Analysis of Third-Party Warehousing Contracts with Commitments

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Abstract

This paper considers multi-period warehousing contracts under random space demand. A typical contract is specified by a starting space commitment plus a certain number of times at which the commitment can be further modified. Three forms of contracts are analyzed: (1) There is a restriction on the range of commitment changes and the schedule for the changes is pre-set by the warehouser; (2) the same as form 1 but there is no restriction on the range; (3) the same as form 2 but the schedule for the changes is chosen by the user. We explore properties and algorithms for the three problems from the user's perspective. Solutions of simple form are obtained for the first two models and an efficient dynamic programming procedure is proposed for the last. A numerical comparison of the total expected leasing costs suggests that under certain demand patterns, contract forms 2 and 3 could be cost effective.

Keywords: Stochastic Process, Dynamic Programming, Warehousing Contracts, Third Party Logistics.

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1 Introduction

Since the late 1980s logistics outsourcing has been recognized as a strategic weapon that can provide competitive advantages and help curtail distribution costs. The sales figures of major third-party-logistics (3PL) service providers in the US reflected this trend: Annual growth rates of 20% to 40% are common. As part of the 3PL service, third party warehousing (3PW) also enjoys a parallel growth and has advanced from being a form of reactive tactic to becoming an integral source of competitive advantages. This growth is augmented by the trend towards the expansion of stocking locations in order to have products positioned closer to end customers because of new services such as continuous replenishment, just-in-time deliveries, vendor-managed inventories and customization postponement (Copacino [2]). It has been estimated that the warehousing cost currently represents the greatest share of total Asia-Pacific logistics costs, about 39 percent, and logistics itself accounts for up to 25 percent of total operating costs in the region (see McAdam [6]).

The 3PW process generally involves a user entering a contract with the 3PW provider for specific services at agreed prices over a fixed contract duration, normally covering multiple periods. The long-term commitment in a contract creates many issues that have significant impacts on management practices and research. One of the important decisions is how large space the user should commit to. On one hand, the 3PW provider would like the user to commit the same size of space over the contract duration so as to ensure stable sales. To entice such a response from the user, the warehouser will provide disincentives so that there will be a premium charge for any requirement of space above the committed size. On the other hand, due to the uncertainty on storage space requirement, it is in the best interest of the user that it makes no firm commitment before the demand realizes in each period. The common ground stands therefore in the middle: During the multiple periods of contract duration the user firm may obligate to certain commitments of space sizes but enjoys some degree of flexibility in that adjustments may be exercised. In this paper we analyze one, perhaps the most important, type of the 3PW contract — warehousing lease with size commitments and certain flexibilities.

The motivation for considering commitments with certain degree of flexibility arises from our experience with an actual contract of this type used by a division of a multi-national corporation (MNC) in Singapore. Currently, the MNC contracts its warehousing operations to a local 3PW. The 3PW bills to the MNC with respect to occupancy charge according to a contract with commitments. The contract can be described as follows: Fairly ahead of the beginning of the year the MNC firm makes an annual commitment (*base commitment*) but can make up to a certain percentage of up-/down-ward adjustment on the quarterly basis. For instance, if the base commitment is $10,000$ pallets and an adjustment cap is 25%, then each of the 4 quarterly commitments can vary between $7,500$ and $12,500$ pallets. The charge on space is essentially according to the peak usage: The daily usage of space is tracked by the end-of-day net space taken by the company and the monthly charge is then determined by the highest daily usage. If the highest usage is greater than the commitment, the extra space is billed at a premium rate, and if it is at or below the commitment, the company is only charged for a basic fee.

The MNC attempts to remove the basic commitment as well as to have flexible commencements for individual commitments. They are concerned with the inflexibility of the present contract because the quarterly adjustment does not necessarily reflect the pattern of its warehousing demand. This leads to the two objectives of this research: First, we investigate the impact of the base commitment in the current contract; i.e., we evaluate the costs of the contracts with/without a base commitment. Second, we investigate a new contract which allows the user to choose the optimal timings to adjust the previous commitment and see the saving potential from such an additional flexibility. These flexibilities — removing the base commitment and relaxing the fixed schedule for individual commitments — may be pursued, however, at higher cost rates. Evaluation procedures are therefore essential for comparing the alternatives.

The literature in warehousing operations is vast. Cormier and Gunn [3] provide a comprehensive review of analytical research on warehouse models from the warehouser's perspective. The work by Lowe, Francis and Reinhardt [5] is probably most relevant to our problem in the warehouse management literature. In their model changes in storage capacity are allowed from period to period, e.g., by leasing additional storage space, procuring additional storage racks or closing a section of the warehouse. The change from one size in one period to a different size in the next period incurs either (linear) "expansion" or "contraction" costs. The current research is also motivated by the work of Hum and Ngoh [4]. Hum and Ngoh provide a first definition of the contract problem and provide insights into the nature of the problem using numerical examples. However, only deterministic demand is considered in [4].

The reader familiar with inventory literature may relate the problems considered here to inventory models. While warehouse space resembles to perishable inventory and committing on a space size is similar to committing on an order-up-to inventory level, the contract for warehouse space is like take-or-pay. In addition, under the inventory management context, it does not make sense for a buyer to commit on any order-up-to level. Finally, the optimal timings for starting individual commitments are not addressed in the literature (For non-perishable inventory models with order quantity commitments and flexibilities, the reader might refer to Tray [8] and Bassok and Anupindi [1]).

The rest of the paper is organized as follows. In the first part of the next section we define the problem and formulate the model. We then discuss the evaluation procedure for each of the three models. We report our computational experience and observations in Section 3 and conclude the paper in Section 4 with a brief summary and some direction for possible future research.

2 Formulation of Models

We first present assumptions and basic notation, then define the cost structure which underlies the interested models.

Demand for space in period $t = 1, 2, ..., N$ is a non-negatively valued random variable, denoted by ξ_t, with a known probability distribution $\Phi_t(\xi_t)$. We require ξ_t to be independent but not necessarily stationary over time. Though it is tempting to include correlation between demands of different periods, here we consider only the independent case. This simplification makes it easier to gain insight into the impact of the changes in the contract since the independence assumption makes the problem computationally more tractable.

The cost incurred in period t consists of the following components:

- A fixed charge cS is paid if the commitment size for the period is S and the cost of per-unit committed space is c.
- The variable – "overflow" – leasing cost is $p\max(0, \xi_t - S)$, where p is the premium charge per-unit space and ξ_t is the demand. Let

$$G_t(S) = E[\max(0, \xi_t - S)] = \int_S^\infty (\xi_t - S)d\Phi_t(\xi_t). \tag{1}$$

  Then $pG_t(S)$ is the expected variable leasing cost when the size of S is committed in period t. Note that $G_t'(S) = \Phi_t(S) - 1$, which shows that $G_t(S)$ is convex in S.

Let m denote the number of adjustment opportunities and k_i be the period at the beginning of which the ith adjustment is to be made: $2 \leq k_1 < k_2 < \cdots < k_m \leq N$. (Hence there are total of $m+1$ commitments.) Associated with k_i is the commitment size S_i. For convenience of notations, define $k_0 = 1$, $k_{m+1} = N+1$, and two vectors $K = (k_1, \cdots, k_m)$ and $S = (S_0, S_1, \cdots, S_m)$. Then the total expected costs is:

$$f(K, S) = c\sum_{i=0}^{m}(k_{i+1} - k_i)S_i + p\sum_{i=0}^{m}\sum_{t=k_i}^{k_{i+1}-1} G_t(S_i). \tag{2}$$

2.1 Model FSB - Fixed Schedule for Commitments around a Base Level

Suppose that the contract requires a base commitment Q, a decision variable, and each commitment should be set within $[(1-\alpha)Q, (1+\beta)Q]$, where $0 \leq \alpha \leq 1, \beta \geq 0$. The schedule for adjustment, K, is set by the warehouser. That is, the user wants to determine Q for minimum cost given α, β and vector K. In each commitment duration, let $C_i(S_i) = c(k_{i+1} - k_i)S_i + p\sum_{t=k_i}^{k_{i+1}-1} G_t(S_i)$. Note that $C_i(S_i)$ is convex in S_i. Then the minimum-cost contract problem can be formulated as

$$\begin{array}{llll}(\mathbf{FSB}) & \min\limits_{Q,S} & \sum\limits_{i=0}^{m} C_i(S_i) & \\ & \text{s.t.} & (1-\alpha)Q \le S_i \le (1+\beta)Q. & \end{array} \tag{3}$$

A remark is in order. The user can choose optimal base size Q^* and commitment sizes S_i^* although the latter are not necessarily documented in the contract. The commitment sizes are in fact set "dynamically" in practice. Since demands for space are serially independent, knowing demands in periods $1, 2, \cdots, i-1$, does not improve choosing S_i^*, which means all S_i^* can be determined "up-front", suggesting that **FSB** is a static multiple period problem.

Proposition 1 *Problem* **FSB** *is equivalent to*

$$(\mathbf{FSB}_0) \qquad \min_Q \pi(Q), \tag{4}$$

where

$$\pi(Q) = \sum_{i=0}^{m} \min_{(1-\alpha)Q \le S_i \le (1+\beta)Q} C_i(S_i).$$

Furthermore, $\pi(Q)$ *is convex in* Q*.*

Proof. The equivalence between **FSB** and $\mathbf{FSB}_0$ is evident because $\sum_{i=0}^{m} C_i(S_i)$ is separable. It is well known that if $C_i(S_i)$ is convex in S_i, then $\min_{(1-\alpha)Q \le S_i \le (1+\beta)Q} C_i(S_i)$ is convex in Q. As a result, $\pi(Q)$ is convex in Q. □

Now we show how to obtain $C_i^*(Q) = \min_{(1-\alpha)Q \le S_i \le (1+\beta)Q} C_i(S_i)$. Let us assume that all values of $C_i(y)$ are available and y_i is the minimizer of $C_i(y)$ without the constraint $(1-\alpha)Q \le y \le (1+\beta)Q$. If Q is between

$$\left[\frac{y_i}{1+\beta}, \frac{y_i}{1-\alpha}\right],$$

then $C_i^*(Q){=}C_i(y_i)$; if Q is below $\frac{y_i}{1+\beta}$, then $C_i^*(Q){=}C_i(Q+\beta Q)$; and if Q is above $\frac{y_i}{1-\alpha}$, then $C_i^*(Q){=}C_i(Q-\alpha Q)$. Therefore once we have all values of $C_i(y_i)$, we automatically have all the values of $C_i^*(Q)$. Moreover, the computation of y_i is not difficult. It reduces to the solution of equation

$$\sum_{t=k_i}^{k_{i+1}-1} \Phi_t(y_i) = \frac{p-c}{p}(k_{i+1} - k_i), \tag{5}$$

for $i = 0, ..., m$, which is obtained by taking derivative of (2) and setting it to zero.

Equation (5) is quite easy to solve. Thus, a simple linear search procedure for $\pi(Q)$ will suffice to find the optimal commitment size Q^*.

2.2 Model FSNB – Fixed Schedule without Base Commitment

Suppose that the schedule of adjustments is predetermined as in **FSB** but there is no restriction on the range of changes, i.e., no base commitment. Then the problem can be formulated as

$$(\mathbf{FSNB}) \qquad \min_S \qquad \sum_{i=0}^{m} C_i(S_i) = \sum_{i=0}^{m} \min_{S_i} C_i(S_i) \tag{6}$$

for $i = 0, ..., m$. Hence we minimize $C_i(S_i)$ separately by letting $S_i = y_i$, where y_i is the solution to (5), $i = 0, ..., m$.

2.3 Model $\mathbf{P_m}$ - Fixed Number of Commitments with Flexible Schedule

The last model is the most flexible, denoted by $\mathbf{P_m}$:

$$(\mathbf{P_m}) \qquad \begin{cases} \min & f(K,S) = c\sum_{i=0}^{m}(k_{i+1} - k_i)S_i + p\sum_{i=0}^{m}\sum_{t=k_i}^{k_{i+1}-1} G_t(S_i) \\ \text{s.t.} & \\ & k_{i-1} < k_i, \quad i = 1, 2, \cdots, m. \end{cases}$$

The difference between **FSNB** and $\mathbf{P_m}$ is that, in addition to S, the latter further allows the user to optimally choose the schedule K for all commitments.

2.3.1 Nature of the General Problem

Problem $\mathbf{P_m}$ is hard to solve since there are potentially total of $\binom{N-1}{m}$ different schedules in designing the contract. The solution procedure now includes a search for the optimal schedule. We had hoped to find some monotonicity property so that the search can proceed in a certain pattern rather than going through an enumeration of all choices. However, it turns out that such a property may not exist, making the solution of $\mathbf{P_m}$ difficult.

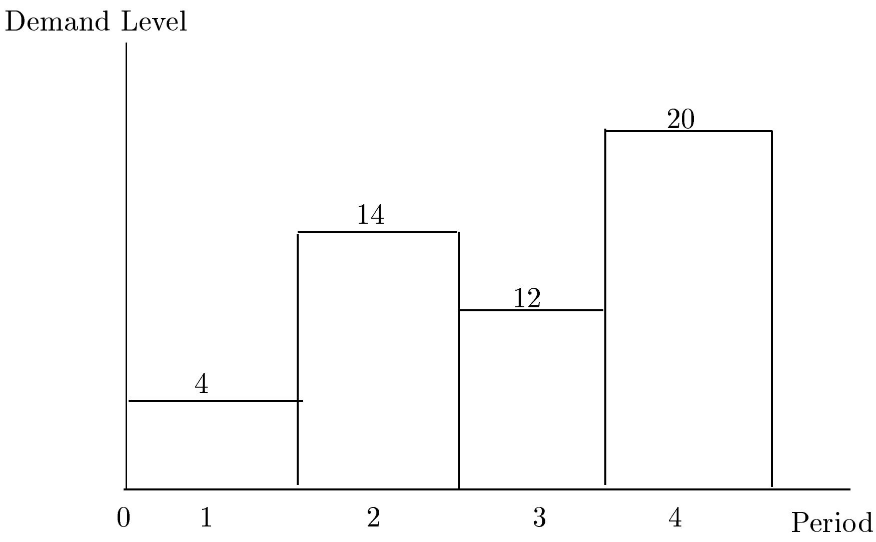

Figure 1. An Example

Consider an example of $\mathbf{P_1}$ with four periods and deterministic demands : Demands for warehouse space in the four periods are 4, 14, 12 and 20, respectively (see Figure 1). Suppose $c = 1$ and p is arbitrarily large. There is one adjustment option provided for between period 2 and period 4, and there is no restriction on the two commitment sizes. If the adjustment period is chosen as period 2 ($L = 1$, $k_L = 2$), then $S_0 = 4$ and $S_1 = 20$ (since $p >> c$) is optimal and the total cost for such a solution is $4 + 3 * 20 = 64$. Now consider $k_L = 3$, then $S_0 = 14$ and $S_1 = 20$ is optimal and the resulting total cost is $2 * 14 + 2 * 20 = 68$, greater than the cost in the former case. Finally, let $k_L = 4$. Then $S_0 = 14$ and $S_1 = 20$ is optimal; the total cost equals $3 * 14 + 20 = 62$, which is the minimum cost. To summarize, the costs associated with the adjustments in periods 2, 3 and 4 are 64, 68 and 62, respectively. We see a non-monotonic pattern in terms of costs. This suggests the same difficulty for the general case (i.e., problem $\mathbf{P_m}$) where multiple adjustment options are provided.

We therefore resort to a dynamic programming (DP) solution procedure that is the subject of the next subsection. A nice property of the problem, which makes the DP approach a reasonable choice, is that optimal commitment sizes are uniformly bounded as described by the following proposition.

Proposition 2 *Let y_t be the minimizer of $cy + pG_t(y)$, $t = 1, 2, \cdots, N$, $y^{\min} = \min\{y_1, y_2, \cdots, y_N\}$, and $y^{\max} = \max\{y_1, y_2, \cdots, y_N\}$. Then an optimal solution to $\mathbf{P_m}$ satisfies $y^{\min} \leq S_j^* \leq y^{\max}$, $j = 0, 1, \cdots, m$.*

Proof. Note that $cy + pG_t(y)$ is decreasing in y when $y < y^{\min}$. Suppose that, to the contrary, $S_j^* < y^{\min}$ for some period k_j. We raise S_j^* to $S_j^* + \epsilon$, where ϵ is a small positive number. We keep all other S_k^* unchanged. As a result, the total cost will be reduced or at least remains the same because $cy + pG_t(y)$ is decreasing when $y < y^{\min}$. If we keep raising the S_j^*, then either it will reach $y^{\min}$ or it will reach some S_i^*. In the latter case we then raise S_j^* and S_i^* together until both of them reach $y^{\min}$ or some third S_k^* join the team. Obviously, when all S_j^* in the team reach $y^{\min}$, which is necessarily the case after a finite number of adjustments, the total cost will be pulled down or at least "stand still" in the process. Thus, an optimal solution must be found to have all $S_j^* \geq y^{\min}$.

Noting that $cy + pG_t(y)$ is increasing over $y \geq y^{\max}$, the relationship $y^{\max} \geq S_t^*$ can be established analogously. □

2.3.2 The Solution Procedure

Now we reformulate $\mathbf{P_m}$ as a dynamic programming problem. To this end, we need to define the state at the beginning of any period. Assume that after a decision has been made on whether the commitment size should be adjusted at the beginning of period t, the number of adjustment options available from periods $t+1$ to period N is ℓ_t and the commitment size becomes S_t. Denote by (ℓ_t, S_t) the state of period t (after the decision on adjustment has been made but before the demand is observed), where $\ell_t \geq 0$. Note now that the state of period 1 is (m, S_0). Let $f_t(\ell_t, S_t)$ denote the achievable minimum cost if period t begins with state (ℓ_t, S_t) (after possible adjustment). Then for $t = 1, 2, ..., N-1$

$$f_t(\ell_t, S_t) = pG_t(S_t) + cS_t + \min\left\{ f_{t+1}(\ell_t, S_t), \min_{S_{t+1} \in [y^{\min}, y^{\max}]} f_{t+1}(\ell_t - 1, S_{t+1}) \right\},$$

where if the outside minimum is attained by $f_{t+1}(\ell_t, S_t)$, then the best strategy is not to adjust the space at time t. Otherwise, the best is to adjust, so we have $\ell_t = \ell_{t+1} + 1$ and S_{t+1} is the minimizer of the inside minimum.

To initialize the algorithm we have

$$f_N(\ell_m, S_m) = pG_N(S_m) + cS_m$$

for any given ℓ_m and S_m. The algorithm will end with

$$f_0(m, S_0^*) = \min_{S_0 \in [y^{\min}, y^{\max}]} f_1(m, S_0).$$

Thus, the minimum total expected leasing cost and optimal commitment sizes S_t can be found by solving $f_0(m, S_0^*)$ through the standard DP backward search algorithm. We denote by $\{S_0^*, S_1^*, \cdots, S_N^*\}$ the sequence of optimal commitment sizes.

The complexity of the DP solution procedure can easily be estimated as follows. Suppose we divide interval $[y^{\min}, y^{\max}]$ into M points. Assume that values of $G_t(y)$ are available for y at all these M points, and $t = 1, 2, \cdots, N$. The complexity of the backward search algorithm is thus bounded by $O(N^2 m^2 M^2)$.

It should be noted that the algorithm can also incorporate range restrictions over possible changes in commitment. For example,

$$(1 - \alpha_i)S_{i-1} \leq S_i \leq (1 + \beta_i)S_{i-1} \quad i = 1, \cdots, m,$$

where $0 \leq \alpha_i < 1$ and $\beta_i \geq 0$ are the downward and upward proportions that bound the change of space commitment, respectively. From the warehouser's perspective, such a restriction is to avoid a sudden change in any two consecutive commitments. Thus it may be incorporated into the contract terms.

2.3.3 A Special Case: Stochastically Increasing/Decreasing Demands

At the end of this section we discuss a special, yet interesting case, which can further reduce the amount of computation in the DP algorithm. A demand sequence $\{\xi_1, \xi_2, \cdots, \xi_N\}$ is said to be stochastically increasing if $1 - \Phi_1(y) \leq 1 - \Phi_2(y) \leq \cdots \leq 1 - \Phi_N(y)$ for any finite y. Evidently, a stochastically increasing demand sequence means that the sequence of corresponding demand means is increasing (see Ross [7]). That is, when the sequence of demand means is denoted by $\{\mu_1, \mu_2, \cdots, \mu_N\}$, then $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_N$ if the demand sequence is stochastically increasing. For a stochastically decreasing demand sequence, all the above relationships are just reversed.

Proposition 3 *If the demand over the contract duration is stochastically increasing (decreasing), then* $S_1^* \leq S_1^* \leq \cdots \leq S_m^*$ *(* $S_0^* \geq S_1^* \geq \cdots \geq S_m^*$*), where* S_i^* *is the optimal commitment size,* $i = 0, \cdots, m$.

Proof. We only prove the case with stochastically increasing demands since the proof for stochastically decreasing demand case can be carried out similarly.

Suppose the optimal solution partitions N periods into $m+1$ segments with breakpoints $k_1, k_2, \cdots, k_m$. That is, for the first $k_1 - 1$ periods, the optimal commitment size is S_0^*; for periods from $k_1 + 1$ to k_2, the optimal commitment size is S_1^*; $\cdots$, for periods from $k_m + 1$ to N, the optimal commitment size is S_m^*.

There might be the case in which the number m of adjustment options is more than what is actually needed. That is, the actual number of adjustments in the optimal solution is less than m. In this case, we arbitrarily insert a point between k_i and k_{i+1} if $k_i + 1 < k_{i+1}$. Repeat the insertion until we have obtained m time points.

According to (5) we have

$$\sum_{t=k_i}^{k_{i+1}-1} \Phi_t(S_i^*) = \frac{p-c}{p}(k_{i+1} - k_i). \tag{7}$$

Note that when the demand is increasing over time, $\Phi_t(y) \geq \Phi_{t+1}(y)$ for any $y \geq 0$. Thus from (7) we obtain

$$\Phi_{k_i+1}(S_i^*) \geq (p-c)/p.$$

Similarly by considering the time interval $[k_{j-1} + 1, k_j]$ we obtain

$$\Phi_{k_i}(S_{i-1}^*) \leq (p-c)/p.$$

Hence we have

$$\Phi_{k_i}(S_{i-1}^*) \leq \Phi_{k_i+1}(S_i^*) \leq \Phi_{k_i}(S_i^*),$$

which implies $S_{i-1}^* \leq S_i^*,\ i = 1, ..., m.$ $\square$

Obviously, if the demand is stochastically increasing, then the upper bound $y^{\max}$ of S_i^* can be replaced by S_{i+1}^* in the DP algorithm. Similarly, the lower bound of S_i^* can be improved if the demand is stochastically decreasing.

3 Computational Experience and Discussion

To evaluate the three forms of contracts, we tested them on three different data sets representing ordinary, divergent and seasonal demand scenarios.

The set of experiments reported here used realistic input data as follows:

$$p/c = 1.5;\ \alpha = 0.25, \beta = .25.$$

We arbitrarily chose $c = 10.0$. All contracts were assumed to cover 12 periods during which 4 commitments could be made. In Model **FSB** and Model **FSNB**, each commitment covered a quarter, while in Model $\mathbf{P_m}$, the timing to start each of the four commitments was optimally chosen.

Demands of space for 12 periods follow the normal distribution. We show three demand scenarios in Table 1, where the numbers without parenthesizes are the mean demands and those in parenthesizes are the corresponding standard deviations. In the case of Ordinary Demand, space requirements over time are relatively steady; in the case of Divergent Demand, they vary significantly but not-seasonally over time, while in the case of Seasonal Demand, they exhibit both divergence and seasonality (as well as stochastic increase). In the case of Divergent Demand, the degree of divergency - the ratio of the highest mean to the lowest mean is: $110/50 = 2.2$, while in the case of Seasonal Demand, this ratio is: $120/30 = 4.0$. The cost evaluation is summarized in Table 2, where the figures in parenthesizes are percentage saving as compared with Model **FSB**.

The DP algorithm is implemented on a Pentium II-300 PC. For each setting, our DP program takes only a fraction of a second in searching for the optimal solution. The same problems were also solved by an integer programming approach. We found that the DP algorithm is substantially faster than the branch-and-bound algorithm used by the commercial integer programming package we used.

Period	1	2	3	4	5	6	7	8	9	10	11	12
Ordinary	50	50	80	75	80	52	52	80	53	53	60	70
	(15)	(15)	(15)	(15)	(15)	(15)	(15)	(15)	(15)	(15)	(15)	(15)
Divergent	50	50	60	80	110	50	60	80	70	50	110	110
	(15)	(15)	(20)	(25)	(30)	(10)	(20)	(20)	(15)	(15)	(30)	(30)
Seasonal	30	30	30	50	50	50	80	80	80	120	120	120
	(6)	(6)	(6)	(8)	(8)	(8)	(15)	(15)	(15)	(30)	(30)	(30)

Table 1 Demand Scenarios

	Ordinary	Divergent	Seasonal
Model **FSB**	8792	11294	12897
Model **FSNB**	8792	11294	12312
Improvement over **FSB**	0%	0%	4.5%
Model $\mathbf{P_m}$	8615	10902	12312
Improvement over **FSB**	2.0%	3.4%	4.5%

Table 2 Cost Evaluation of 3 Contract Forms

In general, Model $\mathbf{P_m}$ could result in saving in warehousing cost. In the case of Seasonal Demand, Model **FSNB** also leads to sizable saving due to the restriction imposed in Model **FSB** (the range for adjustments). The impact of the base commitment is reflected in the case of Seasonal Demand: As it was removed, we observe a cost reduction of 4.5% (from **FSB** to **FSNB**). Comparing between Models **FBNB** and $\mathbf{P_m}$, we can see the impact of the fixed time schedule for adjustments on cost. For example, under Divergent Demand, the flexible schedule ($\mathbf{P_m}$) costs 3.4% less than **FSNB**. The saving potential provides the base for negotiating the terms of new contracts.

We also conduct sensitivity analysis for the case of Divergent Demand. For example, we raise the degree of divergency from 2.2 to 3, which is not unusual from our observation of the MNC data. The changed demand data is as follows:

Period	1	2	3	4	5	6	7	8	9	10	11	12
Divergent	50	50	60	50	150	150	110	80	60	70	120	120
	(15)	(15)	(20)	(15)	(45)	(40)	(10)	(20)	(15)	(15)	(40)	(30)

Table 3 Demand Scenarios

Then the resulted cost for each model: Model **FSB**: 12984; Model **FSNB**: 12972; Model $\mathbf{P_m}$: 12276. The saving from switching contract form of either **FSB** or **FSNB** to form of $\mathbf{P_m}$ is as high as 5.5%.

4 Conclusions

In this paper we provide a framework for analyzing three forms of warehousing contracts with space commitments and adjustment options, which were motivated by a practical situation. The first form allows a number of commitments for prespecified time intervals but imposes that the commitments must fall within a certain range around a base commitment. The next contract form removes the restriction on the range of adjustments (hence also removes the base commitment), while the last goes a step even further by relaxing the times for adjustments as well.

Various procedures are proposed for evaluating different forms of contract. The preliminary computational experience suggests that if requirement for space is highly seasonal and variant, the user firm should pursue the second contract form, while if it varies considerably over time but without clear seasonality, then it is the interest of the user to go after the last contract form. Of course, the ultimate choice depends also on the cost structure associated with each contract alternative. Evaluation procedures could then be applied to aid the selection of the optimal contract.

There is a possible topic for future research. In this study we considered only one type of space while in reality, multiple types of space may be available, for example, nonair-conditioned and air-conditioned storage. Then substitution between different types of storage may exist. It will be of practical interest to know how to contract for each type of space under this situation.

Acknowledgment The authors appreciate the helpful comments of Professor C. P. Teo. This research is partially funded under research grants RP-3981010, RP-3960011, and RP-3972073 from the National University of Singapore.

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