Optimal maintenance policy for mission-oriented systems with continuous degradation and external shocks

Abstract

This paper develops a maintenance model for mission-oriented systems subject to natural degradation and external shocks. For mission-oriented systems which are used to perform safety-critical tasks, maintenance actions need to satisfy a range of constraints such as availability/reliability, maintenance duration and the opportunity of maintenance. Additionally, in developing maintenance policy, one needs to consider the natural degradation due to aging and wearing along with the external shocks due to variations of the operating environment. In this paper, the natural degradation is modeled as a Wiener process and the arrival of random shock as a homogeneous Poisson process. The damage caused by shocks is integrated into the degradation process, according to the cumulative shock model. Improvement factor model is used to characterize the impact of maintenance actions on system restoration. Optimal maintenance policy is obtained by minimizing the long-run cost rate. Finally, an example of subsea blowout preventer system is presented to illustrate the effectiveness of the proposed model.

Key words: Imperfect maintenance, mission-oriented system, natural degradation, external shocks, reliability constraint.

1. Introduction

Many engineering systems are missioned-oriented, which are designed to fulfil a sequence of missions within the service lifetimes. Examples can be found in military systems such as avionics parts of airborne weapon systems and in manufacturing equipment such as manipulator arms working in production lines (Levitin et al., 2013; Guo et al., 2013). A mission has to be aborted if the system fails to complete the mission. Hence, it is critical to ensure that the system is properly maintained and retain a high reliability before performing a mission. Different from general systems, maintenance activities on mission-oriented systems can only be carried out during breaks between successive missions, i.e., no maintenance activity is allowed during mission. Further, one needs to ensure that a range of constraints such as availability/reliability, and maintenance duration are satisfied while performing the actions.

Mission-oriented systems are very common in industry. For example, a power generation unit in a power plant keeps working for nearly a full week until maintenance actions such as minimal repair, preventive maintenance (PM) and overhaul are performed within a stipulated period at the end of the week (Pandey et al., 2013). For high-speed railway, maintenance activities are carried out during maintenance windows scheduled late at night, when no carrying demand is required (Campos and de Rus, 2009). If a train fails, the mission of carrying passengers during that period has to be abandoned. As for aircrafts, the maintenance activities that guarantee the success of the next flight are implemented between flights (Feo and Bard, 1989).

During operation, mission-oriented systems are subject to natural degradation and external shocks. Usually, the natural degradation is modeled as a stochastic process so as to have flexibility in describing the failure-generating mechanism (Singpurwalla, 1995; Lu et al., 2016). Whenever the degradation level of a system exceeds a certain threshold, the system is deemed to have failed. This is commonly referred to as "soft failure" (Ye and Xie, 2015). Among degradation models, Wiener process has become particularly popular with respect to its mathematical properties and physical interpretations (Wang et al., 2014; Zhang et al., 2016). Wiener processes have been used extensively to model a variety of degradation processes encountered in real systems, e.g., fatigue crack dynamics (Si et al., 2013), light-emitting diodes (LED) (Peng and Tseng, 2009), and bridge beams (Wang, 2010).

On the other hand, external shock occurs due to sudden and unexpected variations of the working environment. Four types of random shock models can be found in literature: the extreme shock model, the cumulative shock model, the run shock model, and the δ- shock model (Rafiee et al., 2014).

In literature, the natural degradation and external shocks are modeled as competing failure processes. Several works have been conducted on reliability analysis of the competing failure processes (Keedy and Feng, 2012; Rafiee et al., 2014; Song et al., 2016). Ye et al. (2011) captured both degradation and external shocks into a single degradation model by assuming that the failure time belongs to certain distribution family. Wang and Pham (2012) modeled the dependent relationship of degradation process and random shocks by a time-varying copula.

Huang et al. (2016) developed a condition based maintenance for systems with dependent competing failures due to degradation and external shocks. These maintenance models assume that PM can be carried out immediately after the system state has reached a critical threshold. However, in a mission-oriented system, PM can only be carried out after the mission has been fulfilled. A limitation of previous studies is that they are tailored to systems without mission constraints.

This paper develops an imperfect maintenance model for mission-oriented systems subject to degradation and external shocks. Instead of availability constraint, reliability constraint is adopted as a requirement during mission operation. We select reliability over availability as the constraint because reliability is more appealing than availability for a safety-critical system where failure of a mission leads to huge losses. In particular, we consider multiple dependent competing failure processes where either external shocks or natural degradation can lead to system failure. The degradation process is modeled as a Wiener process and the shocks are assumed to arrive according to Poisson process. The optimal maintenance policy is achieved by minimizing the long-run cost rate.

The rest of this paper is organized as follows. Section 2 presents the general assumptions and details of system degradation process. Maintenance policy is described in Section 3 and the cost model is formulated in Section 4. Section 5 develops a maintenance optimization algorithm in which the optimal PM threshold is obtained by minimizing the long-run cost rate. Section 6 presents a numerical example illustrating the effectiveness of the proposed maintenance policy. Finally, Section 7 summarizes the main conclusions and provides suggestions for future research.

Notation

$B(t)$	Standard Brownian motion
C	Maintenance cost in a renewal cycle
$C^{\infty}(\bullet)$	Long-run cost rate
$D(t)$	Natural degradation level by time t
$F_{T_a}(t, x_0)$	cdf (cumulative distribution function) of time to perform imperfect PM

$F_{T_f}(t, x_0)$	cdf of time to failure
l_a	Threshold associated with imperfect PM
l_f	Failure threshold
$N(t)$	Number of shocks arriving by time t
N_I	Number of inspections in a renewal cycle
N_I^i	Number of inspections in ith PM cycle
N_M	Number of imperfect PM actions in a renewal cycle
P^F	Probability that a renewal cycle ends with a corrective replacement
P^U	Probability that a renewal cycle ends with a preventive replacement
$R(t, x_0)$	System reliability as a function of time and initial system state
U	Maximum number of PM actions within a renewal cycle
T	Length of a renewal cycle
$W(t)$	Cumulative magnitude of shock by time t
W_i	Magnitude of the ith shock, following a normal distribution, $W_i \sim N(\mu_w, \sigma_w^2)$
x_0	Initial degradation level
x_0^i	Initial state of the system in ith PM cycle
$X(t, x_0)$	Overall degradation level
α	Imperfect PM factor
λ	Arrival rate of random shocks
μ	Drift coefficient
$\Phi(\bullet)$	cdf of a standard normal distribution
σ	Diffusion coefficient

τ	Length of a mission
ξ	Reliability constraint for completing the next mission

2. System description

A mission-oriented system operates intermittently to complete a mission. During operation, the system goes through natural degradation process along with cumulative external shocks. Within the service lifetime, imperfect PM is performed when the system state hits a critical threshold at inspection.

2.1 Assumptions

With respect to constructing a specific but realistic model, the following assumptions are made.

1. The system fails when the overall degradation exceeds a critical threshold. The overall degradation is composed of natural degradation and damage caused by external shocks.
2. Shocks arrive according to a Poisson process and have a cumulative impact on system degradation (according to the cumulative shock model).
3. All the missions have the same duration. In practice, the mission duration is usually a random variable. However, for system where the variation of mission duration is negligible (e.g., high-speed railway), it is reasonable to model mission duration as a constant.
4. Inspection is carried out after each mission, which is assumed to be perfect and non-destructive.
5. Compared with the duration of the mission, maintenance actions are assumed to be immediate and instantaneous.

The above assumptions are commonly used in related researches, such as Peng et al. (2010), Chen (2012) and Guo et al. (2013).

2.2 Degradation process and external shocks

The system is subject to a Wiener process during a mission, described as follows:

$$D(t, x_0) = x_0 + \mu t + \sigma B(t) \tag{1}$$

where x_0 is the initial degradation level, $D(t)$ is the natural degradation level, μ is the drift coefficient, σ is the diffusion coefficient, and $B(t)$ is the standard Brownian motion, i.e., $B(t) \sim N(0,t)$. Note that, although the Wiener process is a non-monotone process, the mean degradation amount increases monotonically, i.e., $E[D(t)] = \mu t + x_0$.

The cumulative damage caused by shocks at time t can be expressed as

$$W(t) = \begin{cases} \sum_{i=1}^{N(t)} W_i & if\ N(t) \geq 1 \\ 0, & if\ N(t) = 0 \end{cases} \tag{2}$$

where $N(t)$ is the number of shocks arriving by time t, W_i is the magnitude of the *i*th shock, following a normal distribution, $W_i \sim N(\mu_w, \sigma_w^2)$, where μ_w and σ_w are the mean and standard deviation of shock magnitude.

Figure 1 describes the degradation process of a system subject to external shocks. As shown in the figure, shocks with magnitudes W_1 and W_2 arrive at times t_1 and t_2. The arrival of a shock abruptly changes the degradation level. The system degrades continuously and fails when the degradation level hits the threshold, l_f. The failure threshold l_f denotes the degradation level where the mission cannot be satisfactorily performed. The threshold is usually determined by engineers or experts. The overall degradation of the system can then be expressed as

$$X(t, x_0) = D(t) + W(t) = x_0 + \mu t + \sigma B(t) + \sum_{i=1}^{N(t)} W_i \tag{3}$$

At the beginning of a PM cycle, the initial system state varies as imperfect PM actions are carried out. While analyzing system reliability within a PM cycle, it is important to consider the influence of initial state x_0 and time t. Considering the arrival of random shocks, the reliability of the system is expressed as

$$\begin{aligned} R(t, x_0) &= \sum_{k=0}^{\infty} P\left\{X(t) < l_f \mid N(t) = k\right\} \cdot P\left\{N(t) = k\right\} \\ &= \sum_{k=0}^{\infty} \Phi\left(\frac{l_f - (x_0 + \mu t + k\mu_w)}{\sqrt{\sigma^2 t + k\sigma_w^2}}\right) \cdot \frac{e^{-\lambda t}(\lambda t)^k}{k!} \end{aligned} \tag{4}$$

where $\Phi(\bullet)$ is the cdf of standard normal distribution.

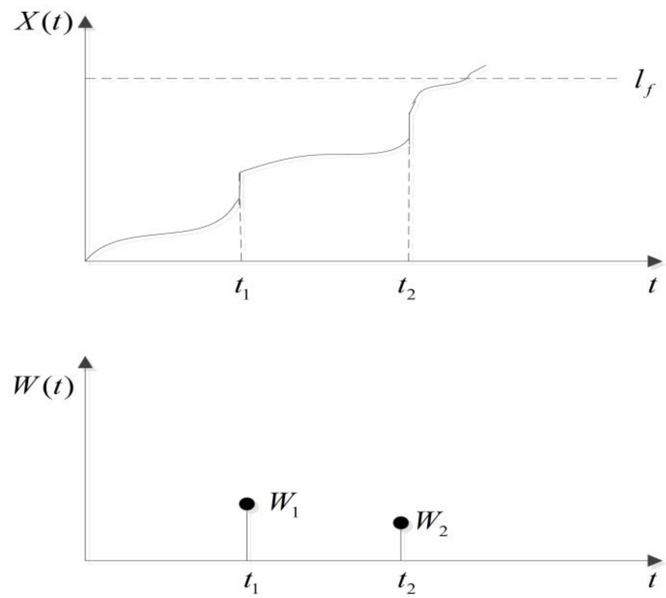

Figure 1 Degradation processes subject to external shocks.

3. Maintenance model for mission-oriented system

Three types of maintenance actions are considered in this paper: imperfect PM, preventive replacement and corrective replacement. In practice, imperfect PM can refer to a simple repair, oiling, cleaning, etc., which does not restore the system to being “as good as new”. Preventive replacement can be an overhaul of the total system while corrective replacement can be a physical replacement of the whole system (Peng et al., 2012; Chen et al., 2013). Although both preventive replacement and corrective replacement can restore the system to the as-good-as-new state, their contexts differ in practice. Corrective replacement is usually unplanned and undertaken whenever the system is either in a state of severe deterioration or total failure (Huynh et al., 2012; (Zhang et al., 2014).). An imperfect PM is carried out each time the overall degradation level exceeds l_a. If the imperfect PM is unable to satisfy the prescribed reliability constraint, a preventive replacement is performed instead.

The PM process and system state evolution are shown in Figure 2. A renewal cycle is defined as the time interval between two consecutive replacements (either preventive replacement or corrective replacement). A PM cycle is defined as the time interval between two consecutive PM actions or that between PM action and replacement (either preventive or corrective). Let S_m be the length of the *m*th renewal cycle, τ the length of each mission, and $t_{m,i}$ the time of the *i*th PM action in the *m*th renewal cycle. Note that, as we assume that each maintenance action is instantaneous, the timing of the last maintenance action is identical with that at the beginning of the next PM cycle.

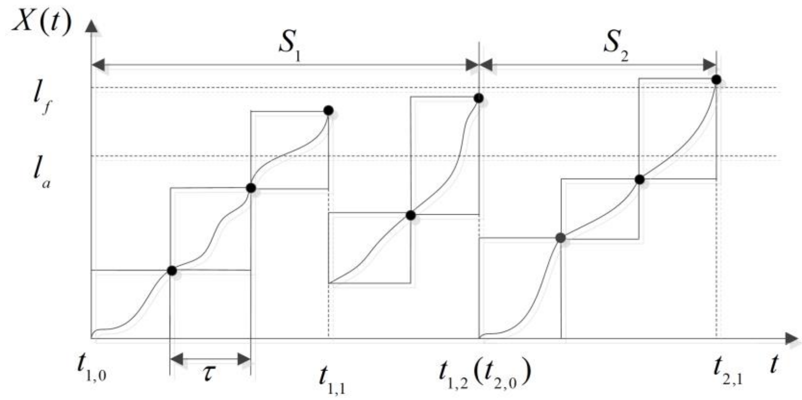

Figure 2 Description of PM process.

3.1 Improvement factor model

The improvement factor model is used to measure the restoration effect of imperfect maintenance (Canfield, 1986). Wang and Pham (2011) utilized an improvement factor that scales improvements in the failure threshold. Power-law relationship was constructed to characterize the influence of the number of maintenance actions on the failure threshold. In this study, we borrow the idea of power-law from Wang and Pham (2011) to measure the effect of imperfect maintenance on system state.

In practice, the capacity of PM activity to improve system health weakens when more and more PM actions have been taken. Moreover, unduly frequent disassembly and assembly can even result in rapid degradation. Considering the above two aspects, the improvement factor model is expressed as

$$X(t^*) = (1-\alpha^i)l_f \tag{5}$$

where $X(t^*)$ is the state of the system after PM , α is the imperfect PM factor and *i* denotes the index of PM action.

As the restored system state equals the starting state of the next PM cycle, for notational simplicity, we denote $x_0^i = (1-\alpha^{i-1})l_f$ as the initial state of the *i*th PM cycle.

3.2 Reliability constraint

Preventive replacement is carried out at the end of the mission when the reliability of completing the next mission reaches an unacceptable level, ξ. The system reliability at the end of the *j*th mission in the *i*th PM cycle can be obtained as

$$R(j\tau, x_0^i) = \sum_{k=0}^{\infty} \Phi\left(\frac{l_f - (x_0^i + \mu j\tau + k\mu_w)}{\sqrt{\sigma^2 j\tau + k\sigma_w^2}} \right) \cdot \frac{e^{-\lambda j\tau}(\lambda j\tau)^k}{k!} \tag{6}$$

Reliability constraint is essential for system operation, especially for safety-critical systems such as high-speed railway and nuclear power plant (Liu et al., 2014).

Proposition 1. For system subject to Wiener degradation with linear drift and cumulative random shocks arriving according to a homogeneous Poisson process, system reliability of completing missions decreases with the number of PM cycles.

Detailed proof is shown in Appendix A. Proposition 1 implies that there is a one-to-one correspondence between the initial system state and the reliability of completing the next mission. Therefore, the threshold for the reliability constraint, ξ, can be transformed into the threshold of the initial state, x_0^*, by using the following expression:

$$x_0^* = \arg\max_{x_0^*} \left\{ R(\tau, x_0^*) \geq \xi \right\} \tag{7}$$

As is shown in Equation 6, system reliability function $R(\bullet)$ decreases monotonously with the initial degradation state if no maintenance actions are performed. By use of the monotonicity property, x_0^* can be obtained as:

$$x_0^* = \left\{ x_0^* \mid R(\tau, x_0^*) = \xi \right\} \tag{8}$$

Due to the complexity of system reliability function $R(\bullet)$, x_0^* cannot be obtained analytically, instead numerical method is used to compute x_0^*. Equation 8 is useful in practice since the system state can be measured at inspection and engineers or managers can make maintenance decisions by observing the system state directly.

Imperfect PM is carried out when the PM action is able to restore the system to a state below x_0^*. Combining this observation with the improvement factor model by Equation 5, we have $(1-\alpha^i)l_f < x_0^*$. The maximum number of imperfect PM actions in a renewal cycle is given by

$$U = \left\lfloor \log_{\alpha}\left(1 - \frac{x_0^*}{l_f}\right) \right\rfloor \tag{9}$$

Equation 9 can be applied to determine the renewal cycle: ending with a preventive replacement or with a corrective replacement. If the number of PM actions reaches U, then preventive replacement is implemented. Otherwise, the renewal cycle ends with corrective replacement. It can be concluded that the maximum number of imperfect PM actions U shows a non-increasing trend with respect to the reliability threshold ξ.

Proposition 2. The maximum number of imperfect PM actions is non-increasing with respect to the reliability threshold.

Detailed proof is given in Appendix B. Proposition 2 indicates that less imperfect PM actions are allowed if high reliability constraint is required.

4. Formulation of long-run cost rate

Cost rate over an infinite horizon is used as the criterion to assess the performance of the proposed maintenance model. After replacement (either corrective or preventive), the system is restored to the state "as good as new", which constitute a renewal cycle (Liu et al., 2015). According to the renewal reward theory, the long-run cost rate is given as (Grall et al., 2002)

$$C^{\infty}(l_a) = \lim_{t\to\infty} CR(t) = \frac{E[C]}{E[T]} \tag{10}$$

where $CR(t)$ is the cost rate over time period $[0,t]$, C is the cost in a renewal cycle and T is the length of a renewal cycle.

The cost items include inspection cost at each inspection time C_I, imperfect PM cost C_M, preventive replacement cost C_P, and corrective replacement cost C_C (including the penalty cost due to mission abandonment). The renewal cycle can be classified into two types: renewal cycle ending with preventive replacement or with corrective replacement. The long-run cost rate can then be expressed as

$$C^{\infty}(l_a) = \frac{C_C P^F + C_P P^U + C_M E[N_M] + C_I E[N_I]}{E[T]} \tag{11}$$

where P^F is the probability that a renewal cycle ends with corrective replacement, P^U is the probability that a renewal cycle ends with preventive replacement, N_M is the number of imperfect PM actions in a renewal cycle and N_I is the number of inspections in a renewal cycle.

4.1 Scenarios of maintenance actions

Evolution of the degradation process $X\left(t, x_0\right)$ is a renewal process with the regenerative times by corrective replacement or preventive replacement. Before we proceed to derive the expression of the cost items, we first need to investigate the scenarios of various maintenance actions at inspection. Within the *i*th PM cycle, there are four scenarios at the j_ith inspection time:

(1) If the degradation level exceeds the failure threshold l_f, then a corrective replacement is performed. The probability for such an event is given as:

$$\begin{aligned} P_1(j_i \mid i) &= P\{X(j_i\tau, x_0^i) > l_f \cap X((j_i-1)\tau, x_0^i) < l_a\} \\ &= \int_{x_0^i}^{l_a} P\{X(j_i\tau, x_0^i) - X((j_i-1)\tau, x_0^i) > l_f - x\} dF_x(x;(j_i-1)\tau, x_0^i) \\ &= \int_{x_0^i}^{l_a} F_{l_f}(\tau; x) f_\phi(x;(j_i-1)\tau) dx \end{aligned} \tag{12}$$

where

$$F_{l_f}(\tau; x) = \sum_{k=0}^{\infty} \Phi\left(\frac{(x + \mu\tau + k\mu_w) - l_f}{\sqrt{\sigma^2\tau + k\sigma_w^2}} \right) \cdot \frac{e^{-\lambda t}(\lambda\tau)^k}{k!}$$

$f_\phi(x;t)$ is the pdf of the degradation level $X(t, x_0) = x$ at time t. Mathematically,

$$f_\phi(x;t) = \sum_{k=0}^{\infty} \frac{1}{\sqrt{2\pi(\sigma^2 t + k\sigma_w^2)}} \exp\left(-\frac{(x - \mu t - k\mu_w - x_0)^2}{2(\sigma^2 t + k\sigma_w^2)} \right) \cdot \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

(2) If the degradation level satisfies $l_a < X(j\tau, x_0^i) < l_f$, given the number of PM cycles exceeds the maximum number U, $i > U$, then preventive replacement is implemented. The probability of such an event is:

$$\begin{aligned} P_2(j_i \mid i) &= P\{l_a < X(j_i\tau, x_0^i) < l_f \cap X((j_i-1)\tau, x_0^i) < l_a \mid i > U\} \\ &= \int_{x_0^{U+1}}^{l_a} (F_{l_a}(\tau; x) - F_{l_f}(\tau; x)) f_\phi(x;(j_i-1)\tau) dx \end{aligned} \tag{13}$$

(3) If the degradation level satisfies $l_a < X(j\tau, x_0^i) < l_f$, given the number of PM cycles is less than U, $i \le U$, then imperfect PM is implemented. The probability of such an event is as follows:

$$\begin{aligned} P_3(j_i \mid i) &= P\{l_a < X(j_i\tau, x_0^i) < l_f \cap X((j_i-1)\tau, x_0^i) < l_a \mid i \le U\} \\ &= \int_{x_0^i}^{l_a} (F_{l_a}(\tau; x) - F_{l_f}(\tau; x)) f_\phi(x; (j_i-1)\tau) dx \end{aligned} \quad (14)$$

(4) If the degradation level is less than l_a, $X(j\tau, x_0^i) < l_a$, then the system is left as it was. The probability of such an event is given as:

$$\begin{aligned} P_4(j_i \mid i) &= P\{X(j_i\tau, x_0^i) < l_a\} \\ &= \sum_{k=0}^{\infty} \Phi\left(\frac{l_a - (x_0^i + \mu j_i \tau + k\mu_w)}{\sqrt{\sigma^2 j_i \tau + k\sigma_w^2}} \right) \cdot \frac{e^{-\lambda j \tau} (\lambda j_i \tau)^k}{k!} \end{aligned} \quad (15)$$

4.2 Maintenance cost and length of a renewal cycle

Based on how a renewal cycle ends, the system renewal cycle can be classified into two types: renewal cycle ending with a corrective replacement and renewal cycle ending with a preventive replacement. In the following, the cost model is formulated separately, based on the type of renewal cycles.

Case 1: Renewal cycle ending with corrective replacement

Corrective replacement is carried out when the system has failed at inspection. Given the number of PM cycles i and the number of inspections in each PM cycle N_I^k, we can have the associated cost and length of a renewal cycle as

$$C_1 = C_I (N_I^i + \sum_{k=0}^{i-1} N_I^k) + C_M (i-1) + C_C \quad (16)$$

and

$$T_1 = \left(\sum_{k=0}^{i-1} N_I^k + N_I^i \right) \tau \quad (17)$$

where $N_I^0 \equiv 0$.

The expected cost can be obtained by considering all the possible combinations of the number of PM cycles and the number of missions completed in each PM cycle. After some calculations, the expected cost can be obtained as

$$\begin{aligned}E[C_1] &= C_I E[N_{I(1)}] + C_M E[N_{M(1)}] + CcP^F \\ &= C_I \sum_{i=1}^{U+1}\left[\sum_{k=1}^{i-1}\sum_{j_k=1}^{\infty} j_k P_3(j_k \mid k) + \sum_{j_i=1}^{\infty} j_i P_1(j_i \mid i)\right]\prod_{k=1}^{i-1}\sum_{j_k} P_3(j_i \mid i)\times\sum_{j_i} P_1(j_i \mid i) \\ &+ C_M \sum_{i=1}^{U+1}(i-1)\prod_{k=1}^{i-1}\sum_{j_k} P_3(j_i \mid i)\times\sum_{j_i} P_1(j_i \mid i) \\ &+ Cc\sum_{i=1}^{U+1}\prod_{k=1}^{i-1}\sum_{j_k} P_3(j_i \mid i)\times\sum_{j_i} P_1(j_i \mid i)\end{aligned} \tag{18}$$

The expected length is given as

$$E[T_1] = \tau E[N_{I(1)}] = \tau\sum_{i=1}^{U+1}\left[\sum_{k=1}^{i-1}\sum_{j_k=1}^{\infty} j_k P_3(j_k \mid k) + \sum_{j_i=1}^{\infty} j_i P_1(j_i \mid i)\right]\prod_{k=1}^{i-1}\sum_{j_k} P_3(j_i \mid i)\times\sum_{j_i} P_1(j_i \mid i) \tag{19}$$

Detailed derivations are provided in Appendix C.

Case 2: Renewal cycle ending with preventive replacement

Preventive replacement is carried out when imperfect PM cannot bring the system back to a state under reliability constraint. Based on the previous discussions, the reliability constraint can be transformed to the limit of number of imperfect PM actions in a renewal cycle. The preventive replacement is carried out at the end of a mission if the number of imperfect PM actions reaches U, i.e., the system has survived the previous U PM cycles.

Given the number of inspections in each PM cycle, the cost of a renewal cycle ending with preventive replacement can be obtained as

$$C_2 = C_I \sum_{i=1}^{U+1} N_I^i + C_M \cdot U + C_P$$

and the length of the renewal cycle is

$$T_2 = \sum_{i=1}^{U+1} N_I^i \cdot \tau$$

We can have the expected cost in a renewal cycle for this case as

$$E[C_2] = C_I E[N_{I(2)}] + C_M E[N_{M(2)}] + C_P P^U$$
$$= \left(C_I \sum_{i=1}^{U+1} \sum_{j_i=1} j_i \int_{x_0^i}^{l_a} (F_{l_a}(\tau;x) - F_{l_f}(\tau;x)) f_\phi(x;(j_i-1)\tau)dx + C_M U + C_P \right) \tag{20}$$
$$\times \prod_{i=1}^{U+1} \int_{x_0^i}^{l_a} (F_{l_a}(\tau;x) - F_{l_f}(\tau;x)) f_\phi(x;(j_i-1)\tau)dx$$

and the expected length of a renewal cycle as

$$E[T_2] = \tau \sum_{i=1}^{U+1} \sum_{j_i} j_i \int_{x_0^i}^{l_a} (F_{l_a}(\tau;x) - F_{l_f}(\tau;x)) f_\phi(x;(j_i-1)\tau)dx$$
$$\times \prod_{i=1}^{U+1} \int_{x_0^i}^{l_a} (F_{l_a}(\tau;x) - F_{l_f}(\tau;x)) f_\phi(x;(j_i-1)\tau)dx \tag{21}$$

Detailed derivations of Equation 20 and 21 are given in Appendix D. Combining the results of the two cases, the long-run cost rate C^∞ can be readily obtained.

Corollary 1. The optimal long run cost rate for mission-oriented system is larger than that without mission constraint.

Proof. The conclusion is intuitive. For general system without mission constraint, the optimal long run cost rate $C^\infty(\kappa_I, l_a)$ is obtained by optimizing the preventive maintenance threshold l_a and the inspection interval κ_I. When the length of mission τ deviates from κ_I^* ($\tau \neq \kappa_I^*$), we can always have $C^\infty(l_a^*) > C^\infty(\kappa_I^*, l_a^*)$. □

5. Maintenance optimization

The objective of the maintenance policy is to minimize the long-run cost rate C^∞. The decision variable is the optimal threshold for imperfect PM action l_a. According to the analyses in Section 4, the optimization model is formulated as

$$\min \; C^\infty(l_a) = \frac{E[C_1] + E[C_2]}{E[T_1] + E[T_2]}$$

Subject to

$$N_M \le U$$
$$0 < l_a < l_f$$

The first constraint indicates that the number of imperfect PM actions cannot exceed the maximum number U, which originates from the reliability constraint. The second constraint implies the domain of the PM threshold l_a. Analytical solution of the optimization model is difficult to obtain owing to the complexity of the cost model. Hence we have to resort to numerical methods. In particular, search algorithm combined with Monte Carlo simulation is adopted to optimize l_a and $C^{\infty}(l_a)$. When the number of simulation histories N_l is large enough, the cost model given by Equation 10 can be expressed as (Huynh et al., 2012)

$$C^{\infty}(l_a) = \lim_{t\to\infty} CR(t) = \frac{E[C]}{E[T]} = \lim_{N\to\infty} \frac{\sum_{n=1}^{N} C^{(n)}}{\sum_{n=1}^{N} T^{(n)}} \simeq \frac{\sum_{n=1}^{N_l} C^{(n)}}{\sum_{n=1}^{N_l} T^{(n)}} \tag{22}$$

where $C^{(n)}$ and $T^{(n)}$ are the maintenance cost and length of a renewal cycle in the *n*th simulation history.

The optimal l_a can be obtained by minimizing the long-run cost rate, i.e.,

$$l_a = \arg\min_{l_a} \left\{ C^{\infty}(l_a) \mid 0 < l_a \le l_f \right\} \tag{23}$$

The following algorithm is used to determine the optimal l_a by searching over the range of $\left(0, l_f\right]$.

Algorithm 1: Optimization algorithm

Step 1: Compute the value of U from Equations 8 and 9.

Step 2: Start with a small value of l_a.

Step 3: Determine the long-run cost rate from Monte Carlo simulation:

3.1: Initialization: set $C = 0$ and $T = 0$.

3.2: Generate a historical account of degradation and shocks to determine the overall degradation level $X(t)$.

3.3: Calculate the length and cost of a renewal cycle, $T^{(n)}$ and $C^{(n)}$, respectively (see algorithm 2 below).

3.4: Compute $C = C + C^{(n)}$, $T = T + T^{(n)}$ and $C^{\infty} = \dfrac{C}{T}$.

3.5: If C^{∞} converges, go to step 4; else, repeat 3.1-3.4.

Step 4: If $l_a < l_f$, increase l_a with a small increment and go back to step 3; else, go to step 5.

Step 5: Determine the optimal l_a associated with the minimal C^{∞}.

The following algorithm is used to determine the values of $T^{(n)}$ and $C^{(n)}$ in a Monte Carlo simulation.

Algorithm 2: **Computation of** $T^{(n)}$ **and** $C^{(n)}$ **in a Monte Carlo simulation**

Start from $i=1$ (the number of PM cycles)

Step 1: Initialization: Set $N_I^i = 0$.

Start from $j=1$ (the number of missions in the *i*th PM cycle)

Step 2: At the end of a mission, decide using the following logic:

If $X\left(j\tau, x_0^i\right) < l_a$, do nothing, let $j = j+1$ and $N_I^i = N_I^i + 1$, and turn back to step 2; else, go to Step 3.

Step 3: if $l_f > X\left(j\tau, x_0^i\right) > l_a$, and

(a) if $i \le U$, do imperfect PM, update $i = i+1$ and $x_0^i = (1-\alpha^{i-1})l_f$, and return to step 1.

(b) if $i > U$, do preventive replacement, get $C^{(n)} = C_I \sum_{k=1}^{U+1} N_I^k + C_M \cdot U + C_P$ and $T^{(n)} = \tau \cdot \sum_{k=1}^{U+1} N_I^k$, and jump to step 5.

Step 4: if the system has already failed, $X\left(j\tau, x_0^i\right) > l_f$, do corrective replacement, get

$C^{(n)} = C_I(j + \sum_{k=0}^{i-1} N_I^k) + C_M(i-1) + C_C$ and $T^{(n)} = \left(\sum_{k=0}^{i-1} N_I^k + j\right)\tau$, and proceed to step 5.

Step 5: Output $T^{(n)}$ and $C^{(n)}$.

In the following, we will present a numerical example to illustrate the effectiveness of the proposed maintenance policy.

6. Application of subsea blowout preventer system

An example of subsea blowout preventer system is presented to illustrate the maintenance procedure. Subsea blowout preventer system is used to seal, control and monitor oil and gas wells to prevent blowout. It plays an important role in assuring safe working conditions for deep-sea drilling activities. Failures of subsea blowout preventer system may lead to catastrophic consequences. For example, a deep-sea petroleum drilling rig "Deepwater Horizon" exploded on

April 20, 2010 and oil contaminated wide area of seawater along the coast of Louisiana (Cai et al., 2013). Thus high reliability has to be guaranteed during the operation of subsea blowout preventer system. A subsea blowout preventer system is activated during deep-sea drilling. When the drilling mission is completed, inspection has to be implemented to detect the health condition of the system. During the drilling mission, the system is subject to internal degradation and external shocks. A subsea blowout preventer system is mainly composed of blowout preventer control system and blowout preventer stack, whose general structure is illustrated in Figure 3 (Cai et al., 2012).

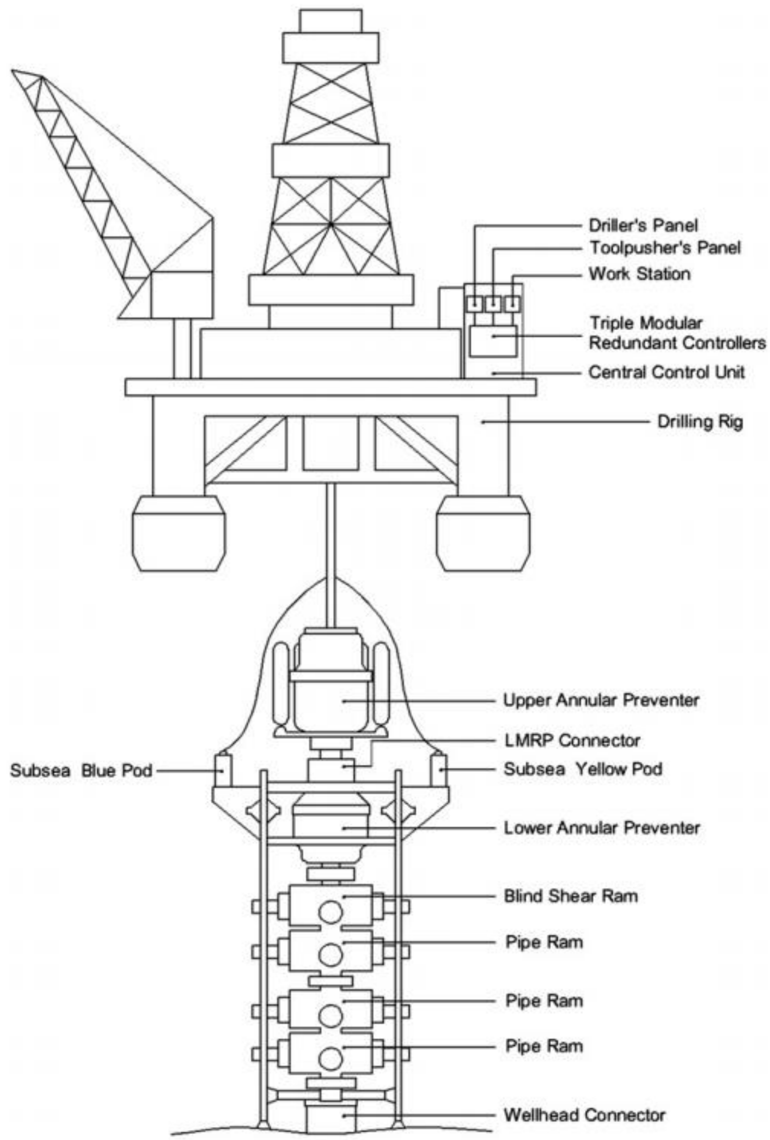

Figure 3 General structure of subsea blowout preventer system.

6.1 Optimal maintenance policy

Suppose that the subsea blowout preventer system suffers a Wiener degradation process, with the drift coefficient $\mu = 0.2$ and diffusion coefficient $\sigma = 0.02$. The initial degradation level is 0. The system is exposed to external shocks arriving according to a Poisson Process with the arrival rate $\lambda = 0.5$. The magnitude of each shock follows a normal distribution with $\mu_w = 0.1$ and $\sigma_w = 0.01$.

Suppose further that the duration of a mission $\tau = 3$ months. After each mission, inspection is carried out with the cost $C_I = 1$ (k\$). After inspection, if the degradation level hits a prescribed threshold l_a, an imperfect PM is carried out, with cost $C_M = 10$ (k\$). The imperfect PM exerts an influence on the system state according to the improvement factor model (see Equation 5). The improvement factor $\alpha = 0.6$. A preventive replacement is carried out at the end of a mission if the reliability of completing the next mission, $\xi = 0.95$, cannot be sustained. The cost associated with the preventive replacement is $C_P = 40$ (k\$).

According to Equation 8 and 9, the maximum number of imperfect PM actions U in a renewal cycle is computed as U = 4. If the overall degradation level of the system exceeds $l_f = 8$, the system is deemed to have failed and corrective replacement is performed, with cost $C_C = 80$ (k\$). The cost parameters are shown here for illustration purpose. In the following, the unit will be suppressed for notational simplicity.

For imperfect PM, the influence of PM action decreases with the increasing PM cycles. Note that the restored system state decreases rapidly with the increase of PM cycles. If the system cannot be restored to a state satisfying the reliability constraint, preventive replacement has to be performed. This implies a limited number of PM cycles within a renewal cycle.

We set the initial PM threshold as $l_a = 5$ and search the optimal PM threshold l_a^* within the range [5, 8]. The step size is 0.02 and the number of repetitions for Monte Carlo simulation is 10,000. The goal of the maintenance policy is to find the optimal threshold for imperfect PM so as to minimize the long-run cost rate. We obtain the minimum long-run cost rate of $C^{\infty *} = 1.5476$ at $l_a^* = 7.14$. Figure 4 shows variation of the long-run cost rate $C^{\infty}(l_a)$ as a function of l_a. In addition, Table 1 shows how the minimum long-run cost rate $C^{\infty *}$ and the optimal PM threshold l_a^* vary with the number of repetitions. The result shows that optimal $C^{\infty *}$ and l_a^* converge when the number of repetitions N_l is larger than 500.

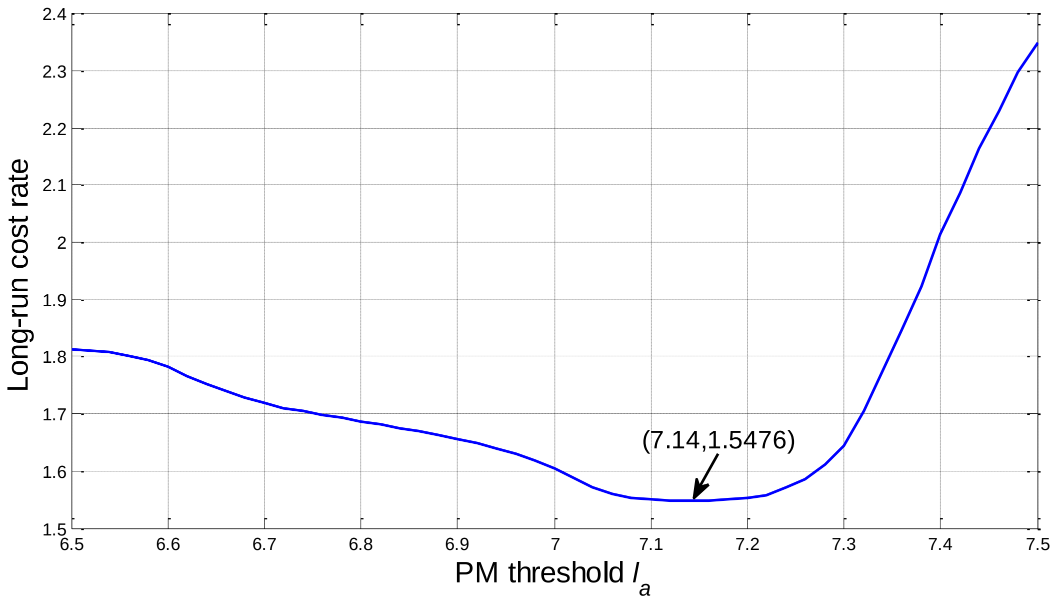

Figure 4 Long-run cost rate *vs* PM threshold.

Table 1 Optimal decisions *vs* number of repetitions

Number of repetitions N_l	Minimum long-run cost rate $C^{\infty *}$	Optimal PM threshold l_a^*
50	1.5455	7.22
100	1.5458	7.20
500	1.5486	7.14
1,000	1.5469	7.14
5,000	1.5473	7.14
10,000	1.5476	7.14

The system deteriorates by natural degradation and external shocks, and improves following maintenance actions. It is therefore of interest to investigate the variation of system reliability as a result of the degradation (external shocks) and maintenance actions. Figure 5 shows the system reliability within a renewal cycle. Note that system reliability has been restored to 1 at times 30, 48, 57 and 63, which means that there are 5 PM cycles in the renewal cycle and the corresponding PM actions are carried out at these time points. Also note that the reliability of the system is different when maintenance actions are carried out at times 30, 48, 57 and 63.This is due to the mission constraint that inspection and imperfect PM can only be performed at the end of a mission. This is different from systems where maintenance actions can be carried out at any time. Note also that, due to the uncertainty of Wiener process and randomness of external shocks, the number of imperfect PM cycles and maintenance times can vary from one renewal cycle to another. For illustration purposes, we only depict the variation of system reliability in a renewal cycle in the case that the renewal cycle ends with a preventive replacement.

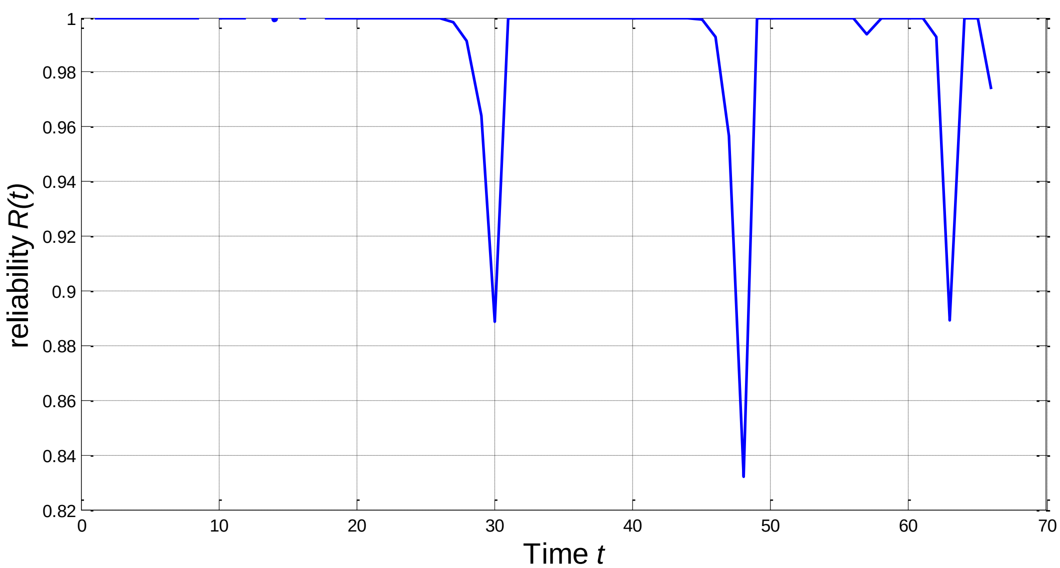

Figure 5 Variation in system reliability over time.

6.2 Sensitivity analysis

Sensitivity analysis is performed to examine the uncertainty of the optimal maintenance policy. The parameters of interest are the length of the mission τ, the imperfect PM factor α, and the arrival rate of the Poisson process λ. In the following, we investigate the influence of the three parameters on the optimum PM threshold. The results shown in Figures 6 to 8 are obtained by varying one parameter at a time while fixing the other parameters.

Figure 6 shows the variation of C^{∞} when τ is increased progressively from 2 to 4. Note that, when $\tau = 4$, the corresponding optimal PM threshold $l_a = 6.5$; when $\tau = 2$ or $\tau = 3$, the associated optimal PM threshold values equal 7. As l_a is the decision variable on which the maintenance policy is based, the results imply that the optimal maintenance policy is affected by the mission length τ . Also note that, when τ changes from 2 to 4, the corresponding optimal long-run cost rate $C^{\infty *}$, decreases from 1.8565 to 1.5, which indicates a decreasing trend with τ .

Actually, the impact of τ on the long-run cost rate is two-fold. A larger value of τ requires lesser inspection within a renewal cycle; thus incurring a lower inspection cost. Meanwhile, a greater penalty cost is incurred if the mission constraint prevents an appropriate maintenance action from being taken in a timely manner. The results shown in Figure 6 verify this. As we can see, when l_a is smaller than the optimal value, the long-run cost rate decreases with τ . This is because when the PM threshold is low, the system needs to be maintained more frequently, so the impact of mission constraint on the maintenance cost rate is not as significant as that with the inspection number. However, when l_a is large, the impacts of mission constraint and inspection number become intricate.

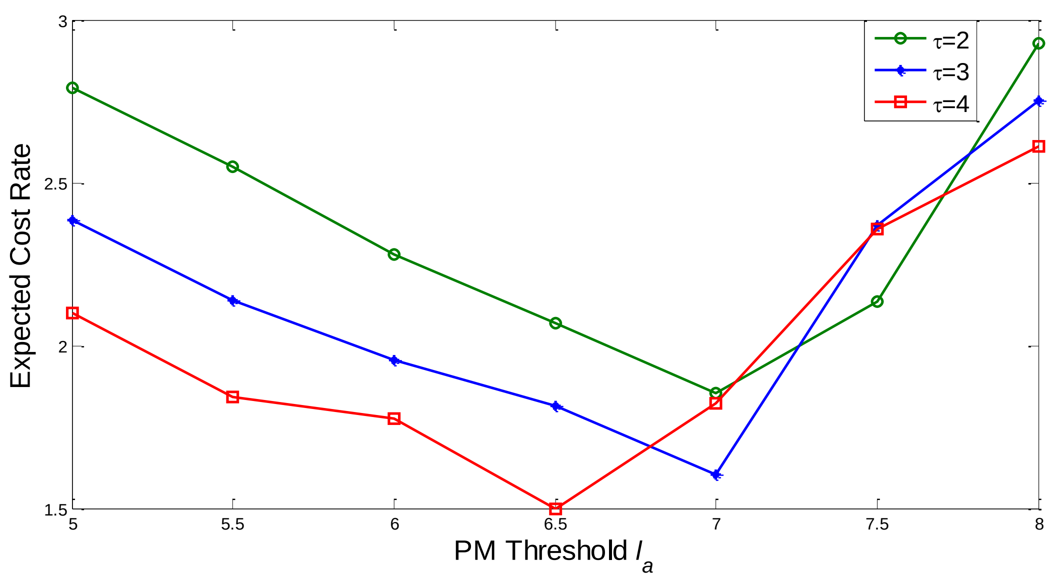

Figure 6 Sensitivity analysis of τ on the long-run cost rate.

As shown in Figure 7, when the imperfect PM factor α increases from 0.5 to 0.7, the minimum long-run cost rate decreases from 1.6692 to 1.4484. When the PM threshold is small (≤ 7), the long-run cost rate shows an obviously decreasing trend with increasing α . However, the trend is not so obvious when l_a becomes quite large. This is due to the fact that, when l_a is larger, imperfect PM is carried out less frequently and, as a result, the influence of imperfect PM factor turns out to be less significant. It is interesting to note that when l_a approaches l_f, the long-run cost rate remains constant. This can be explained by the fact that when the PM threshold is equal to l_f, the imperfect maintenance policy is reduced to a block replacement policy (Beichelt, 1981), which is irrelevant to α. Also note that although the long-run cost rate varies with α, the PM threshold l_a to achieve the minimum cost rate remains invariant. This implies that the optimal maintenance policy is not sensitive to the imperfect PM factor.

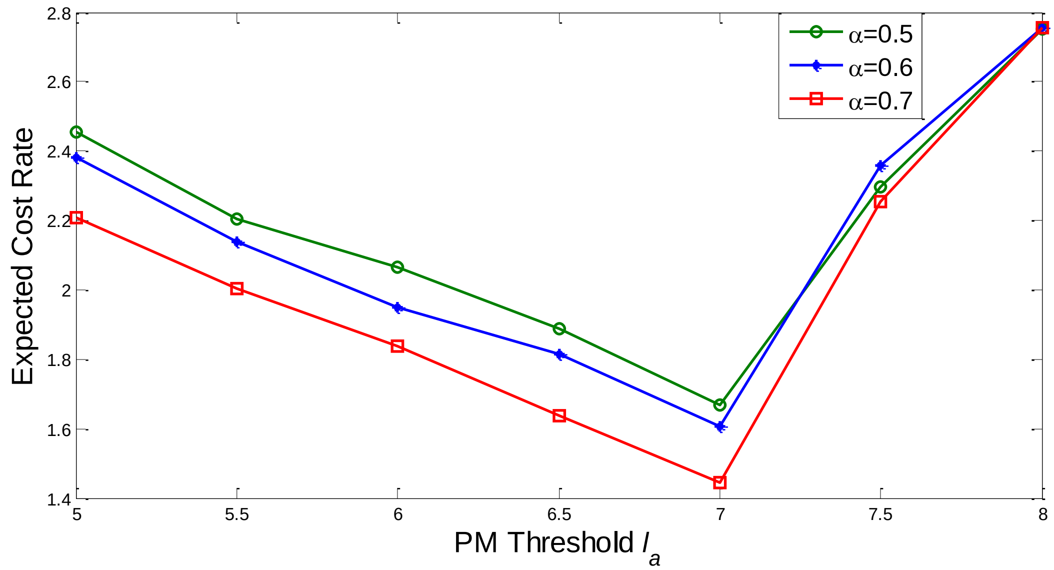

Figure 7 Sensitivity analysis of α on long-run cost rate

The arrival rate λ influences the maintenance policy and the maintenance cost rate in a way that an increase in λ accelerates the overall degradation level of the system. As a result, more frequent maintenance actions are required to keep the system operating. Not surprisingly, the long-run cost rate increases as λ increases. As shown in Figure 8, when λ increases from 0.3 to 0.7, the minimum long-run cost rate increases from 1.5094 to 1.7433. This points to an increasing trend of long-run cost rate with increasing λ. Similar to the results shown in Figure 6, the optimal maintenance policy is insensitive to the arrival rate λ. This demonstrates the robustness of the maintenance policy.

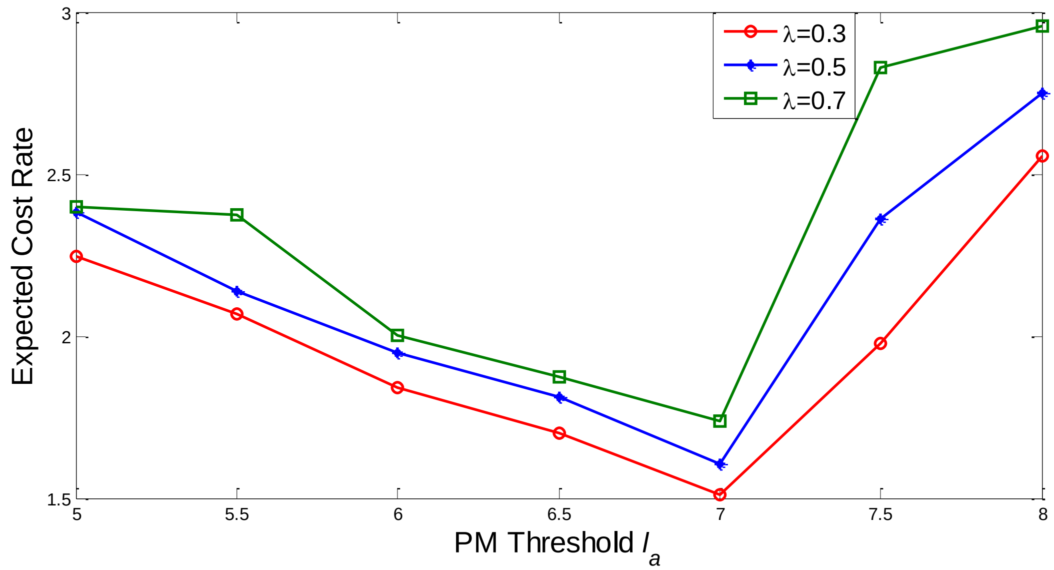

Figure 8 Sensitivity analysis of λ on long-run cost rate.

6.3 Comparison with general system

In order to investigate the effect of mission constraint on maintenance policy, a comparison study is conducted for a general system without mission constraint. The general system for comparison is a continuously operating system subject to natural degradation and external shocks. Yet inspection can be carried out at arbitrary time. Reliability constraint is interpreted as to ensure that the system operates above reliability threshold until next inspection. The objective of the maintenance policy is to determine the inspection interval κ_I and threshold for imperfect PM l_a, so as to minimize the long-run cost rate $C^{\infty}(\kappa_I, l_a)$. The cost parameters and degradation parameters are identical as previously shown. Figure 9 plots the variation of long-run cost rate and optimal threshold for imperfect PM with the inspection interval. The optimal maintenance policy for a general system is obtained at $\kappa_I^* = 6$, $l_a^* = 6.4$, with the minimal cost rate $C^{\infty *} = 1.2278$. Compared with mission-oriented system, maintenance decision for general system implies a smaller optimal PM threshold and associated long-run cost rate.

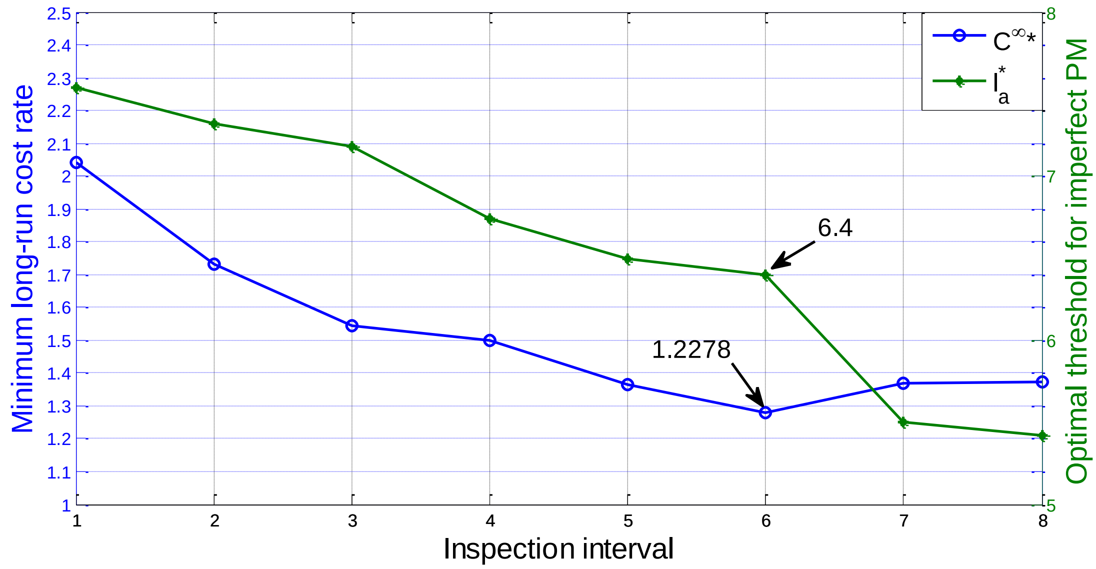

Figure 9 Optimal PM threshold & cost rate for a general system.

7. Conclusions

An imperfect maintenance policy is developed in this paper for mission-oriented systems subject to degradation and external shocks. The reliability of the system is derived using a cumulative shock model depicting the influence of external shocks on the system's degradation level. Both reliability and mission time constraints are taken into account while formulating the maintenance policy. Cost model is developed by classifying a renewal cycle into two types: ending with preventive replacement and with corrective replacement. Optimal solution is obtained by minimizing the long-run maintenance cost rate. Results from a numerical example show that the optimal PM threshold is significantly affected by the length of the mission, thus confirming the importance of mission time constraint and verifying the effectiveness and significance of the proposed maintenance model for mission-oriented systems.

Further advances can be achieved by relaxing some assumptions made in this paper. For example, imperfect inspection can be taken into consideration, as in reality, the true system state is almost impossible to obtain due to the existence of noise or limitations related to inspection techniques. Another possible improvement relates to the improvement factor model used in this paper. Future work based on a more precise imperfect maintenance model should be useful. In addition, one can further investigate the influence of length of a mission on the maintenance policy.

Acknowledgements

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Appendix A

Proof of Proposition 1.

Equation 6 shows the system reliability of completing *j* missions. By taking derivative with respect to x_0^i, we have

$$\frac{\partial R(j\tau, x_0^i)}{\partial x_0^i} = -\sum_{k=0}^{\infty} \phi\left(\frac{l_f - (x_0^i + \mu j\tau + k\mu_w)}{\sqrt{\sigma^2\tau + k\sigma_w^2}}\right) \cdot \frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!} \cdot \frac{x_0^i}{\sqrt{\sigma^2\tau + k\sigma_w^2}} < 0$$

which indicates that system reliability $R(j\tau, x_0^i)$ is a decreasing function with respect to the initial degradation level x_0^i.

From Equation 5, we have

$$x_0^i = (1-\alpha^{i-1})l_f$$

which implies that x_0^i increases monotonically with the number of PM cycles *i*. Hence, it can be readily concluded that $R(j\tau, x_0^i)$ decreases with the number of PM cycles *i*. □

Appendix B

Proof of Proposition 2.

Denote $g(\tau, x_0^*, \xi) = R(\tau, x_0^*) - \xi$. As τ is a constant here, it can be obtained that

$$\frac{\partial g}{\partial \xi} = \frac{\partial g}{\partial x_0^*}\frac{\partial x_0^*}{\partial \xi} - 1 = 0 \Rightarrow \frac{\partial x_0^*}{\partial \xi} = \frac{1}{\partial g / \partial x_0^*}$$

Since $\partial g / \partial x_0^* = \partial R(\tau, x_0^*) / \partial x_0^* < 0$, we have $\partial x_0^* / \partial \xi < 0$, which implies that the threshold of the initial state x_0^* decreases with respect to the reliability threshold ξ.

As $\alpha < 1$, with Equation 9, it can be obtained that U is non-decreasing with x_0^*. Therefore, it can be concluded that the maximum number of imperfect PM actions U is non-increasing with respect to the reliability threshold ξ. □

Appendix C

Derivations of Equation 18 and 19.

Assume that there are i ($i \le U$) PM cycles before corrective replacement in a renewal cycle. $P_1(j_i \mid i)$ denotes the probability that a failure occurs after completing j_i missions, given that a failure has occurred in the ith PM cycle. In the ith PM cycle, the probability that the renewal cycle ends with a corrective replacement can be obtained as

$$
\begin{aligned}
& P^F(i) = \sum_{j_1} P\left(l_f > X(j_1\tau, 0) > l_a \cap X((j_1-1)\tau, 0) < l_a\right) \times \\
& \cdots \times \sum_{j_{i-1}} P\left(l_f > X(j_{i-1}\tau, x_0^{i-1}) > l_a \cap X((j_{i-1}-1)\tau, x_0^{i-1}) < l_a\right) \\
& \times \sum_{j_i} P\left(X(j_i\tau, x_0^i) > l_f \cap X((j_i-1)\tau, x_0^i) < l_a\right) \\
& = \prod_{k=1}^{i-1} \sum_{j_k} P_3(j_i \mid i) \times \sum_{j_i} P_1(j_i \mid i) \\
& = \prod_{k=1}^{i-1} \sum_{j_k} \int_{x_0^k}^{l_a} (F_{l_a}(\tau; x) - F_{l_f}(\tau; x)) f_\phi(x; (j_k-1)\tau) dx \\
& \times \sum_{j_i} \int_{x_0^i}^{l_a} F_{l_f}(\tau; x) f_\phi(x; (j_i-1)\tau) dx
\end{aligned}
$$

The probability that the renewal cycle ends with a corrective replacement can then be obtained by summing all the $P^F(i)$ for $i = 1, 2..., U+1$. Mathematically,

$$
\begin{aligned}
& P^F = \sum_{i=1}^{U+1} P^F(i) = \sum_{i=1}^{U+1} \prod_{k=1}^{i-1} \sum_{j_k} \int_{x_0^k}^{l_a} (F_{l_a}(\tau; x) - F_{l_f}(\tau; x)) f_\phi(x; (j_k-1)\tau) dx \\
& \times \sum_{j_i} \int_{x_0^i}^{l_a} F_{l_f}(\tau; x) f_\phi(x; (j_i-1)\tau) dx
\end{aligned}
$$

The expected number of imperfect PM actions in this case is given as

$$
\begin{aligned}
E[N_{M(1)}] &= \sum_{i=1}^{U+1}(i-1)P^F(i) \\
&= \sum_{i=1}^{U+1}(i-1)\prod_{k=1}^{i-1}\sum_{j_k}\int_{x_0^k}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_k-1)\tau)dx \\
&\times\sum_{j_i}\int_{x_0^i}^{l_a}F_{l_f}(\tau;x)f_\phi(x;(j_i-1)\tau)dx
\end{aligned}
$$

To compute the number of inspections in a renewal cycle $N_{I(1)}$, we first need to determine the number of completed missions j_i in the ith PM cycle. Given that the system fails in the ith PM cycle, we can have the conditional expected number of inspections as

$$
E_{\cdot|i}[N_{I(1)}] = \sum_{k=1}^{i}E[N_I^k] = \sum_{k=1}^{i-1}\sum_{j_k=1}^{\infty} j_k P_3(j_k \mid k) + \sum_{j_i=1}^{\infty} j_i P_1(j_i \mid i)
$$

The expectation of $N_{I(1)}$ can be computed by considering all the possible realizations of the number of PM cycles, which is expressed as

$$
\begin{aligned}
E[N_{I(1)}] &= \sum_{i=1}^{U+1}E_{\cdot|i}[N_{I(1)}]\cdot P^F(i) \\
&= \sum_{i=1}^{U+1}[\sum_{k=1}^{i-1}\sum_{j_k=1}^{\infty} j_k P_3(j_k \mid k) + \sum_{j_i=1}^{\infty} j_i P_1(j_i \mid i)]\cdot P^F(i) \\
&= \sum_{i=1}^{U+1}\{[\sum_{k=1}^{i-1}\sum_{j_k=1}^{\infty} j_k\int_{x_0^k}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_k-1)\tau)dx \\
&+\sum_{j_i=1}^{\infty} j_i\int_{x_0^i}^{l_a}F_{l_f}(\tau;x)f_\phi(x;(j_i-1)\tau)dx] \\
&\cdot\prod_{k=1}^{i-1}\sum_{j_k}\int_{x_0^k}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_k-1)\tau)dx \\
&\times\sum_{j_i}\int_{x_0^i}^{l_a}F_{l_f}(\tau;x)f_\phi(x;(j_i-1)\tau)dx\}
\end{aligned}
$$

Integrating the above the equations, the expected cost in a renewal cycle can be obtained as

$$E[C_1]=C_I E[N_{I(1)}]+C_M E[N_{M(1)}]+CcP^F$$
$$=C_I\sum_{i=1}^{U+1}\{[\sum_{k=1}^{i-1}\sum_{j_k=1}^{\infty} j_k\int_{x_0^k}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_k-1)\tau)dx$$
$$+\sum_{j_i=1}^{\infty} j_i\int_{x_0^i}^{l_a}F_{l_f}(\tau;x)f_\phi(x;(j_i-1)\tau)dx]$$
$$\cdot\prod_{k=1}^{i-1}\sum_{j_k}\int_{x_0^k}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_k-1)\tau)dx$$
$$\times\sum_{j_i}\int_{x_0^i}^{l_a}F_{l_f}(\tau;x)f_\phi(x;(j_i-1)\tau)dx\}$$
$$+C_M\sum_{i=1}^{U+1}(i-1)\prod_{k=1}^{i-1}\sum_{j_k}\int_{x_0^k}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_k-1)\tau)dx$$
$$\times\sum_{j_i}\int_{x_0^i}^{l_a}F_{l_f}(\tau;x)f_\phi(x;(j_i-1)\tau)dx$$
$$+Cc\sum_{i=1}^{U+1}\prod_{k=1}^{i-1}\sum_{j_k}\int_{x_0^k}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_k-1)\tau)dx$$
$$\times\sum_{j_i}\int_{x_0^i}^{l_a}F_{l_f}(\tau;x)f_\phi(x;(j_i-1)\tau)dx$$

and the expected length of a renewal cycle is

$$E[T_1]=\tau\sum_{i=1}^{U+1}\{[\sum_{k=1}^{i-1}\sum_{j_k=1}^{\infty} j_k\int_{x_0^k}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_k-1)\tau)dx$$
$$+\sum_{j_i=1}^{\infty} j_i\int_{x_0^i}^{l_a}F_{l_f}(\tau;x)f_\phi(x;(j_i-1)\tau)dx]$$
$$\cdot\prod_{k=1}^{i-1}\sum_{j_k}\int_{x_0^k}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_k-1)\tau)dx$$
$$\times\sum_{j_i}\int_{x_0^i}^{l_a}F_{l_f}(\tau;x)f_\phi(x;(j_i-1)\tau)dx\}$$

□

Appendix D

Derivations of Equation 20 and 21.

The probability for occurrence of preventive replacement is given as

$$P^U=\prod_{i=1}^{U+1}P\left(l_f>X(j_i\tau,x_0^i)>l_a\cap X((j_i-1)\tau,x_0^i)<l_a\right)$$
$$=\prod_{i=1}^{U+1}\int_{x_0^i}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_i-1)\tau)dx\cdot F_{l_a}\left((j_i-1)\tau,x_0^i\right)$$

The expected number of PM actions in this case is

$$E[N_{M(2)}]=UP^U=U\prod_{i=1}^{U+1}\int_{x_0^i}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_i-1)\tau)dx\cdot F_{l_a}\left((j_i-1)\tau,x_0^i\right)$$

The expected number of inspections is given as

$$E[N_{I(2)}]=\sum_{i=1}^{U+1}E_{\cdot|i}[N_{I(2)}]\times P^U=\sum_{i=1}^{U+1}\sum_{j_i=1}j_iP_3(j_i\,|\,i)\times P^U$$
$$=\sum_{i=1}^{U+1}\sum_{j_i=1}j_i\int_{x_0^i}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_i-1)\tau)dx\cdot F_{l_a}\left((j_i-1)\tau,x_0^i\right)$$
$$\times\prod_{i=1}^{U+1}\int_{x_0^i}^{l_a}(F_{l_a}(\tau;x)-F_{l_f}(\tau;x))f_\phi(x;(j_i-1)\tau)dx\cdot F_{l_a}\left((j_i-1)\tau,x_0^i\right)$$

Combining the above equations, Equation 20 and 21 can be readily obtained. □

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