and thus it is possible to obtain a *SACF* filter for a *M*-CSS based on the architecture for *M*/2-CSS. Table 1 presents the number of resources required (adders, multipliers, memory registers) for implementing the *SACF* of the *M*-CSS with sequences of length L=M^N. Three possible implementations have been considered: straightforward; Efficient Set of Sequences Correlators using Eq. (8) and ESSC [7]; and the new proposed algorithm ESSC^T (15).

The significant reduction of operations and memory resources is clearly demonstrated in Fig. 3 for m=3. In these case when N=2 the total number of multipliers, adders and memory elements and memory bits used in these implementations is: (512, 512, 511) with straightforward correlators, (248, 440, 511) with ESSCs and (24, 49, 63) with ESSC^T, respectively.

4. Conclusions

A new efficient correlator to directly obtain the *SACF* for an *M-CSS* has been presented. This implementation requires a lower computational load than the straightforward implementation and the one based on *ESSCs*. The proposed algorithm benefits from the transposed implementation of the *ESSC* architecture to minimize the number of operations required, and do not penalize the memory requirements, thus allowing its hardware implementation in devices as *FPGAs*, as well as its use with very long sequences based on orthogonal Complementary Set of Sequences.

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References

[1] C.-C. Tseng, C.L. Liu, Complementary Set of Sequences, IEEE Transactions on Information Theory IT-18 (1972) 644–652.
[2] S. Tseng, C. Chiou, A multicarrier DS/SSMA system with reduced multiple access interference and higher data rate in Rician fading channels, Signal Processing 81 (9) (2001) 1889–1897.
[3] V. Diaz, D. Hernanz, D. Lillo, J. Berian, J. Ureña, An emerging technology: orthogonal time division multiplexing (OTDM), in: Proceedings of the IEEE Conference Emerging Technologies and Factory Automation, 2003, pp. 33–36.
[4] K.K. Wong, T. O'Farrell, Spread spectrum techniques for indoor wireless IR communications, IEEE Wireless Communications 10 (2003) 54–63.
[5] F.J. Álvarez, Á. Hernández, J. Ureña, M. Mazo, J.J. García, J.A. Jiménez, A. Jiménez, Real-time implementation of an efficient correlator for complementary sets of four sequences applied to ultrasonic pulse compression systems, Microprocessors and Microsystems 30 (1) (2006) 43–51.
[6] A.T. Fam, I. Sarkar, A new class of interlaced complementary codes based on components with unity peak sidelobes, Signal Processing 88 (2) (2008) 307–314.
[7] C. De Marziani, J. Ureña, A. Hernandez, M. Mazo, F.J. Alvarez, J.J. Garcia, P. Donato, Modular architecture for efficient generation and correlation of complementary set of sequences, IEEE Transactions on Signal Processing 55 (2007) 2323–2337.
[8] M.C. Pérez, J. Ureña, A. Hernández, F.J. Álvarez, A. Jiménez, C.D. Marziani, Efficient correlator for LS codes generated from orthogonal CSS, IEEE Communications Letters 12 (2008) 764–766.
[9] İ. Güvença, H. Arslanb, A review on multiple access interference cancellation and avoidance for IR-UWB, Signal Processing 87 (4) (2007) 623–653.