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A Hamiltonian viewpoint in the modeling of switching power converters[1]

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A systematic modeling procedure of a large class of switching power converters using the Hamiltonian approach.

Abstract

In this paper we show how, using the Hamiltonian formalism, we can systematically derive mathematical models that describe the behaviour of a large class of switching power converters, including the "Boost", "Buck", "Buck-Boost", "Čuk" and "Flyback" converters. We follow the approach proposed by van der Schaft and Maschke and extract from the basic (energy-conserving) LC–circuit the remaining elements, i.e., resistors, switches, diodes and transformers, which we treat as external ports. This method naturally yields a Hamiltonian system with two additional conjugated sets of port variables. This procedure, besides being systematic and very general, has the additional advantage of resulting in equations of a form appropriate for simulation and design of the highly succesful passivity–based controllers. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Switching power converters are complex hybrid devices, which are becoming ubiquitous in many control applications. Since they usually operate at very high frequencies, their dynamic behaviour is typically neglected in the controller design. However, the increasing demand on higher bandwidths and the stiffer constraints on harmonic generation makes it necessary to incorporate their dynamics in modern control schemes.

Power converters may be viewed as a set of voltage (or current) sourced subsystems interconnected through switches. The objective of the switches, which actuate as lossless transformers, is to allow the transfer of energy from one subsystem to another. The subsystems consist of passive elements (like inductors, capacitors, and resistances), power sources, and a load where the desired energy is delivered.[2] Because of the presence of discontinuous elements the behaviour of power converters consists of several modes, corresponding to different circuit topologies. A major stumbling block in the standard modeling approaches is how to combine the various topologies to obtain a unique model. This step, which usually requires the identification of ignorable variables, is far from obvious and demands a lot of ingenuity.

The main objective of the paper is to show how, using the generalized Hamiltonian formalism advocated in Maschke et al. (1995), van der Schaft and Maschke

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[2] For more about DC–DC power converters topologies and physical construction the reader is referred to Wood (1981).

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(1993), we can systematically derive mathematical models that describe the behaviour of a large class of switching power converters, including the "Boost", "Buck", "Buck-Boost", "Čuk" and "Flyback" converters. In this Hamiltonian modeling approach the non-energetic elements such as resistors, transformers, diodes and switches are first extracted from the circuit, thereby leaving an energy-conserving LC circuit with ports corresponding to the various extracted elements. This LC circuit with ports can be represented in an intrinsic way as a Hamiltonian system with port variables. The representation of the original circuit is then obtained by terminating the ports of this Hamiltonian system by the extracted non-energetic elements. This decomposition can be naturally carried out for power converters, with the external ports including transformers and resistive components as well as discontinuous elements such as switches and diodes. The main feature of our approach is that for all operating modes we consider the same state variables, the same Hamiltonian, and the same dissipation functions. The variable topology is captured then in some structure matrices that are easily defined by inspection. A final advantage of the proposed modeling method is that the resulting equations are in a form suitable for simulation and control purposes. In particular, the energy dissipation properties of the circuit — which are exploited in passivity–based control (Sanders and Verghese, 1990; van der Schaft, 1996; Ortega et al., 1998) are clearly revealed.

A further complication in circuits containing switches and diodes is that they may, in principle, lead to algebraic constraints on the energy variables. Characterization of such behaviours is still possible invoking the notion of implicit Hamiltonian systems recently introduced in van der Schaft and Maschke (1997). Moreover, most practical power converters are designed in such a way that such algebraic constraints due to switches do not appear, because otherwise there maybe some physical problems with closing or opening switches like sparks, short circuits, etc. Roughly speaking, we can say that in their functioning, the state variables of the power converter do not exhibit jumps and the generalized Hamiltonian formalism of Maschke et al. (1995) and van der Schaft and Maschke (1995) suffices. We assume also that these circuits are composed only of independent elements, i.e., there are no excess of elements producing algebraic constraints.

Control–oriented modeling of switching power converters is typically addressed using small–signal and averaged models (Kassakian et al., 1991). More recently, generalized averaging procedures were used in Sanders et al., (1991) while a Lagrangian viewpoint is proposed in Sira and Delgado (1997), see also Ortega et al., (1998). In relation with the latter there are several clear advantages of the Hamiltonian approach advocated here. First, besides being more elegant and more systematic, the Hamiltonian approach allows us to introduce the characteristics of diodes and transformers into the model, a feature that does not seem feasible in a Lagrangian formulation. Second, as pointed out above, in the proposed formulation we keep the same state variables, and the same energy and dissipation functions for all opearating modes. In the Lagrangian formulation to handle the topology changes some rather artificial parametrizations (in terms of the switching variables) of these functions are introduced. This raises fundamental questions of uniqueness of these parametrizations, which are not totally clear in Sira and Delgado (1997). Furthermore, in a Lagrangian setting an additional step, which is obviated in the Hamiltonian approach, is needed to obtain a reduced set of differential equations that removes the ignorable coordinates. This step requires some algebraic manipulations, which are guided by inspection, and are far from being systematic.

A somehow related research is reported in Söderman (1995) and Strömberg (1994) where the bond graphs approach was extended to mode switching physical systems, i.e., systems constructed by engineers involving continuous as well as discrete behavioural changes. They refer to this extension as switched bond graphs.

2. Switching devices

All the elements will be represented as ports, for which we adopt the convention of identifying currents as flow variables f, and voltages as effort variables e. (No distinction is made at this point between inputs and outputs, but later on in writting the dynamic equations we will find convenient to do so.)

An *ideal switch* can be considered as a lossless element, due to the fact that it can conduct current at zero voltage, while it is closed, and hold a voltage at zero current, while it is open. Both positions can be controlled by an input u, which takes values from the discrete set $\{0,1\}$. This characteristic allows us to connect or disconnect subsystems, which is essential to move energy from one subsystem to another. The behaviour of the ideal switch is described by the (parametrized) graph of Fig. 1, which corresponds to the following relations:

$$\left.\begin{array}{ll}\text{Mode 1:} & u=0 \Rightarrow e_{\mathrm{SW}} \in \mathbb{R},\ f_{\mathrm{SW}}=0 \\ \text{Mode 2:} & u=1 \Rightarrow f_{\mathrm{SW}} \in \mathbb{R},\ e_{\mathrm{SW}}=0\end{array}\right\} e_{\mathrm{SW}} f_{\mathrm{SW}}=0.$$

Note that such an ideal switch can conduct current in both directions.

An *ideal diode* is a particular case of unidirectional uncontrolled switch. Its input–output curve is depicted in Fig. 2, which represents the following conditions:

$$\left.\begin{array}{ll}\text{Mode 1:} & e_{\mathrm{D}} \leq 0,\ f_{\mathrm{D}}=0 \\ \text{Mode 2:} & f_{\mathrm{D}} \geq 0,\ e_{\mathrm{D}}=0\end{array}\right\} e_{\mathrm{D}} f_{\mathrm{D}}=0.$$

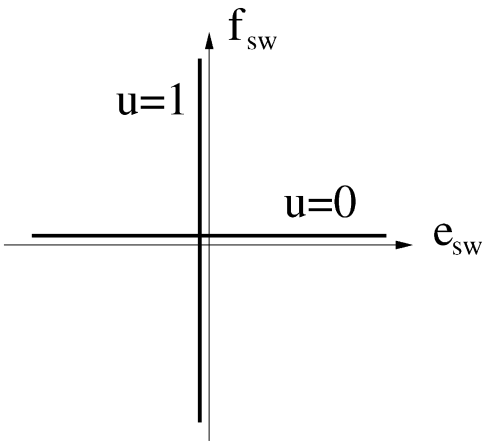


Fig. 1. Ideal switch input–output relation curve.

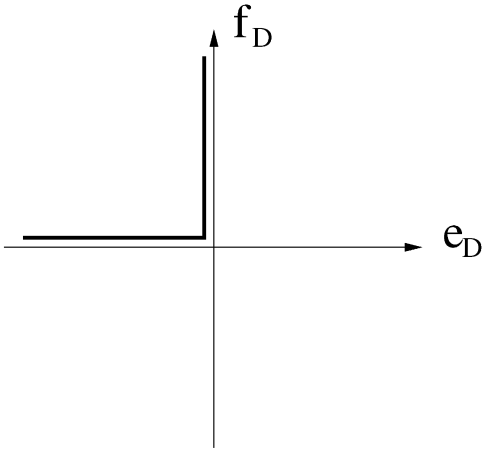


Fig. 2. Ideal diode input–output relation curve.

Inequalities of this type are referred in van der Schaft and Schumacher (1998) as *complementarity conditions* giving rise to a kind of hybrid system called *complementarity systems.*

3. Modeling of continuous circuits

As pointed out in the introduction in our Hamiltonian modeling approach the non-energetic elements such as resistors, transformers, diodes and switches are first extracted from the circuit, thereby leaving an energy-conserving LC circuit with ports corresponding to the various extracted elements. This LC circuit with ports can be represented in an intrinsic way as a Hamiltonian system with port variables. The representation of the original circuit is then obtained by terminating the ports of this Hamiltonian system by the extracted non-energetic elements.

3.1. LC-circuits with external ports

For the LC circuit, it can be shown (Maschke et al., 1995) that an n–element LC circuit with m external ports, and total energy

$$H(x) = \tfrac{1}{2}x^{\mathrm{T}}Qx,$$

where $x \in \Re^n$ is the state vector of the system, consisting of independent (no algebraic constraints due to "excess" elements appear) inductance fluxes ϕ_{L_i} and capacitor charges q_{C_i}, and Q is a diagonal matrix containing the circuit parameters $1/C_i$, $1/L_i$, can always be written in the form

$$\dot{x} = \mathscr{J}Qx + Gu, \tag{1}$$

where $u \in \Re^m$ is the vector of external inputs to the system, $G \in \Re^{n\times m}$ is called the *input matrix* and $\mathscr{J}$ is an $n \times n$ skew-symmetric matrix, which is called the *structure matrix*. The matrices G and $\mathscr{J}$ are determined from Kirchhoff's laws.

It can be shown that the, so–called, "natural" outputs of the generalized system (1) are written in the form

$$y = G^{\mathrm{T}}Qx + Du, \quad y \in \Re^m, \tag{2}$$

where D is a skew-symmetric matrix, called the *throughput matrix*, that appears whenever there are static relations between the ports variables. The skew-symmetric nature of these matrices stems from the fact that the interconnections are all energy conserving. Furthermore, it immediately follows from Eqs. (1) and (2) that along the trajectories of the system,

$$\frac{\mathrm{d}}{\mathrm{d}t}H = u^{\mathrm{T}}y$$

which expresses *energy conservation.* (Note that $u^{\mathrm{T}}y$ is the external power applied to the system.)

4. Modeling of power converters

4.1. Systems with ideal switches

Let us consider the ideal Čuk circuit shown in Fig. 3. This converter provides an output voltage that could be less than or greater to the input voltage. The capacitor C_2 is used to transfer energy from the source to the load. The circuit operation can be divided in two modes, each mode corresponding to the switch position $u \in \{0, 1\}$.

Note that the ideal *two positions switch* presented in Fig. 3 can be modeled as two ideal conjugated switches as in Fig. 4, i.e., both are controlled by the same control signal u but when one of them is closed the other is open. Now using the concept of extracting the switches from

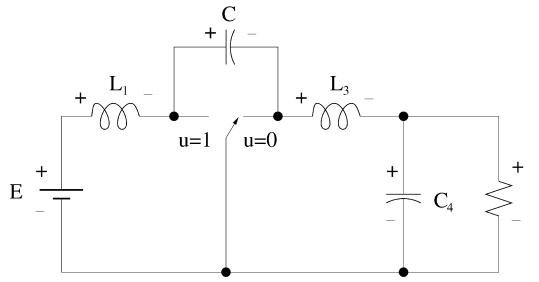


Fig. 3. Ideal Čuk circuit with a two positions switch.

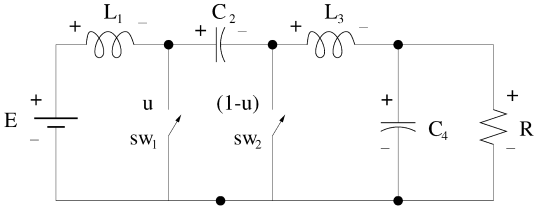


Fig. 4. Ideal Čuk circuit with two single ideal switches.

the system,

$$\begin{bmatrix} \dot{\phi}_{L_1} \\ \dot{q}_{C_2} \\ \dot{\phi}_{L_3} \\ \dot{q}_{C_4} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\phi_{L_1}}{L_1} \\ \dfrac{q_{C_2}}{C_2} \\ \dfrac{\phi_{L_3}}{L_3} \\ \dfrac{q_{C_4}}{C_4} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} E$$

$$+ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} (-f_{\mathrm{SW}1}) + \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} (-e_{\mathrm{SW}2}) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} (-f_{\mathrm{R}}), \quad (3)$$

the corresponding outputs for the source, resistance and switches are given by,

$$\begin{bmatrix} f_{\mathrm{S}} \\ e_{\mathrm{SW}1} \\ f_{\mathrm{SW}2} \\ e_{\mathrm{R}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} Qx$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E \\ -f_{\mathrm{SW}1} \\ -e_{\mathrm{SW}2} \\ -f_{\mathrm{R}} \end{bmatrix} \quad (4)$$

from the constitutive relation for resistances and Eq. (4),

$$f_{\mathrm{R}} = \frac{e_{\mathrm{R}}}{R} = \frac{q_{C_4}}{RC_4} \quad (5)$$

Analyzing now each one of the two positions of the switch, $u \in \{0, 1\}$, we have

Mode 1: $u = 0$

$$\Rightarrow \begin{cases} e_{\mathrm{SW}2} = 0 \\ f_{\mathrm{SW}1} = 0 \end{cases} \Rightarrow \begin{cases} f_{\mathrm{SW}2} = \dfrac{\phi_{L_1}}{L_1} - \dfrac{\phi_{L_3}}{L_3} \\ e_{\mathrm{SW}1} = \dfrac{q_{C_2}}{C_2} \end{cases}$$

Mode 2: $u = 1$

$$\Rightarrow \begin{cases} f_{\mathrm{SW}2} = 0 \\ e_{\mathrm{SW}1} = 0 \end{cases} \Rightarrow \begin{cases} e_{\mathrm{SW}2} = -\dfrac{q_{C_2}}{C_2} \\ f_{\mathrm{SW}1} = \dfrac{\phi_{L_1}}{L_1} - \dfrac{\phi_{L_3}}{L_3}. \end{cases}$$

Note that even if the number of switches is 2, which implies that there would appear 2^2 modes, we only have two modes because they are clearly dependent.

From the derivations above, we can write,

$$e_{\mathrm{SW}2} = -u \frac{q_{C_2}}{C_2},$$

$$f_{\mathrm{SW}1} = u\left(\frac{\phi_{L_1}}{L_1} - \frac{\phi_{L_3}}{L_3}\right).$$

Summarizing, we obtain the dynamical equations

$$\begin{bmatrix} \dot{\phi}_{L_1} \\ \dot{q}_{C_2} \\ \dot{\phi}_{L_3} \\ \dot{q}_{C_4} \end{bmatrix} = \begin{bmatrix} 0 & -(1-u) & 0 & 0 \\ (1-u) & 0 & u & 0 \\ 0 & -u & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\phi_{L_1}}{L_1} \\ \dfrac{q_{C_2}}{C_2} \\ \dfrac{\phi_{L_3}}{L_3} \\ \dfrac{q_{C_4}}{C_4} \end{bmatrix}$$

$$+ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} E + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \left(-\frac{q_{C_4}}{RC_4}\right),$$

$$f_{\mathrm{S}} = \frac{\phi_{L_1}}{L_1}.$$

In the *general case* the dynamical equations of an LCR cicuit with switches will be of the form

$$\dot{x} = \mathcal{J} Qx + G_{\mathrm{S}} u_{\mathrm{S}} + G_{\mathrm{R}} u_{\mathrm{R}} + G_{\mathrm{SW}} u_{\mathrm{SW}}, \quad (6)$$

$$\begin{bmatrix} y_{\mathrm{R}} \\ y_{\mathrm{SW}} \\ y_{\mathrm{S}} \end{bmatrix} = \begin{bmatrix} G_{\mathrm{R}}^{\mathrm{T}} \\ G_{\mathrm{SW}}^{\mathrm{T}} \\ G_{\mathrm{S}}^{\mathrm{T}} \end{bmatrix} Qx + D \begin{bmatrix} u_{\mathrm{R}} \\ u_{\mathrm{SW}} \\ u_{\mathrm{S}} \end{bmatrix}, \quad (7)$$

where u_{SW} is the input vector of the extracted switches (current or voltage in the external port), and y_{SW} is the corresponding outputs vector (voltage or current in the external port, respectively), G_{SW} is the associated input matrix, and D is the throughput skew-symmetric matrix which describes the interaction between switches, resistances and sources.

If the ports assigned to the resistances are independent from the ports of the switches, then the outputs in the model can be written as

$$y_R = G_R^T Q x + D_R u_R, \tag{8}$$

$$y_{SW} = G_{SW}^T Q x + D_{SW} u_{SW}, \tag{9}$$

$$y_S = G_S^T Q x + D_S u_S, \tag{10}$$

where D_{SW} is a throughput skew-symmetric matrix that shows the interaction between the switches, and D_R the throughput skew-symmetric matrix describing the interaction between the resistances. Note that in this case there is no interaction between resistances and switches.

In order to complete the description of the system we should include the ideal switch characteristics showed in the input–output curve in Fig. 1. Now, the output equations and the input–output curves can be written in a single equation parametrized on the switch position signal u. This is always possible because the matrix D_{SW} in Eq. (9) is invertible, which stems from the assumption that the switches do not create algebraic constraints. After substitution of these expressions into the dynamic equations we obtain a more compact model for the circuit which will be parametrized by the switch position signal u.

4.2. Systems with switches and diodes

The ideal switch considered above is conducting in both directions, in real converters this is not the case. To capture this phenomenon we must include diodes in the circuit. This allows us to describe behaviours like the commonly encountered *discontinuous mode*.

We illustrate the procedure with the boost circuit with clamping diode of Fig. 5. This circuit is employed to obtain an output voltage greater than the voltage in the input source, thus it is also commonly called *step-up* converter. In this converter the inductor is the element used to transfer energy from the input source to the output resistance load.

As before, we obtain the model by extracting both, diodes and switches, from the system and treating them as external ports. This procedure leads to

$$\begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} E + \begin{bmatrix} 1 \\ 0 \end{bmatrix} (-f_R) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -f_{SW} \\ -e_D \end{bmatrix} \tag{11}$$

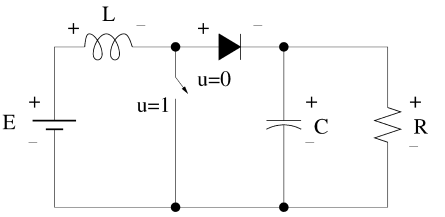


Fig. 5. Boost circuit with clamping diode.

with the corresponding outputs

$$\begin{bmatrix} f_S \\ e_R \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} Q x = \begin{bmatrix} \frac{\phi_L}{L} \\ \frac{q_C}{C} \end{bmatrix}, \tag{12}$$

$$\begin{bmatrix} e_{SW} \\ f_D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} f_{SW} \\ e_D \end{bmatrix}. \tag{13}$$

We have for the resistance,

$$f_R = \frac{e_R}{R} = \frac{q_C}{RC}.$$

Analyzing now each one of the two positions of the switch, $u \in \{0, 1\}$, we have

$$\text{Case 1:} \quad u = 0 \Rightarrow f_{SW} = 0 \Rightarrow \begin{cases} e_D = e_{SW} - \frac{q_C}{C} \\ f_D = \frac{\phi_L}{L} \end{cases}$$

$$\text{Case 2:} \quad u = 1 \Rightarrow e_{SW} = 0 \Rightarrow \begin{cases} e_D = -\frac{q_C}{C} \\ f_{SW} = \frac{\phi_L}{L} - f_D. \end{cases}$$

Substituting this values in the last term of Eq. (11) we obtain for each position,

$$\text{Case 1:} \quad \begin{bmatrix} -f_{SW} \\ -e_D \end{bmatrix} = \begin{bmatrix} 0 \\ -e_D \end{bmatrix},$$

$$\text{Case 2:} \quad \begin{bmatrix} -f_{SW} \\ -e_D \end{bmatrix} = \begin{bmatrix} -\frac{\phi_L}{L} + f_D \\ \frac{q_C}{C} \end{bmatrix}.$$

We can express both cases via the parametrized expression

$$\begin{bmatrix} -f_{SW} \\ -e_D \end{bmatrix} = \begin{bmatrix} -u\frac{\phi_L}{L} + u f_D \\ u\frac{q_C}{C} - (1-u)e_D \end{bmatrix}. \tag{14}$$

Substituting the above result in Eq. (11) yields

$$\begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} 0 & (1-u) \\ -(1-u) & 0 \end{bmatrix} \begin{bmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} E + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(-\frac{q_C}{RC} \right) + \begin{bmatrix} u f_D \\ -(1-u)e_D \end{bmatrix}. \tag{15}$$

For each case analyzed before (each switch position), there exists two modes, depending if the diode is conducting or not, so we can distinguish four modes of operation,

Mode 1: $u = 0,\ e_D = 0$

$$\Rightarrow f_D = \frac{\phi_L}{L} \qquad \Rightarrow \begin{cases} \dot{q}_C = \frac{\phi_L}{L} - \frac{q_C}{RC}, \\ \dot{\phi}_L = -\frac{q_C}{C} + E. \end{cases}$$

Mode 2: $u = 1,\ f_D = 0$

$$\Rightarrow e_D = -\frac{q_C}{C} \qquad \Rightarrow \begin{cases} \dot{q}_C = -\frac{q_C}{RC}, \\ \dot{\phi}_L = E. \end{cases}$$

Mode 3: $u = 0,\ f_D = \frac{\phi_L}{L} = 0$

$$\Rightarrow e_D = E - \frac{q_C}{C} \qquad \Rightarrow \begin{cases} \dot{q}_C = -\frac{q_C}{RC}, \\ \dot{\phi}_L = 0. \end{cases}$$

Mode 4: $u = 1,\ e_D = \frac{q_C}{C} = 0$

$$\Rightarrow f_D = 0 \qquad \Rightarrow \begin{cases} \dot{q}_C = 0, \\ \dot{\phi}_L = E. \end{cases}$$

In order to know the mode where the system is located, we should observe first the position of u, and then we look at the states of the system. For $u = 0$ we have two modes, if the signal $\phi_L/L > 0$ then the system is in mode 1, and if $\phi_L/L = 0$ then it's in mode 3. For position $u = 1$, if $q_C/C > 0$ then the system is in mode 2 but if $q_C/C = 0$ then it's working in mode 4.

Once again, in the *general case* the equations take the form

$$\dot{x} = \mathcal{J}(u)Qx + G_S(u)u_S + G_R(u)u_R + G_D(u)u_D, \tag{16}$$

with respective outputs defined as

$$\begin{bmatrix} y_R \\ y_D \\ y_S \end{bmatrix} = \begin{bmatrix} G_R^T(u) \\ G_D^T(u) \\ G_S^T(u) \end{bmatrix} Qx + D(u) \begin{bmatrix} u_R \\ u_D \\ u_S \end{bmatrix}. \tag{17}$$

4.3. Systems with switches, diodes and transformers

To wrap up this section we consider the circuit in Fig. 6 commonly referred in the literature as *Flyback*. Note that this circuit is very similar to that of the Buck-Boost circuit, with the characteristic that in this case the output is electrically isolated from the source. This characteristic allows the output to be positive, in contrast to that of the normal Buck-Boost.

To show the flexibility of the method let us analyze now the system for each position of the switch, and extract from the model only the transformer terminals and the diode.

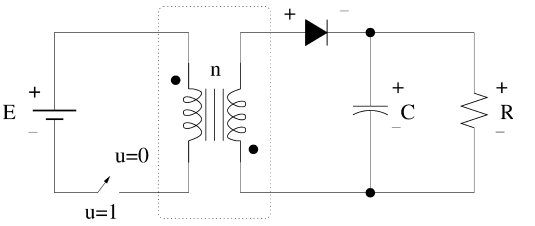


Fig. 6. Flyback circuit.

Case $u = 1$. The equations for this case are

$$\begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e_S + \begin{bmatrix} 1 \\ 0 \end{bmatrix} (-f_R) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} f_D \tag{18}$$

with the corresponding outputs

$$e_R = [1 \quad 0] \begin{bmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{bmatrix} = \frac{q_C}{C}, \tag{19}$$

$$\begin{bmatrix} e_D \\ f_S \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{n} \\ \frac{1}{n} & 0 \end{bmatrix} \begin{bmatrix} f_D \\ e_S \end{bmatrix}, \tag{20}$$

where $e_S = E$ is the voltage source and f_S the corresponding current. For the resistance we have

$$f_R = \frac{e_R}{R} = \frac{q_C}{RC}.$$

As we can see

$$e_D = -\frac{q_C}{C} - E$$

so, if we assume that the converter is in normal operation, i.e., $q_C/C \geq 0$, then $e_D < 0$. This, in turn, implies that $f_D = 0$, and we have only one mode of operation. For this case then, the model results in

Mode 1:

$$\begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} E + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(-\frac{q_C}{RC} \right). \tag{21}$$

Case $u = 0$. The equations are

$$\begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} (-f_R) + \begin{bmatrix} f_D \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ e_{T1} \end{bmatrix}, \tag{22}$$

with corresponding outputs

$$e_R = [1 \quad 0]\, Qx = \frac{q_C}{C}, \tag{23}$$

$$e_D = [-1 \quad 0] Qx + e_{T2} = -\frac{q_C}{C} + e_{T2}, \tag{24}$$

$$f_{T1} = [0 \quad 1] Qx = \frac{\phi_L}{L}. \tag{25}$$

From the relations for the ideal transformer, we know that

$$e_{T1} = -ne_{T2}, \tag{26}$$

$$f_{T2} = nf_{T1}, \tag{27}$$

with this and the fact that

$$f_D = f_{T2},$$

we obtain

$$e_{T1} = -ne_D - n\frac{q_C}{C}, \tag{28}$$

$$f_D = n\frac{\phi_L}{L} \tag{29}$$

and for the resistance

$$f_R = \frac{q_C}{RC}.$$

Substituting these expressions in the model we obtain

$$\begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(-\frac{q_C}{RC} \right) + \begin{bmatrix} 0 \\ -ne_D \end{bmatrix}. \tag{30}$$

As we see, for $u = 0$, there exist two different modes, namely

$$\text{Mode 2:} \quad e_D = 0 \Rightarrow f_D = n\frac{\phi_L}{L} \Rightarrow \begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} 0 & n \\ -n & 0 \end{bmatrix} \begin{bmatrix} \frac{q_C}{C} \\ \frac{\phi_L}{L} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(-\frac{q_C}{RC} \right).$$

$$\text{Mode 3:} \quad f_D = 0 \Rightarrow e_D = -\frac{q_C}{C} \Rightarrow \begin{bmatrix} \dot{q}_C \\ \dot{\phi}_L \end{bmatrix} = \begin{bmatrix} -\frac{q_C}{RC} \\ 0 \end{bmatrix}.$$

In order to decide in which mode the system is located, we should observe the position of u, if it is $u = 0$ then we are in mode 1, but if $u = 1$ then we should observe the state ϕ_L/L, if it is greater than zero, we are in mode 2, but if is exactly zero, then we are in mode 3, which corresponds to the commonly called discontinuous mode in this circuit.

5. Conclusions

In this article we have shown how, using the generalized Hamiltonian formalism of Maschke et al. (1995), and van der Schaft and Maschke (1995), we can systematically derive mathematical models that describe the behaviour of a large class of switching power converters. The methodology allows us to include often encountered devices like diodes and transformers, hence allowing for a more realistic description. The main feature of our approach is that for all operating modes we consider the same state variables, the same Hamiltonian, and the same dissipation functions, hence the variable topology—under which power converters usually operate—is captured in some structure matrices that are easily defined by inspection.

The resulting system has mixed continuous/discrete dynamics, sometimes refered in the literature as hybrid systems. This is an emerging area still at the stage of formalism definition. Power converters are, of course, a (very) particular case of hybrid systems. Nevertheless, the fact that they can exhibit very complex dynamic behaviour, together with their unquestionable practical importance, makes them a suitable paradigm to test and validate these studies. On the other hand, the possibility of incorporating physical intuition in the formulation of a theoretical formalism can hardly be overestimated. In summary, it is our belief that the theoretical community working in hybrid systems and the power electronics practicioners can only benefit from this cross-fertilization.

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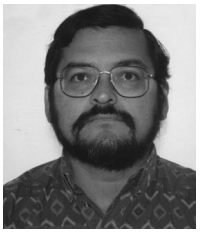
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