Vessel routing and scheduling under uncertainty in the liquefied natural gas business

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Abstract

Liquefied natural gas (LNG) is natural gas transformed into liquid state for the purpose of transportation mainly by specially built LNG vessels. This paper considers a real-life LNG ship routing and scheduling problem where a producer is responsible for transportation from production site to customers all over the world. The aim is to create routes and schedules for the vessel fleet that are more robust with respect to uncertainty such as in sailing times due to changing weather conditions. A solution method and several robustness strategies are proposed and tested on instances with time horizons of 3 to 12 months. The resulting solutions are evaluated using a simulation model with a recourse optimization procedure. The results show that there is a significant improvement potential by adding the proposed robustness approaches.

Keywords: Maritime transportation, Liquefied natural gas, Ship routing and scheduling, Simulation, Uncertainty

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1. Introduction

Natural gas is an energy source vital to the world's energy supply. It is among others used to generate electricity, in domestic homes for cooking and heating, and

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as fuel for vehicles. It is increasing in popularity compared to other alternatives due to its properties of cleaner burning and lower emission. One way of transporting natural gas from the production site to the consumers is by transformation into liquefied natural gas (LNG) followed by sea transportation to consumers by dedicated LNG vessels. This way natural gas can be delivered from one production site to consumers in all corners of the world.

In a previous study, Halvorsen-Weare and Fagerholt (2010) studied a real-life ship routing and scheduling problem from the LNG business, and a solution method based on decomposition of routing and scheduling decisions was proposed. The solution method solves the problem of creating an annual delivery program (ADP). The ADP lists the shipments (cargoes) to deliver to the customers during the year, i.e. the cargoes' pick-up and delivery days and what vessel that are servicing what cargoes. The cargo size is determined by the vessel servicing it as the cargoes usually are full shiploads.

Today, the creation of such an ADP is done by manual spreadsheet procedures. Such planning methods suffer from drawbacks as it may be difficult to create even a feasible solution when the problem size increases. The solution method proposed by Halvorsen-Weare and Fagerholt (2010) can create good (cost-optimal or near cost-optimal), feasible solutions to large problems within short computational time. The problem is considered as deterministic, with all input parameters given. However, the vessels operate in a highly uncertain environment where factors like weather conditions and port congestion easily can influence the sailing times. It is also assumed that the daily LNG production rates are known for the whole year. This is a simplification as unforeseen events can result in fluctuations in the production rates and thus it may not be possible to predict future production rates to such a detailed extent. These uncertain elements can result in delays which will induce extra cost for the LNG producer. These costs can be the outcome of having to increase sailing speed to make deliveries on time, penalty costs to customers or lost goodwill for delayed deliveries, and having to charter-in vessels to be able to service all cargoes within acceptable time.

The purpose of this paper is to create solutions to the LNG ship routing and scheduling problem that are more robust, i.e. solutions that can better withstand deviations in the uncertain parameters. We focus on uncertainties in sailing times and daily LNG production rates as these are the most interesting from a planning perspective in this particular problem. The contributions of this paper consist of a new improved optimization model that solves the same real-life LNG ship routing and scheduling problem as in Halvorsen-Weare and Fagerholt (2010). Robustness strategies are then added to this model with the aim of creating solutions anticipating uncertainties in sailing times and LNG production rates better. It is not given that one strategy will provide better results than others for all planning problems.

Therefore, a third contribution is the analysis of a number of different solutions to give the planners the possibility to choose the solution that overall performs best. For this purpose we have developed a simulation model with a recourse re-route optimization procedure that imitates a real-life re-planning situation. The optimization model, together with the different robustness strategies and the simulation procedure with re-routing, creates a good basis for a complete decision support system.

The problem we study is highly affected by uncertain elements, which is also the case for most other maritime transportation problems. However, uncertainties are often neglected in the literature. Christiansen et al. (2004) and Christiansen et al. (2007) are two recent reviews of literature on ship routing and scheduling, and reveal that most problems are solved in a deterministic setting. However, a few references that incorporate uncertainty exist. Christiansen and Fagerholt (2002) solve a deterministic version of a shipping problem, but create more robust solutions by penalizing solutions that are considered risky. A simulation study for a fleet sizing problem with uncertainty in weather conditions and future spot rates was presented by Shyshou et al. (2010), while Alvarez et al. (2011) propose a robust optimization model for the fleet sizing and deployment problem to deal with the uncertainty in future price and demand.

Two related topics are stochastic and dynamic vehicle routing problems (see e.g. Gendreau et al. (1996) and Psaraftis (1995)), and stochastic airline and aircrew scheduling. While there has been quite an extensive research on stochastic and dynamic vehicle routing problems, airline and aircrew scheduling algorithms used for planning purposes in real-life assume no disruptions and rely on recovery planning (see the discussion by Barnhart et al. (2003)). However, the interest for methods for achieving robustness in schedules has increased the last years (see Clausen et al. (2010)). Two recent references that incorporates disruptions when creating an aircrew schedule are Yen and Birge (2006) and Schaefer et al. (2005). Yen and Birge (2006) propose a stochastic aircrew scheduling model. The approach by Schaefer et al. (2005) has similarities to ours. They suggest two algorithms for finding aircrew schedules that may perform well in operations with disruptions, and evaluate the crew schedules by a simulation program of airline operations with disruptions.

The remaining part of this paper is organized as follows: Section 2 provides a problem description of the LNG ship routing and scheduling problem. Then Section 3 presents the mathematical model formulation. Section 4 gives a brief introduction to the uncertain elements we focus on in this paper, and Section 5 presents four robustness strategies that may be added to the model for the purpose of handling uncertainty more efficient. Section 6 gives a description of the simulation-optimization framework for evaluation of solutions, and Section 7 presents the computational study. Finally, the paper is concluded in Section 8.

2. Problem description

The LNG ship routing and scheduling problem studied in this paper is a real-life tactical planning problem faced by one of the world's largest LNG producers. The annual LNG production capacity for the producer amounts to 42 million tons. The LNG producer is contractually committed to transport LNG from production port to customers that are located all over the world. Every year the producer has to create and present an annual delivery program (ADP) to the customers that specifies when the customers will receive LNG shipments throughout the year (including time of delivery, by what vessel and quantity of LNG). The aim is then to create such an ADP. A thorough problem description of the LNG ship routing and scheduling problem can be found in Halvorsen-Weare and Fagerholt (2010). The major problem features are outlined here.

Long-term contracts state how much LNG that is to be delivered to each customer during the year. The actual delivery dates have to be agreed upon in a process where the LNG producer will create an initial ADP with suggested delivery dates that the customers may accept or decline. It may therefore be necessary to reoptimize an ADP with some delivery dates fixed during the process of creating the ADP.

To transport LNG from the production port to the customers, the LNG producer controls a heterogeneous fleet with vessels of varying loading capacities and sailing speeds. This fleet is fixed during the planning horizon, and some of the vessels are tied up to certain delivery contracts and can therefore only be used to service subsets of the customers.

All LNG deliveries are usually full shiploads as it is not economically beneficial to visit more than one customer on a voyage before returning to the production port. This creates a simple network structure with one pick-up port, several delivery ports and only full shiploads. Each LNG shipment will thus consist of a round-trip from production port to one customer and back to the production port.

One full shipload represents one cargo. Based on the vessels' average loading capacities, the producer initially estimates how many cargoes that should be delivered to each customer during the year, and defines a time window for when the cargoes should be picked up in the production port based on the specifications in the customer contracts. This can be done as the loading capacities for the vessels that may visit a given subset of customers only vary to a small degree (less than 10% difference between smallest and largest vessel capacity). We call this problem *cargo-based* as all cargoes are defined (by pick-up time window, customer and set of vessels that may service them) and need to be serviced. The LNG producer may make an under- or over-delivery (typically not more than 10%) on the yearly contractual volume to deliver to each customer, which allows for vessels' with varying loading capacity to visit the customers. This can for a general problem result in solutions where slightly smaller, cheaper vessels are preferred resulting in regular under-delivery, but this will not be the case for this problem as the vessel fleet's capacity compared with the demand is quite tight so that all vessels need to be utilized.

There is limited berth capacity at the production port. Hence, no more vessels can pick-up a cargo on a given day than there are available berths. There is also limited LNG inventory capacity, requiring LNG inventory levels to be within maximum and minimum levels at all times. Usually the LNG production is higher than the committed LNG delivery volumes to the customers. Consequently, spot cargoes are sold in the open market. These are being picked-up by vessels that are not in the producer's vessel fleet (as these vessels are contractually committed to only be used in customer service), and will therefore only affect the berth capacity and LNG inventory levels. We choose not to consider the profit of spot cargoes to avoid maximizing the number of spot cargoes. They are therefore only to be considered as means of inventory level control.

The LNG ship routing and scheduling problem of creating an ADP is then to minimize the costs of transporting all customer cargoes within the specified time windows, while at the same time ensuring that berth capacity and LNG inventory level constraints at the production port are not violated.

3. Mathematical formulation

This section provides a mathematical cargo-based assignment model that presents and solves the LNG ship routing and scheduling problem described in the previous section. This is a new model formulation that is more effective than the one from Halvorsen-Weare and Fagerholt (2010), but solves the exact same problem. The model formulation from Halvorsen-Weare and Fagerholt (2010) is an arc-flow model where binary flow variables describe directly the flow of the vessels. This demands for a greater number of variables than the assignment model we suggest here, where the binary variables describe an assignment of a cargo to a vessel on a given day. In addition, the arc-flow model formulation requires one more set of constraints: The flow conservation constraints.

In the mathematical modeling formulation, let $\mathcal{V}$ be the set of vessels, and $\mathcal{N}_v$ be the set of customers that vessel $v \in \mathcal{V}$ may service. Then set $\mathcal{N}$ contains all customers. Set $\mathcal{T}$ contains the days in the planning horizon, set $\mathcal{U}$ contains all customer cargoes that must be serviced during the planning horizon, and subset $\mathcal{U}_i \subset \mathcal{U}$ contains all cargoes that are to be shipped to customer i.

Further, let C_{vi} represent the cost for delivering a cargo of LNG to customer i by vessel v. A_{vit^*t} is one if vessel v has not returned to the production port at day t after starting on a voyage to customer i at day t^*, and zero otherwise. R_v^{MX} is the length in days of the longest return-trip from the production port to a customer vessel v can service. F_i is the total number of cargoes to deliver to customer i during the planning horizon. T_u^{MN} and T_u^{MX} represent the first and last day of the time window for start of loading cargo u, respectively. Q_v is the loading capacity of vessel v, while Q^S is the loading capacity of a typical spot vessel. D_i^{MN} and D_i^{MX} are the minimum and maximum volumes of LNG to deliver to customer i during the planning horizon, respectively. B is the number of berths at the production port, and P_t is the production of LNG at day t. S_0 is the inventory level of LNG at the start of the planning horizon and S^{MN} and S^{MX} are the minimum and maximum inventory levels of LNG at the production port, respectively.

The decision variables are:

$$x_{vit} = \begin{cases} 1, & \text{if vessel } v \text{ starts loading a cargo to customer } i \text{ on day } t\ (v \in \mathcal{V}, i \in \mathcal{N}_v, t \in \mathcal{T}) \\ 0, & \text{otherwise} \end{cases}$$

s_t	continuous variable representing the inventory level at the end of day t ($t \in \mathcal{T}$)
z_t	integer variable representing the number of spot cargoes loaded in the production port on day t ($t \in \mathcal{T}$)

The mathematical formulation for the cargo-based assignment model then becomes:

$$min \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}_v} \sum_{t \in \mathcal{T}} C_{vi} x_{vit}, \tag{1}$$

subject to

$$\sum_{t^*=max\{0,t-R_v^{MX}+1\}}^{t} \sum_{i\in\mathcal{N}_v} A_{vit^*t} x_{vit^*} \leq 1, \quad v \in \mathcal{V}, t \in \mathcal{T}, \tag{2}$$

$$\sum_{v\in\mathcal{V}} \sum_{t=T_u^{MN}}^{T_u^{MX}} x_{vit} \geq 1, \quad i \in \mathcal{N}, u \in \mathcal{U}_i, \tag{3}$$

$$\sum_{v\in\mathcal{V}} \sum_{t\in\mathcal{T}} x_{vit} = F_i, \quad i \in \mathcal{N}, \tag{4}$$

$$D_i^{MN} \leq \sum_{v\in\mathcal{V}} \sum_{t\in\mathcal{T}} Q_v x_{vit} \leq D_i^{MX}, \quad i \in \mathcal{N}, \tag{5}$$

$$\sum_{v\in\mathcal{V}} \sum_{i\in\mathcal{N}_v} x_{vit} + z_t \leq B, \quad t \in \mathcal{T}, \tag{6}$$

$$s_t = s_{t-1} + P_t - \sum_{v\in\mathcal{V}} \sum_{i\in\mathcal{N}_v} Q_v x_{vit} - Q^S z_t, \quad t \in \mathcal{T}, \tag{7}$$

$$S^{MN} \leq s_t \leq S^{MX}, \quad t \in \mathcal{T}, \tag{8}$$

$$x_{vit} \in \{0, 1\}, \quad v \in \mathcal{V}, i \in \mathcal{N}_v, t \in \mathcal{T}, \tag{9}$$

$$z_t \in \mathbb{Z}^+, \quad t \in \mathcal{T}. \tag{10}$$

The objective function (1) minimizes the sailing costs for delivering all cargoes. Constraints (2) ensure that a vessel can only service one cargo on any given day, and constraints (3) are the time window constraints for the cargoes. Overlapping time windows for cargoes to deliver to one customer will allow that more than one cargo to that customer is serviced during the overlapping cargoes' time windows. Hence, constraints (3) are formulated as greater than or equal to constraints. Constraints (4) ensure that each customer get the required number of cargoes during the planning horizon. In the case of no overlapping time windows constraints (3) can be modeled as equality constraints and constraints (4) are redundant. Constraints (5) ensure that the total volume of LNG delivered to each customer at the end of the planning horizon is within the predefined minimum and maximum quantities. Constraints (6) are the berth constraints. Constraints (7) determine the volume of LNG at the production port, s_{t-1} being equal to S_0 for $t = 1$, and constraints (8) ensure that the volume is within the inventory's minimum and maximum levels at all times. Finally, constraints (9) set the binary requirements for the x_{vit} variables, and constraints (10) set the integer requirements for the z_t variables.

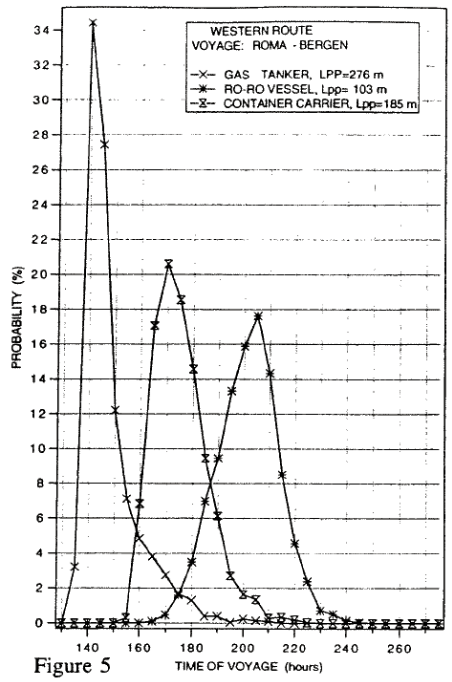

Figure 1: Probability distributions for sailing times. Source: Kauczynski (1994).

4. Uncertainties in the LNG routing and scheduling problem

In general, all maritime transportation problems are exposed to uncertainties although they are often solved by deterministic modeling approaches like the one presented in the previous section. In Halvorsen-Weare and Fagerholt (2010) the LNG ship routing and scheduling problem is solved without embedding any elements that considers such uncertainties. For this problem there are two main uncertain parameters that should be taken into consideration from a planning perspective: Sailing times and daily LNG production rates.

Sailing times for vessels are weather dependent, and it is not possible to predict the weather conditions for much more than a few days ahead. This is a common uncertain element for all maritime transportation problems. Still we observe that for most planning purposes sailing times are considered constant. This can be a realistic simplification for problems considering short-sea shipping in sheltered water. But for many problems, and this LNG ship routing and scheduling problem in particular, voyages last for several days (and up to a month) in a deep-sea shipping environment where vessels can experience large weather variations while sailing a round-trip to a customer.

Table 1: Probability distribution for increased sailing time

Increase (%)	0.0	3.0	7.0	12.0	15.0
Probability (%)	38.8	30.2	16.5	11.0	3.5

Table 2: Probability distribution for changes in daily production rates

Change (%)	85	90	95	100	105	110
Probability (%)	5	10	15	35	20	15

Kauczynski (1994) studied the ship transportation between selected ports in Europe to determine the distribution of speed losses in a realistic operational environment. Figure 1 shows the probability functions for sailing times on a voyage between Rome (Italy) and Bergen (Norway) for a gas tanker, ro-ro vessel and container carrier.

The sailing times for the LNG vessels in the LNG ship routing and scheduling problem we consider, follows a similar curve to the one for the gas tanker in Figure 1: A high likelihood of using approximately the planned sailing time, and a long tail illustrating the probability of delays and break-downs. The curve for the gas tanker in Figure 1 can be fitted to a log logistic probability distribution on the following form (see Palisade Corporation (2010)):

$$f(x) = \frac{\alpha t^{\alpha-1}}{\beta\left(1+t^{\alpha}\right)^2}, \tag{11}$$

$$F(x) = \frac{1}{1+\left(\frac{1}{t}\right)^{\alpha}}, \tag{12}$$

where

$$t = \frac{x-\gamma}{\beta}. \tag{13}$$

Function (11) describes the density function and (12) the cumulative distribution function. For the probability function for the gas tanker, $\alpha = 2.24$, $\beta = 9.79$ and $\gamma = 134.47$, giving an expected sailing time of 148.42 hours.

Table 1 shows the calculated probabilities for some discrete increases in sailing time based on the probability function for the gas tanker when the extreme outcomes (long tail) are cut off.

The LNG producer has a daily LNG production plan for the next year. But chances are that the produced volume for each day will not be exactly as planned. Therefore a good ADP should also allow for some variations in the daily planned

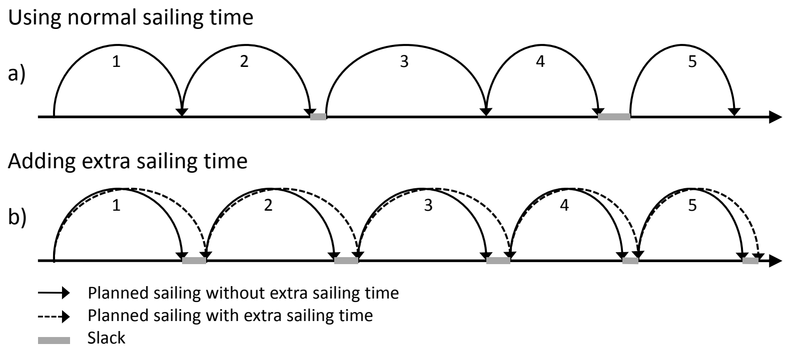

Figure 2: Schedule for a vessel with (b) and without (a) adding extra sailing time

production volumes. Table 2 shows an example of a discrete probability distribution for daily production rates as percent of the planned rates.

5. Robustness strategies

The problem formulation presented in Section 3 can be used to solve the real-life LNG ship routing and scheduling problem as it is described in Section 2. However, the solution obtained when solving this model can be difficult to execute in real-life as it does not take into consideration any of the uncertain parameters in this maritime transportation problem described in the previous section. Here we present four robustness strategies that can be embedded to the model formulation from Section 3 with the intention of creating solutions that are more robust with respect to the uncertainties described in Section 4.

5.1. Adding extra sailing time to each round-trip

A straightforward strategy to add some robustness to a solution is to plan with some slack in the schedule by planning that each round-trip should last longer than under normal conditions. This means, for example, that a round-trip from the production port to a customer that usually takes 30 days when sailing at normal speed is planned to last 32 days.

Figure 2 illustrates what a schedule for one vessel may look like when adding extra sailing time for each round-trip (Figure 2b) compared with a schedule using normal sailing times based on the vessel's service speed (Figure 2a). The figure shows when a vessel is planned to arrive at and depart from the production port during the planning horizon. Because the total required sailing time in a schedule is less than the planning horizon, there may be some slack between the round-trips for a solution based on normal sailing times. This happens after round-trips 2 and

4 in Figure 2a. For the solution with added sailing time to each round-trip there will always be slack between the round-trips due to the difference between planned sailing time and normal sailing time. This planned extra slack can lead to a vessel not being able to service the same customers as in the solution without extra sailing time, as we see in the plan where round-trip number 3 is shorter for the solution with extra sailing time (Figure 2b) than the corresponding one for the solution with normal sailing time (Figure 2a).

Robustness strategies with similarities to this one have been applied to obtain robust aircrew schedules. E.g. Ehrgott and Ryan (2002) construct robust crew schedules by penalizing solutions where aircrew is scheduled to change aircraft for a successive flight and the ground time minus duty ground time (time the crew is obliged to be on ground) is less than the expected delay.

Adding slack to each round-trip in the means of extra sailing time does not require any changes to the model formulation from Section 3. The input data, however, need to be modified by adjusting the values for some of the A_{vit^*t} parameters in the model.

A negative consequence of this robustness strategy arises when the vessel fleet's capacity is close to being fully utilized. This means that a sailing schedule using normal sailing times will have little slack. In this case it may not be possible to find a feasible solution servicing all cargoes if round-trips are planned to last for example 32 days instead of 30 (which reduces the fleet capacity by 6.25%). Therefore a decision maker should be careful when using this approach and not plan with increased sailing times that make the planning problem infeasible.

5.2. Target inventory level

The inventory level in the storage tanks at the production port cannot exceed the maximum level nor be below the minimum level. In general, there are higher risks involved with being close to the maximum level than the minimum level, as exceeding maximum level can result in having to temporarily stop production. The probability of being close to maximum levels is also higher as both increased daily production rates and delayed vessels will result in higher inventory levels than planned. Being close to the minimum level (in this case 0) can happen in the case of lower production volumes than planned, and may result in vessels having to wait some time before being able to load a full cargo.

The planners for the real-life LNG ship routing and scheduling problem we consider are, however, more concerned with having a target inventory level at half of the maximum volume. Therefore we define a target inventory level strategy where any levels below or above the target levels are penalized equally in the objective function. This has similarities with the approach suggested by Christiansen and Nygreen (2005).

The overall goal for the target inventory level strategy is to have inventory level close to half of the maximum volume. Since it will not be possible to have an inventory level exactly at this volume on all days, high and low target inventory levels are defined. These are defined based on the largest vessel in the fleet: The volume within the high and low target levels should equal the loading capacity of the largest vessel (the one with the greatest capacity). Let I^H and I^L be the high and low target inventory levels, S^{MX} be maximum inventory level, and Q^{MX} equal the loading capacity of the largest vessel. Then the high and low target inventory levels are calculated as follows:

$$I^H = \frac{S^{MX} + Q^{MX}}{2}, \tag{14}$$

$$I^L = \frac{S^{MX} - Q^{MX}}{2}. \tag{15}$$

The high and low target inventory levels are soft constraints that can be violated at a penalty cost in the objective function. The following two non-negative variables have to be added to the model formulation from Section 3:

$$s_t^+ \geq s_t - I^H, \tag{16}$$

$$s_t^- \geq I^L - s_t, \tag{17}$$

where s_t^+ equals the amount of inventory above the high target inventory level at time t, and s_t^- equals the amount below the low target inventory level. Both variables equal zero if inventory levels are within high and/or low target levels.

The objective function (1) needs to be replaced by

$$min \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}_v} \sum_{t \in \mathcal{T}} C_{vi} x_{vit} + \sum_{t \in \mathcal{T}} I^P \left(s_t^+ + s_t^- \right), \tag{18}$$

where I^P is a penalty cost per m^3 the inventory level is above or below the high and low target inventory levels.

5.3. Target accumulated berth use

Vessels that are delayed to the production port for the loading of one cargo can affect other cargoes that are to be loaded as there is limited berth capacity. For example, for a problem with one berth there can easily be conflicts when cargoes are planned to be picked-up on several consecutive days. This means that there may be gains by spreading the berth occupation during the planning horizon to avoid solutions where there are time periods with high planned berth activity followed by time periods with low berth activity.

In the target accumulated berth use strategy, soft constraints are added to the mathematical model formulation from Section 3 with the intention that the accumulated berth use should be within a minimum and maximum level. The accumulated berth use on a given day t is given by the sum of vessel visits from day 1 to day t in the planning horizon.

Let b_t^{ACC} be the accumulated berth use on day t, and x_{vit} and z_t be as defined in Section 3. Then the accumulated berth use on day t is calculated as follows:

$$b_t^{ACC} = \sum_{u=1}^{t} \left(\sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}} x_{viu} + z_u \right). \tag{19}$$

We define a high and low target accumulated berth use on day t, B_t^H and B_t^L, respectively. Let U^{TOT} be the estimated total number of cargoes being shipped from the production port during the planning horizon, including estimated number of spot cargoes, and $|T|$ be the total length of the planning horizon. Then the high and low target accumulated berth use on day t are calculated as follows:

$$B_t^H = \lceil \frac{t * U^{TOT}}{|T|} \rceil, \tag{20}$$

$$B_t^L = \lfloor \frac{t * U^{TOT}}{|T|} \rfloor. \tag{21}$$

The following two non-negative variables have to be added to the model formulation:

$$b_t^+ \geq b_t^{ACC} - B_t^H, \tag{22}$$

$$b_t^- \geq B_t^L - b_t^{ACC}, \tag{23}$$

where b_t^+ and b_t^- represent the accumulated berth use above or below the target levels on day t, respectively.

The objective function (1) needs to be replaced by

$$min \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}_v} \sum_{t \in \mathcal{T}} C_{vi} x_{vit} + \sum_{t \in \mathcal{T}} B^P \left(b_t^+ + b_t^- \right), \tag{24}$$

where B^P is the penalty cost for accumulated berth use above or below the high and low target accumulated berth use.

5.4. *Combined strategy*

The combined strategy is a combination of the three robustness strategies from Sections 5.1-5.3. The variables described in (16)-(17), (19) and (22)-(23) are added to the model formulation from Section 3, in addition to adjusting some of the parameters A_{vit^*t} by adding slack to round-trips.

The objective function (1) needs to be replaced by the following combination of (18) and (24):

$$min \sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{N}_v}\sum_{t\in\mathcal{T}} C_{vi}x_{vit} + \sum_{t\in\mathcal{T}} I^P\left(s_t^+ + s_t^-\right) + \sum_{t\in\mathcal{T}} B^P\left(b_t^+ + b_t^-\right). \quad (25)$$

6. A simulation-optimization framework for evaluating solutions

To evaluate a selection of candidate solutions to the LNG ship routing and scheduling problem, a simulation program has been developed. This program considers uncertainties in both sailing times and daily production rates as described in Section 4. It combines simulation with optimization by calling the recourse action of reoptimizing the schedule when given conditions occurs. In Section 6.1 an overview of the simulation program is given. Then follows a description of the reoptimizing (re-route) procedure in Section 6.2.

6.1. The simulation program

The purpose of the simulation program is to evaluate a given solution (or robustness strategy). A solution will in this setting contain which customers to deliver LNG to on which day by which vessel. Embedded in the simulation program is a re-route optimization procedure that can be considered a recourse action: Whenever certain conditions occur in a simulation, the planned schedule is reoptimized, and the new reoptimized schedule is used in the rest of that simulation. This is to capture the essence of the real planning situation. The main focus is that deliveries to customers should ideally be made on the planned days. The vessel making the delivery is not of that great importance. This will be valid for the problem considered in this paper as all vessels that may make delivery to a customer are quite similar with respect to loading capacities.

Figure 3 shows the flow diagram for the simulation program. For each simulation, we start on the first day of the planning horizon. The inventory level is set to the inventory level the previous day (or start inventory if it is the first day in the planning horizon) plus any LNG production on this day. The daily LNG production rate is uncertain and is calculated based on the expected LNG production and the probability distribution for changes in the daily production rate (see Table 2) using a Monte Carlo sampling technique (see e.g. Rubinstein and Kroese (2008)). Further, for any cargoes that are planned to be serviced on this day, the planned vessel is chosen if it is in the production port available for service. The planned

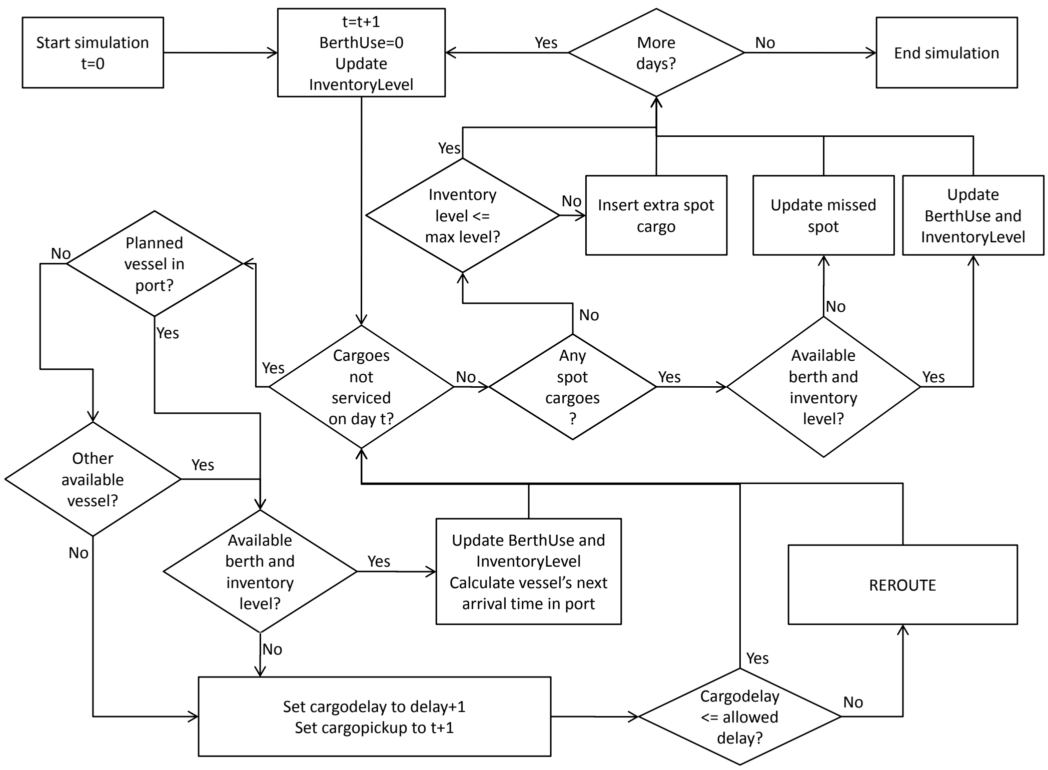

Figure 3: Flow diagram for the simulation program

vessel may not be available if it is delayed to the production port during service of a previous cargo, or if it has been used to service a cargo that was planned serviced by a different vessel. If the planned vessel is not available, a different vessel is chosen if any vessel that can service that cargo is idle. The cargo will then be serviced as long as there are available berths and there is an inventory level that accounts for a full shipload (or close to a full shipload).

If the cargo is serviced, the inventory level and berth use is updated, and the return-time for the vessel is calculated based on the probability distribution for increased sailing time (see Table 1) using a Monte Carlo sampling technique. From this, the vessel's next arrival time in the production port is calculated.

If the cargo cannot be serviced, the delay for that cargo is updated with one day (initially zero days) and the pickup day is set to next day. The user of the simulation program defines a maximum allowed delay for the cargo pickups, and if the delay is greater than this allowed delay, the re-route optimization procedure (described in the next section) is called.

When all cargoes are serviced on a given day or delayed to be serviced the next day, spot cargoes with planned pick-up on that day are serviced if there are sufficient inventory level and available berth capacity. If the inventory level is above maximum level, an extra spot cargo is inserted and the inventory level reduced correspondingly.

After each simulation, the total cost of the sailed schedule is calculated. Also calculated is the total number of pick-up days changed from the originally planned schedule, and the number of times the re-route optimization procedure had to be called. Any other information that a decision maker may find relevant for evaluating a solution to the LNG ship routing and scheduling problem can also be calculated and stored.

After running a user specified number of simulations, average numbers and standard deviation over all simulations are calculated and can be used as decision making criteria to evaluate a given solution (or robustness strategy).

6.2. The re-route optimization procedure

Whenever the re-route optimization procedure is called during a simulation an optimization problem is solved to resemble the real-life planning process. This optimization problem is a modified version of the basic model from Section 3, and will only consider the remainder of the planning horizon at the day where the re-route procedure is called.

The objective for the re-route optimization problem is to create a new minimum cost schedule that is as close to the previous schedule as possible, i.e. it is preferred that the customers get deliveries on the same days if possible. As in the simulation program, no weight is put on what vessel that delivers a cargo to a customer as long as it is a vessel that can make delivery to that customer.

Input from the simulation program to the re-route procedure is the remaining of the planned schedule, consisting of the remaining customer cargoes and the planned days for start of servicing them, and the vessels' positions given as the day they will be available for service at the production port.

Let sets $\mathcal{N}$, $\mathcal{T}$ and $\mathcal{V}$ be as described in Section 3. Then subset $\mathcal{T}^A \subset \mathcal{T}$ is the set of remaining days of the planning horizon when the re-route procedure is called (day t^A). Set $\mathcal{G}$ contains the remaining planned schedule in terms of which customer cargoes that are planned to be serviced on which days, (i, t^*).

The parameters C_{vi}, R_v^{MX}, A_{vit^*t}, B, P_t, Q_v, Q^S, S^{MN} and S^{MX} are as described in Section 3. F_i is now the number of remaining cargoes to deliver to customer i. H_{it} is zero if a cargo to customer i is scheduled to be serviced on day t and one if scheduled to be serviced on day $t-1$ or $t+1$. H^P is the penalty cost for customers not being serviced on the scheduled day.

The decision variables are the same as in Section 3: x_{vit}, z_t and s_t. We introduce a new vessel variable, x_{jit}^S. Index $j \in \{spotvessel, spotcargo\}$ represents either a charter-in spot vessel servicing a customer cargo (the customer cargo is serviced by a vessel that is not in the LNG producer's fleet), or a spot delivery of LNG to a customer (the vessel servicing the customer cargo is not in the LNG producer's fleet and the LNG delivered is bought from some other LNG producer). These are possible real-life recourse actions. The costs for these two options are relatively high compared with utilizing own fleet and LNG (reflecting the market costs for charter-in vessels and spot deliveries), are the same for all customers and represented by C_j. Let $\mathcal{V}^S$ be the set containing these two options. Then variable x_{jit}^S equals 1 if option j is used to service a cargo to customer i starting on day t, and zero otherwise.

Further, the new variable s_t^{MN} represents LNG inventory at the production port below the minimum level on day t. We allow for the inventory level being slightly under the minimum level as the simulation procedure allows for cargoes being close to full shiploads when the inventory level is lower than a full shipload. The re-route optimization problems only allows for full shiploads, thus allowing a small negative inventory level will create solution that will not require an expensive spot delivery of LNG when the inventory level amounts to close to a full shipload. S^{MXS} is the maximum amount of LNG allowed below minimum inventory level, and S^P the penalty cost for each m^3 of LNG the inventory level is below minimum.

The re-route optimization problem then becomes:

$$min \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}^A}\left(\sum_{v\in\mathcal{V}} C_{vi}x_{vit} + \sum_{j\in\mathcal{V}^S} C_j x^S_{jit}\right) + \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}^A} H_{it}H^P\left(\sum_{v\in\mathcal{V}} x_{vit} + \sum_{j\in\mathcal{V}^S} x^S_{jit}\right) + \sum_{t\in\mathcal{T}^A} S^P s^{MN}_t, \tag{26}$$

subject to

$$\sum_{t^*=max\{t^A,t-R^{MX}_v+1\}}^{t}\sum_{i\in\mathcal{N}_v} A_{vit^*t}x_{vit^*} \le 1, \quad v\in\mathcal{V}, t\in\mathcal{T}^A, \tag{27}$$

$$\sum_{v\in\mathcal{V}}\sum_{t=max\{t^A,t^*-1\}}^{t^*+1} x_{vit} + \sum_{j\in\mathcal{V}^S}\sum_{t=max\{t^A,t^*-1\}}^{t^*+1} x^S_{jit} \ge 1, \quad (i,t^*)\in\mathcal{G} \tag{28}$$

$$\sum_{v\in\mathcal{V}}\sum_{t\in\mathcal{T}^A} x_{vit} + \sum_{j\in\mathcal{V}^S}\sum_{t\in\mathcal{T}^A} x^S_{jit} = F_i, \quad i\in\mathcal{N} \tag{29}$$

$$\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{N}_v} x_{vit} + \sum_{j\in\mathcal{V}^S}\sum_{i\in\mathcal{N}} x^S_{jit} + z_t \le B, \quad t\in\mathcal{T}^A, j\backslash\{spotcargo\} \tag{30}$$

$$s_t = s_{t-1} + P_t - \sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{N}_v} x_{vit}Q_v - \sum_{j\in\mathcal{V}^S}\sum_{i\in\mathcal{N}} x^S_{jit}Q^S - z_tQ^S, \quad t\in\mathcal{T}^A, j\backslash\{spotcargo\} \tag{31}$$

$$S^{MN} - s^{MN}_t \le s_t \le S^{MX}, \quad t\in\mathcal{T}^A \tag{32}$$

$$s^{MN}_t \in \left[0, S^{MXS}\right], t\in\mathcal{T}^A \tag{33}$$

$$x_{vit}\in\{0,1\}, \quad v\in\mathcal{V}, i\in\mathcal{N}_v, t\in\mathcal{T}^A, \tag{34}$$

$$x^S_{jit}\in\{0,1\}, \quad j\in\mathcal{V}^S, i\in\mathcal{N}, t\in\mathcal{T}^A, \tag{35}$$

$$z_t\in\mathbb{Z}^+, \quad t\in\mathcal{T}^A. \tag{36}$$

The objective function (26) minimizes the cost of the schedule including sailing costs, charter-in vessels and the cost of spot deliveries. It also minimizes the number of customers receiving deliveries on other days than the ones in the planned input schedule, and the volume of LNG at the production port being below minimum level. Constraints (27) are similar to constraints (2) and ensure that a vessel is only assigned to servicing one cargo at the same time. Constraints (28) ensure that all planned customer cargoes are serviced either on the planned day or one day previous to or after this day. The second term being one if a customer cargo is serviced by a chartered-in vessel or by a spot cargo delivery. Constraints (29) ensure that all remaining customer cargoes are serviced. These are redundant if all cargoes to a given customer is planned to be serviced with at least two days in between each pick-up. But for some customers that are to receive cargoes frequently this may not be the case and constraints (28) alone can result in some cargoes not being serviced. Constraints (30) and (31) are similar to constraints (5) and (6), but are valid only for the remaining days of the simulation. Then constraints (32) are the inventory level constraints. These are formulated as hard constraints for the maximum level, and soft constraints for the minimum level (see the discussion above). Constraints (33) set the bound on the s_t^{MN} variable, and constraints (34)-(36) set the binary and integer requirements on the problem variables.

There are no constraints that ensure that total delivered volume to the customers are within minimum and maximum level, like constraints (4). These constraints are omitted to simplify the re-route optimization model and because the vessels that can sail to a given customer have similar loading capacities so that there should not be much difference in the total delivery. The sum of all deliveries to each customer is calculated in the simulation procedure so that the validity of these constraints can be checked a posteriori.

7. Computational study

Five different strategies for creating solutions to the LNG ship routing and scheduling program are evaluated by the simulation program described in Section 6. These are:

BASIC	Model formulation as described in Section 3
EST	BASIC strategy with added slack on each round-trip to the production port
TIL	BASIC strategy with target inventory levels
TBA	BASIC strategy with target accumulated berth use
COMBINED	Combination of EST, TIL and TBA

Nine problem instances based on the real problem have been created for this purpose. In Section 7.1 the problem instances are described along with the test settings used when solving the optimization problems and evaluating the corresponding solutions. Numerical results are provided in Section 7.2.

7.1. Description of problem instances and test settings

Problem instances are created based on three real planning problems: C1, C2 and C3. An overview of the three planning problems is provided in Table 3.

Three time horizons are defined for each of the planning problems; 90, 180 and 360 days, giving a total of nine problem instances. Table 4 gives the number of customer cargoes to service for each problem instance, and the estimated number of spot cargoes needed to keep the inventory level within the maximum level.

Table 3: Overview of the three planning problems

Planning problem	C1	C2	C3
# Vessels	8	13	11
# Customers	5	12	3
# Berths	1	1	1
Min inventory level [1000 m^3]	0.00	0.00	0.00
Max inventory level [1000 m^3]	510.00	333.36	420.00

Table 4: Number of customer cargoes for each problem instance

Problem instance	C1-90	C1-180	C1-360	C2-90	C2-180	C2-360	C3-90	C3-180	C3-360
# cargoes	24	52	104	37	74	148	46	86	171
# spot	10	18	38	1	0	3	5	6	9

For each problem instance, the initial width of the time windows for picking up the customer cargoes are seven days except when this will lead to overlapping time windows for some customer cargoes. In the case of overlapping time windows, the width is reduced so that they are not overlapping.

The nine problem instances are solved by the five strategies BASIC, EST, TIL, TBA and COMBINED described above.

The shortest duration of a round-trip for a vessel from the production port to a customer is 8 days. The round-trip durations for the other customers vary from 22 to 30 days depending somewhat also on the vessels' sailing speed. For planning problems C1 and C3 round-trips of duration eight days are added one extra day of slack and the longer round-trips are added two days of slack. This is consistent with the probabilities for increased sailing times in Table 1, where a round-trip of duration 8 days will never be longer than 9 days, and for round-trips of durations 22 to 30 days there is a 85.5 % chance that the sailing time will have a maximum increase of two days. For planning problem C2, there are four customers with round-trip durations of 22-25 days depending on which vessel that services them. The round-trip durations for these customers are added only one day of slack because more slack made these instances infeasible (all cargoes could not be serviced by the LNG producer's own vessel fleet).

The target inventory levels and target accumulated berth use are set as described in Section 5. The penalty costs for violating the target inventory levels and target accumulated berth use are set so high that these soft constraints will only be violated when necessary to obtain a feasible integer solution.

The simulation program is running 100 simulations for each planned schedule.

The probabilities for increased sailing time and changes in LNG production rates are as given in Tables 1 and 2, respectively. Allowed delay for the customer cargoes is zero days so that the re-route optimization procedure will be called whenever a customer cargo cannot be serviced on the planned day.

All test results were obtained on a 2.16 GHz Intel Core 2 Duo PC with 2 GB RAM. The basic model formulation with the extensions was implemented in Xpress-IVE 1.19.00 with Xpress-Mosel 2.4.0 and solved by Xpress-Optimizer 19.00.00. The simulation program and re-route procedure was written in C++ using Visual Studio 2005, the re-route optimization problem was modeled with BCL and solved by calling Xpress-Optimizer 19.00.00.

The stopping criteria for the Xpress-Optimizer when getting solutions for the BASIC, EST, TIL and TBA strategies are as follows:

1. Optimal solution (or when gap from best known lower bound is less than 0.1 %)
2. Best integer solution after 3600 seconds
3. If no integer solution is found after 3600 seconds, first integer solution

And for the re-route optimization problem:

1. Optimal solution (or when gap from best known lower bound is less than 1 %)
2. Best integer solution after 600 seconds

7.2. Numerical results

Table 5 shows the planned costs (i.e. without running the simulation program) in percentage of the BASIC solution costs and optimality gaps (gap between solution and best known lower bound reported by the Xpress-Optimizer) for the nine problem instances when solved using the five strategies. The planned costs are only the costs of sailing the planned schedule and do not include any penalty costs for violating target inventory levels or target accumulated berth use. The optimality gap, on the other hand, is the optimality gap for the objective function value that may also include penalty costs for strategies TIL, TBA and COMBINED. No integer solution was found by the Xpress-Optimizer for problem instance C2-360 with strategy COMBINED after a CPU time of 12 hours; therefore no results are shown for C2-360 COMBINED. The bottom row shows the total cost over all problem instances, not including instance C2-360.

We observe from Table 5 that the EST and COMBINED strategies have the highest planned costs. This is as expected as these strategies both include slack in sailing times which allows for less flexibility in the solutions than the other strategies. The EST strategy has a total cost that is higher than the COMBINED

Table 5: Planned cost and optimality gap. The planned cost of the robustness strategies are expressed as % of the BASIC planned cost.

	BASIC	EST		TIL		TBA		COMBINED	
	Opt. gap (%)	Plan. cost (%)	Opt. gap (%)	Plan. cost (%)	Opt. gap (%)	Plan. cost (%)	Opt. gap (%)	Plan. cost (%)	Opt. gap (%)
C1-90	0.00	100.05	0.00	100.06	0.72	100.07	0.12	100.16	1.89
C1-180	0.00	106.25	0.00	100.07	2.68	100.03	1.00	106.31	3.81
C1-360	0.00	115.63	0.02	103.14	7.49	106.27	8.02	121.88	11.42
C2-90	0.00	100.00	0.00	100.00	0.67	100.00	0.08	100.00	0.57
C2-180	0.00	100.00	0.00	100.06	1.78	100.00	0.08	100.21	2.57
C2-360	1.99	98.07	0.03	102.49	9.49	98.09	2.34	-	-
C3-90	0.00	100.00	0.00	100.00	0.08	100.00	0.10	100.00	0.10
C3-180	0.00	106.15	5.78	100.02	2.84	100.04	2.80	100.06	2.71
C3-360	0.00	103.84	3.68	100.02	4.31	100.03	4.31	100.03	4.41
Total[a]		105.51		100.69		101.35		105.36	

[a] Not including C2-360

strategy even though the opposite should occur since the COMBINED strategy is the EST strategy with more constraints. This can happen as the extra constraints and added penalty functions for the COMBINED strategy may guide the Xpress-Optimizer in a different direction than the EST strategy. This can result in lower cost solutions when the optimal integer solution is not found after the CPU time limit of 3600 seconds.

Figure 4 shows the resulting inventory levels for problem instance C1-90 solved for BASIC and TIL. In the figure are also the high and low target inventory levels (IH and IL) and maximum inventory level (SMAX) shown. The figure illustrates how the TIL strategy typically results in inventory volumes further away from maximum and minimum levels.

Tables 6 and 7 show the average simulated costs over 100 simulations and the corresponding standard deviation (in percent) when there is uncertainty in only sailing times and in both sailing times and daily LNG production rates, respectively. For strategy BASIC the simulated cost is given as percentage of the planned cost, while for all other strategies it is given as percentage of the BASIC simulated cost. The simulated costs reflects the expected extra costs due to using more expensive vessels, needing to charter-in vessels to service customer cargoes or needing to buy spot cargoes of LNG to deliver to customers. The last row gives the total

Table 6: Average simulated cost and standard deviation, uncertainty in sailing times only. Simulated cost of BASIC solutions are expressed as % of the BASIC planned cost. Simulated cost of other solutions are expressed as % of BASIC simulated cost.

	BASIC		EST		TIL		TBA		COMBINED	
	Sim. cost (%)[b]	St. dev. (%)	Sim. cost (%)[c]	St. dev. (%)	Sim. cost (%)[c]	St. dev. (%)	Sim. cost (%)[c]	St. dev. (%)	Sim. cost (%)[c]	St. dev. (%)
C1-90	100.12	0.03	102.96	5.65	106.10	6.65	100.36	2.28	104.09	6.80
C1-180	112.62	4.46	99.36	4.62	101.01	5.57	98.71	4.75	98.69	4.37
C1-360	125.27	3.33	101.03	3.44	101.67	3.12	103.75	2.62	101.31	3.17
C2-90	106.09	7.08	100.60	14.42	100.18	5.52	98.61	4.61	98.92	4.75
C2-180	114.96	4.25	92.63	4.39	94.45	3.84	96.27	3.51	91.68	5.23
C2-360	111.34	2.92	103.82	3.25	95.45	2.47	96.65	2.85	-	-
C3-90	106.01	6.31	95.16	2.97	98.84	5.26	96.82	4.82	95.99	3.97
C3-180	117.29	9.34	90.89	6.77	93.55	5.47	99.78	5.39	88.64	5.18
C3-360	102.90	3.38	100.69	1.71	101.48	3.32	100.51	1.92	103.61	6.68
Total[a]	112.49		97.93		99.34		99.93		98.06	

[a]Not including C2-360
[b]Percent of planned cost
[c]Percent of simulated cost for BASIC strategy

average simulated costs over all problem instances (not including C2-360).

For most of the problem instances, the calculated costs of the solutions are lower than the simulated cost, but if the optimal solution is not found for a problem instance, it is also possible that the simulated cost is lower as the re-route optimization procedure can produce lower-costs solutions. This was the case for problem instance C3-360 EST. For all BASIC solutions, the planned costs were lower than the simulated costs. But we observe that the expected extra costs vary for the problem instances; from only 0.12 % for problem instance C1-90, and up to 25.27 % for problem instance C1-360. In total over all problem instances, the expected extra cost is 12.21 %.

Observations from Tables 6 and 7 show that there is not one strategy that provides the lowest cost solutions for all problem instances. When there is only uncertainty in sailing times (Table 6), each strategy produces the lowest cost solution for at least one problem instance. When it comes to total expected cost over all problem instances, EST provides the lowest cost, with a reduction of 2.07% compared with BASIC, closely followed by the COMBINED strategy.

Table 7: Average simulated cost and standard deviation, uncertainty in sailing times and daily LNG production rates.Simulated cost of BASIC solutions are expressed as % of the BASIC planned cost. Simulated cost of other solutions are expressed as % of BASIC simulated cost.

	BASIC		EST		TIL		TBA		COMBINED	
	Sim. cost (%)[b]	St. dev. (%)	Sim. cost (%)[c]	St. dev. (%)	Sim. cost (%)[c]	St. dev. (%)	Sim. cost (%)[c]	St. dev. (%)	Sim. cost (%)[c]	St. dev. (%)
C1-90	100.26	1.42	101.86	4.97	108.09	7.02	100.36	2.66	102.57	5.78
C1-180	111.73	4.77	100.72	4.88	102.74	5.73	100.11	4.78	99.83	4.21
C1-360	124.67	3.43	104.36	4.64	102.02	3.23	104.23	3.15	101.54	2.97
C2-90	108.26	8.04	101.20	10.50	99.60	5.05	97.54	6.38	97.24	5.45
C2-180	120.59	7.41	92.63	6.09	90.25	4.14	97.55	6.15	89.76	6.22
C2-360	119.53	7.59	102.79	6.14	93.87	5.47	94.24	5.24	-	-
C3-90	107.18	7.45	101.34	12.46	98.19	7.37	97.10	6.08	95.87	6.10
C3-180	119.19	9.10	96.85	10.46	94.55	6.69	99.20	5.91	86.61	4.67
C3-360	104.56	4.05	101.92	5.17	100.50	4.17	100.21	4.45	100.72	5.73
Total[a]	114.05		100.10		98.75		100.18		96.84	

[a]Not including C2-360
[b]Percent of planned cost
[c]Percent of simulated cost for BASIC strategy

When there is uncertainty in both sailing times and daily LNG production rates (Table 7), strategies EST and TBA do not give lowest expected cost solutions for any of the problem instances. These strategies also provide higher expected cost over all problem instances than the BASIC strategy. The TIL strategy gives the lowest expected cost solution for problem instance C2-360, while the COMBINED strategy provides the lowest expected cost solutions for five of the remaining eight problem instances. Over all problem instances the COMBINED strategy provides the lowest total expected cost, representing a reduction of 3.16% on average compared with the BASIC strategy.

The simulated costs do not reflect any costs involved with a replanning situation (represented by a call to the re-route optimization procedure) and costs involved with changing delivery dates to customers. These costs are difficult to estimate, and depends on the extent of the replanning (variation from old plan) and the customers' flexibility to changed delivery dates (low flexibility can lead to high penalty costs and/or loss of goodwill). Therefore weight should also be put on these elements.

Table 8: Average number of times the re-route optimization procedure is called (# RR) and average number of cargo pick-up days changed from original plan (# D), uncertainty in sailing times only

	BASIC		EST		TIL		TBA		COMBINED	
	# RR	# D	# RR	# D	# RR	# D	# RR	# D	# RR	# D
C1-90	0.08	0.08	0.24	0.03	0.90	0.81	0.11	0.11	0.50	0.26
C1-180	2.64	2.74	1.74	1.15	3.00	3.31	2.87	4.18	1.44	1.04
C1-360	8.25	12.63	4.87	2.72	8.62	12.13	7.72	9.29	4.49	2.83
C2-90	1.06	0.69	0.93	0.97	1.66	1.29	1.22	1.01	1.12	1.22
C2-180	5.37	9.76	3.61	5.02	2.89	3.40	6.55	10.66	2.28	2.47
C2-360	13.20	26.96	12.89	19.60	8.20	12.43	7.17	8.09	-	-
C3-90	2.63	5.61	0.58	0.74	3.13	7.27	1.91	3.70	0.77	0.95
C3-180	6.76	18.33	2.61	4.35	7.59	33.44	6.91	23.76	1.80	2.16
C3-360	13.98	41.14	1.51	2.36	13.80	54.72	1.50	2.37	2.61	3.88
Total[a]	40.77	90.98	16.09	17.34	41.59	116.37	28.79	55.08	15.01	14.81

[a]Not including C2-360

Tables 8 and 9 show how often the re-route optimization procedure on average had to be called during a simulation (# RR), and the average total number of pick-up days changed from the originally planned schedule (# D). For example, if a cargo was planned to be picked-up on day 137 in the planning horizon, but in a simulation is picked-up on day 139, two is added to this number. These are average numbers over 100 simulations. The last row shows the sum over all problem instances.

The re-route optimization procedure is called whenever a customer cargo cannot be picked-up on the planned day. There is also a direct link between the number of re-route calls and the number of changed cargo pick-up days as the pick-up days can only be changed in the re-route optimization procedure. Therefore, we observe from Tables 8 and 9 that the number of calls to the re-route optimization procedure is lower than the number of changed pick-up days for almost all problem instances and solution strategies.

The number of re-route calls and pick-up days changed vary for the different strategies, with COMBINED and EST being the ones with the lowest numbers. This is not surprising as adding slack to each return trip means that these strategies allow for some delay and thus are also less exposed for replanning.

Since the costs of re-routing and changing cargo pick-up dates are not reflected in the simulation costs in Tables 6 and 7, both simulation costs and number of re-route calls and cargo pick-up days changed should be studied before concluding

Table 9: Average number of times the re-route optimization procedure is called (# RR) and average number of cargo pick-up days changed from original plan (# D), uncertainty in sailing times and daily LNG production rates

	BASIC		EST		TIL		TBA		COMBINED	
	# RR	# D	# RR	# D	# RR	# D	# RR	# D	# RR	# D
C1-90	0.09	0.09	0.26	0.24	0.91	0.69	0.51	0.48	0.44	0.33
C1-180	2.75	3.14	1.76	1.54	3.03	3.03	3.02	4.33	1.46	0.96
C1-360	8.22	10.43	5.26	3.23	8.45	11.15	7.39	8.28	4.40	2.62
C2-90	1.34	1.61	0.89	1.29	1.67	1.04	1.36	1.34	1.09	1.19
C2-180	4.48	7.79	3.66	5.99	2.94	3.46	4.20	6.70	2.57	3.21
C2-360	9.24	18.31	8.37	14.43	7.24	10.02	8.21	10.98	-	-
C3-90	2.44	5.90	0.80	0.85	2.99	6.51	2.13	4.63	0.77	0.94
C3-180	6.61	17.92	2.29	3.15	7.58	31.52	6.91	25.80	1.82	2.58
C3-360	13.09	39.61	2.61	4.36	13.00	52.91	13.68	42.52	2.98	5.06
Total[a]	39.02	86.49	17.53	20.65	40.57	110.31	39.20	94.08	15.53	16.89

[a]Not including C2-360

what solution that overall performs the best.

The results illustrate how the best strategy varies for the different problem instances. However, we observe that in total, over all problem instances, the COMBINED strategy provides the best results. It has both the lowest average simulated costs and shows the best results on average with respect to the number of re-route optimization procedure calls and cargo pick-up day changes. This shows that it will add value to the solution to add some robustness strategies. On the other hand, the BASIC strategy also showed good results for some problem instances, which illustrates the importance for a decision maker to have the opportunity to create more than one solution based on different criteria and having access to a tool that can evaluate them.

8. Concluding remarks

This paper considered a ship routing and scheduling problem arising in the LNG business. A number of customer cargoes with given pick-up time windows need to be serviced by the available vessel fleet while at the same time not violating the production port's berth capacity and inventory level constraints.

As most maritime transportation problems this problem also includes uncertainty. In this paper we proposed and tested different robustness strategies that can be added to an optimization model with the aim of creating solutions that better handles the problem's underlying uncertain parameters: Sailing times and daily LNG production rates. The solutions obtained when solving the optimization model with and without adding robustness strategies were then compared by running a simulation program with a recourse re-route optimization procedure to imitate a real-life planning situation.

In total, five different strategies for creating solutions to the LNG ship routing and scheduling problem was tested: One basic approach where the optimization model was solved without adding any robustness strategies, one with added slack to each sailed round-trip, one with target inventory levels, one with target accumulated berth use and one with a combination of all robustness strategies. The results show that there is none of the robustness strategies that perform better than the others for all problem instances. However, most of the proposed robustness strategies, and the combined one in particular, gave solutions with lower expected costs than the basic approach (without any robustness strategies). In addition, the strategies of adding extra slack and combining all robustness strategies, lead to a significant overall decrease in the number of times a schedule had to be re-planned and changes in pick-up days for the customer cargoes.

The observed results illustrate the importance of addressing uncertainty in maritime transportation problems. The difficulty of creating one solution method that will create solutions that outperform all other solutions can be avoided by creating several solutions by adding various robustness strategies and assessing the results by a simulation program that imitates the real-life situation. The solution strategies proposed in this paper together with the simulation-optimization framework for evaluating solutions form a good foundation for a complete decision support system that will support both the initial planning process and the re-planning activities.

The re-route optimization model does not use any robustness strategies. This means that the replanning activity in the simulation program is solved by a modified version of the basic approach. This was done because it is necessary that the re-route optimization problem is solved within reasonably short CPU time as it will be solved several times during a simulation. We leave to future work to improve the re-route procedure and test the effect of also adding robustness strategies in the replanning situation.

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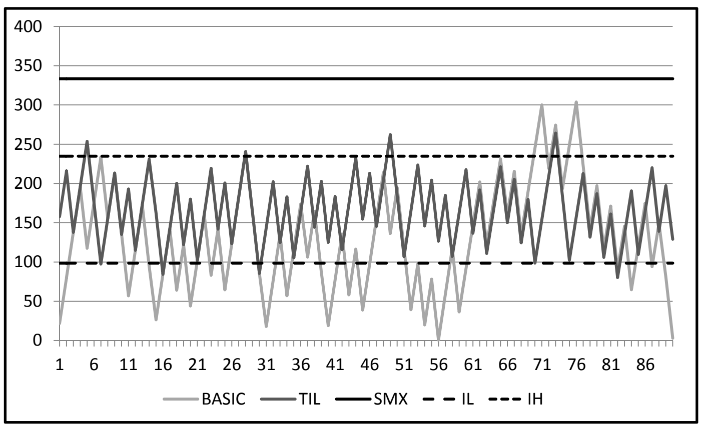

Figure 4: Inventory levels for problem instances C1-90 BASIC and TIL