On validating closed-loop behaviour from noisy frequency response measurements [⋆]

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Abstract

It is shown how noisy closed-loop frequency response measurements can be used to obtain pointwise in frequency bounds on the possible difference between an otherwise unknown closed-loop system and the closed-loop comprising a nominal model of the plant and a stabilising controller. To this end, the Vinnicombe's gap metric framework for robustness analysis plays a central role. Indeed, an optimisation problem and corresponding algorithm are proposed for estimating the chordal distance between the frequency responses of the nominal plant model and a plant that is consistent with the closed-loop data and *a priori* information, when projected onto the Riemann sphere.

Key words: ν-gap metric, robust performance, closed-loop validation

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Notation

Let $\mathbb{C}$ denote the field of complex numbers, $\mathbb{C}^{n\times m}$ the space of $n \times m$ matrices with complex entries, $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$ the unit circle, and $\mathbb{D}_\rho := \{z \in \mathbb{C} : |z| < \rho\}$ the open disc of radius $\rho > 0$. The symbol $\bar{\mathbb{D}}_\rho$ is used to denote the closure of $\mathbb{D}_\rho$ and for convenience, the sets $\mathbb{D}_1$ and $\bar{\mathbb{D}}_1$ are denoted by $\mathbb{D}$ and $\bar{\mathbb{D}}$, respectively. Given $\rho \geq 1$, let $\mathcal{H}_{\infty,\rho} := \{f : \mathbb{C} \to \mathbb{C} : f$ is analytic in $\mathbb{D}_\rho$ and $\|f\|_{\infty,\rho} := \sup_{z\in\mathbb{D}_\rho} |f(z)| < \infty\}$ and for convenience, denote $\mathcal{H}_{\infty,1}$ and $\|f\|_{\infty,1}$ by $\mathcal{H}_\infty$ and $\|f\|_\infty$, respectively. The ball of radius $\gamma > 0$ in $\mathcal{H}_{\infty,\rho}$ is denoted by $\bar{\mathcal{B}}\mathcal{H}_{\infty,\rho}(\gamma) := \{f : \mathbb{C} \to \mathbb{C} : f$ is analytic in $\mathbb{D}_\rho$ and $\|f\|_{\infty,\rho} := \sup_{z\in\mathbb{D}_\rho} |f(z)| \leq \gamma\}$. Given $f \in \bar{\mathcal{B}}\mathcal{H}_{\infty,\rho}(\gamma)$, it can be shown that each term f_k, of the impulse response of the system corresponding to multiplication by the frequency domain symbol f, is bounded as $|f_k| \leq \gamma\rho^{-k}$. Given a matrix Q, the notation Q^T, Q^* and $\bar{\sigma}(Q)$ is used to represent the transpose, complex conjugate transpose and maximum singular value of Q, respectively. Finally, given a set $\{X_i\}_{i=1}^n$ of matrices, the direct sum $\bigoplus_{i=1}^n X_i$ denotes the diagonal matrix $\mathrm{diag}(X_1, X_2, \ldots, X_n)$.

1 Introduction

As many modern techniques for control system design are model based, it is of practical interest to know in what sense a system model should be accurate. Indeed, significant research effort has been devoted to answering such questions over the last few decades. Within the context of feedback compensator design, the gap and ν-gap metric frameworks for robustness analysis are particularly useful [1–3]. In fact, these metrics induce the coarsest topology with respect to which both feedback stability and closed-loop performance are robust properties. This is established within a general linear setting in [4], using the following inequalities:

Given linear systems P_1, P_2 and C, such that the standard feedback configurations $[P_1, C]$ and $[P_2, C]$ are both stable, let

$$H(P_i, C) := \begin{bmatrix} (I - CP_i)^{-1} & -C(I - P_iC)^{-1} \\ P_i(I - CP_i)^{-1} & -P_iC(I - P_iC)^{-1} \end{bmatrix}.$$

Then

$$\mathrm{gap}(P_1, P_2) \leq \|H(P_1, C) - H(P_2, C)\| \leq \|H(P_1, C)\| \, \|H(P_2, C)\| \, \mathrm{gap}(P_1, P_2), \quad (1)$$

where $\mathrm{gap}(P_1, P_2)$ denotes the gap metric distance between P_1 and P_2, and $\|\cdot\|$ denotes the ℓ_2 induced norm.

For linear time-invariant (LTI) systems the bounds in (1) hold pointwise in frequency $\varphi := e^{j\omega}$, with $\mathrm{gap}(P_1, P_2)$ replaced by the chordal distance

$$\kappa(P_1(\varphi), P_2(\varphi)) = \bar{\sigma}\left((I - P_1 P_1^*)^{-1/2}(P_1 - P_2)(I - P_2^* P_2)^{-1/2}(\varphi)\right),$$

between the stereographic projection of the frequency responses $P_i(\varphi)$ $(i = 1, 2)$ onto the Riemann sphere [3,5] – i.e.

$$\kappa(P_1(\varphi), P_2(\varphi)) \leq \bar{\sigma}\left(H(P_1(\varphi), C(\varphi)) - H(P_2(\varphi), C(\varphi))\right) \leq \frac{\kappa(P_1(\varphi), P_2(\varphi))}{\rho_1(\varphi) \cdot \rho_2(\varphi)}, \qquad (2)$$

where $\rho_i(\varphi) := \rho(P_i(\varphi), C(\varphi)) := 1/\bar{\sigma}\left(H(P_i(\varphi), C(\varphi))\right) \leq 1$. Furthermore,

$$\arcsin \rho(P_2(\varphi), C(\varphi)) \geq \arcsin \rho(P_1(\varphi), C(\varphi)) - \arcsin \kappa(P_1(\varphi), P_2(\varphi)) \qquad (3)$$

for all $\varphi = \mathbb{T}$. Indeed, $\sup_{\varphi \in \mathbb{T}} \rho(P_i(\varphi), C(\varphi)) =: b(P_i, C)$ is the generic measure of performance and robustness in the $\mathcal{H}_\infty$ loop-shaping paradigm for design [6,5]. Observe that the bounds in (1), (2) and (3) imply that gap-like metrics capture the important difference between open-loop systems from the perspective of closed-loop behaviour.

Given a nominal model P_m of a true plant P_t, suppose that a feedback compensator C is known to stabilise both the nominal model and the true plant.[1] In addition to stability, a handle on the actual behaviour/performance of P_t in closed-loop with C is typically of interest. To this end, two approaches could be taken: (*i*) One could try to identify $H(P_t, C)$ at frequencies of interest from closed-loop measurements; or (*ii*) since the nominal closed-loop $[P_m, C]$ is known, one could estimate $\kappa(P_m(\varphi), P_t(\varphi))$ at frequencies of interest and then use the bounds in (2) and (3). In approach (*i*), the technique for identifying $H(P_t, C)$ should involve constraints to reflect the relationships between the blockwise elements of $H(P_t, C)$; for example, the quotient of the 12-block and the 11-block, which is known *a priori* to be C. Such constraints and noisy data makes this a difficult problem. Furthermore, identifying $H(P_t, C)$ only yields information that is directly pertinent to the closed-loop behaviour of P_t with the particular controller C. Approach (*ii*), on the other hand, can be posed in terms of a tractable optimisation problem (as will be shown shortly). Moreover, and most importantly, if the estimate of $\kappa(P_m(\varphi), P_t(\varphi))$ is large (close to 1) at a particular

[1] Observe that if P_t and P_m are known to be stable, then $C = 0$ will do.

frequency, the following conclusion can be made: For *any* controller C_1 that stabilises both the true plant and the nominal model, the closed-loop $[P_t, C_1]$ differs significantly from the nominal $[P_m, C_1]$. That is, a better nominal model of the plant may be required for model-based feedback compensator design. Motivated by this, the following sections are dedicated to presenting a numerical technique for determining a sensible estimate of $\kappa(P_m(\varphi), P_t(\varphi))$, from noisy closed-loop measurements. Work that is related in terms of assessing closed-loop performance from measured-data/identified-sets, but distinct in terms of the approach taken, can be found in [7,8] and the references therein.

2 Estimating the chordal distance at sample frequencies

For the sake of notational simplicity, the plant and controller are taken to be SISO systems. The MIMO case follows similarly with appropriate notational modifications.

The *a priori* information for the problem introduced above includes a model P_m of an unknown true system P_t, and a controller C which stabilises both P_m and P_t. Since C stabilises P_t, it is possible to obtain a sample of

$$X_t = \begin{bmatrix} X_{t,1} \\ \\ X_{t,2} \end{bmatrix} := \begin{bmatrix} I \\ \\ P_t \end{bmatrix} (I - CP_t)^{-1}$$

at frequencies of interest [9,10]. Note that, unless C is itself stable, the factorisation $P_t = X_{t,1}^{-1} X_{t,2}$ is not necessarily coprime over $\mathcal{H}_\infty$, and hence, X_t is not necessarily a graph symbol in the usual sense [5]. However, if $\varphi \in \mathbb{T}$ is not a pole of C, then $X_t(\varphi)$ is left-invertible by $[\, I \;\; -C(\varphi) \,]$. Thus, the range of $X_t(\varphi)$ is the graph of $P_t(\varphi)$ and correspondingly,

$$\kappa(P_m(\varphi), P_t(\varphi)) = \min_{Q \in \mathbb{C}} \bar{\sigma}(G_m(\varphi) - X_t(\varphi)Q), \tag{4}$$

where G_m denotes any *normalised* right graph symbol for P_m [5]. Such a graph symbol can be constructed as $G_m = \left[\begin{smallmatrix} D_m \\ N_m \end{smallmatrix}\right]$, from any normalised right coprime factorisation $N_m D_m^{-1}$ of P_m.

The *a posteriori* information is a set of *noisy* frequency response samples $\{X_i\}_{i=1}^{N} \subset \mathbb{C}^{2\times 1}$ of X_t; i.e. each $X_i = X_t(\varphi_i) + V_i$, for some $V_i \in \mathbb{C}^{2\times 1}$ and $\varphi_i \in \mathbb{T}$. It is assumed that these, not necessarily uniform over frequency, samples are consistent with the following additional *a* *priori* information: (i) $X_t \in \bar{\mathcal{B}}\mathcal{H}_{\infty,\rho}(\gamma)$, for specified constants $\gamma > 0$ and $\rho > 1$, which reflect the allowable behaviour of the true system;[2] and (ii) Each $\|V_i\| \leq \epsilon_i$ for a specified $\epsilon_i > 0$, which reflects the level of noise one is prepared to associate with the sample of $X_t(\varphi_i)$.

Towards explaining the observed data in terms of noise and true closed-loop behaviour, consider the following constrained optimisation problem:

$$\min_{V \in \mathbb{C}^{2\times n}} \max_{i \in [1,n]} \left(\min_{Q_i \in \mathbb{C}} \bar{\sigma}(G_m(\varphi_i) - (X_i - V_i)Q_i) \right) \tag{5}$$

where V_i donotes the i-th column of $V \in \mathbb{C}^{2\times n}$, subject to

$$\|V_i\| \leq \epsilon_i \text{ and } X_i - V_i = \hat{X}_t(\varphi_i) \text{ for } i = 1, \ldots, n \text{ and an } \hat{X}_t \in \bar{\mathcal{B}}\mathcal{H}_{\infty,\rho}(\gamma). \tag{6}$$

Now, let $\lambda^\star$ denote the globally optimal cost. Then from (4), it follows that $\lambda^\star$ is the *smallest* number for which there exists an $\hat{X}_t \in \bar{\mathcal{B}}\mathcal{H}_{\infty,\rho}(\gamma)$, and ϵ_i-bounded noise terms V_i, so that for each $i = 1, 2, \ldots, n$:

(1) the measured data is interpolated as $X_i = \hat{X}_t(\varphi_i) + V_i$;
(2) $\kappa(P_m(\varphi_i), \mathrm{Quot}(\hat{X}_t(\varphi_i))) \leq \lambda^\star$, where $\mathrm{Quot}([\begin{smallmatrix} X_d \\ X_n \end{smallmatrix}]) := X_d^{-1} X_n$.

In other words, any plant that is consistent with both the *a priori* assumptions (i.e. γ, ρ, $\{\epsilon_i\}_{i=1}^n$ and C) and the *a posteriori* data (i.e. the closed-loop frequency response samples $\{X_i\}_{i=1}^n$), lies no closer than $\lambda^\star$ to the nominal model P_m, in terms of worst case chordal distance over all sample frequencies.

An algorithm for solving the optimisation problem (5-6) is presented in the next section. It is closely related to an algorithm proposed in [11] and the Pick interpolation based worst case identification algorithms presented in [12,13]. Before proceeding, however, it is worth noting that since the manifestation of noise is captured in terms of a bound on the level only, solving the optimisation problem always yields and estimate that is less than the true chordal distance. As such, one should ensure that the bounds used for the noise are not overly conservative. Furthermore, this implies that an estimate obtained via the optimisation problem is only useful if it is large. In this case, the estimate is evidence that the nominal model P_m may not be suitable for model based design. In particular, for any controller C_1 that stabilises both P_m and P_t, the difference between $H(P_m, C_1)$ and $H(P_t, C_1)$ would be larger than the estimate of the chordal distance at the corresponding frequency.

[2] Recall that the k-th term of the impulse response of a function in $\bar{\mathcal{B}}\mathcal{H}_{\infty,\rho}(\gamma)$ is bounded by $\gamma\rho^{-k}$.

3 A co-ordinate decent approach

The optimisation problem (5-6) is not convex and as such, it is difficult to obtain a global solution. In fact, it is equivalent to a linear cost problem with bi-affine constraints (cf. Step 2 of Algorithm 1 below), which is known to be NP-hard [14]. Although various branch and bound type algorithms have been proposed for solving these problems (e.g. [15]), they are known to be useful only when there is a small number of variables which need to be fixed to yield affine constraints [16]. In the context of the problem considered here, the number of such variables corresponds to the number of frequency response samples, which could be large. In view of this, a so-called co-ordinate decent approach is presented in Algorithm 1 below.

Algorithm 1

(1) Given the set $\{\varphi_i\}_{i=1}^n \subset \mathbb{T}$ *of sample frequencies, let* $E \in \mathbb{C}^{n\times n}$ *denote the matrix with entries*

$$E_{k,l} = 1 \Bigg/ \left(1 - \frac{e^{\arg(\varphi_k)-\arg(\varphi_l)}}{\rho^2}\right).$$

Note that $\det(E) \neq 0$*. Then for* $i = 1, 2, \ldots, n$*, let* $Q^\star_{i,0} = 1$*. Finally, set* $k = 1$*.*

(2) Solve[3]

$$V_k^\star = \left[V_{1,k}^\star \ V_{2,k}^\star \ \cdots \ V_{n,k}^\star\right] := \operatorname*{argmin}_{[V_1 \ V_2 \ \cdots \ V_n]\in\mathbb{C}^{2\times n}} \lambda \tag{7}$$

subject to the affine matrix inequality constraints

$$\bigoplus_{i=1}^{n} \begin{bmatrix} \lambda I & \left(G_m(\varphi_i) - (X_i - V_i)\, Q^\star_{i,k-1}\right) \\ \left(G_m(\varphi_i) - (X_i - V_i)\, Q^\star_{i,k-1}\right)^* & \lambda \end{bmatrix} \geq 0, \tag{8}$$

$$\bigoplus_{i=1}^{n} \begin{bmatrix} \epsilon_i I & V_i \\ V_i^* & \epsilon_i \end{bmatrix} \geq 0, \tag{9}$$

$$\bigoplus_{i=1}^{n} \begin{bmatrix} 1 & \frac{x^*_{i,1} - v^*_{i,1}}{\gamma} \\ \frac{x_{i,1} - v_{i,1}}{\gamma} & 1 \end{bmatrix} \geq 0, \tag{10}$$

[3] If the set of V_is satisfying (9-13) is empty, then (6) is infeasible and the *a priori* information (ρ, γ, $\{\epsilon_i\}_{i=1}^n$) would need to be adjusted accordingly. The constraint (8) is always feasible for large enough λ.

$$\begin{bmatrix} E^{-1} & \bigoplus_{i=1}^{n}\left(\frac{x_{i,1}^*-v_{i,1}^*}{\gamma}\right) \\ \bigoplus_{i=1}^{n}\left(\frac{x_{i,1}-v_{i,1}}{\gamma}\right) & E \end{bmatrix} \geq 0, \tag{11}$$

$$\bigoplus_{i=1}^{n}\left(\begin{bmatrix} 1 & \frac{x_{i,2}^*-v_{i,2}^*}{\gamma} \\ \frac{x_{i,2}-v_{i,2}}{\gamma} & 1 \end{bmatrix}\right) \geq 0, \tag{12}$$

and

$$\begin{bmatrix} E^{-1} & \bigoplus_{i=1}^{n}\left(\frac{x_{i,2}^*-v_{i,2}^*}{\gamma}\right) \\ \bigoplus_{i=1}^{n}\left(\frac{x_{i,2}-v_{i,2}}{\gamma}\right) & E \end{bmatrix} \geq 0, \tag{13}$$

where for each $i = 1, 2, \ldots, n$,

$$\begin{bmatrix} x_{i,1} \\ x_{i,2} \end{bmatrix} := X_i \text{ and } \begin{bmatrix} v_{i,1} \\ v_{i,2} \end{bmatrix} := V_i.$$

Then define

$$\lambda_{k,k-1}^{\star,i} := \bar{\sigma}(G_m(\varphi_i) - (X_i - V_{i,k}^{\star})Q_{i,(k-1)}^{\star}) \tag{14}$$

and $\lambda_{k,(k-1)}^{\star} := \max_{i\in[0,n]} \lambda_{k,(k-1)}^{\star,i}$.

(3) *For $i = 1, 2, \ldots, n$, solve the unconstrained linear least-squares problem*

$$Q_{i,k}^{\star} := \operatorname*{argmin}_{Q_i\in\mathbb{C}} \bar{\sigma}(G_m(\varphi_i) - (X_i - V_{i,k}^{\star})Q_i).$$

Then define

$$\lambda_{k,k}^{\star,i} := \bar{\sigma}(G_m(\varphi_i) - (X_i - V_{i,(k-1)}^{\star})Q_{i,k}^{\star}), \tag{15}$$

and $\lambda_{k,k}^{\star} := \max_{i\in[0,n]} \lambda_{k,k}^{\star,i}$.

(4) *If $k > 1$ and $|\lambda_{k,k}^{\star} - \lambda_{(k-1),(k-1)}^{\star}|$ is less than some desired tolerance, then stop. Otherwise, set $k = k + 1$ and return to Step 2.*

By virtue of Pick's interpolation theorem (see [17,12] for example) the constraints (10–13) are equivalent to the existence of analytic interpolants $f_1, f_2 : \mathbb{D} \mapsto \bar{\mathbb{D}}$ such that

$$f_1(\frac{\varphi_i}{\rho}) = \frac{x_{i,1} + v_{i,1}}{\gamma} \quad \text{and} \quad f_2(\frac{\varphi_i}{\rho}) = \frac{x_{i,2} + v_{i,2}}{\gamma}.$$

Correspondingly,

$$\begin{bmatrix} \gamma f_1(\frac{z}{\rho}) \\ \gamma f_2(\frac{z}{\rho}) \end{bmatrix} \in \bar{\mathcal{B}}\mathcal{H}_{\infty,\rho}(\gamma)$$

interpolates each $(X_i - V_i)$, as required. Another important property of Algorithm 1, from the perspective of the stopping condition, is that the cost is always non-increasing.

Lemma 1 $\lambda^\star_{k+1,k+1} \leq \lambda^\star_{k+1,k} \leq \lambda^\star_{k,k}$ *for all* $k \geq 1$.

Proof : The proof follows from the definition of $\lambda^\star_{k+1,k}$ and $\lambda^\star_{k,k}$ via (14) and (15), respectively, and the fact that $V^\star_k$ satisfies the constraints (8–13) with $\lambda = \lambda^\star_{k,k}$, for the first part (i.e. Step 2) of the $(k+1)$-th iteration, for all $k \geq 1$. ∎

Remark 1 *Co-ordinate decent algorithms, of the kind detailed in Algorithm 1 above, are widely used – e.g. D-K iteration for μ-synthesis [18], the dual iteration for fixed order synthesis [19], iterative identification and control re-design [20]. However, in such algorithms decent directions are essentially constrained to be aligned with a strict subset of the overall co-ordinates at each step. Correspondingly, convergence to a locally optimal solution cannot be guaranteed [15]. In particular, the algorithm can converge to a saddle point when one exists. An alternative iterative approach to linear cost optimisation problems with bi-affine constraints was recently established in [21]. The approach involves the local optimisation of suitable non-convex functions at each step and enjoys local convergence under certain conditions.*

4 Numerical Example

To illustrate the closed-loop validation procedure described above, consider the following example. Suppose that the model

$$P_m(s) = \frac{2(s-1)}{s^2 - 0.4s + 2}$$

is used to design a feedback controller C for the "true" SISO plant

$$P_t(s) = \frac{2.1s - 2}{s^2 - 0.5s + 1.1}.$$

Although an explicit expression for the transfer function of P_t is specified here to provide sufficient context, it should be considered to be unknown. Furthermore, for the purpose of constructing closed-loop frequency response data below, consider the controller

$$C(s) = \frac{4.4s - 3.75}{s + 15.25},$$

which stabilises both P_m and P_t, and achieves $b(P_m, C) = 0.22$.

Let $\{\varphi_i\}_{i=1}^{25}$ denote the measurement frequencies, spaced logarithmically between 0.1 rad/s and 40 rad/s, at which samples of $X_t = [\begin{smallmatrix} I \\ P_t \end{smallmatrix}](I - CP_t)^{-1}$ are taken. Gaussian-distributed complex noise, with variance equal to 10% of the norm of $X_t(\varphi_i)$, is then added to each frequency-response sample. Finally, the measurement frequencies are mapped to the unit circle, via the (conformal) bilinear transform $z = \frac{(1+sT/2)}{(1-sT/2)}$ for $T = \frac{0.9\pi}{40}$. This final step simply maps the continuous-time problem under consideration to the discrete-time setting of the paper.

Applying Algorithm 1, with $\rho = 1.01$, $\gamma = 13$ and $\epsilon_i = 1.04$ (for $i = 1, 2, \ldots, 25$),[4] yields Figure 1. This shows the estimate of the smallest frequency-by-frequency chordal distance from the nominal model P_m to a system that is consistent with both the *a priori* assumptions and the data constructed above (see the solid curve). As expected, this is less than the actual chordal distance at each frequency (see the dashed curve), due to the freedom in the level of noise that can be used to explain the data. Despite this, it can be seen that the worst case chordal distance is always greater than 0.14 at low frequencies. As such, P_m may not be a suitable model for P_t from the perspective of closed-loop system design. Indeed, by (2), the difference between the closed-loops $H(P_m, C_1)$ and $H(P_t, C_1)$ is no smaller than 0.14 at low frequency, for *any* stabilising controller C_1. This could be significant if the design objectives include low frequency performance objectives (e.g. tracking, input disturbance rejection).

5 Conclusion

An algorithm is proposed for estimating the smallest pointwise chordal distance between a nominal model of a plant and a system that is consistent with noisy closed-loop frequency response samples and *a priori* assumptions on the closed-loop behaviour *only*. These estimates allow the difference between nominal closed-loop behaviour and achieved closed-loop behaviour to be quantified.

[4] Recall that appropriate values for ρ and γ may be determined from the impulse response of the closed-loop. The constants ϵ_i, on the other hand, are to be determined from knowledge of the experiment used to obtain the frequency response samples.

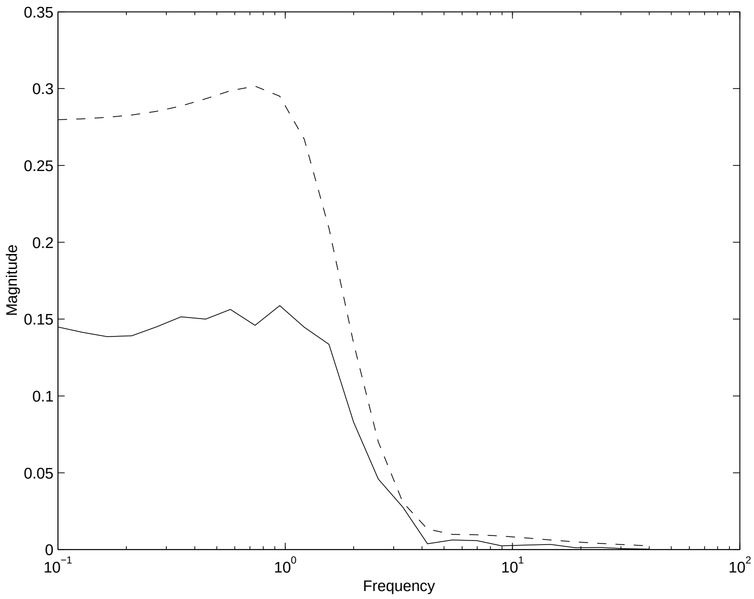

Fig. 1. Chordal distance to a system consistent with data and *a priori* assumptions (solid); Actual $\kappa(P_m, P_t)$ based on model used above (dashed).

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