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TITLE: **TOWARDS A FORMAL TAXONOMY OF HYBRID UNCERTAINTY REPRESENTATIONS**

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Towards a Formal Taxonomy of Hybrid Uncertainty Representations*

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Recent years have seen a proliferation of methods in addition to probability theory to represent information and uncertainty, including fuzzy sets and systems [6], fuzzy measures [10], rough sets [7], random sets [4] (Dempster-Shafer bodies of evidence [2]), possibility distributions[1], imprecise probabilities [9], etc. We can identify these fields collectively as General Information Theory (GIT) [5].

The components of GIT represent information according to different axiomatic bases, and are thus capable of capturing different semantic aspects of uncertainty. Traditionally, these semantic criteria include such categories as fuzziness, vagueness, nonspecificity, conflict, and randomness.

So it is clear that there is a pressing need for the GIT community to synthesize these methods, searching out larger formal frameworks within which to place these various components with respect to each other. Ideally, syntactic (mathematical) generalization can both aid and be aided by the semantic analysis available in terms of the conceptual categories outlined above.

In this paper we present some preliminary ideas about how to formally relate various uncertainty representations together in a taxonomic lattice, capturing both syntactic and semantic generalization. Some partial and provisional results are shown in Fig. 1.

Assume a simple finite universe of discourse $\Omega = \{a, b, c\}$. We want to describe a situation in which we ask a question of the sort "what is the value of a variable x which takes values in Ω?". When there is no uncertainty, we have a single alternative, say $x = a$. In logical terms, we would say that the proposition $\mathbf{p}$: "the value of x is a" is TRUE.

Our approach begins with two primitive concepts which can change our knowledge of x, each of which represents a different form of uncertainty:

Nonspecificity: We can have more than one alternative, creating a homogeneous collection of possible entities. Introducing pure nonspecificity to the situation described above implies creating more propositions (more alternative), while maintaining the truth-value representation. We now have a set P of propositions $\mathbf{p}$, one for each of some group of the $\omega \in \Omega$, for example a and b. The value of each proposition is TRUE, and so the situation is represented as the crisp subset $\{a, b\}$.

Fuzziness: We can have more than one possible truth-value, allowing a graduated weighting of the entities. Introducing pure fuzziness to the situation described above implies that x is a, but to a degree, maintaining the proposition but changing the truth-value representation. $\mathbf{p}$ is still "the value of x is a", but it's truth is now $\mu(\mathbf{p}) \in [0, 1]$, for example .3, and so the situation is represented as the "fuzzy element" $\langle a, .3 \rangle$.

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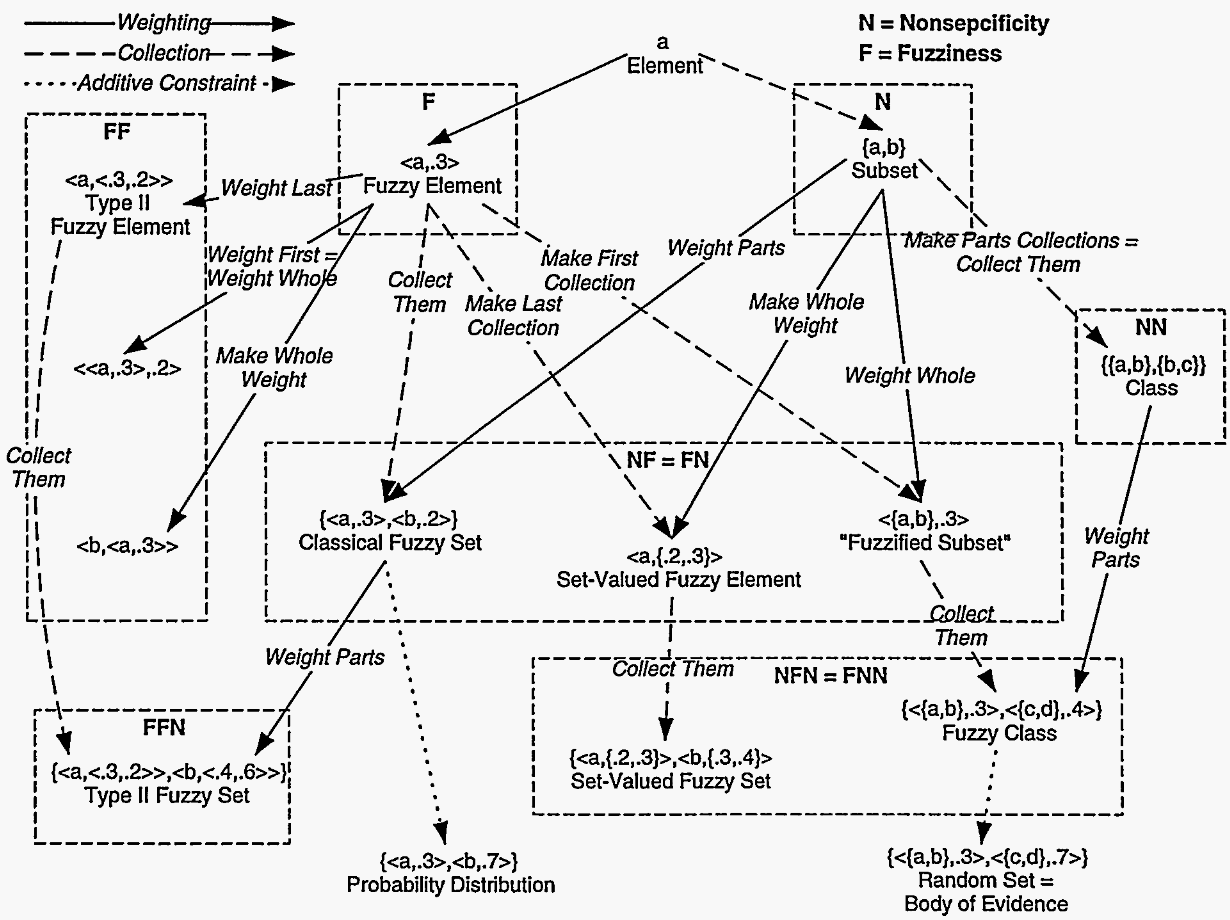

Figure 1: Partial lattice of uncertainty representations.

These forms can then interact to create more complex representations, where nonspecificity, fuzziness, and other constraints discussed below, are all present simultaneously in different ways.

Our overall method consists of distinct logical stages:

1. Our first motivation is to construct all the possible hybrid forms for uncertainty representation on a purely formal, or syntactic, basis, beginning with the most general expression of the concepts of nonspecificity and fuzziness.

   This step itself proceeds in two ways:

   - From first principles, considering all possible ways to both weight and collect entities and their parts. For example, the parts of a weighted entity can be made collections, or the elements of a collection can be given weights.
   - By applying transformations of weighting and collecting consistently to all entities seen. For example, weighting the parts of a collection $\{a, b\}$, yielding a classical fuzzy set, suggests weighting the parts of a weighted element separately.

   The value of this approach can be seen in the feedback between these two methods: both the transformations from first principles and the transformations applied consistently produce the same results. In this way equivalent sets of transformations are identified, for example making a whole collection a weight is equivalent to making the weight of a weighted element a collection.

   So using just these two concepts of fuzziness and nonspecificity, we can describe a variety of foundational representations, including a variety of simple sets, weighted elements, and classical fuzzy sets. After another iteration, with another level of fuzziness or nonspecificity added in, more complex forms such as type II fuzzy sets, set-valued fuzzy sets, and fuzzy classes, appear.

2. The next step is to apply meaningful constraints on the weightings, including:

   - Additive normalization to achieve probability distributions and random sets (Dempster-Shafer bodies of evidence);
   - Maximal normalization to possibility distributions and (crisp) interval representations such as interval-valued fuzzy elements and sets.

   These additional mathematical constraints allow greater semantic expressibility. Specifically, the additivity of probability allows the expression of conflict, or randomness. And the maxitivity of possibility allows the expression of ordinal concepts surrounding distances and capacities.

3. The next step is to consider whether all these structures are truly distinct, and if not to identify equivalent structures. For example, making the parts of a collection a collection is equivalent to combining separate collections into a new collection.

4. The final step is to consider the meaningfullness and usefullness of the remaining structures, and eliminate irrelavent forms. In Fig. 1, only meaningful forms have been given labels.

A longer-term goal of this process is to arrive at the most complex forms of uncertainty representation, where fuzziness, nonspecificity, and conflict are all present in complex, intermingled ways. These include type-2 and level-2 fuzzy sets, evidence sets [8], random intervals [3], etc.