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Generalized Detectability for Discrete Event Systems

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Abstract

In our previous work, we investigated detectability of discrete event systems, which is defined as the ability to determine the current and subsequent states of a system based on observation. For different applications, we defined four types of detectabilities: (weak) detectability, strong detectability, (weak) periodic detectability, and strong periodic detectability. In this paper, we extend our results in three aspects. (1) We extend detectability from deterministic systems to nondeterministic systems. Such a generalization is necessary because there are many systems that need to be modeled as nondeterministic discrete event systems. (2) We develop polynomial algorithms to check strong detectability. The previous algorithms are based on observer whose construction is of exponential complexity, while the new algorithms are based on a new automaton called detector. (3) We extend detectability to D-detectability. While detectability requires determining the exact state of a system, D-detectability relaxes this requirement by asking only to distinguish certain pairs of states. With these extensions, the theory on detectability of discrete event systems becomes more applicable in solving many practical problems.

Keywords

Discrete Event Systems; State Estimation; Detectability; D-detectability

1. Introduction

In many applications, we need to determine the state of a system. For example, when an assembly line has a failure, we need to diagnose its fault before the assembly line can be repaired and return to normal operation. Also, before we make a treatment plan for a patient, it is necessary to determine the disease status of the patient. Obviously, there are many such examples. In many situations, the system under consideration can be modeled as a discrete event system [1,2,3,4,5,6] and the problems can be translated into state estimation problems in a discrete event system framework.

State estimation problems in discrete event systems are important and interesting. They are first investigated in [7,8] and followed by [9]. In [7,8] current state estimation and initial state estimation are discussed. [8] discusses how to construct an observer which can describe the ability to uniquely estimate the state for any state output sequence. In [9] the authors discuss whether the current state can be determined periodically and call such property stability. We first present a full investigation of state estimation in [10] by introducing

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detectability. Detectability is defined as the ability to estimate the current and subsequent states of a system based on observation. We define four types of detectabilities for different applications: (weak) detectability, strong detectability, (weak) periodic detectability and strong periodic detectability. We derive necessary and sufficient conditions and algorithms to check these four detectabilities by constructing observers.

However, systems considered in [10] are deterministic in the sense that if a system is in a given state and a particular event occurs, then the next state is determined. In the current paper, we first extend our results to nondeterministic discrete event systems. Even though the ways to solve detectability of nondeterministic discrete event systems and detectability of deterministic discrete event systems with partial event observation are similar in terms of methodology, it is still necessary to extend the detectability results to nondeterministic discrete event systems for more realistic applications of detectability.

Checking detectabilities using methods in [10] requires the construction of an observer. Since in the worst case, the cardinality of the state space of the observer is exponential with respect to the cardinality of the state space of the original system, the computational complexity of checking detectabilities using methods in [10] is exponential with respect to the cardinality of the state space of the original system in the worst case. So it is important to search for more efficient algorithms for checking detectabilities. As the second contribution of this paper, we develop a method to check strong detectability and strong periodic detectability with only polynomial complexity in terms of the cardinality of the state space of the original system. The method is based on the construction of a “detector”. The detector is nondeterministic and its state space is quadratically bounded.

In our previous paper [10], the goal is to know exactly the current and subsequent states after some finite number of observations. However, this goal may be too restrictive for many applications. In some applications, we may want to know whether a system can enter a subset of states periodically (see [9]). In other applications, we may want to know whether the system always stays in a subset of states after some finite occurrences of events. For a diagnosis problem based on states [13,14], the state set is divided into several subsets. We want to know which subset the system is in. In supervisory control of discrete event systems based on state feedback, we need to distinguish the states that have different control actions [15]. In all these applications, the requirements can be translated to distinguishing certain pairs of states. To use detectability in these applications, we need a more general definition of detectability. The third contribution of this paper is to extend the detectability to D-detectability. While detectability requires determining the exact state of a system, D-detectability relaxes the requirement by asking only to distinguish certain pairs of states. We derive necessary and sufficient conditions for checking D-detectability.

In the rest of the paper, we will generalize detectability results to nondeterministic discrete event systems in Section 2; derive polynomial algorithms to check strong detectability and strong periodic detectability in Section 3; and investigate D-detectability in Section 4.

2. Nondeterministic Discrete Event Systems

We use an automaton to describe a nondeterministic discrete event system [3,16],

$$G=(Q, \Sigma, f, q_0)$$

where Q is the finite state set, $\sum$ is the finite event set, $f: Q \times \Sigma \to 2^Q$ is the (nondeterministic) transition function, and q_0 is the initial state. The transition function f describes the dynamic behavior of the system: if the system is in state q and event σ occurs,

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then the system moves (non-deterministically) to one of the states in $f(q,\sigma)$. An equivalent way to define the transition function is to specify the set of all possible transitions: $\{(q,\sigma,q') : q' \in f(q,\sigma)\}$. With a slight abuse of notation, we will also use f to denote the set of all possible transitions and write $(q,\sigma,q') \in f$ if $q' \in f(q,\sigma)$.

For different state estimation problems, we will assume different knowledge on the initial state. We use Q_0 ($\subseteq Q$) to denote the set of possible initial states. If $Q_0 = \{q_0\}$, then we know exactly the initial state q_0; and if $Q_0 = Q$, then we know nothing about the initial state. More generally, we can have $Q_0 \subseteq Q$. So the automation can be re-written as:

$$G=(Q, \Sigma, f, Q_0)$$

The event set is divided into two disjoint parts: the observable part $\sum_o$ and the unobservable part $\sum_{uo}$. The state estimation is based on event observation, which is described by the natural projection:

$$P(\epsilon)=\epsilon, P(s\sigma)=\begin{cases} P(s)\sigma & if \sigma \in \Sigma_0 \\ P(s) & if \sigma \notin \Sigma_0 \end{cases}$$

where ε denotes the empty string. P^{-1} denotes the inverse of natural projection P :

$$P^{-1}=\{t: (\forall t \in L(G))\, P(t)=s\}.$$

As in [9,10], we make the following two assumptions. (1) G is deadlock free, that is, for any state of the system, at least one event is defined at that state: $(\forall q \in Q)(\exists \sigma \in \Sigma)\, f(q,\sigma) \neq \phi$. (2) No loops in G contain only unobservable events:

$$\neg(\exists q \in Q)(\exists s \in \Sigma_{uo}^*)\, s \neq \epsilon \wedge q \in f(q,s).$$

A possible trajectory of the system is represented by an infinite sequence of events that the system may generate. The set of all possible trajectories of G is denoted by the ω-language $L^\omega(G)$ [17]. Because of the above assumptions, we know that for an infinite sequence of events, the length of the projected observable event sequence should also be infinite.

Suppose that the discrete event system G is in a set of possible states $Q' \subseteq Q$, then the set of all possible states after observing $t \in \Sigma_o^*$ is denoted by

$$R\left(Q', t\right)=\left\{q \in Q: \left(\exists q' \in Q'\right)(\exists s \in \Sigma^*)\, P(s)=t \wedge q \in f\left(q', s\right)\right\}$$

In particular, the unobservable reach is defined as:

$$UR\left(Q'\right)=R\left(Q', \epsilon\right).$$

For $s \in L^\omega(G)$, denote the set of all its prefixes by $\Pr(s)$. Let us also denote the set of positive integers by N . If t is a string, then $|t|$ denotes its length. If x is a set, $|x|$ denotes its cardinality (number of elements). With these notations, we can extend the definitions of detectabilities defined in [10] to nondeterministic systems in a formal way.

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Definition 1 (Strong Detectability)

A nondeterministic discrete event system G is strongly detectable with respect to P if we can determine, after a finite number of observations, the current state and subsequent states of the system for all trajectories of the system. That is,

$$(\exists n \in N)(\forall s \in L^{w}(G))(\forall t \in Pr(s))\,|P(t)|>n \Rightarrow |R(Q_0, P(t))|=1$$

Definition 2 (Detectability)

A nondeterministic discrete event system G is detectable with respect to P if we can determine, after a finite number of observations, the current state and subsequent states of the system for some trajectories of the system. That is,

$$(\exists n \in N)(\exists s \in L^{w}(G))(\forall t \in Pr(s))\,|P(t)|>n \Rightarrow |R(Q_0, P(t))|=1$$

Definition 3 (Strong Periodic Detectability)

A nondeterministic discrete event system G is strongly periodically detectable with respect to P if we can periodically determine the current state of the system for all trajectories of the system. That is,

$$(\exists n \in N)(\forall s \in L^{w}(G))(\forall t \in Pr(s))\left(\exists t' \in \Sigma^{*}\right) tt' \in Pr(s) \wedge |P\left(t'\right)|<n \wedge |R(Q_0, P(t))|=1$$

Definition 4 (Periodic Detectability)

A nondeterministic discrete event system G is periodically detectable with respect to P if we can periodically determine the current state of the system for some trajectories of the system. That is,

$$(\exists n \in N)(\exists s \in L^{w}(G))(\forall t \in Pr(s))\left(\exists t' \in \Sigma^{*}\right) tt' \in Pr(s) \wedge |P\left(t'\right)|<n \wedge |R(Q_0, P(t))|=1$$

The intuition and usefulness of the above definitions of four types of detectabilities are discussed in [10] and briefly summarized here: Strong detectability is the strongest among four; it requires that the state can be determined after some finite observations for all trajectories and at all time. Periodic detectability is the weakest; it requires that the state can be determined for some trajectories and at some time. In some application, such as ensuring safety of a nuclear reactor, we need a strong version of detectability; while in some other applications, a weaker version may be sufficient.

The problem of checking these detectabilities can be solved by constructing an observer. The observer can be constructed by first replacing all unobservable events in G by the empty string ε and then converting the nondeterministic automaton into a deterministic automaton in the usual way [3,10]. Denote the observer by

$$G_{obs}=(X, \Sigma_0, \delta, x_0)=Ac\left(2^{Q}, \Sigma_0, \delta, UR(Q_0)\right)$$

where $Ac(.)$ denotes the accessible part and $UR(Q_0)$ is the unobservable reach of Q_0. Note that a state $x \in X$ is a subset of Q ($x \subseteq Q$). The transition function $\delta: X \times \Sigma_o \to X$ is defined, for $x \subseteq Q$ and $\sigma \in \sum_o$, as

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$$\delta(x,\sigma)=UR\left(\left\{q\in Q:\left(\exists q^{'}\in x\right)q\in f\left(q^{'},\sigma\right)\right\}\right).$$

If the above set is empty, we say that $\delta(x,\sigma)$ is undefined. We extend δ to strings in the usual way. As shown in [3,10], $L(G_{obs}) = P(L(G))$ and for $s \in L(G_{obs})$, $\delta(x_o,s) = R(Q_0,s)$, that is, the observer describes the estimation of possible states of the system.

To present necessary and sufficient conditions for detectabilities, we first mark the states in G_{obs} that contain singleton state and denote the set by:

$$X_m=\{x\in X:|x|=1\}.$$

When G_{obs} is in X_m , we know exactly which state G is in. For detectability, states involved in loops are of particular interest because G_{obs} can stay in these states forever. Therefore, let us denote the set of all loops in G_{obs} as

$$Loop=\left\{(x,u)\in X\times\Sigma_0^*:|u|\geq 1\wedge\delta(x,u)=x\right\}.$$

Theorem 1

A nondeterministic discrete event system G is strongly detectable with respect to P if and only if in observer G_{obs} , $(\forall(x,u)\in Loop)(\forall w\in\Sigma_o^*)\,\delta(x,w)\in X_m$, that is, any state reachable from any loop in G_{obs} is in X_m .

Proof:

(If) Assume that G is not strongly detectable with respect to P , that is,

$$(\forall n\in N)\left(\exists s\in L^w(G)\right)(\exists t\in Pr(s))\,|P(t)|>n\wedge|R(Q_0,P(t))|\neq 1.$$

Let $n =| X |$. Then such an s must go through at least one loop in G_{obs} . Denote the first loop by $(x,u) \in Loop$. For this s , $(\exists t\in\Pr(s))\,|P(t)|>|X|\wedge|R(Q_o,P(t))|\neq 1$. Since (x,u) is the first loop and $|P(t)|>| X |$, $P(t)$ will pass x first, that is, $(\exists w\in\Sigma_o^*)\,\delta(x_0,P(t))=\delta(x,w)$. Furthermore,

$$|R(Q_0,P(t))|\neq 1\Rightarrow\delta(x,w)\notin X_m.$$

Therefore, $(\exists(x,u)\in Loop)(\exists w\in\Sigma_o^*)\,\delta(x,w)\notin X_m$.

(Only If) Assume $(\forall(x,u)\in Loop)(\forall w\in\Sigma_o^*)\,\delta(x,w)\in X_m$ is not true, that is, $(\exists(x,u)\in Loop)(\exists w\in\Sigma_o^*)\,\delta(x,w)\notin X_m$. Let v be any string heading to x from the initial state, that is, $\delta(x_o,v) = x$. For any $n \in N$, there exists $s\in P^{-1}(vu^nw\ldots)\cap L^w(G)$ and $t=\Pr(s)\cap P^{-1}(vu^nw)$ such that

$$\begin{aligned}&(\forall n\in N)(\exists s\in L^w(G))(\exists t\in Pr(s))\,|P(t)|>n\wedge\delta(x_0,P(t))\notin X_m\\ \Rightarrow\ &(\forall n\in N)(\exists s\in L^w(G))(\exists t\in Pr(s))\,|P(t)|>n\wedge R(Q_0,P(t))\notin X_m\\ \Rightarrow\ &(\forall n\in N)(\exists s\in L^w(G))(\exists t\in Pr(s))\,|P(t)|>n\wedge|R(Q_0,P(t))|\neq X_m\end{aligned}$$

That is, G is not strongly detectable with respect to P .

Q.E.D

Theorem 2

A nondeterministic discrete event system G is detectable with respect to P if and only if in observer G_{obs} , $(\exists (x,u) \in Loop)(\forall w \in \Pr(u))\,\delta(x,w) \in X_m$, that is, there are loops in G_{obs} that are entirely within X_m .

Proof:

(If) Assume $(\exists (x,u) \in Loop)(\forall w \in \Pr(u))\,\delta(x,w) \in X_m$. Let v be any string leads to x from the initial state, that is, $\delta(x_o,v) = x$. Then there exists $s \in P^{-1}(vuuu\ldots) \cap L^{w}(G)$ and $n = |v| \in N$ such that

$$
\begin{aligned}
&(\exists n \in N)(\exists s \in L^{w}(G))(\exists t \in Pr(s))\,|P(t)|>n \Rightarrow \delta(x_0, P(t)) \notin X_m \\
\Rightarrow\ &(\exists n \in N)(\exists s \in L^{w}(G))(\exists t \in Pr(s))\,|P(t)|>n \Rightarrow R(Q_0, P(t)) \notin X_m \\
\Rightarrow\ &(\exists n \in N)(\exists s \in L^{w}(G))(\exists t \in Pr(s))\,|P(t)|>n \Rightarrow |R(Q_0, P(t))| \neq X_m
\end{aligned}
$$

That is, G is detectable with respect to P .

(Only If) Assume that G is detectable with respect to P , that is,

$$
(\exists n \in N)(\exists s \in L^{w}(G))(\forall t \in Pr(s))\,|P(t)|>n \Rightarrow |R(Q_0, P(t))|=1.
$$

Then such a s must go through at least one loop in G_{obs} infinitely often. Denote this loop by $(x,u) \in Loop$. From the above equation,

$$
|R(Q_0, P(t))|=1 \Rightarrow (\forall w \in Pr(u))\,\delta(x,w) \in x_m
$$

Therefore, $(\exists (x,u) \in Loop)(\forall w \in \Pr(u))\,\delta(x,w) \in X_m$.

Q.E.D

Theorem 3

A nondeterministic discrete event system G is strongly periodically detectable with respect to P if and only if in observer G_{obs} , $(\forall (x,u) \in Loop)(\exists w \in \Pr(u))\,\delta(x,w) \in X_m$, that is, all loops in G_{obs} must include at least one state belonging to X_m .

Proof:

(If) Assume that G is not strongly periodically detectable with respect to P , that is,

$$
(\forall n \in N)(\exists s \in L^{w}(G))(\exists t \in Pr(s))\left(\forall t' \in \Sigma^*\right)tt' \in Pr(s) \wedge |P\left(t'\right)|<n \Rightarrow |R\left(Q_0, P\left(tt'\right)\right)| \neq 1.
$$

Let $n =| X | +1$. Then such a s must go through at least one loop in G_{obs} such that it is the last loop passed by $P(tt')$ with $P(t') =| X |$. Denote this loop by $(x,u) \in Loop$.

since $\left(\forall t' \in \Sigma^*\right)tt' \in \Pr(s) \quad \wedge |P\left(t'\right)|<|X|+1 \Rightarrow |R\left(Q_o, P\left(tt'\right)\right)| \neq 1,$

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$$(\forall w \in Pr(u))\, \delta(x, w) \notin X_m.$$

Therefore, $(\exists (x, u) \in Loop)\, (\forall w \in \Pr(u))\, \delta(x, w) \notin X_m$.

(Only If) Assume $(\forall (x, u) \in Loop)\, (\exists w \in \Pr(u))\, \delta(x, w) \in X_m$ is not true, that is, $(\exists (x, u) \in Loop)\, (\forall w \in \Pr(u))\, \delta(x, w) \notin X_m$. Let v be any string leads to x from the initial state, that is, $\delta(x_o,v) = x$. Then there exists $s \in P^{-1}(vuuu\ldots) \cap L^w(G)$ and $t \in \Pr(s) \cap P^{-1}(v)$ such that

$$\begin{aligned}
&(\forall n \in N)\, (\exists s \in L^w(G))\, (\exists t \in Pr(s)) \left(\forall t' \in \Sigma^*\right) tt' \in Pr(s) \wedge |P\left(t'\right)| < n \Rightarrow \delta\left(x_0, P\left(tt'\right)\right) \notin X_m \\
\Rightarrow\ &(\forall n \in N)\, (\exists s \in L^w(G))\, (\exists t \in Pr(s)) \left(\forall t' \in \Sigma^*\right) tt' \in Pr(s) \wedge |P\left(t'\right)| < n \Rightarrow R\left(Q_0, P\left(tt'\right)\right) \notin X_m \\
\Rightarrow\ &(\forall n \in N)\, (\exists s \in L^w(G))\, (\exists t \in Pr(s)) \left(\forall t' \in \Sigma^*\right) tt' \in Pr(s) \wedge |P\left(t'\right)| < n \Rightarrow |R\left(Q_0, P\left(tt'\right)\right)| \neq 1
\end{aligned}$$

That is, G is not strongly periodically detectable with respect to P .

Q.E.D

Theorem 4

A nondeterministic discrete event system G is periodically detectable with respect to P if and only if in observer G_{obs} , $(\exists (x, u) \in Loop)\, (\forall w \in \Pr(u))\, \delta(x, w) \in X_m$, that is, there are loops in G_{obs} which include at least one state belonging to X_m .

Proof:

(If) Assume that G is periodically detectable with respect to P , that is,

$$(\exists n \in N)\, (\exists s \in L^w(G))\, (\forall t \in Pr(s)) \left(\exists t' \in \Sigma^*\right) tt' \in Pr(s) \wedge |p\left(t'\right)| < n \wedge |R\left(Q_0, P\left(tt'\right)\right)| = 1$$

Then such a s must go through at least one loop in G_{obs} in which $|R(Q_0,P(tt'))| = 1$ is true for some t' . Denote this loop by $(x,u) \in Loop$. From the above equation,

$$|R\left(Q_0, P\left(tt'\right)\right)| = 1 \Rightarrow (\exists w \in Pr(u))\, \delta(x, w) \in X_m.$$

Therefore, $(\exists (x, u) \in Loop)\, (\forall w \in \Pr(u))\, \delta(x, w) \in X_m$.

(Only If) Assume $(\exists (x, u) \in Loop)\, (\forall w \in \Pr(u))\, \delta(x, w) \in X_m$ is true. Let v be any string leads to x from the initial state, that is, $\delta(x_o,v) = x$. Then there exists $s \in P^{-1}(vuuu\ldots) \cap L^w(G)$ and $n = |vu| \in N$ such that

$$\begin{aligned}
&(\exists n \in N)\, (\exists s \in L^w(G))\, (\forall t \in Pr(s)) \left(\exists t' \in \Sigma^*\right) tt' \in Pr(s) \wedge |P\left(t'\right)| < n \wedge \delta\left(x_0, P\left(tt'\right)\right) \in X_m \\
\Rightarrow\ &(\exists n \in N)\, (\exists s \in L^w(G))\, (\forall t \in Pr(s)) \left(\exists t' \in \Sigma^*\right) tt' \in Pr(s) \wedge |P\left(t'\right)| < n \wedge R\left(Q_0, P\left(tt'\right)\right) \in X_m \\
\Rightarrow\ &(\exists n \in N)\, (\exists s \in L^w(G))\, (\forall t \in Pr(s)) \left(\exists t' \in \Sigma^*\right) tt' \in Pr(s) \wedge |P\left(t'\right)| < n \wedge |R\left(Q_0, P\left(tt'\right)\right)| = 1
\end{aligned}$$

That is, G is periodically detectable with respect to P .

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Q.E.D

Remark 1

The above necessary and sufficient conditions for four detectabilities are similar to those obtained in [10]. Here we state them more formally and prove them for nondeterministic discrete event systems. In [10], we made a mistake in stating Theorem 1 for checking strong detectability of deterministic discrete event system. Note that deterministic discrete event systems can be viewed as special case of nondeterministic discrete event systems. Therefore, the correct version of the theorem is as stated in Theorem 1 above.

3. Polynomial Algorithms

Checking detectabilities using Theorems 1-4 requires the construction of an observer. Since in the worst case, the cardinality of the state space of the observer is $2^{|Q|}$, the computational complexity of constructing the observer is exponential. Therefore, it is useful and important to find some more efficient ways to check detectabilities. So far, we have not found more efficient ways to check detectability and periodic detectability. However, we have found a method to check strong detectability and strong periodic detectability with polynomial complexity. The method is based on the construction of a "detector", which is defined as follows:

$$G_{det} = (Y, \Sigma_0, \xi, y_0) = Ac\left(Y', \Sigma_0, \xi, UR(Q_0)\right).$$

The initial state of $G_{\det}$ is $y_0 = UR(Q_0)$ (same as x_0 of G_{obs}). The other states of $G_{\det}$ are subsets of Q which contain at most two elements, that is,

$$Y' = \{y : y \subseteq Q \wedge |y| \le 2\} \cup \{y_0\}$$

The transition function $\xi : Y' \times \Sigma_o \to 2^{r}$ is defined, for $y \in Y'$, $y \subseteq Q$ and $\sigma \in \sum_o$ as follows. First, define an intermediate or temporary variable $Temp = UR\left(\left\{q \in Q : \left(\exists q' \in y\right) q \in f\left(q', \sigma\right)\right\}\right)$. Then according to the number of elements in $Temp$, we have

$$\xi(y, \sigma) = \begin{cases} \{Temp\} & if |Temp| = 1 \\ \left\{y' : y' \subseteq Temp \wedge |y'| = 2\right\} & if |Temp| \ge 2 \\ \text{undefined} & if |Temp| = 0. \end{cases}$$

We extend ξ to strings in the usual way.

The difference between G_{obs} and $G_{\det}$ can be viewed as follows. Whenever a state in G_{obs} contains more than two elements, it is split into several states and each contains two elements. This makes $G_{\det}$ nondeterministic. Since the cardinality of the state space of $G_{\det}$ is bounded by $|Y'| \le 1 + |Q| + |Q|(|Q| - 1) / 2$, the complexity of constructing $G_{\det}$ is polynomial.

Remark 2

Polynomial algorithms to check event-based diagnosability are presented in [11,12]. [11] checks diagnosability by constructing a verifier and [12] checks diagnosability by

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constructing a new automaton G_d . There are significant differences between the results of [11,12] and our results. First, we study detectability while [11,12] investigate diagnosability. In particular, the initial state in [11,12] is certain, but in our paper it is uncertain. Furthermore, in terms of technology, the transitions defined for verifier and G_d are totally different from the transitions defined for our detector. Our transitions are obtained by splitting the state *Temp* after we observe an observable event.

The following lemma relates observer G_{obs} and detector G_{det} and will be used in the proofs of the results later.

Lemma 1

The relation between observer G_{obs} and detector G_{det} is as follows.

1. For any $s \in \Sigma_o^*$, $\xi(y_0, s) = \{\delta(x_0, s)\}$ if $|\delta(x_0,s)| = 1$.
2. For any $s \in \Sigma_o^*$ and any $y \subseteq Q$ such that $|y| = 2$, $y \in \xi(y_0, s) \iff y \subseteq \delta(x_0, s)$.

Proof:

Since the definitions of $\delta(y,\sigma)$ and *Temp* are the same, from the definition of ξ , we can conclude the following.

(C1) If $\delta(y',\sigma)$ satisfies $|\delta(y',\sigma)| = 1$, then

$$\xi(y', \sigma) = \{\delta(y', \sigma)\}.$$

(C2) If $\delta(y',\sigma)$ satisfies $|\delta(y',\sigma)| \geq 2$, then for any state $y \subseteq Q$ such that $|y| = 2$,

$$y \in \xi(y', \sigma) \iff y \subseteq \delta(y', \sigma)$$

Using (C1) and (C2), we can prove the lemma by induction on the length $|s|$ of $s \in \Sigma_o^*$.

Base: For $|s| = 1$, that is, $s = \sigma \in \sum_o$, we can prove Part 1 of the lemma by letting $y' = y_0$ in (C1) and prove Part 2 of the lemma by letting $y' = y_0 = x_0$ in (C2).

Induction Hypothesis (IH): Suppose that for any $s \in \Sigma_o^*$ such that $|s| \leq n$ Lemma 1 is true.

Induction Step: we need to prove that Lemma 1 is true for any $s\sigma \in \Sigma_o^*$ such that $|s\sigma| = n + 1$. Since the size of $\delta(y_0,s)$ and $\delta(y_0,s\sigma)$ may be one or more than one, we need to discuss four possible cases as follows.

Case 1: $|\delta(x_0,s)| = 1$ and $|\delta(x_0,s\sigma)| = 1$.

Since $|\delta(x_0,s\sigma)| = 1$, we need to prove $\xi(y_0,s\sigma) = \{\delta(x_0,s\sigma)\}$, which can be done as follows.

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$$\begin{aligned}
& \xi(y_0, s\sigma) \\
= & \ \xi(\xi(y_0, s), \sigma) \\
= & \ \left\{y \in Y: \left(\exists y' \in \xi(y_0, s)\right) y \in \xi\left(y', \sigma\right)\right\} \\
= & \ \left\{y \in Y: \left(\exists y' \in \delta(x_0, s)\right) y \in \xi\left(y', \sigma\right)\right\} && \text{by IH and} |\delta(x_0, s)| = 1 \\
= & \ \{y \in Y: y \in \xi(\delta(x_0, s), \sigma)\} \\
= & \ \{y \in Y: y \in \{\delta(\delta(x_0, s), \sigma)\}\} && \text{by (C1) and } (\delta(x_0, s\sigma)) = 1 \\
= & \ \{\delta(\delta(x_0, s), \sigma)\} \\
= & \ (\delta(x_0, s\sigma)).
\end{aligned}$$

Case 2: $|\delta(x_0,s)| = 1$ and $|\delta(x_0,s\sigma)| \geq 2$.

Since $|\delta(x_0,s\sigma)| \geq 2$, we need to prove for any $y \subseteq Q$ such that $|y| = 2$, $y \in \xi(y_0, s\sigma) \iff y \subseteq \delta(x_0, s\sigma)$. This can be done as follows.

$$\begin{aligned}
& y \in \xi(y_0, s\sigma) \\
\iff & y \in \xi(\xi(y_0, s), \sigma) \\
\iff & y \in \left\{y \in Y: \left(\exists y' \in \xi(y_0, s)\right) y \in \xi\left(y', \sigma\right)\right\} \\
\iff & \left(\exists y' \in \xi(y_0, s)\right) y \in \xi\left(y', \sigma\right) \\
\iff & \left(\exists y' \in \{\delta(x_0, s)\}\right) y \in \xi\left(y', \sigma\right) && \text{by IH and} |\delta(x_0, s)| = 1 \\
\iff & y \in \xi(\delta(x_0, s), \sigma) \\
\iff & y \subseteq \delta(\delta(x_0, s), \sigma) && \text{by (C2) and } (\delta(x_0, s\sigma)) \geq 2 \\
\iff & y \subseteq \delta(x_0, s\sigma).
\end{aligned}$$

Case 3: $|\delta(x_0,s)| \geq 2$ and $|\delta(x_0,s\sigma)| = 1$.

Since $|\delta(x_0,s\sigma) = 1$, we need to prove $\xi(y_0,s\sigma) = \{\delta(x_0,s\sigma)\}$, which can be done as follows.

$$\begin{aligned}
& \xi(y_0, s\sigma) \\
= & \ \xi(\xi(y_0, s), \sigma) \\
= & \ \left\{y \in Y: \left(\exists y' \in \xi(y_0, s)\right) y \in \xi\left(y', \sigma\right)\right\} \\
= & \ \left\{y \in Y: \left(\exists y' \subseteq \delta(x_0, s)\right) y \in \xi\left(y', \sigma\right)\right\} && \text{by IH and} |\delta(x_0, s)| \geq 2 \\
= & \ \left\{y \in Y: \left(\exists y' \subseteq (\delta(x_0, s), \sigma)\right) y \in \left\{\delta\left(y', \sigma\right)\right\}\right\} && \text{by (C1) and} |\delta(x_0, s\sigma)| = 1 \\
= & \ \left\{y \in Y: \left(\exists y' \subseteq \delta(x_0, s)\right) y = \delta\left(y', \sigma\right)\right\} \\
= & \ \{\delta(\delta(x_0, s), \sigma)\} \\
= & \ (\delta(x_0, s\sigma)).
\end{aligned}$$

Case 4: $|\delta(x_0,s)| \geq 2$ and $|\delta(x_0,s\sigma)| \geq 2$.

Since $|\delta(x_0,s\sigma)| \geq 2$, we need to prove for any $y \subseteq Q$ such that $|y| = 2$, $y \in \xi(y_0, s\sigma) \iff y \subseteq \delta(x_0, s\sigma)$. This can be done as follows.

$$
\begin{aligned}
& y \in \xi(y_0, s\sigma) \\
\Longleftrightarrow\; & y \in \xi(\xi(y_0, s), \sigma) \\
\Longleftrightarrow\; & y \in \left\{y \in Y: \left(\exists y' \in \xi(y_0, s)\right) y \in \xi\left(y', \sigma\right)\right\} \\
\Longleftrightarrow\; & \left(\exists y' \in \xi(y_0, s)\right) y \in \xi\left(y', \sigma\right) \\
\Longleftrightarrow\; & \left(\exists y' \subseteq \delta(x_0, s)\right) y \in \xi\left(y', \sigma\right) \quad \text{by IH and} |\delta(x_0, s)| \geq 2 \\
\Longleftrightarrow\; & \left(\exists y' \subseteq \delta(x_0, s)\right) y \subseteq \delta\left(y', \sigma\right) \quad \text{by (C2) and} |\delta(x_0, s\sigma)| \geq 2 \\
\Longleftrightarrow\; & y \subseteq \delta(\delta(x_0, s), \sigma) \\
\Longleftrightarrow\; & y \subseteq \delta(x_0, s\sigma)
\end{aligned}
$$

Detector G_{det} can be used to check strong detectability and strong periodic detectability. To do this, let us mark singleton states in G_{det} as:

$$Y_m = \{y \in Y : |y| = 1\}.$$

Similar to G_{obs} , let us denote the set of all loops in G_{det} as

$$YLoop = \left\{(y, u) \in Y \times \Sigma_0^* : |u| \geq 1 \wedge y \in \xi(y, u)\right\}$$

The necessary and sufficient conditions to check strong detectability and strong periodic detectability are given in the following theorems.

Theorem 5

A nondeterministic discrete event system G is strongly detectable with respect to P if and only if in detector G_{det} , $(\forall (y, u) \in YLoop)(\forall w \in \Sigma_o^*)\, \xi(y, w) \subseteq Y_m$, that is, any state reachable from any loop in G_{det} is in Y_m .

Proof:

By Theorem 1, we only need to show that $(\forall (x, u) \in Loop)(\forall w \in \Sigma_o^*)\, \delta(x, w) \in X_m$ if and only if $(\forall (y, u) \in YLoop)(\forall w \in \Sigma_o^*)\, \xi(y, w) \subseteq Y_m$.

(Only If) If $(\forall (x, u) \in Loop)(\forall w \in \Sigma_o^*)\, \delta(x, w) \in X_m$ is true, then in particular, any state in any loop in G_{obs} belongs to X_m , that is, $(\forall (x, u) \in Loop)\, x \in X_m$. Hence, none of these states will be split in G_{det} . Formally, for any $(x,u) \in Loop$, let v be any string that leads to x from the initial state, that is, $\delta(x_o,v) = x$. By Lemma 1, $\xi(y_o,v) = \{\delta(x_o,v)\}$. Therefore, the set of loops in G_{obs} , $Loop$, is isomorphic to the set of loops in G_{det} , $YLoop$. Hence, by Lemma 1,

$$
\begin{aligned}
& (\forall (x, u) \in Loop)\left(\forall w \in \Sigma_0^*\right) \delta(x, w) \in X_m \\
\Rightarrow\; & (\forall (x, u) \in YLoop)\left(\forall w \in \Sigma_0^*\right) \xi(x, w) \subseteq Y_m
\end{aligned}
$$

(If) If $(\forall (y, u) \in YLoop)(\forall w \in \Sigma_o^*)\, \xi(y, w) \subseteq Y_m$ is true, then in particular, any state in any loop in G_{det} belongs to Y_m . Hence, none of these states is obtained by splitting states in G_{obs} . Therefore, as in the Only If part of the proof, the set of loops in G_{det} , $YLoop$, is isomorphic to the set of loops in G_{obs} , $Loop$. Hence, by Lemma 1,

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$$\begin{array}{ll} & (\forall (x,u) \in YLoop) \left(\forall w \in \Sigma_0^*\right) \xi(x,w) \subseteq Y_m \\ \Rightarrow & (\forall (x,u) \in Loop) \left(\forall w \in \Sigma_0^*\right) \delta(x,w) \in X_m \end{array}$$

Q.E.D

Theorem 6

A nondeterministic discrete event system G is strongly periodically detectable with respect to P if and only if in detector $G_{\det}$, $(\forall (y,u) \in YLoop)\,(\exists w \in \Pr(u))\,\xi(y,w) \subseteq Y_m$, that is, all loops in $G_{\det}$ must include at least one state belonging to Y_m .

Proof:

By Theorem 3, we only need to show that $(\forall (x,u) \in Loop)\,(\exists w \in \Pr(u))\,\delta(x,w) \in X_m$ if and only if $(\forall (y,u) \in YLoop)\,(\exists w \in \Pr(u))\,\xi(y,w) \subseteq Y_m$.

(Only If) If $(\forall (x,u) \in Loop)\,(\exists w \in \Pr(u))\,\delta(x,w) \in X_m$ is true, then any loop will include at least one state $\delta(x,w) \in X_m$. Such a singleton state $\delta(x,w)$ will not be split in $G_{\det}$. Formally, for any $(x,u) \in Loop$, let v be any string that leads to x from the initial state, that is, $\delta(x_o,v) = x$. By Lemma 1, $\xi(y_o,vw) = \{\delta(x_o,vw)\}$. Therefore, although $G_{\det}$ may have more loops than G_{obs} , but all the loops in $G_{\det}$ will include at least one singleton state. Hence,

$$\begin{array}{ll} & (\forall (x,u) \in Loop)\,(\exists w \in Pr(u))\,\delta(x,w) \in X_m \\ \Rightarrow & (\forall (y,u) \in YLoop)\,(\exists w \in Pr(u))\,\xi(y,w) \subseteq Y_m \end{array}$$

(If) Suppose $(\forall (y,u) \in YLoop)\,(\exists w \in \Pr(u))\,\xi(y,w) \subseteq Y_m$ is true, then any loop will include at least one singleton state $\xi(y,w) \in Y_m$, which is not obtained by splitting states in G_{obs} . Therefore, as in the Only If part of the proof, although $G_{\det}$ may have more loops than G_{obs} , but all the loops in G_{obs} will include at least one singleton state. Hence,

$$\begin{array}{ll} & (\forall (y,u) \in YLoop)\,(\exists w \in Pr(u))\,\xi(y,w) \subseteq Y_m \\ \Rightarrow & (\forall (x,u) \in Loop)\,(\exists w \in Pr(u))\,\delta(x,w) \in X_m \end{array}$$

Q.E.D

To illustrate the above results on how to check detectabilities, let us consider the following example.

Example 1

Consider a discrete event system shown in Figure 1. We assume that all the events are observable and we know nothing about the initial state, that is: $\sum_o = \sum$ and $Q_0 = Q$.

To check detectabilities, we construct the observer, which is shown in Figure 2. The marked states $X_m = \{x \in X : |x| = 1\}$ are those in the last row in Figure 2. Because not all the loops in G_{obs} are entirely within X_m , by Theorem 1, the system is not strongly detectable. Because there are loops in X_m , by Theorem 2, the system is detectable. Because there are loops in G_{obs} which are entirely within $X - X_m$, by Theorem 3, the system is not strongly periodically detectable. Because there are loops in G_{obs} which include at least one state belonging to X_m , by Theorem 4, the system is periodically detectable. Observer G_{obs} has $2^{|Q|} - 1 = 2^4 - 1 = 15$ states, which is exponential with respect to the number of states in Q .

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To avoid the exponential complexity, let us construct detector $G_{\det}$ as shown in Figure 3. Detector $G_{\det}$ has $|Q|(|Q|-1)/2+|Q|+1 = 4{\cdot}3/2 + 4 + 1 = 11$ states, which is polynomial with respect to the number of states in Q.

For detector $G_{\det}$, the marked states $Y_m = \{y \in Y : |y| = 1\}$ are those in the last row in Figure 3. Because not all the loops in $G_{\det}$ are entirely within Y_m, by Theorem 5, the system is not strongly detectable. Because there are loops in $G_{\det}$ which are entirely within $Y - Y_m$, by Theorem 6, the system is not strongly periodically detectable.

4. D-Detectability

Detectabilities discussed in the previous sections require that we can exactly determine the current and subsequent states after a finite number of observations. This requirement is useful in some applications but may be too strong in others. In this section, we relax this requirement and ask only to distinguish certain pairs of states. The requirement of distinguishing certain pairs of states is used in communication problems and sensor activation problems [15]. However, it has not been studied in terms of detectability before. Detectabilities in terms of distinguish certain pairs of states will be called D-detectabilities (D for “distinguish”). To define D-detectabilities, we define the set of state-pairs T, for a given set of states Q, as

$$T=\left\{\left(q,q'\right):q\in Q\wedge q'\in Q\right\}.$$

Noted that (q,q’) and (q’,q) are the same state-pair. We specify the set of state-pairs to be distinguished as a subset of T, that is,

$$T_{spec}\subseteq T.$$

We call T_{spec} a specification. D-detectability requires that any state pair (q,q’) in specification T_{spec} is distinguishable after a finite number of observations.

If the estimation of the states is a subset $Q' \subseteq Q$, then the set of undistinguishable state pairs is given by:

$$SP\left(Q'\right)=\left\{\left(q,q'\right):q\in Q'\wedge q'\in Q'\right\}$$

Since the set of all possible states after observing $t \in \Sigma^*$ is denoted $R(Q_0,t)$, when we observe event sequence $t \in \Sigma^*$, the set of undistinguishable state pairs is given by:

$$SP\left(R\left(Q_0,t\right)\right)=\left\{\left(q,q'\right):q\in R\left(Q_0,t\right)\wedge q'\in R\left(Q_0,t\right)\right\}.$$

Therefore, we have the following definitions of D-detectabilities.

Definition 5 (Strong D-detectability)

A nondeterministic discrete event system G is strongly D-detectable with respect to P and T_{spec} if we can distinguish state pairs in T_{spec} all the time, after a finite number of observations, for all trajectories of the system. That is,

$$(\exists n \in N)\,(\forall s \in L^{w}(G))\,(\forall t \in Pr(s))\,|P(t)|>n \Rightarrow SP(R(Q_0, P(t))) \cap T_{spec}=\phi$$

Definition 6 (D-detectability)

A nondeterministic discrete event system G is D-detectable with respect to P and T_{spec} if we can distinguish state pairs in T_{spec} all the time, after a finite number of observations, for some trajectories of the system. That is,

$$(\exists n \in N)\,(\exists s \in L^{w}(G))\,(\forall t \in Pr(s))\,|P(t)|>n \Rightarrow SP(R(Q_0, P(t))) \cap T_{spec}=\phi$$

Definition 7 (Strong Periodic D-detectability)

A nondeterministic discrete event system G is strongly periodically D-detectable with respect to P and T_{spec} if we can distinguish state pairs in T_{spec} periodically for all trajectories of the system. That is,

$$(\exists n \in N)\,(\forall s \in L^{w}(G))\,(\forall t \in Pr(s))\left(\exists t' \in \Sigma^*\right) tt' \in Pr(s) \wedge |P\left(t'\right)|<n \wedge SP\left(R\left(Q_0, P\left(tt'\right)\right)\right) \cap T_{spec}=\phi$$

Definition 8 (Periodic D-detectability)

A nondeterministic discrete event system G is periodically D-detectable with respect to P and T_{spec} if we can distinguish state pairs in T_{spec} periodically for some trajectories of the system. That is,

$$(\exists n \in N)\,(\exists s \in L^{w}(G))\,(\forall t \in Pr(s))\left(\exists t' \in \Sigma^*\right) tt' \in Pr(s) \wedge |P\left(t'\right)|<n \wedge SP\left(R\left(Q_0, P\left(tt'\right)\right)\right) \cap T_{spec}=\phi$$

To check D-detectabilities, we construct an observer G_{obs} as in Section 2. However, we mark the states differently as follows:

$$X_D=\left\{x \in X: SP(x) \cap T_{spec}=\phi\right\}$$

In other words, when the system is in $x \in X_D$, we can distinguish all the state pairs in T_{spec} .

Based on observer G_{obs} , the following necessary and sufficient conditions for D-detectability can now be derived as shown in the following theorems, whose proofs are omitted because they are similar to the proofs in the previous sections.

Theorem 7

A nondeterministic discrete event system G is strongly D-detectable with respect to P and T_{spec} if and only if in observer G_{obs} , $(\forall (x, u) \in Loop)\,(\forall w \in \Sigma_o^*)\,\delta(x, w) \in X_D$, that is, any state reachable from any loop in G_{obs} is in X_D .

Theorem 8

A nondeterministic discrete event system G is D-detectable with respect to P and T_{spec} if and only if in observer G_{obs} , $(\exists (x, u) \in Loop)\,(\forall w \in \Pr(u))\,\delta(x, w) \in X_D$, that is, there are loops in G_{obs} that are entirely within X_D .

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Theorem 9

A nondeterministic discrete event system G is strongly periodically D-detectable with respect to P and T_{spec} if and only if in observer G_{obs} , $(\forall (x, u) \in Loop)(\exists w \in \Pr(u))\, \delta(x, w) \in X_D$, that is, all loops in G_{obs} must include at least one state belonging to X_D .

Theorem 10

A nondeterministic discrete event system G is periodically D-detectable with respect to P and T_{spec} if and only if in observer G_{obs} , $(\exists (x, u) \in Loop)(\exists w \in \Pr(u))\, \delta(x, w) \in X_D$, that is, there are loops in G_{obs} which include at least one state belonging to X_D .

In Section 3 we propose a method to check strong detectability and strong periodic detectability with polynomial complexity by constructing a detector. This method also works for strong D-detectability. Let $G_{\det}$ be the detector for G as in Section 2. Mark the states in $Y_D = \left\{y \in Y : SP(y) \cap T_{spec} = \phi\right\}$. We have the following theorem to check strong D-detectability with polynomial complexity.

Theorem 11

A nondeterministic discrete event system G is strongly D-detectable with respect to P and T_{spec} if and only if in detector $G_{\det}$, $(\forall (y, u) \in YLoop)(\forall w \in \Sigma_o^*)\, \xi(y, w) \subseteq Y_D$, that is, any state reachable from any loop in $G_{\det}$ is in Y_D .

Proof:

By Theorem 7, we only need to show that $(\forall (x, u) \in Loop)(\forall w \in \Sigma_o^*)\, \delta(x, w) \in X_D$ if and only if $(\forall (y, u) \in YLoop)(\forall w \in \Sigma_o^*)\, \xi(y, w) \subseteq Y_D$.

For a state in G_{obs} , $x \in X$, if $|x| > 2$, then let us split it into states in $G_{\det}$, that is,

$$x \leftrightarrow Y_x = \{y : y \subset x \wedge |y| = 2\}.$$

It is not difficult to show that

$$x \in X_D \iff (\forall y \in Y_x)\, y \in Y_D.$$

On the other hand, if $|x| \le 2$, then x will remain the same and clearly

$$x \in X_D \iff x \in Y_D.$$

With this in mind, we can prove the theorem as follow.

(Only If) If $(\forall (x, u) \in Loop)(\forall w \in \Sigma_o^*)\, \delta(x, w) \in X_D$ is true, the all states in all loops are in X_D . Some states will be split in $G_{\det}$ and possibly forms more loops. By the above observation, all those split states must be in Y_D . Hence,

$$\begin{aligned} & (\forall (x, u) \in Loop)\left(\forall w \in \Sigma_0^*\right) \delta(x, w) \in X_D \\ \Rightarrow\ & (\forall (x, u) \in YLoop)\left(\forall w \in \Sigma_0^*\right) \xi(x, w) \subseteq Y_D \end{aligned}$$

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(If) By the same observation, if all split states in all loops in $G_{\det}$ are in Y_D , then all the original states in all loops in G_{obs} are in X_D . Hence,

$$\begin{aligned} & (\forall (x,u) \in YLoop)\left(\forall w \in \Sigma_0^*\right)\xi(x,w) \subseteq Y_D \\ \Rightarrow\quad & (\forall (x,u) \in Loop)\left(\forall w \in \Sigma_0^*\right)\delta(x,w) \in X_D \end{aligned}$$

Q.E.D

Note that the proof of the above theorem is also different than the corresponding Theorem 5. Also, unlike detectability, detector $G_{\det}$ cannot be used to check strong periodic D-detectability. Let us take an example to illustrate it.

Example 2

Consider a discrete event system shown in Figure 4(a). We assume that all the events are observable and we know nothing about the initial state, that is: $\sum_o = \sum$ and $Q_0 = Q$. The specification is $\{(q_1,q_3)\}$. Then the observer is constructed as in Figure 4(b) and the detector is constructed as in Figure 5.

From the observe we can see that the system is not strongly periodically detectable because there is one loop $(q_1, q_2, q_3) \xrightarrow{\alpha} (q_1, q_2, q_3)$. However, there are no loops in $Y - Y_D$.

Remark 3

Detectability is a special case of D-detectability. If we let $T_{spec} = Q \times Q - \{(q,q) : q \in Q\}$, then we want to distinguish all states. Hence D-detectability reduces to detectability.

Remark 4

Stability of discrete event systems has been investigated before, including in [18,19,20]. According to [18,20], a discrete event system is stable if the system will eventually enter and stay in a subset of states denoted by $Q_s \subseteq Q$. A weak stability is defined in [19], which says that discrete event system is weak stable if the system will periodically enter a subset of states denoted by $Q_s \subseteq Q$. If we define the specification T_{spec} as $T_{spec} = (Q - Q_s) \times Q$, then the stability becomes strong D-detectability and weak stability becomes strong periodic D-detectability.

5. Conclusion

In this paper, we extended the results on detectability of discrete event systems in [10] in several directions. First, we considered nondeterministic discrete event systems rather than deterministic discrete event systems. We then developed polynomial algorithms to check detectability for nondeterministic systems (and hence for deterministic systems as well) by introducing a new tool called detector. We also extended detectability to D-detectability for more applications.

Acknowledgments

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References

[1]. Ramadge PJ, Wonham WM. Supervisory control of a class of discrete event processes. SIAM J. Control and Optimization. 1987; 25(1):206–230.

[2]. Ramadge PJ, Wonham WM. The control of discrete event systems. Proceedings of the IEEE. 1989; 77(1):81–98.

[3]. Cassandras, CG.; Lafortune, S. Introduction to discrete event systems. Kluwer; 1999.

[4]. Lin F, Wonham WM. On observability of discrete event systems. Information Sciences. 1988; 44(3):173–198.

[5]. Lin F, Wonham WM. Decentralized supervisory control of discrete-event systems. Information Sciences. 1988; 44(3):199–224.

[6]. Cieslak R, Desclaux C, Fawaz A, Varaiya P. Supervisory control of discrete event processes with partial observations. IEEE Transactions on Automatic Control. 1988; 33(3):249–260.

[7]. Ramadge PJ. Observability of discrete event systems. Proceedings of CDC. 1986:1108–1112.

[8]. Caines PE, Greiner R, Wang S. Dynamical Logic Observers for Finite Automata. Proceedings of CDC. 1988:226–233.

[9]. Ozveren CM, Willsky AS. Observability of discrete event dynamic systems. IEEE Transactions on Automatic Control. 1990; 35(7):797–806.

[10]. Shu S, Lin F, Ying H. Detectability of discrete event systems. IEEE Transactions on Automatic Control. 2007; 52(12):2356–2359.

[11]. Yoo TS, Lafortune S. Polynomial-Time Verification of Diagnosability of Partially Observed Discrete-Event Systems. IEEE Transactions on Automatic Control. Sep.; 2002 47(9):1491–1495.

[12]. Jiang S, Huang Z, Ch V, Kumar R. A Polynomial Algorithm for Testing Diagnosability of Discrete Event Systems. IEEE Transactions on Automatic Control. 2001; 46:1318–1321.

[13]. Lin F. Diagnosability of discrete event systems and its applications. Discrete Event Dynamic Systems: Theory and Applications. 1994; 4(1):197–212.

[14]. Zad SH, Kwong RH, Wonham WM. Fault diagnosis in discrete-event systems: Framework and model reduction. IEEE Transactions on Automatic Control. 2003; 48(7):1199–1212.

[15]. Wang W, Lafortune S, Lin F. An algorithm for calculating indistinguishable states and clusters in finite-state automata with partially observable transitions. Systems & Control Letters. 2007; 56(9):656–661.

[16]. Heymann M, Lin F. Discrete event control of nondeterministic systems. IEEE Transactions on Automatic Control. 1998; 43(1):3–17.

[17]. Thistle JG, Wonham WM. Control of infinite behaviour of finite automata. SIAM J, Control and Optimization. 1994; 32(4):1075–1097.

[18]. Brave Y. On stabilization of discrete event processes. Proceedings of CDC. 1989:2737–2743.

[19]. Ozveren CM, Wilsky AS. Output stabilizability of discrete-event dynamic systems. IEEE Transactions on Automatic Control. 1991; 36:925–935.

[20]. Brave Y, Shayman M. Stabilization of discrete event processes. International Journal of Control. 1990; 51(5)

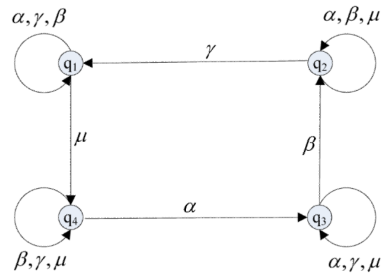

Figure 1.
Discrete Event Systems

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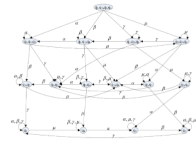

Figure 2.
the observer G_{obs} for the system in Figure 1

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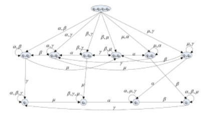

Figure 3.
The Detector G_{det} for the system in Figure 1

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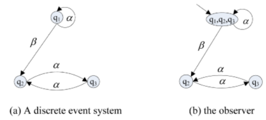

Figure 4.
A discrete event system and its observer G_{obs}

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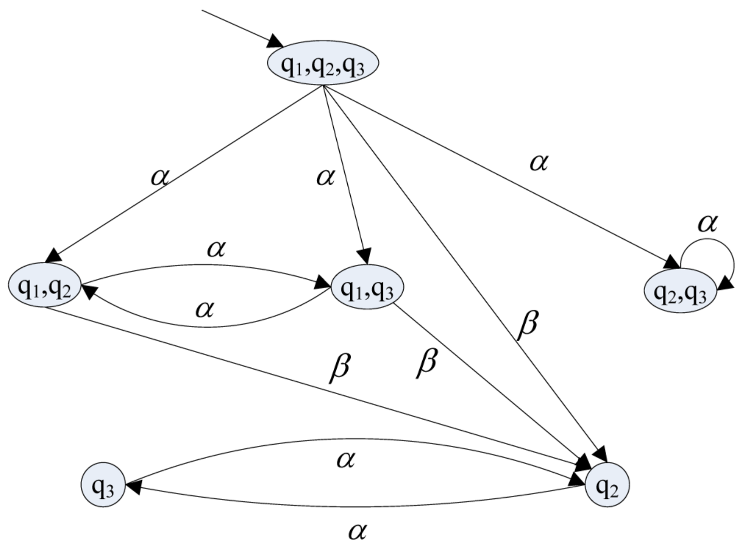

Figure 5.
The detector G_{det} for the system shown in figure 4(a)