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Networked Stabilization for Multi-Input Systems Over Quantized Fading Channels*

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Abstract

This paper studies feedback stabilization for networked control systems (NCSs) over quantized fading channels placed at the plant input, which cover both logarithmic quantization and packet drop in the actuator channel. The notion of mean-square (MS) stability is developed in the input-output setting, and the MS stabilizability is studied for both single-input (SI) and multi-input (MI) systems under state feedback. A necessary and sufficient condition is derived for the MS stabilizability of the NCS over the quantized fading channel by using channel resource allocation. Our result improves the known MS stabilizability condition and complement the existing work in the NCS area.

1 Introduction

This paper considers the actuator channel in an NCS which involves both quantization and fading. Two different logarithmic quantization methods are considered. The first one is proposed in [4], and induces a sector bounded uncertainty in multiplicative form [7], which will be referred to as multiplicative logarithmic quantization (MLQ). The second one is proposed in [13] and induces a sector bounded uncertainty in relative form, which will be referred to as relative logarithmic quantization (RLQ). We assume that the state feedback control signal is first quantized via either the MLQ or RLQ method and then transmitted over a fading channel prior to being applied at the plant input as the true actuating signal. The fading channel model from [3, 18] is adopted, which is described

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by a set of m independent multiplicative random processes with m the input dimension of the plant. They are independent and identically distributed (i.i.d.) stationary processes with fixed means and variances, and cover packet drop as a special case. If the Gauss distribution is assumed, then the fading channel resembles the Ricean channel widely used in digital communications. Due to the presence of the fading channel, the NCS under consideration represents a stochastic control system. This paper is aimed at deriving the MS stabilizability of the NCS in terms of the quantization error bounds and signal-to-noise ratios (SNRs) of the MI fading channel. Our result shows that the RLQ method is preferred for which the MS stabilizability problem admits an analytic solution.

Quantization and packet drop in networked control systems are two of the major sources for the information loss that limits the feedback stabilization and control performance. Both have been investigated extensively. Elia and Mitter are the first to study logarithmic quantization [4], who obtain the quadratic stabilizability for SI systems under state feedback. The same problem is studied in [7, 9] for MI systems, and a nice analytic solution is obtained in [13]. It is now known that the least number of bits required for the quadratic stabilizability is exactly the topological entropy of the system [19]. On the other hand, packet drop is initially studied in [2, 14]. The results available in [3, 18] indicate that the fading channel is more general and covers the packet drop as a special case. More importantly the MS stabilizability for MI NCSs over the fading channel depends on roughly the SNR to be clarified later.

The NCS in presence of both quantization and packet drop leads to a stabilization problem which is not a simple combination of the above two problems. Rather, it is more involved and requires a deeper insight into the problem. This can be seen from the expression of channel capacity obtained in our main result. The problem is studied first in [16] for SI systems and an inequality is derived for the feedback stabilizability. It turns out that the MLQ employed in [16] for quantization weakens its MS stabilizability results. Indeed recent conference papers [6, 17] show that a stronger result can be obtained if the RLQ method is used for quantization. However the MS stabilizability for MI systems remained unknown, which will be studied and a complete solution will be derived in this paper.

We wish to emphasize at this point that all the results for MI systems mentioned above, e.g. those in [13, 18], are obtained by an important technique called *channel resource allocation*, which will be also employed in this paper. The technique avoids an NP-hard problem on structured uncertainties by imposing only a total information constraint on the overall channel, rather than on each of the individual channels. The technique not only mitigates the difficulty in solving the problem for MI systems, but also suggests a wiser way to do synthesis, which can be referred to as *channel/controller co-design*. This powerful tool is crucial to achieve the minimum required channel capacity for stabilization. Moreover, it leads to an analytically solvable problem and the solution

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is constructive. The technique was first proposed in the conference paper [8] and have been proved to be efficient for various channel models. Detailed explanation can be found in [13]. We will also demonstrate the technique when proving our main result.

The notation in this paper is standard with $\mathbb{R}/\mathbb{C}$ standing for the set of real/complex numbers. A matrix $M \in \mathbb{R}^{n\times m}/\mathbb{C}^{n\times m}$ has $n \times m$ elements in $\mathbb{R}/\mathbb{C}$. Its transpose is denoted by M' and conjugate transpose by M^*. The ith singular value of M is denoted by $\sigma_i(M)$ arranged in descending order with $\overline{\sigma}(M) = \sigma_1(M)$. If $n = m$, its ith eigenvalue is denoted by $\lambda_i(M)$, its spectral radius by $\rho(M)$, its determinant by $\det(M)$ and its trace by $\mathrm{Tr}\{M\}$. For a transfer function matrix $G(z)$ of dimension $p \times m$, its $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms are defined respectively by

$$\|G\|_{\mathcal{H}_\infty} := \sup_{|z|>1} \overline{\sigma}[G(z)], \quad \|G\|_{\mathcal{H}_2} := \sup_{\rho>1} \sqrt{\mathrm{Tr}\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} G(\rho e^{j\omega})^* G(\rho e^{j\omega})\, d\omega\right\}}. \tag{1}$$

For stable and rational $G(z)$, $\rho = 1$ can be taken in computing $\|G\|_{\mathcal{H}_\infty}$ and $\|G\|_{\mathcal{H}_2}$.

2 Problem Formulation

The discrete-time plant model in consideration admits a state space description

$$x(t+1) = Ax(t) + Bu(t), \quad x(0) = x_0, \tag{2}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and thus $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ with time index t non-negative integer valued. Due to the existence of the communication network at the plant input, a traditional state feedback control law $u(t) = Fx(t)$ cannot be applied directly. Instead $u(t) = g[Fx(t)]$ has to be employed with $g[\cdot]$ modeling the network effect. This paper assumes that multiple independent channels are used, and during transmission, considers only quantization and packet drop that are the two major sources of information loss in a digital network, i.e. each component $s_k(t)$ of the control signal $s(t) = Fx(t)$ is first quantized and then transmitted in a packet over the network independently from others. The quantization error and possible packet drop present a significant challenge to feedback stabilization.

This stabilization problem is studied in [16] for single input (SI) systems (with $m = 1$), which provides an interesting inequality derived for the stabilizability of the NCS. A similar result is available in [6, 17] with the former focusing on SISO output feedback systems, and the latter on state feedback for multi-input (MI) systems. Both provide only sufficient conditions for networked stabilizability. It turns out that the inequality derived first in [16] and generalized to MI systems in [17] has a far reaching implication. In fact it provides an equivalent condition for the networked stabilizability over quantized fading channels which will be shown in this paper.

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Consider first logarithmic quantization, denoted by $\mathcal{Q}(\cdot)$ and proposed in [4] for the case $m = 1$. It is a scalar nonlinear mapping $\mathcal{Q} : \mathbb{R} \to \mathbb{R}$ satisfying $\mathcal{Q}(s) = -\mathcal{Q}(-s)$ and is defined by [7]

$$\mathcal{Q}(s) = v_{(i)} \;\; \forall\; s \in \left(\frac{v_{(i)}}{1+\delta}, \frac{v_{(i)}}{1-\delta} \right] \tag{3}$$

where $v_{(i)} = \left(\frac{1-\delta}{1+\delta}\right)^i v_{(0)} > 0$ for some $v_{(0)} > 0$, $i = 0, \pm 1, \pm 2, \cdots$, and $0 < \delta < 1$. If $m > 1$, then the kth component of s can be quantized independently as in (3) with δ replaced by δ_k. In light of [7], such a quantization induces a sector bounded time-varying uncertainty in the form of

$$v(t) = [I + \Delta(t)]s(t), \quad |\Delta_k(t)| \le \delta_k < 1 \; \forall \; t \tag{4}$$

where $\Delta(t) = \text{diag}[\Delta_1(t), \cdots, \Delta_m(t)]$ and the signal $s(t) = Fx(t)$. The above quantization method is referred to as multiplicative logarithmic quantization (MLQ). A different quantization method, referred to as relative logarithmic quantization (RLQ), is proposed in [13] by taking

$$\mathcal{Q}(s) = v_{(i)} \;\; \forall\; s \in \left(v_{(i)}(1-\delta),\; v_{(i)}(1+\delta)\right] \tag{5}$$

with all the parameters defined identically to those of (3). The above $\mathcal{Q}[\cdot]$ for each component of $s(t)$ leads to the sector bounded time-varying uncertainty

$$v(t) = [I + \Delta(t)]^{-1}s(t), \quad |\Delta_k(t)| \le \delta_k < 1 \; \forall \; t \tag{6}$$

where $\Delta(t) = \text{diag}[\Delta_1(t), \cdots, \Delta_m(t)]$ as well. Different from [4, 7], the sector bounded uncertainty will be assumed to be nonlinear time-varying and dynamic (NTVD) with the unique equilibrium point at the origin having norm bounds (under zero initial condition)

$$\|\Delta_k\|_{\mathcal{H}_\infty} := \sup_{\|s\|_2 \neq 0} \frac{\|\Delta_k s_k\|_2}{\|s_k\|_2} \le \delta_k, \qquad \|s_k(t)\|_2 := \left(\sum_{\tau=0}^{\infty} |s_k(\tau)|^2 \right)^{1/2}, \tag{7}$$

for $1 \le k \le m$. The use of NTVD uncertainty Δ can take time delay into account.

In contrast to the stabilizability results in [13], the aforementioned two quantization methods result in rather different stabilizability conditions when the quantized signals $\{v(t)\}$ are transmitted in packets over the network to be shown in the next section. A packet drop model is proposed in [2] which is extended to stochastic multiplicative fading channels [18]. Let $v_k(t)$ be the kth component of $v(t)$, which is transmitted over the fading channel. The received signal is $u_k(t) = \frac{1}{\mu_k}\mathcal{F}_k(t)v_k(t)$ that is the kth component of the control signal. The fading channel in [18] assumes that $\{\mathcal{F}_k(t)\}$ are i.i.d. stationary random processes with mean $\{\mu_k\}$ and variance $\{\sigma_k^2\}$. Such a fading channel resembles the Ricean channel in digital communications, and covers packet drop as a special case [3, 18]. Indeed if the kth packet arrival rate is γ_k satisfying $0 < \gamma_k < 1$, then the

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fading channel covers the stationary packet drop model with $\mu_k = \gamma_k$ and $\sigma_k^2 = \gamma_k(1-\gamma_k)$. For $1 \le k \le m$, denote

$$\mathcal{N}_k(t) = \frac{1}{\mu_k}\mathcal{F}_k(t) - 1, \quad \nu_k^2 = \frac{\sigma_k^2}{\mu_k^2} = \gamma_k^{-1} - 1. \tag{8}$$

Then $\{\mathcal{N}_k(t)\}$ are also i.i.d. stationary processes with mean zero and variance $\{\nu_k^2\}$, respectively.

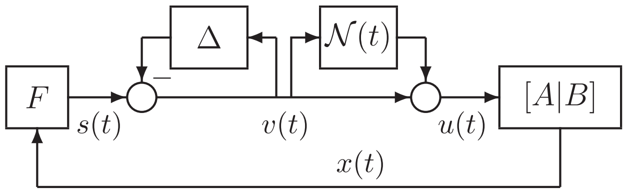

Fig. 1 The NCS over the (RLQ) quantized fading channel

The NCS over the quantized fading channel is shown in Fig. 1 in which the RLQ method is employed for quantization and $[A|B]$ stands for the system (2) or equivalently the transfer matrix $(zI-A)^{-1}B$. By keeping the signal $v(t)$ and outputs of Δ and $\mathcal{N}(t)$ intact, an equivalent block diagram is obtained next.

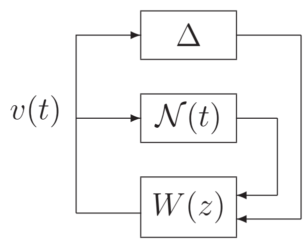

Fig. 2 An equivalent NCS

Due to the use of RLQ in Fig. 1, the above figure admits the transfer matrix

$$W(z) = \begin{bmatrix} T(z) & S(z) \end{bmatrix} \tag{9}$$

with $S(z)$ as the sensitivity, and $T(z)$ as the complementary sensitivity, specified by

$$T(z) = F(zI - A - BF)^{-1}B, \quad S(z) = I + T(z). \tag{10}$$

The uncertainty blocks are represented by

$$\Delta = \mathrm{diag}[\Delta_1, \cdots, \Delta_m], \quad \mathcal{N}(t) = \mathrm{diag}[\mathcal{N}_1(t), \cdots, \mathcal{N}_m(t)]. \tag{11}$$

The NCS in Fig. 2 is in effect a stochastic control system. Assume that its initial condition $x(0) = x_0 \neq 0$ with $x(t)$ the state vector associated with $W(z)$, which can also be random but

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is independent of $\mathcal{N}(t)$ for all t. The MS stability is the stability notion commonly used in the literature that is concerned with $\mathrm{E}\{x(t)x(t)'\}$ as $t \to \infty$ where $\mathrm{E}\{\cdot\}$ denotes the operation of expectation. Such a notion is equivalent to the input/output MS stability as defined next.

Definition 1 Suppose that the system represented by transfer matrix $W(z)$ is internally stable. Let the uncertainty blocks Δ and $\mathcal{N}(t)$ be given in (11) and described as earlier, and $\|\cdot\|$ be the Euclidean norm. The feedback system in Fig. 2 is said to be MS stable, if $\mathrm{E}\left\{\|v(t)\|^2\right\} \to 0$ as $t \to \infty$ for any mean-power bounded initial condition, i.e., $\mathrm{E}\{\|x_0\|^2\} < \infty$, and in presence of all $\mathcal{H}_\infty$-norm bounded uncertainties $\{\Delta_k\}$ and all multiplicative random processes $\{\mathcal{N}_k(t)\}$ with mean 0 and variance $\{\nu_k^2\}$.

It is important to point out that when the feedback system in Fig. 2 is MS stable, then it describes an asymptotically wide-sense stationary (WSS) process $v(t)$ even if the initial condition $x(0) = x_0 \neq 0$. Let the (mean) power norm of $v(t)$ be defined as

$$\|v\|_{\mathcal{P}} := \lim_{N\to\infty}\left(\frac{1}{N}\sum_{\tau=0}^{N-1}\mathrm{E}\{\|v(\tau)\|^2\}\right)^{\frac{1}{2}}. \tag{12}$$

Then the MS stability for the closed-loop system in Fig. 2 is equivalent to $\|v\|_{\mathcal{P}} = 0$.

For the NCS in Fig. 2, the uncertainty bounds $\{\delta_k\}$ of $\{\Delta_k\}$ measure roughly the information loss in quantization, and variances $\{\nu_k^2\}$ are roughly the inverse of the SNR in packet transmission by $\nu_k^2 = \sigma_k^2/\mu_k^2$. As $\{\delta_k\}$ and $\{\nu_k^2\}$ increase, the network becomes less reliable and the underlying NCS is less likely to be stabilizable in the MS sense. For this reason, $\{\delta_k^{-1}\}$ and $\{\nu_k^{-2}\}$ represent the network resource qualitatively. Our goal is to characterize the minimum network resource required to stabilize the NCS in Fig. 2, and study how to design controller F and allocate the network resource jointly in achieving the MS stabilization.

Let (A, B) be stabilizable and A have no eigenvalue on the unit circle. Then there exists a unique stabilizing solution $X \geq 0$ to the algebraic Riccati equation (ARE)

$$X = A'X(I + BB'X)^{-1}A. \tag{13}$$

The following lists some useful facts when $(A + BF)$ is a Schur matrix, i.e. $\rho(A + BF) < 1$:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} S(e^{j\omega})^* S(e^{j\omega})\, d\omega = I + \frac{1}{2\pi}\int_{-\pi}^{\pi} T(e^{j\omega})^* T(e^{j\omega})\, d\omega, \tag{14}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} S(e^{j\omega})^* S(e^{j\omega})\, d\omega \geq I + B'XB. \tag{15}$$

Equality holds for (15) if [13]

$$F = -(I + B'XB)^{-1}B'XA. \tag{16}$$

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In fact the use of the above state feedback gain yields the optimal sensitivity [13]

$$S(e^{j\omega})^* S(e^{j\omega}) = I + B'XB \ \ \forall\, \omega \in \mathbb{R}. \tag{17}$$

There holds identity $\det(I + B'XB) = M(A)^2$ [4, 7, 13] with $M(A)$ the Mahleh measure defined as

$$M(A) := \prod_{i=1}^{n} \max_i \{1, |\lambda_i(A)|\}. \tag{18}$$

Finally consider a vector-valued WSS random process $h(t)$. Its autocorrelation and power spectral density (PSD) are defined respectively by

$$R_h(\tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N-1} \mathrm{E}\{h(i)h(i-\tau)'\}, \quad \Psi_h(\omega) = \sum_{\tau=-\infty}^{\infty} R_h(\tau) e^{-j\omega\tau}.$$

The mean-power of $h(t)$ has an alternative expression

$$\mathcal{P}_h = \|h\|_{\mathcal{P}}^2 = \mathrm{Tr}\{R_h(0)\} = \mathrm{Tr}\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} \Psi_h(\omega)\, d\omega\right\}.$$

Let $P(z)$ be a rational transfer matrix for a stable system. If $h(t)$ is used as input, then its output $g(t)$ admits a PSD $\Psi_g(\omega) = P(e^{j\omega})\Psi_h(\omega)P(e^{j\omega})^*$ and

$$R_g(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \Psi_g(\omega)\, d\omega = P(e^{j\omega_0})R_h(0)P(e^{j\omega_0})^*, \tag{19}$$

if the PSD of $h(t)$ is chosen as $\delta_D(\omega - \omega_0)$ with $\delta_D(\cdot)$ the Dirac Delta function. Note that although $\delta_D(\omega)$ is unbounded at $\omega = 0$, a WSS process with $\delta_D(\omega)$ as the PSD has a constant and bounded mean-power at each time index t. If $g(t) = \mathcal{N}(t)h(t)$ with $\mathcal{N}(t)$ in (11) described earlier, then

$$R_g(\tau) = \delta_K(\tau)R_h(0) \implies \Psi_g(\omega) = R_h(0) \ \ \forall\, \omega \in \mathbb{R} \tag{20}$$

where $\delta_K(\cdot)$ is the Kronecker Delta function and "$\Longrightarrow$" stands for "implies".

3 SI System Stablization

We first present our result on SI systems. In this case $m = 1$. Although [6, 17] contain two different proofs in the case $m = 1$, a more direct derivation, extendable to MI systems, is provided here. Assume from now on that RLQ is used for quantization, i.e. $W(z)$ is given by (9).

Theorem 1 *The* NCS *in* Fig. 2 *with* $m = 1$ *and* $W(z)$ *in* (9) *is* MS *stabilizable for all* $\|\Delta\|_{\mathcal{H}_\infty} \leq \delta$ *and* $\mathrm{E}\{|\mathcal{N}(t)|^2\} \leq \nu^2$, *if and only if* (A, B) *is stabilizable and*

$$\mathfrak{C} := \frac{1}{2}\log\left(\frac{1+\nu^2}{\nu^2+\delta^2}\right) > h(A) := \log M(A). \tag{21}$$

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Proof: Note that (21) is equivalent to

$$\frac{1+\nu^2}{\nu^2+\delta^2} > M(A)^2 = \det(1+B'XB) \tag{22}$$

with $X \geq 0$ the stabilizing solution to ARE (13). We assume that A has no eigenvalues on the unit circle throughout the paper. If A has eigenvalues on the unit circle, a standard trick (see for example [4, 7, 13]) can be used to modify the proof. The inequality (22) can be written as

$$\left[M(A)^2 - 1\right]\nu^2 + M(A)^2\delta^2 < 1.$$

By taking F to be the same as in (16), the inequality (21) is in turn equivalent to

$$\|T\|_{\mathcal{H}_2}^2\nu^2 + \|S\|_{\mathcal{H}_\infty}^2\delta^2 < 1. \tag{23}$$

It is commented that $\|T\|_{\mathcal{H}_2}^2 \geq M(A)^2 - 1$ and $\|S\|_{\mathcal{H}_\infty} \geq M(A)$ for any $F \in \mathbb{R}^{m\times n}$ with equality achieved by taking F in (16) [1, 4, 7, 13]. Next note that for the NCS in Fig. 2 with $m = 1$, there holds

$$v(t) = T(q)\mathcal{N}(t)v(t) + S(q)\Delta[v(t)] \tag{24}$$

by the nonlinearity of Δ. The i.i.d. assumption on $\mathcal{N}(t)$ and the equality (20) yields

$$\|T(q)\mathcal{N}(t)v(t)\|_{\mathcal{P}}^2 = \|T\|_{\mathcal{H}_2}^2\nu^2\mathcal{P}_v. \tag{25}$$

The property of the $\mathcal{H}_\infty$ norm implies that

$$\|S(q)\Delta[v(t)]\|_{\mathcal{P}}^2 \leq \|S\|_{\mathcal{H}_\infty}^2\delta^2\mathcal{P}_v.$$

Finally computing the mean power for both sides of (24), noting the independence of $v(t)$ and $\mathcal{N}(t)$, and taking F to be the same as in (16) lead to

$$\mathcal{P}_v \leq \left(\|T\|_{\mathcal{H}_2}^2\nu^2 + \|S\|_{\mathcal{H}_\infty}^2\delta^2\right)\mathcal{P}_v < \mathcal{P}_v$$

in light of (23) when $\mathcal{P}_v \neq 0$, which is a contradiction. Thus (21) implies $\mathcal{P}_v = 0$ for some controller F, concluding the MS stabilizability.

Conversely if (21) is violated, then a deterministic uncertainty Δ and random uncertainty $\mathcal{N}(t)$ can be constructed with $\|\Delta\|_{\mathcal{H}_\infty} \leq \delta$ and $\mathrm{E}\{|\mathcal{N}(t)|^2\} \leq \nu^2$ such that the feedback system in Fig. 2 admits a solution $v(t)$ with $\mathcal{P}_v = \|v\|_{\mathcal{P}}^2 > 0$ no matter which stabilizing controller F is used. Specifically it is noted that a linear time-invariant (LTI) $\Delta(z)$ is a special case of NTVD uncertainty. Taking $\Delta(z)$ with $|\Delta(z)| = \delta$ for all $|z| = 1$ implies the existence of $v(t)$ such that

$$\|S(q)\Delta(q)v(t)\|_{\mathcal{P}}^2 = \|S\|_{\mathcal{H}_\infty}^2\delta^2\mathcal{P}_v.$$

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Such a $v(t)$ admits a PSD equal to $\mathcal{P}_v\delta_D(\omega-\omega_m)$ that is a multiple of the Dirac Delta function having the peak at $\omega=\omega_m$ at which $|S(e^{j\omega_m})|$ achieves the global maximum, i.e., $\|S\|_{\mathcal{H}_\infty}=|S(e^{j\omega_m})|$. In connection with (25), it results in the equality

$$\mathcal{P}_v = \left(\|T\|_{\mathcal{H}_2}^2\nu^2 + \|S\|_{\mathcal{H}_\infty}^2\delta^2\right)\mathcal{P}_v. \tag{26}$$

Since (21) does not hold, $\|T\|_{\mathcal{H}_2}^2\nu^2 + \|S\|_{\mathcal{H}_\infty}^2\delta^2 \geq 1$ in light of (23) and the subsequent comment in the sufficiency proof. Denote $\Longleftrightarrow$ for equivalence. Only the critical case of

$$\inf_F\left\{\|T\|_{\mathcal{H}_2}^2\nu^2 + \|S\|_{\mathcal{H}_\infty}^2\delta^2\right\} = 1 \iff \frac{1+\nu^2}{\nu^2+\delta^2} = M(A)^2$$

needs to be considered, because the case "> 1" includes the critical case by taking smaller value of δ and ν. Recall that the MS stability has to hold for not only the case of $\|\Delta\|_{\mathcal{H}_\infty}=\delta$ and $\mathrm{E}\{|\mathcal{N}(t)|^2\}=\nu^2$ but also for the case $\|\Delta\|_{\mathcal{H}_\infty}<\delta$ and $\mathrm{E}\{|\mathcal{N}(t)|^2\}<\nu^2$. Clearly the equality (26) admits a solution $\mathcal{P}_v>0$, if the optimal F is used. On the other hand if a different stabilizing F is used, the quantization error $\delta_0<\delta$ or SNR of the fading channel $\nu_0^2<\nu^2$ exists such that $\|T\|_{\mathcal{H}_2}^2\nu_0^2+\|S\|_{\mathcal{H}_\infty}^2\delta_0^2=1$. Thus a nonzero solution $\mathcal{P}_v>0$ to $\mathcal{P}_v=\left(\|T\|_{\mathcal{H}_2}^2\nu_0^2+\|S\|_{\mathcal{H}_\infty}^2\delta_0^2\right)\mathcal{P}_v$ exists again. Consequently the NCS in Fig. 2 is not MS stable for any stabilizing F which concludes the proof. $\square$

The proof of Theorem 1 shows that the MS stabilizability is hinged on

$$\inf_F\left\{\|T\|_{\mathcal{H}_2}^2\nu^2 + \|S\|_{\mathcal{H}_\infty}^2\delta^2\right\} < 1. \tag{27}$$

Since the optimal F achieving the infimum of $\|T\|_{\mathcal{H}_2}$ coincides with that of $\|S\|_{\mathcal{H}_\infty}$ [13] (see (15) and (17) also), the inequality (21) is equivalent to the MS stabilizability. It is important to observe that in the case when quantization is absent, the MS stabilizability condition (21) reduces to $1+\nu^{-2}>M(A)^2$ that is the same as reported in [2]. When the fading channel is absent, the condition (21) reduces to $\delta^{-1}>M(A)$ that is first obtained in [4] under the MLQ method for quantization, and in [13] under the RLQ method for quantization.

Remark 1 The inequality (21) is also derived in [16] for the NCS using MLQ for quantization. However [16] claims only that the underlying NCS is MS stabilizable, if and only if (21) holds for some $\delta>0$ and $\nu>0$, contrasting to Theorem 1 that applies to any $\delta>0$ and $\nu>0$ satisfying (21). In fact the MS stabilizability under MLQ and packet drop may require that either δ be close to zero ($\delta=0$ corresponds to absence of quantization) or ν close to zero ($\nu=0$ corresponds to absence of packet drop) in addition to satisfying the inequality (21). For this reason the RLQ method in [13] is preferred for quantization if the quantized signal is to be transmitted in packets, which will be able to offer a true trade-off between the quantization error and packet drop rate. Nonetheless,

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we still conjecture that the corresponding result to Theorem 1 for MLQ can be obtained by our approach and is extendable to the MI case. We will continue to work on this. □

4 MI System Stablization

In this section we consider MI systems. Let $\nu = \det(D_\nu)$ and $\delta = \det(D_\delta)$ be the total uncertainty bounds, where

$$D_\nu = \mathrm{diag}(\nu_1, \cdots, \nu_m), \qquad D_\delta = \mathrm{diag}(\delta_1, \cdots, \delta_m). \tag{28}$$

Then $\nu = \det(D_\nu)$ and $\delta = \det(D_\delta)$ represent the total uncertainty bounds of $\{\mathcal{N}_i(t)\}$ and $\{\Delta_i\}$, respectively. Still assume that $W(z)$ is given by (9).

As commented at the beginning of the paper, channel resource allocation will be used to pursue the minimum channel capacity required for stabilization. In (21) we have defined the capacity for a single channel. In the MI case, since multiple independent channels are used, we define the capacity of the kth channel $\mathfrak{C}_k$ in the same way as in (21), i.e.

$$\mathfrak{C}_k := \frac{1}{2}\log\left(\frac{1+\nu_k^2}{\nu_k^2+\delta_k^2}\right)$$

and the capacity of the whole channel is defined to be their sum $\mathfrak{C} := \sum_{k=1}^m \mathfrak{C}_k$. Note that $\mathfrak{C}_k$ for each k is determined by δ_k and ν_k, which can be influenced by the resource allocated to the kth channel. For instance, channels with more resource could have finer quantizer and introduce less fading effect, and hence have higher capacity. Assume that the information constraint is given in terms of only $\mathfrak{C}$ instead of each $\mathfrak{C}_k$. In this case, the controller designer has extra freedom and can pursue the minimum channel capacity by allocating $\mathfrak{C}_k$ judiciously among the channels so that a stabilizing controller can be constructed. We refer to this procedure as *channel/controller co-design*, which is demonstrated in the proof to the following theorem.

Theorem 2 *The* NCS *in* Fig. 2 *with* $W(z)$ *in* (9) *and* $m > 1$ *can be* MS *stabilized by some allocation* $\{\mathfrak{C}_k\}$ *and some feedback controller* F *for all* $\|\Delta_k\|_{\mathcal{H}_\infty} \le \delta_k$ *and* $\mathrm{E}\{|\mathcal{N}_k(t)|^2\} \le \nu_k^2$ *where* $1 \le k \le m$*, if and only if* (A, B) *is stabilizable and*

$$\mathfrak{C} > h(A). \tag{29}$$

Proof: The sufficiency will be proved constructively by using channel resource allocation as explained above. Assume that (29) holds and that (A, B) are in Wonham decomposition form:

$$A = \begin{bmatrix} A_1 & * & \cdots & * \\ 0 & A_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \cdots & 0 & A_m \end{bmatrix}, \qquad B = \begin{bmatrix} b_1 & * & \cdots & * \\ 0 & b_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \cdots & 0 & b_m \end{bmatrix}.$$

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Since (A,B) is stabilizable, each pair (A_k, b_k) is stabilizable. Hence $M(A) = \prod_{k=1}^m M(A_k)$, and

$$\mathfrak{C} = \sum_{k=1}^m \mathfrak{C}_k > h(A) \implies \mathfrak{C}_k := \frac{1}{2}\log\left(\frac{1+\nu_k^2}{\nu_k^2+\delta_k^2}\right) > h(A_k) \tag{30}$$

can be made true. Let $n_i \times n_i$ be dimension of A_i. Denote

$$D_\epsilon = \text{diag}(1, \epsilon, \cdots, \epsilon^{m-1}), \quad S_\epsilon = \text{diag}(I_{n_1}, \epsilon I_{n_2}, \cdots, \epsilon^{m-1} I_{n_m}),$$

with $\epsilon > 0$. Note that constant diagonal scalings are allowed, which do not change the feedback stability in light of the μ analysis [12]. Thus the MS stability of the NCS in Fig. 2 remains the same when $W(z)$ is replaced by

$$W_\epsilon(z) = D_\epsilon^{-1}\left[\begin{array}{cc} T(z)D_\epsilon & S(z)D_\epsilon \end{array}\right] = \left[\begin{array}{cc} 0 & I \end{array}\right] + D_\epsilon^{-1}T(z)D_\epsilon\left[\begin{array}{cc} I & I \end{array}\right].$$

It can be verified that under the similarity transform matrix S_ϵ for the state vector of $W(z)$ or equivalently $T(z)$,

$$D_\epsilon^{-1}T(z)D_\epsilon = D_\epsilon^{-1}FS_\epsilon[sI - S_\epsilon^{-1}(A+BF)S_\epsilon]^{-1}S_\epsilon^{-1}BD_\epsilon.$$

By taking $D_\epsilon^{-1}FS_\epsilon = \text{diag}(f_1, \cdots, f_m)$ and $\epsilon \to 0$, there holds [13]

$$W_\epsilon(z) \;\to\; \text{diag}[W_1(z), \cdots, W_m(z)], \quad D_\epsilon^{-1}T(z)D_\epsilon \;\to\; \text{diag}[T_1(z), \cdots, T_m(z)]$$

where $T_k(z) = f_k(zI - A_k - b_k f_k)^{-1}b_k$. Hence the MI feedback system in Fig. 2 is asymptotically diagonalizable by tweaking ϵ. Because $\mathfrak{C} > h(A)$, $\mathfrak{C}_k > h(A_k)$ can be made true for each k. See (30). In light of Theorem 1, $\mathfrak{C}_k > h(A_k)$ in turn implies the existence of the stabilizing f_k such that $\|S_k\|_{\mathcal{H}_\infty}^2\delta_k^2 + \|T_k\|_{\mathcal{H}_2}^2\nu_k^2 < 1$ for $1 \le k \le m$ where $S_k(z) = 1 + T_k(z)$. Consequently $\mathcal{P}_{v_k} = 0$ is the only possible solution for each k to $\mathcal{P}_{v_k} = (\|S_k\|_{\mathcal{H}_\infty}^2\delta_k^2 + \|T_k\|_{\mathcal{H}_2}^2\nu_k^2)\mathcal{P}_{v_k}$, which concludes the MS stability and thus the sufficiency proof for (29).

For the necessity proof, assume that (29) is violated. An LTI $\Delta(z) = \text{diag}[\Delta_1(z), \cdots, \Delta_m(z)]$ satisfying $\|\Delta_k\|_{\mathcal{H}_\infty} \le \delta_k$ and a diagonal i.i.d. $\mathcal{N}(t)$ satisfying $\text{E}\{|\mathcal{N}_k(t)|^2\} \le \nu_k^2$ will be constructed such that the NCS in Fig. 2 admits a solution $v(t)$ with $\mathcal{P}_v > 0$ no matter what stabilizing controller F is used, i.e., the NCS is not MS stabilizable. For this purpose assume that F is stabilizing and thus $(A+BF)$ is a Schur matrix. Note that for the feedback system in Fig. 2 with Δ replaced by an LTI uncertainty $\Delta(z)$, there holds

$$v(t) = T(q)\mathcal{N}(t)v(t) + S(q)\Delta(q)v(t). \tag{31}$$

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Recall D_ν and D_δ in (28). The independence of the i.i.d. processes $\mathcal{N}(t)$ and $v(t)$ yields

$$\begin{aligned}\mathcal{P}_v = \|v\|_{\mathcal{P}}^2 &= \|T(q)\mathcal{N}(t)v(t) + S(q)\Delta(q)v(t)\|_{\mathcal{P}}^2 \\ &\leq \operatorname{Tr}\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} T(e^{j\omega})D_\nu R_v(0) D_\nu T(e^{j\omega})^*\, d\omega\right\} \\ &\quad + \sup_\omega \operatorname{Tr}\left\{S(e^{j\omega})D_\delta R_v(0)D_\delta S(e^{j\omega})^*\right\}.\end{aligned}$$

Equality is achievable by constructing an appropriate LTI dynamic uncertainty $\Delta(z)$. For instance a stable LTI $\Delta(z) = \operatorname{diag}[\Delta_1(z), \cdots, \Delta_m(z)]$ can be constructed satisfying $|\Delta_k(e^{j\omega})| = \delta_k$ for all ω and $1 \leq k \leq m$, leading to

$$\operatorname{Tr}\left\{\left[I - \left(D_\delta S(e^{j\omega_m})^* S(e^{j\omega_m}) D_\delta + \frac{1}{2\pi}\int_{-\pi}^{\pi} D_\nu T(e^{j\omega})^* T(e^{j\omega}) D_\nu\, d\omega\right)\right] R_v(0)\right\} = 0 \tag{32}$$

by taking $v(t)$ whose PSD is an appropriate multiple of Dirac Delta having the peak at ω_m and using (19). The above can be written as $\operatorname{Tr}\{(I - \Pi_1)R_v(0)\} = 0$ by denoting

$$\Pi_1 = D_\delta S(e^{j\omega_m})^* S(e^{j\omega_m}) D_\delta + \frac{1}{2\pi}\int_{-\pi}^{\pi} D_\nu T(e^{j\omega})^* T(e^{j\omega}) D_\nu\, d\omega.$$

Clearly eigenvalues of Π_1 decrease as δ_k and ν_k^2 decrease. On the other hand

$$M_\Pi := I - \Pi_1 = (I + D_\nu^2) - \Pi_2$$

by the property in (14) where

$$\Pi_2 = D_\delta S(e^{j\omega_m})^* S(e^{j\omega_m}) D_\delta + \frac{1}{2\pi}\int_{-\pi}^{\pi} D_\nu S(e^{j\omega})^* S(e^{j\omega}) D_\nu\, d\omega. \tag{33}$$

The equality (32) is now equivalent to $\operatorname{Tr}\left\{\left[\left(I + D_\nu^2\right) - \Pi_2\right] R_v(0)\right\} = 0$. If the optimal stabilizing F in (16) is used so that equality holds for (15), and (17) is also true, then

$$\Pi_2 = D_\delta(I + B'XB)D_\delta + D_\nu(I + B'XB)D_\nu.$$

It is claimed that $M_\Pi = (I - \Pi_1) = \left[\left(I + D_\nu^2\right) - \Pi_2\right]$ is an indefinite matrix, i.e., M_Π has both positive and negative eigenvalues, by the hypothesis that (29) is violated. To prove the claim with the argument of contradiction, assume that M_Π is a definite matrix. If $M_\Pi > 0$, it is equivalent to

$$\left(I + D_\nu^2\right) > \Pi_2 = D_\delta(I + B'XB)D_\delta + D_\nu(I + B'XB)D_\nu. \tag{34}$$

In light of Minkowski's inequality (page 482 of [10]), $\det(P_1 + P_2) \geq \det(P_1) + \det(P_2)$ for any two positive definite matrices P_1 and P_2. Taking determinant on both sides of (34) with $P_1 = D_\delta(I + B'XB)D_\delta$ and $P_2 = D_\nu(I + B'XB)D_\nu$ yields

$$\det(I + D_\nu^2) > \det(D_\nu^2 + D_\delta^2)\det(I + B'XB) = M(A)^2 \det(D_\nu^2 + D_\delta^2).$$

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The above is the same as condition (29), thereby contradicting the hypothesis that (29) is violated. The case $M_\Pi < 0$ is not meaningful because as $\{\delta_k\}$ and $\{\nu_k^2\}$ decrease, $M_\Pi > 0$ is true eventually, leading to the same contradiction. Recall that $M_\Pi = I - \Pi_1$ and the eigenvalues of Π_1 decrease as δ_k and ν_k^2 decrease with limit $\Pi_1 = 0$ if $\delta_k \to 0$ and $\nu_k^2 \to 0$ for all k. See also the necessity proof of Theorem 1. It follows that M_Π is indefinite and there is a nonzero solution $R_v(0) \geq 0$ satisfying $\mathcal{P}_v > 0$ to $M_\Pi R_v(0) = 0$ which concludes the instability of the NCS in Fig. 2. Now if F is stabilizing but F is different from the optimal one in (16), then the strict inequality (34) cannot hold anymore, i.e., $M_\Pi > 0$ is not true. Because $M_\Pi < 0$ is not possible (which otherwise contradicts $M_\Pi \to I$ and $\Pi_1 \to 0$ as $\delta_k \to 0$ and $\nu_k^2 \to 0$ for all k), M_Π must be indefinite for some $\{\Delta_k\}$ and $\{\mathcal{N}_k(t)\}$ satisfying $\|\Delta_k\|_{\mathcal{H}_\infty} \leq \delta_k$ $\mathrm{E}\{|\mathcal{N}_k(t)|^2\} \leq \nu_k^2$. Hence there again exists some WSS process $v(t)$ satisfying $R_v(0) \geq 0$ and $\mathcal{P}_v > 0$ to $\mathrm{Tr}\{M_\Pi R_v(0)\} = \mathrm{Tr}\{(I - \Pi_1)R_v(0)\} = 0$, thereby concluding the necessity proof. $\square$

The sufficiency part of the above proof is constructive, and detailed procedure of the channel/controller co-design is showcased by using channel resource allocation. Indeed for a given total capacity $\mathfrak{C} > h(A) = \sum_{k=1}^m h(A_k)$, it is always possible to find an allocation of $\mathfrak{C}_1, \mathfrak{C}_2, \ldots, \mathfrak{C}_m$ such that $\mathfrak{C} = \sum_{k=1}^m \mathfrak{C}_k$ and $\mathfrak{C}_k > h(A_k)$ for all k, where all A_k's are given by the Wonham decomposition in the proof. Now with the allocated $\mathfrak{C}_1$, a controller f_1 can be designed to stabilize all unstable modes controllable from the first input; similarly with $\mathfrak{C}_2$, a controller f_2 can be designed to stabilize all remaining unstable modes controllable from the second input; $\cdots$; finally with $\mathfrak{C}_m$, a controller f_m can be designed to stabilizing all remaining unstable modes. It's easy to see from the Wonham decomposition that there is no other unstable modes left, and thus the channel/controller co-design is accomplished and the underlying NCS is stabilized.

5 Numerical Examples

We present two numerical examples with the first for the SI case and the second for the MI case.

Example 1 Consider the second-order system $x(t+1) = Ax(t) + Bu(t)$ where

$$A = \begin{bmatrix} 0 & 1 \\ 1.8 & -0.3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Assume $\mathcal{F}(t)$ is a Bernoulli process satisfying $P\{\mathcal{F}(t) = 0\} = 0.25$ and $P\{\mathcal{F}(t) = 1\} = 0.75$ for all time index t. Hence $\mu = 0.75$. This example is adopted from [16] in which the MLQ quantizer with $\delta = 0.2565$ and packet drop with equivalent SNR $\nu^2 = 1/0.75 - 1 = 0.3333$ (see (8)) are both

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present. Our design uses RLQ with the same δ and ν^2. The capacity is thus given by

$$\mathfrak{C} = \frac{1}{2}\log\frac{1+1/0.75-1}{0.2565^2+1/0.75-1} = 0.8701 > h(A) = 0.8480.$$

It follows that the underlying NCS is MS stabilizable. In [16] the input $u(t) = \mathcal{F}(t)\mathcal{Q}[Fx(t)]$ where the controller F is

$$[\ -1.800 \quad 0.656\ ],$$

while in our method $u(t) = \frac{1}{\mu}\mathcal{F}(t)\mathcal{Q}[Fx(t)] = [1+\mathcal{N}(t)]\mathcal{Q}[Fx(t)]$ with the controller F computed according to Theorem 1 being

$$[\ -1.2444 \quad 0.4667\ ].$$

The simulation result is shown in Fig. 1. The two NCSs begin from the same initial condition $x(0) = [\ 1 \quad 1\ ]'$, and also share the same $\mathcal{F}(t)$, i.e. the packets arrive or drop exactly the same at each time sample in both NCSs. Note also that although the state $x(t) := [\ x_1(t) \quad x_2(t)\ ]'$ is 2-dimensional, only one of the state variables is plotted, because for the given (A, B), there holds $x_1(t+1) = x_2(t)$ for all $t \geq 0$. Hence it suffices to observe only one of the two state variables.

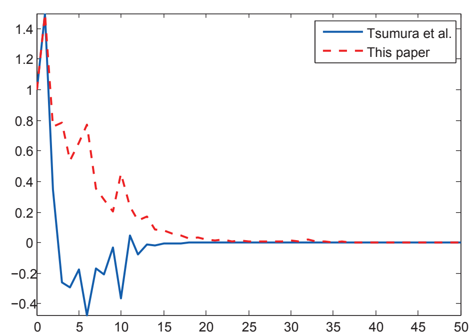

Figure 1: Simulation Result ($\delta = 0.2565, \nu^2 = 0.3333$)

The figure shows that both the controller in [16] and our controller stabilize the NCS.

Next a multiple input system is used to demonstrate how channel resource is allocated in the channel/controller co-design. We use the same example as the one in [13] in which only one channel uncertainty is considered, contrasting to multiple channel uncertainties studied in this paper.

Example 2 Consider the unstable system (A, B) with

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

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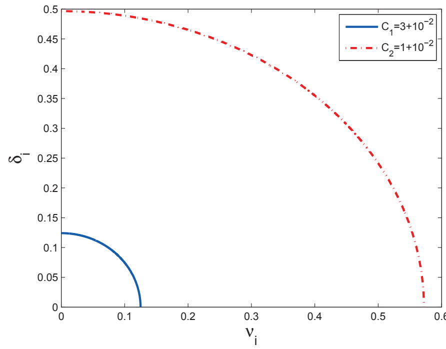

Figure 2: All possible realizations of the allocated capacity

Obviously (A, B) is stabilizable and is already in the Wonham decomposition form with

$$A_1 = \mathrm{diag}(4, 2), \quad b_1 = [\, 1 \quad 1 \,]', \quad A_2 = 2, \quad b_2 = 1.$$

As a result $h(A) = h(A_1) + h(A_2) = 3 + 1 = 4$. Hence if the total given capacity is $\mathfrak{C} = 4 + 2 \times 10^{-2}$, then it is possible to allocate sufficient resource to each channel such that $\mathfrak{C}_i > h(A_i)$, in light of Theorem 2. One possible allocation sets $\mathfrak{C}_1 = 3 + 10^{-2}$ and $\mathfrak{C}_2 = 1 + 10^{-2}$. It is worth to mentioning that for the NCS problems dealing solely with quantization [13] or fading errors [18], $\{\delta_i\}$ or $\{\nu_i\}$ is determined once the channel capacity is allocated. However in this paper, there exists an additional trade-off between $\{\delta_i\}$ and $\{\nu_i\}$ for a fixed capacity. That is, realizations of the allocated capacity are not unique, nor is there any preferred choice among them. Fig. 2 shows all possible combinations of $\{\delta_i\}$ and $\{\nu_i\}$ satisfying the required capacity for each channel.

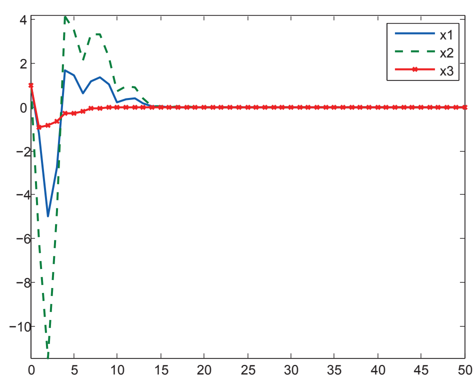

Figure 3: Simulation Result

Take $\nu_1 = \nu_2 = 0.1$ for instance, then $\delta_1 = 0.0746$ and $\delta_2 = 0.4889$. Solve the optimization

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problem described in (27) for $[A_1|b_1]$ and $[A_2|b_2]$, respectively, to obtain f_1 and f_2. They are exactly given by the standard expensive controllers [13], i.e. $f_1 = [\ -6.5625 \quad 1.3125\ ]$ and $f_2 = -1.5$, i.e.,

$$F = \mathrm{diag}(f_1, f_2) = \begin{bmatrix} -6.5625 & 1.3125 & 0 \\ 0 & 0 & -1.5 \end{bmatrix}.$$

With the above channel/controller co-design, the closed-loop evolution of the plant states starting from the initial condition $x(0) = [\ 1 \quad 1 \quad 1\ ]'$ is shown in Fig. 3. Clearly all the system state variables converge to zero asymptotically. It is commented that if the resource is not allocated correctly, thereby violating $\mathfrak{C}_i > h(A_i)$ for some i, then the stabilizing controller does not exist for the corresponding $[A_i|b_i]$, and the underlying NCS cannot be stabilized.

6 Conclusion

This paper studies the MS stabilizability for NCSs over the quantized fading (actuator) channel. By assuming the RLQ method for logarithmic quantization and the NTVD uncertainty for the quantization error, we are able to provide a satisfactory solution and derive a necessary and sufficient condition for the MS stabilizability for MI systems under state feedback. Our result generalizes the one in [6, 17] for SI systems to MI systems, and strengthens the one in [16] to enable the true tradeoffs between the quantization error and packet drop rate for design of the state feedback controller. The problem of MS stabilizability under output feedback control is more difficult. While a result is available in [6] for SISO systems, it contains only a sufficient condition. Hence the MS stabilizability for MIMO systems over quantized fading channels poses a major challenge to the NCS, which deserves further study in the future.

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