Decomposing school and school-type efficiency

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Abstract

This paper puts forward a Data Envelopment Analysis (DEA) approach to decomposing a pupil's under-attainment at school. Under-attainment is attributed to the pupil, the school and the type of funding regime under which the school operates. A pupil-level analysis is used firstly on a within school and secondly on a between school basis, grouping schools by type such as state-funded, independent and so on. Overall measures of each pupil's efficiency are thus disentangled into pupil, school and school-type efficiencies. This approach provides schools with a set of efficiency measures, each one conveying different information.

Introduction

This paper puts forward a Data Envelopment Analysis (DEA) approach to identify the root sources of a pupil's under-attainment at school. Potential sources investigated are the effort (or lack of) by the pupil, school effectiveness and the type of funding regime under which the school operates.

School efficiency has traditionally been measured using various techniques. Regression analysis is one of these, where pupils' outcomes (usually aggregated at the school-level) are explained by a set of variables concerning the school and also its pupils. Such studies can be found in Levitt and Joyce (1987, Ch. 10), Gray (1981) and Sammons et al. (1996). Although regression analysis does not directly provide efficiency measures, the efficiency of schools can be inferred and schools ranked through the residuals. Regression equations, however, do not explore the variations in pupils' outcomes inside the same school as this variation is hidden behind an average (e.g. see Goldstein (1997) who used an example to show the possibly misleading nature of an aggregate level analysis). Hence, another approach, known as *multilevel modelling*, or *variance components analysis* or *hierarchical linear models* (Gray et al., 1990) is often used. In simple terms, this approach estimates a

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regression equation for each school based on its own pupil level data. The regression models are then compared in terms of slopes and intercepts, and conclusions about schools' effectiveness can be reached. Multilevel analysis can determine the percentage of variation of pupil attainment explained by different variables at different organisational levels (class, school, etc.). The most interesting characteristic of multilevel modelling is that it recognises the existence of within and between school variations, therefore separating school from pupil effects in the assessment of schools' efficiency. For more details on this technique see Raudenbush and Willms (1991) and Goldstein (1995), or the several studies applying it, such as Gray et al. (1986), Sammons et al. (1993), Fitz-Gibbon (1991), Paterson (1991) and Tymms (1992).

More recently DEA has increasingly been used in school assessments. DEA, developed by Charnes et al. (1978), is a linear programming based method used generally to assess productive efficiencies of 'units of assessment' such as schools, bank branches, or retail outlets. The interested reader can find introductions to DEA in Cooper et al. (2000), Norman and Stoker (1991), Land (1991), Boussofiane et al. (1991) and Charnes et al. (1994).

DEA has been used by a number of authors as a means to assess school efficiency using school-level data (see for example Bessent et al., 1982; Mayston and Jesson, 1988; Jesson et al., 1987; Färe et al., 1989; Thanassoulis and Dunstan, 1994). The use of pupil-level data in DEA assessments is quite recent, having started, to the authors' knowledge, with the work of Thanassoulis (1999). He uses pupil level data to estimate targets of attainment for pupils, identifying separate components dependent on the pupil's own effort and components dependent on improved school effectiveness. The work of Thanassoulis (1999) is extended here to identify not only the pupil and school but also the school-type effects.

In most countries secondary schools can be categorised on a number of criteria such as whether they admit pupils of one or both genders, whether their emphasis is on academic or skills training and so on. Here we investigate three broad categories of UK secondary schools by funding:

Comprehensive schools funded by the tax-payer through Local Education Authorities;

Grant Maintained schools funded by the tax-payer through the Funding Agency for Schools, and

Independent schools funded by fees charged on pupils.

While all three types of school aim to enhance as far as possible pupil attainment, most notably academic attainment, the funding regime has implications for the school's management and operation and these in turn could impact pupil attainment. Our aim is to investigate the extent and direction of such impacts. The effects of school type on school performance have hitherto been investigated mainly through multilevel modelling. For example, Gray et al. (1984) found no evidence of differences between the attainment of pupils attending more selective and fully comprehensive schools. Tymms (1992) on the other hand concludes that there are small but significant differences on the performance of pupils attending different types of school. It is often argued, e.g. Fitz-Gibbon (1985) and Tymms (1992), that a selective environment can foster better achievements because it potentially creates some beneficial competition and co-operation amongst pupils.

Our approach in this paper is based on a decomposition of *radial* efficiencies computed by means of DEA models. The decomposition of efficiency measures computed by DEA models is certainly not new. Indeed, Farrell (1957), whose paper is the basis for the seminal work on DEA by Charnes et al. (1978), introduced the decomposition of an overall measure of efficiency into *technical* and *price* components (see Farrell, 1957, p. 255). Färe et al. (1985, Chapters 3 and 4) on the other hand divided a set of radial efficiency measures into two groups referred to, respectively, as *primary* and *derived*. The latter were derived from the primary measures by a decomposition approach. The primary measures are *the technical input* (or *output*) *efficiency*, in the sense of Farrell (1957), *the weak input* (or *output*) *efficiency*, and the *overall input* (or *output*) *efficiency*. The efficiency measures derived from the primary ones are the *input congestion measure* (or the *output loss measure*) and the *allocative input* (*or output*) *efficiency* measure. Each decomposed measure conveys its own information on performance. See Färe et al. (1985) for further details.

Decompositions of DEA efficiencies can be found in many other contexts too. For example Färe and Primont (1984) decompose the efficiency of multiplant firms into within-firms and between-firms efficiency. Färe et al. (1990), decomposed an overall measure of efficiency of rice farms in California into actual efficiency and financial efficiency, while Sueyoshi et al. (1998) decompose efficiencies of Japanese agricultural co-operatives.

Our approach was inspired by Charnes et al. (1981), who computed with respect to a set of schools *managerial* and *policy* efficiencies. However, our approach differs in two ways from that of Charnes et al. (1981). It uses pupil rather than aggregate level data and it uses radial rather than non-radial measures of efficiency. Grosskopf et al. (1999) use the ratio of *direct* and *indirect* (budget constrained) efficiency measures in the form of distance functions to estimate the gain schools would achieve from educational reform, whereby schools will be given a budget but be free to choose their staff mix. This differs from our approach in that our aim is to identify the impact on pupil attainment attributable to given but different funding regimes rather than estimate an optimal, in a sense, use of funding.

The paper is laid out as follows. The Section 2 presents a graphical illustration of the method used to decompose pupil efficiency into school and school-type effects. This is followed by a section that generalises the method developed. The application of the method and the results obtained are presented in the subsequent section. The paper ends with a summary of the conclusions reached.

Decomposition of efficiencies - a graphical illustration

In order to illustrate the decomposition approach, two hypothetical British secondary schools, labelled school 1 and school 2, are used. Two variables are considered one representing an outcome variable (A-level score) and another representing a contextual variable (GCSE score). For those unfamiliar with the British educational system, pupils usually take 'A-level' (General Certificate of Education Advanced Level) courses on three subjects after they have taken their General Certificate of Secondary Education (GCSE) examinations at the age of 16. The A-level courses typically last two years.

For each one of these schools (1 and 2) pupils will first be compared amongst pupils attending the same school. For this reason, any differences in pupil attainments on exit will be the result of differences in effort by pupils, differential school effectiveness or random noise as systematic school effects are identical across pupils. Fig. 1 illustrates how we can disentangle pupil and school effects. Each dot represents a pupil of school 1 and each cross a pupil of school 2.

The piece-wise linear segment BC, in Fig. 1, corresponds to the DEA-efficient boundary of school 1, and FG to the DEA-efficient boundary of school 2. We shall refer to such boundaries as '*pupil within-school efficient boundary*'. The segments BA, FE, GH, and CD are boundary but do not reflect efficient segments as they are dominated by B, F, G, and C, respectively. The frontiers shown for each school are based on DEA under *variable returns to scale* (VRS). A pupil lying on its school's efficient frontier will be said to be '*pupil-within-school-efficient*'. Such a pupil has best observed exit attainment for his/her GCSE score on entry.

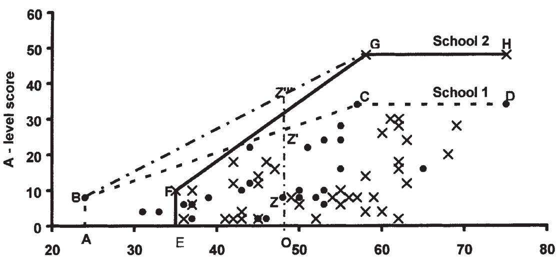

Fig. 1. Disentangling pupil and school effects.

When we consider all the pupils in Fig. 1 irrespective of the school attended, the piece-wise linear segment BG is the locus of DEA efficient attainments. We shall refer to BG as the '*pupil within-all-schools efficient boundary*'.

Different measures of efficiency can be computed depending on the efficient boundary we are referring to. Two different efficiency measures with respect to pupil Z in Fig. 1 are:

1. The pupil-within-school-efficiency measure of pupil Z, OZ/OZ'.
2. The pupil-within-all-schools-efficiency measure of pupil Z OZ/OZ'''.

The ratio OZ/OZ' reflects the proportion of A-level score pupil Z is achieving relative to the best he/she could achieve in school 1, given his/her GCSE score. The ratio OZ/OZ''' reflects the proportion of A-level score pupil Z is achieving relative to the best he/she could achieve within the two schools considered and given his/her GCSE score. (We ignore at this stage noise on attainment levels recorded.)

In the ratio OZ/OZ' school effects are absent as one is comparing pupils in the same school. The distance of pupil Z from the efficient boundary (point Z') is attributed to the pupil's effort as the school has demonstrated, through the achievements of other pupils, that point Z' represents a feasible attainment level at the school. Thus the component OZ/OZ' of the pupil-within-all-schools-efficiency (OZ/OZ''') is attributable to pupil Z and not to his/her school.

If pupil Z had been efficient within her/his own school, she/he would have been achieving the score at point Z'. Then the pupil-within-all-schools-efficiency of pupil Z would have been OZ'/OZ'''. As OZ'/OZ''' is less than 1 there is some inefficiency. This inefficiency cannot be attributed to the pupil as she/he is attaining the best-observed performance in school 1. This inefficiency is thus attributed to school 1. Thus, for pupil Z we have the following decomposition:

$$OZ/OZ''' = (OZ/OZ') \times (OZ'/OZ'''). \qquad (1)$$

That is, a global measure of efficiency (pupil-within-all-schools-efficiency), at the input levels of pupil Z, is a composite of two measures of efficiency: The pupil-within-school-efficiency (OZ/OZ'), and the *school-within-all-schools-efficiency* (OZ'/OZ'''). The first measure relates to the pupil and the second to the school. It is important to note, however, that each of these measures is computed for each pupil.

The approach outlined so far can be used to decompose efficiency at as many levels as desired. We can thus add a third level of decomposition, that of school type. We use for this purpose Fig. 2 which is as Fig. 1, except that school 3 has been added, its pupils being represented by triangles. It is assumed that schools 1 and 2 belong to the same type (type A), whereas school 3 belongs to type B.

For pupils attending schools type A (school 1 and 2) the locus of DEA-efficient attainments is the piece-wise linear segment BG while for pupils attending schools type B (school 3) the locus of DEA-efficient attainments is JKL. We shall refer to BG and JKL as '*pupil-within-school-type efficient boundaries*'. The piece-wise linear segment JKG becomes now the *pupil-within-all-schools efficient boundary*.

The component of the pupil-within-all-schools-efficiency attributable to the school at the input level of pupil Z is, as outlined previously, OZ'/OZ'''. This score will now be further decomposed into components attributable to the school itself and to its type.

When all pupils being compared attend schools of the same type we are computing the pupil-within-school-type-efficiency. For pupil Z, attending a school of type A, this efficiency measure is given by OZ/OZ''. It is easy to verify that

$$OZ/OZ'' = (OZ/OZ') \times (OZ'/OZ''). \qquad (2)$$

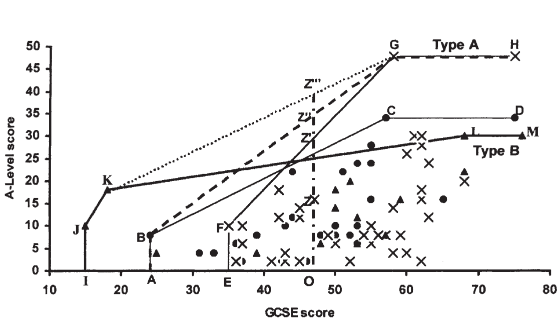

Fig. 2. Disentangling school-type effects.

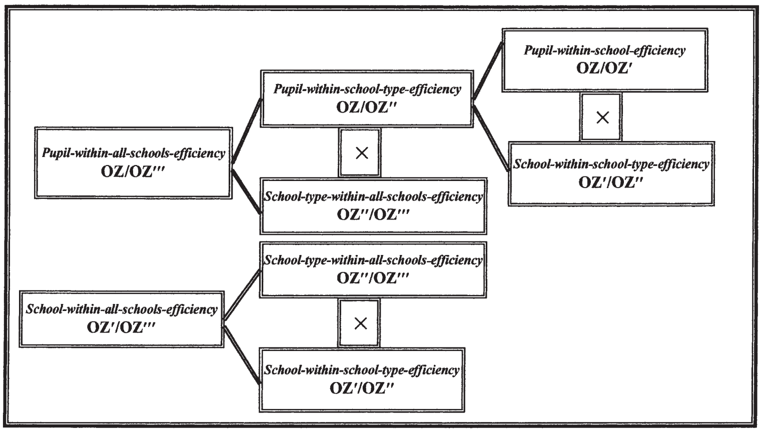

Fig. 3. Summary of school and pupil efficiency decompositions.

The expression in (2) decomposes, at the input level of pupil Z, the pupil-within-school-type-efficiency, OZ/OZ″, into the pupil-within-school-efficiency, OZ/OZ′, and a measure of efficiency attributable to the school excluding any possible effects of its type, OZ′/OZ″. We shall refer to OZ′/OZ″ as the school-within-school-type-efficiency measured at the input level of pupil Z. This efficiency score measures the distance between the pupil-within-school efficient boundary and the corresponding pupil-within-school-type efficient boundary at the input level of pupil Z.

Taking the above decomposition further, we see that the pupil-within-all-schools-efficiency at the input level of pupil Z, OZ/OZ‴, can be decomposed as follows:

$$\mathrm{OZ}/\mathrm{OZ}''' = (\mathrm{OZ}/\mathrm{OZ}'') \times (\mathrm{OZ}''/\mathrm{OZ}'''). \quad (3)$$

That is, at the input level of pupil Z, the pupil-within-all-schools-efficiency equals the product of the pupil-within-school-type-efficiency, OZ/OZ″, and a component of school efficiency that is attributed to its type, OZ″/OZ‴, as can be deduced from Fig. 2. The component OZ″/OZ‴ in (3) is referred to as the school-type-within-all-schools-efficiency, measured at the input level of pupil Z. This component measures the distance between the pupil-within-school-type efficient boundary and the pupil-within-all-schools efficient boundary, and reflects the shortfall of the pupil attainment that is attributable to the school type the pupil attends.

Finally, it can be seen from Fig. 2 that the school-within-all-schools-efficiency, OZ′/OZ‴, can be decomposed into the school-within-school-type-efficiency and the school-type-within-all-schools-efficiency, in the form

$$\mathrm{OZ}'/\mathrm{OZ}''' = (\mathrm{OZ}'/\mathrm{OZ}'') \times (\mathrm{OZ}''/\mathrm{OZ}'''). \quad (4)$$

Fig. 3 summarises the decompositions constructed in respect of Fig. 2.

Before proceeding it is important to recall that the efficiencies being decomposed here are radial or “Farrell” efficiency measures. Therefore our decompositions differ from those of Charnes et al. (1981), who would project radially and then move if necessary a pupil to the efficient boundary making slack adjustments, before computing school level efficiencies. No such slack adjustments are being proposed here.

Decomposition of efficiencies - an application

The procedure outlined in Section 2 will be applied to a real set of data on schools. Implicit in the procedure is the identification of contextual and outcome variables appropriate to assess schools and the estimation of DEA-efficiencies.

Choice of contextual and outcome variables

Children undertaking A-level courses in the British secondary education system are the object of this study. The pupils took their GCSE exams at the age of 16 (in 1992) and two years later (in 1994) they took A-level or AS-level examinations. The AS-levels are a variant of A-levels with reduced syllabus content but to the same standard as A-levels.

The outcome variables to be used are intended to capture pupil attainment at A- and AS-level. Pupil attainment is typically measured through academic results as these are readily available and are objective. It is, however, recognised (see for example Gray, 1981; Ray, 1991; Thanassoulis and Dunstan, 1994) that academic results are only one of the multiple outcomes of education. Indeed, schools may prepare pupils not only to achieve high academic scores but also other goals such as employment, good inter-personal skills, sports and so on. However, we only have data on academic attainments.

Contextual variables are those which schools cannot control, but can affect pupil attainment. There is an extensive body of research aiming to identify the factors influencing pupil performance, including controllable factors. These studies usually concern junior schools for pupils in the age group 11–16 (see for example Gray et al., 1986, 1990; Sammons et al., 1993, 1996; Mayston and Jesson, 1988; Jesson et al., 1987), but some concern A-level studies (Fitz-Gibbon, 1985, 1991; Tymms, 1992; O'Donoghue et al., 1997).

Table 1 contains the input and output variables chosen in our assessment. The two contextual variables are proxies for the innate ability of the pupil. GCSEpts reflect the total number of subjects attempted at GCSE and the grades obtained.

Table 1
Outcome and contextual variables

Contextual variables	Outcome variables
Total GCSE points (GCSEpts)	Total A and AS points (Apts)
GCSE points per attempt (GCSEpts_att)	A and AS points per attempt (Apts_att)

This variable will fail to reflect innate ability of pupils who attempt a small number of subjects at GCSE. (This could be because their school did not support a wider range of subjects.) The GCSE points per attempt will reflect pupil ability, when the pupil attempts only a few subjects but achieves good grades. Our input variables capture academic performance rather than innate ability and the strength of correlation between the two may vary across schools. In the general case our approach can be used with the best variables available on innate ability. For the analysis at hand, the GCSE results were the best variable on innate ability at our disposal as they do explain by far the largest variation in pupil A-level attainment. (O'Donoghue et al., 1997; Fitz-Gibbon, 1985, 1991; Tymms, 1992). There is no strong evidence that other factors such as socio-economic background, gender or ethnicity have influence on the attainment of A-level pupils (Fitz-Gibbon, 1985, 1991; Tymms, 1992), although they might have some significant influence in prior stages of schooling (Sammons et al., 1993, 1996; Levitt and Joyce, 1987, Ch. 10; Gray et al., 1986).

The outcome variables in Table 1 were chosen following the same line of reasoning as the input variables. High outcomes on exit can follow from an industrious pupil or an able pupil. Thus, in essence, the variables in Table 1 reflect both the industry and the ability of the pupil on entry and on exit and measure value – added by the school.

The estimation and decomposition of DEA efficiencies

DEA models were solved to assess pupils within schools, within schools of the same type, and within schools of all types. In general, given a comparative set of n pupils the radial DEA-efficiency of pupil j_0 within that set is $100/\Theta^*_{j_0}\%$, where $\Theta^*_{j_0}$ is the optimal value of Θ in the following model:

$$\mathrm{Max}\,\Theta + \varepsilon(\mathrm{SGCSEpts} + \mathrm{SGCSEpts\_att} + \mathrm{SApts} + \mathrm{SApts\_att}),$$

s.t.

$$
\begin{aligned}
&\sum_{j=1}^{n}\mathrm{GCSEpts}_j\lambda_j + \mathrm{SGCSEpts} = \mathrm{GCSEpts}j_0,\\
&\sum_{j=1}^{n}\mathrm{GCSEpts\_att}_j\lambda_j + \mathrm{SGCSEpts\_att}\\
&\quad = \mathrm{GCSEpts\_att}j_0,\\
&\sum_{j=1}^{n}\mathrm{Apts}_j\lambda_j - \mathrm{SApts} = \Theta\,\mathrm{Apts}j_0,\\
&\sum_{j=1}^{n}\mathrm{Apts\_att}_j\lambda_j - \mathrm{SApts\_att} = \Theta\mathrm{Apts\_att}j_0,\\
&\sum_{j=1}^{n}\lambda_j = 1,\\
&\lambda_j,\ j=1,\ldots,n,\ \mathrm{SGCSEpts},\ \mathrm{SGCSEpts\_att},\\
&\ \mathrm{SApts},\ \mathrm{SApts\_att} \geqslant 0,\quad \Theta\,\text{free},
\end{aligned}
\tag{M1}
$$

where GCSEpts, GCSEpts_att, Apts, and Apts_att are as defined in Table 1, j subscripting the pupil. SGCSEpts, SGCSEpts_att, SApts, SApts_att, Θ and λ_j are variables in the model. The generic model on which (M1) is based was developed by Banker et al. (1984), and assumes VRS. In the educational context measurement scales are arbitrary and so we cannot assume constant returns to scale hold between contextual and attainment factors. Model (M1) would typically be applied to cases, where the contextual and the outcome variables are measured in numerical values. In the UK context, however, we have used numerical equivalent values to translate GCSE and A-level grades into numerical data. GCSE and A-level grades were converted to points according to the following correspondences:

	Grade						
	A	B	C	D	E	F	G
Points at A-level	10	8	6	4	2	–	–
Points at AS-level	5	4	3	2	1	–	–
Points at GCSE	7	6	5	4	3	2	1

The foregoing letter grade numerical correspondences are used very widely by schools, government, universities and others in the UK and provide de facto a numerically based system for measuring the contextual and outcome variables in Table 1. To the extent that numerical scales used to reflect academic attainment are always arbitrary, the results of model M1 are sensitive to the particular scale used as follows. Noting that (M1) is output-oriented the efficiency measures derived will be unaffected if we simply *translate* the input data by adding some numerical constant (assuming the input data remains non-negative). (See Cooper et al., 2000, Section 4.3.2 on translation invariance under VRS.) If not only the input but also the output data is translated by the addition of an arbitrary constant the efficiency rating of pupils will generally change but the classification of pupils into efficient and inefficient will not change. (This is readily deduced from Cooper et al. (2000, Theorem 4.6).) Thus in general when using model M1 we need to be aware that the efficiency results are with reference to the particular numerical system used to measure attainment levels on the outcome and contextual variables.

Model (M1) controls for the contextual values of pupil j_0 and identifies the largest factor $\Theta^*_{j_0}$ by which his/her attainment levels can be expanded *radially*. This expansion is based on the observed performance of those pupils within the comparative set of n pupils, who correspond to positive λ-values at the optimal solution of (M1). Such pupils are known as *efficient peers* to pupil j_0 and they have 100% efficiency under model (M1).

The pupil-within-school, pupil-within-school-type, and pupil-within-all-schools radial DEA-efficiencies are derived using model (M1) by adjusting the set of comparative pupils n using the steps summarised in Table 2.

In order to allow for random impacts on pupil attainment in Step 1 we dropped certain pupils from the comparative set. The approach we followed was in its essence that detailed in Thanassoulis (1999). At the heart of this procedure is the notion that if a *sufficiently large* proportion of the pupil body of a school reaches a certain level of attainment, after we control for contextual variables, then that level of attainment is genuinely feasible within the school and is not deemed to be the result of stochastic impacts. What constitutes a sufficiently large proportion of the pupil body in

Table 2
Required steps to calculate a set of efficiency measures

Step 1. Adjust the set of pupils of each school to arrive at a subset n to compute *pupil-within-school-efficiencies* (EFF_{wj_0}). The adjustment concerns allowing for random impacts on pupil attainment as detailed later
Step 2. Using the subsets of pupils of Step 1 relating to schools of a given type and the corresponding '*adjusted data*', construct a combined set of n pupils to compute *pupil-within-school-type-efficiencies* (EFF_{tj_0}). The adjusted data is to allow for random noise on pupil attainment as detailed later
Step 3. Combine across all school types the subsets of pupils used for each school type in Step 2. Compute *pupil-within-all-schools-efficiencies* (EFF_{ij_0}) using the amalgamated set of data
Step 4. Decomposition of efficiencies computed. At the input levels of pupil j_0 we have: *School-within-all-schools-efficiency* (EFF_{sj_0}) $= \mathrm{EFF}_{ij_0}/\mathrm{EFF}_{wj_0}$ [according to (1)] *School-within-school-type-efficiency* (EFF_{mj_0}) $= \mathrm{EFF}_{tj_0}/\mathrm{EFF}_{wj_0}$ [according to (2)] *School-type-within-all-schools-efficiency* (EFF_{pj_0}) $= \mathrm{EFF}_{ij_0}/\mathrm{EFF}_{tj_0}$ or $\mathrm{EFF}_{pj_0} = \mathrm{EFF}_{sj_0}/\mathrm{EFF}_{mj_0}$ [according to (3) or (4)]

this context is subjective in the same manner as the choice of a level of significance is in hypothesis tests. The procedure developed by Thanassoulis (1999) begins with the full set of pupils of the school and progressively eliminates pupils whose observed attainment is not to be used as a benchmark for other pupils because it is 'exceptional' given the contextual values the pupil offers. (Exceptional pupils were identified making use of the concept of super-efficiency first introduced by Andersen and Petersen (1993).) In our implementation of this procedure we dropped pupils who had exceptional performance, but only if their performance influenced heavily the estimated pupil-within-school efficiencies of the rest of the pupils of that school. [This was ascertained by counting the number of times a pupil appeared in the peer reference set, and by measuring the average change in measured efficiency after deleting a pupil (see for example Wilson, 1995; Dusansky and Wilson, 1994)]. The procedure terminates when a subjectively defined proportion of the pupils of the school have been eliminated as offering exceptional performance. (It should be noted that the subjectively defined proportion of exceptional pupils can be varied to test the sensitivity of the results ultimately derived.) *The pupils left at this stage constitute in the context of* (*M1*) *the comparative set for computing the pupil-within-school-efficiencies.* (The full detail of the underlying basis of this procedure can be found in Thanassoulis (1999).)

Exceptional pupils not permitted to define the within-school efficient boundary in Step 1 were included in Step 2 after first scaling radially their attainments down to move them to their respective within-school efficient boundary. This constitutes the 'adjusted data' referred to in Step 2 above. This process renders the exceptional pupils 100% within-school efficient. This approach is consistent with discounting a component of the attainment of exceptional pupils as not indicating school effectiveness but rather *stochastic impacts*.

In Step 3 the comparative set consists of all pupils of all school types, including all adjusted data of Step 2 referred to above. Since this set of pupils is a super set which includes the set of pupils used to compute the pupil-within-school-efficiencies, it will always be the case that $\mathrm{EFF}_{ij_0} \leqslant \mathrm{EFF}_{wj_0}$. Thus we will always have $\mathrm{EFF}_{sj_0} = \mathrm{EFF}_{ij_0}/\mathrm{EFF}_{wj_0} \leqslant 1$. When $\mathrm{EFF}_{sj_0} = 1$, the pupil-within-all-schools and pupil-within-school efficiencies of pupil j_0 are identical. In this situation, if pupil j_0 does not lie on the within-school-efficient boundary then it is because this pupil lacks application, as we have accounted for the pupil's attainment on entry, and for stochastic impacts. When $\mathrm{EFF}_{sj_0} < 1$ and $\mathrm{EFF}_{wj_0} = 1$, pupil j_0 will have pupil-within-all-schools-efficiency rating below 1 and his/her inefficiency will be attributable to the school the pupil attends as within the school itself the pupil has efficiency rating of 100%.

The school-within-school-type-efficiency, EFF_{mj_0}, at a pupil's input levels reflects the shortfall of her/his attainment which is attributable to the practices of the school attended, given its type. As the within-school-type set of comparative pupils always includes the pupils used to compute pupil-within-school-efficiencies it will always be the case that $\mathrm{EFF}_{tj_0} \leqslant \mathrm{EFF}_{wj_0}$ and so $\mathrm{EFF}_{mj_0} = \mathrm{EFF}_{tj_0}/\mathrm{EFF}_{wj_0} \leqslant 1$. The school-type-within-all-schools-efficiency, EFF_{pj_0}, reflects the shortfall, if any, of pupil j_0's attainment that is attributable to the funding regime of the school the pupil attends. The all-schools set of pupils includes the pupils used to compute pupil-within-school-type-efficiencies, and so we have $\text{EFF}_{ij_0} \leqslant \text{EFF}_{tj_0}$. Thus we will always have $\text{EFF}_{pj_0} = \text{EFF}_{ij_0}/\text{EFF}_{tj_0} \leqslant 1$.

It is perhaps worth clarifying at this point that in decomposing pupil efficiencies in the foregoing manner and thereby ascribing different components of inefficiency to school type, school and pupil we are in essence making an initial diagnosis as to where potential inefficiencies lie. More precisely, we are saying that when we control for pupils' attainment on entry and stochastic impacts, shortfalls in attainment on exit appear to originate from school, school type and pupil in the decomposition measures derived. It is a matter for further analysis, at pupil, school type and school level, as to what might explain the apparent shortfalls in attainment at each decomposition level and how they might be reduced.

The results of decomposing efficiencies in a large random sample of pupils

We decomposed the efficiencies of pupils taking A-levels in a random sample of 122 schools of three different types: *Comprehensive*, *Independent* and *Grant Maintained*. The data were provided by the Department for Education and Employment (DfEE). We had data on GCSE and A-level attempts and results for 6700 pupils. The number of schools of each type, and the average values of the variables used in the assessment are presented in Table 3.

Following the steps outlined in Table 2, six efficiency measures were calculated for each pupil in each school. A detailed analysis of these scores can prove useful as for each one of its pupils the school would be able to identify different targets of attainment combining pupil and school effects. For example pupil-within-school targets will indicate the shortfall in A-level points and A-level points per attempt of a pupil due to his/her own lack of effort. Knowing these targets, teachers can attempt to motivate pupils to achieve what they are believed to be capable of through their own efforts alone. Pupil-within-all-schools targets on the other hand will give a measure of the additional attainment a pupil can expect if their school's effectiveness were to improve to the level of the best performing schools within those considered. Such information will be useful in indicating, where the school itself needs to improve performance. Take for example school 64 in our sample, where pupil 112673 had $\text{EFF}_w = 43.24\%$, $\text{EFF}_i = 30\%$, $\text{EFF}_s = 69.38\%$, $\text{EFF}_t = 30\%$, $\text{EFF}_m = 69.38\%$, and $\text{EFF}_p = 100\%$. The radial targets for this pupil are shown in Table 4.

The difference between the actual attainment levels of the pupil and the targets in (1) in Table 4 give an idea of the extent of outputs the pupil is

Table 3
Number of schools of each type

Code	Type of school	Number of schools	Average Apts	Average Apts_att	Average GCSEpts	Average GCSEpts_att
1	Comprehensive	76	15.65	2.49	50.79	5.68
2	Grant Maintained	19	15.86	2.63	52.16	5.87
3	Independent	27	21.11	3.27	56.95	6.45

Table 4
Within-school and within-all-schools targets for pupil 112673 of school 64

	GCSEpts	GCSEpts/att	Apts	Apts/att
Actual values	51	5.67	6	1.5
Within-school radial targets (1)	51	5.67	13.87	3.5
Within-all schools radial targets (2)	51	5.67	20	5

Table 5
Decomposed average efficiencies

Sch.	Type	Average EFF_{ij_0}	Average EFF_{wj_0}	Average EFF_{sj_0}	Average EFF_{mj_0}	Average EFF_{pj_0}	Sch.	Type	Average EFF_{ij_0}	Average EFF_{wj_0}	Average EFF_{sj_0}	Average EFF_{mj_0}	Average EFF_{pj_0}
121	1	52.01	56.45	93.80	94.72	98.93	28	1	38.69	63.33	63.61	64.81	97.82
124	1	58.68	65.61	89.68	91.59	97.80	86	1	32.55	54.05	62.19	63.87	97.21
25	1	50.97	57.28	89.45	90.32	98.93	84	1	33.49	59.98	61.80	62.51	98.69
55	1	48.90	56.99	87.71	88.69	98.63	5	1	37.40	68.93	61.57	66.45	93.05
93	1	49.23	56.25	87.05	87.89	98.87	17	1	40.78	74.46	60.72	61.84	97.86
83	1	53.10	59.96	86.67	87.98	98.31	22	1	33.33	57.27	58.25	59.63	97.28
72	1	51.50	59.69	86.05	88.53	97.06	58	1	30.83	60.72	57.88	59.19	97.26
2	1	49.81	59.46	85.24	88.17	95.82	31	1	31.13	63.99	56.45	57.08	98.01
14	1	40.87	50.44	84.79	85.76	98.79	109	1	32.31	64.16	55.08	56.01	98.09
56	1	49.70	59.74	84.60	85.03	99.37	32	1	24.38	56.12	54.38	55.15	98.23
21	1	47.08	56.15	84.16	86.82	96.79	19	1	23.99	54.96	53.40	54.71	97.38
40	1	45.16	53.60	83.53	85.97	97.04	24	1	29.01	63.29	51.27	52.44	97.32
127	1	46.49	55.04	83.24	87.88	94.50	30	1	30.78	59.89	50.61	52.07	97.14
78	1	54.05	64.77	83.05	83.80	98.95	34	1	31.67	62.57	49.46	50.59	97.51
26	1	49.43	58.34	82.83	84.43	97.89	4	1	34.53	73.19	46.60	47.63	97.50
100	1	46.69	57.04	81.06	81.52	99.29	116	2	55.66	61.68	90.55	92.36	97.70
13	1	43.16	54.65	80.99	81.95	98.39	107	2	62.57	70.18	88.07	89.41	98.24
99	1	55.14	69.27	80.58	81.72	98.48	41	2	51.95	58.78	87.64	90.01	96.69
53	1	22.08	28.90	80.55	81.10	99.12	7	2	40.05	48.13	85.64	89.20	95.25
63	1	46.71	57.95	80.54	81.89	98.03	117	2	51.41	60.72	85.35	87.22	97.26
94	1	49.95	62.65	80.21	81.62	98.19	103	2	54.55	66.19	83.02	87.42	94.56
16	1	35.72	48.20	80.00	81.08	98.47	118	2	37.62	47.72	82.80	92.41	89.21
80	1	34.49	45.85	79.26	81.56	96.93	111	2	54.72	67.24	81.00	83.78	96.37
62	1	52.22	65.23	78.67	80.82	97.01	113	2	52.25	65.11	80.62	84.80	94.63
120	1	49.43	64.10	78.33	79.39	98.55	108	2	47.68	61.78	78.38	82.68	94.11
33	1	47.81	62.64	78.29	79.21	98.54	110	2	46.04	60.24	78.04	82.03	94.33
79	1	48.14	61.91	77.40	78.54	98.19	102	2	51.63	68.86	76.06	80.87	92.94
126	1	48.32	63.28	77.35	77.98	99.00	112	2	44.89	61.31	72.57	74.53	96.39
35	1	44.58	58.80	77.24	77.69	99.25	8	2	40.63	56.84	72.45	75.76	94.62
97	1	47.92	61.45	76.75	77.73	98.41	104	2	31.50	49.51	70.68	77.47	90.28

98	1	50.14	65.70	76.25	76.59	99.40	12	2	43.84	67.16	66.51	69.13	94.81
27	1	45.98	59.21	76.12	78.05	97.10	69	2	32.88	63.42	59.80	66.14	89.25
73	1	45.29	61.39	75.30	76.88	97.64	115	2	36.68	62.13	58.26	67.30	85.96
114	1	48.48	63.49	74.61	75.53	98.55	9	2	36.63	68.15	57.78	61.90	92.62
122	1	46.95	64.34	74.49	76.14	97.49	48	3	76.76	83.45	91.69	92.45	98.86
51	1	44.09	62.21	74.36	75.14	98.63	23	3	67.19	72.61	91.51	93.69	97.05
71	1	41.26	57.37	73.97	75.24	97.80	75	3	70.41	76.79	91.40	94.38	96.42
38	1	40.47	56.07	73.70	75.90	96.75	87	3	73.01	79.37	91.20	91.64	99.33
18	1	45.62	61.90	73.04	75.77	96.26	65	3	63.79	71.49	89.36	93.48	94.92
46	1	45.05	63.11	72.57	74.12	97.29	61	3	63.67	73.28	86.37	89.96	95.26
36	1	41.57	59.14	72.51	72.90	99.15	29	3	61.92	73.38	83.78	87.09	95.15
119	1	39.46	55.97	72.37	73.57	98.23	88	3	68.07	82.04	82.92	85.36	96.31
105	1	40.54	56.96	72.22	73.22	98.60	70	3	64.76	80.01	81.65	83.61	94.26
77	1	43.34	61.09	72.05	73.35	97.77	50	3	57.64	71.39	81.57	82.41	98.06
20	1	48.77	68.40	71.64	71.90	99.49	89	3	55.30	68.64	81.46	83.89	96.36
123	1	27.88	45.55	71.08	71.83	98.70	45	3	57.63	70.81	81.18	85.27	93.86
90	1	43.57	63.33	70.94	71.69	98.49	44	3	50.96	65.29	78.60	84.40	92.11
125	1	45.64	64.77	70.74	71.77	98.45	59	3	60.06	78.90	78.20	80.06	96.47
49	1	37.45	55.54	69.71	70.79	98.26	74	3	53.71	69.82	76.77	79.88	95.79
92	1	45.20	64.68	69.14	70.05	98.05	6	3	65.46	89.74	74.07	79.77	92.12
64	1	43.95	68.30	68.07	68.71	98.80	39	3	51.64	70.64	73.78	76.39	95.08
47	1	47.26	69.85	67.13	75.42	88.97	15	3	50.56	70.48	72.41	76.09	95.02
54	1	19.77	41.67	66.76	67.10	99.21	3	3	53.87	75.21	71.36	77.24	91.86
101	1	34.00	54.01	66.56	67.23	98.89	42	3	53.30	75.02	70.17	72.20	96.95
52	1	23.38	42.91	65.93	66.92	98.05	43	3	51.04	78.63	64.48	65.82	96.48
1	1	39.49	61.90	65.89	66.42	98.75	57	3	48.17	77.10	60.91	65.86	90.93
85	1	40.37	65.86	65.21	66.15	98.36	60	3	48.80	82.42	60.56	63.01	94.53
76	1	42.79	67.02	64.91	67.32	96.73	95	3	45.19	78.22	56.56	61.90	90.98
91	1	41.62	67.29	64.47	66.30	96.59	96	3	34.85	65.14	52.21	56.23	92.46
82	1	48.12	75.10	64.01	65.58	97.54	11	3	12.65	66.67	45.98	48.48	87.89
37	1	35.30	59.86	64.00	64.58	98.88	10	3	38.47	90.00	44.47	48.00	90.71
							Average		45.57	63.56	73.54	75.74	96.53

not achieving due to his own lack of effort. On the other hand the difference between the targets in (1) and (2) in Table 4 gives an idea of the extent of outputs the pupil is not achieving due to school effects. Indeed, for this pupil, school 64 is only making possible the achievement of 69.38% of the output levels the most effective pupils in the sample across all schools would achieve. This measure of school efficiency cannot be justified by the funding regime of school 64 (i.e. the type of school it is) as when this pupil is assessed against pupils belonging to the same type of school his/her efficiency measure is exactly the same as when all the pupils of all schools in the sample are considered ($EFF_t = EFF_i = 30\%$).

The components of efficiency resulting from decomposing the pupil-within-all-schools efficiency measure were computed at the input levels of each pupil in our sample. Table 5 shows the individual components averaged at school level. The averaging is done merely to indicate the central tendency of each measure at school level. Information is naturally lost in the averaging process, however, individual pupil efficiencies can be examined at school level for a better insight into the school's performance. Such detailed school-level analysis is beyond the scope of this paper. The next section discusses how the measures can lead to insights about each school and pupil.

Insights gained from decomposing pupil efficiencies

A more precise determination of school impacts

If efficiency scores were computed using pupil level data but without considering school effects, then the measure of efficiency one would calculate for each pupil would be that defined as pupil-within-all-schools efficiency. Such measures would clearly under-estimate pupils' efficiencies as one would be ignoring that the attained score is a composite of pupil and school effects. By aggregating pupils at school level and assessing pupil-within-school efficiencies we in effect control for school effects and assess the pupil's own impact on their attainment or lack thereof.

The same principle holds as we aggregate to levels above school to assess efficiency. For example aggregating pupils attending schools funded in a certain way (e.g. Comprehensive schools funded through Local Education Authorities) we control for the type of school and assess the impact on efficiency due to pupil and school. It is important that in aggregations of this type the units being compared (in our case pupils) are homogeneous in terms of attainments pursued and factors impacting those pursuits so that we compare like with like. In our context pupils engaging in A-level studies almost invariably aim for the highest possible academic attainments as the A-levels are very much the passport to higher education for them. The pupils simply differ in the facilitators (schools) they opted for (or had to go for due to financial or family reasons) in pursuit of their academic objectives. For example a wealthy pupil may be taking A-levels at an Independent school and a poorer pupil at a Comprehensive school but both wish to attain the best A-level scores they can and in both cases innate ability as reflected in the input variables will impact attainment. (As noted earlier research shows that by this stage of education much self selection has taken place and factors such as socio-economic background are less influential on attainment.)

There may, however, be scope for disaggregation of pupils from school to 'teacher units' (classes). Indeed classes can, in some cases, present as much variation as schools themselves. Unfortunately we had no data on classes mainly because in "secondary schools self contained classrooms my not exist, with students being allocated to different teachers for different subjects" (Goldstein, 1997).

Table 6 illustrates differences between efficiency components. It shows some statistics for efficiency components of all the pupils in the sample irrespective of the school attended. As expected the pupil-within-all-schools-efficiency scores are clearly lower (in terms of mean, median and mode) than all other efficiency measures presented. This confirms that an efficiency assessment based solely on these measures would indeed under-estimate pupils' efficiency.

Table 6
Statistics on efficiencies calculated at pupil-level

	Mode	Median	Mean
Pupil-within-all-schools-efficiency (EFF_{ij_0})	]50,60]	46.6	48.4
Pupil-within-school-efficiency (EFF_{wj_0})	]90,100]	65.0	62.4
School-within-all-schools-efficiency (EFF_{sj_0})	]90,100]	82.8	78.4
School-within-school-type-efficiency (EFF_{mj_0})	]90,100]	85.5	80.5
School-type-within-all-schools-efficiency (EFF_{pj_0})	]90,100]	93.8	96.9

Pupil-within-school, EFF_{wj_0}, and school-within-all-schools, EFF_{sj_0}, efficiencies are generally high, the modal interval in each case being in the 90–100% range. However, school-within-all-schools-efficiency scores tend to be higher than pupil-within-school-efficiency scores (see median and mean values in Table 6), meaning that generally, the pupil-within-all-schools efficiency measures (EFF_{ij_0}) are mostly attributable to the pupils themselves rather than to the schools they attend. It is also clear from Table 6 that the school-within-all-schools-efficiencies, EFF_{sj_0}, are largely determined by the school-within-school-type-efficiencies. It seems, thus, that the (funding) type a school belongs to does not affect significantly the school's effectiveness. It is noteworthy that for 50% of the pupils in our set the school-type-within-all-schools-efficiency is higher than 93.8% as shown by the relevant median in Table 6.

If we look further, at the school level, additional interesting insights are gained. Take for example school 121, which is a Comprehensive school (listed first in Table 5). The average pupil-within-all-schools-efficiency of its pupils is 52.01% and their average pupil-within-school-efficiency is 56.45%. The similarity between these values means that on average the within-all-schools, and the within-school-121 efficient boundaries are very close to one another and for this reason the school-within-all-schools-efficiency is on average very high, 93.80%. That is, if all pupils of school 121 had been efficient within their own school, then on average they would be achieving 93.80% of the outcome scores of the most effective pupils across all schools. Turning to the type of school, it is seen that when school 121 is compared with schools of the same type its average efficiency (94.72%) is not much different from the school-within-all-schools average efficiency of 93.80%. The difference between these two measures of efficiency is accounted for by the type of school (Comprehensive) that school 121 belongs to. The average school-type-within-all-schools-efficiency of school 121 is almost 99%, meaning that the fact that school 121 is Comprehensive does not in itself impact much its efficiency.

Looking at results by school type, the summary values in Table 7 are obtained. It is clear that on average schools have high values of school-type-within-all-schools-efficiency (EFF_{pj_0}), meaning that the type of a school is not the main cause of the pupil inefficiencies at schools. Nevertheless, Comprehensive schools have, on average, higher EFF_{pj_0} than the other school types. Grant Maintained schools have the highest average efficiency both when compared with schools of the same type (EFF_{mj_0}), and when compared with all the schools irrespective of the type (EFF_{sj_0}). Therefore, it seems that Comprehensive schools' efficiencies, although less affected by the type of school, are especially negatively influenced by the schools themselves (lower EFF_{sj_0}, and EFF_{mj_0} averages), whereas Grant Maintained schools seem to be

Table 7
Average efficiencies by school type

	EFF_{ij_0}	EFF_{wj_0}	EFF_{sj_0}	EFF_{mj_0}	EFF_{pj_0}
1. Comprehensive	41.93	59.90	72.40	73.78	*97.85*
2. Grant Maintained	47.62	61.32	*76.59*	80.76	93.60
3. Independent	*55.51*	75.43	74.62	77.72	94.64

better run in themselves, (higher EFF_{sj_0}, and EFF_{mj_0} averages), but their efficiency might be negatively influenced by their type (lower average EFF_{pj_0}).

Although Independent schools are more selective on entry (as can be verified by the values in Table 3), this does not necessarily mean that this type of school is more efficient. Pupils in independent schools have the highest pupil efficiency scores, both when compared with all the pupils in the sample (EFF_{ij_0}) and within their own schools (EFF_{wj_0}). Interestingly school-within-all-schools (EFF_{sj_0}) efficiency measures for independent schools are not the highest, suggesting that despite their pupils' high efficiencies, these schools have slightly more scope to raise the attainments of their pupils than do Grant Maintained schools.

Identifying differential school effectiveness

The spread of school-within-all-schools-efficiency scores across the pupils of a school can be seen as an indicator of *differential effectiveness* by the school, in the sense that the school has varying effectiveness between different groups of pupils. We can use a graphical means to look for differential effectiveness of schools as illustrated in Fig. 4.

The school-within-all-schools-efficiency scores of pupils from schools 121 and 2 are plotted in Fig. 4 against the GCSE points per attempt. The GCSE points per attempt can be taken as indicative of the innate academic ability of the pupil.

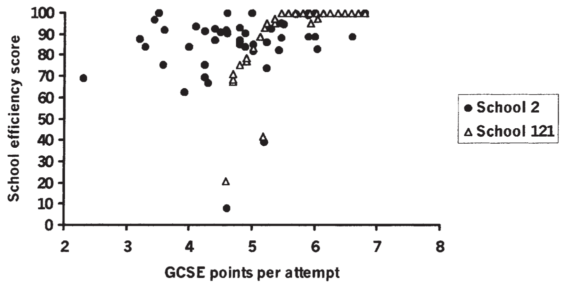

Fig. 4. Differencial effectiveness illustration through schools 2 and 121.

The graph suggests that school 121 is more effective with the stronger than with the weaker of its pupils as can be ascertained through the existence of several school-within-all-schools-efficiency scores of 100% for pupils with high prior attainment levels (taken to be 5 or above). In contrast, there is no evidence of differential effectiveness at school 2.

The graphical approach sketched above can be complemented or indeed replaced by a numerical approach to identify evidence of differential school effectiveness. Some relevant statistics concerning the school-within-all-schools-efficiency scores for pupils in various ranges of attainment on entry to A-level studies can be computed, and thereby evidence of differential effectiveness can be identified. In the case of schools 121 and 2 in Fig. 4 we have the results in Table 8.

Clearly the difference between the median school-within-all-schools-efficiency scores for the bottom and top cohorts on entry is much larger in school 121 than in school 2. The same goes for

Table 8
School-within-all-schools efficiencies by pupils grouped on ability

	School 121		School 2	
	Bottom 50% on entry[a]	Top 50% on entry[b]	Bottom 50% on entry[a]	Top 50% on entry[b]
Min	20.80	95.00	8.20	39.10
Q1	78.70	100.00	75.40	84.60
Median	95.00	100.00	90.60	88.80
Mean	84.90	99.90	82.30	88.00
Q3	100.00	100.00	92.10	96.00
Max	100.00	100.00	100.00	100.00

[a] Pupils offering below the median GCSE points per attempt, the median relating only to pupils of the school concerned.
[b] Pupils offering above the median GCSE points per attempt, the median relating only to pupils of the school concerned.

mean school-within-all-schools efficiencies. Testing these differences using a Mann–Whitney test and a t-test, respectively, it was found that indeed the median and the mean of school 121 were different for the two groups of pupils considered in Table 8, whereas for school 2 these were considered equal at the 5% significance level.

Pupil targets and peers

The decomposition of school and pupil efficiency also makes it possible to set pupils targets of achievement which can incorporate separate components of efficiency improvement for pupils and schools. The approach is detailed in Thanassoulis (1999) for the case, where pupils at the younger stage of 11–16 years of age are concerned. In the context of the assessment undertaken in this paper, the target levels and peers for each pupil j_0 flow directly from the solution of model (M1). Depending on the comparative set of pupils n used in (M1), these targets will reflect expectations based on within-school, within-all schools, or within-school-type attainments. See also Table 4.

Conclusion

This paper has decomposed an overall measure of efficiency, calculated at the pupil level, into components attributable to the pupil and to the school. The component of efficiency attributable to the school was further decomposed into school-within-school-type-efficiency and school-type-within-all-schools-efficiency. The approach outlined here provides useful information to schools. Indeed, the disentangling of school and pupil efficiencies makes it possible to get a more incisive view of the performance of a school both in terms of its overall effectiveness and its differential effectiveness over pupils. Such information is of value both for current and prospective pupils of the school and for those involved with the management and running of the school. The information can also help teachers in the day to day management of pupil expectations on attainment by identifying the overall measure of improvement that can be expected of a pupil, and the extent to which that improvement can be secured by pressing the pupils and/or by pressing the school to improve its performance. At the same time the outputs provided by this approach are also valuable for external parties, such as government institutions, interested in augmenting the national quality of the education system, or parents, interested in choosing the best school for their children. If one type of school is proved to be better than another, or if a school is proved to be differentially effective with stronger pupils, then parents' choice and governmental action can be guided by such information.

The main element of this approach is that it recognises that pupils have in themselves inefficiencies that will contribute to lower school efficiency measures when these are measured through examination outcomes, as it is generally the case. For this reason, the comparison of pupils within schools is the means to ascertain the efficiency of pupils, as school effects are absent. Then the comparison of pupils coming from different schools allows the isolation of school effects as distinct from pupil effects. Extensions of this approach to consider more levels are quite straightforward. In this paper the extension considered was the type of school. Another extension could be the consideration of teacher units (classes), which can be, in some cases, as different as schools.

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