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VIP-club phenomenon: emergence of elites and masterminds in social networks

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Abstract

Hubs, or vertices with large degrees, play massive roles in, for example, epidemic dynamics, innovation diffusion, and synchronization on networks. However, costs of owning edges can motivate agents to decrease their degrees and avoid becoming hubs, whereas they would somehow like to keep access to a major part of the network. By analyzing a model and tennis players' partnership networks, we show that combination of vertex fitness and homophily yields a VIP club made of elite vertices that are influential but not easily accessed from the majority. Intentionally formed VIP members can even serve as masterminds, which manipulate hubs to control the entire network without exposing themselves to a large mass. From conventional viewpoints based on network topology and edge direction, elites are not distinguished from many other vertices. Understanding network data is far from sufficient; individualistic factors greatly affect network structure and functions per se.

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1 Introduction

Real networks are neither regular nor completely random. They are random to some extent and equipped with short diameters and the clustering property (Watts and Strogatz, 1998). Many networks are also scale-free. In other words, the number of edges per vertex, which is denoted by k, follows the power-law distribution: $p(k) \propto k^{-\gamma}$ ($\gamma > 0$). This implies the existence of a considerable number of hubs, or vertices with huge degrees, beyond what is expected of a stereotypical bell-shaped $p(k)$ such as the Gaussian distribution. Hubs play central roles in, for example, robustness against random and intentional attacks against vertices or edges, propagating innovations, disease spreading, and synchronizing dynamical agents placed on vertices (Albert and Barabási, 2002; Newman, 2003; Pastor-Satorras and Vespignani, 2001). Naturally, real data analysis has focused on finding and characterizing hubs, including the evaluation of γ (Albert and Barabási, 2002; Newman, 2003; Zhou and Mondragon, 2004).

However, vertices other than hubs are also important on other occasions. As an example, let us imagine a one-shot transmission of innovation or rumor starting from an arbitrarily chosen vertex. Diffusion studies establish that very first adopters are regarded too radical or immature to directly communicate the new information to a majority. Instead, so-called early adopters or opinion leaders, presumably with high social statuses, credibility, or high connectivity as represented by hubs, receive the new information from the first adopters and boost its propagation (Rogers, 2003). Apart from the obvious role of hubs, however, properties of the first adopters determine the dominant timescale and even the success probability of diffusion, which are important in applications such as marketing and epidemics. In infinite particle systems used in physics and mathematics, such as the percolation and the contact process, the role of hubs (resp. early adopters) are manifested when there are initially many (resp. few) vertices.

If connectivity is flexible, agents can even be motivated to avoid becoming hubs and take advantage of other hubs. For example, computer viruses on scale-free networks proliferate by spreading through hubs since they are more accessible from others (Pastor-Satorras and Vespignani, 2001). Then, it is better for the system cracker to hide behind hubs and exploit them, than to expose themselves to a major part of networks as hubs, which raises a risk to be detected by the authority or other vertices. Similarly, intention of manipulating hubs may be present in economical behavior, politics, and marketing. The tradeoff between a cost of directly spanning edges and a benefit of having direct and indirect access to others can be formalized by a utility function such as

$$\sum_{l=1}^{\infty} k_l \delta^l - Ck, \tag{1}$$

where k_l is the number of vertices at distance l from a reference vertex, $k \equiv k_1$, $0 < \delta < 1$ is a discount factor (Jackson and Wolinsky, 1996; Bala and Goyal, 2000; Watts, 2001), and Ck ($C > 0$) is the cost of maintaining edges or being exposed to others. Equation (1) is also related to the growing scale-free network model, in which new vertices with a typically small constant k are consecutively added to a network. Based on the assumption of the preferential attachment, newcomers attempt to get linked to hubs (Albert and Barabási, 2002; Newman, 2003). If edges are dynamically created and removed according to Eq. (1) or similar utility functions, networks typically end up with wheels, stars, or complete networks (Jackson and Wolinsky, 1996; Bala and Goyal, 2000; Watts, 2001). These mathematical results imply realistic network structures as a result of evolution; for example, stars indicate hubs. However, real networks seem to deviate from wheels, stars and complete graphs that the utility model, which usually accompanies strong constraints, predicts. We call a vertex with a large utility value an elite or a mastermind.

In this paper, we explore how hubs and elites emerge, function, and interact, which has been neglected except in a few studies (Lau et al., 2000; Anghel et al., 2004).

Based on thresholding and homophily explained in Sec. 2, we propose a network model in Sec. 3. In Sec. 4, we show that combination of thresholding and homophily naturally generates elites in networks. We also analyze tennis tournament data in Sec. 5.

2 Thresholding and Homophily

Let us introduce the intrinsic weight of the ith vertex denoted by w_i. It quantifies the potential to win edges, such as physical ability, fame, and social status (Bianconi and Barabási, 2001; Caldarelli et al., 2002; Goh et al., 2001; Boguñá and Pastor-Satorras, 2003; Masuda et al., 2004; Barrat et al., 2004; Masuda et al., 2005). Depending on situations, the direction of influence from vertices with larger weights to ones with smaller weights can be deduced independently of the predefined edge direction (Anghel et al., 2004). For instance, whether a computer virus at a host can invade another depends on the relative security level of hosts, or vertices. Networks with weight-driven edge direction also underly human relationships.

Another key element is homophily, which means that similar agents, particularly humans, tend to flock together. Many real data from individual questionnaires (Marsden, 1988; McPherson et al., 2001; McPherson and Smith-Lovin, 1987), diffusion studies (Rogers, 2003), and analysis of online communities (Adamic and Adar, 2003; Adamic et al., 2003) support homophily according to nominal (e.g. race, hobbies, sex, religious preference, personal traits) and graduated (e.g. physical distance, age, education, social status) parameters. Homophily in terms of vertex degrees, or rates of social contacts, underlies the fact that the degrees of adjacent vertices are positively correlated in social networks (Newman, 2003). Various models of real networks and social interactions incorporate homophily. For example, Granovetter (1973) proposed that one is mainly connected to similar others, and some weak ties also exist to bridge heterophilous individuals. Then, the small-world property results because weak ties shorten the network diameter and abundant homophilous connectivity enhances clus-

tering (Watts and Strogatz, 1998). Hierarchical networks in which vertices with closer hierarchical levels are more likely to be adjacent are used to address search and congestion problems on networks (Watts et al., 2002; Dodds et al., 2003). We note that hierarchy is also reminiscent of weight-driven edge direction mentioned before. Other examples include the cultural exchange models with general homophily (Axelrod, 1997) and the gravity models with spatial homophily (Zipf, 1949; Barrat et al., 2004; Masuda et al., 2005). We focus on homophily based on graduated parameters because vertices are equipped with graduated intrinsic weights.

3 Model

We show in the context of scale-free networks that combination of thresholding and homophily yields networks with elites. Let us prepare n vertices and choose w_i ($1 \leq i \leq n$) randomly and independently from a distribution $f(w)$. We start with the threshold graph, in which two vertices with weights w and w' are connected if $w + w' \geq \theta$ (Caldarelli et al., 2002; Boguñá and Pastor-Satorras, 2003; Masuda et al., 2004). A larger w induces a larger vertex degree k, which is the so-called rich-club phenomenon (Zhou and Mondragon, 2004). Scale-free networks with the small-world properties actually result from various $f(w)$; $p(k) \propto k^{-2}$ from the exponential distribution $f(w) = \lambda e^{-\lambda w}$ ($w \geq 0$) (Caldarelli et al., 2002; Boguñá and Pastor-Satorras, 2003; Masuda et al., 2004) and $p(k) \propto k^{-(a+1)/a}$ from the Pareto distribution $f(w) \propto w^{-a-1}$ ($w \geq w_0$, $\exists w_0 > 0$) (Masuda et al., 2004). We now supply a homophily rule by making the connection probability decreasing in $|w' - w|$. For simplicity, an edge is assumed to be created only when $|w' - w| \leq c$. Together with $w + w' \geq \theta$, a vertex with w is adjacent to vertices whose weights satisfy

$$w' \in \begin{cases} \emptyset, & (w < \frac{\theta - c}{2}) \\ [\theta - w, w + c], & (\frac{\theta - c}{2} \leq w < \frac{\theta + c}{2}) \\ [w - c, w + c]. & (w \geq \frac{\theta + c}{2}) \end{cases} \tag{2}$$

In the limit $c \to \infty$, Eq. (2) becomes $\theta - w \leq w' < \infty$, returning to the original threshold graph. We obtain k as a function of w by integrating $f(w')$ over the range given in Eq. (2). If $f(w)$ is monotone decreasing for $w > w_c \equiv (\theta + c)/2$, $k(w)$ is maximized at $w = w_c$. Even if not, sufficiently large w with

$$1 - F(w - c) < F\left(\frac{\theta + 3c}{2}\right) - F\left(\frac{\theta - c}{2}\right) \tag{3}$$

satisfies $k(w) < k\left((\theta + c)/2\right)$, meaning that $k(w)$ takes the maximum at $w = w_c \in (\frac{\theta+c}{2}, \infty)$. In both cases, vertices with $w \cong w_c$ are hubs. Elites are vertices with $w \gg w_c$ and not exposed via direct edges to the major group of vertices with small w.

The psychological Weber-Fechner law dictates that vertices may sense relative rather than absolute differences in weights. To mimic this, let us modify the homophily condition to $|w' - w|/(w + w') < c$ $(c < 1)$. Then, the counterpart of Eq. (2) reads

$$w' \in \begin{cases} \emptyset, & (w < \frac{1-c}{2}\theta) \\ [\theta - w, \frac{1+c}{1-c}w], & (\frac{1-c}{2}\theta \leq w < \frac{1+c}{2}\theta) \\ [\frac{1-c}{1+c}w, \frac{1+c}{1-c}w]. & (w \geq \frac{1+c}{2}\theta) \end{cases} \tag{4}$$

For $k(w)$ to be maximized at $w = (1 + c)\theta/2$, $f(w)$ fulfilling

$$\left(\frac{1+c}{1-c}\right)^2 f\left(\left(\frac{1+c}{1-c}\right)^2 w\right) < f(w) \quad (w > (1-c)/2\theta) \tag{5}$$

is required. However, as in the previous case, the maximum of $k(w)$ at $w = w_c > (1 + c)\theta/2$ is assured even if Eq. (5) is violated.

To consider weight-driven edge direction, we again apply $|w' - w| \leq c$. Since a directed edge $w \to w'$ may form only when $w > w'$, a vertex with w sends directed edges to ones with

$$w' \in \begin{cases} \emptyset, & (w < \frac{\theta}{2}) \\ [\theta - w, w], & (\frac{\theta}{2} \leq w < \frac{\theta+c}{2}) \\ [w - c, w], & (w \geq \frac{\theta+c}{2}) \end{cases} \tag{6}$$

which is essentially the same as Eq. (2).

4 VIP-club Phenomenon

For concreteness, we set $f(w) = \lambda e^{-\lambda w}$ $(w \geq 0)$. The following results hold as long as $f(w)$ largely decreases when $w \geq w_c$, which is supported by real data (Zipf, 1949; Masuda et al., 2004; Masuda et al., 2005). Based on Eq. (6), the vertex degree as a function of the weight is represented by

$$k(w) = \begin{cases} 0, & (w < \frac{\theta}{2}) \\ e^{-\lambda(\theta - w)} - e^{-\lambda w}, & (\frac{\theta}{2} \leq w < \frac{\theta + c}{2}) \\ e^{-\lambda(w-c)} - e^{-\lambda w}, & (w \geq \frac{\theta + c}{2}) \end{cases} \tag{7}$$

for $\theta \geq c$, and

$$k(w) = \begin{cases} 0, & (w < \frac{\theta}{2}) \\ e^{-\lambda(\theta - w)} - e^{-\lambda w}, & (\frac{\theta}{2} \leq w < \theta) \\ 1 - e^{-\lambda w}, & (\theta \leq w < c) \\ e^{-\lambda(w-c)} - e^{-\lambda w}, & (w \geq c) \end{cases} \tag{8}$$

for $\theta < c$. As shown in Fig. 1(a) for $(\theta, c) = (6, 5)$ [Eq. (7)], $k(w)$ (solid line) has a single peak. Similar upshots result from Eq. (2), (4), or (8) if the thresholding and the homophily are roughly balanced.

With a utility function like Eq. (1) in mind, we derive $k_2(w)$, which is the number of the vertices within two hops from a vertex with weight w. Although we exclude the reference vertex itself, this subtlety does not matter when the network is large enough. The neighbor's weight w' satisfies Eq. (6). The weight of the neighbor's neighbor, which is denoted by w'', similarly satisfies

$$w'' \in \begin{cases} \emptyset, & (w' < \frac{\theta}{2}) \\ [\theta - w', w'], & (\frac{\theta}{2} \leq w' < \frac{\theta + c}{2}) \\ [w' - c, w']. & (w' \geq \frac{\theta + c}{2}) \end{cases} \tag{9}$$

For a given w, we integrate the density of vertices $f(w'')$ over the range compatible with Eqs. (6) and (9) to obtain

$$k_2(w) \propto \begin{cases} 0, & (w < \frac{\theta}{2}) \\ \lambda e^{-\lambda\theta}\left(w - \frac{\theta}{2}\right) + \frac{e^{-2\lambda w} - e^{-\lambda\theta}}{2}, & (\frac{\theta}{2} \leq w < \frac{\theta + c}{2}) \\ \frac{\lambda c e^{-\lambda\theta} - (e^{\lambda c} - 1)e^{-2\lambda w}}{2}, & (\frac{\theta + c}{2} \leq w < \frac{\theta + 2c}{2}) \\ \frac{\lambda(\theta - 2w + 3c) + 1}{2} e^{-\lambda\theta} + \frac{e^{-2\lambda w}(-e^{2\lambda c} - e^{\lambda c} + 1)}{2}, & (\frac{\theta + 2c}{2} \leq w < \frac{\theta + 3c}{2}) \\ \frac{(e^{\lambda c} - 1)(e^{2\lambda c} - 1)}{2} e^{-2\lambda w}, & (w \geq \frac{\theta + 3c}{2}) \end{cases} \tag{10}$$

for $\theta \geq c$, and

$$k_2(w) \propto \begin{cases} 0, & (w < \frac{\theta}{2}) \\ \lambda e^{-\lambda\theta}\left(w - \frac{\theta}{2}\right) + \frac{e^{-2\lambda w} - e^{-\lambda\theta}}{2}, & (\frac{\theta}{2} \leq w < \theta) \\ \frac{\lambda\theta+1}{2} e^{-\lambda\theta} - e^{-\lambda w} - \frac{e^{-2\lambda w}}{2}, & (\theta \leq w < c) \\ \frac{\lambda\theta+1}{2} e^{-\lambda\theta} + \frac{-e^{-\lambda c} + e^{-2\lambda w} - e^{(-2w+c)\lambda}}{2}, & (c \leq w < c + \frac{\theta}{2}) \\ (1 - \lambda(w - c - \theta))\, e^{-\lambda\theta} + \frac{1}{2} e^{-2\lambda w}\left(-e^{2\lambda c} - e^{\lambda c} + 1\right) - \frac{1}{2} e^{-\lambda c}, & (c + \frac{\theta}{2} \leq w < c + \theta) \\ \frac{1}{2} e^{-2\lambda w}\left(-e^{2\lambda c} - e^{-\lambda c} + 1\right) - \frac{1}{2} e^{-\lambda c} + e^{-\lambda(w-c)}, & (c + \theta \leq w < 2c) \\ \frac{(e^{\lambda c} - 1)(e^{2\lambda c} - 1)}{2} e^{-2\lambda w}, & (w \geq 2c) \end{cases} \tag{11}$$

for $\theta < c$. Figure 1(a) shows that $k_2(w)$ (dotted line) has a unique peak at $w = \overline{w}_c > w_c$ and that $k_2(w)$ decays more slowly in w than $k(w)$ (solid line) does. Under Eq. (1) or a simpler utility function with only two terms involving k and k_2 (Jackson and Wolinsky, 1996), vertices with $w \cong \overline{w}_c$ are elites or masterminds, whereas vertices with $w \cong w_c$ are hubs. The masterminds are linked to the majority of vertices with small w and presumably large $f(w)$ only indirectly via hubs. Owing to homophily, they flock together with others with $w \cong \overline{w}_c$, which are usually rare. We call it VIP-club phenomenon in contrast to the rich-club phenomenon (Zhou and Mondragon, 2004), in which larger w simply means larger $k(w)$.

Figure 1(b) shows $k(w)$ (solid line) and $k_2(w)$ (dotted line) for $(\theta, c) = (6, 100)$ with which homophily is practically absent. In accordance with the standard threshold model, $k(w)$ increases monotonically in w, and vertices with large w serve as hubs (Caldarelli et al., 2002; Boguñá and Pastor-Satorras, 2003; Masuda et al., 2004). This is the rich-club but not VIP-club phenomenon.

With homophily only (Fig. 1(c), $(\theta, c) = (0, 5)$), the majority vertices, which have small w, own large degrees because homophily simply interconnects these vertices in the absence of thresholding. What reduces $k(w)$ for small w in Fig. 1(c) is the lower bound of the exponential weight distribution at $w = 0$, which is nonessential. In consequence, hubs abound in the network, and the VIP-club does not form. Moreover, the homophily-only configuration is unrealistic for two reasons. First, $p(k)$ becomes

flat (circles in Fig. 1(d)), which contradicts real data (Watts and Strogatz, 1998; Albert and Barabási, 2002; Newman, 2003). In contrast, although homophily prohibits huge hubs, $p(k)$ of a network with both homophily and thresholding is scale-free (crosses), as is $p(k)$ of the the threshold graph without homophily (squares). This is because elites are scarce even if they exist. Second, too strong homophily mars communication between vertices with distant weights. With the density of edges given, the diameter becomes too large in a strongly homophilous network due to the scarceness of shortcuts bridging heterophilous vertices (Granovetter, 1973; Watts and Strogatz, 1998; Rogers, 2003; Adamic et al., 2003). The thresholding effect counteracts the homophily effect to render a network small-world. In sum, the VIP-club phenomenon requires both homophily and thresholding in our framework.

Since the analysis developed so far is for deterministic dense networks, let us numerically examine sparse networks with stochasticity. An edge is assumed to form between vertices with weights w and w' with probability proportional to $e^{-\beta_2|w'-w|}/(1+e^{-\beta_1(w+w'-\theta)})$ where β_1 and β_2 are the inverse temperatures. We set $n = 50000$, $\lambda = 1$, $\theta = 6$, and the mean degree 10. Figures 2(a) and 2(b) show $k(w)$ and $p(k)$, respectively, for $(\beta_1, \beta_2) = (1.5, 0.5)$ (both homophily and thresholding, plotted by crosses), $(1.5, 0)$ (thresholding only, squares), and $(0, 0.5)$ (homophily only, circles). These results are roughly consistent with Fig. 1.

Let us next examine a scale-free network in which $w_i = i^{-\alpha}$ is assigned to the ith vertex (Goh et al., 2001). Then, a pair of vertices is picked according to the distribution $p(i) = w_i / \sum_{i=1}^{n} w_i$, and an edge is created if they are not yet adjacent. This procedure is repeated until the mean degree 10 is reached. As a result, we obtain $p(k) \propto k^{-\gamma}$ with $\gamma = (1+\alpha)/\alpha$ (Goh et al., 2001). A type of thresholding is embedded in this algorithm. We implement homophily by supposing that an edge forms with probability $e^{-\beta_2|w'-w|}$ after two vertices with weights w and w' are selected. The results shown in Fig. 3 for $n = 50000$ and $\alpha = 0.5$ with homophily present ($\beta_2 = 5$, crosses) and absent

($\beta_2 = 0$, squares) are consistent with Figs. 1 and 2. Although we have presented just two examples, our framework is applicable to other networks with vertex fitness.

5 Analysis of Tennis Players' Networks

In social networks with homophily, graduated weight variables often exist but are difficult to measure. In the professional tennis community, players are ranked based on the scores, which serve as w, gained by winning the singles games of official tournaments. We analyze networks of tennis players, which are indicative of the VIP-club phenomenon. For each sex, the score distribution obeys a power law with an exponential cutoff, as the rank-score plots in Fig. 4 indicate. The fact that the probability density decreases in the score w means that better players are scarcer. The players constitute undirected networks by doubles partnership, and the edges are defined by two players pairing up in any of the doubles tournaments in a year. Our model cannot account for some aspects of the data, such as the homophily in nationality and score-independent individuality in the frequency of participation in the tournaments. Nevertheless, two players should be strong in total to be successful, which provides thresholding. Simultaneously, a strong player is expected to stick to a small number of partners because of the paucity of similarly strong players and the aversion to tying with weak players. This is equivalent to homophily.

The women's network based on all the WTA doubles tournaments in 2003 [1] has $n = 366$ excluding the isolated vertices. Similarly, the principal connected component of the men's network based on the ATP doubles tournaments in 2003 [2] has $n = 367$. We do not consider mixed doubles in which a man and a woman team up. Since the ranking is updated every week, players are differently aligned according to the rankings at the ends of 2002 and of 2003. To circumvent noise in the data, we add k, or the

[1] http://www.wtatour.com/rankings/singles_numeric.asp
[2] http://www.stevegtennis.com/

number of doubles partners, of five players with consecutive ranks. The results are shown for women and men in Figs. 5(a) and 5(b), respectively. Although noise is yet large and the degree distributions are not scale-free, players with intermediate ranks are somewhat more capable of encountering partners. The data in other years also have this tendency (data not shown). Players with high scores form a loose VIP club. Players with intermediate ranks are popular among both stronger and weaker players.

The VIP-club phenomenon is also expected in other social situations. For example, when choosing a partner of the life, one may set a threshold on social statuses or incomes. Since they are graduated quantities subject to homophily (Marsden, 1988; McPherson et al., 2001; McPherson and Smith-Lovin, 1987), maybe for people to communicate efficiently and live in comfort, those in upper-middle classes may be more promising in finding partners than those too close to the top. Similarly, scientific collaboration networks may exhibit the VIP-club phenomenon particularly in the fields like mathematics where coauthorship is rather strict. A group of strong researchers generally publishes their work in nice journals (thresholding), and researchers with close abilities may flock together to coauthor (homophily). The age also shows homophily (Marsden, 1988; McPherson et al., 2001; McPherson and Smith-Lovin, 1987), and the population density, or $f(w)$, naturally decreases in the age, or w. Consequently, business or social activities that accompany thresholding on ages can result in VIP clubs formed by the old. More generally, hierarchical organizations of the pyramid type, such as companies and bureaucracies (Watts et al., 2002; Dodds et al., 2003), are likely to have VIP clubs.

6 Conclusions

We have shown that the combination of homophily and thresholding on graduated vertex weights induces networks with elites. Loss of homophily leads to the rich-club phenomenon (Zhou and Mondragon, 2004), while unrealistically many hubs emerge if

without thresholding. A VIP club is invisible to the majority of vertices with small weights. They can even escape the eyes of network analyzers unless the weight-driven edge direction in addition to the predefined edge direction, which are more readily obtained, are actually inspected (Anghel et al., 2004). Actually, elites and the majority of vertices with small weights remain undistinguished if based on vertex properties such as k, the closeness centrality, the reach centrality, the betweenness centrality, or the local clustering coefficient (Newman, 2003; Scott, 2000). They occupy structurally equivalent or similar locations of a network (Scott, 2000). Paradoxically, complete understanding of connectivity and edge direction is not necessarily sufficient for knowing a network (Anghel et al., 2004). To understand the nature of a network, intrinsic properties of individual vertices must be taken into account.

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Figure captions

Figure 1: Theoretically evaluated $k(w)$ (solid lines) and $k_2(w)$ (dotted lines) for the threshold graph supplemented by homophily. We set $\lambda = 1$, and (a) $(\theta, c) = (6, 5)$, (b) $(6, 100)$, and (c) $(0, 5)$. For clarity, $k_2(w)$ is normalized so that the maximum is 1. (d) $p(k)$ for $(\theta, c) = (6, 5)$ (crosses), $(\theta, c) = (6, 100)$ (squares), $(\theta, c) = (0, 5)$ (circles), and $p(k) \propto k^{-2}$ (line).

Figure 2: Numerically obtained (a) $k(w)$ and (b) $p(k)$ for the modified threshold graph. We set $n = 50000$, $\lambda = 1$, $\theta = 6$, and $(\beta_1, \beta_2) = (1.5, 0.5)$ (crosses), $(\beta_1, \beta_2) = (1.5, 0)$ (squares), and $(\beta_1, \beta_2) = (0, 0.5)$ (circles). The lines in (a) and (b) correspond to $k(w) \propto e^{\lambda w}$ and $p(k) \propto k^{-2}$, respectively, which are the predictions by the threshold graph without homophily.

Figure 3: (a) The results for the model by Goh et al. (2001) with $n = 50000$ and $\alpha = 0.5$. (a) $k(w)$ and (b) $p(k)$ for $\beta_2 = 5$ (crosses) and $\beta_2 = 0$ (squares). The theoretical estimates $k(w) \propto w^{-\alpha}$ and $p(k) = k^{-(\alpha+1)/\alpha} = k^{-3}$ are indicated by lines.

Figure 4: Dependence of the score (w) on the rank of female (crosses) and male (circles) tennis players, based on the ranking at the end of 2003.

Figure 5: Vertex degrees of the (a) women's and (b) men's tennis networks in 2003. The number of partners summed over five players with consecutive ranks are plotted. The players are arranged according to the ranking at the ends of 2002 (dotted lines) and of 2003 (solid lines).

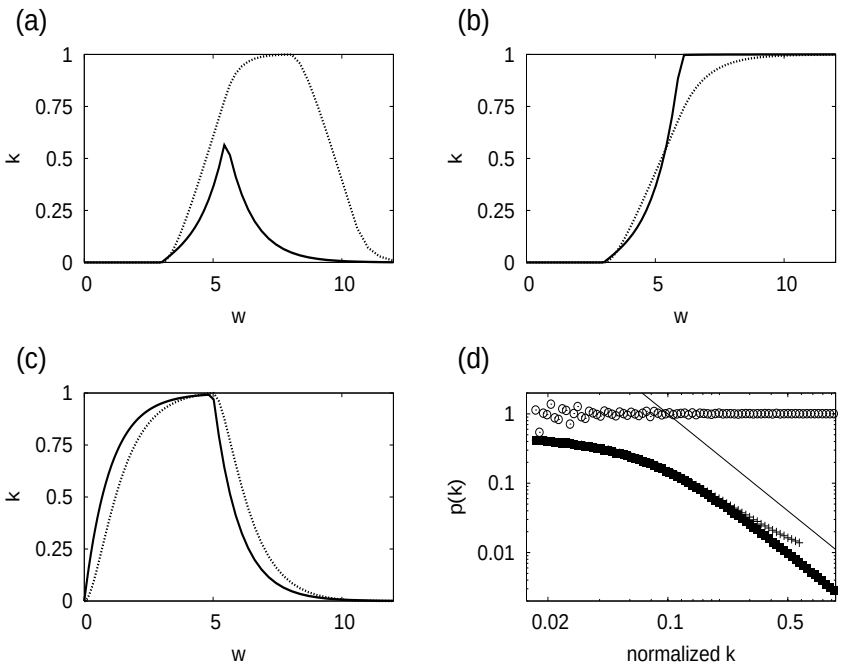

Figure 1:

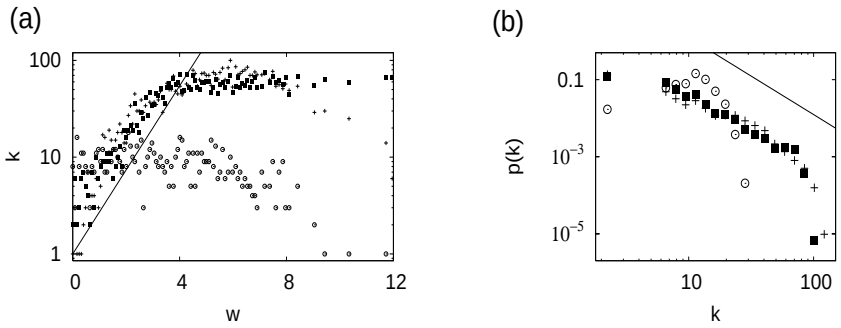

Figure 2:

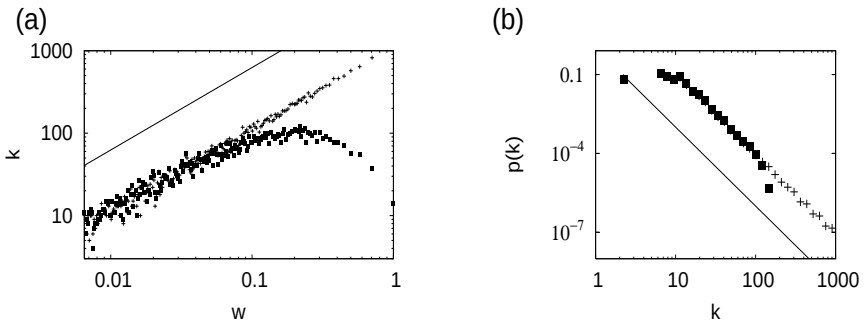

Figure 3:

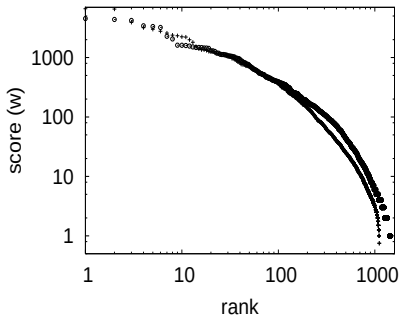

Figure 4:

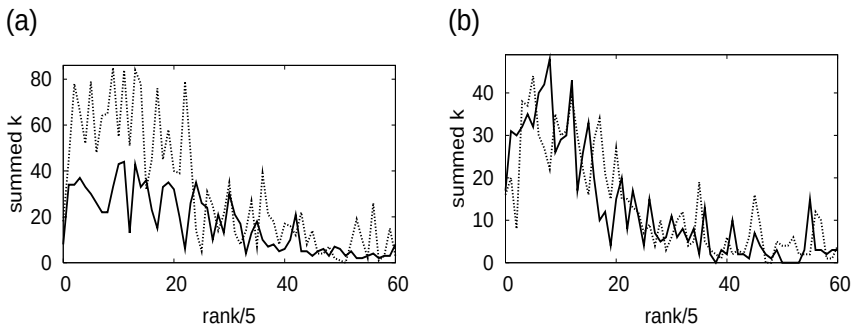

Figure 5: