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Frontier-based performance analysis models for supply chain management: state of the art and research directions

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Abstract

Effective supply chain management relies on information integration and implementation of best practice techniques across the chain. Supply chains are examples of complex multi-stage systems with temporal and causal interrelations, operating multi-input and multi-output production and services under utilization of fixed and variable resources as well as potentially environmental exposure. Acknowledging the lack of system's view, the need to identify system-wide as well as individual effects, as well as the incorporation of a coherent set of performance metrics, the recent literature reports on an increasing, but yet limited, number of applications of frontier analysis models (e.g. DEA) for the performance assessment of supply chains or networks. The relevant models in this respect are multi-stage models with various assumptions on the intermediate outputs and inputs, enabling the derivation of metrics for technical and cost efficiencies for the system as well as the autonomous links. This paper reviews the state of the art in multi-stage or network DEA modeling, along with a critical review of the advanced applications that are reported in terms of the consistency of the underlying assumptions and the results derived. Consolidating the current work in this range using a unified notation and by comparing the properties of the models presented, the paper is closed with recommendations for future research in terms of both theory and application.

Keywords: supply chain management, data envelopment analysis, two-stage, game theory, bi level programming.

JEL Classification: C14, M11, C79

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1. Introduction

Supply chain management (SCM) was introduced as a common scientific and managerial term in 1982 (cf. Oliver and Webber, 1992) to describe a hierarchical control system for material, information and financial flows in a potentially multidirectional network of autonomous decision making entities. Although there is a lack of universally accepted definition (Otto and Kotzab, 1999), a well-used and typical definition of a supply chain is '*a network of organizations that are involved, through upstream and downstream linkages in the different processes and activities that produce value in the form of products and services in the hand of the ultimate consumer*.' Christopher (1998, p.15). The management activity is consequently the coordination of this network, or 'chain', of independent processes as to achieve the overall goal in terms of value creation. Three elements are important in our context: the *multi-level character* of the network, the *interdependency* and the *competitive* objective. First, the underlying system is constituted of multiple layers, both horizontally (sequential processing) and vertically (control layers, levels of integration into firms, business units, joint ventures, information sharing, etc.). This implies that the systematic analysis of a supply chain must take into account the level of processing as well as the locus of control in order to understand the organization and its performance. Second, the 'links' in the chain form sequential processing stages that are interdependent with respect to potentially all three types of flows; material flows in progressive processing, information flows specifying types and quantity of processes to be performed, and financial flows to reimburse or incentivize the units to devote time and resources to the joint activity. Third, a supply chain is not an arbitrary processing plan but involves multiple independent organizations (conventionally at least three) cooperating under commercial conditions and subject to actual or potential future competition, both as a common endeavor and individually for each processing stage. Taken together, the three observations underline that *performance evaluation* is of highest importance to assure continuity, competitiveness and, ultimately, survival of the network, but that this evaluation must take into account the specificities of the network character and the decision-making autonomy of the evaluated units.

A wide range of metrics for supply chain performance have been proposed (cf. Neely et al., 1995, Melnyk et al., 2004) using an equally diverse portfolio of methodologies (cf. Estampe et al., 2010). Whereas most SCM literature has been devoted to the elaboration and evaluation of absolute metrics, usually linked to the dimensions cost (profit), time (rates) and flexibility (change of rate), there has also been a growing awareness of the need to perform external benchmarking (Beamon, 1999), the lack of integration of metrics (Beamon, 1999, Chan, 2003), the lack of system's view (Holmberg, 2000) and the lack of non-cost indicators (Beamon, 1999, De Toni and Tonchia, 2001). In response to this critique, several applications of non-parametric frontier analysis, such as Data Envelopment Analysis (DEA), have been proposed for supply chain management. The production-economic foundations and the capacity to derive a consistent set of informative performance metrics for a multi-input and multi-output setting qualify the frontier analysis as a useful tool for operation management assessments. However, the interdependencies among evaluated units call for specific frontier models, in particular the multi-stage or network models (cf. Färe and Grosskopf, 1996b). These models explicitly take into account the network structure in the evaluation, deriving metrics that can evaluate both individual unit and chain-wide performance in the long and the short run. However, the rapid development of such models (e.g. Färe and Grosskopf, 2000, Chen and Zhu, 2004; Chen et al., 2006a, Chen et al., 2009a, 2009b; Zha and Liang, 2010) and their relevance to supply chain performance assessment have not yet been critically reviewed.

It is to fill this need that this paper summarizes the state-of-the-art in frontier analysis models for supply chain management and their applications, along with identification of future research directions. Special emphasis is put on the special case of multi-level DEA that is called the two-stage process. The outline of this paper is organized as follows. In section 2, we first present the definition of the term SCM and then we discuss performance assessment in SCM. Section 3 is a short recapitulation of DEA definitions for readers not familiar with the models. In Section 4, we present a generic activity model for supply chain evaluation. In Section 5 we review the DEA-models applied to two-stage structures, including models based on cooperative and non-cooperative game theory, in particular bi-level programming. The paper is concluded in section 6 with critical analysis of the reviewed work as some directions for future research.

2. Performance evaluation in supply chain management

In the late 1980s, the term Supply Chain Management (SCM) arose and came into widespread use in the 1990s. SCM has been increasingly developed in theory and practice (e.g. Houlihan, 1985; Jones and Riley, 1987). There have been a large number of definitions of SCM (see e.g. Mentzer et al., 2001) but unfortunately, there is no explicit and generally accepted description of SCM in the literature. The term supply chain management is composed of a "*supply chain*" as the object of control and "*management*" as the scope of activity. Some definitions of a supply chain are proposed below (Ganeshan and Terry 1996; Lambert et al. 1998):

- *A supply chain is the alignment of firms that bring products or services to market.*
- *A supply chain consists of all stages involved, directly or indirectly, in fulfilling a customer request. The supply chain not only includes the manufacturer and suppliers, but also transporters, warehouses, retailers, and customers themselves.*
- *A supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers.*

Generally, supply chain is a system of organizations, people, technology, activities, information and resources involved in moving a product or service from supplier to customer. Supply chain activities transform natural resources, raw materials and components into a finished product that is delivered to the end customer. In many cases a supply chain consists of multiple suppliers, manufacturers, wholesalers, retailers. The management of a supply chain can be defined as (Bidgoli, 2010):

The systemic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole.

Supply chain management takes an integrated system's view on the design, monitoring and control of the chain. This approach serves to arbitrate the potential conflicts of individual agents in the chain in order to coordinate the flow of products and services to best serve the ultimate customer. We refer to this framework as "centralized", in that it represents the objective of a hypothetical benevolent supply chain coordinator with authority to implement any necessary decision throughout the chain.

Performance measurement is intrinsically anchored in SCM as both a predictive and normative paradigm. Predictive in the sense that performance management provides data and estimates necessary for the management of material and information flows in order to meet stochastic demand, product and process changes or changes in the price/cost structure for inputs and outputs. Normative in the sense that the supply chain management interfaces with both operations and sourcing, providing targets for improvement as well as potentially credible threats of substitution or volume reductions in case of poor [relative] performance. A seminal paper in performance measurement design is Neely et al. (1995), defining the scope of performance assessment as the quantification of effectiveness and efficiency of action. The paper also offers an overview over a wide range of techniques and metrics used as well as their limitations and areas for future research. Conventionally, the operations management literature limited the attention to performance measurement to the mere definition of absolute (e.g. cost per unit) and partial productivity (e.g. labor hours per unit produced) metrics (Cf. Melnyk et al. 2004 for a critique of this approach or Lambert et al., 2001 for an example) without paying attention to their systemic or economic integration, or even to their value as predictors of future profitability or survival in the market place. Neely et al. (1995) provide greater nuance to the analysis of supply chain performance by distinguishing the type of measurement, metric and method based on an analysis of organizational level, integration, organizational support, managerial application and hierarchical level. The authors document empirically that firms frequently neglect non-financial data, use internal cost data of varying quality, deploy methods with no or poor connection to organizational strategy and globally are dissatisfied with their performance assessment system. Shepherd and Gunter (2006) review 362 scientific papers on supply chain performance measurement and conclude that the findings of Neely et al. in many aspects are still valid. Alternative qualitative approaches exist using tools such as balanced scorecards (cf. Bhagwat and Sharma, 2007), however, the information made available from such models is limited in terms of e.g. decomposing productive and cost efficiency. Nevertheless, the need to identify performance in supply chain can be of strategic as well as operational value, cf Gunasekaran et al. (2004) and Olugu and Wong (2009), leading us to require consistency in the evaluation methodology between the two levels.

Applications using frontier methods to complex multi-stage systems, normally the non-parametric DEA method that is the focus here, are relatively rare. An early application to US Army recruitment in Charnes et al. (1986) used a two-stage approach with intermediate outputs that forms the basis of later network models. Ross and Droge (2002) proposed an integrated benchmarking approach for measuring temporal efficiency using some extensions to DEA methodology and then applying their approach into real data including 102 distribution centers in the petroleum business. Talluri et al. (1999) proposed a framework based on DEA and multi-criteria decision models for value chain network design, primarily aiming at the identification of an optimal supplier-manufacturer dyad. Löthgren and Tambour (1999) used the network DEA model introduced by Färe and Grosskopf (1996a) to estimate efficiency and productivity for a set of Swedish pharmacies. Hoopes, Triantis, and Partangel (2000) developed a goal-programming DEA formulation that models serial manufacturing processes and applied it to data on circuit board manufacturing. Talluri and Baker (2002) proposed an interesting three-phase approach for designing an effective supply chain using a DEA framework. Phase I evaluates potential suppliers, manufacturers, and distributors in determining their efficiencies using a combination of a DEA models and the pair-wise efficiency game. Phase II contains an integer programming model, which optimally selects candidates for supply chain design using a combination of the efficiencies obtained in phase I, demand and capacity requirements, and location constraints. Phase III includes the identification of optimal routing decisions for all entities in the network by solving a minimum-cost transshipment model. Sexton and Lewis (2003) evaluated managers' management efficiency of 30 Major League Baseball teams in 1999 under two-stage model. Their model distinguishes inefficiency of the first stage from the second stage, allowing managers to target inefficient stages of the production process. Lewis and Sexton (2004) viewed the network as a baseball team and extended Sexton and Lewis (2003) to consider efficiency at each node of a network. Narasimhan et al. (2004) considered a two-stage framework, namely flexibility competency and execution competency, for discussing the relationship between manufacturing flexibilities and manufacturing performance of a set of firms. Their model used the reduced CRS-DEA model proposed by Andersen and Petersen (1993) to measure the efficiency of each stage independently. Sheth et al. (2007) evaluate the overall performance of an agency's bus routes by using network DEA (Färe and Grosskopf, 2000) and goal programming (Athanassopoulos, 1995) with environmental factors from the supplier, consumer, and society viewpoints. Yu and Lin (2008) used a multi-activity network DEA model for estimating passengers and freight technical efficiency, service effectiveness and technical effectiveness for 20 railway firms in the world. Yu (2008a) used a multi-activity DEA model for measuring the efficiency of multi-mode bus transit under highway and urban bus services in the presence of environmental variables, also used for a shared output-model in Yu and Fan (2006) and and an enhanced network system with consideration of consumption in addition to highway and urban bus services in Yu and Fan (2009). Yu (2008b) presents a network DEA approach consisting of two stages, the production and Consumption stages, to evaluate the technical efficiency, the service and technical effectiveness of a selected sample of 40 global railways. Vaz et al. (2010) proposed a method to measure Portuguese retail stores performance based on the network DEA (Färe et al., 1997), which takes into account the interdependencies of the sections composing the store.

Supply chain management involves decision on a multi-level decision network structure. Application of conventional DEA models considers the supply chain as a black box and considers only the inputs from the beginning of the upstream components and final outputs at the very end of downstream components in the performance evaluation. Thus, those intermediate measures are ignored. The efficiency scores will result in ambiguous or too optimistic estimates of the SCM.

3. Data envelopment analysis

The data envelopment analysis (DEA) approach to efficiency measurement is a deterministic method that does not require the definition of a functional relationship between inputs and outputs. In economic terms, DEA utilizes the non-parametric mathematical programming approach to estimate best practice production frontiers (envelope). The basic DEA model as introduced by Debreu (1951) and Farrell (1957) and later developed by Charnes et al. (1978) is a data-driven method for evaluating the relative efficiency of a set of entities with multi-inputs and multi-outputs. DEA has rapid and continuous growth in different areas since 1978. Emrouznejad et al. (2008) reported more than 4000 DEA research studies published in journals or book chapters. A taxonomy and general model frameworks for DEA also can be found in Cook and Seiford (2009).

Let us introduce the *technology set T* or *production possibilities set (PPS)*

$$T = \{(x, y) \in R_+^m \times R_+^s \,|\, x \, can \, produce \, y\}$$

The background of the DEA is production theory, and the idea is that the DMUs have a common underlying technology *T*. In reality, we usually could not specify the *technology set* but DEA deals with the problem by estimating PPS, *T**, from observed data on actual production activities according to the *minimal extrapolation principle.*

The mathematical programs can be obtained when we combine the idea of minimal extrapolation with Farrell's idea of measuring efficiency as a proportional improvement.

Assume that there are *n* DMUs to be evaluated where every DMU$_j$, $j = 1, 2, ..., n$, produces *s* outputs, $Y^j = (y_1^j, ..., y_s^j) \in R_+^s$, using *m* inputs, $X^j = (x_1^j, ..., x_m^j) \in R_+^m$. The $s \times n$ matrix of output measures is denoted by *Y*, and the $m \times n$ matrix of input measures is denoted by *X*.

The efficiency of a specific DMU$^{\circ}$ is denoted by $\theta^{\circ}(X,Y,\gamma)$ in output-oriented and $\varphi^{\circ}(X,Y,\gamma)$ in input-oriented where γ represents the returns to scale. Therefore, $\theta^{\circ}(X,Y,\gamma)$ is calculated by using the following mathematical DEA model

$$\begin{aligned} \min \quad & \theta^{\circ} \\ s.t. \quad & X\lambda \le \theta^{\circ} X^{\circ}, \\ & Y\lambda \ge Y^{\circ}, \\ & \lambda \in \Omega(\gamma). \end{aligned} \tag{1}$$

where $\Omega(\gamma)$ specify the shape of the frontier. In other words, $\Omega(\gamma)$ differentiates between the models based on the returns to scale assumption. In short, we can define six following classical DEA models as follows (cf. Bogetoft and Otto, 2010):

The constant returns to scale (CRS) model when $\Omega(crs) = \{\lambda \in R_{+} \mid \lambda \ \ free\} = R_{+}$

The decreasing returns to scale (DRS) model when $\Omega(drs) = \{\lambda \in R_{+} \mid 1\lambda \le 1\}$

The increasing returns to scale (IRS) model when $\Omega(irs) = \{\lambda \in R_{+} \mid 1\lambda \ge 1\}$

The varying returns to scale (VRS) model when $\Omega(vrs) = \{\lambda \in R_{+} \mid 1\lambda = 1\}$

The free disposability hull (FDH) model when $\Omega(fdh) = \{\lambda \in N_{+} \mid 1\lambda = 1\}$

The free replicability hull (FRH) model when $\Omega(frh) = \{\lambda \in N_{+} \mid \lambda \ \ free\} = Z_{+}$

where Z_{+} is set of the non-negative integers.

The CRS, DRS, IRS and VRS models are linear programming (LP) problems while FDH and FRH are mixed integer problems (MIP). The dual problem of (1) is:

$$\begin{aligned} \max \quad & \theta^{\circ} = uY^{\circ} \\ s.t. \quad & vX^{\circ} = 1, \\ & uY - vX + u_0 \le 0, \\ & u, v \ge 0, \\ & u_0 \in \Phi(\gamma). \end{aligned} \tag{2}$$

where $\Phi(CRS) = \{0\}$, $\Phi(DRS) = R_{-}, \Phi(IRS) = R_{+}$ and $\Phi(VRS) = R$. In model (2), u and v are the weight vectors assigned to the output and input vectors, respectively. Note that in the dual program of CRS, $u_0 = 0$ since there are no restrictions on λ in the primal model, therefore, it becomes

$$\begin{aligned} \max \quad & \theta^{\circ} = uY^{\circ} \\ s.t. \quad & vX^{\circ} = 1, \\ & uY - vX \le 0, \\ & u, v \ge 0. \end{aligned} \tag{3}$$

A DMU with $\theta^{\circ *}(X,Y,\gamma)=1$, is called efficient with respect to the technology set $T(X,Y)$ and the returns to scale γ, otherwise, $\theta^{\circ *}(X,Y,\gamma)\neq 1$ is called inefficient. Problem (1) is referred to as the envelopment or primal problem, and (3) the multiplier or dual problem.

4. A generic SCM model

The inefficient DMUs are notably interested in the factors that cause the inefficiency, although it is obvious that either reducing inputs or increasing outputs will improve their performance. To answer this question, much effort has been devoted to breaking down the overall efficiency into components so that the sources of inefficiency can be identified. One type of decomposition focuses on the structure of the DEA models. The general multi-level/multi-stage structure for performance evaluation in the complex and real environment is illustrated in Figure 1. This model involves the direct inputs and outputs for each stage, the intermediate flows between two stages and the common inputs among all levels of the system and shared inputs among stages of each level.

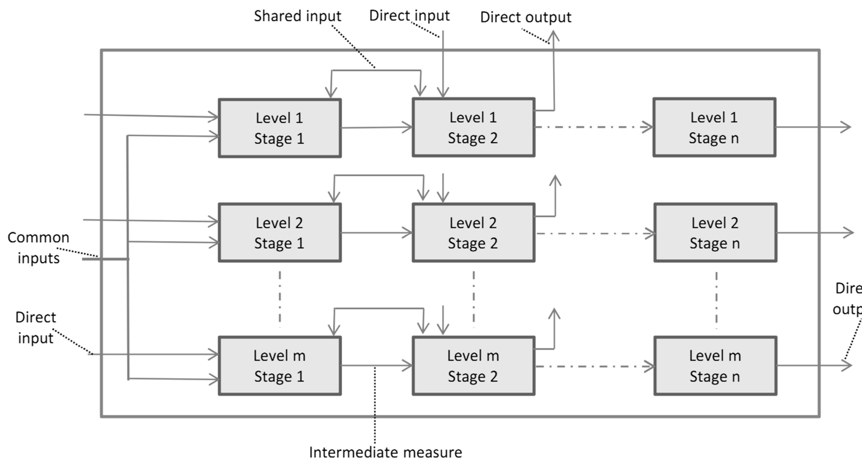

Fig. 1: The common multi-stage activity model

Figure 2 shows a simplified model of a two-stage process with the shared resource, where each DMU is composed of two sub-DMUs in series, and intermediate products by the sub-DMU in stage 1 is consumed by the sub-DMU in stage 2.

Suppose that stage 1 of each DMU_j $(j=1,...,n)$ has *m* direct inputs $X_1=(x_1^j,...,x_m^j)$ and two sets of direct outputs: *p* outputs $Y_1=(y_1^j,...,y_p^j)$ and *q* outputs $Z=(z_1^j,...,z_q^j)$, while stage 2 of each DMU_j $(j=1,...,n)$ consists of *s* direct outputs $Y_2=(y_1^j,...,y_s^j)$ and two sets of direct inputs: *t* inputs $X_2=(x_1^j,...,x_t^j)$ and *q* inputs $Z=(z_1^j,...,z_q^j)$. We also assume *k* shared inputs $X_3=(x_1^j,...,x_k^j)$ which are allocated among two stages. Note that Z is the intermediate measure e.g. the outputs of one stage become inputs to a later stage. The generic model is usually not analyzed in the current literature where most instructions are special cases where some sets are empty. In this study, we denote the efficiencies of stage 1 and stage 2 by $E_k^{\circ}(X_p, Y_q, Z, \gamma)$, k=1, 2; p=1,2,3; q=1,2 and the overall efficiency is denoted by $E^{\circ}(X_p, Y_q, Z, \gamma)$ in input oriented while we use $F_k^{\circ}(X_p, Y_q, Z, \gamma)$ and $F^{\circ}(X_p, Y_q, Z, \gamma)$ notations in output oriented.

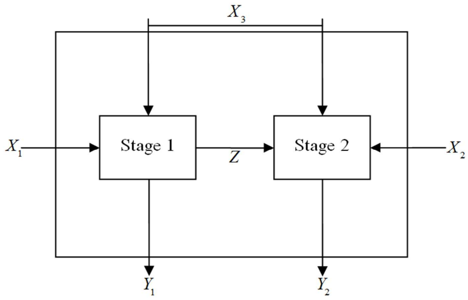

Fig. 2: The common two-stage activity model.

5. Literature Review

In the black-box approach of conventional DEA, the internal structures of DMUs are generally ignored, and the performance of a DMU is assumed to be a function of the chosen inputs and outputs. In the mid-1980s, Färe and Primont (1984) started working on performance evaluation of DMUs with known internal structures. They constructed multi-plant efficiency measures and illustrated their models by analyzing utility firms each of whom operated several electric generation plants. Although Färe (1991), Färe and Whittaker (1995) and Färe and Grosskopf (1996a) further expanded this modeling approach, the studies of Färe and Grosskopf (1996b, 2000) in the literature are known as a pioneered line of research at developing a general multi-stage model with intermediate inputs-outputs which is commonly called *network DEA*. Cook et al. (1998) discussed a general framework for hierarchies in DEA, grouping DMUs and their individual and aggregate performance indexes. Cook et al. (2000) presented a non-linear DEA model for measuring the efficiency of two components (i.e., service and sales) in banking system in the presence of shared resources. Cook and Green (2004) modified the DEA model developed by Cook et al. (2000) in order to specify the core business performance in multi-plant firms. Jahanshahloo et al. (2004a) determined the progress and regress of each component of a DMU upon the basis of Cook et al. (2000). Jahanshahloo et al. (2004b) linearized the model proposed in Cook et al. (2000) in the presence of discretionary and non-discretionary shared resources. Yang et al. (2000) proposed a DEA evaluation model for multiple independent parallel subsystems in which the efficiency of the overall process equals to the maximum of the efficiencies of all sub-processes. Castelli et al. (2001, 2004) and Amirteimoori and Shafiei (2006) discussed on some types of the network structure using DEA-like models. Golany et al. (2006) simultaneously measured the efficiency of the whole system and each sub-system as a special case of the Färe and

Grosskopf network framework (2000). Chen (2009) developed a dynamic production network DEA model by introducing an alternative efficiency measure for evaluating the performance of various hierarchical levels in the dynamic environment along with discussing on some returns to scale properties of production network. Chen and Yan (2011) recently developed three different network DEA models based on the concept of centralized, decentralized and mixed organization systems along with discussing on the relationship between their efficiencies. A later complementary network DEA formulation is a non-radial slacks based approach in Tone and Tsutsui (2009). This approach has applications in Fukuyama and Weber (2009), Fukuyama and Weber (2010) for bad outputs and, Avkiran (2009) to a banking setting and Yu (2010) to airport operations. An important modeling contribution for the Tone and Tsutsui (2009) model is made in Chang et al. (2011), where the focus is on the ownership-control for the formulation of a full set of efficiency metrics.

A two-stage process which is a special case of Färe and Grosskopf's multi-stage framework involves a large number of real evaluation problems. Therefore, DMUs may have a two-stage structure in which the first stage uses inputs to produce outputs that become inputs of the second stage and then the second stage uses these first stage outputs to generate its own outputs. An excellent review of DEA models exploring internal structure in general, including some of our work, is found in Castelli et al. (2010). In this section, we present a literature review on models relevant to supply chain management. We review different DEA approaches organized with respect to methods in the two-stage process, game theory and bi level programming etc. to measure efficiency of supply chains.

5.1. *Two-stage DEA*

Wang et al. (1997) were the first, to the best of our knowledge, to apply a two-stage structure for measuring the performance. Their model was composed of X_1, Z and Y_2 which are the input vector of stage 1, the intermediate vector and the output vector of stage 2, respectively (see Figure 2). Wang et al. (1997) ignored the intermediate measures and obtained an overall efficiency with the inputs of the first stage and the outputs of the second stage (see model (5)). Similarly, Seiford and Zhu (1999) proposed a two-stage method to obtain the profitability and marketability of the top 55 U.S. commercial banks, consisting of X_1, Z and Y_2 presented in Figure 1. Seiford and Zhu (1999) used independent CRS models (4), (5) and (6) to measure the overall efficiency and the efficiencies of stage 1 and stage 2:

$$\begin{aligned} \max \quad & E^{\circ}(X_1, Y_2, Z, crs)^{sz} = uY_2^{\circ} \\ s.t. \quad & vX_1^{\circ} = 1, \\ & uY_2 - vX_1 \le 0, \\ & u, v \ge 0. \end{aligned} \tag{4}$$

Stage 1
$$\begin{aligned}\max\quad & E_1^{\circ}(X_1, Y_2, Z, crs)^{sz} = wZ^{\circ} \\ s.t.\quad & vX_1^{\circ} = 1, \\ & wZ - vX_1^{\circ} \le 0, \\ & w, v \ge 0.\end{aligned} \tag{5}$$

Stage 2
$$\begin{aligned}\max\quad & E_2^{\circ}(X_1, Y_2, Z, crs)^{sz} = uY_2^{\circ} \\ s.t.\quad & wZ^{\circ} = 1, \\ & uY_2 - wZ \le 0, \\ & u, w \ge 0.\end{aligned} \tag{6}$$

The intermediate measures can arise the potential conflicts between two stages. For example, the second stage may reduce its inputs (intermediate measures) to achieve an efficient status. Such an action would, however, imply a reduction in the first stage outputs, thereby reducing the efficiency of the first stage. Zhu (2000) applied a method similar to that of Seiford and Zhu (1999) to the Fortune Global 500 companies.

When the model consists of X_1, Z and Y_2, another conventional CRS model is to use the intermediate measure as an output ($Z + Y_2$) for measuring the overall efficiency. Chen and Zhu (2004) demonstrated that such DEA model fails to correctly characterize the two-stage process and the improvement to the DEA frontier can be distorted, i.e., the performance improvement of one stage affects the efficiency status of the other, because of the presence of intermediate measures. Zhu (2003) and Chen and Zhu (2004) also demonstrated that DEA model (4) does not correctly characterize the performance of the two stages, because it only considers the inputs and outputs of the whole process and ignores intermediate measures Z associated with two stages. Alternatively, one can consider the following DEA model that is the average efficiency of two stages:

$$\begin{aligned}\max\quad & E^{\circ}(X_1, Y_2, Z, crs) = \frac{1}{2}\left[\frac{w_1 Z^{\circ}}{vX_1^{\circ}} + \frac{uY_2^{\circ}}{w_2 Z^{\circ}}\right] \\ s.t.\quad & w_1 Z - vX_1 \le 0, \\ & uY_2 - w_2 Z \le 0, \\ & w, v \ge 0.\end{aligned} \tag{7}$$

Although model (7) includes intermediate measures Z, it does not consider the relationship between the first and second stages owing to issue of decouples of the two stages. This does not show an ideal supply chain system (Liang et al. (2006)). Chen and Zhu (2004) suggested the following linear model for the two-stage process based upon the envelopment form of VRS consisting of X_1, Z and Y_2 presented in Figure 2:

$$
\begin{aligned}
\min \quad & E^{\circ}(X_1, Y_2, Z, vrs)^{cz} = \gamma_1\alpha - \gamma_2\beta \\
s.t. \quad & \left.\begin{aligned} & X_1\lambda \le \alpha X_1^{\circ} \\ & Z\lambda \ge \hat{Z}^{\circ} \\ & 1\lambda = 1 \\ & \lambda \ge 0 \end{aligned}\right\} Stage\,1, \\
& \left.\begin{aligned} & Z\mu \le \hat{Z}^{\circ} \\ & Y\mu \ge \beta Y_2^{\circ} \\ & 1\mu = 1 \\ & \mu \ge 0 \end{aligned}\right\} Stage\,2.
\end{aligned} \tag{8}
$$

where γ_1 and γ_2 are the predetermined weights reflecting the preference over the two stages' performance and $\hat{Z}^{\circ}$ which is unknown decision variables represents an intermediate measure for a specific DMU under assessment. According to model (8), if each stage is efficient, (that is $\alpha^* = \beta^* = 1$) then the two-stage process also is efficient. Note that model (8) not only measures the overall efficiency, but also obtains optimized values on the intermediate measures for a DMU under evaluation. They claimed that model (8) can determine the DEA frontier for the two-stage process so as to project the inefficient observations onto the efficient frontier. Chen et al. (2006a) applied a DEA model to assess the IT impact on firm performance by considering both stages of the scenario studied in Wang et al. (1997) and Chen and Zhu (2004). They decomposed some inputs in the first stage into the second stage. Chen et al. (2006a) developed a shared two-stage DEA with respect to X_3, Z and Y_2. Assume, therefore, that X_3 is split into two parts αX_3 and $(1-\alpha)X_3$. The average CRS ratios of stages 1 and 2 in the program (9) is used to measure the overall efficiency with common input and output weights for the two stages.

$$
\begin{aligned}
\max \quad & E^{\circ}(X_3, Y_2, Z, crs)^{clyz} = \frac{1}{2}\left[\frac{wZ^{\circ}}{v_3\alpha X_3^{\circ}} + \frac{uY_2^{\circ}}{wZ^{\circ} + v_3(1-\alpha)X_3^{\circ}}\right] \\
s.t. \quad & \frac{wZ}{v_3\alpha X_3} \le 1, \\
& \frac{uY_2}{wZ + v_3(1-\alpha)X_3} \le 1, \\
& u, v_1, v_3, w \ge 0.
\end{aligned} \tag{9}
$$

Model (9) is a non-linear fractional programming that can be transformed into (10).

$$\begin{aligned} \max \quad & E^{\circ}(X_3, Y_2, Z, crs)^{clyz} = \frac{1}{2}(w'Z^{\circ} + u'Y_2^{\circ}) \\ s.t. \quad & v_3'X_3 - w'Z \geq 0, \\ & v_3'X_3^{\circ} = 1, \\ & (v_3'' - v_3')X_3 + w'Z - ku'Y_2 \geq 0, \\ & v_3''X_3^{\circ} + w'Z^{\circ} - k = 1, \\ & v_3'' - v_3' \geq 0, \\ & v_3'', v_3', k, w', u' \geq 0. \end{aligned} \tag{10}$$

Due to the $ku'Y_2$ term, model (10) is a non-linear program. For a given k ($\geq w'Z^{\circ}$), however, the model can be treated as a linear parametric program. The efficiencies of the first and second stages can be then attained, respectively, via $w'^{*}Z^{\circ}$ and $u'^{*}Y^{\circ}$ where w'^{*} and u'^{*} are optimal measures obtained from (10). The overall efficiency is the average efficiency of the two-stage process $1/2(w'^{*}Z^{\circ} + u'^{*}Y^{\circ})$. Furthermore, $\alpha = v_3'^{*}/v_3''^{*}$ demonstrates how to allocate the resource (X_3) to two stages so as to maximize the average efficiency of whole process.

Remark 1. *If there is only one intermediate input, then the non-linear DEA model (9) becomes a linear program [Chen et al. 2006a].*

Remark 2. *The optimal α^* in model (9) is always equal to unity and the optimal β^* represents the overall efficiency for the entire process [Chen et al. 2009a].*

Saranga and Moser (2010) utilized the two-stage model developed by Chen and Zhu (2004) to evaluate purchasing and supply management (PSM) performance.

Contrary to previous studies (e.g. Seiford and Zhu (1999)), which treated the whole process and the two sub-processes as independent, Kao and Hwang (2008) considered a series of relationship between the whole process and the two sub-processes in measuring the efficiencies when a production process is composed of X_1, Z and Y_2 as depicted in Figure 2. The overall efficiency is decomposed into the product of the two individual efficiencies, namely

$$E^{\circ} = E_1^{\circ} \times E_2^{\circ} = \frac{wZ^{\circ}}{vX_1^{\circ}} \times \frac{uY_2^{\circ}}{wZ^{\circ}} = \frac{uY_2^{\circ}}{vX_1^{\circ}} \tag{11}$$

Consequently, the overall efficiency $E_{\circ}$ under the CRS assumption calculates as:

$$\begin{aligned}
\max \quad & E^{\circ}(X_1, Y_2, Z, crs)^{kh} = uY_2^{\circ} \\
s.t. \quad & vX_1^{\circ} = 1 \\
& uY_2 - vX_1 \leq 0, \\
& wZ - vX_1 \leq 0, \\
& uY_2 - wZ \leq 0, \\
& u, v, w \geq 0.
\end{aligned} \tag{12}$$

The constraint set of (12) is the envelope of those of models (4), (5) and (6). Note that the weight associated with Z in the constraints are assumed to be the common. It means that it does not matter whether the intermediate measures play the role of output or input. This assumption permits the conversion of their original non-linear program into a linear programming problem. This assumption also links the two stages. Note also that the constraint $uY_2 - vX_1 \leq 0$ is redundant in model (12) because of existing two constraints $wZ - vX_1 \leq 0$ and $uY_2 - wZ \leq 0$. If u^*, v^* and w^* be the optimal multipliers of (13), the overall efficiency, the efficiencies of stages 1 and 2 are calculated by $E^{\circ} = u^* Y_2^{\circ}$, $E_1^{\circ} = w^* Z^{\circ} \big/ v^* X_1^{\circ}$, $E_2^{\circ} = u^* Y_2^{\circ} \big/ w^* Z^{\circ}$, respectively. The optimal multipliers of (12) may not be unique; hence, the decomposition of $E^{\circ} = E_1^{\circ} \times E_2^{\circ}$ would not be unique. Kao and Hwang (2008) proposed the following model so as to find the set of multipliers which produces the largest E_1° while maintaining the overall efficiency score at E° calculated from (12):

$$\begin{aligned}
\max \quad & E_1^{\circ}(X_1, Y_2, Z, crs)^{kh} = wZ^{\circ}, \\
s.t. \quad & vX_1^{\circ} = 1, \\
& uY_2^{\circ} - E^{\circ}(vX_1^{\circ}) = 0, \\
& wZ - vX_1 \leq 0, \\
& uY_2 - wZ \leq 0, \\
& u, v, w \geq 0.
\end{aligned} \tag{13}$$

The relationship $E^{\circ} = E_1^{\circ} \times E_2^{\circ}$ enables us to obtain the efficiency of the second stage.

Chen et al. (2009a) investigated the relationship between the approaches of Chen and Zhu (2004) and Kao and Hwang (2008) for evaluation performance of two-stage processes. Note that Kao and Hwang (2008)'s model was developed under the CRS technology in the multiplier DEA model (see model (12)), while Chen and Zhu (2004)'s model was developed under the VRS technology in the envelopment DEA model (see model (8)).

Remark 3. *The CRS version of the Chen and Zhu (2004) model under* $\gamma_1 = \gamma_2 = 1$ *is equivalent to the Kao and Hwang's (2008) output-oriented model i.e.,* $F^\circ(X_1, Y_2, Z, crs)^{kh} = E^\circ(X_1, Y_2, Z, crs)^{cz}$ *[Chen et al. 2009a].*

According to Kao and Hwang (2008) approach, Chen et al. (2009b) used the additive efficiency decomposition approach to calculate the overall efficiency, expressed as a weighted sum of the efficiencies of the individual stages. In fact, Chen et al. (2009b) claimed that the two-stage DEA model of Kao and Hwang (2008) cannot be extended to VRS assumption because $E^\circ = ((wZ^\circ + u_1)/vX_1^\circ) \times ((uY_2^\circ + u_2)/wZ^\circ)$ could not be transformed into a linear program even if assuming the same weights on the intermediate measures for the two stages. However, Chen et al. (2009b) approach can be applied under both CRS and VRS assumptions while the method proposed by Kao and Hwang (2008) restricted to the CRS assumption. Chen et al. (2009b) used a weighted additive (arithmetic mean) approach to calculate the overall efficiency of the process under the VRS assumption by solving the following problem instead of combining the stages in a multiplicative (geometric) way proposed in Kao and Hwang (2008):

$$\begin{aligned}
\max \quad & E^\circ(X_1, Y_2, Z, vrs)^{cclz} = \lambda_1 \cdot \left(\frac{wZ^\circ + u_1}{vX_1^\circ} \right) + \lambda_2 \cdot \left(\frac{uY_2^\circ + u_2}{wZ^\circ} \right) \\
s.t. \quad & \frac{(wZ + u_1)}{vX_1} \le 1, \\
& \frac{(uY_2 + u_2)}{wZ} \le 1, \\
& u, v, w \ge 0.
\end{aligned} \tag{14}$$

where $\lambda_1 = \dfrac{vX_1^\circ}{vX_1^\circ + wZ^\circ}$ and $\lambda_2 = \dfrac{wZ^\circ}{vX_1^\circ + wZ^\circ}$ are the relative importance of the performance of stages 1 and 2, respectively, by means of the 'relative sizes' of two stages for measuring the overall performance of the process. By putting the above weights, λ_1 and λ_2, assigned to two stages in the objective function of (14) and using the Charnes–Cooper transformation, the linear model (15) can be obtained.

$$\begin{aligned}
\max \quad & E^\circ(X_1, Y_2, Z, vrs)^{cclz} = wZ^\circ + u_1 + uY_2^\circ + u_2 \\
s.t. \quad & vX_1^\circ + wZ^\circ = 1, \\
& wZ - vX_1 + u_1 \le 0, \\
& uY_2 - wZ + u_2 \le 0, \\
& u, v, w \ge 0.
\end{aligned} \tag{15}$$

Analogous to Kao and Hwang (2008), the weights (or multipliers) on the intermediate measures are the same for the two stages. Once an optimal solution to (15) is obtained, the efficiency scores for the two individual stages can be calculated in the same way as in Kao and Hwang (2008) (see model (13)). In other words, Chen et al. (2009b) used Kao and Hwang's (2008) approach to find a set of multipliers which produces the largest first (or second) stage efficiency score whilst maintaining the overall efficiency score computed from model (15). In case the first stage is to be given pre-emptive priority, the following model determines its efficiency, while maintaining the overall efficiency score at E° computed from model (15).

$$
\begin{aligned}
\max \quad & E_1^{\circ *}(X_1, Y_2, Z, vrs)^{cclz} = wZ^{\circ} + u_1 \\
s.t. \quad & vX_1^{\circ} = 1, \\
& wZ - vX_1 + u_1 \leq 0, \\
& uY_2 - wZ + u_2 \leq 0, \\
& (1 - E^{\circ})wZ^{\circ} + uY_2^{\circ} + u_1 + u_2 = E^{\circ}, \\
& u, v, w \geq 0.
\end{aligned}
\tag{16}
$$

The efficiency for the second stage is then attained as $E_2^{\circ} = (E^{\circ} - \lambda_1^*.E_{\circ}^{1*}) / \lambda_2^*$ where λ_1^* and λ_2^* are optimal weights which can be obtained using model (15). In the same way, Chen et al. (2009b) approach can be easily applied under the CRS assumption to evaluate the overall efficiency and the individual stages' efficiencies.

Wang and Chin (2010) demonstrated that a two-stage DEA model with a weighted harmonic mean of the efficiencies of two individual stages is equivalent to Chen et al. (2009b)'s model.

Remark 4. *The overall efficiency of Chen et al. (2009b) is greater than or equal to that of Kao and Hwang (2008) under the CRS assumption i.e.,* $E^{\circ}(X_1, Y_2, Z, crs)^{cclz} \geq E^{\circ}(X_1, Y_2, Z, crs)^{kh}$ *[Wang and Chin 2010].*

Furthermore, Wang and Chin (2010) extended Kao and Hwang (2008) to the VRS assumption and also generalized Chen et al. (2009b)'s model. Even though Kao and Hwang (2008) computed the efficiency of a series system, Kao (2009a) proposed a parallel DEA model with the individual components for measuring the efficiency of a DMU which is consisted of independent units connected in parallel. His model minimizes the inefficiency slacks of a DMU as well as inefficiency slacks of its units in order to determine the inefficient units. Kao (2009b) developed alternative relational network DEA model by defining dummy processes to transform a network system into a series system (a multi-stage system), where each stage is composed of a parallel structure with a set of processes. Kao (2010) then built a relational network CRS-DEA (series-parallel systems) in both envelopment and multiplier forms. Hsieh and Lin (2010) applied the relational network DEA introduced by Kao (2009b) to evaluate the performance of a set of hotels.

Wang and Chin (2010) used a weighted harmonic mean of the efficiencies of the two individual stages to obtain the overall efficiency of the process by solving the following problem instead of the approaches of Kao and Hwang (2008) and Chen et al. (2009b):

$$\begin{aligned} \max \quad & E^{\circ}(X_1, Y_2, Z, crs)^{wc} = \frac{1}{\lambda_1.\left(vX_1^{\circ} / wZ^{\circ}\right) + \lambda_2.\left(wZ^{\circ} / uY_2^{\circ}\right)} \\ s.t. \quad & \frac{wZ}{vX_1} \leq 1, \\ & \frac{uY_2}{wZ} \leq 1, \\ & u, v, w \geq 0. \end{aligned} \tag{17}$$

where $\lambda_1 = wZ^{\circ} \big/ \left(uY_2^{\circ} + wZ^{\circ}\right)$ and $\lambda_2 = uY_2^{\circ} \big/ \left(uY_2^{\circ} + wZ^{\circ}\right)$ are the relative importance of the performances of stages 1 and 2, respectively. These weights, similar to those of Chen et al. (2009b), are the relative sizes of the two stages. By substituting λ_1 and λ_2 into the objective function of (17), we achieve exactly the same model proposed by Chen et al. (2009b) in order to get the overall efficiency of two-stage process under the CRS assumption. Likewise, the overall efficiency of two-stage process under the VRS condition can be modeled as follows:

$$\begin{aligned} \max \quad & E^{\circ}(X_1, Y_2, Z, vrs)^{wc} = \frac{1}{\lambda_1.\left(vX_1^{\circ} / wZ^{\circ} + u_1\right) + \lambda_2.\left(wZ^{\circ} / uY_2^{\circ} + u_2\right)} \\ s.t. \quad & \frac{wZ + u_1}{vX_1} \leq 1, \\ & \frac{uY_2 + u_2}{wZ} \leq 1, \\ & u, v, w \geq 0. \end{aligned} \tag{18}$$

Similarly, λ_1 and λ_2 which are the weights assigned to stages 1 and 2 can be defined as $\left(wZ^{\circ} + u_1\right) \big/ \left(wZ^{\circ} + u_1 + uY_2^{\circ} + u_2\right)$ and $uY_2^{\circ} + u_2 \big/ \left(wZ^{\circ} + u_1 + uY_2^{\circ} + u_2\right)$, respectively. By setting these weights into the objective function of (18), the same model defined by Chen et al. (2009b) can be obtained for evaluation of the overall efficiency under the VRS assumption. Once E_1° or E_2° is obtained, using the proposed approach by Chen et al. (2009b), the another one can be determined by $E_2^{\circ} = \lambda_2^* \big/ \left((1/E^{\circ *}) - (\lambda_1^* / E_1^{\circ *})\right)$ or $E_1^{\circ} = \lambda_1^* \big/ \left((1/E^{\circ *}) - (\lambda_2^* / E_2^{\circ *})\right)$, where λ_1 and λ_2 are harmonic mean weights.

Remark 5. *$E^{\circ}(X_1, Y_2, Z, \gamma)^{cclz} = E^{\circ}(X_1, Y_2, Z, \gamma)^{wc}$ where γ =crs and vrs.*

Chen et al. (2010) suggested the DEA model under VRS for measuring the efficiency of two-stage system with the shared inputs. The structure is consisted of X_1, X_3, Z and Y_2 .(see Figure 2). The shared inputs, X_3, can be split into two parts αX_3 and $(1-\alpha)X_3$ where $L_1 \le \alpha \le L_2$.
Overall efficiency was defined in Chen et al. (2006a) as the average efficiency of stages 1 and 2 under CRS (see model (9)) An alternative definition in Chen et al. (2010) and Chen et al. (2009b), draws on a weighted average of the efficiencies of the two stages as follows:

$$\lambda_1 \cdot \left(\frac{wZ^{\circ} + u_1}{v_1 X_1^{\circ} + v_3 \alpha X_3^{\circ}} \right) + \lambda_2 \cdot \left(\frac{uY_2^{\circ} + u_2}{v_3 (1-\alpha) X_3^{\circ} + wZ^{\circ}} \right) \tag{19}$$

where $\lambda_1 = \dfrac{v_1 X_1^{\circ} + v_3 \alpha X_3^{\circ}}{v_1 X_1^{\circ} + v_3 X_3^{\circ} + wZ^{\circ}}$ and $\lambda_2 = \dfrac{v_3 (1-\alpha) X_3^{\circ} wZ^{\circ}}{v_1 X_1^{\circ} + v_3 X_3^{\circ} + wZ^{\circ}}$. The non-linear program (20) is created by substituting λ_1 and λ_2 in (*) and using the Charnes–Cooper transformation.

$$\begin{aligned}
\max \quad & E^{\circ}(X_1, X_3, Y_2, Z, vrs)^{cdsz} = wZ^{\circ} + u_1 + uY_2^{\circ} + u_2 \\
s.t. \quad & v_1 X_1^{\circ} + v_3 X_3^{\circ} + wZ^{\circ} = 1, \\
& wZ - (v_1 X_1 + v_3 \alpha X_3) + u_1 \le 0, \\
& uY_2 - (v_3 (1-\alpha) X_3 + wZ) + u_2 \le 0, \\
& L_1 \le \alpha \le L_2, \quad u, v, w \ge 0.
\end{aligned} \tag{20}$$

Model (20) can be transformed to a linear form using the alternation variable $v_3 \alpha = \gamma$. Once we attain an optimal solution of the linear model, the efficiencies of the two stages can be consequently computed. Besides, Chen et al. (2010) applied Kao and Hwang's (2008) approach to deal with the problem of multiple optimal solutions.

Remark 6. *The overall efficiency proposed by Chen et al. (2009b) and Chen et al. (2010) are equal if we take the shared inputs defined in (Chen et al., 2010) away from performance evaluation, i.e.,* $E^{\circ}(X_1, Y_2, Z, vrs)^{cclz} = E^{\circ}(X_1, X_3, Y_2, Z, vrs)^{cdsz}$ *iff* $X_3 = 0$.

Wang and Chin (2010) additionally extended the Kao and Hwang (2008)'s model to the VRS assumption. We should note that the first stage is evaluated with the input-oriented VRS model and the second stage with the output-oriented VRS model. Kao and Hwang (2008)'s model under the VRS assumption can therefore be expressed as

$$
\begin{aligned}
\max \quad & E^{\circ}(X_1, Y_2, Z, vrs)^{kw} = uY_2^{\circ} + u_2 \\
s.t. \quad & vX_1^{\circ} - u_1 = 1, \\
& wZ - vX_1 + u_1 \le 0, \\
& uY_1 - wZ + u_2 \le 0, \\
& u, v, w \ge 0.
\end{aligned}
\tag{21}
$$

The optimal multipliers of (21) may not be unique; hence, the decomposition of E° = $E_1^{\circ} \times E_2^{\circ}$ would not be unique. Hence, similar to Kao and Hwang (2008) the set of multipliers which produce the largest E_1° (E_2°) can be obtained while preserving the overall efficiency score at E° calculated from (21).

Remark 7. *The overall efficiency under the assumption of VRS is decomposed into the product of the two individual efficiencies i.e.,*

$$E^{\circ}(X_1, Y_2, Z, vrs)^{kw} = E_1^{\circ}(X_1, Y_2, Z, vrs)^{kw} \times F_2^{\circ}(X_1, Y_2, Z, vrs)^{kw} =$$

$$\frac{wZ^{\circ}}{vX_1^{\circ} - u_1} \times \frac{uY_2^{\circ} + u_2}{wZ^{\circ}} = \frac{uY_2^{\circ} + u_2}{vX_1^{\circ} - u_1}$$

Furthermore, Wang and Chin (2010) generalized Chen et al. (2009b)'s models to taking into consideration the relative importance weights of two individual stages. To do, a two-stage process is transformed to a single process in which the two stages are treated equally. In other words, the single process considers stage 1's input (X_1) and an intermediate measure (Z) as inputs, and stage 2's output (Y_2) and an intermediate measure (Z) as outputs. The generalized overall efficiency of Chen et al. (2009b)'s model under the VRS assumption is formulated in model (20).

$$
\begin{aligned}
\max \quad & E^{\circ}(X_1, Y_2, Z, vrs)^{wc(g)} = \frac{\lambda_1(wZ^{\circ} + u_1) + \lambda_2(uY_2^{\circ} + u_2)}{\lambda_1 vX_1^{\circ} + \lambda_2 wZ^{\circ}} \\
s.t. \quad & \frac{wZ + u_1}{vX_1} \le 1, \\
& \frac{uY_2 + u_2}{wZ} \le 1, \\
& u, v, w \ge 0.
\end{aligned}
\tag{22}
$$

The objective function of (22) can be transformed into a weighted harmonic mean as

$$E^{\circ} = \frac{1}{\gamma_1 \cdot \dfrac{vX_1^{\circ}}{wZ^{\circ} + u_1} + \gamma_2 \cdot \dfrac{wZ^{\circ}}{uY_2^{\circ} + u_2}}$$

where

$$\gamma_1 = \frac{\lambda_1(wZ^\circ + u_1)}{\lambda_1(wZ^\circ + u_1) + \lambda_2(uY_2^\circ + u_2)} \text{ and } \gamma_1 = \frac{\lambda_2(uY_2^\circ + u_2)}{\lambda_1(wZ^\circ + u_1) + \lambda_2(uY_2^\circ + u_2)}.$$

Remark 8. *Model (14) by Chen et al. (2009b) is a special case of (22) with* $\lambda_1 = \lambda_2 = 1/2$ *i.e.,* $E^\circ(X_1, Y_2, Z, vrs)^{cclz} = E^\circ(X_1, Y_2, Z, vrs)^{wc(g)}$. *(Proof in Wang and Chin (2010))*

Remark 9. *If* $u_1 = u_2 = 0$ *in model (22), the generalized overall efficiency* $E^\circ(X_1, Y_2, Z, crs)^{wc(g)}$ *under the CRS assumption can be derived.*

Chen et al. (2010) proposed an approach to specify the frontier points for inefficient DMUs based upon the Kao and Hwang (2008)'s model. The dual of model (12) proposed by Kao and Hwang (2008) can be expressed as

$$\begin{aligned} \min \quad & DE^\circ(X_1, Y_2, Z, crs)^{kw} = \theta \\ s.t. \quad & X_1\lambda \le \theta X_1^\circ, \\ & Y\mu \ge Y_2^\circ, \\ & Z(\lambda - \mu) \ge 0, \\ & \lambda, \mu \ge 0, \quad \theta \le 1. \end{aligned} \tag{23}$$

Model (12) can just obtain an overall efficiency score under the assumption of CRS, but would not be able to identify how to project inefficient DMUs on to the DEA frontier. Chen et al. (2010), therefore, put forward the following model that is equivalent to the model (23):

$$\begin{aligned} \min \quad & DE^\circ(X_1, Y_2, Z, crs)^{kw} = \theta \\ s.t. \quad & X_1\lambda \le \theta X_1^\circ, \\ & Y_2\mu \ge Y_2^\circ, \\ & Z\lambda \ge \widehat{Z}^\circ, \\ & Z\mu \le \widehat{Z}^\circ, \\ & \lambda, \mu, \widehat{Z}^\circ \ge 0, \quad \theta \le 1. \end{aligned} \tag{24}$$

where the decision variable $\widehat{Z}^\circ$ in the constraints $Z\lambda \ge \widehat{Z}^\circ$ and $Z\mu \le \widehat{Z}^\circ$ treats as output and input, respectively, for the intermediate measure. According to model (24), the projection point for DMU$^\circ$ is given by $(\theta^* X_1^{\circ *}, \widehat{Z}^{\circ *}, Y^\circ)$ which is efficient under models (24) and (23).

5.2. *DEA using game theory*

Game theory allows us to explicitly model the sequence of bargaining and the strategic interaction present in decentralized decision making, such as supply chain management. Game theory has been successfully applied both to supply chain management coordination in general, and to normative applications of frontier models. Liang et al. (2006) proposed two DEA-based models for evaluation the efficiency of a supply chain and its members (stages 1 and 2) using the concept of non-cooperative and cooperative games in game theory. The models are, therefore, described in a seller-buyer supply chain context, when the relationship between the seller and buyer is treated first as one of leader-follower, and second as one that is cooperative. In the non-cooperative (leader-follower) approach, the leader is first assessed, and then the follower is evaluated using the leader's efficiency. In the cooperative structure, the overall efficiency which is modeled as an average of the two stages' efficiencies is maximized, and both supply chain members are evaluated simultaneously. The resulting cooperative game model is a non-linear DEA model which can be solved as a parametric linear programming problem. Figure 2 without Y_1 shows a buyer-seller supply chain examined by Liang et al. (2006). They assumed that the first and second stages are the seller (leader) and the buyer (follower), respectively, the efficiency of the first stage (E_1°) is obtained using the standard input oriented CRS model. If the optimal value $E_{\circ}^{1*}$ holds when assessing the efficiency of stage 1, the efficiency of stage 2 is calculated. It means dominating stage 2 by stage 1. The second stage's efficiency, therefore, can be obtained as

$$
\begin{aligned}
\max \quad & E_2^{\circ} = \frac{u_2 Y_2^{\circ}}{v_2 X_2^{\circ} + D \times w Z^{\circ}} \\
s.t. \quad & \frac{u Y_2}{v_2 X_2 + D \times w Z} \le 0, \\
& wZ - v_1 X_1 \le 0, \\
& v_1 X_1^{\circ} = 1, \\
& wZ^{\circ} = E_1^{\circ *}, \\
& u_2, v_1, v_2, w, D \ge 0.
\end{aligned}
\tag{25}
$$

Model (25) can be converted into the following non-linear program:

$$
\begin{aligned}
\max \quad & E_2^{\circ} = u_2 Y_2^{\circ} \\
s.t. \quad & v_2 X_2^{\circ} + DwZ^{\circ} = 1 \\
& u_2 Y_2 - v_2 X_2 - DwZ \le 0, \\
& wZ - v_1 X_1 \le 0, \\
& v_1 X_1^{\circ} = 1, \\
& wZ^{\circ} = E_1^{\circ *}, \\
& u_2, v_1, v_2, w, D \ge 0.
\end{aligned}
\tag{26}
$$

where $0 \le D < 1 \big/ E_1^{\circ *}$ and therefore D can be treated as a parameter. That is, model (26) can be considered as a parametric linear program. Once the first and second stage's efficiency are obtained by the conventional CRS and (26), respectively, the overall efficiency was then calculated via $E^{\circ} = 1/2(E_1^{\circ *} + E_2^{\circ *})$.

Likewise, we can apply the above procedure for the situation in which stage 1 (follower) is entirely dominated by stage 2 (leader). Liang et al. (2006) also deemed the situation where two stages have the same degree of power to influence the supply chain system. The following cooperative game model, hence, seeks to maximize the average of the first and second's efficiency when the weights on the intermediate measures must be equal.

$$\begin{aligned} \max \quad & E^{\circ} = \frac{1}{2}\left[\frac{wZ^{\circ}}{v_1 X_1^{\circ}} + \frac{u_2 Y_2^{\circ}}{v_2 X_2^{\circ} + wZ^{\circ}}\right] \\ s.t. \quad & wZ - v_1 X_1 \leq 0, \\ & u_2 Y_2 - v_2 X_2 - wZ \leq 0, \\ & u_2, v_1, v_2, w \geq 0. \end{aligned} \tag{27}$$

Model (27) can be transformed into the following non-linear program:

$$\begin{aligned} \max \quad & E^{\circ} = 1/2\,(wZ^{\circ} + u_2 Y_2^{\circ}) \\ s.t. \quad & v_1 X_1^{\circ} = 1, \qquad v_2 X_2^{\circ} + k \times wZ^{\circ} = 1, \\ & wZ - v_1 X_1 \leq 0, \\ & u_2 Y_2 - v_2 X_2 - k \times wZ \leq 0, \\ & u_2, v_1, v_2, w, k \geq 0. \end{aligned} \tag{28}$$

Note that in model (24) $0 \leq k < 1/w^* Z^{\circ}$ where w^* is the optimal value to model (28). That is to say, model (28) can be treated as a parametric linear program. The efficiency of stages 1 and 2 are then equivalent to $w^* Z^{\circ}$ and $u_2^* Y_2^{\circ}$, respectively, at the optima. The remarkable conclusion in Liang et al. (2006) shows that the supply chain efficiency under the assumption of cooperation generally will not be less than the efficiency under the assumption of non-cooperation. The model in Liang et al. (2006) was extended for the multi-stage process in Cook et al. (2010).

Chen et al. (2006b) showed that there exist numerous Nash equilibria in two-stage (supplier-manufacturer) game. They used a bargaining-DEA-Game model under CRS technology to analyze the relationships among the two stages as well as defining two efficiency functions for the first and second stages.

Figure 2 without Y_1 and X_3 presents the supplier-manufacturer (two-stage) proceeded by Liang et al. (2006). The efficiency of each stage can be defined as

$$E_1^j = \frac{wZ^j}{v_1 X_1^j}$$
$$E_2^j = \frac{uY_2^j}{wZ^j + v_2 X_2^j} \tag{29}$$

Based on the decentralized control system, the non-linear programs 1 and 2 can be used for the supplier (stage 1) and the manufacturer (stage 2), respectively.

$$S(E_2) = \max\left\{E_1^\circ \mid E_1 \le 1, E_2 \le 1, E_2 \le E_2^\circ\right\}$$
$$M(E_1) = \max\left\{E_2^\circ \mid E_1 \le 1, E_2 \le 1, E_1 \le E_1^\circ\right\} \tag{30}$$

Chen et al. (2006b) determined the obvious Nash equilibriums in the existing game between the supplier and the manufacturer. Notice that $S(E_2)$ and $M(E_1)$ are functions of E_1 and E_1, respectively. If $E_2 = M(S(E_2))$, (E_1, E_1) is a Nash equilibrium, otherwise, Nash equilibriums does not exist. Likewise, Nash equilibriums exist if $E_1 = S(M(E_1))$. In addition, Chen et al. (2006b) mentioned some properties on the two efficiency functions as well as extending their method to the centralized control system.

Cook et al. (2010) extended Liang et al. (2006) to take in account multi-stage structures i.e., more than two stages in the CRS and VRS technologies. They calculated the overall efficiency as an additive weighted average of the efficiencies of the individual stages. In addition, the developed model in (Cook et al., 2010) was a LP while Liang et al. (2006) model used a heuristic search algorithm after converting the non-linear model into a parametric linear model.

Using the geometric mean of the efficiencies of the two stages, Zha et al. (2008) proposed a two-stage cooperative efficiency to calculate the overall efficiency under DEA-VRS model. They suggest that the efficiency of the first stage is evaluated with the input-oriented VRS model and the second stage with the output-oriented VRS model. Then, the overall efficiency is evaluated in a cooperative framework. The upper and the lower bounds are reached when non-cooperative framework is considered. The non-linear model is transformed into a parametric one, where optimal solution of the overall efficiency is reached easily. If input-oriented VRS model is suggested for performance evaluation, inconsistency of the intermediate outputs exists between the two stages. Specifically, the two stages are cooperative for the reason that they are in series in an organization. They considered the non-cooperative setting in order to determine the upper and lower bounds of the efficiencies of the sub-DMUs in different stages. Two conditions are examined as follows.

A) Sub-DMU in stage 1 dominates the system, while the sub-DMU in stage 2 follows.

B) Sub-DMU in stage 2 dominates the system, while the sub-DMU stage 1 follows.

In both conditions, intermediate outputs need to be consistent in two stages. So an input-oriented VRS model is suggested when evaluating the efficiency of the sub-DMU in stage 1, and an output-oriented VRS model is suggested when evaluating the efficiency of the sub-DMU in stage 2.

The upper bound of the efficiency of stage 1 is expressed as follows:

$$\begin{aligned} \min \quad & E_1^{U\circ} = vX_1^\circ + u_1 \\ s.t. \quad & wZ^\circ = 1, \\ & vX_1 + u_1 - wZ \geq 0, \\ & v, w \geq 0. \end{aligned} \tag{31}$$

The lower bound of the efficiency of stage 2 can be calculated by the following model if the optimal value $E_1^{U\circ}$ obtained from (31) holds when evaluating the efficiency of stage 2:

$$\begin{aligned} \max \quad & E_2^{L\circ} = uY^\circ - u_2 \\ s.t. \quad & wZ^\circ = 1, \qquad vX_1^\circ + u_1 = E_1^L, \\ & vX_1 + u_1 - wZ \geq 0, \\ & uY_2 - wZ - u_2 \leq 0, \\ & u, v, w \geq 0. \end{aligned} \tag{32}$$

Note that stage 2 is entirely dominated by stage 1. Likewise, the upper bound of the efficiency of stage 2, denoted by $E_2^{U\circ}$, is first acquired, then, with holding $E_2^{U\circ}$, the lower bound of the efficiency of stage 1, denoted by $E_1^{L\circ}$, is calculated. Zha et al. (2008) considered the overall efficiency as the geometric mean of the efficiencies the two-stages. Hence, they assume that the efficiency of stage 1 and stage 2 are evaluated using the input-oriented and the output-oriented models, respectively. Geometric average cooperative efficiency of the two stages is obtained by the following model

$$\begin{aligned} \max \quad & E^\circ = \frac{wZ^\circ}{vX_1^\circ + u_1} \times \frac{uY_2^\circ - u_2}{wZ^\circ} \\ s.t. \quad & \frac{wZ}{vX_1 + u_1} \leq 1, \qquad \frac{uY_2^\circ - u_2}{wZ} \leq 1, \\ & \frac{1}{E_1^{L\circ}} \leq \frac{wZ^\circ}{vX_1^\circ + u_1} \leq \frac{1}{E_1^{U\circ}} \\ & u, v, w \geq 0. \end{aligned} \tag{33}$$

(33) can be transformed into

$$
\begin{aligned}
\max \quad & E^{\circ} = uY_2^{\circ} - u_2 \\
s.t. \quad & vX_1^{\circ} + u_1 = 1, \\
& wZ - vX_1 - u_1 \le 0, \qquad uY_2 - wZ - u_2 \le 0, \\
& 1/E_1^{L\circ} \le wZ^{\circ} \le 1/E_1^{U\circ}, \\
& u, v, w \ge 0,
\end{aligned} \tag{34}
$$

Liang et al. (2008) developed a two-stage model using non-cooperative and cooperative concepts in game theory. In non-cooperative approach, they assume that one of the stages is the leader that seeks to maximize its DEA efficiency. Then the efficiency of the other stage (the follower) is calculated subject to the leader-stage maintaining its DEA efficiency. In other words, the leader stage can be viewed as being more important than the other stage(s) in improving its efficiency. In cooperative approach, they assumed that initially both stages' efficiency scores are maximized simultaneously, while determining a set of optimal (common) weights assigned to the intermediate measures.

Consider Figure 2 without X_2 and Y_1. It is assumed that the first and second stages are the leader and the follower, respectively, the efficiency of the first stage (E_1°) is obtained using the standard input oriented CRS model. If the optimal value $E_1^{\circ *}$ holds when assessing the efficiency of stage 1, the efficiency of stage 2 is calculated. It means that stage 2 is entirely dominated by stage 1. The second stage's efficiency, therefore, can be obtained as

$$
\begin{aligned}
\max \quad & E_2^{\circ} = \frac{1}{E_1^{\circ *}} uY_2^{\circ} \\
s.t. \quad & uY_2 - wZ \le 0, \\
& wZ - vX_1 \le 0, \\
& vX_1^{\circ} = 1, \qquad wZ^{\circ} = E_1^{\circ *}, \\
& u, v, w \ge 0.
\end{aligned} \tag{35}
$$

Likewise, the second stage can be the leader and then one obtains the first stage (follower) model with regard to holding the efficiency of stage 2, $E_{\circ}^{2*}$. Finally, the overall efficiency can be calculated as $E^{\circ *} = E_1^{\circ *} . E_2^{\circ *} = E_1^{\circ *} .(1/E_1^{\circ *}) u^* Y_2^{\circ} = u^* Y_2^{\circ}$. Or, in other words, $E^{\circ *} = E_1^{\circ *} . E_2^{\circ *} = u^* Y_2^{\circ} / v^* X_1^{\circ}$ and, as a result, $E^{\circ *}$ is equal to $u^* Y_2^{\circ}$ because of $v^* X_1^{\circ} = 1$.

An alternative method proposed by Liang et al. (2008) to measuring the efficiency of the two stage process is to view them from a cooperative perspective. The cooperative approach is characterized by letting the same weights for intermediate data in two stage models. Note that because of the same weights for intermediate data, the overall efficiency ($E_1^{\circ} . E_2^{\circ}$) becomes $uY_2^{\circ} / vX_1^{\circ}$ which it can be modeled as follows:

$$\begin{aligned} &\max \quad E^{\circ} = E_1^{\circ}.\, E_2^{\circ} = uY_2^{\circ} \big/ vX_1^{\circ} \\ &s.t. \quad E_1^{j} \le 1 \text{ and } E_2^{j} \le 1, \qquad \forall j. \end{aligned} \tag{36}$$

The linear program of (36) is:

$$\begin{aligned} &\max \quad E_{\circ} = uY_2^{\circ} \\ &s.t. \quad uY_2 - wZ \le 0, \\ &\qquad wZ - vX_1 \le 0, \\ &\qquad vX_1^{\circ} = 1, \\ &\qquad u, v, w \ge 0. \end{aligned} \tag{37}$$

The efficiencies of the first and second stages can be then calculated as $E_1^{\circ *} = w^* Z^{\circ} \big/ v^* X_1^{\circ} = w^* Z^{\circ}$, and $E_2^{\circ *} = u^* Y_2^{\circ} \big/ w^* Z^{\circ}$

Note that optimal multipliers from model (37) may not be unique, as a result, E_1° and E_2° may not be unique. To discover for uniqueness, the maximum achievable value of E_1° is firstly calculated using the following model:

$$\begin{aligned} &\max \quad E_{1+}^{\circ} = wZ^{\circ} \\ &s.t. \quad uY_1^{\circ} = E_{\circ} \\ &\qquad uY_2 - wZ \le 0, \\ &\qquad wZ - vX_1 \le 0, \\ &\qquad vX_1^{\circ} = 1, \\ &\qquad u, v, w \ge 0. \end{aligned} \tag{38}$$

The minimum of E_2° is then calculated by $E_{2-}^{\circ} = E^{\circ} \big/ E_{1+}^{\circ}$. In a similar way, the maximum of E_2° and the minimum of E_1°, denoted by E_{2+}° and E_{1-}°, respectively, can be obtained. Note that $E_{1-}^{\circ} = E_{1+}^{\circ}$ if and only if $E_{2-}^{\circ} = E_{2+}^{\circ}$. If $E_{1-}^{\circ} = E_{1+}^{\circ}$ or $E_{2-}^{\circ} = E_{2+}^{\circ}$, E_1° and E_2° are uniquely determined using model (37), otherwise, E_1° and E_2° lead to multiple optimal solutions. In the case of $E_{1-}^{\circ} \ne E_{1+}^{\circ}$ or $E_{2-}^{\circ} \ne E_{2+}^{\circ}$ Liang et al. (2008) proposed a procedure to achieve a fair and alternative distribution of E_1° and E_2° between the two stages.

Yang et al. (2009) proposed a CRS DEA approach to measure the overall efficiency of the entire supply chain using a predefined PPS. By comparing the obtained supply chain frontier with other supply chains, chain-level performance can be identified as efficient or inefficient. The efficiency perspective and corresponding improvement strategies for inefficient supply chains can be given at the same time. Figure 2 without Y_1 and X_2 shows the two-stage structure (supplier-manufacturer) developed in Yang et al.

(2009). It is assume that all supply chains are separable and their members can be aggregated with other supply chain members so as to make a virtual supply chain. The PPS can be characterized by all existing supply chains and some virtual supply chains. Thus, the sub-perfect supply chain CRS PPS is defined as follows:

$$T = \{(X_1^{\circ}, Y_2^{\circ}) \big| \lambda X_1 E_1^{\circ *} \le X_1^{\circ},\ \lambda Z \ge Z^{\circ},\ \lambda Z \le Z^{\circ},\ \lambda(Y_2 / E_2^{\circ *}) \ge Y_2^{\circ},\ \lambda \ge 0\}$$

where $E_1^{\circ *}$ and $E_2^{\circ *}$, which can be obtained from models (6), are the CRS efficiencies of stages 1 and 2, respectively. Note that in the proposed PPS the corresponding envelopment coefficients for each DMU are the same λ i.e. the members of each virtual supply chain are restricted in the same actual supply chain. Note also that $(XE_1^{\circ *}, Z)$ and $(Z, Y / E_2^{\circ *})$ are projections of stage 1 and stage 2 for DMU°. Based upon PPS, the overall efficiency of a supply chain is modeled as follows:

$$\begin{aligned} \min \quad & E^{\circ} = \theta \\ s.t. \quad & X_1^{*} \lambda \le \theta X_1^{\circ}, \\ & Y_2^{*} \lambda \le Y_2^{\circ}, \\ & (X_1^{*}, Y_2^{*}) \in T, \\ & \lambda \ge 0. \end{aligned} \tag{39}$$

where $(X_1^{*}, Y_2^{*}, Z^{*})$ points located at the frontier enveloped by the sub-perfect supply chain CRS PPS (see *T*). The following model is equivalent to (39):

$$\begin{aligned} \min \quad & E^{\circ} \\ s.t. \quad & X_1^{*} \lambda \le E^{\circ} X_1^{\circ}, \qquad Y_2^{*} \lambda \le Y_2^{\circ}, \\ & \bar{\lambda} X_1 E_1^{\circ *} \le X_1^{\circ *}, \\ & \bar{\lambda} Z \ge Z^{\circ *}, \qquad \bar{\lambda} Z \le Z^{\circ *}, \\ & \bar{\lambda}(Y_2^{*} / E_2^{\circ *}) \ge Y_2^{\circ *}, \\ & \lambda, \bar{\lambda} \ge 0. \end{aligned} \tag{40}$$

Yang et al. (2009) also proved that $E^{\circ *}$ computed from model (40) is always smaller than or equal to $E^{\circ *}$ obtained from model (1) under CRS. Also, they demonstrated that the optimal value of (40) is always smaller than or equal to $E_1^{\circ *} \times E_2^{\circ *}$. Their proposed approach can be applied to evaluate the efficiency of multiple-member supply chains.

Zha and Liang (2010) developed an approach to measure the performance of a two-stage process in a non-cooperative and cooperative manner within the framework of game theory, where the shared inputs can be allocated among different stages. Similar to Chen et al. (2006a), Zha and Liang (2010) used Figure 2 as a two-stage process with shared inputs, e.g. all inputs (denoted as X_3) are directly associated with two stages. To do this, it is assumed that X_3 is divided into two parts αX_3 and $(1-\alpha)X_3$. Zha and

Liang (2010) utilized the product of two stages to evaluate the overall efficiency of each DMU while the average of two stages was only used in the Chen el al. (2006)'s cooperative model. Let us assume that the first and second stages are the leader and the follower, respectively, for the non-cooperative evaluation. First the efficiency of the first stage (E_1°) can be calculated using the input oriented CRS model as follows:

$$\begin{aligned} \max \quad & E_1^\circ = wZ^\circ \\ s.t. \quad & wZ - v_3 \alpha X_3 \le 0, \\ & v_3 \alpha X_3^\circ = 1, \\ & v_3, w \ge 0. \end{aligned} \tag{41}$$

The second stage's efficiency can be then obtained from the following program subject to the restriction that the efficiency of the first stage remains at optimal value $E_1^{\circ *}$:

$$\begin{aligned} \max \quad & E_2^\circ = uY_2^\circ \\ s.t. \quad & uY_2 - (\delta wZ + v_3^2 X_3) \le 0, \\ & wZ - v_3^1 X_3 \le 0, \\ & v_3^1 X_3^\circ = 1, \\ & wZ^\circ = E_1^{\circ *}, \\ & v_3^2 X_3^\circ + \delta E_1^{\circ *} = 1, \\ & u, v_3^1, v_3^2, w \ge 0. \end{aligned} \tag{42}$$

Model (42) is a non-linear program because of δw in the first constraint. However, this model can be treated as a parametric linear program since in specifying the optimum $E_2^{\circ *}$, $\delta \in [0,1]$ (with regard to the interval $0 < \delta \le 1 / E_1^{\circ *}$) is considered as a parameter. On the other hand, the second stage can be the leader and then one obtains the efficiency of the first stage (follower) model based on stage 2. Therefore, the efficiency of stage 2 can be first calculated as in the following model:

$$\begin{aligned} \max \quad & E_2^\circ = uY_2^\circ \\ s.t. \quad & uY_2 - (wZ + v_3 X_3) \le 0, \\ & wZ^\circ + v_3 X_3^\circ = 1, \\ & u, v_3, w \ge 0. \end{aligned} \tag{43}$$

Note that model (43) corresponds to the conventional CRS DEA model. Assume that u^*, v_3^* and w^* are optimal solution for (43). We must investigate three cases to obtain the efficiency of stage 1. In the first case, if there exists a given d (d=1,…,p) satisfying $w_d^* \ne 0$, the efficiency of stage 1 is equivalent to

$$\begin{aligned} \max \quad & E_1^{\circ} = \delta w^* Z^{\circ} \\ s.t. \quad & \delta w^* Z - v_3 X_3^{\circ} \le 0, \\ & v_3 X_3^{\circ} = 1, \\ & v_3, \delta \ge 0. \end{aligned} \tag{44}$$

In the second case, $w^* = 0$, accordingly, the efficiency of stage 1 becomes zero and, lastly, if there exists multiple optimum values in model (41), the efficiency of stage 1 dominated by stage 2 can be expressed as

$$\begin{aligned} \max \quad & E_1^{\circ} = \delta w Z^{\circ} \\ s.t. \quad & \delta w Z - v_3^1 X_3 \le 0, \\ & v_3^1 X_3^{\circ} = 1, \qquad u Y_2^{\circ} - (v_3^2 X_3^{\circ} + wZ) \le 0, \\ & u Y_2^{\circ} = E_2^{\circ *}, \qquad v_3^2 X_3^{\circ} + w Z^{\circ} = 1, \\ & u, v_3^1, v_3^2, w \ge 0. \end{aligned} \tag{45}$$

Note that the efficiency of stage 1 obtained from (45) is less than (44). In a special situation, when intermediate product is single the optimal values of the objective function (42) and (43) are equal.

In the cooperative efficiency, while the weights of the intermediate outputs in stage 1 are equal to the weights of the corresponding intermediate inputs in stage 2, the product of stages 1 and 2 for measuring the overall efficiency can be expressed as

$$\begin{aligned} \max \quad & E^{\circ} = \left[\frac{w Z^{\circ}}{v_3 \alpha X_3^{\circ}} \times \frac{u Y_2^{\circ}}{w Z^{\circ} + v_3 (1-\alpha) X_3^{\circ}} \right] \\ s.t. \quad & \frac{wZ}{v_3 \alpha X_3} \le 1, \\ & \frac{u Y_2^{\circ}}{wZ + v_3 (1-\alpha) X_3} \le 1, \\ & u, v_1, w \ge 0 \end{aligned} \tag{46}$$

Model (46) is a non-linear programming and we can rewrite it as:

$$\begin{aligned}
\max \quad & E^{\circ} = wZ^{\circ} \times uY_2^{\circ} \\
s.t. \quad & wZ^{\circ} - v_3 X_3^{\circ} \leq 0, \\
& v_3 X_3^{\circ} = 1 \\
& \delta\left[(h - v_3) X_3^{\circ} + wZ^{\circ}\right] = 1, \\
& uY_2 - \delta\left[(h - v_3) X_3^{\circ} + wZ^{\circ}\right] \leq 0, \\
& h \geq u \geq 0 \\
& v_3, w \geq 0
\end{aligned} \tag{47}$$

Based on the non-cooperative approach, let $L \leq k \leq U$ where L is the efficiency of stage 1 when stage 2 treats as a leader and U the efficiency of stage 1 when stage 1 consider as a leader. Also, assume that $\delta(h - v_3) = v$, $wZ^{\circ} = k$. Accordingly, (47) is transformed into

$$\begin{aligned}
\max \quad & E^{\circ} = k \times uY_2^{\circ} \\
s.t. \quad & wZ^{\circ} = k \\
& L \leq k \leq U \\
& wZ^{\circ} - v_3 X_3^{\circ} \leq 0, \\
& v_3 X_3^{\circ} = 1 \\
& vX_3^{\circ} + \delta wZ^{\circ} = 1, \\
& uY_2 - (vX_3^{\circ} + \delta wZ^{\circ}) \leq 0, \\
& u, v, v_3, w \geq 0
\end{aligned} \tag{48}$$

Model (48) can be considered as a parametric linear programming since we can gain $\delta \leq 1/k$ from a given $k \in [L, U]$ and the constraint $vX_3^{\circ} + \delta wZ^{\circ} = 1$.

Du et al. (2011) created a Nash bargaining game model (cooperative game model) under a two-stage structure with X_1, Z and Y_2 (see Figure 2) to measure the efficiency of DMUs and sub-DMUs. The input-oriented DEA bargaining model of Du et al. (2011) was constructed as

$$\begin{aligned}
\max \quad & E^{\circ} = \left[\frac{wZ^{\circ}}{vX_1^{\circ}} - E_1^{-}\right]\left[\frac{uY_2^{\circ}}{wZ^{\circ}} - E_2^{-}\right] \times \\
s.t. \quad & \frac{wZ^{\circ}}{vX_1^{\circ}} \geq E_1^{-}, \qquad \frac{uY_2^{\circ}}{wZ^{\circ}} \geq E_2^{-}, \\
& \frac{wZ}{vX_1} \leq 1, \qquad \frac{uY_2}{wZ} \leq 1, \\
& u, v_1, w \geq 0.
\end{aligned} \tag{49}$$

In model (49), E_1^- and E_2^- are the CRS efficiency scores of the two least ideal DMUs as a breakdown point where the least ideal DMUs for the for the first and second stages are defined as ($(X_1^{\max} = \max\{X_1\}, Z^{\min} = \min\{Z\})$ and $(Z^{\max} = \max\{Z\}, Y_1^{\min} = \min\{Y_1\})$, respectively. The equivalent non-linear model is

$$\begin{aligned}
\max \quad & E_\alpha^\circ = \alpha u Y_2^\circ - E_1^- u Y_2^\circ - E_2^- w Z^\circ + E_1^- . E_2^- \\
s.t. \quad & wZ^\circ \geq E_1^-, \qquad uY_2^\circ \geq E_2^-, \\
& vX_1^\circ = 1, \qquad wZ^\circ = \alpha, \\
& wZ - vX_1 \leq 0, \qquad \alpha u Y_2 - wZ \leq 0, \\
& \alpha, u, v_1, w > 0.
\end{aligned} \tag{50}$$

where α can be behaved as a parameter within [E_1^- , 1], therefore, model (50) can be a parametric linear model. After setting an initial value for α and obtaining corresponding objective function of model (50), E_α , we decrease α by a very small positive number until $\alpha = E_1^-$. Accordingly, the optimal solutions are associated with a given α^* when $E_{\alpha^*}^* = \max\{E_\alpha\}$. Thus, the efficiency scores of the first and second stages and the overall process are $E_1^{\circ *} = \alpha^* = w^* Z^\circ$, $E_2^{\circ *} = u^* Y_2^\circ$ and $E^{\circ *} = E_1^{\circ *} . E_2^{\circ *}$, respectively. It is interesting to note that when only one intermediate measure exists between the two stages $E_1^{\circ *}$ and $E_2^{\circ *}$ are equal to the efficiency scores of the two stages calculated from the standard DEA approach separately (see models (5) and (6)).

Remark 10. *The Nash bargaining game model proposed by Du et al. (2011) is equivalent to the cooperative model of Liang et al. (2008) when* $E_1^- = E_2^- = 0$.

5.3. *DEA using bilevel programming*

Wu (2010) was first to explore a bi-level programming DEA approach by combining DEA cost efficiency proposed by Cooper et al. (2000) into the bi-level programming framework in order to evaluate the a two-stage process performance in decentralized decisions. In their study, each DMU includes two decentralized subsystems: a leader (stage 1) and a follower (stage 2) as it is depicted in Figure 2.

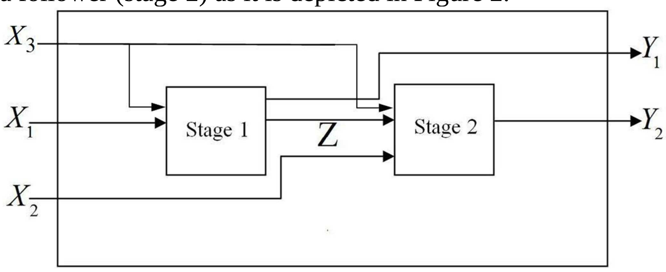

Fig. 2. A leader-follower structure

The leader uses two types of inputs, i.e., the shared input X_3^1 and the direct input X_1, to produce two different types of outputs: the intermediate measure Z and the direct output Y_1. The follower uses three types of inputs, i.e., the shared input X_3^2 and the direct input X_2 and the intermediate measure Z, to produce the output Y_2. Furthermore, assume that C_3, C_1, C_Z and C_2 are the input unit cost vectors associated with X_3^1 (X_3^2), X_1, Z and X_2, respectively. In fact, the exact fixed value and maximum fixed value are two separate cases for the total amount of the shared resource that we can take into account as an extra constraint. According, when the total amount of the shared input is fixed the bi-level programming cost efficiency DEA model can be expressed as:

$$
\begin{aligned}
&(E_1^{\circ}) \ \min_{\hat{X}_3^1, \hat{X}_1, \lambda} (C_3\hat{X}_3^1 + C_1\hat{X}_1) + (C_3\hat{X}_3^2 + C_2\hat{X}_2 + C_z\hat{Z}) \\
&s.t. \quad \hat{X}_3^1 \geq X_3^1\lambda, \\
&\qquad \hat{X}_1 \geq X_1\lambda, \\
&\qquad Y_1^{\circ} \leq Y_1\lambda, \\
&\qquad Z^{\circ} \leq Z\lambda, \\
&\qquad \hat{X}_3^1 + \hat{X}_3^2 = F \\
&(E_2^{\circ}) \ \min_{\hat{X}_2, \hat{Z}, \mu} C_3\hat{X}_3^2 + C_2\hat{X}_2 + C_z\hat{Z} \\
&s.t. \quad \hat{X}_3^2 \geq X_3^2\mu, \\
&\qquad \hat{X}_2 \geq X_2\mu, \\
&\qquad Z \geq Z\mu, \\
&\qquad Y_2^{\circ} \leq Y_2\mu, \\
&\qquad \hat{X}_3^1, \hat{X}_1, \hat{X}_2, \hat{Z}, \lambda, \mu \geq 0.
\end{aligned}
\tag{51}
$$

The shared input $(\hat{X}_3^1)$, the direct input $(\hat{X}_1)$ and an optimal multiplier λ can be calculated by the first level of model (51) so as to minimize the total costs for the leader. As a result, $\hat{X}_3^2$ is simply obtained for the follower using $\hat{X}_3^1 + \hat{X}_3^2 = F$. Note that in the above bi-level programming cost efficiency DEA model intermediate measure is output for the leader in the upper level and also input for the follower in the lower level. The second case is when the total amount of the shared input has the fixed maximum value. To do, we substitute $\hat{X}_3^1 + \hat{X}_3^2 \leq F$ for $\hat{X}_3^1 + \hat{X}_3^2 = F$ in model (51). Wu (2010) applied the branch and bound algorithm proposed by Shi et al. (2006) to solve the model (51). Once the optimal value of $(\hat{X}_3^{1*}, \hat{X}_3^{1*}, \hat{X}_1^*, \hat{X}_2^*, \hat{Z}^*, \lambda^*, \mu^*)$ is obtained from model (45) the cost efficiency of the *j*th leader (CE_1^j), the *j*th follower (CE_2^j) and the *j*th system (CE^j) are defined as

$$CE_1^j = \frac{C_3\widehat{X}_3^{1*} + C_1\widehat{X}_1^*}{C_3X_3^1 + C_1X_1}$$

$$CE_2^j = \frac{C_3\widehat{X}_3^{2*} + C_2\widehat{X}_2^* + C_z\widehat{Z}^*}{C_3X_3^2 + C_2X_2 + C_zZ}$$

$$CE^j = \frac{(C_3\widehat{X}_3^{2*} + C_2\widehat{X}_2^* + C_z\widehat{Z}^*) + (C_3\widehat{X}_3^{1*} + C_1\widehat{X}_1^*)}{(C_3X_3^2 + C_2X_2 + C_zZ) + (C_3X_3^1 + C_1X_1)}$$

The *j*th leader, the *j*th follower– and the *j*th system are cost efficient if and only if $CE_1^j = 1$, $CE_2^j = 1$ and $CE^j = 1$, respectively. In addition, Wu (2010) similar to Cooper et al. (2000) used the reference units to rank the efficient DMUs.

6. Conclusions and future research directions

Supply chain management (SCM) covers several disciplines and is growing rapidly. Performance measurement is an important activity, especially in the multi-dimensional case of international supply chains. DEA as a non-parametric technique for measuring efficiency continues to enjoy increasing popularity. Reviewing the multi- and two-level extensions published in the DEA literature reveals a considerable wealth of different models, based either on restrictions in the reference set, the weight system or the sequence of optimization of the DMU problems.

However, the analysis also shows several open problems in the application of DEA to supply chain performance measurement.

First, the *limitations and rigidity in model specification*. Whereas supply chains by definition involves several stages (normally at least three) interacting independently with markets for raw materials and intermediate outputs, bulk of the extensions are limited by explicit or implicit restrictions to two-stage processes with no third-party interaction. In practice, this implies a strict dyadic buyer-seller dichotomy in which all intermediate outputs are consumed by a single entity. The assumption is very strong and in open contradiction to standard results in multi-stage supply chain planning models, where intermediate plants and distribution centers are expected to serve multiple downstream units, within and/or without the focal enterprise. Moreover, the lack of flexibility in the model structure is commonly motivated by the solution approach, derivations of joint metrics etc., that consequently hamper the generalization of the results to a realistic situation. Further work is necessary on this fundamental point to allow applications of frontier-based methods to real multi-stage supply chains.

Second, the *lack of motivation for the intermediate measures.* Besides the multi-stage property, one of the underlying features distinguishing supply chain management from general operations management is the prevalence of decentralized decision making. In economics and management science, we tend to attribute these decision makers with some procedural rationality that renders them susceptible to mathematical modeling. A common assumption is that the decision makers maximize some profit or objective function subject to some rationally imposed constraints, e.g. resource allocation across a group. It is therefore necessary for any performance assessment to take into account the objectives of the underlying units in their assessment, if the resulting estimate is to have any relevance as an indication for the effectiveness of their decision making. We note that some suggested models tend to abstract from the economic or preferential reality of the evaluated units in assuming that their objectives *per se* should be related to, or even centered on, the very metric that analysts propose for their evaluation. In fact, most models dispose of this step by simply assuming that the objectives of the unit correspond to the maximization of some single-stage evaluation problem, such as the conventional CRS formulation. Already in a single-stage setting, the interpretation of productivity measures is associated with many limitations, cf. Agrell and West (2001). In the supply chain setting, with the interdependencies between levels and the ambiguous character of the input resource restrictions challenge this perception and prompt for a careful and well justified behavioral motivation for the submodels, as well as for the centralized models. Further consolidation of the literature based on game-theoretical approaches may be way to address this shortcoming.

Third, *modeling of the power or governance structures* within the supply chain. Given the absence of a centralized decision maker, the modeler faces a hierarchical multi-criteria problem without any clear preferential structure. Whereas conventional approaches in economics would use Stackelberg-type bilevel games or Nash bargaining concepts, the supply chain management literature frequently employs non-cooperative and cooperative game theoretical approaches. Although some models are founded on elements hereof, there is need of stringent models unifying the evaluation model structure with the underlying assumptions about the power or governance structure within the chain. Such work, founded on economic theory and decision theory, may also eliminate the too frequent resort to *ad hoc* technical and scaling parameters in the models without any methodological foundation.

Fourth, predominance of *multiplicative models*. Multi-product networks, especially for dynamic approaches, involve relatively large dimensional output vectors and likely (correctly) zero-valued observations. Multiplicative approaches (radial efficiency metrics) here yield computationally poor results with efficiency scores in the presence of significant slack, i.e. weak technical efficiency. Additive models (seminal work by Charnes et al., 1985) are traditionally viewed as inferior, lacking translation and unit invariance (cf. Ali and Seiford, 1990) and difficult to decompose in relevant submeasures. The special structure for supply chain problems, however, where units often can be homogenous (value, weight, energy contents, pieces) and decompositions can be consistent and informative using simple transformations as in Agrell and Bogetoft (2005). The use of additive approaches also opens for relevant substitutions and analyses of cost- versus technical efficiency for more realistic dimensions. However, more work is necessary to determine the properties and robustness of such models in generalized multi-stage settings. The work by Chang et al. (2011) based on the non-radial Tone and Tsutsui (2009) model is here particularly interesting, also from a conceptual viewpoint.

Stating these areas of desired progress is in no way negating the positive and productive wealth of work in the areas of two-stage non-parametric frontier models. On the contrary, it is this energy and thrust that will unlock the force of the models to attack the so far unsolved, frustrating and decisive problems found in supply chain performance measurement.

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