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Recursive subspace identification of linear and non-linear Wiener state-space models☆

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The MOESP class of identification algorithms are made recursive on the basis of various updating schemes for subspace tracking

Abstract

The problem of MIMO recursive identification is analyzed within the framework of subspace model identification (SMI) and the use of recent signal processing algorithms for the recursive update of the singular value decomposition (SVD) is proposed. To accommodate for arbitrary correlation of the disturbances, an instrumental variable (IV) approach is followed. In particular, recursive formulations for the subspace identification algorithms of the multivariable output-error state space (MOESP) class are given. A recursive algorithm for the identification of non-linear models of the Wiener type is also obtained. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Subspace model identification (SMI) methods have attracted an increasing attention in the last few years (Van Overschee & De Moor, 1996; Verhaegen, 1994; Viberg, 1995). Among the advantages of such methods we mention the ability to deal with MIMO identification in a straightforward way, and the ease of use due to the small number of parameters which have to be chosen by the user. Also, unlike prediction error methods (PEM), SMI algorithms do not require non-linear searches in the parameter space but are based on computational tools such as the QR factorization and the singular-value decomposition (SVD), which make them intrinsically robust from a numerical point of view.

Part of the research in SMI has been dedicated to the problem of deriving recursive versions of such algorithms. Various solutions to the problem of recursive SMI (RSMI) have been proposed in the literature (see, e.g., Cho & Kailath, 1995; Gustafsson, 1997a,b,c; Lovera & Verhaegen, 1998; Lovera, 1987; Verhaegen & Deprettere, 1991), with different characteristics in terms of computational load, tracking performance, etc. The main obstacle with implementation of RSMI is that the SVD is computationally burdensome to update. Consequently, all of the above cited RSMI algorithms apply certain updating techniques that avoid direct application of the SVD.

Furthermore, a drawback of Cho and Kailath (1995) and Verhaegen and Deprettere (1991) is that the disturbances acting on the system output are required to be spatially and temporally white. When this restrictive assumption is not fulfilled, biased estimates are obtained.

The aims of this work are to provide an overview of the recursive implementations of the SMI algorithms of the MOESP class, and to present some recent developments. First of all, a faster version (compared with Verhaegen & Deprettere, 1991) of the recursive (ordinary) MOESP

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is proposed, based on subspace tracking ideas for the update of the SVD. See Comon and Golub (1990) for a review of subspace tracking algorithms. The recursive ordinary MOESP algorithm, however, suffers from the same assumptions as Cho and Kailath (1995) and Verhaegen and Deprettere (1991). To overcome this problem, recursive versions of the instrumental variable based algorithms past inputs (PI) MOESP and past outputs (PO) MOESP (Verhaegen, 1993, 1994) are investigated. The recently developed errors-in-variables (EIV) MOESP algorithm (see Chou & Verhaegen, 1997) is also given a recursive formulation.

The idea of applying subspace tracking algorithms to the RSMI problem was originally introduced in Gustafsson (1997). The basic idea of that algorithm is to use the close relationship between SMI and sensor array signal processing (SAP) problems. The main advantage of this algorithm is its low complexity, as it is comparable with the algorithm in Cho and Kailath (1995), but can deliver consistent estimates also when process noise is present. However, this approach relies on successive subtractions involving an estimated quantity, and hence its numerical reliability can be questioned. These drawbacks can be overcome by a combination of the approaches studied in Gustafsson (1997a,b,c), Lovera and Verhaegen (1998) and Lovera (1998), and such an approach is proposed in the present paper.

Finally, PI MOESP can also be applied to the estimation of the linear part of Wiener-type nonlinear models, see Westwick and Verhaegen (1996). Therefore, a recursive algorithm for identification of this kind of models has also been developed, by combining the above-mentioned RSMI algorithms with recursive estimators of static nonlinearities, using orthogonal polynomials.

2. Overview of the MOESP family of SMI algorithms

The MOESP family of SMI algorithms is studied in this paper for its combination of numerical simplicity and accuracy. Currently, the MOESP family consists of five different variants:

1. The *Elementary MOESP* (EM) scheme (Verhaegen & Dewilde, 1992).
2. The *Ordinary MOESP* (OM) scheme (Verhaegen & Dewilde, 1992).
3. The *PI* MOESP scheme (Verhaegen, 1993).
4. The *PO* MOESP scheme (Verhaegen, 1994).
5. The *PO* extension for the errors-in-variables identification problem, indicated by *PO_EIV* in Chou and Verhaegen (1997).

These variants reflect the type of perturbations that can be tolerated on the recorded input and output samples, the class of systems, and the nature of the input signal. In this section, we review the computational procedures and the identification problems addressed by these schemes. The latter are all variants of the following general identification problem:

Consider the finite dimensional, linear time-invariant (LTI) state-space model:

$$\begin{aligned} x_{t+1} &= Ax_t + B\tilde{u}_t + f_t, \qquad u_t = \tilde{u}_t + w_t, \\ \tilde{y}_t &= Cx_t + D\tilde{u}_t, \qquad y_t = \tilde{y}_t + v_t, \end{aligned} \tag{1}$$

where $x_t \in \mathbb{R}^n$, $u_t \in \mathbb{R}^m$, $y_t \in \mathbb{R}^p$ and the triples $\{f_t, v_t, w_t\}$ are additive perturbations to be defined in more detail for the different variants individually. The **key** *problem is the consistent estimation of the column space of the extended observability matrix Γ, defined as*

$$\Gamma = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{bmatrix}$$

from measured input–output (i–o) samples $\{u_t, y_t\}$.

One important equation in the derivation of SMI algorithms is a data equation relating (block) Hankel matrices constructed from the i–o data samples. Let the following block-Hankel matrices be defined:

$$Y_{t,i,j} = \begin{bmatrix} y_t & y_{t+1} & \cdots & y_{t+j-1} \\ y_{t+1} & y_{t+2} & \cdots & y_{t+j} \\ \vdots & \vdots & \ddots & \vdots \\ y_{t+i-1} & y_{t+i} & \cdots & y_{t+i+j-2} \end{bmatrix},$$

$$U_{t,i,j} = \begin{bmatrix} u_t & u_{t+1} & \cdots & u_{t+j-1} \\ u_{t+1} & u_{t+2} & \cdots & u_{t+j} \\ \vdots & \vdots & \ddots & \vdots \\ u_{t+i-1} & u_{t+i} & \cdots & u_{t+i+j-2} \end{bmatrix},$$

$$X_{t,j} = [x_t \quad x_{t+1} \quad \cdots \quad x_{t+j-1}]$$

and let the following block-Toeplitz matrix be defined:

$$H = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & D \end{bmatrix}. \tag{2}$$

Consider the special case of absence of f_t, v_t, w_t. Then, the data equation is denoted as

$$Y_{t,i,j} = \Gamma X_{t,j} + HU_{t,i,j}. \tag{3}$$

A summary of the computational scheme and the necessary additional assumptions on f_t, v_t, w_t required for the scheme to produce a consistent estimate of $span_{\mathrm{col}}(\Gamma)$ is given in the following. The consistency results imply assumptions on the input u_t (some kind of persistence of excitation notion) and on the true system. We restrict making additional remarks and refrain a precise definition of the required degree of persistence of excitation for the SMI schemes, since the latter notion is still not well understood for a number of them (cf. Jansson, 1997; Verhaegen & Dewilde, 1992).

(1) *The EM scheme*: The EM scheme relies on the (restrictive) assumption that an estimate of the quadruple of system matrices $[A, B, C, D]$ (up to a similarity transformation) is available. In that case, we can construct an estimate $\hat{H}$ of H, and form the modified data matrix

$$Z_{t,i,j} \stackrel{\text{def}}{=} Y_{t,i,j} - \hat{H} U_{t,i,j}. \tag{4}$$

If we let $E_{t,i,j}$ collect the effect of all perturbations in the data equation, then

$$Z_{t,i,j} = \Gamma X_{t,j} + E_{t,i,j} + (H - \hat{H}) U_{t,i,j}. \tag{5}$$

If f_t, w_t are zero, and v_t is a zero-mean white noise independent of the input u_t, and a consistent estimate of $[A, B, C, D]$ is available, a consistent estimate of $span_{\text{col}}(\Gamma)$ can be obtained via an SVD of $Z_{t,i,j}$.

(2) *The OM scheme*: This scheme considers the factorization of the matrix

$$\begin{bmatrix} U_{t,i,j} \\ Y_{t,i,j} \end{bmatrix}$$

into a lower triangular matrix and a matrix with orthogonal rows, denoted as

$$\begin{bmatrix} U_{t,i,j} \\ Y_{t,i,j} \end{bmatrix} = \begin{bmatrix} R_{11}(\bar{t}) & 0 \\ R_{21}(\bar{t}) & R_{22}(\bar{t}) \end{bmatrix} \begin{bmatrix} Q_1(\bar{t}) \\ Q_2(\bar{t}) \end{bmatrix}, \tag{6}$$

with $\bar{t} = t + i + j - 2$. Then a consistent estimate of $span_{\text{col}}(\Gamma)$ is provided via an SVD of $R_{22}(\bar{t})$ under the same assumptions on the perturbations for the EM scheme.

Remark 1. Both the EM and the OM scheme attempt to eliminate the term $HU_{t,i,j}$ from $Y_{t,i,j}$. The difference lies in the way the estimate $\hat{H}$ is defined. The left singular vectors of $Y_{t,i,j} Q_2(\bar{t})^{\mathrm{T}}$ are identical to those of

$$Y_{t,i,j} Q_2(\bar{t})^{\mathrm{T}} Q_2(\bar{t}) = Y_{t,i,j} \Pi_U^{\perp}, \tag{7}$$

where $\Pi_U^{\perp}$ denotes the orthogonal projection onto the null-space of $U_{t,i,j}$. However,

$$Y_{t,i,j} \Pi_U^{\perp} = Y_{t,i,j} - \hat{H}_{LS} U_{t,i,j}, \tag{8}$$

where $\hat{H}_{LS}$ denotes the unconstrained minimizing argument of $\|Y_{t,i,j} - HU_{t,i,j}\|_F^2$. Here $\|\cdot\|_F^2$ denotes the Frobenius norm. One appealing feature of the EM algorithm is that the structure of H is taken into account, in contrast to the OM approach. On the other hand, in the EM scheme we did not specify how the estimates of the system matrices were obtained.

(3) *The PI scheme*: This scheme considers the RQ factorization

$$\begin{bmatrix} U_{t+i,i,j} \\ U_{t,i,j} \\ Y_{t+i,i,j} \end{bmatrix} = \begin{bmatrix} R_{11}(\bar{t}) & 0 & 0 \\ R_{21}(\bar{t}) & R_{22}(\bar{t}) & 0 \\ R_{31}(\bar{t}) & R_{32}(\bar{t}) & R_{33}(\bar{t}) \end{bmatrix} \begin{bmatrix} Q_1(\bar{t}) \\ Q_2(\bar{t}) \\ Q_3(\bar{t}) \end{bmatrix} \tag{9}$$

with $\bar{t} = t + 2i + j - 1$. A consistent estimate of $span_{\text{col}}(\Gamma)$ is provided via an SVD of $R_{32}(\bar{t})$ under the assumptions that $f_t \equiv 0$, $w_t \equiv 0$ and v_t is an ergodic sequence of finite variance, satisfying $E[u_t v_s^{\mathrm{T}}] = 0 \ \forall t, s$.

(4) *The PO scheme*: This scheme considers the RQ factorization

$$\begin{bmatrix} U_{t+i,i,j} \\ \begin{bmatrix} Y_{t,i,j} \\ Y_{t,i,j} \end{bmatrix} \\ Y_{t+i,i,j} \end{bmatrix} = \begin{bmatrix} R_{11}(\bar{t}) & 0 & 0 & 0 \\ R_{21}(\bar{t}) & R_{22}(\bar{t}) & 0 & 0 \\ R_{31}(\bar{t}) & R_{32}(\bar{t}) & R_{33}(\bar{t}) & 0 \\ R_{41}(\bar{t}) & R_{42}(\bar{t}) & R_{43}(\bar{t}) & R_{44}(\bar{t}) \end{bmatrix} \times \begin{bmatrix} Q_1(\bar{t}) \\ Q_2(\bar{t}) \\ Q_3(\bar{t}) \\ Q_4(\bar{t}) \end{bmatrix} \tag{10}$$

with $\bar{t} = t + 2i + j - 1$. Then a consistent estimate of $span_{\text{col}}(\Gamma)$ is provided via an SVD of $[R_{42}(\bar{t}) \ R_{43}(\bar{t})]$ under the assumptions that $w_t \equiv 0$ and f_t, v_t are ergodic sequences of finite variances satisfying

$$E\left[\begin{bmatrix} f_t \\ v_t \end{bmatrix} [f_s^{\mathrm{T}} \quad v_s^{\mathrm{T}}]\right] = \begin{bmatrix} Q & S \\ S^{\mathrm{T}} & R \end{bmatrix} \delta_{s,t}$$

which are furthermore independent of u_t.

(5) *The PO_EIV scheme*: This scheme considers the RQ factorization:

$$\begin{bmatrix} U_{t+i,i,j} U_{t,i,j}^{\mathrm{T}} & U_{t+i,i,j} Y_{t,i,j}^{\mathrm{T}} \\ Y_{t+i,i,j} U_{t,i,j}^{\mathrm{T}} & Y_{t+i,i,j} Y_{t,i,j}^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} R_{11}(\bar{t}) & 0 \\ R_{21}(\bar{t}) & R_{22}(\bar{t}) \end{bmatrix} \times \begin{bmatrix} Q_1(\bar{t}) \\ Q_2(\bar{t}) \end{bmatrix} \tag{11}$$

with $\bar{t} = t + 2i + j - 1$. Then a consistent estimate of $span_{\text{col}}(\Gamma)$ is provided via an SVD of the matrix $R_{22}(\bar{t})$ under the assumptions that f_t, v_t, w_t are ergodic sequences of finite variance satisfying

$$E\left[\begin{bmatrix} f_t \\ w_t \\ v_t \end{bmatrix} [f_s^{\mathrm{T}} \quad w_s^{\mathrm{T}} \quad v_s^{\mathrm{T}}]\right] = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \Sigma_{12}^{\mathrm{T}} & \Sigma_{22} & \Sigma_{23} \\ \Sigma_{13}^{\mathrm{T}} & \Sigma_{23}^{\mathrm{T}} & \Sigma_{33} \end{bmatrix} \delta_{s,t}.$$

Once an estimate of $span_{\rm col}(\Gamma)$ has been obtained, we can derive an estimate of A and C by exploiting the shift invariance of $span_{\rm col}(\Gamma)$. As for the estimation of B and D, it can be based on the minimization of the simulation error over the identification data set (see McKelvey, 1994; Van Overschee & De Moor, 1996; Westwick & Verhaegen, 1996).

3. Review of PAST and IV-PAST

The update of the SVD is of particular relevance when it comes to recursive implementations of SMI schemes, as was highlighted for the OM scheme in Verhaegen and Deprettere (1991), where the computational complexity of the update of the SVD was $O((pi)^3)$. In the literature of sensor array signal processing, a large number of methods for partial update of the SVD are available. In this paper, we will exploit the close relationship between array signal processing and SMI to derive efficient update schemes for the SVD. More precisely, the Projection Approximation Subspace Tracking (PAST) algorithm (Yang, 1995, 1996) and its instrumental variables modification IV-PAST (Gustafsson, 1997a,b,c) are used and modified to derive an efficient partial SVD update in the different SMI schemes considered in Section 2.

3.1. The PAST scheme

Consider a random vector $x \in \mathbb{R}^m$, and study the *unconstrained* criterion

$$V(W) = E\|x - WW^{\rm T}x\|^2 \tag{12}$$

with a full rank ($=n$) matrix argument $W \in \mathbb{R}^{m\times n}$, $m > n$. Let the eigenvalue decomposition (EVD) of $R_x = E[xx^{\rm T}]$ be given as

$$R_x = Q\Lambda Q^{\rm H} \tag{13}$$

with $Q = [q_1, \dots, q_m]$, $\Lambda = \mathrm{diag}(\lambda_1, \dots, \lambda_m)$. Furthermore, assume that R_x is positive definite. The eigenvalues are ordered as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m$. Now, consider the following theorem (Yang, 1995):

Theorem 1. *The global minimum of $V(W)$ is attained if and only if $W = \bar{Q}T$ where $\bar{Q}$ contains the n dominating eigenvectors of R_x. Here T is an arbitrary unitary matrix. Furthermore, all other stationary points are saddle points.*

If the expectation operator in (12) is replaced with a finite summation, one gets

$$V(W(t)) = \sum_{k=1}^{t} \beta^{t-k}\|x(k) - W(t)W^{\rm H}(t)x(k)\|^2, \tag{14}$$

and a recursive algorithm is obtained, as follows. Theorem 1 is applicable also if R_x is replaced with

$$\hat{R}_x(t) = \sum_{k=1}^{t} \beta^{t-k}x(k)x^{\rm T}(k). \tag{15}$$

The key idea of PAST is to replace $W^{\rm T}(t)x(k)$ in (14) with

$$h(k) = W^{\rm T}(k-1)x(k). \tag{16}$$

This so-called *projection approximation* results in the criterion

$$\bar{V}(W(t)) = \sum_{k=1}^{t} \beta^{t-k}\|x(k) - W(t)h(k)\|^2 \tag{17}$$

which is quadratic in $W(t)$, and is minimized by

$$W(t) = \hat{R}_{xh}(t)\hat{R}_h^{-1}(t), \tag{18}$$

with obvious definitions of the involved matrices. When the matrix inversion lemma is applied to (18), an RLS-like algorithm of $O(mn)$ complexity is derived, see Yang (1995).

3.2. The IV-PAST scheme

Recently, an IV generalization of PAST was proposed, see Gustafsson (1997a,b,c). In this scenario it is assumed that the cross-correlation matrix $R_{xy} = E[xy^{\rm T}]$ has a low rank ($=n$) structure:

$$E[xy^{\rm T}] = \Gamma\Psi \tag{19}$$

where $\Gamma \in \mathbb{R}^{m\times n}$, $\Psi \in \mathbb{R}^{n\times l}$ both have full rank n, $m, l \geq n$. Introduce the following criterion:

$$V(W(t)) = \|\hat{R}_{xy}(t) - W(t)W^{\rm H}(t)\hat{R}_{xy}(t)\|_F^2, \tag{20}$$

where

$$\hat{R}_{xy}(t) = \sum_{k=1}^{t} \beta^{t-k}x(k)y^{\rm T}(k). \tag{21}$$

Then the following result can be shown:

Theorem 2. *Let $\hat{R}_{xy}(t)$ have the SVD*

$$\hat{R}_{xy}(t) = \hat{Q}\hat{\Sigma}\hat{V}^{\rm T}. \tag{22}$$

Then the global minimum of $V(W(t))$ is obtained if and only if $W(t) = \hat{Q}T$ where $\hat{Q}$ contains the n dominating left singular vectors of $\hat{R}_{xy}(t)$ and T is an arbitrary unitary matrix. All other stationary points are saddle points.

Applying the previous projection approximation, a recursive $O(ml)$ algorithm can be derived, see Gustafsson (1997a,b,c).

4. Recursive updating of $span_{\mathrm{col}}(\Gamma)$

To apply the PAST schemes to RSMI, the essential step is to derive a low rank update/downdate of the matrix from which the $span_{\mathrm{col}}(\Gamma)$ is estimated. This will be outlined in the subsequent sections.

4.1. The update in the EM scheme

Let $\bar{t} = t + i + j - 2$. Then we assume the following new set of i–o data vectors to become available:

$$\phi_u(\bar{t}+1) = [u_{t+j}^{\mathrm{T}} \dots u_{t+i+j-1}^{\mathrm{T}}]^{\mathrm{T}},$$
$$\phi_y(\bar{t}+1) = [y_{t+j}^{\mathrm{T}} \dots y_{t+i+j-1}^{\mathrm{T}}]^{\mathrm{T}}. \tag{23}$$

Assume further that an estimate $\hat{\Gamma}(\bar{t})$ and $[\hat{B}(\bar{t})\ \hat{D}(\bar{t})]$ of Γ and of $[B\ D]$ is available. Then $\hat{H}(\bar{t})$ can be computed by noticing that the first block column of $\hat{H}(\bar{t})$ is given by

$$[\hat{D}^{\mathrm{T}}(\bar{t})(\hat{\Gamma}_{1:p(i-1),:}(\bar{t})\hat{B}(\bar{t}))^{\mathrm{T}}]^{\mathrm{T}}, \tag{24}$$

where a MATLAB-like notation has been used. This is simpler than evaluating (2), since $\hat{A}^k$ is not explicitly computed. The remaining columns of $\hat{H}(\bar{t})$ are obtained from partitions of this (block) vector.

The vector to be fed to the PAST algorithm is

$$\phi(\bar{t}+1) = \phi_y(\bar{t}+1) - \hat{H}(\bar{t})\phi_u(\bar{t}+1). \tag{25}$$

4.2. The update in the OM scheme

Following Verhaegen and Deprettere (1991), let the new i–o data vectors be defined as in Eq. (23). Then we can restrict to a *partial* update of the RQ factorization required in the OM scheme, making use of the classical method of Givens rotations (see Golub & Van Loan, 1989).

The appropriate sequence of Givens rotations, represented by the matrix $P(\bar{t}+1)$, makes the following matrix reduction:

$$\begin{bmatrix} R_{11}(\bar{t}) & 0 & \phi_u(\bar{t}+1) \\ R_{21}(\bar{t}) & R_{22}(\bar{t}) & \phi_y(\bar{t}+1) \end{bmatrix} P(\bar{t}+1) = \begin{bmatrix} R_{11}(\bar{t}+1) & 0 & 0 \\ R_{21}(\bar{t}+1) & R_{22}(\bar{t}) & \phi(\bar{t}+1) \end{bmatrix}. \tag{26}$$

Then by stopping halfway in the update of the RQ factorization, the information contained in $[\phi_u^{\mathrm{T}}(\bar{t}+1)\ \phi_y^{\mathrm{T}}(\bar{t}+1)]^{\mathrm{T}}$ is condensed into a rank-one modification of $R_{22}(\bar{t})$. From the above RQ update we derive

$$R_{22}(\bar{t}+1)R_{22}(\bar{t}+1)^{\mathrm{T}} = R_{22}(\bar{t})R_{22}(\bar{t})^{\mathrm{T}} + \phi(\bar{t}+1)\phi(\bar{t}+1)^{\mathrm{T}}. \tag{27}$$

Notice that the "covariance" matrices $R_{22}(\bar{t}+1)R_{22}(\bar{t}+1)^{\mathrm{T}}$ and $\phi(\bar{t}+1)\phi(\bar{t}+1)^{\mathrm{T}}$ do not have to be computed explicitly, only $\phi(\bar{t})$ is used as input for the PAST algorithm.

Remark 2. Exponential forgetting can be included by modifying (27) as

$$R_{22}(\bar{t}+1)R_{22}(\bar{t}+1)^{\mathrm{T}} = \beta R_{22}(\bar{t})R_{22}(\bar{t})^{\mathrm{T}} + \phi(\bar{t}+1)\phi(\bar{t}+1)^{\mathrm{T}},$$

where $0 < \beta \leq 1$.

4.3. The update and downdate in the PI and PO schemes

We outline the update only for the PI scheme. Let $\bar{t} = t + 2i + j - 2$ and assume the following new set of i–o data vectors to become available:

$$\phi_{u_f}(\bar{t}+1) = [u_{t+i+j}^{\mathrm{T}} \dots u_{t+2i+j-1}^{\mathrm{T}}]^{\mathrm{T}},$$
$$\phi_{y_f}(\bar{t}+1) = [y_{t+i+j}^{\mathrm{T}} \dots y_{t+2i+j-1}^{\mathrm{T}}]^{\mathrm{T}} \tag{28}$$

and define $\phi_{u_p}(\bar{t}+1) = [u_{t+j}^{\mathrm{T}} \dots u_{t+i+j-1}^{\mathrm{T}}]^{\mathrm{T}}$.

According to Section 2, the factorization

$$\begin{bmatrix} R_{11}(\bar{t}+1) & 0 & 0 \\ R_{21}(\bar{t}+1) & R_{22}(\bar{t}+1) & 0 \\ R_{31}(\bar{t}+1) & R_{32}(\bar{t}+1) & R_{33}(\bar{t}+1) \end{bmatrix} \begin{bmatrix} Q_1(\bar{t}+1) \\ Q_2(\bar{t}+1) \\ Q_3(\bar{t}+1) \end{bmatrix} \tag{29}$$

should be computed. The problem is to work out a perturbation which relates the column space of $R_{32}(\bar{t})$ to the column space of $R_{32}(\bar{t}+1)$. By a first sequence $P_1(\bar{t}+1)$ of Givens rotations, we perform the matrix reduction

$$\begin{bmatrix} R_{11}(\bar{t}) & 0 & 0 & \phi_{u_f}(\bar{t}+1) \\ R_{21}(\bar{t}) & R_{22}(\bar{t}) & 0 & \phi_{u_p}(\bar{t}+1) \\ R_{31}(\bar{t}) & R_{32}(\bar{t}) & R_{33}(\bar{t}) & \phi_{y_f}(\bar{t}+1) \end{bmatrix} P_1(\bar{t}+1) = \begin{bmatrix} R_{11}(\bar{t}+1) & 0 & 0 & 0 \\ R_{21}(\bar{t}+1) & R_{22}(\bar{t}) & 0 & \bar{\phi}_{u_p}(\bar{t}+1) \\ R_{31}(\bar{t}+1) & R_{32}(\bar{t}) & R_{33}(\bar{t}) & \bar{\phi}_{y_f}(\bar{t}+1) \end{bmatrix}.$$

A second sequence of rotations zeros out the elements of $\bar{\phi}_{u_p}(\bar{t}+1)$:

$$\begin{bmatrix} R_{11}(\bar{t}+1) & 0 & 0 & 0 \\ R_{21}(\bar{t}+1) & R_{22}(\bar{t}) & 0 & \bar{\phi}_{u_p}(\bar{t}+1) \\ R_{31}(\bar{t}+1) & R_{32}(\bar{t}) & R_{33}(\bar{t}) & \bar{\phi}_{y_f}(\bar{t}+1) \end{bmatrix} P_2(\bar{t}+1) = \begin{bmatrix} R_{11}(\bar{t}+1) & 0 & 0 & 0 \\ R_{21}(\bar{t}+1) & R_{22}(\bar{t}+1) & 0 & 0 \\ R_{31}(\bar{t}+1) & R_{32}(\bar{t}+1) & R_{33}(\bar{t}) & \bar{\bar{\phi}}_{y_f}(\bar{t}+1) \end{bmatrix}.$$

In analogy with Eq. (27), the update in the PI scheme is characterized by the following update and downdate:

$$\begin{aligned} R_{32}(\bar{t}+1)R_{32}(\bar{t}+1)^{\mathrm{T}} &= R_{32}(\bar{t})R_{32}(\bar{t})^{\mathrm{T}} \\ &+ \bar{\phi}_{y_f}(\bar{t}+1)\bar{\phi}_{y_f}(\bar{t}+1)^{\mathrm{T}} \\ &- \bar{\bar{\phi}}_{y_f}(\bar{t}+1)\bar{\bar{\phi}}_{y_f}(\bar{t}+1)^{\mathrm{T}}. \end{aligned} \tag{30}$$

In order to derive a PAST-like algorithm for this rank two perturbation, consider the matrix

$$C(\bar{t}) = \sum_{k=1}^{\bar{t}} (\bar{\phi}_{y_f}(k)\bar{\phi}_{y_f}(k)^{\mathrm{T}} - \bar{\bar{\phi}}_{y_f}(k)\bar{\bar{\phi}}_{y_f}(k)^{\mathrm{T}}), \tag{31}$$

and introduce the criterion

$$\begin{aligned} J(W(\bar{t})) &= Tr[C(\bar{t}) - W(\bar{t})W^{\mathrm{T}}(\bar{t})C(\bar{t})] \\ &= \sum_{i=1}^{\bar{t}} \|\bar{\phi}_{Y_f}(i) - W(\bar{t})W(\bar{t})^{\mathrm{T}}\bar{\phi}_{Y_f}(i)\|^2 \\ &\quad - \sum_{i=1}^{\bar{t}} \|\bar{\bar{\phi}}_{Y_f}(i) - W(\bar{t})W(\bar{t})^{\mathrm{T}}\bar{\bar{\phi}}_{Y_f}(i)\|^2. \end{aligned} \tag{32}$$

By introducing the projection approximations

$$\bar{h}(i) = W^{\mathrm{T}}(i-1)\bar{\phi}_{Y_f}(i), \qquad \bar{\bar{h}}(i) = W^{\mathrm{T}}(i-1)\bar{\bar{\phi}}_{Y_f}(i)$$

one has the new criterion

$$\begin{aligned} \bar{J}(W(\bar{t})) &= \sum_{i=1}^{\bar{t}} \|\bar{\phi}_{Y_f}(i) - W(\bar{t})\bar{h}(i)\|^2 \\ &\quad - \sum_{i=1}^{\bar{t}} \|\bar{\bar{\phi}}_{Y_f}(i) - W(\bar{t})\bar{\bar{h}}(i)\|^2 \end{aligned} \tag{33}$$

which is minimized by

$$W(\bar{t}) = [\hat{R}_{\bar{\phi}_{Y_f},\bar{h}} - \hat{R}_{\bar{\bar{\phi}}_{Y_f},\bar{\bar{h}}}][\hat{R}_{\bar{h}} - \hat{R}_{\bar{\bar{h}}}]^{-1} \tag{34}$$

provided one can ensure the positive definiteness of the matrix the inverse of which appears in the above equation. We conjecture that these inverses exists under appropriate assumptions on the persistence of excitation of the applied input and the scaling of the available data. However, the precise requirements that assure the existence of the matrix inverse in (34) are still a matter of investigation. The resulting algorithm for subspace tracking will in the sequel be referred to as UD-PAST.

An alternative update for the PI/PO scheme can be obtained by using IV-PAST instead of UD-PAST. For that purpose, we consider the explicit form of the data equation, namely

$$\begin{aligned} Y_{t+i,i,j+1} &= \Gamma[X_{t+i,j}\ x_{t+i+j}] + HU_{t+i,i,j+1} \\ &\quad + [E_{t+i,i,j}\ \phi_{e_f}(\bar{t}+1)]. \end{aligned}$$

Then we consider the following factorization, as in the OM scheme:

$$\begin{bmatrix} U_{t+i,i,j} \\ Y_{t+i,i,j} \end{bmatrix} = \begin{bmatrix} R_{11}(\bar{t}) & 0 \\ R_{21}(\bar{t}) & R_{22}(\bar{t}) \end{bmatrix} \begin{bmatrix} Q_1(\bar{t}) \\ Q_2(\bar{t}) \end{bmatrix}.$$

When a new data point is obtained, the decomposition must be updated as

$$\begin{bmatrix} U_{t+i,i,j+1} \\ Y_{t+i,i,j+1} \end{bmatrix} = \begin{bmatrix} R_{11}(\bar{t}) & 0 & \phi_{u_f}(\bar{t}+1) \\ R_{21}(\bar{t}) & R_{22}(\bar{t}) & \phi_{y_f}(\bar{t}+1) \end{bmatrix} \begin{bmatrix} Q_1(\bar{t}) & 0 \\ Q_2(\bar{t}) & 0 \\ 0 & 1 \end{bmatrix},$$

where, in particular

$$R_{21}(\bar{t}) = \Gamma X_{t+i,j}Q_1(\bar{t})^{\mathrm{T}} + HR_{11}(\bar{t}) + E_{t+i,i,j}Q_1(\bar{t})^{\mathrm{T}}.$$

The sequence of Givens rotations necessary to annihilate $\phi_{u_f}(\bar{t}+1)$ is

$$P(\bar{t}+1) = \begin{bmatrix} P_{11}(\bar{t}+1) & 0 & P_{12}(\bar{t}+1) \\ 0 & I & 0 \\ P_{21}^{\mathrm{T}}(\bar{t}+1) & 0 & P_{22}(\bar{t}+1) \end{bmatrix} \tag{35}$$

and the above factorization becomes

$$\begin{aligned} &\begin{bmatrix} R_{11}(\bar{t}) & 0 & \phi_{u_f}(\bar{t}+1) \\ R_{21}(\bar{t}) & R_{22}(\bar{t}) & \phi_{y_f}(\bar{t}+1) \end{bmatrix} \\ &\quad\times \begin{bmatrix} P_{11}(\bar{t}+1) & 0 & P_{12}(\bar{t}+1) \\ 0 & I & 0 \\ P_{21}^{\mathrm{T}}(\bar{t}+1) & 0 & P_{22}(\bar{t}+1) \end{bmatrix} \\ &\quad\times \begin{bmatrix} P_{11}(\bar{t}+1)^{\mathrm{T}}Q_1(\bar{t}) & P_{21}(\bar{t}+1) \\ Q_2(\bar{t}) & 0 \\ P_{12}(\bar{t}+1)Q_1(\bar{t}) & P_{22}(\bar{t}+1) \end{bmatrix}. \end{aligned}$$

Using the fact that the input u_t is independent from the v_t and f_t and letting

$$\begin{aligned} R_{11}(\bar{t}+1) &= R_{11}(\bar{t})P_{11}(\bar{t}+1) \\ &\quad + \phi_{u_f}(\bar{t}+1)P_{21}^{\mathrm{T}}(\bar{t}+1), \end{aligned} \tag{36}$$

$$\begin{aligned} R_{21}(\bar{t}+1) &= R_{21}(\bar{t})P_{11}(\bar{t}+1) \\ &\quad + \phi_{y_f}(\bar{t}+1)P_{21}^{\mathrm{T}}(\bar{t}+1), \end{aligned} \tag{37}$$

we obtain for the R factor:

$$\begin{bmatrix} R_{11}(\bar{t}+1) & 0 & 0 \\ R_{21}(\bar{t}+1) & R_{22}(\bar{t}) & \bar{\phi}_{y_f}(\bar{t}+1) \end{bmatrix},$$

where

$$\begin{aligned} \bar{\phi}_{y_f}(\bar{t}+1) &= \Gamma(X_{t+i,j}Q_1(\bar{t})^{\mathrm{T}}P_{12}(\bar{t}+1) \\ &\quad + x_{t+i+j}P_{22}(\bar{t}+1)) \\ &\quad + E_{t+i,i,j}Q_1^{\mathrm{T}}(\bar{t})P_{12}(\bar{t}+1) \\ &\quad + \phi_{e_f}(\bar{t}+1)P_{22}(\bar{t}+1). \end{aligned} \tag{38}$$

By feeding $\bar{\phi}_{y_f}(\bar{t}+1)$ to the PAST algorithm to update the column space of the matrix $R_{22}(\bar{t})$ we would in general obtain a biased estimate. A rescue to this problem is the introduction of IVs:

$$\xi(\bar{t}+1) = F\phi_{u_p}(\bar{t}+1), \tag{39}$$

$$\xi(\bar{t}+1) = F\begin{bmatrix} \phi_{u_p}(\bar{t}+1) \\ \phi_{y_p}(\bar{t}+1) \end{bmatrix}, \tag{40}$$

where F is a user-defined matrix (or filter operator), see Gustafsson (1997) for the design of F. A necessary requirement on the dimension of $\xi(\bar{t})$ is that it is larger than or equal to n, the system order.

For updating the approximation of $span_{\mathrm{col}}(\Gamma)$, we use the IV-PAST algorithm of Section 3.2 with x and y taken equal to $\bar{\phi}_{y_f}(\bar{t}+1)$ and $\xi(\bar{t}+1)$, respectively.

4.4. The update in the PO_EIV scheme

Let $\bar{t} = t + 2i + j - 2$ and assume the new i–o data vectors be available and define $\phi_{y_p}(\bar{t}+1)$ accordingly to $\phi_{u_p}(\bar{t}+1)$. Then the update of the left-hand side of Eq. (11) reads

$$\begin{bmatrix} U_{t+i,i,j+1}U^{\mathrm{T}}_{t,i,j+1} & U_{t+i,i,j+1}Y^{\mathrm{T}}_{t,i,j+1} \\ Y_{t+i,i,j+1}U^{\mathrm{T}}_{t,i,j+1} & Y_{t+i,i,j+1}Y^{\mathrm{T}}_{t,i,j+1} \end{bmatrix} = \begin{bmatrix} U_{t+i,i,j}U^{\mathrm{T}}_{t,i,j} & U_{t+i,i,j}Y^{\mathrm{T}}_{t,i,j} \\ Y_{t+i,i,j}U^{\mathrm{T}}_{t,i,j} & Y_{t+i,i,j}Y^{\mathrm{T}}_{t,i,j} \end{bmatrix} + \begin{bmatrix} \phi_{u,f}(\bar{t}+1) \\ \phi_{y_f}(\bar{t}+1) \end{bmatrix} [\phi_{u_p}(\bar{t}+1)^{\mathrm{T}}\phi_{y_p}(\bar{t}+1)^{\mathrm{T}}].$$

To get a simple expression for the low rank update of the matrix containing an estimate of the column space of Γ, we consider the EM-like rank-one update:

$$\begin{aligned}(Y_{t+i,i,j+1} - \hat{H}(\bar{t})U_{t+i,i,j+1})[U^{\mathrm{T}}_{t,i,j+1}\, Y^{\mathrm{T}}_{t,i,j+1}] &= (Y_{t+i,i,j} - \hat{H}(\bar{t})U_{t+i,i,j})[U^{\mathrm{T}}_{t,i,j}\, Y^{\mathrm{T}}_{t,i,j}] \\ &\quad + (\phi_{y_f}(\bar{t}+1) - \hat{H}(\bar{t})\phi_{u_f}(\bar{t}+1)) \\ &\quad \times [\phi_{u_p}(\bar{t}+1)^{\mathrm{T}}\bar{\phi}_{y_p}(\bar{t}+1)^{\mathrm{T}}]. \end{aligned} \tag{41}$$

For updating the approximation of $span_{\mathrm{col}}(\Gamma)$, we again use the IV-PAST algorithm of Section 3.2 with x and y now replaced by $(\phi_{y_f}(\bar{t}+1) - \hat{H}(\bar{t})\phi_{u_f}(\bar{t}+1))$ and $[\phi_{u_p}(\bar{t}+1)^{\mathrm{T}}\bar{\phi}_{y_p}(\bar{t}+1)^{\mathrm{T}}]$.

4.5. Comments and discussion

We will refer to the different algorithms as recursive implementations of their off-line counterparts, abbreviated by putting an R before their corresponding acronym. An exception is made for the two variants of the PI and PO implementations. Here the first variant based on the update and downdate is referred to as the $\mathrm{RPI}_1/\mathrm{RPO}_1$ scheme, while the second variant based on the IV-PAST algorithms is referred to as the $\mathrm{RPI}_2/\mathrm{RPO}_2$ variant. The aim of this section is to discuss the relative merits of the algorithms.

REM's main drawback is that the subtraction $\phi_y(\bar{t}+1) - \hat{H}(\bar{t})\phi_u(\bar{t}+1)$ may turn out to be not fully reliable; it is an approximate solution, the accuracy of which is very difficult to analyze in a precise way. On the other hand, this approach is attractive from a computational point of view and in our simulations reliable estimates were obtained in all the considered cases. $\mathrm{ROM}/\mathrm{RPI}_1$ and RPO_1 are based on the update of the RQ factorization, so they provide an exact recursive version of the OM, PI and PO methods, up to the approximation introduced by the subspace tracking algorithm. This theoretical solidity is to be paid with a higher computational cost. The above considerations lead then to consider the RPI_2 and RPO_2 variants. This approach can be considered as a sound compromise between the first two classes of methods. In particular, IVs are introduced in the PAST algorithm, at a low computational cost, and the update of the RQ factorization is performed on the same data matrix as in the ROM method. That is, on a data matrix which has smaller dimensions than the ones for the RPI_1 and RPO_1 methods.

In Table 1 the major steps of the different algorithms are summarized and the involved complexities are stated. In terms of execution time, our experience shows that in these conditions one iteration can usually be performed in less than 0.1 s on a Pentium PC.

Table 1
The major steps of the proposed algorithms. The complexity column denotes the number of multiplications necessary in each step

Algorithm	Computations	Complexity
ROM	QR-update	$O((\frac{m}{2}+p)mi^2)$
	PAST	$O(pin)$
REM	Subtraction	$O(mpi^2)$
	IV-PAST	$O(mpi^2)$
RPI_1	QR-update	$O((2m+p)^2i^2)$
	UD-PAST	$O(2pin)$
RPO_1	QR-update	$O((2m+2p)^2i^2)$
	UD-PAST	$O(2pin)$
RPI_2	QR-update	$O((\frac{m}{2}+p)mi^2)$
	IV-PAST	$O(mpi^2)$
RPO_2	QR-update	$O((\frac{m}{2}+p)mi^2)$
	IV-PAST	$O(p(m+p)i^2)$
RPO_EIV	Subtraction	$O(mpi^2)$
	EIV-PAST	$O((m+p)pi^2)$

5. Recursive update of the system matrices

When $\hat{\Gamma}(\bar{t})$ is found, we estimate the system matrices $\hat{A}(\bar{t})$ and $\hat{C}(\bar{t})$ as in the non-recursive case. Common to all SMI schemes, we discuss the update of the estimates of B and D.

When $\bar{t} \gg 1$ and when the eigenvalues of $\hat{A}(\bar{t})$ lie strictly inside the unit-circle, the linear regression

$$y_{\bar{t}} = \varphi^{\mathrm{T}}(\bar{t})\theta + e_{\bar{t}}, \tag{42}$$

where

$$\varphi^{\mathrm{T}}(\bar{t}) = \left[u^{\mathrm{T}}_{\bar{t}} \otimes I_p \sum_{s=1}^{\bar{t}-1} (u^{\mathrm{T}}_s \otimes CA^{\bar{t}-s-1}) \right] \tag{43}$$

is applicable. Here, I_p denotes the $p \times p$ identity matrix, and $\otimes$ denotes the Kronecker product. Partition $\varphi^{\mathrm{T}}(\bar{t}) = [\varphi_1^{\mathrm{T}}(\bar{t})\ \varphi_2^{\mathrm{T}}(\bar{t})]$ conformably with

$$\theta = [\mathrm{vec}^{\mathrm{T}}(D)\ \mathrm{vec}^{\mathrm{T}}(B)]^{\mathrm{T}}, \tag{44}$$

where $\mathrm{vec}(\cdot)$ denotes the vectorization operator. It follows that

$$\begin{aligned}\varphi_2^{\mathrm{T}}(\bar{t}+1) &= \sum_{s=1}^{\bar{t}} (u_s^{\mathrm{T}} \otimes C A^{\bar{t}-s}) \\ &= \varphi_2^{\mathrm{T}}(\bar{t})(I_m \otimes A) + (u_t^{\mathrm{T}} \otimes C).\end{aligned} \tag{45}$$

Hence, there is a recursion in one of the regressors. The parameter vector θ can be computed at time $\bar{t}$, by using (42) and updating $\varphi(\bar{t})$ with $\hat{A}(\bar{t})$ and $\hat{C}(\bar{t})$, for example with the multivariate RLS-algorithm [Ljung, 1987, Section 11]. Notice that when dealing with EIV problems, a recursive IV-algorithm should be considered. Such an algorithm can be derived from a recursive implementation of the one given in Chou and Verhaegen (1997), which is not reported here for brevity. When the current RLS variant is run using the exact A and C matrix, it would provide consistent estimates of the B and D matrices.

6. Recursive identification of Wiener-type nonlinear models

A Wiener system is given by the cascade interconnection of a linear time-invariant system with a static nonlinearity:

$$\begin{aligned} x_{t+1} &= A x_t + B u_t, \\ y_t &= C x_t + D u_t, \\ z_t &= \Phi(y_t) + v_t, \end{aligned} \tag{46}$$

where $\Phi(\cdot)$ is a nonlinear function and v_t is a zero mean stochastic process with arbitrary color.

Wiener models are often useful in practical applications (for a recent example see Verhaegen, 1998). For this kind of models, the identification problem can be formulated as follows: given data sequences of u and z, find consistent estimates of the linear part of the model and of the nonlinear mapping $\Phi(\cdot)$.

In the literature, the problem of Wiener model identification has been mostly analyzed in the prediction error framework (see, e.g., Wigren, 1993, 1994 and the references therein). Recently, in Westwick and Verhaegen (1996), a subspace-based approach to the problem was proposed, making use of the PI MOESP method for the estimation of the system matrices of the linear part.

In this section a recursive identification method for Wiener models is discussed, based on the above proposed recursive versions of PI MOESP.

6.1. The PI scheme for Wiener systems

The PI scheme provides a consistent estimate of the linear dynamic part of Wiener models when the input is a stochastic sequence with a Gaussian distribution (see Westwick & Verhaegen, 1996). More precisely, some additional assumptions are required, concerning the nature of the nonlinear mapping $\Phi(\cdot)$: consistent estimates of the linear part can be obtained when the nonlinear mapping $\Phi(\cdot)$ is odd, i.e., it is an odd function of its arguments. While extensions of the basic algorithm to the case when $\Phi(\cdot)$ contains even terms have been derived (see Westwick & Verhaegen, 1996 for details), we will restrict the present analysis to the case with an odd nonlinearity, for which the following Theorem holds:

Theorem 3. *For the Wiener system* (46) *affected by Gaussian measurement noise v of arbitrary color and independent of the Gaussian input u, consider the RQ factorization*

$$\begin{bmatrix} U_{t+i,i,j} \\ U_{t,i,j} \\ Z_{t+i,i,j} \end{bmatrix} = \begin{bmatrix} R_{11}(\bar{t}) & 0 & 0 \\ R_{21}(\bar{t}) & R_{22}(\bar{t}) & 0 \\ R_{31}(\bar{t}) & R_{32}(\bar{t}) & R_{33}(\bar{t}) \end{bmatrix} \begin{bmatrix} Q_1(\bar{t}) \\ Q_2(\bar{t}) \\ Q_3(\bar{t}) \end{bmatrix}. \tag{47}$$

Then the R_{32} block in the factorization provides a consistent estimate of the observability subspace of the linear part of the model.

On the basis of the above theorem, the system matrices can be consistently recovered. Once the linear part is known, one can estimate the data sequence y by simulating the estimated linear model and subsequently obtain an estimate of the nonlinearity $\Phi(\cdot)$ by estimating the parameters of some appropriate model class.

6.2. A recursive subspace algorithm for Wiener systems

Given the similarity between the PI algorithm for linear systems and the identification procedure for Wiener systems, one can consider the development of a recursive identification method for such class of nonlinear systems.

In particular, if the chosen parameterization for the static nonlinearity is linear in the parameters, one can use RLS for their update. The generic iteration (time $\bar{t}$) of the algorithm is structured as follows:

(1) Use of the recursive PI scheme described in Section 4.3, to update the estimates of A, B, C and D.
(2) Use the updated estimates of the system matrices to compute an estimate $\hat{y}(t)$ of $y(t)$.
(3) Using $\hat{y}(t)$ and the output measurement $z(t)$, update the estimate of the static nonlinearity $\Phi(\cdot)$ by means, e.g., of an RLS algorithm.

Concerning the choice of a parameterization for the static nonlinearities, there are many possibilities. In Wigren (1993) (where however only the SISO case is treated), a piecewise linear approximation is used. In this paper the MIMO case is dealt with, so more appropriate parameterizations have been considered. In particular, the MIMO Tchebiceff polynomials described in Westwick and Verhaegen (1996) have been used in the simulation examples, while the use of neural networks is currently being investigated.

The class of Tchebiceff polynomials $\Psi(\cdot)$ is defined recursively as

$$\Psi_{k+1}(y) = 2y\Psi_k(y) - \Psi_{k-1}(y), \quad k = 3, \dots \tag{48}$$

with

$$\Psi_1(y) = 1, \qquad \Psi_2(y) = y. \tag{49}$$

The considered nonlinearity is hence approximated as

$$\Phi(y) = \sum_{i=1}^{M} w_i \Psi_i(y), \tag{50}$$

where w_i is the set of coefficients in the Tchebiceff expansion to be estimated, and M is a predefined constant. The above model for the nonlinearity is linear in the parameters w_i so a least squares estimator can be used for their recursive update.

7. Simulation examples

The RSMI algorithms proposed in this paper have been applied in a series of simulated experiments. The simulations have been performed using Matlab and batch versions of the algorithms from the SMI Toolbox (Haverkamp & Verhaegen, 1997) have been used for initialisation.

7.1. A MIMO linear time-invariant system

We considered an $n = 10$ state space model of the Minimast structure, which is a 20 m long deployable-retractable truss located at NASA Langley research Center. The model considers the first two bending modes, the first torsional mode and the second two bending modes (see Abdelghani & Verhaegen, 1995 for additional details) and has two inputs and two outputs. This is a difficult model to identify, because of closely spaced eigenfrequencies and lightly damped poles.

The model of Abdelghani and Verhaegen (1995) is converted to discrete time using zero-order hold with sampling frequency 30 Hz. The system and measurement noise covariances are given as follows: (a) Perform a noise-free simulation using a white-noise input sequence with standard deviation 10. (b) Determine the noise-free sequences Bu_t and y_t. (c) The standard deviation of the process noise is chosen to be 10% of the standard deviation of Bu_t, and similarly for the measurement noise. Both the process noise and the measurement noise are stationary zero-mean Gaussian random processes. Furthermore, the following user-defined quantities have been applied: $i = 20, \beta = 0.999$.

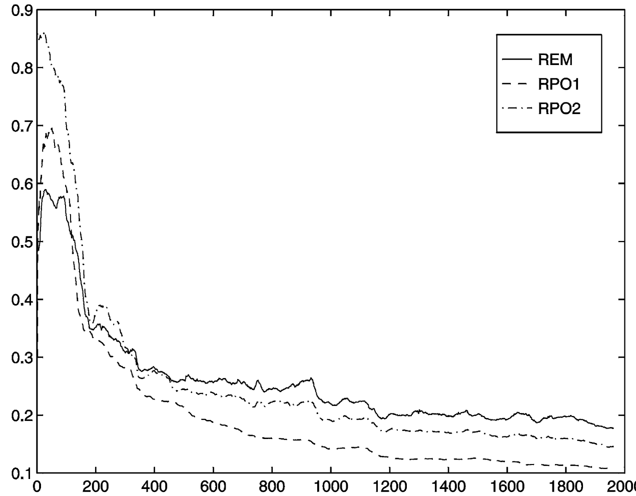

Fig. 1. Minimast model. Mean trajectory of subspace angles in Fig. 1. $\beta = 0.999$.

Table 2
Matlab flop count relative to REM

	REM	RPO_1	RPO_2
MIMO	100%	210%	120%
SISO	100%	201%	161%

In Fig. 1 the subspace angle (Golub & Van Loan, 1989, Section 12.4.3) (in degrees) between $\hat{\Gamma}$ and Γ is shown; from the figure, one sees the accuracy of RPO_1 is the best, and the accuracy of *REM* is the worst, still surprisingly accurate in this difficult identification scenario. We explain the superiority of RPO_1 with its built-in pre-whitening of the instruments. It is conjectured that the accuracy of RPO_2 is improved when a pre-whitening of the instruments is applied. However, all of the studied algorithms are capable of improving the initial accuracy provided by the corresponding batch-version. Finally, in Table 2 the flop count from our specific Matlab implementations are given.

7.2. A SISO time-varying system

As a second example, consider the following SISO system similar to the one studied in Cho and Kailath (1995):

$$A_t = \begin{bmatrix} 0.7 + 0.1^{\frac{\exp(-t/1000)-1}{\exp(-1)-1}} & 0 \\ 0 & 0.2 - 0.15^{\frac{\exp(-t/1000)-1}{\exp(-1)-1}} \end{bmatrix},$$

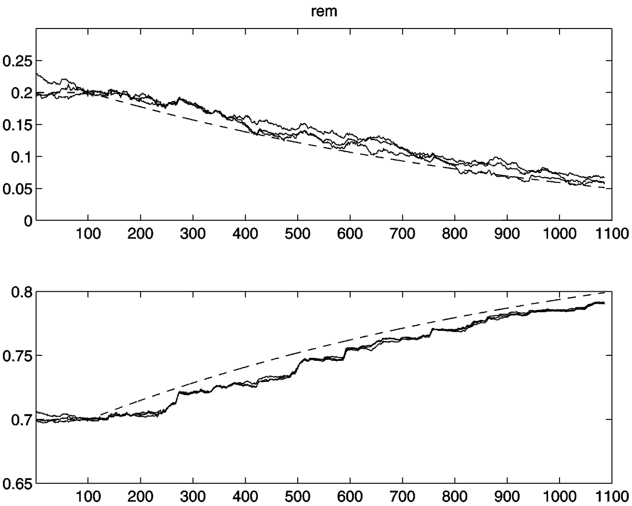

Fig. 2. SISO model. Estimated poles using REM: $\beta = 0.99$.

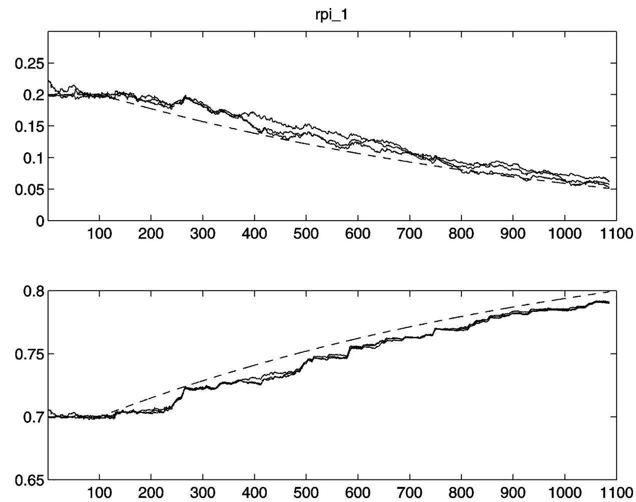

Fig. 3. SISO model. Estimated poles using RPI_1: $\beta = 0.99$.

$$B_t = \begin{bmatrix} 2 \\ 1 \end{bmatrix},$$

$$C_t = [1 \quad 2], \quad D_t = 0.05. \tag{51}$$

The eigenvalues of A drift from $\{0.7,0.2\}$ to $\{0.8,0.05\}$ during the simulation. The same PRBS (pseudo-random binary sequence) was applied in all simulations. Colored measurement noise was added to the noise-free output, generated as

$$e_t = \frac{1}{1 + 0.8q^{-1}}\varepsilon_t, \tag{52}$$

where ε_t is white Gaussian noise. The noise level was adjusted so that the output SNR was 25 dB. In this example we are not interested in estimating the noise dynamics. Therefore, the PI approach is applied, i.e. only delayed inputs are used as IVs.

The eigenvalue trajectories for three different noise realizations are shown in Figs. 2–4. Note, all of the proposed algorithms produce eigenvalue estimates that are essentially equally accurate. In Table 2 the Matlab flop counts for the proposed algorithms are given.

7.3. A Wiener system

Consider the following system:

$$x_1(t+1) = 0.8x_1(t) + u(t), \tag{53}$$

$$x_2(t+1) = 0.1x_1(t) + u(t), \tag{54}$$

$$y(t) = x_1(t) + x_2(t), \tag{55}$$

$$z(t) = \sin(2y(t)) + v(t), \tag{56}$$

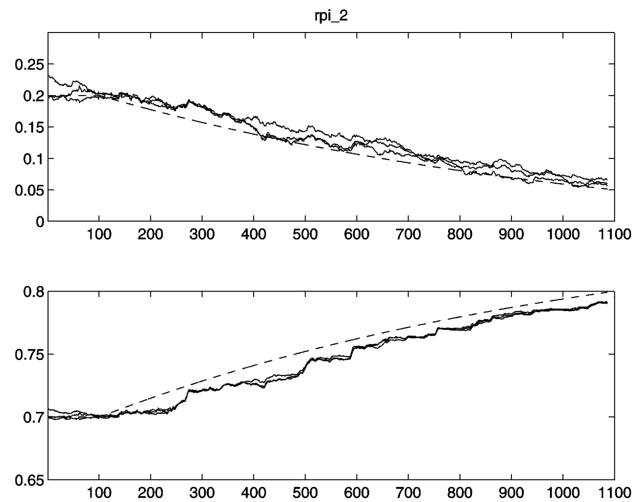

Fig. 4. SISO model. Estimated poles using RPI_2: $\beta = 0.99$.

where v and u both are realizations of white Gaussian noise, of variance 0.01 and 0.1, respectively.

RPI_1 was applied to 500 samples of input-output data, with $i = 10$ and no forgetting. Fifth-order Tchebicheff polynomials were used for the approximation of the nonlinear map.

The results obtained in the estimation of the eigenvalues of the system are given in Fig. 5, while a comparison between the true and the estimated nonlinear functions is given in Fig. 6.

As in the previous case, the initial estimate for the initialization of the recursive algorithm has been obtained by applying the batch version of PI to the first 50 data samples and the function tchebest.m from the SMI Toolbox (Haverkamp & Verhaegen, 1997).

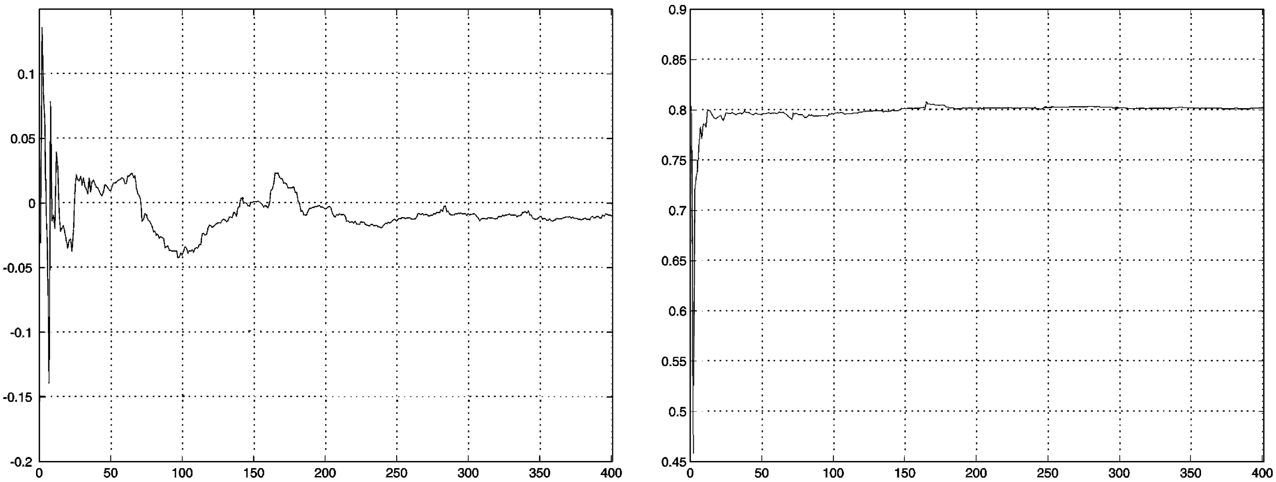

Fig. 5. Wiener system: Estimated poles using RPI_1. True poles at $z = 0$ and $z = 0.8$.

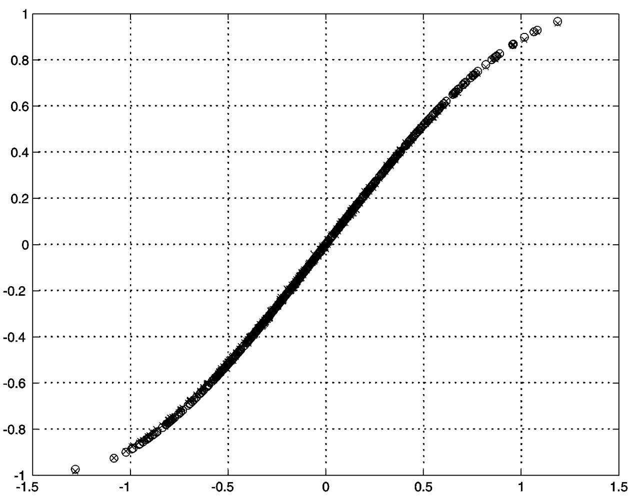

Fig. 6. True (x) and estimated (o) nonlinearity.

8. Concluding remarks

The problem of recursive identification in the framework of subspace methods has been considered and several recursive formulations for the algorithms of the MOESP class have been derived. The proposed algorithms are based on IV ideas and on the use of subspace tracking for the update of the SVD. A recursive algorithm for the identification of nonlinear models of the Wiener type has also been obtained, by exploiting the similarities existing in the subspace formulation between the linear and the Wiener identification problems. Simulation examples illustrate the performance of the proposed algorithms. The outlined approach of making recursive many of the MOESP family of subspace identification schemes is very general.

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