# Intuitionistic fuzzy parametrized soft set theory and its decision making

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### Abstract

In this work, we first define intuitionistic fuzzy parametrized soft sets (intuitionistic FP-soft sets) and study some of their properties. We then introduce an adjustable approaches to intuitionistic FP-soft sets based decision making. We also give an example which shows that they can be successfully applied to problems that contain uncertainties.

*Keywords:* Soft sets, fuzzy sets, FP-soft sets, intuitionistic FP-soft sets, soft level sets, decision making.

#### 1. Introduction

Many fields deal with the uncertain data which may not be successfully modeled by the classical mathematics, probability theory, fuzzy sets [33], rough sets [30], and other mathematical tools. In 1999, Molodtsov [28] proposed a completely new approach so-called *soft set theory* that is more universal for modeling vagueness and uncertainty.

After definitions of the operations of soft sets [3, 11, 26, 32], the properties and applications on the soft set theory have been studied increasingly (e.g. [3, 7, 9]). The algebraic structure of soft set theory has also been studied increasingly (e.g. [1, 2, 5, 13, 17, 18, 19, 20, 31]). In recent years, many interesting applications of soft set theory have been expanded by embedding

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the ideas of fuzzy sets (e.g. [4, 6, 10, 14, 20, 22, 25, 27]), rough sets (e.g. [4, 15, 16]) and intuitionistic fuzzy sets (e.g. [23, 24, 29])

Çağman *et al.*[8] defined FP-soft sets and constructed an FP-soft set decision making method. In this paper, we first define intuitionistic fuzzy parametrized soft sets (intuitionistic FP-soft sets), and study their operations and properties. We then introduce a decision making method based on intuitionistic FP-soft sets. This method is more practical and can be successfully applied to many problems that contain uncertainties.

#### 2. Preliminary

In this section, we present the basic definitions of soft set theory [28], fuzzy set theory [33], intuitionistic fuzzy set theory [12] and FP-soft set theory [8] that are useful for subsequent discussions.

**Definition 1.** Let U be an initial universe, P(U) be the power set of U, E is a set of parameters and  $A \subseteq E$ . Then, a soft set  $F_A$  over U is defined as follows:

$$F_A = \{(x, f_A(x)) : x \in E\}$$

where  $f_A : E \to P(U)$  such that  $f_A(x) = \emptyset$  if  $x \notin A$ .

Here,  $f_A$  is called approximate function of the soft set  $F_A$ , and the value  $f_A(x)$  is a set called x-element of the soft set for all  $x \in E$ . It is worth noting that the sets  $f_A(x)$  may be arbitrary.

**Definition 2.** Let U be a universe. Then a fuzzy set X over U is a function defined as follows:

$$X = \{(\mu_X(u)/u) : u \in U\}$$

where  $\mu_X: U \to [0.1]$ 

Here,  $\mu_X$  called membership function of X, and the value  $\mu_X(u)$  is called the grade of membership of  $u \in U$ . The value represents the degree of u belonging to the fuzzy set X.

**Definition 3.** Let E be a universe. An intuitionistic fuzzy set A on E can be defined as follows:

$$A = \{ < x, \mu_A(x), \gamma_A(x) > : x \in E \}$$

where,  $\mu_A : E \to [0, 1]$  and  $\gamma_A : E \to [0, 1]$  such that  $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for any  $x \in E$ . Here,  $\mu_A(x)$  and  $\gamma_A(x)$  is the degree of membership and degree of nonmembership of the element x, respectively.

If A and B are two intuitionistic fuzzy sets on E, then

- i.  $A \subset B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\gamma_A(x) \geq \gamma_B(x)$  for  $\forall x \in E$
- ii. A = B if and only if  $\mu_A(x) = \mu_B(x)$  and  $\gamma_A(x) = \gamma_B(x) \ \forall x \in E$

iii. 
$$A^c = \{ < x, \gamma_A(x), \mu_A(x) > : x \in E \}$$

- iv.  $A \cup B = \{ \langle x, max(\mu_A(x), \mu_B(x)), min(\gamma_A(x), \gamma_B(x)) \rangle : x \in E \},\$
- v.  $A \cap B = \{ < x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x)) > : x \in E \},$

vi. 
$$A + B = \{ \langle x, \mu_X(x) + \mu_Y(x) - \mu_X(x)\mu_Y(x), \gamma_X(x)\gamma_Y(x) \rangle : x \in E \},\$$

vii. 
$$A \cdot B = \{ \langle x, \mu_A(x)\mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x)\gamma_B(x) \rangle : x \in E \}.$$

**Definition 4.** Let U be an initial universe, P(U) be the power set of U, E be a set of all parameters and X be a fuzzy set over E. Then a FP-soft set  $(f_X, E)$  on the universe U is defined as follows:

$$(f_X, E) = \{(\mu_X(x)/x, f_X(x)) : x \in E\}$$

where  $\mu_X : E \to [0.1]$  and  $f_X : E \to P(U)$  such that  $f_X(x) = \emptyset$  if  $\mu_X(x) = 0$ .

Here  $f_X$  called approximate function and  $\mu_X$  called membership function of FP-soft sets.

#### 3. Intuitionistic FP-soft sets

In this section, we define intuitionistic fuzzy parametrized soft sets (intuitionistic FP-soft sets) and their operations.

**Definition 5.** Let U be an initial universe, P(U) be the power set of U, E be a set of all parameters and K be an intuitionistic fuzzy set over E. An intuitionistic FP-soft sets  $\coprod_K$  over U is defined as follows:

$$\amalg_K = \left\{ (\langle x, \alpha_K(x), \beta_K(x) \rangle, f_K(x)) : x \in E \right\}$$

where  $\alpha_K : E \to [0.1], \ \beta_K : E \to [0.1]$  and  $f_K : E \to P(U)$  with the property  $f_K(x) = \emptyset$  if  $\alpha_K(x) = 0$  and  $\beta_K(x) = 1$ .

Here, the function  $\alpha_K$  and  $\beta_K$  called membership function and non-membership of intuitionistic FP-soft set, respectively. The value  $\alpha_K(x)$  and  $\beta_K(x)$  is the degree of importance and unimportant of the parameter x. Obviously, each ordinary FP-soft set can be written as

$$\Pi_K = \left\{ (\langle x, \alpha_K(x), 1 - \alpha_K(x) \rangle, f_K(x)) : x \in E \right\}$$

Note that the sets of all intuitionistic FP-soft sets over U will be denoted by IFPS(U).

**Definition 6.** Let  $\amalg_K \in IFPS(U)$ . If  $\alpha_K(x) = 0$  and  $\beta_K(x) = 1$  for all  $x \in E$ , then  $\amalg_K$  is called a empty intuitionistic FP-soft sets, denoted by  $\amalg_{\Phi}$ .

**Definition 7.** Let  $\coprod_K \in IFPS(U)$ . If  $\alpha_K(x) = 1$ ,  $\beta_K(x) = 0$  and  $f_K(x) = U$  for all  $x \in E$ , then  $\amalg_K$  is called universal intuitionistic FP-soft set, denoted by  $\amalg_{\tilde{E}}$ .

**Example 1.** Assume that  $U = \{u_1, u_2, u_3, u_4, u_5\}$  is a universal set and  $E = \{x_1, x_2, x_3, x_4\}$  is a set of parameters. If

$$K = \{ \langle x_2, 0.2, 0.5 \rangle, \langle x_3, 0.5, 0.5 \rangle, \langle x_4, 0.6, 0.3 \rangle \}$$

and

$$f_K(x_2) = \{u_2, u_4\}, f_K(x_3) = \emptyset, f_K(x_4) = U$$

then an intuitionistic FP-soft set  $\amalg_K$  is written by

$$\mathbf{H}_{K} = \{(\langle x_{2}, 0.2, 0.5 \rangle, \{u_{2}, u_{4}\}), (\langle x_{3}, 0.5, 0.5 \rangle, \emptyset), (\langle x_{4}, 0.6, 0.3 \rangle, U)\}$$

If  $L = \{ \langle x_1, 0, 1 \rangle, \langle x_2, 0, 1 \rangle, \langle x_3, 0, 1 \rangle, \langle x_4, 0, 1 \rangle \}$ , then the intuitionistic FP-soft set  $\coprod_L$  is an empty intuitionistic FP soft set.

If  $M = \{ \langle x_1, 1, 0 \rangle, \langle x_2, 1, 0 \rangle, \langle x_3, 1, 0 \rangle, \langle x_4, 1, 0 \rangle \}$  and  $f_M(x_1) = U$ ,  $f_M(x_2) = U$ ,  $f_M(x_3) = U$  and  $f_M(x_4) = U$ , then the intuitionistic FP-soft set  $\coprod_M$  is a universal intuitionistic FP-soft set.

**Definition 8.**  $\amalg_K, \amalg_L \in IFPS(U)$ . Then  $\amalg_K$  is a intuitionistic FP-soft subset of  $\amalg_L$ , denoted by  $\amalg_K \subseteq \amalg_L$ , if and only if  $\alpha_K(x) \leq \alpha_L(x)$ ,  $\beta_K(x) \geq \beta_L(x)$  and  $f_K(x) \subseteq f_L(x)$  for all  $x \in E$ .

**Remark 1.**  $\amalg_K \subseteq \amalg_L$  does not imply that every element of  $\amalg_K$  is an element of  $\amalg_L$  as in the definition of classical subset. For example, assume that  $U = \{u_1, u_2, u_3, u_4\}$  is a universal set of objects and  $E = \{x_1, x_2, x_3\}$  is a set of all parameters. If  $K = \{< x_1, 0.4, 0.6 >\}$  and  $L = \{< x_1, 0.5, 0.5 >, < x_3, 0.4, 0.5 >\}$ , and  $\amalg_K = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(< x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, \{u_2, u_4\})\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, (u_2, u_4)\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, (u_2, u_4)\}$ ,  $\amalg_L = \{(= x_1, 0.4, 0.6 >, (u_2, u_4)\}$ ,  $\sqcup_L = \{(= x_1, 0.4, 0.6 >, (u_2, u_4)\}$ ,  $\sqcup_L = \{(= x_1, 0.4, 0.6 >, (u_2, u_4)\}$ ,  $\sqcup_L = \{(= x_1, 0.4, 0.6 >, (u_2, u_4)\}$ ,  $\sqcup_L = \{(= x_1, 0.4, 0.$ 

 $x_1, 0.5, 0.5 >, \{u_2, u_3, u_4\}), (< x_3, 0.4, 0.5 >, \{u_1, u_5\})\}, \text{ then for all } x \in E, \\ \alpha_K(x) \leq \alpha_L(x), \ \beta_K(x) \geq \beta_L(x) \text{ and } \amalg_K(x) \subseteq \amalg_L(x) \text{ is valid. Hence} \\ \amalg_K \subseteq \amalg_L. \text{ It is clear that } (< x_1, 0.4, 0.6 >, \{u_2, u_4\}) \in \amalg_K \text{ but } (< x_1, 0.4, 0.6 >, \{u_2, u_4\}) \notin \amalg_L.$ 

**Proposition 1.** Let  $\coprod_K, \amalg_L \in IFPS(U)$ . Then

- *i.*  $\coprod_K \widetilde{\subseteq} \amalg_{\tilde{E}}$
- *ii.*  $\coprod_{\Phi} \widetilde{\subseteq} \amalg_K$
- *iii.*  $\coprod_K \widetilde{\subseteq} \amalg_K$

**Definition 9.**  $\Pi_K, \Pi_L \in IFPS(U)$ . Then  $\Pi_K$  and  $\Pi_L$  are intuitionistic FPsoft-equal, written by  $\Pi_K = \Pi_L$ , if and only if  $\alpha_K(x) = \alpha_L(x)$ ,  $\beta_K(x) = \beta_L(x)$  and  $f_K(x) = f_L(x)$  for all  $x \in E$ .

**Proposition 2.** Let  $\amalg_K, \amalg_L, \amalg_M \in IFPS(U)$ . Then

- *i.*  $\amalg_K = \amalg_L$  and  $\amalg_L = \amalg_M \Leftrightarrow \amalg_K = \amalg_M$
- ii.  $\amalg_K \widetilde{\subseteq} \amalg_L$  and  $\amalg_L \widetilde{\subseteq} \amalg_K \Leftrightarrow \amalg_K = \amalg_L$
- *iii.*  $\amalg_K \cong \amalg_L$  and  $\amalg_L \cong \amalg_M \Rightarrow \amalg_K \cong \amalg_M$

**Definition 10.**  $\amalg_K \in IFPS(U)$ . Then complement of  $\amalg_K$ , denoted by  $\amalg_K^c$ , is a intuitionistic FP-soft set defined by

$$\amalg_K^c = \left\{ (\langle x, \beta_K(x), \alpha_K(x) \rangle, f_{K^c}(x)) : x \in K \right\}$$

where  $f_{K^c}(x) = U \setminus f_K(x)$ .

**Proposition 3.** Let  $\coprod_K \in IFPS(U)$ . Then

- *i.*  $(\amalg_K^c)^c = \amalg_K$
- *ii.*  $\amalg^c_{\Phi} = \amalg_{\tilde{E}}$

*iii*.  $\coprod_{\tilde{E}}^c = \coprod_{\Phi}$ 

**Definition 11.**  $\amalg_K, \amalg_L \in IFPS(U)$ . Then union of  $\amalg_K$  and  $\amalg_L$ , denoted by  $\amalg_K \widetilde{\cup} \amalg_L$ , is defined by

$$\Pi_{K}\widetilde{\cup}\Pi_{L} = \left\{ (\langle x, max(\alpha_{K}(x), \alpha_{L}(x)), min(\beta_{K}(x), \beta_{L}(x)) \rangle, f_{K\widetilde{\cup}L}(x)) : x \in E \right\}$$

where  $f_{K \widetilde{\cup} L}(x) = f_K(x) \cup f_L(x)$ .

**Proposition 4.** Let  $\amalg_K, \amalg_L, \amalg_M \in IFPS(U)$ . Then

- *i.*  $\coprod_K \widetilde{\cup} \amalg_K = \amalg_K$
- *ii.*  $\coprod_K \widetilde{\cup} \amalg_\Phi = \amalg_K$
- *iii.*  $\coprod_K \widetilde{\cup} \amalg_{\tilde{E}} = \amalg_{\tilde{E}}$
- *iv.*  $\coprod_K \widetilde{\cup} \amalg_L = \amalg_L \widetilde{\cup} \amalg_K$
- v.  $(\amalg_K \widetilde{\cup} \amalg_L) \widetilde{\cup} \amalg_M = \amalg_K \widetilde{\cup} (\amalg_L \widetilde{\cup} \amalg_M)$

**Definition 12.**  $\amalg_K, \amalg_L \in IFPS(U)$ . Then intersection of  $\amalg_K$  and  $\amalg_L$ , denoted by  $\amalg_K \cap \amalg_L$ , is a intuitionistic FP-soft sets defined by

$$\amalg_{K} \widetilde{\cap} \amalg_{L} = \left\{ \langle x, \min(\alpha_{K}(x), \alpha_{L}(x)), \max(\beta_{K}(x), \beta_{L}(x)), f_{K \widetilde{\cap} L}(x)) : x \in E \right\}$$
  
where  $f_{K \widetilde{\cap} L}(x) = f_{K}(x) \cap f_{L}(x).$ 

**Proposition 5.** Let  $\amalg_K, \amalg_L, \amalg_M \in IFPS(U)$ . Then

- *i*.  $\amalg_K \widetilde{\cap} \amalg_K = \amalg_K$
- *ii.*  $\amalg_K \widetilde{\cap} \amalg_\Phi = \amalg_\Phi$
- *iii.*  $\amalg_K \widetilde{\cap} \amalg_{\tilde{E}} = \amalg_K$
- *iv.*  $\amalg_K \widetilde{\cap} \amalg_L = \amalg_L \widetilde{\cap} \amalg_K$
- v.  $(\amalg_K \widetilde{\cap} \amalg_L) \widetilde{\cap} \amalg_M = \amalg_K \widetilde{\cap} (\amalg_L \widetilde{\cap} \amalg_M)$

**Remark 2.** Let  $\amalg_K \in IFPS(U)$ . If  $\amalg_K \neq \amalg_{\Phi}$  or  $\amalg_K \neq \amalg_{\tilde{E}}$ , then  $\amalg_K \widetilde{\cup} \amalg_K^c \neq \amalg_{\tilde{E}}$  and  $\amalg_K \widetilde{\cap} \amalg_K^c \neq \amalg_{\Phi}$ .

**Proposition 6.** Let  $\amalg_K, \amalg_L, \amalg_M \in IFPS(U)$ . Then

*i.* 
$$\amalg_K \widetilde{\cup} (\amalg_L \widetilde{\cap} \amalg_M) = (\amalg_K \widetilde{\cup} \amalg_L) \widetilde{\cap} (\amalg_K \widetilde{\cup} \amalg_M)$$
  
*ii.*  $\amalg_K \widetilde{\cap} (\amalg_L \widetilde{\cup} \amalg_M) = (\amalg_K \widetilde{\cap} \amalg_L) \widetilde{\cup} (\amalg_K \widetilde{\cap} \amalg_M)$ 

**Proposition 7.** Let  $\amalg_K, \amalg_L IFPS(U)$ . Then following DeMorgans types of results are true.

- *i.*  $(\coprod_K \widetilde{\cup} \amalg_L)^c = \coprod_K^c \widetilde{\cap} \amalg_L^c$
- *ii.*  $(\coprod_K \widetilde{\cap} \amalg_L)^c = \amalg_K^c \widetilde{\cup} \amalg_L^c$

**Definition 13.**  $\amalg_K, \amalg_L \in IFPS(U)$ . Then OR-sum of  $\amalg_K$  and  $\amalg_L$ , denoted by  $\amalg_K \vee^+ \amalg_L$ , is defined by

$$\Pi_{K} \vee^{+} \Pi_{L}(x) = \left\{ (\langle x, \alpha_{K}(x) + \alpha_{L}(x) - \alpha_{K}(x)\alpha_{L}(x), \beta_{K}(x)\beta_{L}(x) \rangle, f_{K\widetilde{\cup}L}(x)) : x \in E \right\}$$

where  $f_{K\widetilde{\cup}L}(x) = f_K(x) \cup f_L(x)$ .

**Definition 14.**  $\amalg_K, \amalg_L \in IFPS(U)$ . Then AND-sum of  $\amalg_K$  and  $\amalg_L$ , denoted by  $\amalg_K \wedge^+ \amalg_L$ , is defined by

$$\Pi_{K} \wedge^{+} \Pi_{L}(x) = \left\{ (\langle x, \alpha_{K}(x) + \alpha_{L}(x) - \alpha_{K}(x)\alpha_{L}(x), \beta_{K}(x)\beta_{L}(x) \rangle, f_{K \cap L}(x)) : x \in E \right\}$$

where  $f_{K \cap L}(x) = f_K(x) \cap f_L(x)$ .

**Proposition 8.** Let  $\amalg_K, \amalg_L, \amalg_M \in IFPS(U)$ . Then

*i.*  $\amalg_K \vee^+ \amalg_\Phi = \amalg_K$ 

*ii.* 
$$\amalg_K \vee^+ \amalg_{\tilde{E}} = \amalg_{\tilde{E}}$$

*iii.* 
$$\amalg_K \lor^+ \amalg_L = \amalg_L \lor^+ \amalg_K$$

*iv.* 
$$\amalg_K \wedge^+ \amalg_L = \amalg_L \wedge^+ \amalg_K$$

$$v. \ (\amalg_K \vee^+ \amalg_L) \vee^+ \amalg_M = \amalg_K \vee^+ (\amalg_L \vee^+ \amalg_M)$$

vi.  $(\amalg_K \wedge^+ \amalg_L) \wedge^+ \amalg_M = \amalg_K \wedge^+ (\amalg_L \wedge^+ \amalg_M)$ 

**Definition 15.**  $\amalg_K, \amalg_L \in IFPS(U)$ . Then OR-product  $\amalg_K$  and  $\amalg_L$ , denoted by  $\amalg_K \vee^{\times} \amalg_L$ , is defined by

$$\begin{aligned} \Pi_{K} \vee^{\times} \Pi_{L}(x) &= \\ \left\{ < x, \alpha_{K}(x)\alpha_{L}(x), \beta_{K}(x) + \beta_{L}(x) - \beta_{K}(x)\beta_{L}(x) >, f_{K\widetilde{\cup}L}(x)) : x \in E \right\} \end{aligned}$$

where  $f_{K \cup L}(x) = f_K(x) \cup f_L(x)$ .

**Definition 16.**  $\amalg_K, \amalg_L \in IFPS(U)$ . Then AND-product  $\amalg_K$  and  $\amalg_L$ , denoted by  $\amalg_K \wedge^{\times} \amalg_L$ , is defined by

$$\begin{aligned} \Pi_{K} \wedge^{\times} \Pi_{L}(x) &= \\ \left\{ < x, \alpha_{K}(x)\alpha_{L}(x), \beta_{K}(x) + \beta_{L}(x) - \beta_{K}(x)\beta_{L}(x) >, f_{K \cap L}(x)) : x \in E \right\} \end{aligned}$$

where  $f_{K \cap L}(x) = f_K(x) \cap f_L(x)$ .

**Proposition 9.** Let  $\amalg_K, \amalg_L, \amalg_M \in IFPS(U)$ . Then

*i.*  $\amalg_K \wedge^{\times} \amalg_\Phi = \amalg_\Phi$ 

*ii.* 
$$\coprod_K \wedge^{\times} \coprod_{\tilde{E}} = \coprod_K$$

- *iii.*  $\amalg_K \wedge^{\times} \amalg_L = \amalg_L \wedge^{\times} \amalg_K$
- *iv.*  $\amalg_K \lor^{\times} \amalg_L = \amalg_L \lor^{\times} \amalg_K$
- v.  $(\amalg_K \wedge^{\times} \amalg_L) \wedge^{\times} \amalg_M = \amalg_K \wedge^{\times} (\amalg_L \wedge^{\times} \amalg_M)$
- vi.  $(\amalg_K \lor^{\times} \amalg_L) \lor^{\times} \amalg_M = \amalg_K \lor^{\times} (\amalg_L \lor^{\times} \amalg_M)$

## 4. Intuitionistic FP-soft decision making method

In this section, we have defined a reduced intuitionistic fuzzy set of an intuitionistic FP-soft set, that produce an intuitionistic fuzzy set from an intuitionistic FP-soft set. We then have defined a reduced fuzzy set of an intuitionistic fuzzy set, that produce a fuzzy set from an intuitionistic fuzzy set. These sets present an adjustable approach to intuitionistic FP-soft sets based decision making problems.

**Definition 17.** Let  $\amalg_K$  be an intuitionistic FP-soft set. Then, a reduced intuitionistic fuzzy set of  $\amalg_K$ , denoted by  $K_{rif}$ , defined as follows

$$K_{rif} = \left\{ \langle u, \alpha_{K_{rif}}(u), \beta_{K_{rif}}(u) \rangle : u \in U \right\}$$

where

$$\alpha_{K_{rif}} : U \to [0, 1], \ \alpha_{K_{rif}}(u) = \frac{1}{|U|} \sum_{x \in E, u \in U} \alpha_K(x) \chi_{f_K(x)}(u)$$
  
$$\beta_{K_{rif}} : U \to [0, 1], \ \beta_{K_{rif}}(u) = \frac{1}{|U|} \sum_{x \in E, u \in U} \beta_K(x) \chi_{f_K(x)}(u)$$

where

$$\chi_{f_K(x)}(u) = \begin{cases} 1, & u \in f_K(x) \\ 0, & u \notin f_K(x) \end{cases}$$

Here,  $\alpha_{K_{rif}}$  and  $\beta_{K_{rif}}$  are called reduced-set operators of  $K_{rif}$ . It is clear that  $K_{rif}$  is an intuitionistic fuzzy set over U.

**Definition 18.**  $\coprod_{K} \in IFPS(U)$  and  $K_{rif}$  be reduced intuitionistic fuzzy set of  $\amalg_{K}$ . Then, a reduced fuzzy set of  $K_{rif}$  is a fuzzy set over U, denoted by  $K_{rf}$ , defined as follows

$$K_{rf} = \left\{ \mu_{K_{rf}}(u) / u : u \in U \right\}$$

where

$$\mu_{K_{rf}}: U \to [0, 1], \ \mu_{K_{rf}}(u) = \alpha_{K_{rif}}(u)(1 - \beta_{K_{rif}}(u))$$

Now, we construct an intuitionistic FP-soft decision making method by the following algorithm to produce a decision fuzzy set from a crisp set of the alternatives.

According to the problem, decision maker;

- i. constructs a feasible intuitionistic fuzzy subsets K over the parameters set E,
- ii. constructs an intuitionistic FP-soft set  $\coprod_K$  over the alternatives set U,
- iii. computes the reduced intuitionistic fuzzy set  $K_{rif}$  of  $\coprod_K$ ,
- iv. computes the reduced fuzzy set  $K_{rf}$  of  $K_{rif}$ ,

v. chooses the element of  $K_{rf}$  that has maximum membership degree.

Now, we can give an example for the intuitionistic FP-soft decision making method. Some of it is quoted from example in [11].

**Example 2.** Assume that a company wants to fill a position. There are 5 candidates who fill in a form in order to apply formally for the position. There is a decision maker (DM), that is from the department of human resources. He want to interview the candidates, but it is very difficult to make it all of them. Therefore, by using the intuitionistic FP-soft decision making method, the number of candidates are reduced to a suitable one. Assume that the set of candidates  $U = \{u_1, u_2, u_3, u_4, u_5\}$  which may be characterized by a set of parameters  $E = \{a_1, a_2, a_3, a_4\}$ . For i = 1, 2, 3, 4 the parameters  $a_i$  stand for experience, computer knowledge, training and young age, respectively. Now, we can apply the method as follows:

Step i. Assume that DM constructs a feasible intuitionistic fuzzy subsets K over the parameters set E as follows;

$$K = \{ \langle x_1, 0.7, 0.3 \rangle, \langle x_2, 0.2, 0.5 \rangle, \langle x_3, 0.5, 0.5 \rangle, \langle x_4, 0.6, 0.3 \rangle \}$$

Step ii. DM constructs an intuitionistic FP-soft set  $\amalg_K$  over the alternatives set U as follows;

$$\Pi_{K} = \left\{ (\langle x_{1}, 0.7, 0.3 \rangle, \{u_{1}, u_{2}, u_{4}\}), (\langle x_{2}, 0.2, 0.5 \rangle, U), (\langle x_{3}, 0.5, 0.5 \rangle, \{u_{1}, u_{2}, u_{4}\}), (\langle x_{4}, 0.6, 0.3 \rangle, \{u_{2}, u_{3}\}) \right\}$$

Step *iii.* DM computes the reduced intuitionistic fuzzy set  $K_{rif}$  of  $\coprod_K$  as follows;

$$K_{rif} = \left\{ (< u_1, 0.28, 0.26 >, < u_2, 0.40, 0.32 >, < u_3, 0.16, 0.16 >, < u_4, 0.28, 0.32 >, < u_5, 0.04, 0.10 > \right\}$$

Step iv. DM computes the reduced fuzzy set  $K_{rf}$  of  $K_{rif}$  as follows;

$$K_{rf} = \left\{ 0.2072/u_1, 0.2720/u_2, 0.1344/u_3, 0.1904/u_4, 0.0360/u_5 \right\}$$

Step v. Finally, DM chooses  $u_2$  for the position from  $K_{rf}$  since it has the maximum degree 0.2720 among the others.

Note that this decision making method can be applied for group decision making easily with help of the Definition 13, Definition 14 Definition 15 and Definition 16.

# 5. Conclusion

In this paper, we first defined intuitionistic FP-soft sets and their operations. We then presented the decision making method on the intuitionistic FP-soft set theory. Finally, we provided an example that demonstrated that the decision making method can successfully work. It can be applied to problems of many fields that contain uncertainty such as computer science, game theory, and so on.

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