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STABILITY ANALYSIS OF PERTURBED INFINITE-DIMENSIONAL SAMPLED-DATA SYSTEMS*

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Abstract. This paper addresses the stability analysis of infinite-dimensional sampled-data systems under unbounded perturbations. We present two classes of unbounded perturbations preserving the exponential stability of sampled-data systems. To this end, we investigate the continuity of strongly continuous semigroups with respect to their generators, considering the uniform operator topology.

 ${\bf Key}$ words. Exponential stability, infinite-dimensional systems, sampled-data systems, unbounded perturbation

1. Introduction. In this paper, we study the robustness of the exponential stability of infinite-dimensional sampled-data systems under unbounded perturbations. Modern control systems employ digital technology for the implementation of controllers, and sampled-data control plays an important role in cyber-physical systems.

It is also worth emphasizing that the analysis and synthesis of sampled-data systems are of great theoretical interest due to the interaction between continuoustime and discrete-time dynamics. One particular approach to overcome the difficulty arising from this hybrid property is the lifting technique developed in [2, 27], which treats the intersample behavior of sampled-data systems in a unified, time-invariant framework. For infinite-dimensional systems, various sampled-data control problem have been studied, for example, stabilization [9, 10, 15, 17, 20, 23], robustness analysis with respect to sampling for stabilization [16, 21, 22], and output regulation [12–14, 18, 25]. However, relatively little work has been done on the stability analysis of perturbed infinite-dimensional sampled-data systems.

Consider the following sampled-data system with state space X and input space U (both Banach spaces):

(1.1a)
$$\dot{x}(t) = Ax(t) + B_0 u(t) \quad \forall t \ge 0; \qquad x(0) = x^0 \in X_t$$

(1.1b)
$$u(k\tau + t) = Fx(k\tau) \quad \forall t \in [0,\tau), \ \forall k \in \mathbb{Z}_+,$$

where $x(t) \in X$, $u(t) \in U$, $\tau > 0$ is a sampling period, A is the generator of a strongly continuous semigroup T(t) on X, and the input operator B_0 and the feedback operator F are bounded linear operators from U to X and from X to U, respectively. Since we here give only stability analysis and do not go over to feedback design, we define the bounded operator $B := B_0 F$ on X for simplicity of notation. By the standard theory of strongly continuous semigroups (see, e.g., [1, 4, 7, 8]), this abstract evolution equation (1.1) has the unique solution defined recursively by

(1.2a)
$$x(0) = x^0$$
,

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(1.2b)
$$x(k\tau+t) = T(\tau)x(k\tau) + \int_0^t T(s)Bx(k\tau)ds \quad \forall t \in (0,\tau], \ \forall k \in \mathbb{Z}_+.$$

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We assume that the nominal sample-data system (1.1) is exponentially stable, which is defined as follows:

DEFINITION 1.1 (Exponential stability). The sampled-data system (1.1) is exponentially stable with decay rate greater than ω if there exist $M \ge 1$ and $\tilde{\omega} > \omega$ such that the solution x(t) given by (1.2) satisfies

(1.3)
$$||x(t)|| \le M e^{-\widetilde{\omega}t} ||x^0|| \qquad \forall x^0 \in X, \ \forall t \ge 0.$$

Consider a linear operator $D : \text{dom}(D) \subset X \to X$, and let the generator A in (1.1a) be perturbed by this operator D. Consequently, the perturbed sampled-data system is written as

(1.4a)
$$\dot{x}(t) = (A+D)x(t) + B_0u(t) \quad \forall t \ge 0; \qquad x(0) = x^0 \in X,$$

(1.4b)
$$u(k\tau + t) = Fx(k\tau) \quad \forall t \in [0, \tau), \ \forall k \in \mathbb{Z}_+.$$

The difficulty in analyzing the stability of the perturbed system (1.4) arises from the unboundedness of the operator D. In this paper, we focus on two typical classes of unbounded perturbations and provide conditions for preservation of exponential stability. We denote by X_1 the Banach space $(\text{dom}(A), \|\cdot\|_A)$, where $\|\cdot\|_A$ is the graph norm for A, i.e., $\|x\|_A := \|x\| + \|Ax\|$ for $x \in \text{dom}(A)$. The first theorem shows that the exponential stability of the sampled-data system is robust against unbounded perturbations called Miyadera-Voigt perturbations.

THEOREM 1.2. Suppose that the nominal sampled-data system (1.1) is exponentially stable with decay rate greater than ω . Choose $t_0 > 0$ arbitrary. There exists 0 < q < 1 such that for every bounded linear operator $D: X_1 \to X$ satisfying

$$\int_0^{t_0} \|DT(s)x\| ds \le q \|x\| \qquad \forall x \in \operatorname{dom}(A),$$

the perturbed sampled-data system (1.4) is also exponentially stable with decay rate greater than ω .

In the second theorem, we consider only the case where T(t) is an analytic semigroup (see, e.g., Chapter XVII in [8], Section II.4.a in [7], and Section 3.7 in [1] for analytic semigroups). This restriction allows us to deal with a larger class of unbounded perturbations, called relatively A-bounded perturbations, than in the first theorem.

THEOREM 1.3. Suppose that the nominal sampled-data system (1.1) is exponentially stable with decay rate greater than ω and that T(t) is an analytic semigroup. There exist $\alpha, \beta > 0$ such that for every linear operator $D : \operatorname{dom}(D) \subset X \to X$ satisfying $\operatorname{dom}(A) \subset \operatorname{dom}(D)$ and

$$||Dx|| \le \alpha ||Ax|| + \beta ||x|| \qquad \forall x \in \operatorname{dom}(A),$$

the perturbed sampled-data system (1.4) is also exponentially stable with decay rate greater than ω .

Note that the theorems above implicitly assume the existence of the strongly continuous semigroup generated by the perturbed operator A + D. The existence of the semigroup has been proved in [19] for Miyadera-Voigt perturbations and in [8, Chapter XIII] for relatively A-bounded perturbations, respectively; see Theorems 3.1 and 4.2 below for more rigorous statements. Let $T_D(t)$ denote the strongly continuous semigroup generated by A + D. In the beginning of the proofs of Theorems 1.2 and 1.3, we show that if $||T(t) - T_D(t)||$ is sufficiently small on $[0, \tau]$, then the perturbed sampled-data system (1.4) is also exponentially stable. To evaluate $||T(t) - T_D(t)||$ on $[0, \tau]$, we use the variation of parameter formula for Miyadera-Voigt perturbations and the integral representation of an analytic semigroup for relatively A-bounded perturbations, respectively.

In Section 2, we present basic results on the exponential stability of sampled-data systems and the robustness of power stability. Section 3 is devoted to the proof of Theorem 1.2. In Section 4, we prove Theorem 1.3 and then provide an example to compare the perturbation classes in Theorems 1.2 and 1.3. In Section 5, we present several examples to illustrate the obtained results.

Notation and terminology. We denote by \mathbb{Z}_+ the set of nonnegative integers. For r > 0, we define $\mathbb{D}_r := \{z \in \mathbb{C} : |z| < r\}$ and $\mathbb{E}_r := \{z \in \mathbb{C} : |z| > r\}$. For a set $\Omega \subset \mathbb{C}$, its closure is denoted by $\overline{\Omega}$. For $\delta \in (0, \pi/2]$, we define $\Sigma_{\delta} := \{z \in \mathbb{C} \setminus \{0\} : |\arg z| \le \delta\}$. Let X and Y be Banach spaces. Let us denote by $\mathcal{L}(X, Y)$ the space of all bounded linear operators from X to Y. We write $\mathcal{L}(X)$ for $\mathcal{L}(X, X)$. An operator $\Delta \in \mathcal{L}(X)$ is said to be *power stable* if there exists constants $M \ge 1$ and $\theta \in (0, 1)$ such that $\|\Delta^k\| \le M\theta^k$ for every $k \in \mathbb{Z}_+$. For a linear operator A from X to Y, we denote by dom(A) the domain of A. The resolvent set of a linear operator $A : \operatorname{dom}(A) \subset X \to X$ is denoted by $\varrho(A)$, and $R(\lambda, A) := (\lambda I - A)^{-1}$ for $\lambda \in \varrho(A)$.

2. Preliminaries. For the nominal sampled-data system (1.1), define the operator $\Delta_{\tau} \in \mathcal{L}(X)$ by

(2.1)
$$\Delta_{\tau} := T(\tau) + \int_0^{\tau} T(s) B ds.$$

The state x defined by (1.2) satisfies $x((k+1)\tau) = \Delta_{\tau} x(k\tau)$ for every $k \in \mathbb{Z}_+$.

Lemma 2.1 below provides the relationship between the exponential stability of the sampled-data system and the power stability of Δ_{τ} . One can obtain this result by slightly modifying the proof of Proposition 2.1 in [20].

LEMMA 2.1. For any $\tau > 0$, the sampled-data system (1.1) is exponentially stable with decay rate greater than ω if and only if $e^{\omega \tau} \Delta_{\tau}$ is power stable.

By Corollary 4.5 of [26], which gives the stability radius of an infinite-dimensional discrete-time system, one can also immediately obtain the following lemma:

LEMMA 2.2. Let X be a Banach space, and let $\Delta_1 \in \mathcal{L}(X)$ and $\kappa > 0$. If $\kappa \Delta_1$ is power stable, then there exists $\epsilon > 0$ such that $\kappa \Delta_2$ is also power stable for every $\Delta_2 \in \mathcal{L}(X)$ satisfying $\|\Delta_1 - \Delta_2\| < \epsilon$.

Applying Lemma 2.2 to the operator Δ_{τ} defined by (2.1), we obtain the following simple result.

PROPOSITION 2.3. For $j \in \{1,2\}$, let T_j be strongly continuous semigroups and define

$$\Delta_{\tau,j} := T_j(\tau) + \int_0^\tau T_j(s) B ds.$$

For a given $\kappa > 0$, suppose that $\kappa \Delta_{\tau,1}$ is power stable. There exists $\epsilon > 0$ such that the inequality

$$||T_1(t) - T_2(t)|| < \epsilon \qquad \forall t \in [0, \tau]$$

implies that $\kappa \Delta_{\tau,2}$ is also power stable.

Proof. Suppose that (2.2) holds for some $\epsilon > 0$. Since

$$\Delta_{\tau,1} - \Delta_{\tau,2} = T_1(\tau) - T_2(\tau) + \int_0^\tau \left[T_1(s) - T_2(s) \right] B ds,$$

it follows that

$$\|\Delta_{\tau,1} - \Delta_{\tau,2}\| \le \|T_1(\tau) - T_2(\tau)\| + \int_0^\tau \|T_1(s) - T_2(s)\| \cdot \|B\| ds$$

$$\le \epsilon (1 + \tau \|B\|).$$

Thus, Lemma 2.2 yields the desired conclusion.

Motivated by Lemma 2.1 and Proposition 2.3, we will study the continuity of strongly continuous semigroups with respect to their generators in Sections 3 and 4. For the strong operator topology, this continuity has been obtained as the Trotter-Kato Approximation theorem in [11, 24]; see also Section III.4.b in [7] and Section 3.6 in [1]. However, we here need the continuity for the uniform operator topology. Theorems 13.5.8 and 13.7.3 of [8] show that strongly continuous semigroups have such a continuity property uniformly with respect to t in a compact interval of $(0, \infty)$. These results are not quite sufficient for our purpose, because the continuity property in the interval $[0, \tau]$ is required in Proposition 2.3. The main results in Sections 3 and 4, Theorems 3.2 and 4.3, can be regarded as extensions of Theorems 13.5.8 and 13.7.3 of [8], respectively.

3. General semigroup and Miyadera-Voigt perturbations. The objective of this section is to prove Theorem 1.2. We first recall the perturbation theorem of Miyadera-Voigt. This theorem guarantees that for the class of perturbations D in Theorem 1.2, the perturbed operator A+D generates a strongly continuous semigroup.

THEOREM 3.1 ([19], Corollary III.3.16 of [7]). Let A be the generator of a strongly continuous semigroup T(t) on a Banach space X, and let $D \in \mathcal{L}(X_1, X)$ satisfy

(3.1)
$$\int_0^{t_0} \|DT(s)x\| ds \le q \|x\| \qquad \forall x \in \operatorname{dom}(A)$$

for some $0 \le q < 1$ and $t_0 > 0$. Then the sum A + D with domain dom(A + D) := dom(A) generates a strongly continuous semigroup $T_D(t)$ on X. Moreover, for every $x \in$ dom(A) and every $t \ge 0$, $T_D(t)$ satisfies

(3.2a)
$$T_D(t)x = T(t)x + \int_0^t T(t-s)DT_D(s)xds,$$

(3.2b)
$$\int_0^{t_0} \|DT_D(s)x\| ds \le \frac{q}{1-q} \|x\|.$$

We call (3.2a) the variation of parameter formula for the perturbed semigroup $T_D(t)$. Using Theorem 3, we obtain the following result:

THEOREM 3.2. Let A be the generator of a strongly continuous semigroup T(t)on a Banach space X, and choose $t_0 > 0$ arbitrarily. For every $\epsilon, \tau > 0$, there exists 0 < q < 1 such that for every $D \in \mathcal{L}(X_1, X)$ satisfying (3.1), the perturbed semigroup $T_D(t)$ generated by A + D satisfies

(3.3)
$$||T(t) - T_D(t)|| < \epsilon \qquad \forall t \in [0, \tau].$$

Proof. Let $t_0, \tau > 0$ be given, and suppose that $D \in \mathcal{L}(X_1, X)$ satisfies (3.1) for some 0 < q < 1. By (3.2a), we obtain

(3.4)
$$T(t)x - T_D(t)x = \int_0^t T(t-s)DT_D(s)xds \quad \forall x \in \operatorname{dom}(A), \ \forall t \ge 0.$$

Since every strongly continuous semigroup is uniformly bounded on a compact interval, there exists $M \ge 1$ such that $||T(t)|| \le M$ for every $t \in [0, \tau]$. Let $nt_0 \le \tau < (n+1)t_0$ with $n \in \mathbb{Z}_+$. In the case n = 0, we obtain $\tau < t_0$. Therefore, (3.2b) and (3.4) yield

$$\begin{aligned} \|T(t)x - T_D(t)x\| &\leq M \int_0^t \|DT_D(s)x\| ds \leq M \int_0^{t_0} \|DT_D(s)x\| ds \\ &\leq \frac{Mq}{1-q} \|x\| \qquad \forall x \in \operatorname{dom}(A), \ \forall t \in [0,\tau]. \end{aligned}$$

Since dom(A) is dense in X, we obtain

(3.5)
$$||T(t) - T_D(t)|| \le \frac{Mq}{1-q} \quad \forall t \in [0, \tau].$$

Next consider the case $n \ge 1$. Similarly to the case n = 0, we have

$$||T(t) - T_D(t)|| \le \frac{Mq}{1-q} =: q_0 \qquad \forall t \in [0, t_0]$$

For every $k \in \mathbb{N}$ and every $t \in [0, t_0]$,

$$\begin{aligned} \|T(kt_0+t) - T_D(kt_0+t)\| \\ &\leq \|T(kt_0+t) - T(t)T_D(kt_0)\| + \|T(t)T_D(kt_0) - T_D(kt_0+t)\| \\ &\leq \|T(t)\| \cdot \|T(kt_0) - T_D(kt_0)\| + \|T_D(kt_0)\| \cdot \|T(t) - T_D(t)\|. \end{aligned}$$

Hence, for every $k \in \{1, \ldots, n\}$, if $q_{k-1} > 0$ satisfies

$$||T((k-1)t_0+t) - T_D((k-1)t_0+t)|| \le q_{k-1} \quad \forall t \in [0, t_0],$$

then

$$||T(kt_0+t) - T_D(kt_0+t)|| \le (M+q_0)q_{k-1} + Mq_0 \qquad \forall t \in [0,t_0].$$

Consider the sequence $\{q_k\}_{k\in\mathbb{Z}_+}$ constructed by

$$q_k = (M+q_0)q_{k-1} + Mq_0 \quad \forall k \in \mathbb{N}; \qquad q_0 = \frac{Mq}{1-q}$$

Then $\{q_k\}_{k\in\mathbb{Z}_+}$ is increasing and satisfies

$$||T(kt_0+t) - T_D(kt_0+t)|| \le q_k \qquad \forall t \in [0, t_0], \ \forall k \in \{0, \dots, n\}.$$

Hence, we obtain $||T(t) - T_D(t)|| \le q_n$ for all $t \in [0, \tau]$. By definition, q_n is continuous with respect to $q \in (0, 1)$ and decreases to 0 as $q \to 0$. Thus for every $\epsilon, \tau > 0$, there exists 0 < q < 1 such that (3.3) holds. This completes the proof.

From Theorem 3.2, we can easily obtain the following result on bounded perturbations.

COROLLARY 3.3. Let A be the generator of a strongly continuous semigroup T(t)on a Banach space X. For every $\epsilon, \tau > 0$, there exists d > 0 such that for every $D \in \mathcal{L}(X, X)$ satisfying $||D|| \leq d$, the perturbed semigroup $T_D(t)$ generated by A + Dsatisfies $||T(t) - T_D(t)|| < \epsilon$ for all $t \in [0, \tau]$.

Proof. By the strong continuity of T, there exists $M \ge 1$ such that $||T(t)|| \le M$ for every $t \in [0, \tau]$. For every $q \in (0, 1)$ and every $t_0 > 0$, $||D|| \le q/(t_0 M)$ yields (3.1). Therefore, the assertion follows from Theorem 3.2.

Proof of Theorem 1.2. Suppose that the sampled-data system (1.1) is exponentially stable with decay rate greater than ω . Lemma 2.1 shows that $e^{\omega\tau}\Delta_{\tau}$ is power stable. For the strongly continuous semigroup T_D generated by A + D, we define

$$\Delta_{\tau,D} := T_D(\tau) + \int_0^\tau T_D(s) B ds$$

Let $t_0 > 0$ be given. By Proposition 2.3 and Theorem 3.2, there exists 0 < q < 1such that $e^{\omega \tau} \Delta_{\tau,D}$ is power stable for every $D \in \mathcal{L}(X_1, X)$ satisfying (3.1). Using Lemma 2.1 again, the perturbed sampled-data system (1.4) is also exponentially stable with decay rate greater than ω .

4. Analytic semigroup and relatively bounded perturbations. We next prove Theorem 1.3. In this theorem, we consider the following class of perturbations:

DEFINITION 4.1. Let $A : \operatorname{dom}(A) \subset X \to X$ be a linear operator on the Banach space X. A linear operator $D : \operatorname{dom}(D) \subset X \to X$ is called a *relatively A-bounded perturbation* with constants $\alpha, \beta \geq 0$ if $\operatorname{dom}(A) \subset \operatorname{dom}(D)$ and if

$$||Dx|| \le \alpha ||Ax|| + \beta ||x|| \qquad \forall x \in \operatorname{dom}(A)$$

Moreover, let $\mathcal{P}_{\alpha,\beta}(A)$ represent the set of all relatively A-bounded perturbations with constants $\alpha, \beta \geq 0$.

In Definition 4.1, we deal with a larger class of perturbations than in Section 3, which is illustrated in Example 4.4 at the end of this section.

Let us assume that A generates an analytic semigroup. We refer the readers to Chapter XVII in [8], Section II.4.a in [7], and Section 3.7 in [1] for the definition and properties of analytic semigroups. The following theorem guarantees that the perturbed operator A + D also generates an analytic semigroup if D is a relatively A-bounded perturbation with sufficiently small constants.

THEOREM 4.2 (Theorem 13.7.1 of [8], Theorem III.2.10 of [7]). Let A generate an analytic semigroup of angle $\delta \in (0, \pi/2]$ on a Banach space. There exists $\alpha, \beta > 0$ such that the sum A + D with domain dom(A + D) := dom(A) generates an analytic semigroup of angle at least δ for every $D \in \mathcal{P}_{\alpha, \beta}(A)$.

The next result shows that if the original semigroup is analytic and if a relatively A-bounded perturbation satisfies (4.1) with sufficiently small constants $\alpha, \beta > 0$, then the difference between the original and perturbed semigroups is also small.

THEOREM 4.3. Suppose that A generates an analytic semigroup T(z) of angle $\delta \in (0, \pi/2]$. For every $\epsilon, r > 0$ and every $0 < \delta_1 < \delta$, there exist constants $\alpha, \beta > 0$ such that for every $D \in \mathcal{P}_{\alpha, \beta}(A)$, the perturbed semigroup $T_D(z)$ generated by A + D satisfies

(4.2)
$$||T(z) - T_D(z)|| < \epsilon \qquad \forall z \in \Sigma_{\delta_1} \cap \mathbb{D}_r.$$

Proof. First we assume that T(z) is bounded, i.e., ||T(z)|| is bounded in $\Sigma_{\delta'}$ for every $0 < \delta' < \delta$. For every $\lambda \in \varrho(A)$ and every relatively A-bounded perturbation D, we obtain

$$\lambda - A - D = \left[I - DR(\lambda, A)\right](\lambda I - A),$$

and hence if $||DR(\lambda, A)|| < 1$, then $\lambda \in \rho(A + D)$ and

(4.3)
$$R(\lambda, A+D) = R(\lambda, A) \left[I - DR(\lambda, A) \right]^{-1}$$

Choose $\nu \in (0,1)$, $\delta_1 \in (0,\delta)$, $\delta_2 \in (\delta_1,\delta)$, and r > 0 arbitrarily. One can show that there exist $\alpha, \beta > 0$ such that

(4.4)
$$||DR(\lambda, A)|| \le \nu \quad \forall D \in \mathcal{P}_{\alpha, \beta}(A), \ \forall \lambda \in \overline{\Sigma}_{\pi/2+\delta_2} \cap \overline{\mathbb{E}}_{1/r} =: \Sigma,$$

using the standard techniques as in the proofs of Lemmas III.2.5, III.2.6 and Theorem III.2.10 of [7]. Let $\alpha, \beta > 0$ satisfy (4.4). By Theorem II.4.6 in [7], there exists $C = C(\delta_2) > 0$ such that for every $\lambda \in \overline{\Sigma}_{\pi/2+\delta_2} \setminus \{0\}$, we obtain $\lambda \in \rho(A)$ and

(4.5)
$$||R(\lambda, A)|| \le \frac{C}{|\lambda|}.$$

Therefore, $\Sigma \subset \rho(A+D)$ for every $D \in \mathcal{P}_{\alpha,\beta}(A)$, and (4.3) yield

(4.6)
$$||R(\lambda, A + D)|| \le \frac{C}{(1-\nu)|\lambda|} \quad \forall D \in \mathcal{P}_{\alpha, \beta}(A), \ \forall \lambda \in \Sigma$$

This inequality implies that for every $D \in \mathcal{P}_{\alpha,\beta}(A)$, A + D generates an analytic semigroup of angle at least δ ; see, e.g., Exercise III.2.18 (3) of [7]. Moreover, since

$$R(\lambda, A) - R(\lambda, A + D) = R(\lambda, A) \left(I - \left[I - DR(\lambda, A) \right]^{-1} \right)$$
$$= -R(\lambda, A) DR(\lambda, A) \left[I - DR(\lambda, A) \right]^{-1},$$

it follows from (4.3) and (4.5) that

(4.7)
$$\|R(\lambda, A) - R(\lambda, A + D)\| \le \frac{C\nu}{(1-\nu)|\lambda|} \qquad \forall D \in \mathcal{P}_{\alpha, \beta}(A), \ \forall \lambda \in \Sigma$$

Let $z \in \Sigma_{\delta_1} \cap \mathbb{D}_r$ and $D \in \mathcal{P}_{\alpha, \beta}(A)$ be given, where $\alpha, \beta > 0$ satisfy (4.4). Choose any piecewise smooth curve γ in $\Sigma_{\pi/2+\delta}$ going from $\infty e^{-i(\pi/2+\delta')}$ to $\infty e^{i(\pi/2+\delta')}$ for some $\delta' \in (|\arg z|, \delta)$. Since A + D generates an analytic semigroup T_D of angle δ by the argument above, there exists $w \ge 0$ such that A + D - wI generates a bounded analytic semigroup $e^{-wz}T_D(z)$ of angle δ . Hence the integral representation of T_D is given by (see, e.g., Definition II.4.2 of [7])

(4.8)
$$T_D(z) = \frac{1}{2\pi i} \int_{\gamma_w} e^{\mu z} R(\mu, A + D) d\mu_z$$

where γ_w is a piecewise smooth curve similar to γ but shifted to the right by w, that is, $\gamma_w := \{\mu \in \mathbb{C} : \mu = w + \mu_1, \ \mu_1 \in \gamma\}$. Consider an integral path $\gamma_z \subset \Sigma$ that consists of the three parts

$$\begin{aligned} \gamma_{z,1} &: \{ -\rho e^{-i(\pi/2 + (\delta_1 + \delta_2)/2)} : -\infty \le \rho \le -1/|z| \}, \\ \gamma_{z,2} &: \{ e^{i\alpha}/|z| : -(\pi/2 + (\delta_1 + \delta_2)/2) \le \alpha \le \pi/2 + (\delta_1 + \delta_2)/2 \}, \\ \gamma_{z,3} &: \{ -\rho e^{i(\pi/2 + (\delta_1 + \delta_2)/2)} : 1/|z| \le \rho \le \infty \}. \end{aligned}$$



Fig. 1: Integral path γ_z .

Figure 1 illustrates the integral path γ_z .

Since $\Sigma \subset \varrho(A + D)$, it follows from Cauchy's integral theorem and (4.8) that

$$T_D(z) = \frac{1}{2\pi i} \int_{\gamma_z} e^{\mu z} R(\mu, A + D) d\mu.$$

The integral representation of T is also given by

$$T(z) = \frac{1}{2\pi i} \int_{\gamma_z} e^{\mu z} R(\mu, A) d\mu,$$

and hence

$$T(z) - T_D(z) = \frac{1}{2\pi i} \int_{\gamma_z} e^{\mu z} \left[R(\mu, A) - R(\mu, A + D) \right] d\mu.$$

Hence (4.7) yields

$$\|T(z) - T_D(z)\| \le \frac{C\nu}{2\pi(1-\nu)} \int_{\gamma_z} \frac{e^{\operatorname{Re}(\mu z)}}{|\mu|} d\mu \qquad \forall D \in \mathcal{P}_{\alpha, \beta}(A).$$

Since a calculation similar to that in the proof of Proposition II.4.3 of [7] shows that

$$\frac{1}{2\pi} \int_{\gamma_z} \frac{e^{\operatorname{Re}(\mu z)}}{|\mu|} d\mu \le \frac{1}{\pi} \int_1^\infty \frac{e^{-\rho \sin(\delta_2 - \delta_1)/2}}{\rho} d\rho + e =: L(\delta_1, \delta_2) < \infty,$$

it follows that

$$\|T(z) - T_D(z)\| \le \frac{CL(\delta_1, \delta_2)\nu}{1 - \nu} \qquad \forall D \in \mathcal{P}_{\alpha, \beta}(A).$$

For every $\epsilon > 0$, if we choose $\nu \in (0, 1)$ satisfying

$$\nu < \frac{\epsilon}{CL(\delta_1, \delta_2) + \epsilon},$$

then (4.2) holds.

To treat the general case, we take $w_0 \ge 0$ such that $A - w_0 I$ generates a bounded analytic semigroup of angle δ . Since

$$||Dx|| \le \alpha ||Ax|| + \beta ||x|| \le \alpha ||(A - w_0 I)x|| + (\alpha w_0 + \beta) ||x|| \qquad \forall x \in \operatorname{dom}(A),$$

the operator D is also relatively $(A - w_0 I)$ -bounded and the coefficients α and $\alpha w_0 + \beta$ can be arbitrarily small for suitable α, β . Take $z \in \Sigma_{\delta_1} \cap \mathbb{D}_r$. Replacing A by $A - w_0 I$ in the argument above, we obtain

$$\|e^{-w_0 z} T(z) - e^{-w_0 z} T_D(z)\| \le \frac{\nu L(\delta_1, \delta_2)}{2\pi (1-\nu)} \qquad \forall D \in \mathcal{P}_{\alpha, \beta}(A).$$

and hence

$$\|T(z) - T_D(z)\| \le \frac{e^{w_0 r} C L(\delta_1, \delta_2) \nu}{1 - \nu} \qquad \forall D \in \mathcal{P}_{\alpha, \beta}(A).$$

Thus, for every $\epsilon > 0$, if $\nu \in (0, 1)$ satisfies

$$\nu < \frac{\epsilon}{e^{w_0 r} CL(\delta_1, \delta_2) + \epsilon},$$

then we obtain (4.2). This completes the proof.

Proof of Theorem 1.3. If we use Theorem 4.3 instead of Theorem 3.2, then we obtain the proof of Theorem 1.3, by proceeding as in the proof of Theorem 1.2 above. We therefore omit the detail. \Box

We conclude this section with the comparison of the perturbation classes in Theorems 1.2 and 1.3. Let A be a generator on a Banach space X. If the perturbation D is small in the sense of (3.1), then D is small in the sense of (4.1) as well. More precisely, for every $\alpha, \beta > 0, t_0 > 0$, and $D \in \mathcal{L}(X_1, X)$, there exists q > 0 such that (3.1) implies (4.1); see, e.g., Exercise III.2.18 (2) of [7]. However, the converse is false as shown below.

EXAMPLE 4.4. Here we provide a perturbation that is small in the sense of (4.1) but large in the sense of (3.1). Since the semigroup we consider in this example is not analytic, Theorem 4.3 is not applicable. We will therefore see that even if the perturbation is sufficiently small in the sense of (4.1), the difference between the original and perturbed semigroups is not small.

Consider the Banach space

$$X := \left\{ (x_n)_{n \in \mathbb{N}} : x_n \in \mathbb{C} \text{ for every } n \in \mathbb{N} \text{ and } \lim_{n \to \infty} x_n = 0 \right\}$$

with norm $||(x_n)_{n\in\mathbb{N}}|| := \sup_{n\in\mathbb{N}} |x_n|$. On the Banach space X, we take the strongly continuous (semi)group T(t) given by

$$T(t)(x_n)_{n\in\mathbb{N}} = \left(e^{(-1+in)t}x_n\right)_{n\in\mathbb{N}} \qquad \forall t \ge 0.$$

This semigroup T(t) is generated by

$$A(x_n)_{n \in \mathbb{N}} := \left((-1 + in) x_n \right)_{n \in \mathbb{N}}$$

with domain

$$\operatorname{dom}(A) := \left\{ (x_n)_{n \in \mathbb{N}} \in X : \left((-1 + in) x_n \right)_{n \in \mathbb{N}} \in X \right\}$$

see, e.g., Lemma in Section II.2.9 of [7]. The semigroup T(t) is not analytic.

Consider the perturbation operator $D_m: X \to X, m \in \mathbb{N}$, defined by

$$D_m(x_n)_{n\in\mathbb{N}} := (\underbrace{0,\ldots,0}_{m-1},\sqrt{m}x_m,0,\ldots).$$

The operator D_m is bounded for each $m \in \mathbb{N}$, but the norm of D_m diverges as $m \to \infty$. (a) First we investigate the conditions (3.1) and (4.1) for D_m . If we take

(4.9)
$$(z_n)_{n \in \mathbb{N}} = (\underbrace{0, \dots, 0}_{m-1}, z_m, 0, \dots) \in \operatorname{dom}(A),$$

then every $t_0 > 0$,

$$\int_0^{t_0} \left\| D_m T(t)(z_n)_{n \in \mathbb{N}} \right\| dt = \int_0^{t_0} \left| \sqrt{m} e^{(-1+im)t} z_m \right| dt = \sqrt{m} \left(1 - e^{-t_0} \right) \| (z_n)_{n \in \mathbb{N}} \|.$$

On the other hand, for every $(x_n)_{n \in \mathbb{N}} \in \text{dom}(A)$,

$$||A(x_n)_{n \in \mathbb{N}}|| \ge |-1 + im| \cdot |x_m| \ge m|x_m|$$

and hence

$$\left\| D_m(x_n)_{n \in \mathbb{N}} \right\| = \sqrt{m} \left| x_m \right| \le \frac{\left\| A(x_n)_{n \in \mathbb{N}} \right\|}{\sqrt{m}}.$$

Therefore, as $m \to \infty$, the perturbation D_m becomes larger in the sense of (3.1) but smaller in the sense of (4.1).

(b) Let us next examine the difference between the semigroup and the perturbed semigroup. Since D_m is bounded, it follows that each $A + D_m$ is the generator of the strongly continuous semigroup $T_m(t)$, which is given by

$$T_m(t)(x_n)_{n \in \mathbb{N}} = (e^{(-1+i)t}x_1, \dots, e^{(-1+i(m-1))t}x_{m-1}, e^{(-1+im)t + \sqrt{m}t}x_m, e^{(-1+i(m+1))t}x_{m+1}, \dots)$$

for every $t \ge 0$. Hence, $(z_n)_{n \in \mathbb{N}}$ in (4.9) satisfies

$$\begin{aligned} \left\| T(t)(z_n)_{n \in \mathbb{N}} - T_m(t)(z_n)_{n \in \mathbb{N}} \right\| &= \left| e^{(-1+im)t} z_m - e^{(-1+im)t + \sqrt{m}t} z_m \right| \\ &= e^{-t} (e^{\sqrt{m}t} - 1) \left\| (z_n)_{n \in \mathbb{N}} \right\| \quad \forall t \ge 0, \end{aligned}$$

which implies that $||T(t) - T_m(t)|| \ge e^{-t}(e^{\sqrt{mt}} - 1)$ for every $m \in \mathbb{N}$ and every $t \ge 0$. From this example, we find that even if the perturbation is sufficiently small in the sense of (4.1), there exists a strongly continuous semigroup such that the difference (in the uniform operator topology) between the original and perturbed semigroups is not small. Moreover, a similar calculation shows that $||T_m(t)|| \ge e^{(\sqrt{m}-1)t}$ for every $m \in \mathbb{N}$ and $t \ge 0$. Since $||T(t)|| = e^{-t}$, the open-loop system $\dot{x} = Ax$ is exponentially stable. However, the perturbed open-loop system $\dot{x} = (A + D_m)x$ is not exponentially stable for any $m \in \mathbb{N}$, although D_m becomes sufficiently small in the sense of (4.1) as $m \to \infty$.

5. Examples. In this section, we illustrate the proposed robustness analysis with delayed perturbations and low-order differential perturbations. First, we apply Theorem 1.2 to delayed perturbations, employing the techniques developed in Chapter 3 in [3]. To analyze the robust stability of diffusion systems, we exploit Theorem 1.3 together with the results obtained in Example III.2.12 of [7] and Example 3.7.34 of [1].

Before presenting examples, we recall the definitions of vector-valued function spaces. For an interval $J \subset \mathbb{R}$, we define the vector-valued Lebesgue space $L^2(J, X)$ and Sobolev space $H^1(J, X)$ by

$$L^{2}(J,X) := \left\{ f: J \to X : f \text{ is measurable and } \int_{J} \|f(s)\|^{2} dx < \infty \right\},$$
$$H^{1}(J,X) := \left\{ f \in L^{2}(J,X) : \exists s_{0} \in J, \ g \in L^{2}(J,X) \text{ s.t. } f(s) = f(s_{0}) + \int_{s_{0}}^{s} g(s) ds \right\}.$$

The spaces $L^2(J, X)$ and $H^1(J, X)$ are endowed, respectively, with the norms

$$||f||_{L^2} := \sqrt{\int_J ||f(s)||^2 dx}, \quad ||f||_{H^1} := ||f||_{L^2} + ||g||_{L^2},$$

where g satisfies the condition in the definition of $H^1(J, X)$. We set $L^2(J) := L^2(J, \mathbb{C})$ and $H^1(J) := H^1(J, \mathbb{C})$. Similarly, $L^p(J)$, $p \in [1, \infty]$, and $H^2(J)$ are denoted by the standard scalar-valued Lebesgue space and second-order Sobolev space, respectively.

EXAMPLE 5.1 (Delayed perturbations). For a Banach space X, define a bounded linear operator $\Phi: H^1([-1,0], X) \to X$ in the form of a Riemann-Stieltjes integral:

(5.1)
$$\Phi(f) := \int_{-1}^{0} d\eta(s) f(s).$$

where $\eta : [-1, 0] \to \mathcal{L}(X)$ is of bounded variation. We refer the readers to Chapter 2 of [5] for the Riemann-Stieltjes integral of a vector-valued function. For $k \in \{1, \ldots, n\}$, let $D_k \in \mathcal{L}(X), h_k \in [0, 1]$, and $\mathbb{1}_{[-h_k, 0]}$ be the characteristic function of $[-h_k, 0]$. If we set $\eta := \sum_{k=1}^n D_k \mathbb{1}_{[-h_k, 0]}$, then Φ is the discrete delay operator given by

$$\Phi(f) = \sum_{k=1}^{n} D_k f(-h_k).$$

For a function $z : [-1, \infty) \to X$ and $t \ge 0$, we define $z_t : [-1, 0] \to X$ by $z_t(\sigma) := z(t + \sigma)$. Let A be the generator of a strongly continuous semigroup T(t) on X. For a Banach space U, let $B_0 \in \mathcal{L}(U, X)$ and $F \in \mathcal{L}(X, U)$. Consider the following sampled-data system with delayed perturbation:

(5.2a) $\dot{z}(t) = Az(t) + \Phi z_t + B_0 u(t)$ $t \ge 0;$ $z(0) = \zeta, z_0 = f,$

(5.2b)
$$u(k\tau + t) = Fz(k\tau) \qquad t \in [0,\tau), \ k \in \mathbb{Z}_+.$$

where $\zeta \in X$, $f \in L^2([-1,0], X)$, and $\tau > 0$. Define $\mathcal{X} := X \times L^2([-1,0], X)$ with norm

$$\left\| \begin{bmatrix} z \\ f \end{bmatrix} \right\|_{\mathcal{X}} := \|z\|_X + \|f\|_{L^2}, \quad \text{where } \|\cdot\|_X \text{ is the norm in } X.$$

 Set

$$\mathcal{A} := \begin{bmatrix} A & 0 \\ 0 & d/d\theta \end{bmatrix} \quad \text{with } \operatorname{dom}(\mathcal{A}) := \left\{ \begin{bmatrix} \zeta \\ f \end{bmatrix} \in \operatorname{dom}(\mathcal{A}) \times H^1([-1,0],X) : f(0) = \zeta \right\},$$

where $d/d\theta$ denotes the distributional derivative. We denote by $\|\cdot\|_{\mathcal{A}}$ the graph norm for \mathcal{A} , i.e., $\|x\|_{\mathcal{A}} := \|x\|_{\mathcal{X}} + \|\mathcal{A}x\|_{\mathcal{X}}$ for $x \in \text{dom}(\mathcal{A})$, and set $\mathcal{X}_1 := (\text{dom}(\mathcal{A}), \|\cdot\|_{\mathcal{A}})$. Define

$$\mathcal{D} := \begin{bmatrix} 0 & \Phi \\ 0 & 0 \end{bmatrix} \in \mathcal{L}(\mathcal{X}_1, \mathcal{X}), \quad \mathcal{B} := \begin{bmatrix} B \\ 0 \end{bmatrix} \in \mathcal{L}(U, \mathcal{X}), \quad \mathcal{F} := \begin{bmatrix} F & 0 \end{bmatrix} \in \mathcal{L}(\mathcal{X}, U).$$

Setting

$$x(t) := \begin{bmatrix} z(t) \\ z_t \end{bmatrix},$$

we find that (5.2) can be written in the abstract form

$$\dot{x}(t) = (\mathcal{A} + \mathcal{D})x + \mathcal{B}u(t) \quad t \ge 0; \qquad x(0) = \begin{bmatrix} \zeta \\ f \end{bmatrix}$$
$$u(k\tau + t) = \mathcal{F}x(k\tau) \qquad t \in [0, \tau), \ k \in \mathbb{Z}_+.$$

see Section 3.1 of [3] for details. Moreover, \mathcal{A} generates a strongly continuous semigroup $\mathcal{T}(t)$ by Theorem 3.2.5 of [3]. For every $t_0 \in (0, 1)$,

$$\int_{0}^{t_{0}} \|\mathcal{DT}(s)x\|_{\mathcal{X}} \, ds \le t_{0}^{1/2} \cdot \sup_{t \in [0,1]} \|T(t)\| \cdot v(\eta; [-1,0]) \cdot \|x\|_{\mathcal{X}} \qquad \forall x \in \operatorname{dom}(\mathcal{A})$$

by (3.41) of [3], where $v(\eta; [-1, 0])$ is the total variation of η on [-1, 0]. By Theorem 1.2, if the total variation $v(\eta; [-1, 0])$ is sufficiently small and if the nominal sampled-data system is exponentially stable with decay rate greater than ω , then the perturbed sampled-data system is also exponentially stable with decay rate greater than ω .

EXAMPLE 5.2 (Perturbations by general low-order differential operators). Consider the second-order derivative operator A on $X := L^2[0, 1]$

$$Af := \frac{d^2 f}{d\xi^2} \quad \text{with } \operatorname{dom}(A) := \big\{ f \in H^2[0,1] : f(0) = f(1) \big\}.$$

This operator generates an analytic semigroup on X, and the first-order derivative $\Gamma_0 := d/d\xi$ with maximal domain $H^1[0, 1]$ is relatively A-bounded, i.e., there exist constants $\alpha_0, \beta_0 \geq 0$ such that

$$\|\Gamma_0 f\|_{L^2} \le \alpha_0 \|Af\|_{L^2} + \beta_0 \|f\|_{L^2} \qquad \forall f \in \text{dom}(A);$$

see Example III.2.12 (i) of [7]. Take $D \in \mathcal{L}(H^1[0,1], L^2[0,1])$. Using Poincare's inequality (see, e.g., Sec. 1.7.2 in [6]),

$$||f||_{L^2} \le \frac{1}{\pi} ||\Gamma_0 f||_{L^2} \quad \forall f \in \text{dom}(A),$$

we have that

$$\begin{split} \|Df\|_{L^{2}} &\leq \|D\| \cdot \|f\|_{H^{1}} \leq \frac{1+\pi}{\pi} \|D\| \cdot \|\Gamma_{0}f\|_{L^{2}} \\ &\leq \frac{1+\pi}{\pi} \|D\| \left(\alpha_{0} \|Ax\|_{L^{2}} + \beta_{0} \|x\|_{L^{2}}\right) \quad \forall f \in \operatorname{dom}(A), \end{split}$$

By Theorem 1.3, if ||D|| is sufficiently small and if the nominal sampled-data system with generator A is exponentially stable with decay rate greater than ω , then the sampled-data system perturbed by D is also exponential stability with decay rate greater than ω .

EXAMPLE 5.3 (Perturbations by low-order differential operators and multiplication operators). We consider the *n*-dimensional diffusion semigroup on $X := L^1(\mathbb{R}^n)$ given by

$$(T(t)f)(x) := (4\pi t)^{-n/2} \int_{\mathbb{R}^n} e^{-|x-r|^2/(4t)} f(r) dr$$

for every t > 0 and every $x \in \mathbb{R}^n$. Let us denote by A the generator of T(t). By Proposition in Paragraph II.2.13 of [7], the generator A is the closure of the Laplacian defined on the Schwartz space, i.e., the space of rapidly decreasing, infinitely differentiable functions. For $j \in \{1, \ldots, n\}$, let U_j be the strongly continuous group on X defined by

$$(U_j(t)f)(x) := f(x_1, \dots, x_{j-1}, x_j + t, \dots, x_n)$$

for every $t \in \mathbb{R}$ and every $x \in \mathbb{R}^n$. For $j \in \{1, \ldots, n\}$, and the generator of U_j be denoted by Γ_j , which is a first-order differential operator. Let $d_j \in L^{\infty}(\mathbb{R}^n)$ and define

$$(D_1f)(x) := \sum_{j=1}^n d_j(x)(\Gamma_j f)(x)$$

for every $x \in \mathbb{R}^n$ with domain

$$\operatorname{dom}(D_1) := \left\{ f \in L^1(\mathbb{R}^n) : f \in \operatorname{dom}(\Gamma_j) \text{ for } j \in \{1, \dots, n\} \text{ with } d_j \neq 0 \right\}.$$

Let $p > \max\{1, n/2\}$ and $q \in L^p(\mathbb{R}^n)$, and define

$$(D_2f)(x) := q(x)f(x)$$

for every $x \in \mathbb{R}^n$ with domain dom $(D_2) := \{f \in L^1(\mathbb{R}^n) : qf \in L^1(\mathbb{R}^n)\}$. It is shown in Example 3.7.34 of [1] that D_1 is relatively A-bounded and satisfies

(5.3)
$$\|D_1 f\|_{L^1} \le \sum_{j=1}^n \|d_j\|_{L^\infty} \left(\|Af\|_{L^1} + \|f\|_{L^1} \right) \quad \forall f \in \operatorname{dom}(A).$$

As shown in Example III.2.12 (ii) of [7], D_2 is also relatively A-bounded and satisfies

(5.4)
$$\|D_2 f\|_{L^1} \le c \|q\|_{L^p} (\|Af\|_{L^1} + \|f\|_{L^1}) \qquad \forall f \in \operatorname{dom}(A),$$

where the constant c > 0 depends only on p and n. Together with (5.3) and (5.4), Theorem 1.3 shows that if $||d_1||_{L^{\infty}}, \ldots, ||d_n||_{L^{\infty}}$ and $||q||_{L^p}$ are sufficiently small and if the nominal sampled-data system with generator A is exponentially stable with decay rate greater than ω , then the sampled-data system perturbed by $D := D_1 + D_2$ is also exponentially stable with decay rate greater than ω .

6. Concluding remarks. We have analyzed the exponential stability of infinitedimensional sampled-data systems under unbounded perturbations. First we have showed that exponential stability is preserved if the perturbed semigroup is sufficiently close to the original semigroup in the uniform operator topology. Then we have presented two results on the continuity of strongly continuous semigroups with respect to their generators. The first result is based on the perturbation theorem of Miyadera-Voigt. The second result is obtained from the results on analytic semigroups and relatively bounded perturbations. In this study, we have considered only bounded control operators in this study, and future work will involve extending the obtained results to systems with unbounded control operators.

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