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Author:
Zaman, F; Elsayed, SM; Ray, T; Sarker, RA

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Co-evolutionary Approach for Strategic Bidding in Competitive Electricity Markets

Forhad Zaman, Saber M. Elsayed, Tapabrata Ray and Ruhul A. Sarker
School of Engineering and Information Technology
University of New South Wales Canberra, Australia
(E-mails: md.zaman@student.adfa.edu.au, s.elsayed@unsw.edu.au, t.ray@unsw.edu.au, r.sarker@unsw.edu.au)

Abstract— Determining optimal bidding strategies in a competitive electricity market to maximize the profit of each bidder is a challenging economic game problem. In this paper, it is formulated as a bi-level optimization problem in which, in the lower level, the community's social welfare is maximized by solving a power flow problem while, in the upper level, the profits of individual bidders are maximized. In this bidders' game, instead of using a set of discrete strategies as is usual, we consider continuous functions as strategies. To solve the upper-level problem, two co-evolutionary approaches are proposed and, for the lower level, an interior point algorithm is applied. Three *IEEE* benchmark problems in four different scenarios are solved and their results compared with those obtained from two conventional approaches and the literature which indicate that the proposed approaches have some merit regarding quality and efficiency.

Keywords— bidding problem, energy market, co-evolutionary algorithm, genetic algorithm, differential evolution.

NOMENCLATURE

i, j, n, p, g	Indices of GENCO, consumer, player, individual of a sub-population, and current generation number, respectively;
K	Total number of transmission grid nodes ($K > 0$);
I	Total number of generators ($I > 0$);
J	Total number of loads (customers) ($J > 0$);
N	Total number of bidders in market ($N = I + J$);
I_k	Number of generators at node k;
J_k	Number of loads at node k;
P_i	Real power injection by generator $i, \forall i$;
P_i^{min}, P_i^{max}	Minimum and maximum real power limits of ith generator;
q_j	Real power demand for load $j \in i$ at node i;
δ_1	Voltage angle at reference bus with fixed value of 0;
δ_k	Voltage angle (in radians) at node $k, \forall k$; $k \neq 1$;
F_{km}	Real power flow through branch connection from nodes k to m;
BR	Set of all distinct branches of km, $k < m$;
$PNetInject_k$	Net injected real power at each node k;
x_{km}	Reactance for branches k to m;
B_{km}	Susceptance ($1/x_{km}$) for branches k to m;
F_{km}^U, F_{km}^L	Lower and upper limits of real power flow for branches k to m;
a_i, b_i, c_i	Cost coefficients of i^{th} generator;
$\acute{b}_i, \acute{c}_i$	Quiescent coefficients of marginal cost coefficient at i^{th} generator;
d_j, e_j	Coefficients of j^{th} consumers' utility function;
$\acute{d}_j, \acute{e}_j$	Quiescent coefficients of j^{th} demand curve;
k_{g_i}	Bidding coefficient of i^{th} generator;
k_{d_j}	Bidding coefficient of j^{th} consumer;
$k_{g_i}^{min}, k_{g_i}^{max}$	Lower and upper limits of i^{th} generator;
$k_{d_j}^{min}, k_{d_j}^{max}$	Lower and upper limits of j^{th} consumer;
λ_k	Locational marginal price (LMP) at k^{th} node;
$\lambda_{P_i}, \lambda_{d_j}$	LMPs of i^{th} generator and j^{th} consumer, respectively.

I. INTRODUCTION

Over the last decade, the electricity markets in many countries have become more decentralized and deregulated in order to increase their economic efficiency and reduce costs. As a consequence, they are no longer monopolistic but being opened up to competition among both suppliers and consumers [1-3]. In this situation, suppliers, *i.e.,* generator companies (GENCOs) and consumers (e.g., large industries, distributor companies, residential loads, etc.) simultaneously submit their bids to an independent system operator (ISO) that determines the market clearing price (MCP) and power dispatch (PD) of each winning bidder by solving an optimal power flow (OPF) problem, with the aim of finding an optimal operating point of a power system by maximizing its community social welfare (CSW) subject to its network and physical constraints. The CSW is defined as the difference between the profits obtained by trading electricity to consumers and the expenses of purchasing it from GENCOs. Once a winning bidder is informed about the MCP and its allocated quantity of PD, its profit is calculated based on its actual cost and revenue. Note that the MCPs of all bidders are the same when transmission congestions (TCs) are ignored but, if they are considered, the MCPs vary significantly from location (or node) to location which is called the locational market price (LMP) [4].

As the profit of a bidder depends on both its own submitted bid and those of its rivals, each bidder plays a game by optimizing its own bidding behavior with respect to those of its competitors as well as power system constraints. An excessively high bid by a player may not be selected by the ISO while a lower one may not cover its own costs. Therefore, it is a challenging optimization problem to select an appropriate bidding strategy for maximizing the profits of all bidders [5].

During the last decade, numerous studies have been conducted to determine the optimal bidding strategy based on different market models, of which optimization and game theory-based equilibrium models are the most popular [2, 6]. In optimization, the problem is solved for a particular player by ignoring other players' bidding behaviors [1]. In this process, a GENCO or consumer first forecasts the MCP and rivals' bidding strategies, and then solves a profit maximization problem using an appropriate algorithm, such as dynamic, fuzzy linear or stochastic dynamic programming [7]. However, estimating the MCP and rivals' bidding strategies is very difficult and, even after doing it, the actual profits may significantly vary from predictions as it is assumed that the LMP is independent of the players' submitted bids [8].

On the contrary, in a game theory-based equilibrium model, a player optimizes its bidding strategy by investigating the interactions of its rivals' bidding behaviors. In it, a GENCO or consumer is represented as a player, economic benefits constitute payoffs and players' options are treated as strategies while it is assumed that all players are rational and have some common knowledge of the actual cost function of each bidder from historical data. Each player ultimately chooses one strategy from a set of known ones which, as each has a payoff assigned to it by the profit function, means that the optimal solution can be reached via the Nash equilibrium (NE). A NE is based on the strategies of all players in which one player cannot increase its payoff by changing its own strategy while the others' strategies remain the same, with the solution known as a saddle point of the equilibrium model. This approach is very popular among researchers and practitioners for solving energy market problems [6, 9, 10].

An equilibrium model is classified as a: *(i)* Bertrand game; *(ii)* Cournot game; *(iii)* Stackelberg model; and *(iv)* supply function equilibrium (SFE). In the Bertrand game, the market price is considered a bidding parameter in which it is assumed that all players have a constant unit cost, with capacity constraints ignored when competing on the price offered to consumers. In both the Cournot and Stackelberg models, the amount of power to be produced by each player is considered a strategic variable, with the difference between these approaches being that the former allows the strategic variables of all players to be simultaneously improved while, in the latter, the leader improves its strategic variable first and then the followers sequentially change theirs. As a consequence, because all players in the Stackelberg model do not choose their quantities simultaneously, the largest one acts as the leader and can manipulate the market. In the SFE model, a linear function is used for each bidder's strategic variable, where the coefficients of the supply function are simultaneously improved to reach the maximum profit [11].

Apart from the above classifications, the players in an equilibrium model can be either cooperative or non-cooperative. In the former, the participants coordinate their strategies in order to maximize the profits of all players while, in the latter, a player maximizes its own profit regardless of those of its rivals, with no commitment to coordinating their strategies [12]. Of the above methods, the non-cooperative SFE game model is more appealing due to its realistic characterisations of the strategic variables which reflect real-life bidding rules in the electricity market [1, 13]. It is widely used both in literature and practice [11]; for example, English and Welsh wholesale electricity spot markets [6].

Solving a non-cooperative SFE model has gained a great deal of attention over the last decade, with bi-level programming techniques widely used. In it, each independent player maximizes its profit in the upper level while the ISO's CSW is maximized in the lower level by solving a nonlinear OPF optimization problem [14-16]. Each decision entity independently optimizes its own objective but is affected by the actions of other entities in a hierarchy. However, this bi-level problem is a challenging optimization problem because it contains a nested optimization task within the constraints of another optimization problem [14]. It becomes more complex in the presence of difficult mathematical properties of the problem, such as multi-modality, non-convexity, non-differentially and others. This problem is inherently harder to solve than traditional mathematical programs, as pointed out in [17]. Therefore, compared with classical techniques, various evolutionary algorithms (EAs), such as genetic algorithms (GAs) [18-24], differential evolution (DE) [25-28], evolutionary programming (EP) [29] and a bat-inspired algorithm [1, 30] are now generating interest in the research community for solving this problem. In these algorithms, a conventional iterative (IT) approach is used to determine the optimal bidding strategies of all participating players, with the bidding strategy of each updated sequentially by one in an iteration to maximize its profit while those of its rivals remain unchanged. This process continues until the bidding strategy of a player improves, with the algorithm terminated as soon as the NE is reached [1]. However, as it solves the bidding problem of each bidder one after the other, it may take too long when there are many bidders which is one of the issues addressed in this paper. Moreover, as most of the abovementioned methods, a game based bidding strategies were used that the bids were represented as discrete quantities such as bidding high, bidding medium or bidding low, the payoff matrices were easily determined by computing all possible combinations of strategies. However, in reality, a player in the energy market submits its bid within a given range [31] that results to the size of payoff matrix becomes infinite and impossible to evaluate all the combinations [32].

In this paper, a non-cooperative bi-level SFE model of an electricity market is considered, in which the bidding strategies represented as the supply functions of the bidders instead of a set of known discrete strategies as is usually applied. We develop two co-evolutionary (CE) approaches for solving the upper-level problem, the first is based on a real-coded GA and the other on a self-adaptive DE. In both variants, each bidder's strategies are evolved in a sub-population with exchanging information among these subpopulations to find the overall best solutions. The lower-level problem is formulated as an OPF problem and solved using an interior point (IP) algorithm with the aim of maximizing the CSW considering power system constraints. In addition, the non-convex OPF problem is formulated as a strictly convex quadratic programming (SCQP) using the linear formulation of the power flow constraints with the quadratic cost function. In the SFE model, both GENCOs and consumers act as independent players that maximize their own profits considering the interactions of their rivals. As the proposed CE algorithm determines the bidding actions of all players simultaneously under the N-subpopulations for N-players which results in a reduction of the computational time significantly. The performances of the proposed CE approaches for solving three well-known benchmark problems are compared with those of two conventional iterative ones [4, 33] and results from the literature. We also analyze the effects of different components on their performances and demonstrate that these methods outperform those of all the other algorithms with which they are compared.

The rest of this paper is organized as follows: Section II presents the formulation of the competitive electricity market; Section III a brief literature review, Section IV solution approaches using the proposed methodology; section V the experimental results; Section VI key discussions; and Section VI conclusions and suggestions for future work.

II. SFE MODEL

Generally, a competitive electricity market is represented as an equilibrium model to determine the NE for a bid-based pool market [11]. It is formulated as a bi-level optimization problem with the objective in the upper level to maximize the profit of each bidder of either GENCOs or consumers by anticipating the profit-maximization actions of its rivals [15, 16]. In the lower level, the electrical power network is represented using an OPF optimization problem with the aim of maximizing the CSW subject to the nonlinear balance, branch flows and capacity constraints of the real and reactive powers. However, in practice, the OPF problem is typically approximated by a more tractable 'DC-OPF' problem that focuses exclusively on real power constraints in a linearized form by simplifying some restrictions regarding voltage magnitudes, voltage angles, admittances and the reactive power [34, 35]. We use DC-OPF to represent the electrical power network which includes the constraints of active transmission power flows, transmission line (TL) capacities, active power generation, nodal voltage angle, and active power demands.

In the following subsections, we discuss (i) formulations of the SFE model with different bidding strategies of GENCOs and consumers, (ii) the ISO's DC-OPF problem and (iii) the profit functions of GENCOs and consumers. It is also noted that, for the variable notations, a boldface one indicates that it is a matrix or vector and a normal italic one a scalar.

A. Electricity market parameters

As previously mentioned, the electricity market we consider has both strategic generating firms (*i.e.*, GENCOs) and consumers (*i.e.*, loads) that are profit maximizers with respect to their bids and rivals. Its structure is based on a single-period bidding model in which each participant submits a bid to the ISO in terms of its supply function. Once the market pool is closed, the ISO solves the DC-OPF and declares the MCP and amount of power to be generated by each winning bidder. Then, each bidder calculates its profit based on its revenue and actual cost.

To maximize its individual profit, each bidder optimizes its bidding function with respect to possible bids from its rivals. Some assumptions made are that each player *(i)* knows the market rules, *(ii)* has complete information of the actual generation costs of itself and its rivals, *(iii)* knows the range of its bidding parameters and those of its rivals from historical data, and *(iv)* knows the capacities of the TLs connected to the market [14].

It is also assumed that each GENCO has a single generator with the quadratic cost function [36]:

$$C_i = a_i + b_i P_i + c_i P_i^2, \ \forall i \in I \tag{1}$$

The marginal cost of the i^{th} generator is:

$$MC_i = \frac{dC_i}{dP_i} = b_i + 2c_i P_i,\ \forall i \in I \quad (2)$$

Since each GENCO plays a game in the market, rather than submitting an actual marginal cost, a strategic quasi-function (Eqn. (2)) called a linear supply function is submitted to the ISO as [1, 11]:

$$B_{g_i} = b'_i + c'_i P_i,\quad \forall i \in I \quad (3)$$

The consumers' utility cost function is the quadratic inverse form [11]:

$$D_j = d_j q_j - e_j q_j^2\ \ \forall j \in J \quad (4)$$

Subsequently, the load demand function of the linear form is the inverse function with a negative gradient:

$$MD_j = d_j - 2e_j q_j\ \ \forall j \in J \quad (5)$$

Again, as a strategic consumer, it plays with the quasi-function in Eqn. (5):

$$B_{L_j} = d'_j - e'_j q_j\ \ \forall j \in J \quad (6)$$

Based on [1, 11, 37], the bidding parameterizations can be selected in the following four ways.

1) *Intercept parameterization*: the strategic players adjust the intercepts of $\acute{b}_i, \forall i$ and $\acute{d}_j, \forall j$ of their marginal cost functions in Eqns. (3) and (6), respectively, to construct their profit-maximizing bids for submission to the ISO while keeping the slope constant as $\acute{c}_i = 2c_i, \forall i$ and $\acute{e}_j = 2e_j, \forall j$.

2) *Slope parameterization*: the strategic bids of the players are modeled by varying the slope of the marginal cost functions in Eqns. (3) and (6) with the values of $\acute{c}_i \forall i$ and $\acute{e}_j \forall j$, respectively, while keeping the intercepts constant as $\acute{b}_i = b_i, \forall i$ and $\acute{d}_j = d_j, \forall j$, respectively.

3) *Slope-and-intercept parameterization*: the players adjust both intercepts ($\acute{b}_i, \forall i$ and $\acute{d}_j, \forall j$) and slopes ($\acute{c}_i, \forall i$ and $\acute{e}_j, \forall j$) independently and simultaneously as their strategic variables to allow more degrees of freedom for choosing the strategic supply function.

4) *Slope intercept parameterization*: the strategic players adjust both the slopes and intercepts in the supply functions in Eqns. (3) and (6) for GENCOs and consumers, respectively, but in a fixed linear relationship between the true and quasi-values of the marginal cost function. This can be interpreted as multiplying the marginal cost functions by arbitrary non-negative constants, say $k_{g_i} \forall i$ and $k_{d_j} \forall j$, in order to construct the supply function bids shown in Eqns. (7) and (8), respectively.

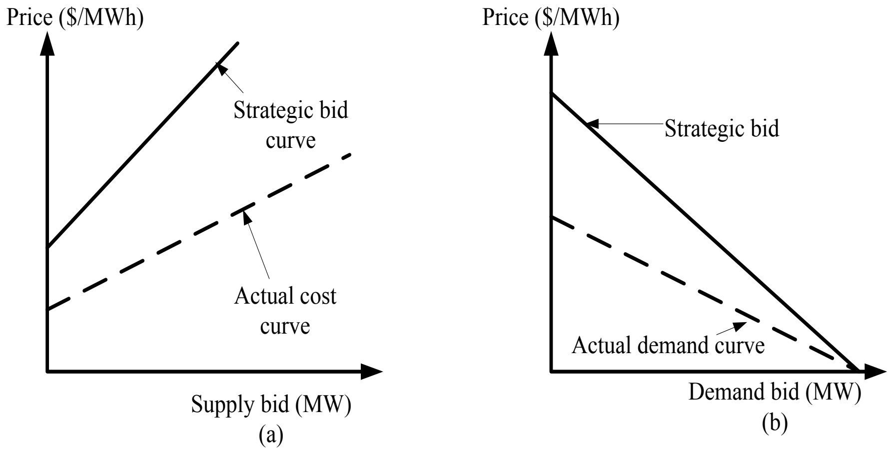

Fig. 1. Strategic bidding for supply and demand

Due to the effectiveness of '*slope intercept parameterization'* in real life, we use it with $k_{g_i} \forall i$ and $k_{d_j} \forall j$ the strategic variables for GENCOs and consumers, respectively. The strategic functions for GENCOs and consumers, respectively, as shown in Fig. 1 by dotted lines, represent the true supply functions and the solid lines the strategic supply ones obtained by multiplying the factors of $k_{g_i} \forall i$ and $k_{d_j} \forall j$ as:

$$B_{g_i} = k_{g_i}\left(b_i + 2c_i P_i\right) = b_i' + c_i' P,\ \forall i \in I$$
$$b_i' = k_{g_i} b_i;\ \ c_i' = 2k_{g_i} c_i\ \ \forall i \tag{7}$$

$$B_{Lj} = k_{d_j}\left(d_j - 2e_j q_j\right) = d_j' - e_j' q_j,\ \forall j \in J$$
$$d_j' = k_{d_j} d_j;\ \ e_j' = 2k_{d_j} e_j\ \ \forall j \tag{8}$$

$$\text{subject to: } k_{g_i}^{\min} \le k_{g_i} \le k_{g_i}^{\max}\ \forall i \text{ and } k_{d_j}^{\min} \le k_{d_j} \le k_{d_j}^{\max}\ \forall j \tag{9}$$

B. Formulation of ISO's optimization problem

Once the participants in the market submit their strategic supply functions, the ISO runs a DC-OPF problem to maximize the CSW subject to the power system's transmission constraints. The ramp rate constraints are ignored in this formulation as it is assumed that they are sufficiently high. Also, start-up and shut-down decisions are not considered as it is assumed that the on/off status of a unit is known a priori at the time of constructing bidding strategies [38]. Since the ISO receives strategic bids from each player, its objective function is represented by the quasi CSW that incorporates the strategic variables $k_{g_i} \forall i$ and $k_{d_j} \forall j$ as:

$$\text{Max}:\ \Pi_{\text{ISO}} = \sum_{j=1}^{J}\left(d_j' q_j - \frac{1}{2} e_j' q_j^2\right) - \sum_{i=1}^{I}\left(b_i' P_i + \frac{1}{2} c_i' P_i^2\right) \tag{10}$$
$$\text{subject to: } P_i, q_j, \delta_k, i = 1,2,\ldots,I; j = 1,2,\ldots,J; k = 1,2,\ldots,K$$

Eqn. (10) represents the objective function of the ISO's DC-OPF problem in which it is equal to the consumers' benefit minus the generation costs considering the strategic bids. The DC-OPF problem has the following constraints when TCs are included.

i. Real power balance [39] constraints for each node ($k = 1,2, \ldots, K$):

$$\sum_{i=1}^{I_k} P_i - \sum_{j=1}^{J_k} q_j - PNetInject_k = 0 \tag{11}$$

where
$$PNetInject_k = \sum_{km/mk \in BR} F_{km} \tag{12}$$

$$F_{km} = B_{km}\left(\delta_k - \delta_m\right) \tag{13}$$

ii. Limits of real power flow through each branch ($km \in BR$):

$$\left|F_{km}\right| \le F_{km}^{U} \tag{14}$$

iii. Limits of real power of each generator ($i = 1,2, \ldots, I$):

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{15}$$

The above DC-OPF problem is a nonlinear and non-convex single-objective optimization problem, and generally represented by its first-order Karush–Kuhn–Tucker (KKT) conditions [11]. Consequently, the LMPs, power output from each GENCO, utility demand of each consumer, transmission flows and nodal voltage angles are calculated to simultaneously satisfy each market participant's first-order optimality conditions for maximizing their net benefits (KKT conditions) while clearing the market (supply = demand).

However, the lower level DC-OPF problem has to be optimized in every generation with the best solution is used in the evaluation of the objective function in the upper level profit maximization problem. Therefore, an efficient technique is desired in solving the lower level problem to reduce the computational complexity of the bi-level optimization problem. This can be achieved by using an SCQP-based technique for solving the lower level optimization problem [34, 35, 40]. Note that, such a technique can be only applied under the assumption that the objective function is quadratic, which has been satisfied of Eqn. (10). The SCQP-based DC-OPF problem is presented in the following subsection.

C. Formulation of ISO's optimization problem based on SCQP

The DC-OPF problem can be represented using an SCQP-based technique by eliminating the voltage angles using substitution in which the ISO's objective function (Eqn. (10)) is subject to the equality constraints of the real power balance in Eqn. (11), is expressed as an SCQP and accumulates a soft penalty function of the sum of squared voltage angle differences as [34, 35, 40]:

$$\text{Min}: \sum_{i=1}^{I}\left(\frac{1}{2}c_i'P_i^2+b_i'P_i\right)-\sum_{j=1}^{J}\left(d_j'q_j-\frac{1}{2}e_j'q_j^2\right)-\pi\left[\sum_{km\in BR}(\delta_k-\delta_m)^2\right] \tag{16}$$

Assuming a reference bus voltage angle of $\delta_1 = 0$, Eqn. (16) is reduced as:

$$\text{Min}: \sum_{i=1}^{I}\left(\frac{1}{2}c_i'P_i^2+b_i'P_i\right)-\sum_{j=1}^{J}\left(d_j'q_j-\frac{1}{2}e_j'q_j^2\right)-\pi\left[\sum_{1m\in BR}\delta_m^2+\sum_{km\in BR,k\geq 2}(\delta_k-\delta_m)^2\right] \tag{17}$$

subject to:

$$\sum_{i=1}^{I_k}P_i-\sum_{km/mk\in BR}B_{km}(\delta_k-\delta_m)=\sum_{j=1}^{J_k}q_j \quad \forall k=1,2,\cdots,K \tag{18}$$

$$-B_{km}(\delta_k-\delta_m)\geq -F_{km}^U \quad \forall k,m=1,2,\cdots,K \tag{19}$$

$$B_{km}(\delta_k-\delta_m)\geq -F_{km}^U \quad \forall k,m=1,2,\cdots,K \tag{20}$$

$$P_i\geq P_i^{\min} \quad \forall i=1,2,\cdots,I \tag{21}$$

$$-P_i\geq -P_i^{\max} \quad \forall i=1,2,\cdots,I \tag{22}$$

To efficiently solve the above optimization problem, we develop a matrix representation of the objective function in Eqn. (17) and constraints in Eqns. (18) to (22) as:

$$\text{Min:} \qquad f(\mathbf{x})=\frac{1}{2}\mathbf{x}^T\mathbf{G}\mathbf{x}+\mathbf{f}^T\mathbf{x} \tag{23}$$

$$\text{subject to:} \qquad \mathbf{C_{in}}\mathbf{x}^T\geq \mathbf{b_{in}} \tag{24}$$

$$\mathbf{C_{eq}}\mathbf{x}^T=\mathbf{b_{eq}} \tag{25}$$

Details of the coefficients of Eq. (23) to (25) are shown in Appendix A.

D. Formulating optimization problem of each player

Once the lower-level SCQP problem is solved, the values of P_i, q_j, λ_{P_i}, λ_{d_j} and F_{km} $\forall i,j,k$ are calculated based on their primal and dual variables in the KKT representation shown in Appendix B. Then, the upper-level optimization problems of the strategic firms, in which individual profits are maximized, are solved given the revenue minus the true generation cost as:

$$\begin{aligned} &Max: \textstyle\prod_{P_i}=\lambda_{P_i}P_i-(a_i+b_iP_i+c_iP_i^2) \quad \forall i\in I \\ &\text{subjec to}: k_{g_i}^{\min}\leq k_{g_i}\leq k_{g_i}^{\max} \end{aligned} \tag{26}$$

$$\begin{aligned} &Max: \textstyle\prod_{d_j}=\left(d_jq_j-e_jq_j^2\right)-\lambda_{d_j}q_j \quad \forall j\in J \\ &\text{subjec to}: k_{d_j}^{\min}\leq k_{d_j}\leq k_{d_j}^{\max} \end{aligned} \tag{27}$$

where $\prod_P$ and $\prod_d$ are the profits of the GENCO and consumer, respectively. As the variables λ_P, λ_d, P and q are produced by the SCQP problem given in Eqns. (23) to (25), they can be expressed as implicit functions of the players' bidding strategies as $\{\mathbf{k_p}=k_{P_1},k_{P_2},\dots,k_{P_I}\}$ and $\{\mathbf{k_d}=k_{d_1},k_{d_2},\dots,k_{d_J}\}$. Therefore, they should satisfy the KKT conditions of the ISO optimization problem.

III. Brief Review of EAs in SFE

The SFE game model is a nonlinear, non-convex, bi-level optimization problem in which each GENCO and consumer maximizes its profit by optimizing its bidding coefficients ($k_{g_i}\forall i$ and $k_{d_j}\forall j$ in Eqns. (26) and (27), respectively) considering the interactions of its rivals [41]. The profit maximization of a player is subject to a set of variables (λ_{p_i}, λ_{d_j}, P_i and q_j, $\forall i,j$) determined after solving the nested optimization problem in Eqns. (10) to (15). Therefore, the optimal bidding strategy issue of an electricity market problem becomes a two-level programming one which is difficult to solve using conventional mathematical approaches [26, 42].

During the last decade, a significant number of research studies has used different meta-heuristic algorithms to develop for solving the bi-level SFE model; for example, Azadeh *et al.* [18] applied a GA to determine the optimal bidding strategies of GENCOs in both cooperative and non-cooperative electricity markets. In [19], another GA was used to solve a scenario-based bi-level strategic game in which a player optimizes its bidding strategy by predicting the possible bidding scenarios of its rivals determined from historical data. However, as there are risk factors associated with assuming opponents' bids when a player optimizes its own, an information gap decision theory (IGDT) has been used to formulate a risk-based optimal bidding strategy optimization model, with a modified particle swarm optimization (MPSO) used to solve it [43]. Gountis and Bakirtzis [20] developed a solution approach based on a GA and Monte Carlo (MC) simulation technique [44] for the bi-level SFE model in which rivals' bidding strategies are estimated using the MC technique and the GA determines each bidder's optimal bidding strategy. However, in many of the above methods, consumers are considered non-strategic, *i.e.,* they have no option to participate in the market. Considering a consumer strategy, in [11, 37], the SFE model is solved using four different types of bidding parameters, with the one between the slope and intercept of the bidding curve a strategic variable to achieve a definite equilibrium. To obtain the NE for a competitive energy market, IT solution approaches based on a GA [45] and bat-inspired algorithm [1], in which the bidding strategy of each player is iteratively updated, have been developed. However, as the bidding strategy of each player is updated sequentially in each iteration, approaching the NE is very difficult, even for a small problem, and requires a long computational effort for a large one.

The CE algorithm is an alternative approach that simultaneously determines the optimal bidding strategies of all players and results in a significantly reduced computational time compared with those required for iterative methods. In the literature, several CE approaches for solving different competitive energy markets have been successfully applied [46-48]; for example, a CE approach based on a GA was developed to determine the multi-period optimal bidding strategy for an oligopoly electricity market [49]. In it, each agent (sub-population) uses a reinforcement learning algorithm to increase its profit from one trading period to the next based on experience from past trading hours. Chen *et al.* [27] developed a CE approach based on a GA for solving the real-world electricity market, in which two different SFE models, the affine and piece-wise affine cost functions, are solved and analyzed, which rapidly converges to the affine one. In order to obtain a NE, another CE approach was tested in two different competitive electricity markets, spot and settlement, with the simulation results indicating the effectiveness of the CE algorithm for finding optimal strategies in both markets [50].

However, although in most of the abovementioned approaches, an EA is used in the upper level to determine the optimal bidding strategy of a bi-level problem, simple linear programming or Lagrange multipliers are used to solve the lower-level one to determine the dispatch quantity and LMP of each GENCO. However, since the lower-level problem contains a non-convex and nonlinear objective function and conventional techniques may fail to determine optimal solutions, the solutions of the upper level are either local or sub-optimal [51]. To overcome this drawback, two EAs, GA and DE, were used in the lower and upper levels, respectively, to solve a simple competitive energy market in [4] which showed that the quality of solutions could be further improved if an EA was used in the lower level although the computational time increased. As this procedure does not consider consumers' strategies and TC constraints, this problem is simple and easy to solve using an EA. However, existing procedures cannot guarantee convergence to the optimal solution when the NE does not exist or provide a local solution in the presence of multiple NEs. The number of NEs depends on the number of strategic GENCOs and consumers present in the model; for example, a two-player game is formulated as a linear complementary system, an NE point is easily determined but, when three or more players participate in a bidding market, it is very difficult to solve as the model is no longer linear [1].

Therefore, a new and efficient algorithm for solving a real-world electricity game model is required as existing ones provide only a local or sub-optimal solution which could be further improved [1, 4]. Also, to the best of our knowledge, using CE approaches to solve a highly complex and TC-constrained SFE model for a large electricity market has not yet been explored.

IV. PROPOSED METHOD

In this research, the electricity market considers both non-competitive and competitive participants. In the former market, the players are non-strategic and the ISO model considered a single-objective DC-OPF optimization problem (Eqns. (23) to (25)) solved using a classical optimization technique. However, for comparison purposes, it is also solved using one of two EAs, such as, a GA and DE.

The competitive electricity market is represented as an SFE by formulating it as a bi-level optimization problem with the upper level addressing the individual profits of GENCOs and consumers in Eqns. (26) and (27), respectively, and the lower one describing the ISO's nonlinear DC-OPF problem in Eqns. (23) to (25). As a consequence, the market is a nonlinear, bi-level, nested optimization problem which is extremely difficult to solve using conventional mathematical approaches [52]. Therefore, in this paper, two proposed CE approaches based on two EA variants, a real-coded GA and a self-adaptive DE called CE-GA and CE-DE, respectively, for solving the competitive SFE model. In them, either the GA or DE is used to maximize the individual profit of each strategic firm in the upper level and, in the lower one, an IP method to maximize the CSW because of its superior performance for solving the ISO optimization problem [11, 37].

In the following subsections, we discuss the proposed CE solution approaches for the competitive electricity market and the IP algorithm for solving the ISO's DC-OPF problem.

A. CE approaches

The CE algorithm is an extension of a traditional EA in which more than one population, called sub-populations, are simultaneously evolved. The main difference between the CE and EA algorithms is that the individual fitness function evaluation of the former depends not only on its performance but reflects its interactions with other individuals in other subpopulations [50]. Since the participants in a competitive electricity market interact with each other to maximize their profits, the individuals in the CE algorithms are simultaneously evolved to obtain an optimal bidding strategy for a player considering the possible bids of its rivals and, consequently, achieve an NE [48].

As previously mentioned, the bi-level SFE model is formulated and solved using the proposed CE approaches, the upper-level optimization problem by either a GA or DE and the lower one using a classical optimization technique (the IP method). A multi-population concept is used in the upper level in which each player's individuals compete under a sub-population, starting with some random individuals with the decision variables of which represent the respective player's bidding coefficients ($k_{g_i} i \in I$ and $k_{d_j} j \in J$). To evaluate an individual of a GENCO ($k_{g_i} i \in I$) in Eqn. (26) or a consumer ($k_{d_j} j \in J$) in Eqn. (27), it is necessary to solve the nested lower-level optimization problem in Eqns. (23) to (25) which require all the $k_{g_i} \forall i$ and $k_{d_j} \forall j$. In this case, the fitness functions of a sub-population are evaluated by comprising them with those of the current best individuals from other sub-populations. Therefore, all the individuals in a sub-population are constantly being explored to obtain fitter individuals by interacting with their opponents' best individuals. Once all the parents and offspring are evaluated, the selection operator selects the best individuals in each sub-population. Therefore, the process leads to an incremental improvement as each sub-population continually evolves to meet the increasing pressure from the others. The flowchart and pseudo code of the proposed solution approaches based on the CE algorithms are shown in Fig. 2 and Algorithm I, respectively.

ALGORITHM I: CE APPROACHES

1. Set the number of sub-populations (i.e., the number of players) as $N \in (I + J)$ and $g = 0$.
2. Require: number of maximum generations ($N_G > 1$) and sub-population size (N_P).
3. Generate the N_P number of random individuals of each player as: $k_{p,n} \in \left\{k_{g_i}, k_{d_j}\right\}, \forall i = 1,2,..,I; j = 1,2,...,J;\ p = 1,2,...,N_P,$, using Eqn. (28).
4. $for\ n = 1{:}N$
5. Obtain $\mathbf{k}_{-n}^{best} \leftarrow$ the best individuals found so far from other sub-populations while those considered randomly at $g = 0$.
6. $for\ p = 1{:}N_P$
7. Based on $k_{p,n}$ and $\mathbf{k}_{-n}^{best}$, update the coefficients of $\acute{b}_i, \acute{c}_i, \forall i$ and $\acute{d}_j \acute{e}_j, \forall j$ using Eqns. (7) and (8).
8. Solve the lower-level ISO problem in Eqns. (23) to (25) using the IP method described in section IV B.
9. Calculate $\mathbf{P}, \mathbf{q}, \boldsymbol{\lambda}_\mathbf{p}$ and $\boldsymbol{\lambda}_\mathbf{d}$, as described in Appendix A.
10. Evaluate the profit functions of the upper level using Eqns. (26) and (27) for GENCOs and consumers, respectively.
11. $end\ for\ (p)$
12. $end\ for\ (n)$
13. Sort the individuals in all sub-populations based on their respective upper-level fitness values found in step 10.
14. $for\ g = 1{:}N_G$
15. $for\ n = 1{:}N$
16. Generate each offspring by performing crossover and mutation based on either GA or DE operators.
17. Evaluate the fitness functions of all the offspring using steps 5 to 11.
18. Determine the best N_P individuals from both the parent and child populations as in step 13.
19. $end\ for\ (n)$
20. The algorithm is terminated when $k_{g_i} \forall i$ and $k_{d_j} \forall j$ are unable to change from the last generation.
21. $end\ for\ (g)$

1. Initialization

A CE algorithm starts with some subpopulations, each of which has N_P random individuals. Considering that $N \in I + J$ players participate in the competitive electricity market, each of which has its own sub-population, an individual in a sub-population is initialized in the search space as:

$$k_{p,n} = k_{\min}^{n} + \left(k_{\max}^{n} - k_{\min}^{n}\right)\mathrm{LHS}(N_P),\ \ p \in N_P, n \in N \tag{28}$$

where $k_n \in \{k_{g_i}, k_{d_j}\}$, $k_{\min}^n \in \{k_{g_i}^{min}, k_{d_j}^{min}\}$ and $k_{\max}^n \in \left\{k_{g_i}^{max}, k_{d_j}^{max}\right\} \forall i, j$, and $\mathrm{LHS}(N_P)$ is the N_P of random individuals generated using Latin hypercube sampling (LHS). As the initial values of the decision variables

($k_n \forall n$) are highly significant for achieving a global NE, the initial $k_n \forall n$ are generated using LHS rules which ensure that each probability distribution is evenly sampled within the area of optimization [53].

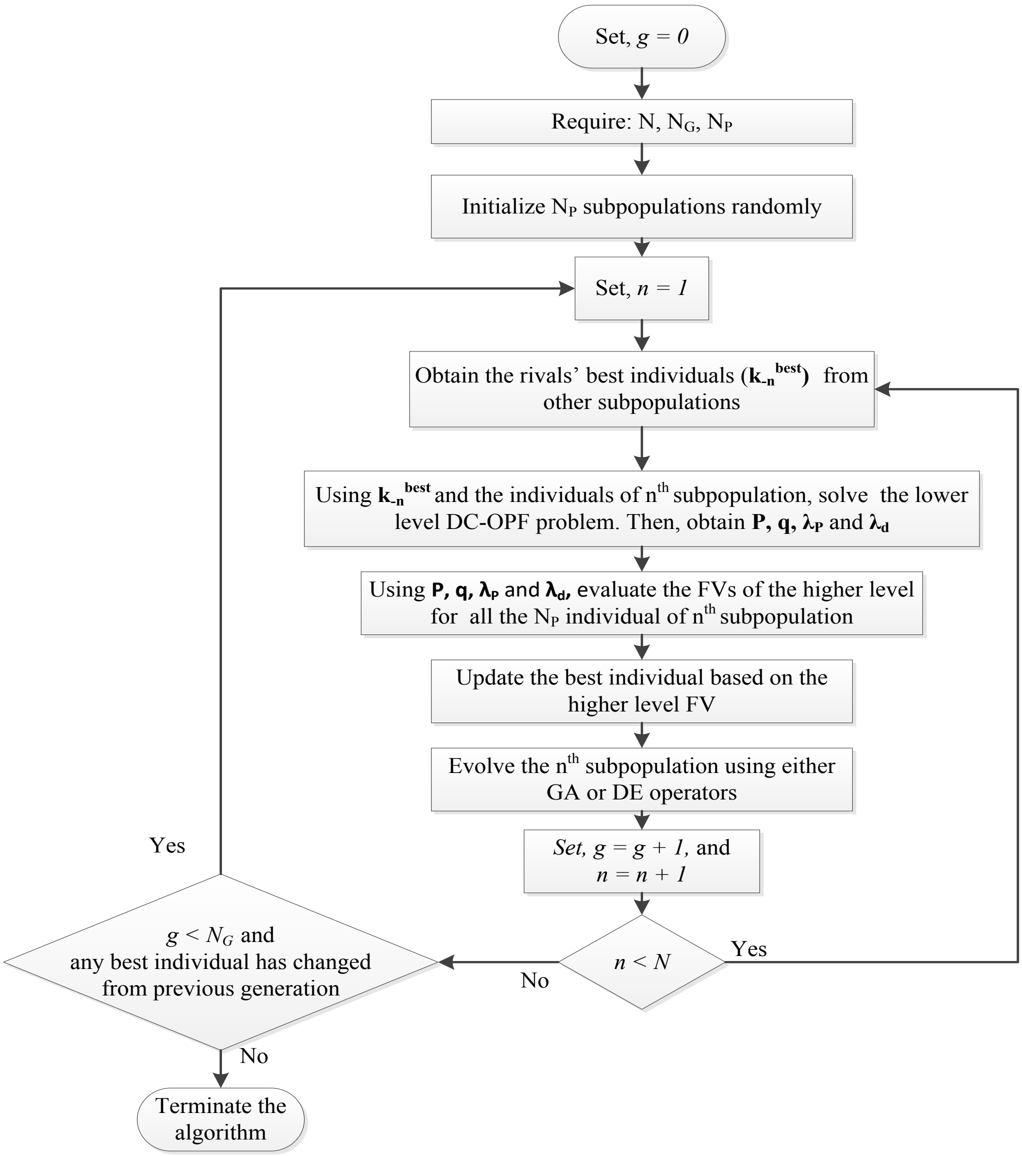

Fig. 2. Flowchart of the proposed CE based algorithms

2. Evaluation

It is already known that the objective function of a player is to maximize its profit by modifying its bidding strategy according to the interactions of its rivals. From the profit functions in Eqns. (26) and (27), it can be seen that, when a player evaluates its fitness function, it must know the values of λ_{P_i} and $P_i, \forall i$ if it is a GENCO and those of λ_{d_j} and $q_j, \forall j$ if it is a consumer. However, to determine either of these parameters, another optimization problem (DC-OPF), in which the objective function in Eqn. (10) involves the bidding coefficients of all players, is required to solve. Therefore, the fitness function evaluation of an individual in a sub-population ($p \in N_P$) for a player ($n \in N$) depends on both its own and its opponents' bidding strategies. Since selecting rivals' bidding strategies is difficult, we use the best bidding strategies found in previous generations. This process is illustrated by an example that assumes the market has two strategic players. The individuals of player 1 in generation $g+1$ are evaluated by taking the best strategy of player 2 from its previous generation (g). It should also be mentioned that, to evaluate the N_P individuals of player 1 in generation $g+1$, its rivals' strategies remain the same as their best ones. For *n*-player, the fitness evaluation of an individual (p) is mathematically expressed as:

$$f_{p,n}(k_{p,n}) = f(k_{p,n}, \mathbf{k}_{-n}^{best}) \;\; p \in N_P, i \in n \tag{29}$$

where $f_{p,n}(k_{p,n})$ is a dummy function representing the actual profit function evaluation for the p^{th} individual of the n^{th} player, $k_{p,n}$ the bidding strategy of the p^{th} individual of the n^{th} player and $\mathbf{k}_{-n}^{best}$ a vector which contains the best bidding strategies of its rivals found so far.

3. Update **k**

To update the bidding strategy of each GENCO ($k_{g_i} \forall i$) and consumer ($k_{d_j} \forall j$), a GA and DE are used for the CE-GA and CE-DE algorithms, respectively, as briefly described in the following subsections.

3.1. GA search operators

As, of the various GA search operators, simulated binary crossover (SBX) and non-uniform mutation (NUM) have shown admirable performances for solving different power system optimization problems [26], they are considered in this research.

3.1.1. SBX

In an SBX operation, firstly, through two tournament operations, the best two parents (k_1 and k_2) are selected from two random ones based on their maximum fitness values (FVs). Then, two offspring (κ_1 and κ_2) are generated as:

$$\kappa_1 = 0.5\left[(1+\beta)k_1 + (1-\beta)k_2\right] \tag{30}$$

$$\kappa_1 = 0.5\left[(1-\beta)k_1 + (1+\beta)k_2\right] \tag{31}$$

$$\text{where } \beta = \begin{cases} (2u)^{1/\eta_c+1} & u \le 0.5, \\ \left(\dfrac{1}{2(1-u)}\right)^{1/\eta_c+1} & u > 0.5, \end{cases} \tag{32}$$

where $u \in [0,1]$ is a random number and η_c a pre-defined parameter of the distribution index set to 3, as in [26].

3.1.2. NUM

The NUM operator is used to increase the diversity of an individual from its original value as:

$$\kappa_i = \kappa_i + \Delta\kappa_{i,g} \tag{33}$$

$$\Delta\kappa_{i,g} = \begin{cases} \left(k_i^{\max} - \kappa_i\right)\left(1 - u^{1-(g/N_G)^b}\right) & u \le 0.5 \\ \left(x_k^{\min} - \kappa_i\right)\left(1 - u^{1-(g/N_G)^b}\right) & u > 0.5 \end{cases} \tag{34}$$

The step length (b), which is used to control the speed of convergence, is set to 5 [54].

3.1.3. Selection

In the GA, a greedy selection scheme is used in which the best N_P individuals are selected from both parents and offspring based on their FVs.

3.2. DE search operators

Like the GA, DE has three search operators, mutation, crossover, and selection, with its performance highly dependent on the settings of their control parameters. Therefore, we employ a self-adaptive mechanism to determine the best control parameters for the crossover and mutation operators during each generation of the evolution [55], as described below.

3.2.1. Mutation and crossover

Of the various DE search operators, we use two mutations and one binomial crossover, as widely used in the literature [55]. In this process, an offspring (κ) is generated from three random parents (k_{r1}, k_{r2} and $k_{r3}, r_1 \neq r_2 \neq r_3$) and the current best parent (k_{best}) as:

$$\kappa_p = \begin{cases} k_{r3} + Fa_p\left(k_{r1} - k_{r2}\right), & \text{if } rand_1 \le Cr_z \,\&\, rand_2 \le prob_1, \\ k_{r3} + Fa_p\left\{\left(k_{r1} - k_{r2}\right) + \left(k_{best} - k_p\right)\right\}, & \text{if } rand_1 \le Cr_z \,\&\, rand_2 > prob_1, \quad p \in N_p \\ k_p, & \text{otherwise;} \end{cases} \tag{35}$$

where $Prob_1 = 0.5$ (as in [26]) is a pre-defined value that determines the methodology for generating new individuals from the current ones, Fa_p the amplification factor for the mutation operators and Cr_p the crossover rate of the pth individual. The Fa and Cr are initially assumed as two sets of parameters ($\mathbf{Fa} \in \mathrm{N}(0.5,0.1)$ and $\mathbf{Cr} \in \mathrm{N}(0.5,0.1)$, respectively) with normal distributions and mean and standard deviation values of 0.5 and 0.1, respectively. Then, their values are updated in each generation to obtain a new offspring as per Eqn. (35) as:

$$Fa_p = \begin{cases} Fa_{r_1} + rand_1(Fa_{r2} - Fa_{r3}), \text{ if } rand_2 \le \tau_1 \\ rand_3, \ otherwise \end{cases} \tag{36}$$

$$Cr_p = \begin{cases} Cr_{r_1} + rand_3(Cr_{r2} - Cr_{r3}), \text{ if } rand_5 \le \tau_1 \\ rand_6, \ otherwise \end{cases} \tag{37}$$

where $\tau_1 = 0.75$ as in [56] and $rand_q \in [0,1]$ for $q = 1,2,..,6$. However, to enhance efficiency, the limits of these two parameters are set as [26]:

$$Fa_p = \min[1, \max(0.1, Fa_p)] \tag{38}$$

$$Cr_p = \min[1, \max(0.1, Cr_p)] \tag{39}$$

where 0.1 and 1 are the lower and upper limits of these two variables, respectively [26].

3.2.2. Selection

Like that in the GA, the selection process in DE follows a greedy scheme in which the fittest (according to the FVs) of two candidates is selected [57]. In addition, the parent's corresponding Fa_p and Cr_p are replaced by the offspring's contributed Fa_p and Cr_p. This process is repeated until all the individuals are selected.

B. Lower-level optimization problem

As seen in previous sections, to evaluate the upper-level fitness functions in Eqs. (26) and (27), it is necessary to solve another nonlinear DC-OPF optimization problem. This problem is used in the lower level of the competitive bi-level SFE model and treated as a single-objective optimization problem in a non-competitive market. For verification purposes, it is solved using a classical IP optimization technique with two EAs, (i) a real-coded GA and (ii) self-adaptive DE. A description of the nonlinear IP algorithm is found in [58] while the GA and DE procedures are described in sections 3.1 and 3.2, respectively. Note that, for the verification purpose, the original lower level problem (Eqns. (10) to (15)) is solved using both GA and DE, while the developed SCQP formulation (Eqns. (23) to (25)) solved using the gradient based IP method.

V. Experimental Results

To test the effectiveness of the proposed approaches, three *IEEE* benchmark problems up to 118 buses with and without TCs are considered [1, 59, 60], with the models solved in both competitive and non-competitive environments. In the competitive market, it is assumed that the participating players are non-cooperative, with a NE obtained using our proposed CE-GA and CE-DE algorithms. The DC-OPF problem for both the competitive and non-competitive markets is solved using three algorithms, (i) GA, (ii) DE and (iii) the IP method. Each test problem has the following four cases.

Case I. GENCOs and consumers are non-strategic with TC.
Case II. GENCOs are strategic but consumers are non-strategic without TC.
Case III. GENCOs are strategic but consumers are non-strategic with TC.
Case IV. GENCOs and consumers are strategic with TC.

In case I, both suppliers (GENCOs) and consumers are treated as non-strategic players which means the bidding problem solves as a dispatch problem, considering the true generation cost of each generator. In both cases II and III, GENCOs and consumers are considered as strategic, and non-strategic players, respectively. However, the difference between these two cases is that the TL constraints are ignored in case II while it is considered in case III. If a bidding problem is solved without considering the TL constraints, the LMP of each node might be the same where the individual profit only depends on the amount of power generation. On the other hand, if the problem considers the TL constraints, the LMPs of all nodes will be significantly different, which results to change the profit of a participant in the system. In case IV, both GENCOs and consumers are considered as strategic players, and the TL constraints also applied. Note that, a strategic customer is one that can participate in the bidding process which increases the number of players in that game while, in a non-strategic customer game, the number of players is reduced to the number of GENCOs with customers only able to buy a predefined amount of electricity from the market [1].

To validate the proposed CE-GA and CE-DE methods, we also implement two other conventional IT-based solution approaches using GA and DE algorithms known as IT-GA and IT-DE, respectively, for the competitive electricity market [4] which are also used to obtain an NE. Here, at each iteration, a player optimizes its bidding strategy knowing that the other players have chosen and fixed their actions. To start the game, each player randomly chooses its action assuming that true marginal strategies are used by the other players and then optimizes its action with respect to those of the other players. To update its strategy, a player uses either the GA or DE search operators described in sections 3.1 and 3.2, respectively. Once a player obtains its optimal bidding strategy, the first iteration is completed. Then, the next player begins to optimize its strategy in the same process while considering the best bidding strategies of its rivals found so far. This iterative process is terminated when no player can change its action, with this stopping point the desired NE of the game.

In addition, as previously mentioned, as each algorithm provides a unique and local NE, we cannot compare an equilibrium point of one with those of the others [61]. Therefore, we use a ranking procedure based on a non-parametric test, called the Friedman test, to evaluate the overall performance of an algorithm [62]. The rankings are calculated based on the relative performances of the algorithms for achieving the maximum profits of each GENCO and load, as illustrated in the following example.

Consider an electricity market with six competitive objectives, four GENCOs (G) and two loads (L), which is solved using the four algorithms (i) IT-DE, (ii) IT-GA, (iii) CE-DE and (iv) CE-GA. Sample solutions obtained by each algorithm for the profits of GENCOs and loads are presented in Table 1.

Table 1: Rankings of algorithms using Friedman test

Objectives	Expected profits ($)				Relative ranks			
	IT-DE	IT-GA	CE-DE	CE-GA	IT-DE	IT-GA	CE-DE	CE-GA
G-1	12.50	12.10	13.70	13.10	3	4	1	2
G-2	8.20	10.10	9.20	9.00	4	1	2	3
G-3	20.12	19.20	20.23	21.20	3	4	2	1
G-4	4.50	3.20	5.20	5.00	3	4	1	2
L-1	14.20	13.02	14.20	12.50	1	2	1	3
L-2	12.30	12.15	11.80	12.00	1	2	4	3
				MR:	**2.50**	**2.83**	**1.83**	**2.33**

Each objective's profits are sorted in descending order and each individual assigned a relative rank, starting with rank 1 for the solution with the maximum profit, with solutions with the same values allocated the same rank. This procedure is repeated for all the objectives and the rank of each algorithm then calculated according to the mean values of the relative ranks of all the objectives. Based on the mean ranks (MRs) in Table 1, it can be seen that CE-DE is the best algorithm followed by CE-GA, IT-DE, and IT-GA.

For all cases, the probabilities of crossover and mutation parameters of the GA are set to 0.9 and 0.1, respectively according to ref. [26]. Based on our empirical results (discussed later), the subpopulation sizes (N_P) of the upper and lower levels are set to 8 and 20, respectively, and the maximum numbers of iterations ($MaxIt$) and generations (N_G) to 100. However, to solve the DC-OPF problem using the IP algorithm, the $MaxIt$ is set to 1000. Thirty independent runs are performed for each test case and the solutions recorded and compared with the results from state-of-the-art techniques.

The algorithms are implemented on a desktop personal computer with a 3.4 GHZ Intel Core i7 processor with 16 GB of RAM using the MATLAB (R2012b) environment. They run until the number of generations is greater than N_G(=100) (criterion 1) or the best FVs are no longer improved in θ $(= 5)$ generations (criterion 2).

A. IEEE 3-bus test system

The IEEE 3-bus test system [1] depicted in Fig. 3 contains two GENCOs at buses (nodes) 1 and 3, two consumers at nodes 1 and 2, and three TLs considered lossless with equal reactance ($x = 0.002$). The capacities of all the TLs are considered infinity except for the maximum capacity of line F_{12} which is set to 500 MW for case II and 25 MW for cases I, III and IV. During the competition, all the GENCOs and consumers are allowed to change their bidding coefficients from 1 to 2.5, and 0.1 to 1, respectively. These cases are described in the following sections.

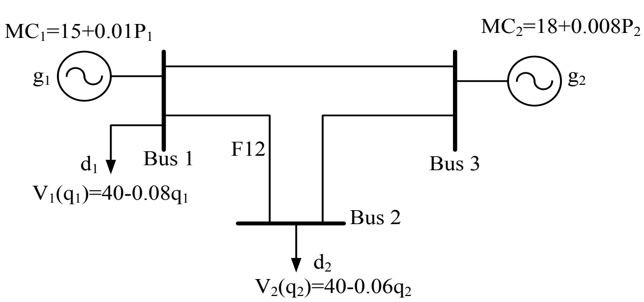

Fig. 3. IEEE 3-bus test system

A.1. Case I

As, in this case, the market is considered non-competitive, all the participating GENCOs and consumers are considered non-strategic with their bidding coefficients set to 1 [1]. In other words, neither GENCOs nor consumers are allowed to maximize their profits but, rather, always play with their true marginal cost functions in Eqs. (2) and (4) for GENCOs and consumers, respectively. Therefore, as this bi-level problem no longer has an upper level, it becomes a single-objective optimization one, i.e., DC-OPF, which aims to maximize the CSW, and is solved using the (i) GA, (ii) DE and (iii) IP algorithms.

After 30 independent runs of each algorithm, their average results and the results in the literature, such as those of EBA [1], BA [1], PSO [1], GA [1] and NCP [63], including the profit and PD of each GENCO and consumer, LMP, CSW and computational time, are presented in Table 2. Note that 'NR' means that the results are not reported in the literature.

As can be seen, the profits of each algorithm are quite similar while the computational time of IP is much lower than those of the others. Also, as the problem considers TC, as expected, the market prices are not the same for all nodes. Moreover, based on the ranking procedure presented in Table 1, the MR of each algorithm is determined considering the objectives of the profits of GENCOs and loads, CSW and computational time. The results demonstrate that the MR for the IP algorithm is the best.

Table 2: Summary of 3-bus system for case I

	Expected profits ($)				Market prices			Dispatches (MW)				CSW ($)	Time (sec)	MR
Alg.	G-1	G-2	L-1	L-2	λ_1	λ_2	λ_3	P_1	P_2	d_1	d_2			
EBA [1]	92.64	620.06	3492.40	164.77	16.36	25.93	21.14	136.12	393.72	295.48	234.36	6212.05	0.96	4.50
BA [1]	92.64	620.06	3492.40	164.77	16.36	25.93	21.14	136.12	393.72	295.48	234.36	6212.05	0.96	4.50
PSO [1]	92.64	620.06	3492.40	164.77	16.36	25.93	21.14	136.12	393.72	295.48	234.36	6212.05	0.96	4.50
GA [1]	92.64	620.06	3492.40	164.77	16.36	25.93	21.14	136.12	393.72	295.48	234.36	6212.05	0.96	4.50
NCP [63]	92.60	620.10	NR	NR	16.40	25.90	21.10	136.10	393.70	295.50	234.40	6212.10	NR	NR
GA	92.68	619.92	3492.38	164.78	16.36	25.94	21.15	136.15	393.67	295.48	234.34	6212.06	0.08	3.33
DE	92.68	619.92	3492.38	164.78	16.36	25.94	21.15	136.15	393.67	295.48	234.34	6212.06	0.09	3.50
IP	92.68	619.92	3492.38	164.78	16.36	25.94	21.15	136.15	393.67	295.48	234.34	6212.06	**0.01**	**3.17**

A.2. Cases II to IV

For cases II and III, although the market is open for competition for GENCOs and consumers, consumers are still considered non-strategic ($i.e., k_d = 1$). The TCs of lines F_{13} and F_{23} are ignored while the capacity of line F_{12} is set to a large value of 500 MW in case II and a smaller one of 25 MW in case III.

As, nowadays, the electricity market is not only open to suppliers (*i.e.,* GENCOs) but also consumers, in case IV, we assume that both consumers and suppliers participate in the market, with each GENCO and consumer choosing a strategic variable from 1 to 2.5, and 0.1 to 1, respectively, to maximize their own profits while considering the interactions of their rivals. Note that the line capacity of F_{12} in case IV is set to its previous value of 25 MW.

All three cases are solved using our proposed CE-GA and CE-DE algorithms as well as the conventional IT-GA and IT-DE ones, with the IP algorithm used in the lower level for all four approaches to solve the DC-OPF problem due to its superior performance in case I. The median results of 30 random runs are compared with those found in recent literature from the EBA [1], BA [1], PSO [1], GA [1] and NCP [63] in Table 3 which contains the expected profits and MRs, and computational times for each algorithm for each case. Detailed results for the expected profit and PD of each GENCO and load, bidding strategies of GENCOs, LMPs, line flows and CSWs for all three cases are presented in Tables 17 to 22 in Appendix C.

It is clear in Table 17 that the market prices of all nodes are equal which is because, as the capacities of the TLs are assumed to be high and there is no limit on the power generated from each GENCO, the cheaper G-1 gains more profit than G-2, as shown in Table 3. When the capacity of F_{12} is reduced to 25 MW in case III, a TC appears in lines 1 to 2, as shown in Table 20, and then the market prices of the nodes vary significantly from those reported in Table 19. As a consequence, G-1 loses its profit and, conversely, G-2 gains more, as shown in Table 3. On the other hand, the simulation results for case IV in Table 3 indicate that the profits of the consumers increase greatly compared with those of the non-elastic demands in cases II and III and its CSW is much lower (Table 22).

However, it is obvious from Table 3 that the market does not have a pure strategy but mixed ones with multiple local NEs, with that of each algorithm providing a local solution very close to those of the others. Based on the profits of the GENCOs and loads shown in Table 3 for all three cases, none of the algorithms performs consistently best regarding obtaining maximum profits. However, based on the ranking method used, the MR of each algorithm indicates that the proposed CE-DE one is ranked 1st and also has the fastest computational time.

Table 3: Summary of simulation results for IEEE 3-bus system for cases II, III and IV

Alg.	Case II Profits ($) G-1	Case II Profits ($) G-2	Case II MR	Case III Profits ($) G-1	Case III Profits ($) G-2	Case III MR	Case IV Profits ($) G-1	Case IV Profits ($) G-2	Case IV Profits ($) L-1	Case IV Profits ($) L-2	Case IV MR	CPU time (seconds)
EBA [1]	1557.07	455.55	4.67	741.59	1789.58	4.67	767.57	859.46	2116.94	767.57	4.50	108.60
BA [1]	1540.17	440.16	6.00	690.43	1586.22	6.00	764.78	892.27	2096.01	764.78	4.25	72.60
PSO [1]	1523.11	429.44	6.33	684.26	1580.74	6.33	761.35	891.46	2096.01	761.35	5.25	60.60
GA [1]	1494.39	413.18	6.67	747.10	1799.00	6.67	734.80	887.30	2109.49	734.80	6.17	138.60
NCP [63]	1560.00	446.50	NR	749.55	1798.61	NR	767.83	862.40	2115.77	767.83	NR	NR
IT-DE	1560.39	447.21	4.00	796.93	1816.66	4.00	762.36	1093.81	1988.80	762.36	4.00	5.45
IT-GA	1835.62	430.34	4.67	749.12	1807.19	4.67	771.71	869.66	2105.91	771.71	4.50	5.39
CE-DE	1581.98	454.55	**3.67**	772.84	1839.50	**3.67**	850.57	892.37	2062.95	850.57	**2.67**	**1.21**
CE-GA	1666.26	468.94	4.00	747.97	1799.02	4.00	767.57	859.46	2116.94	767.57	4.67	1.39

B. IEEE 30-bus test system

To demonstrate the effectiveness of the proposed methods, the moderate IEEE 30-bus test system which contains 6 GENCOs, 20 loads, 41 lines and 30 buses is considered with all the TLs lossless with their reactance set to 0.001. The system data and schematic diagram are described in [59]. There are no TCs for case II while, for the others, the capacity limits of lines 10 ($F_{6,8}$), 17 ($F_{17,12}$) and 26 ($F_{10,17}$) are set to 10, 8 and 10, respectively. As previously, the GENCOs and consumers can change their bidding coefficients from 1 to 2.5, and 0.1 to 1, respectively. The cases for the 30-bus test system are described in the following sections.

B.1. Case I

In this case, the IEEE 30-bus test system with TC is tested with all 6 GENCOs and 20 consumers considered non-strategic, and it is assumed that all the participants submit their marginal bids to the ISO. Since there is no strategy for the bidding coefficients, the model has only a lower-level DC-OPF problem which is solved using the (i) GA, (ii) DE and (iii) IP methods. The results obtained for the expected profits of all GENCOs, and their CSWs, computational times, MRs and line flows are listed in Table 4, with detailed results shown in Tables 23 and 24 in Appendix C. Note that the parentheses in each GENCO's heading represent that GENCO's location (node number). It can be seen in Table 4 that the expected profits of each node obtained from each algorithm are almost the same and TCs appear in lines 17 and 26. However, based on the MRs, the IP is ranked 1st and also obtains solutions in the least computational time.

Table 4: Summary of simulation results for IEEE 30-bus system for case I

Alg.	Profits ($) G-1(#1)	Profits ($) G-2(#2)	Profits ($) G-3(#13)	Profits ($) G-4(#22)	Profits ($) G-5(#23)	Profits ($) G-6(#27)	CSW ($)	Time (Sec)	MR	$(F_{6,8}, F_{17,12}, F_{10,17})$
GA	397.68	360.72	166.95	325.67	945.17	715.17	18453.88	10.09	2.13	(8.90,8.00,10.00)
DE	397.68	360.72	166.95	325.67	945.17	715.17	18453.88	2.28	2.00	(8.90,8.00,10.00)
IP	397.68	360.72	166.95	325.67	945.17	715.17	18453.88	0.08	**1.88**	(8.90,8.00,10.00)

B.2. Cases II to IV

In these cases, the market is open for competition among GENCOs and consumers while their strategic variables are varied from 1 to 2.5, and 0.1 to 1, respectively. In cases II and III, the GENCOs are considered strategic while, in case IV, both they and consumers both strategic. TCs are ignored in case II but those of lines 10, 17 and 26 are set to values of 10, 8 and 10, respectively, in cases III and IV to demonstrate their impact on the market transactions and profits of each bidder.

As previously, for all three cases, the competitive market is solved using the (i) CE-GA, (ii) CE-DE, (iii) IT-GA and (iv) IT-DE methods, with the IP algorithm used to solve the lower-level DC-OPF problem due to its superior performance in case I. After 30 random runs are performed for each test case, the average results for the expected profits, simulation times and MRs are calculated and presented in Table 5, with detailed results provided in Tables 25 to 30 in Appendix C.

In Table 25 as, in case II, the nodal prices for all GENCOs are similar because TCs are not considered, GENCOs 3 and 4 earn more profits and GENCOs 5 and 6 less. As the market has no pure strategy, the algorithms provide local NE solutions which are almost analogous. However, based on their MRs, CE-DE is ranked 1st followed by CE-GA, IT-DE, IT-GA, EBA, BA, PSO and GA.

As the capacity limits of TLs are considered in case III, TCs appear in lines 10 and 26, as shown in Table 28, and the market prices of different nodes subsequently differ from those in Table 27. As a consequence, the profits of the GENCOs are different from those for case II with those of GENCOs 5 and six increased, as shown in Table 5. Based on a comparison of profits, it can be seen that none of the algorithms is consistently better for all GENCOs, but the MRs demonstrate that CE-DE is the best.

Table 5: Summary of expected profits, MRs and simulation times of IEEE 30-bus system for cases II, III and IV

		Expected profits ($)							
		EBA	BA	PSO	GA	IT-DE	IT-GA	CE-DE	CE-GA
Case-II	G-1(#1)	622.27	621.7	611.6	607.9	713.2	706.32	724.94	716.73
	G-2(#2)	605.97	605.31	594.11	590.31	703.22	698.6	714.65	708.35
	G-3(#13)	280.85	280.42	273.25	270.61	356.46	348.26	367.58	361.97
	G-4(#22)	461.11	460.6	450.65	447.22	548.88	542.44	559.02	553.8
	G-5(#23)	461.1	460.6	450.74	447.41	508.51	503.17	518.4	513.48
	G-6(#27)	771.3	770.61	759.47	755.39	869.82	864.83	882.83	877.04
	MR	5.43	6.14	6.86	7.57	2.71	3.86	**1.43**	2.00
Case-III	G-1(#1)	632.45	615.9	614.52	611.66	793.47	795.19	794.38	791.97
	G-2(#2)	609.69	591.12	589.43	586.84	758.1	765.48	760.34	759.09
	G-3(#13)	352.91	354.78	354.23	352.95	396.34	395.04	397.02	410.47
	G-4(#22)	387.45	385.27	384.64	382.99	710.09	704.8	711.07	719.6
	G-5(#23)	985.8	977.02	976.41	975.1	1206.34	1205.08	1206.93	1196.94
	G-6(#27)	1296.02	1276.09	1274.91	1272.46	1191.89	1193.03	1192.42	1177.74
	MR	4.83	5.17	6.17	7.17	3.67	3.00	**2.50**	3.50
Case-IV	NR	NR	NR	NR	NR	2.65	2.42	**2.12**	2.81
CPU time (Sec)		361.8	289.2	253.8	400.2	27.27	29.09	**4.60**	8.59

As there are so many players in case IV, their expected profits are not listed in Table 5 but presented in Table 29 in Appendix C which indicate that, as each consumer optimizes its bidding coefficients, its profit is greater than in case III. The MRs for this case shown in Table 5 demonstrate that CE-DE is again ranked 1st. Furthermore, the computational times of the CE approaches are far better than those of the IT and other methods, with CE-DE the fastest algorithm.

C. IEEE 118-bus test system

The *IEEE* 118-bus test system is a large power system network consisting of 118 nodes, 54 GENCOs, 99 loads and 186 TLs. Its data and diagram can be found in [60]. The system is modified by considering a TC at TL 163 $(F_{100,103})$ with a value of 20 MW for cases I and III and the TLs lossless with equal reactance values of $x = 0.001$. Because of the large number of loads present in this system, we solve only the first three cases with the GENCOs' strategic variables varied from 1 to 2.5 using the proposed CE and conventional IT approaches which are described in the following sections.

C.1. Case I

The GENCOs and loads for this case are considered non-strategic and submit their actual marginal costs to the ISO, with a TC applied in only line 163. The ISO's DC-OPF problem is solved using one of the three algorithms, (i) GA, (ii) DE or (iii) IP. The average solutions for the objective values (CSW), computational times and MRs are calculated after 30 random runs and presented in Table 6. Based on comparisons, it can be seen that the objective values for all three algorithms are the same while the computational time of IP is much less than those of the other two and its MR is the best.

Table 6: Summary of IEEE 118-bus system for case I

	GA	DE	IP
CSW ($)	52235.08	52235.08	52235.08
Time (sec.)	0.44	0.24	0.09
MR	2.50	2.00	**1.50**

Table 7: MRs and computational times of IEEE 118-bus system for cases II and III

	IT-DE	IT-GA	CE-DE	CE-GA
Case II	2.50	3.01	**2.01**	2.48
Case III	2.62	3.01	**2.01**	2.36
Time (sec.)	813.37	946.36	293.21	321.45

C.2. Cases II and III

In this section, the algorithms' performances for solving the largest test system considering strategic GENCOs, in which TCs are ignored in case II and a TC of TL 163 set at a value of 25 MW, are evaluated. Each GENCO varies its strategic variables $(k_i \forall i = 1,2,\dots,54)$ between 1 and 2.5 to maximize its individual profits. As previously, this test system is solved using the (i) IT-DE, (ii) IT-GA, (iii) CE-DE and (iv) CE-GA methods, and the IP algorithm for the lower-level DC-OPF problem. Detailed results (median runs) obtained after 30 independent runs are presented in Tables 31 to 34 in Appendix C and the MRs and computational times in Table 7.

As shown in both Tables 31 and 33, the market prices of all nodes are almost analogous, despite a TC being applied to TL 163, and the power flows reach their maximum limits (Table 31). This is due to the number of GENCOs playing similar bidding strategies in this large energy market because, even if a line is congested, the electricity of that node comes from other generators through other TLs. Regarding obtaining the profits of various nodes, the proposed CE approaches are better than the IT ones.

The computational times in Table 7 demonstrate that the proposed CE-DE algorithm is the fastest and the MRs that it is superior to the other algorithms.

VI. DISCUSSION

In this section, the robustness of the proposed CE approaches is evaluated by analyzing statistically and the effects of parameters (i) N_P, (ii) the stopping criteria and (iii) convergence rates of the CE and IT approaches. To do this, the *IEEE* 3-bus test problem for cases III and IV is solved by following a ceteris paribus strategy in which only one parameter is varied while all the others remain fixed to their best values [64].

A. Statistical analysis

In this section, we perform the statistical tests of the algorithms considered in this paper. Firstly, an ANOVA test in randomized complete block designs procedures is performed as the number of objective functions (treatments) is more than one. Here, the treatments (samples) are considered the profits of all the players, CSWs and computational times for all considered problems with their case studies, and four algorithms, such as IT-GA, IT-DE, CE-GA and CE-DE considered as blocks. The null and alternative hypothesises are defined, respectively, as:

$$H_0\colon \alpha_1 = \alpha_2 = \alpha_3 = \cdots = \alpha_k = 0$$
$$H_1\colon At\ least\ one\ of\ the\ \alpha_i\ is\ not\ equal\ to\ zero$$

where α_i is the effect of the i^{th} treatment. The null hypothesis H_0 is tested at the 5% level of significance, that assumes, all the algorithms provide the solutions with the same mean value. The test results of 'sum of squares (SS)', 'degree of freedom (df)', 'mean square (MS)', computed F and critical F_{crit} with the P-values for both algorithms and treatments are presented in Table 8. From the results, the smaller the P-values (< 0.05) and the meeting the constraint $F > F_{crit}$ demonastrate the evidence to against the null hypothesis , H_O. Therefore, the solutions from the all algorithms are not equal, at least one of the mean is different. However, the ANOVA test does not provide the information where the actually differences lies or which one is better [65].

Table 8: ANOVA analysis for all the considered test problems

	SS	df	MS	F	$P-value$	F_{crit}	Comments
Algorithms	4.01E+04	3	1.34E+04	4.36	0.0047	2.62	$F > F_{crit}$
Treatments	2.41E+10	196	1.23E+08	40136.57	0.00	1.21	$F > F_{crit}$
Error	1.80E+06	588	3.06E+03				
Total	2.41E+10	787	1.23E+08				

To determine the individual algorithm's effect, a Wilcoxon sign test is performed in 200 samples solutions of all players from all considered problems for four algorithms. The comparisons are performed based on the average profits of all players, using a 5% significance level. The results are shown in Table 9, in which found that the P values of all the sets of comparisons are less than 0.05, indicating that there is a significant difference between the solutions from any two algorithms. Also, it is found that the CE-based algorithms obtain better solutions than those of IT-based ones.

In addition, the Friedman test is carried out to rank all the algorithms, as shown in Tables 10, with the results demonstrating that the proposed CE-DE algorithm is ranked 1st, followed by CE-GA, IT-DE and IT-GA. Furthermore, a sample box plot for the 30-bus (case II) system for player 1 is depicted in Fig. 4 that illustrates the performance of the proposed CE-DE algorithm for obtaining highest mean results with a smaller standard deviation.

Table 9: Wilcoxon test results for IT-DE, IT-GA, CE-DE and CE-GA

Comparisons	**Better**	**Similar**	**Worse**	**P-values**
IT-DE vs. IT-GA	88	70	42	0.001
IT-DE vs. CE-DE	51	70	79	0.018
IT-DE vs. CE-GA	52	75	73	0.044
IT-GA vs. CE-DE	46	70	84	0.001
IT-GA vs. CE-GA	44	70	86	0.000
CE-DE vs. CE-GA	84	70	46	0.001

Table 10: Ranks of IT-DE, IT-GA, CE-DE and CE-GA from Friedman test results

IT-DE	IT-GA	CE-DE	CE-GA
2.51	2.82	**2.24**	2.44

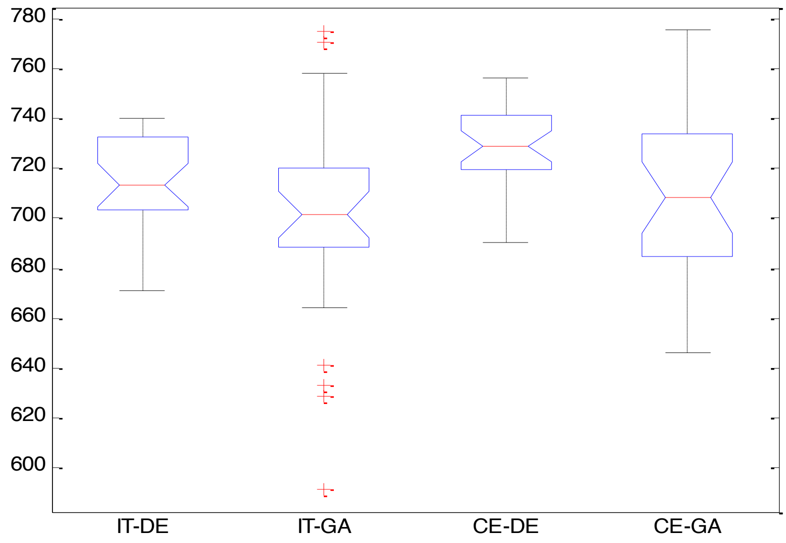

Fig. 4. Sample boxplot of the profits of player-1 of *IEEE* 30-bus system (case II)

B. Effect of N_P

The robustness of the CE algorithms is validated using three different N_P values to solve the *IEEE* 3-bus test problem for cases III and IV in which the GENCOs and consumers act strategically, with the value of N_G set to 100 for both cases. After 30 independent runs for each N_P, the average results are recorded and presented in Tables 11 and 12 for cases III and IV, respectively. It is found that, for each N_P, both CE-GA and CE-DE provide unique local NEs which are not comparable to the others as at least one of the objectives is found to be better. Nevertheless, the profits for each instance are very close to each other, with the best acceptable values, including computational time, found when $N_P = 8$.

Table 11: Effect of $\boldsymbol{N_P}$ for solving *IEEE* 3-bus system for case III

	CE-GA			CE-DE		
	$N_P = 4$	$N_P = 8$	$N_P = 12$	$N_P = 4$	$N_P = 8$	$N_P = 12$
G-1	744.84	**772.84**	779.53	747.49	**749.12**	747.15
G-2	1805.62	**1839.50**	1792.70	1795.06	**1807.19**	1799.04
L-1	2085.66	2024.37	**2085.70**	**2099.67**	2081.62	2094.12
L-2	**990.71**	963.26	922.92	978.87	984.11	**984.16**
Time (sec.)	1.23	1.38	1.98	1.18	1.36	1.51

Table 12: Effect of $\boldsymbol{N_P}$ for solving *IEEE* 3-bus system for case IV

	CE-GA			CE-DE		
	$N_P = 4$	$N_P = 8$	$N_P = 12$	$N_P = 4$	$N_P = 8$	$N_P = 12$
G-1	717.99	**850.57**	748.49	761.63	**771.71**	766.55
G-2	**918.21**	892.37	885.11	**872.15**	869.66	863.01
L-1	2016.74	2062.95	**2124.69**	2105.14	2105.91	**2114.74**
L-2	1417.44	**1439.46**	1439.38	1651.77	**1665.02**	1662.13
Time (sec.)	1.15	1.92	4.01	1.30	1.80	3.28

C. Effect of stopping criteria

In this paper, the two stopping criteria used are that an algorithm is run until *(i)* the number of generations is greater than N_G or *(ii)* the best FVs are no longer improved in the last θ generations. In this section, the robustness of the algorithms are investigated by varying the values of N_G and θ as 50, 100 and 150, and 2, 5 and 10, respectively.

The values of N_G are changed first and then the 3-bus test problem for both cases III and IV solved using the CE-GA and CE-DE algorithms, with their average profits presented in Tables 13 and 14, respectively. It is found that the solutions for all the instances considered are the same which indicate that the algorithms never reach N_G but stop due to stopping criterion (*ii*).

Table 13: Effect of $\boldsymbol{N_G}$ for solving *IEEE* 3-bus system for case III

	CE-GA			CE-DE		
	$N_G = 50$	$N_G = 100$	$N_G = 150$	$N_G = 50$	$N_G = 100$	$N_G = 150$
G-1	772.84	772.84	772.84	749.12	749.12	749.12
G-2	1839.50	1839.50	1839.50	1807.19	1807.19	1807.19
L-1	2024.37	2024.37	2024.37	2081.62	2081.62	2081.62
L-2	963.26	963.26	963.26	984.11	984.11	984.11
Time (sec.)	1.38	1.38	1.38	1.36	1.36	1.36

Table 14: Effect of $\boldsymbol{N_G}$ for solving *IEEE* 3-bus system for case-IV

	CE-GA			CE-DE		
	$N_G = 50$	$N_G = 100$	$N_G = 150$	$N_G = 50$	$N_G = 100$	$N_G = 150$
G-1	850.57	850.57	850.57	771.71	771.71	771.71
G-2	892.37	892.37	892.37	869.66	869.66	869.66
L-1	2062.95	2062.95	2062.95	2105.91	2105.91	2105.91
L-2	1439.46	1439.46	1439.46	1665.02	1665.02	1665.02
Time (sec.)	1.92	1.92	1.92	1.80	1.80	1.80

The average results for cases III and IV after changing the θ values for the same test problem are presented in Tables 15 and 16, respectively. It is clear that the algorithms prematurely converge when $\theta = 2$, and the quality of solutions does not significantly improve if this value is increased from 5 to 10 but the computational time increases. Therefore, the algorithms behave the same for values of $\theta \geq 5$, with $\theta = 5$ the best value for obtaining quality solutions in the least amount of computational time.

Table 15: Effect of $\boldsymbol{\theta}$ for solving *IEEE* 3-bus system for case III

	CE-GA			CE-DE		
	$\theta = 2$	$\theta = 5$	$\theta = 10$	$\theta = 2$	$\theta = 5$	$\theta = 10$
G-1	757.90	**772.84**	772.12	706.57	**749.12**	747.45
G-2	1782.41	**1839.50**	1755.53	1700.26	**1807.19**	1799.29
L-1	2103.06	2024.37	**2137.48**	2053.62	2081.62	**2093.64**
L-2	958.49	**963.26**	897.25	975.38	**984.11**	981.84
Time (sec.)	0.68	1.38	2.12	1.01	1.36	2.05

Table 16: Effect of $\boldsymbol{\theta}$ for solving *IEEE* 3-bus system for case IV

	CE-GA			CE-DE		
	$\theta = 2$	$\theta = 5$	$\theta = 10$	$\theta = 2$	$\theta = 5$	$\theta = 10$
G-1	778.86	**850.57**	850.49	728.24	**771.71**	766.97
G-2	842.88	892.37	**892.96**	912.22	**869.66**	861.40
L-1	2009.20	2062.95	**2062.99**	1981.94	2105.91	**2115.76**
L-2	1436.00	**1439.46**	1438.18	1754.62	1665.02	**1666.71**
Time (sec.)	1.60	1.92	3.52	1.03	1.8	2.16

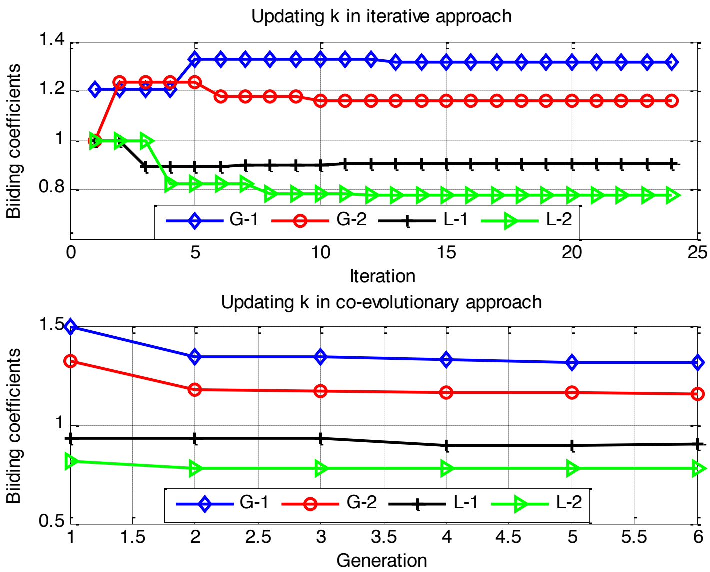

Fig. 5. Convergence characteristics of $\mathbf{k}_{g_i} \forall \boldsymbol{i}$ and $\mathbf{k}_{d_j} \forall \boldsymbol{j}$ in IT-DE and CE-DE, respectively

D. Convergence characteristics

In this section, the performances of the proposed CE approaches are explored by comparing their convergence characteristics with those of the IT methods. The *IEEE* 3-bus test problem for case IV is solved using the CE-DE and IT-DE algorithms with the convergence patterns of their bidding coefficients shown in Fig. 5. It can be seen that the IT-DE takes 25 iterations to converge while the CE-DE converges in only six generations and, even after 4, obtains the best solution. This is because an IT approach determines the best $k_n \forall n$ for each bidder sequentially while the CE one determines them for all bidders simultaneously.

VII. Conclusion and Future Works

In this paper, two solution approaches based on CE algorithms are developed to solve the competitive electricity market in which both strategic GENCOs and consumers participate to maximize their individual profits by optimizing their bidding behaviors while anticipating those of others. The market is represented as an SFE with a bi-level optimization problem in which the lower level maximizes the CSW by solving a DC-OPF problem using a classical optimization technique that satisfies the KKT conditions. In the upper level, one of two EAs, GA or DE, is used to maximize an individual bidder's profit, with multi-populations considered for multiple bidders in which each sub-population represents a player (bidder) that co-evolves with the others and seeks its best propagation considering the best individuals from the other sub-populations. In addition, two well-known conventional solution methods, IT-DE and IT-GA, are also implemented to analyze the effectiveness of the proposed CE algorithms. The performances are validated by solving four different cases of three *IEEE* benchmarks with up to 118 buses. Comparisons of the simulation results with both each other and those in the literature reveal that the CE approaches have merit in terms of quality and reliability, with CE-DE the best method for a competitive energy market.

Possible future work could consider more complex problems, such as multi-period auctions over 24 hours with 5-minute intervals instead of the single period used in this paper. Also, the cost function of the generator, which is assumed to be quadratic, could be represented as piecewise nonlinear, non-convex and non-smooth to align with reality. Moreover, in this paper, it is assumed that each bidder has incomplete information about the bidding strategies of its rivals but perfect information about their cost structures. This could be further extended by considering that they have incomplete information of both. Furthermore, solving the problem as a complete distributed manner is another possible direction.

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Appendices

A. ISO's SCQP problem

In this section, for the DC-OPF SCQP problem, the coefficients of the objective function in Eqn. (23) and constraints in Eqns. (24) and (25) are described.

1. Depiction of objective function

The decision variable of the objective function in Eqn. (23) is:

$$\mathbf{x} = \left[P_1, P_2, \ldots, P_I, \delta_2, \delta_2, \ldots, \delta_K, q_1, q_2, \ldots, q_J\right]^T_{(I+K+J-1)\times 1} \tag{40}$$

and the coefficient: $$\mathbf{G} = \begin{bmatrix} \mathbf{U_g} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{W_{rr}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{U_d} \end{bmatrix} \in \Re_{(I+K+J-1)\times(I+K+J-1)} \tag{41}$$

where $\mathbf{U_g}$ and $\mathbf{U_d}$ are the generators' and consumers' quadratic matrices, respectively, as:

$$\left[\mathbf{U}_{\mathbf{g}_{m,n}}\right]_{(I\times I)} = \begin{cases} c'_m & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}; \forall m, n = 1, 2, \ldots, I \tag{42}$$

$$\left[\mathbf{U}_{d_{m,n}}\right]_{(J\times J)} = \begin{cases} e'_m & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}; \forall m, n = 1, 2, \ldots, J \tag{43}$$

Parameter $\mathbf{W_{rr}}$ is the reduced form of the weight matrix $(\mathbf{W})$ that can be defined as the voltage and angle difference of each node as:

$$\mathbf{W} = 2\pi(w_{mn}) \in \Re_{(K\times K)} \tag{44}$$

$$\text{where } w_{mn} = \begin{cases} -E_{mn} & \text{if } m \neq n \\ \sum\limits_{\substack{k=1 \\ k\neq m}}^{K} E_{mk} & \text{if } m \equiv n \end{cases} \tag{45}$$

where $\mathbf{E} \in \Re_{(K\times K)}$ is the branch connection matrix defined as:

$$\mathbf{E} = \begin{cases} 1 & \text{if either } km \text{ or } mk \in BR \\ 0 & otherwise \end{cases} \tag{46}$$

Once the $\mathbf{W}$ matrix is found, the reduced weight matrix ($\mathbf{W_{rr}}$) is determined after removing the first row and column, *i.e.,* excluding the slack bus ($k = 1$).

The linear argument $(\mathbf{f})$ in Eqn. (23) is determined as:

$$\mathbf{f} = \left[\mathbf{b}' \quad \mathbf{0} \quad \mathbf{d}'\right] \in \Re_{1\times(I+J+K-1)} \tag{47}$$

Comparing the original (17) and quasi (23) objective functions, it is seen that the latter provides a positive definite quadratic form, with at least one non-zero component strictly positive scalar which may indicate that the optimization problem could be satisfied by the first-order optimality of KKT conditions [34]. The KKT representation of the problem is shown in Appendix B.

2. Depiction of Constraints

The coefficients of the inequality constraint in Eqn. (24) are defined as:

$$\mathbf{C_{in}} = \begin{bmatrix} \mathbf{C_g} & \mathbf{O_g} \\ \mathbf{O_d} & \mathbf{C_d} \end{bmatrix} \in \Re_{(2K+2I+J)\times(I+J+K-1)} \tag{48}$$

where

$$\mathbf{C_g} = \begin{bmatrix} \mathbf{O_t} & -\mathbf{DA_r} \\ -\mathbf{O_t} & \mathbf{DA_r} \\ \mathbf{I_p} & \mathbf{O_p} \\ -\mathbf{I_P} & -\mathbf{O_p} \end{bmatrix} \in \Re_{(2K+2I)\times(I+K-1)} \tag{49}$$

where $\mathbf{O_t}$, $\mathbf{O_p}$, $\mathbf{O_d}$ and $\mathbf{O_g}$ are zero matrices of sizes $K \times I$, $I \times (k-1)$, $J \times (I+K-1)$ and $(2K+2I) \times J$ respectively, while $\mathbf{I_p}$ is the identity matrix of size $(I \times k - 1)$. The diagonal matrix $(\mathbf{D})$, reduced adjacency matrix ($\mathbf{A_r}$) and diagonal adjacent matrix of the load demand $(\mathbf{C_d})$ are determined as:

$$\left[D_{m,n}\right]_{(K\times K)} = \begin{cases} B_{m,n} & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}, \ \forall m,n = 1,2,\ldots,K \tag{50}$$

$$\mathbf{A} = \begin{bmatrix} \hbar(1,\mathbf{BI}_1) & \hbar(2,\mathbf{BI}_1) & \ldots & \hbar(K,\mathbf{BI}_1) \\ \hbar(1,\mathbf{BI}_2) & \hbar(2,\mathbf{BI}_2) & \cdots & \hbar(K,\mathbf{BI}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \hbar(1,\mathbf{BI}_N) & \hbar(2,\mathbf{BI}_N) & \cdots & \hbar(K,\mathbf{BI}_N) \end{bmatrix} = \Re_{(I\times K)} \tag{51}$$

$$\hbar(i,\mathbf{BI}_n) = \begin{cases} +1 & \text{if } BI_n \text{ takes the form } ij \in BR \text{ for some node } j > i \\ -1 & \text{if } BI_n \text{ takes the form } ji \in BR \text{ for some node } j < i; i = 1,\ldots,K; n = 1,\ldots,I \\ 0 & \text{otherwise} \end{cases} \tag{52}$$

Then, the reduced adjacency matrix $(\mathbf{A_r})$ is calculated after deleting the first row and column of the **A** matrix, with the load adjacent to the diagonal matrix determined as:

$$\left[C_{d_{m,n}}\right]_{(J\times J)} = \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}, \ \forall m,n = 1,2,\ldots,J \tag{53}$$

The coefficient on the right-hand side in Eqn. (24) $(\mathbf{b_{in}})$ can be calculated as:

$$\mathbf{b_{in}} = \left[\mathbf{F} \quad \mathbf{F} \quad \mathbf{P_{min}} \quad \mathbf{P_{max}}\right]^T \tag{54}$$

where

$$\mathbf{F} = \left[F_1^U, \cdots, F_K^U\right]_{(1\times K)} \tag{55}$$

$$\mathbf{P_{min}} = \left[P_1^{\min}, P_2^{\min}, \cdots, P_I^{\min}\right]_{(1\times I)} \tag{56}$$

$$\mathbf{P}_{\mathbf{m}ax} = \left[P_1^{\max}, P_2^{\max}, \cdots, P_I^{\max}\right]_{(1\times I)} \tag{57}$$

The coefficients of the equality constraints in Eqn. (25) ($\mathbf{C_{eq}}$ and $\mathbf{b_{eq}}$) are determined as:

$$\mathbf{C_{iq}} = \left[\amalg_g \quad \mathbf{Y_{bus}^r} \quad \amalg_d\right] \in \Re_{(K,I+J+K-1)} \tag{58}$$

where $\amalg_g$ and $\amalg_g$ are the matrices indicating the locations of the generators and loads, respectively, which are defined as:

$$\amalg_g = \begin{bmatrix} \exists(1 \in I_1) & \exists(2 \in I_1) & \cdots & \exists(I \in I_1) \\ \exists(1 \in I_2) & \exists(2 \in I_2) & \cdots & \exists(I \in I_2) \\ \vdots & \vdots & \ddots & \vdots \\ \exists(1 \in I_K) & \exists(2 \in I_K) & \cdots & \exists(I \in I_K) \end{bmatrix}_{(K\times I)} \tag{59}$$

$$\text{where } \exists(i \in I_k) = \begin{cases} 1 & \text{if } i \in I_k \\ 0 & \text{if } i \notin I_k \end{cases} \tag{60}$$

$$\text{Ш}_d = \begin{bmatrix} \perp(1 \in J_1) & \perp(2 \in J_1) & \cdots & \perp(J \in J_1) \\ \perp(1 \in J_2) & \perp(2 \in J_2) & \cdots & \perp(J \in J_2) \\ \vdots & \vdots & \ddots & \vdots \\ \perp(1 \in J_K) & \perp(2 \in J_K) & \cdots & \perp(J \in J_K) \end{bmatrix}_{(K \times J)} \tag{61}$$

$$\text{where } \perp(j \in J_k) = \begin{cases} -1 & \text{if } j \in J_k \\ 0 & \text{if } j \notin J_k \end{cases} \tag{62}$$

The impedance matrix (Y_{bus}) is calculated as:

$$\mathbf{Y_{bus}} = \begin{bmatrix} \sum_{k \neq 1}^{K} B_{1,k} & -B_{1,2} & \cdots & -B_{1,K} \\ -B_{2,1} & \sum_{k \neq 2}^{K} B_{2,k} & \cdots & -B_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ -B_{K,1} & -B_{K,2} & \cdots & \sum_{k \neq K}^{K} B_{K,k} \end{bmatrix} \in \Re_{(K \times K)} \tag{63}$$

Then, the reduced impedance matrix (Y_{bus}^r) is obtained from the Y_{bus} matrix after removing the first row. Note that the coefficient of the equality constraints in Eqn. (25) ($\mathbf{b_{eq}}$) is a zero matrix of size ($K \times 1$).

B. KKT conditions of ISO SCQP problem

The ISO SCQP problem can be summarized as:

$$\text{Minimize:} \quad f(\mathrm{x}) \tag{64}$$

$$\text{subject to } g_{in}(x) = \mathbf{b_{in}} - \mathbf{C_{in}} \mathbf{x}^T \leq 0 \ \forall in \in IN \tag{65}$$

$$h_{eq}(x) = \mathbf{b_{eq}} - \mathbf{C_{eq}} \mathbf{x}^T = 0 \ \ \forall eq \in EQ \tag{66}$$

where g and h are the inequality and equality constraints, respectively, and IN and EQ their active numbers, respectively. Note that the number of equality constraints is exactly the same as the number of buses for a power system network. Based on [66], the KKT conditions are:

$$\nabla f(x) + \sum_{in=1}^{IN} u_{in} \nabla g_{in}(x) + \sum_{eq=1}^{EQ} \lambda_k \nabla h_{eq}(x) = 0 \tag{67}$$

$$u_{in} g_{in}(x) = 0 \ \ \forall in = 1,2,\cdots,IN \tag{68}$$

where $u_{in} \forall in$ is a non-negative scalar and λ the scalar dual variables for the inequality and equality constraints, respectively. After differentiating Eqn. (67) with respect to all the primal and dual variables, a well-known matrix equation ($\mathbf{Ay} = \mathbf{b}$) is found, where $\mathbf{y} = [\mathbf{x}^\mathbf{T}, u_1, \ldots, u_{IN}, \lambda_1, \ldots, \lambda_{EQ}]^T$. The process for determining $\mathbf{y}$ is explicity described in [60, 66].

Once $\mathbf{x}$ and $\lambda_k, \forall k = 1,2,\ldots,K$ are known, the PD by each generator (P_i), load dispatch by each consumer (q_j), LMP of each node ($\lambda_{P_i} \forall i, \lambda_{d_j} \forall j$) and branch flows ($F_{km}$) are calculated as:

$$P_i = \mathbf{x}_i \ \ \forall i = 1,2,\cdots,I \tag{69}$$

$$q_j = \mathbf{x}_{I+K+j} \ \ \forall j = 1,2,\cdots,J \tag{70}$$

$$\lambda_{P_i} = \lambda_i \ \ \forall i \in I_K \tag{71}$$

$$\lambda_{d_j} = \lambda_j \ \ \forall j \in J_K \tag{72}$$

$$\mathbf{F} = \mathbf{S} * \mathbf{PNetInject} \tag{73}$$

where

$$\mathbf{S} = (\mathbf{D} * \mathbf{A_r}) * \mathbf{B}_{rr}^{-1} \tag{74}$$

$$\mathbf{PNetInject} = \mathbf{B}_{rr} * \boldsymbol{\delta} \tag{75}$$

$$\delta_k = \mathbf{x}_{I+k} \ \ \forall k = 1,2,\cdots K \tag{76}$$

where $\mathbf{B_{rr}}$ is found from the $\mathbf{Y_{bus}}$ matrix after deleting the first row and column.

C. *Simulation results*

In this section, detailed results for all the three test problems considered are described.

1. IEEE 3-bus system

The results for the IEEE 3-bus test system for cases II, III and V are shown in Tables 17 and 18, 19 and 20, and 21 and 22, respectively. Note that the minus sign of the line flow indicates that the electricity flow is in the reverse direction, for example, $F_{13} = -21.08$ means that the actual electricity flows from nodes 3 to 1.

Table 17: Detailed summary of *IEEE* 3-bus system for case II

	Expected profit ($)				BS		Market price ($/MW)			Dispatch (MW)				Line flow (MW)		
Alg.	G-1	G-2	L-1	L-2	k_{g1}	k_{g2}	λ_1	λ_2	λ_3	P_1	P_2	d_1	d_2	F_{12}	F_{13}	F_{23}
EBA [1]	1557.07	455.55	1557.07	455.55	1.13	1.08	21.11	21.11	21.11	361.79	189.08	236.09	361.79	NR	NR	NR
BA [1]	1540.17	440.16	2239.75	2986.34	1.13	1.08	21.07	21.07	21.07	361.26	190.86	236.63	361.26	NR	NR	NR
PSO [1]	1523.11	429.44	2253.33	3004.44	1.13	1.08	21.01	21.01	21.01	362.79	191.01	237.34	362.79	NR	NR	NR
GA [1]	1494.39	413.18	2274.95	3033.27	1.12	1.07	20.92	20.92	20.92	364.65	191.8	238.48	364.65	NR	NR	NR
NCP [63]	1560.00	446.50	NR	NR	1.13	1.08	21.10	21.10	21.10	361.80	188.8	236.00	361.80	146.8	-21.00	-167.80
IT-DE	1560.39	447.21	2227.14	2969.24	1.13	1.08	21.12	21.12	21.12	361.64	188.92	235.96	314.60	146.76	-21.08	-167.84
IT-GA	1835.62	430.34	2111.37	2814.86	1.14	1.13	21.62	21.62	21.62	395.30	140.76	229.75	306.31	157.29	8.26	-149.03
CE-DE	1581.98	454.55	2213.71	2951.32	1.14	1.09	21.18	21.18	21.18	362.02	186.88	235.25	313.65	146.81	-20.04	-166.84
CE-GA	1666.26	468.94	2168.97	2891.68	1.14	1.10	21.37	21.37	21.37	367.56	175.77	232.86	310.47	148.39	-13.69	-162.08

Table 18: CSW, computational time and Friedman test results for *IEEE* 3-bus test system for case II

	EBA [1]	BA [1]	PSO [1]	GA [1]	IT-DE	IT-GA	CE-DE	CE-GA
CSW ($)	6099.2	6122.05	6157.14	6211.8	6093.22	5907.37	6062.5	5970.04
Time (sec.)	108.6	72.6	60.6	138.6	5.45	5.39	**1.21**	1.39
MR	5.67	4.33	4.17	4.5	4.17	5.33	**3.83**	4.00

Table 19: Detailed summary of *IEEE* 3-bus system for case III

	Expected profit ($)				BS		Market price ($/MW)			Dispatch (MW)				Line flow (MW)		
Alg.	G-1	G-2	L-1	L-2	k_{g1}	k_{g2}	λ_1	λ_2	λ_3	P_1	P_2	d_1	d_2	F_{12}	F_{13}	F_{23}
EBA [1]	747.97	1799.02	2094.20	980.54	1.34	1.25	21.68	25.41	29.13	122.95	287.02	228.96	181.01	NR	NR	NR
BA [1]	741.59	1789.58	2110.39	985.47	1.33	1.25	21.62	25.37	29.12	123.45	287.48	229.69	181.24	NR	NR	NR
PSO [1]	690.43	1586.22	2424.86	859.21	1.23	1.25	20.30	25.07	29.84	151.97	263.46	246.21	169.23	NR	NR	NR
GA [1]	684.26	1580.74	2436.34	862.65	1.23	1.25	20.25	25.04	29.82	152.22	264.14	246.79	169.57	NR	NR	NR
NCP [63]	747.10	1799.00	NA	NA	1.34	1.25	21.70	25.40	29.10	122.90	286.90	228.80	180.90	25.00	-130.90	-155.9
IT-DE	749.55	1798.61	2093.70	977.65	1.34	1.25	21.70	29.17	25.43	123.26	286.04	228.78	180.52	25.00	-130.52	-155.52
IT-GA	796.93	1816.66	2041.20	911.46	1.35	1.27	21.93	29.54	25.73	126.59	273.61	225.90	174.30	25.00	-124.30	-149.30
CE-DE	749.12	1807.19	2081.62	984.11	1.34	1.25	21.75	29.13	25.44	122.01	287.24	228.12	181.12	25.00	-131.12	-156.12
CE-GA	772.84	1839.50	2024.37	963.26	1.36	1.26	22.00	29.25	25.62	120.78	283.38	224.97	179.19	25.00	-129.19	-154.19

Table 20: CSW, computational time and Friedman test results for *IEEE* 3-bus test system for case III

	EBA [1]	BA [1]	PSO [1]	GA [1]	IT-DE	IT-GA	CE-DE	CE-GA
CSW ($)	3873.21	3891.90	4130.12	4147.37	3863.59	3728.12	3856.28	3764.74
Time (sec.)	130.8	84.6	70.8	140.4	5.43	5.42	**1.36**	1.38
MR	4.67	6.00	6.33	6.67	4.00	4.67	**3.67**	4.00

Table 21: Detailed summary of *IEEE* 3-bus system for case IV

	Expected profit ($)				BS				Market price ($/MW)			Dispatch (MW)				Line flow (MW)		
Alg.	G-1	G-2	L-1	L-2	k_{g1}	k_{g2}	k_{d1}	k_{d2}	λ_1	λ_2	λ_3	P_1	P_2	d_1	d_2	F_{12}	F_{13}	F_{23}
EBA [1]	767.57	859.46	2116.94	1658.21	1.31	1.16	0.90	0.78	21.45	24.23	22.84	132.58	215.91	203.04	145.45	NR	NR	NR
BA [1]	764.78	892.27	2096.01	1657.30	1.32	1.16	0.90	0.78	21.40	24.24	22.82	129.70	219.76	202.09	147.38	NR	NR	NR
PSO [1]	761.35	891.46	2096.01	1656.24	1.32	1.16	0.90	0.78	21.54	24.33	22.93	129.21	219.61	201.52	147.30	NR	NR	NR
GA [1]	734.80	887.30	2109.49	1647.35	1.32	1.16	0.89	0.78	21.43	24.40	22.91	126.57	219.65	198.90	147.32	NR	NR	NR
IT-DE	767.83	862.40	2115.77	1655.34	1.31	1.16	0.90	0.78	21.46	24.26	22.86	132.50	215.98	202.99	145.49	25.00	-95.49	-120.49
IT-GA	762.36	1093.81	1988.80	1634.91	1.36	1.18	0.92	0.82	22.07	24.92	23.49	117.70	241.61	201.01	158.30	25.00	-108.30	-133.30
CE-DE	771.71	869.66	2105.91	1665.02	1.32	1.16	0.91	0.78	21.51	24.23	22.87	131.95	217.54	203.21	146.27	25.00	-96.27	-121.27
CE-GA	850.57	892.37	2062.95	1439.46	1.32	1.20	0.90	0.79	21.69	25.19	23.44	142.31	190.97	200.29	132.98	25.00	-82.98	-107.98

Table 22: CSW, computational time and Friedman test results of *IEEE* 3 bus test system for case-IV

	EBA [1]	BA [1]	PSO [1]	GA [1]	IT-DE	IT-GA	CE-DE	CE-GA
CSW ($)	2416.84	2425.83	2412.94	2358.71	2417.78	**2582.24**	2430.62	2308.88
Time (sec.)	146.40	99.00	82.20	177.60	10.83	36.17	**1.80**	1.92
MR	4.5	4.25	5.25	6.17	4	4.5	**2.67**	4.67

2. IEEE 30-bus system

The results for the *IEEE* 30-bus test system for cases I, II, III and IV are shown in Tables 23 and 24, 25 and 26, 27 and 28, and 29 and 30, respectively.

Table 23: Summary of *IEEE* 30-bus system for case I

	Alg.	G-1(#1)	G-2(#2)	G-3(#13)	G-4(#22)	G-5(#23)	G-6(#27)
Average nodal price ($/MWh)	GA	32.10	32.01	33.17	33.41	45.90	34.91
	DE	32.10	32.01	33.17	33.41	45.90	34.91
	IP	32.10	32.01	33.17	33.41	45.90	34.91
Average power production (MW)	GA	56.40	60.06	40.86	57.07	50.00	75.64
	DE	56.40	60.06	40.86	57.07	50.00	75.64
	IP	56.40	60.06	40.86	57.07	50.00	75.64
Average expected profit ($)	GA	397.68	360.72	166.95	325.67	945.17	715.17
	DE	397.68	360.72	166.95	325.67	945.17	715.17
	IP	397.68	360.72	166.95	325.67	945.17	715.17

Table 24: Line flow, CSW, computational time and Friedman test results for *IEEE* 30-bus test system for case I

	GA	DE	IP
$(F_{6,8}, F_{17,12}, F_{10,17})$ (MW)	(8.90,8.00,10.00)	(8.90,8.00,10.00)	(8.90,8.00,10.00)
CSW ($)	18453.88	18453.88	18453.88
Time (sec.)	10.09	2.28	0.08
MR	2.13	2.00	**1.88**

Table 25: Detailed summary of *IEEE* 30-bus system for case II

	Alg.	G-1(#1)	G-2(#2)	G-3(#13)	G-4(#22)	G-5(#23)	G-6(#27)
Average nodal price ($/MWh)	EBA	36.18	36.18	36.18	36.18	36.18	36.18
	BA	36.17	36.17	36.17	36.17	36.17	36.17
	PSO	36.00	36.00	36.00	36.00	36.00	36.00
	GA	35.94	35.94	35.94	35.94	35.94	35.94
	IT-DE	37.17	37.17	37.17	37.17	37.17	37.17
	IT-GA	37.06	37.06	37.06	37.06	37.06	37.06
	CE-DE	37.37	37.37	37.37	37.37	37.37	37.37
	CE-GA	37.27	37.27	37.27	37.27	37.27	37.27
Average bidding coefficients	EBA	1.07	1.08	1.05	1.07	1.07	1.08
	BA	1.07	1.08	1.05	1.07	1.07	1.08
	PSO	1.06	1.07	1.04	1.06	1.06	1.07
	GA	1.06	1.07	1.04	1.06	1.06	1.07
	IT-DE	1.10	1.11	1.04	1.10	1.01	1.11
	IT-GA	1.09	1.10	1.06	1.09	1.04	1.10
	CE-DE	1.10	1.12	1.02	1.10	1.01	1.12
	CE-GA	1.10	1.11	1.03	1.10	1.01	1.11
Average power production (MW)	EBA	61.52	65.72	44.70	57.33	57.33	68.53
	BA	61.53	65.72	44.71	57.35	57.35	68.52
	PSO	61.75	65.75	45.04	57.29	57.34	68.78
	GA	61.76	65.83	45.09	57.31	57.41	68.82
	IT-DE	63.53	67.54	49.02	59.65	50.00	70.17
	IT-GA	63.52	68.28	48.10	59.63	50.00	70.88
	CE-DE	63.28	67.03	49.62	59.15	50.00	69.93
	CE-GA	63.03	67.19	49.21	59.63	50.00	70.39
Average expected profit ($)	EBA	622.27	605.97	280.85	461.11	461.10	771.30
	BA	621.70	605.31	280.42	460.60	460.60	770.61
	PSO	611.60	594.11	273.25	450.65	450.74	759.47
	GA	607.90	590.31	270.61	447.22	447.41	755.39
	IT-DE	713.20	703.22	356.46	548.88	508.51	869.82
	IT-GA	706.32	698.60	348.26	542.44	503.17	864.83
	CE-DE	724.94	714.65	367.58	559.02	518.40	882.83
	CE-GA	716.73	708.35	361.97	553.80	513.48	877.04

Table 26: Line flow, CSW, computational time and Friedman test results for *IEEE* 30-bus test system for case II

	EBA	BA	PSO	GA	IT-DE	IT-GA	CE-DE	CE-GA
$(F_{6,8}, F_{17,12}, F_{10,17})$ (MW)	NR	NR	NR	NR	(9.78,16.98,16.08)	(9.69,16.92,16.28)	(9.77,16.99,15.88)	(9.69,16.94,16.03)
CSW ($)	NR	NR	NR	NR	18225.32	18201.44	18200.8	18216.83
Time (sec.)	261.6	213	185.4	288.6	27.66	23.39	**4.60**	8.67
MR	5.43	6.14	6.86	7.57	2.71	3.86	**1.43**	2.00

Table 27: Summary of *IEEE* 30-bus system for case III

	Alg.	G-1(#1)	G-2(#2)	G-3(#13)	G-4(#22)	G-5(#23)	G-6(#27)
Average nodal price ($/MWh)	EBA	36.58	36.58	40.13	36.15	47.12	43.44
	BA	36.33	36.33	40.04	35.81	46.95	43.25
	PSO	36.30	36.30	40.00	35.79	46.93	43.23
	GA	36.25	36.25	39.95	35.75	46.91	43.19
	IT-DE	38.57	38.39	40.77	40.47	51.13	46.55
	IT-GA	38.62	38.42	40.98	40.42	51.10	46.61
	CE-DE	38.60	38.41	40.81	40.49	51.14	46.57
	CE-GA	38.69	38.47	41.29	40.26	50.94	46.40
Average bidding coefficients	EBA	1.12	1.13	1.27	1.19	1.40	1.26
	BA	1.11	1.13	1.27	1.16	1.40	1.26
	PSO	1.11	1.13	1.27	1.16	1.40	1.26
	GA	1.11	1.13	1.26	1.16	1.40	1.26
	IT-DE	1.15	1.18	1.30	1.23	1.45	1.65
	IT-GA	1.16	1.18	1.32	1.23	1.31	1.66
	CE-DE	1.16	1.18	1.31	1.23	1.41	1.65
	CE-GA	1.17	1.19	1.32	1.20	1.39	1.65
Average power production (MW)	EBA	57.13	59.46	30.19	39.41	50.00	71.43
	BA	56.58	58.62	30.72	41.57	50.00	70.78
	PSO	56.60	58.57	30.79	41.58	50.00	70.79
	GA	56.59	58.65	30.84	41.61	50.00	70.79
	IT-DE	61.72	62.41	31.36	54.58	50.00	48.72
	IT-GA	61.51	63.31	30.60	54.51	50.00	48.66
	CE-DE	61.59	62.53	31.33	54.57	50.00	48.69
	CE-GA	60.11	61.71	31.15	57.56	50.00	48.35
Average expected profit ($)	EBA	632.45	609.69	352.91	387.45	985.80	1296.02
	BA	615.90	591.12	354.78	385.27	977.02	1276.09
	PSO	614.52	589.43	354.23	384.64	976.41	1274.91
	GA	611.66	586.84	352.95	382.99	975.10	1272.46
	IT-DE	793.47	758.10	396.34	710.09	1206.34	1191.89
	IT-GA	795.19	765.48	395.04	704.80	1205.08	1193.03
	CE-DE	794.38	760.34	397.02	711.07	1206.93	1192.42
	CE-GA	791.97	759.09	410.47	719.60	1196.94	1177.74

Table 28: Line flow, CSW, computational time and Friedman test results for *IEEE* 30-bus test system for case III

	EBA	BA	PSO	GA	IT-DE	IT-GA	CE-DE	CE-GA
$(F_{6,8}, F_{17,12}, F_{10,17})$ (MW)	NR	NR	NR	NR	(10,8,10)	(10,8,10)	(10,8,10)	(10,8,10)
CSW ($)	NR	NR	NR	NR	15731.54	15901.73	15778.71	15795.78
Time (sec.)	361.8	289.2	253.8	400.2	27.27	29.09	**4.60**	8.59
MR	4.83	5.17	6.17	7.17	3.67	3.00	**2.50**	3.50

Table 29: Summary of *IEEE* 30-bus system for cases III and IV

	Expected profits ($) at case III				Expected profits ($) at case IV				Bidding coefficients at case IV			
	IT-DE	IT-GA	CE-DE	CE-GA	IT-DE	IT-GA	CE-DE	CE-GA	IT-DE	IT-GA	CE-DE	CE-GA
G-1(#1)	793.47	795.19	794.38	791.97	783.94	768.22	789.52	782.49	1.14	1.14	1.13	1.13
G-2(#2)	758.1	765.48	760.34	759.09	754.07	737.85	768.40	766.86	1.17	1.17	1.16	1.14
G-3(#13)	396.34	395.04	397.02	410.47	376.09	368.92	369.49	361.56	1.25	1.24	1.25	1.24
G-4(#22)	710.09	704.8	711.07	719.6	506.58	510.41	560.16	556.91	1.21	1.19	1.19	1.19
G-5(#23)	1206.34	1205.08	1206.93	1196.94	611.66	625.52	657.67	658.65	1.11	1.12	1.11	1.12
G-6(#27)	1191.89	1193.03	1192.42	1177.74	941.64	936.28	880.76	866.85	1.13	1.13	1.33	1.34
L-1(#2)	666.06	665.55	665.65	664.68	668.20	671.25	667.86	669.83	0.98	0.98	0.98	0.98
L-2(#3)	756.88	755.91	756.45	754.37	761.61	764.80	761.77	763.93	0.98	0.98	0.98	0.98
L-3(#4)	730.1	728.81	729.63	726.78	736.60	740.11	737.06	739.49	0.98	0.97	0.97	0.98
L-4(#7)	940.5	940.42	940.04	940.15	940.05	943.61	939.03	941.20	0.97	0.97	0.97	0.97
L-5(#8)	880.33	876.09	879.8	884.53	1254.90	1258.51	1141.07	1150.41	0.97	0.97	0.66	0.66
L-6(#10)	564.25	565.73	563.84	568.61	557.35	561.56	551.35	553.77	0.95	0.96	0.95	0.95
L-7(#12)	1084.56	1080.45	1083.94	1074.3	1106.55	1110.99	1110.18	1113.73	0.94	0.94	0.94	0.94
L-8(#14)	232.35	235.21	232.64	241.94	527.07	523.22	492.76	484.83	0.45	0.47	0.47	0.47
L-9(#15)	212.14	212.44	212.07	213.96	397.96	391.43	384.62	383.18	0.94	0.94	0.94	0.94
L-10(#16)	947.49	940.73	946.95	930.78	1150.10	1148.37	1151.41	1153.26	0.81	0.83	0.81	0.81
L-11(#17)	92.55	89.13	92.37	83.99	250.36	241.75	249.10	248.50	0.73	0.73	0.73	0.73
L-12(#18)	271.16	271.68	271.03	273.61	441.82	436.85	428.63	427.81	0.95	0.96	0.95	0.95
L-13(#19)	269.57	270.24	269.4	272.13	377.00	374.86	367.39	367.45	0.97	0.97	0.97	0.97
L-14(#20)	333.44	334.34	333.2	336.42	387.95	388.42	380.81	381.72	0.97	0.97	0.97	0.97
L-15(#21)	1227.32	1228.62	1226.91	1231.72	1252.61	1255.86	1244.15	1246.39	0.95	0.96	0.95	0.95
L-16(#23)	474.35	474.71	474.19	476.95	652.05	647.66	637.33	637.00	0.96	0.97	0.96	0.96
L-17(#24)	902.22	902.58	901.93	905.51	1038.28	1037.28	1023.05	1024.34	0.97	0.97	0.97	0.97
L-18(#26)	321	320.8	320.77	323.02	423.58	423.66	409.55	411.34	0.98	0.98	0.97	0.98
L-19(#29)	335.31	334.56	335	337.38	461.90	463.04	441.68	444.79	0.98	0.97	0.96	0.96
L-20(#30)	683.78	682.83	683.39	686.38	838.35	839.76	814.03	817.56	0.97	0.97	0.95	0.94

Table 30: Line flow, CSW, computational time and Friedman test results for *IEEE* 30-bus test system for case IV

	IT-DE	IT-GA	CE-DE	CE-GA
$(F_{6,8}, F_{17,12}, F_{10,17})$ (MW)	(9.18,8,10)	(9.11,8,10)	(9.05,8,10)	(9.42,8,10)
CSW ($)	15146.98	15236.98	14238.36	14271.38
Time (Sec)	219.78	247.98	**35.18**	120.83
MR	2.65	2.42	**2.12**	2.81

3. IEEE 118-bus system

The results for the *IEEE* 118-bus test system for cases II and III are shown in Tables 31 and 32, and 33 and 34, respectively.

Table 31: Summary of *IEEE* 118-bus system for case II

Gen-bus	Expected profits ($)				Power production (MW)				Bidding coefficients			
	IT-DE	IT-GA	CE-DE	CE-GA	IT-DE	IT-GA	CE-DE	CE-GA	IT-DE	IT-GA	CE-DE	CE-GA
G-1(#1)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-2(#4)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-3(#6)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-4(#8)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-5(#10)	167.49	165.10	168.60	167.15	114.62	110.52	112.11	111.33	1.01	1.01	1.01	1.01
G-6(#12)	31.89	31.52	32.16	31.91	23.01	22.67	22.89	22.87	1.00	1.00	1.00	1.00
G-7(#15)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.47	2.50	2.50	2.50
G-8(#18)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-9(#19)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-10(#24)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-11(#25)	82.42	81.33	83.04	82.57	58.53	56.61	57.50	60.27	1.00	1.01	1.01	1.00
G-12(#26)	117.45	115.83	118.11	117.30	82.47	79.42	80.21	80.23	1.00	1.01	1.01	1.01
G-13(#27)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.46	2.50	2.49	2.50
G-14(#31)	2.63	2.59	2.65	2.63	1.92	1.80	1.91	1.92	1.00	1.01	1.00	1.00
G-15(#32)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.49	2.50
G-16(#34)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-17(#36)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-18(#40)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-19(#42)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-20(#46)	7.13	7.02	7.19	7.13	5.19	4.89	5.18	5.18	1.00	1.01	1.00	1.00
G-21(#49)	76.41	75.50	77.02	76.42	53.88	53.06	53.56	53.46	1.00	1.01	1.01	1.01
G-22(#54)	18.00	17.80	18.16	18.01	13.07	12.98	13.03	13.02	1.00	1.00	1.00	1.00
G-23(#55)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.42	2.50	2.49	2.50
G-24(#56)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-25(#59)	58.09	57.42	58.57	58.11	41.42	41.00	41.13	41.07	1.00	1.00	1.00	1.00
G-26(#61)	59.94	59.27	60.05	59.97	42.42	42.32	41.49	42.34	1.00	1.00	1.01	1.00
G-27(#62)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.40	2.50	2.42	2.50
G-28(#65)	145.81	144.26	147.12	145.72	99.76	99.08	99.72	98.25	1.01	1.01	1.01	1.01
G-29(#66)	146.24	144.63	145.83	146.00	100.40	99.34	96.66	98.10	1.01	1.01	1.01	1.01
G-30(#69)	191.80	190.26	193.22	191.51	128.46	129.43	127.07	126.07	1.01	1.01	1.01	1.01
G-31(#70)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.48	2.50	2.47	2.50
G-32(#72)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-33(#73)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-34(#74)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-35(#76)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.49	2.50
G-36(#77)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-37(#80)	177.38	175.18	178.39	177.14	119.65	117.21	117.34	117.30	1.01	1.01	1.01	1.01
G-38(#85)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.45	2.50	2.47	2.50
G-39(#87)	1.50	1.48	1.51	1.50	1.09	1.03	1.10	1.10	1.00	1.01	1.00	1.00
G-40(#89)	195.80	222.15	225.74	223.85	129.91	146.61	145.31	144.48	1.08	1.01	1.02	1.02
G-41(#90)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.38	2.50	2.50	1.46
G-42(#91)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-43(#92)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-44(#99)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-45(#100)	94.38	93.17	95.11	94.30	66.46	64.85	65.73	65.27	1.00	1.01	1.01	1.01
G-46(#103)	15.01	14.83	15.13	15.01	10.92	10.81	10.81	10.87	1.00	1.00	1.00	1.00
G-47(#104)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.49	2.49	2.50	2.47
G-48(#105)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-49(#107)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.49	2.46	2.50
G-50(#110)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.50
G-51(#111)	13.51	13.35	13.62	13.51	9.83	9.73	9.81	9.78	1.00	1.00	1.00	1.00
G-52(#112)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-53(#113)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.50	2.50	2.50	2.49
G-54(#116)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.47	2.50	2.50	2.50

Table 32: Line flow, LMP, CSW, computational time and Friedman test results for *IEEE* 118-bus test system for case II

	IT-DE	IT-GA	CE-DE	CE-GA
$F_{100,103}$ (MW)	24.56	24.66	24.61	24.60
λ (\$/MWh)	22.74	22.72	22.75	22.74
CSW (\$)	52038.40	52031.63	52002.34	52014.12
Time (sec.)	813.37	946.36	293.21	321.45
MR	2.50	3.01	**2.01**	2.48

Table 33: Summary of *IEEE* 118-bus system for case III

Gen-bus	Market clearing prices ((\$/MWh))				Expected profits (\$)				Bidding coefficients			
	IT-DE	IT-GA	CE-DE	CE-GA	IT-DE	IT-GA	CE-DE	CE-GA	IT-DE	IT-GA	CE-DE	CE-GA
G-1(#1)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-2(#4)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-3(#6)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-4(#8)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-5(#10)	22.69	22.68	22.72	22.72	161.25	160.51	165.40	164.85	1.01	1.01	1.01	1.01
G-6(#12)	22.69	22.68	22.73	22.72	30.78	30.55	31.59	31.32	1.00	1.00	1.00	1.01
G-7(#15)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-8(#18)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-9(#19)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.49
G-10(#24)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-11(#25)	22.69	22.68	22.73	22.72	79.56	78.89	81.59	80.65	1.00	1.01	1.01	1.01
G-12(#26)	22.69	22.68	22.73	22.72	113.26	112.11	116.14	115.68	1.01	1.01	1.01	1.01
G-13(#27)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-14(#31)	22.69	22.68	22.73	22.72	2.53	2.52	2.60	2.58	1.00	1.00	1.00	1.00
G-15(#32)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.49
G-16(#34)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.49
G-17(#36)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-18(#40)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-19(#42)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-20(#46)	22.69	22.68	22.73	22.72	6.88	6.83	7.06	7.03	1.00	1.00	1.00	1.00
G-21(#49)	22.69	22.68	22.73	22.72	73.69	73.22	75.67	75.40	1.01	1.00	1.01	1.00
G-22(#54)	22.69	22.68	22.73	22.72	17.37	17.25	17.84	17.77	1.00	1.00	1.00	1.00
G-23(#55)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-24(#56)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-25(#59)	22.69	22.68	22.73	22.72	56.07	55.63	57.53	57.32	1.00	1.00	1.00	1.00
G-26(#61)	22.69	22.68	22.73	22.72	57.87	57.42	59.37	59.17	1.00	1.00	1.00	1.00
G-27(#62)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-28(#65)	22.69	22.68	22.73	22.72	140.54	139.58	144.18	143.82	1.01	1.01	1.01	1.01
G-29(#66)	22.69	22.68	22.73	22.72	140.92	139.94	144.56	144.24	1.01	1.01	1.01	1.01
G-30(#69)	22.69	22.68	22.73	22.72	184.96	184.37	189.38	188.75	1.01	1.01	1.01	1.01
G-31(#70)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.47
G-32(#72)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.47
G-33(#73)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-34(#74)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-35(#76)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-36(#77)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-37(#80)	22.69	22.68	22.73	22.72	171.09	170.30	175.26	174.55	1.01	1.01	1.01	1.01
G-38(#85)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-39(#87)	22.69	22.68	22.73	22.72	1.45	1.44	1.49	1.48	1.00	1.00	1.00	1.00
G-40(#89)	22.69	22.68	22.73	22.72	215.70	216.65	221.43	220.78	1.02	1.01	1.02	1.02
G-41(#90)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-42(#91)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-43(#92)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-44(#99)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-45(#100)	22.69	22.68	22.73	22.72	91.04	90.37	93.40	93.06	1.00	1.01	1.01	1.01
G-46(#103)	26.48	26.17	26.47	26.37	61.53	55.76	61.41	60.20	1.14	1.14	1.14	1.14
G-47(#104)	24.75	24.57	24.76	24.71	0.00	0.00	0.00	0.00	2.46	2.49	2.50	2.50
G-48(#105)	25.07	24.87	25.08	25.02	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-49(#107)	24.60	24.44	24.61	24.56	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-50(#110)	26.13	25.84	26.12	26.04	0.00	0.00	0.00	0.00	1.80	2.49	2.50	2.50
G-51(#111)	26.13	25.84	26.12	26.04	57.07	55.31	57.04	56.09	1.10	1.08	1.10	1.10
G-52(#112)	26.13	25.84	26.12	26.04	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-53(#113)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50
G-54(#116)	22.69	22.68	22.73	22.72	0.00	0.00	0.00	0.00	2.50	2.49	2.50	2.50

Table 34: Line flow, CSW, computational time and Friedman test results for *IEEE* 118-bus test system for case III

	IT-DE	IT-GA	CE-DE	CE-GA
$F_{100,103}$ (MW)	20.00	20.00	20.00	20.00
CSW (\$)	51966.17	51983.37	51926.68	51935.34
Time (sec.)	673.68	843.60	174.78	256.03
MR	2.62	3.01	**2.01**	2.36