

# Network-Level Performance Evaluation of a Two-Relay Cooperative Random Access Wireless System

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## Abstract

In wireless networks relay nodes can be used to assist the users' transmissions to reach their destination. Work on relay cooperation, from a physical layer perspective, has up to now yielded well-known results. This paper takes a different stance focusing on network-level cooperation. Extending previous results for a single relay, we investigate here the benefits from the deployment of a second one. We assume that the two relays do not generate packets of their own and the system employs random access to the medium; we further consider slotted time and that the users have saturated queues. We obtain analytical expressions for the arrival and service rates of the queues of the two relays and the stability conditions. We investigate a model of the system, in which the users are divided into clusters, each being served by one relay, and show its advantages in terms of aggregate and throughput per user. We quantify the above, analytically for the case of the collision channel and through simulations for the case of Multi-Packet Reception (MPR), and we provide insight on when the deployment of a second relay in the system can yield significant advantages.

## I. INTRODUCTION

Cooperative communications have gained significant attention lately. Cooperation can take place in difference communication layers, with the bulk of interest focusing on physical layer performance [2],

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[3]. In that level, cooperation benefits are self-evident, since the explored systems typically belong to a single actor with interest to maximize a specific utility [4]. Promoting cooperation at higher layers, has also drawn significant attention due to the potential benefits from operators and users. Focusing on the purely network layer the benefits of utilizing cooperative techniques have been recently shown to be multi-fold, with respect to system performance in terms of throughput [5]–[10], reliability [11] and delay [8]. In that regard the use of dedicated relays has been introduced in many practical systems, such as wi-fi (known as range extenders) and in LTE.

#### *A. Related Work*

The notion of cooperative communications was introduced by information theory with the relay channel. The relay channel is the basic building block for the implementation of cooperative communications, which are widely acknowledged to provide higher communication rates and reliability in a wireless network with time varying channels [3]. It was initially proposed by van der Meulen [12], and its first information-theoretic characterizations were presented in [13].

Recently, the study of the relay channel has gained significant interest in the wireless communications community. In [14] for the classic relay channel a protocol is presented for selection of reception and transmission time slots adaptively and based on the quality of the involved links. Considering full-duplex and half-duplex relaying [15] shows that if the numbers of antennas at source and destination are equal to or larger than the number of antennas at the relay, half-duplex relaying can achieve in some cases higher throughput than ideal full-duplex relaying. With beamforming and taking inter-relay interference [16] proposes two buffer-aided relay selection schemes. Interference cancellation is employed in [17] to allow opportunistic relaying selection maximising the average capacity of the network. For a practical system, OFDMA based cellular resource allocation schemes are proposed in [18] for multiple relay stations (RS) with adaptive RS activation.

As mentioned, the majority of the works in this area focus on potential gains by cooperation on the physical layer. Recent works [5] and [6] suggest that similar gains can be achieved by network-layer cooperation. By network-layer cooperation they consider relaying to be taking place at a protocol level avoiding physical layer considerations. Random multiple access schemes in these works use the

collision channel model with erasures, where concurrent transmissions will fail [6], [19], [20]. The collision channel however is not the appropriate model for wireless networks.

Random access with Multi-Packet Reception (MPR) capabilities has attracted attention recently [21]–[24]. The seminal paper [21] was the first to examine MPR as an interaction between the physical and medium access control layers for a wireless random access network. In [22], the notion of MPR was introduced and two important theorems for the slotted ALOHA network with MPR are provided. They consider the effect of MPR on stability and delay of slotted Aloha based random-access system and it is shown that the stability region undergoes a phase transition from a concave region to a convex polyhedral region as the MPR capability improves in a two-user system. In [23], the authors specify a general asymmetric MPR model and the medium access control capacity region. In [24], the impact of a relay node to a network with a finite number of users-sources and a destination node is investigated. In this network the relay and the destination nodes have MPR capabilities. Analytical expressions for the characteristics of the relay node queue such as average queue length, stability conditions etc. were obtained. Finally, an overview of MPR-related research work covering the theoretically proved impacts and advantages of using MPR from a channel perspective to network capacity and throughput, the various technologies that enable MPR from transmitter, transceiver, and receiver perspectives and previous work on protocol improvement to better exploit MPR, is provided in [25].

### *B. Contribution*

In this work, we provide a thorough study of the impact of using two relay nodes in a network to assist with relaying packets from a number of users to a destination node. We first investigate the system analytically, assuming the collision channel; then we move to assume that the system is MPR enabled and we conduct a thorough, system-level simulation study. Our common assumptions in both models are that (i) users have saturated queues and random access to the medium with slotted time<sup>1</sup>; (ii) the transmission of a packet takes the duration of exactly one time slot; (iii) the two relays are dedicated, i.e. do not have packets of their own, but assist the users by relaying their packets when necessary; (iv)

<sup>1</sup>Dealing with analytical performance evaluation of random access systems above three users with random arrivals is mathematically intractable. Specifically, assuming the sources saturated, the so-called saturated throughput can be obtained and is an inner bound of the stable throughput [26], [27].

the wireless link between any two nodes of the network is a Rayleigh narrowband flat-fading channel with additive Gaussian noise.

In the first part, we obtain analytical expressions for the arrival and service rates of the queues of the two relays, and for the stability conditions. In doing so we use the stochastic dominance technique [28] because the two queues are coupled (i.e., the service process of each queue depends on the other queue having a packet to send or not). We also look into a topology of the system in which the users are divided into two clusters. In this scenario, we consider that the users of one cluster do not interfere with the users or the relay of the other cluster, still, the relays are interfering with each other. This corresponds to the case of having the users in two distant areas. However, since the location of the users is captured by the link success probability, this scenario can cover any similar case, in which a system practitioner could leverage sophisticated clustering techniques to approach our results, even in an on-line fashion. In general, clustering can deliver results depending on the topology of the users [29]. For both scenarios (with and without clustering) we study the impact of the two relay nodes of the two cases on the aggregate throughput and the throughput per user when the queues of the two relays are stable. We show that the probabilities of the two relays to attempt transmission do not depend on each other when the queues are stable. The insertion of the second relay offers a significant performance gain (higher throughput) when the users are divided into clusters and each cluster is assigned to one relay, though in the general un-clustered scenario the gains are not as significant.

Under the MPR model, the transmission of a node  $j$  is successful if the received Signal to Interference plus Noise Ratio (SINR) is above a threshold  $\gamma_j$ . Here, due to queue coupling the stability analysis and the derivation of analytical expressions for the characteristics of the relays' queues such as arrival and service rates, are not tractable. We therefore conduct extensive simulations to provide a comprehensive insight into the performance of the two-relay system. We show that the use of two relays offers significant advantage in terms of aggregate and throughput per user compared to systems with one and no relay, for values of SINR threshold  $\gamma > 1$ . Under the clustering scenario employed in the first part we study the impact on the aggregate and throughput per user compared to the cases of no relay, one relay and two variations of two relay nodes' operation: a packet received by both relays is either kept by (i) both nodes or (ii) by the one with the smallest queue of the two. Finally, we provide insight for the average

queue size and the average delay per packet of the systems presented.

The paper is structured as follows: in Section II we describe the system model. In Section III we derive the analytical expressions for: (A) the arrival and service rate of the relays' queues, (B) the stability conditions of the queues, (C) the stability region and (D) the throughput per user along with the upper and lower bounds. In Section IV we present the numerical and simulation study of the analytically obtained results of Section III, while in section V we conduct a thorough simulation study for the MPR model. We give our conclusions in Section VI.

## II. SYSTEM MODEL

### A. Network Model

We consider a network with  $N$  source users, two relay nodes  $R_1$  and  $R_2$  and a common destination node  $d$ , a case for  $N = 2$  is depicted in Fig. 1. The sources transmit packets to the destination with the cooperation of the two relays. We assume that the queues of the users are saturated. The users have random access to the medium with no coordination among them. The channel is slotted in time and the transmission of a packet takes the duration of exactly one time slot. We assume fixed packet size, which could be viewed as an average packet size, since taking into account variable packet sizes would severely complicate the analysis. The acknowledgements (ACKs) of successful transmissions are instantaneous and error free. With this set of assumptions, and especially random access of the medium, a host of system parameters that could be available, such as channel state information for the links is not required nor considered in our work.

The relays do not generate packets of their own. If a transmission of a user's packet to the destination fails, the relays store it in their queues and try to forward it to the destination at a subsequent time slot. In case that both relays receive the same packet from a user, they choose randomly and with equal probability which will store it in its queue. The queues at the relays have infinite size.

In this work we consider two cases for the relays and the destination, either that they are equipped with single transceivers thus, a simultaneous transmission attempts by two or more nodes (source-users or relays) result in a collision, or that they are equipped with multiuser detectors, so that they may decode packets successfully from more than one transmitter at a time. The specifics for these are given in the following section.

The notation we consider throughout this paper is the following: The users attempt to transmit with probabilities  $q_i$ , where  $i = 1, 2, \dots, N$ . Each of the relays, having not saturated queues, attempts to transmit with probability  $q_{R_j}$ ,  $j = 1, 2$  if its queue is not empty. Thus probability that a relay will transmit a packet at a time slot  $t$  is  $q_{R_i} \Pr(Q_{R_i}^t > 0)$ , where  $i = 1, 2$  and  $Q_{R_i}^t$  indicates the size of the queue at time slot  $t$ .

### B. Physical Layer Model

1) *Collision Channel*: We model the link between two nodes  $i$  and  $j$  of the network as a Rayleigh narrowband flat-fading channel with additive Gaussian noise. The outage probability of that link with SNR threshold  $\gamma_j$  is known [30] to be  $\Pr(SNR_{ij} < \gamma_j) = 1 - \exp(-\gamma_j n_j r_{ij}^\alpha / P_{tx}(i))$  where  $P_{tx}(i)$  is the transmission power of node  $i$ ,  $r_{ij}$  is the distance between nodes  $i$  and  $j$ ,  $\alpha$  is the path loss exponent and  $n_j$  is the power of the additive white Gaussian noise at  $j$ . So, by  $p_{ij}$  we denote the success probability of a transmission between nodes  $i$  and  $j$ , which is  $p_{ij} = \exp(-\gamma_j n_j r_{ij}^\alpha / P_{tx}(i))$ .

The average service rate seen by the relay  $R_1$  is

$$\mu_{R_1} = q_{R_1} p_{R_1 d} [1 - q_{R_2} \Pr(Q_{R_2} > 0)] \prod_{i=1}^N (1 - q_i). \quad (1)$$

Because of the collision channel, all the users should remain silent, which is with probability  $\prod_{i=1}^N (1 - q_i)$ , also the relay  $R_2$  should remain silent, with probability  $1 - q_{R_2} \Pr(Q_{R_2} > 0)$ . Furthermore, relay  $R_1$  has to be active, with probability  $q_{R_1}$  and the transmission to the destination successful with probability  $p_{R_1 d}$ .

Similarly, the average service rate seen by relay  $R_2$  is

$$\mu_{R_2} = q_{R_2} p_{R_2 d} [1 - q_{R_1} \Pr(Q_{R_1} > 0)] \prod_{i=1}^N (1 - q_i). \quad (2)$$

Since the average service rate of each queue depends on the state of the other, the problem of coupled queues arises. Thus, we will apply the stochastic dominance approach to bypass this difficulty.

2) *MPR*: In the wireless environment, the collision channel is restrictive, since we can not have more than one successful transmissions simultaneously. Thus, we also consider the MPR channel model, which is a generalized form of the packet erasure model [24]. In the MPR case, a node's transmission

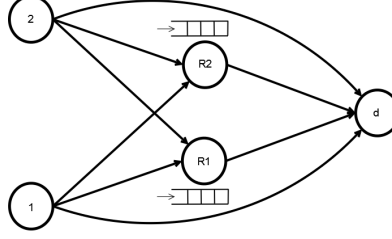


Fig. 1: Two relay nodes with  $N = 2$  user nodes.

is successful if the SINR is above a certain threshold. More specifically, if there exists a set of  $T$  nodes transmitting in the same time slot and  $P_{rx}(i, j)$  is the signal power received from node  $i$  at node  $j$  (when  $i$  transmits), then the  $\text{SINR}(i, j)$  determined by node  $j$  is given by

$$\text{SINR}(i, j) = \frac{P_{rx}(i, j)}{n_j + \sum_{k \in T \setminus \{i\}} P_{rx}(k, j)},$$

where  $n_j$  is the receiver noise power at  $j$ .

We assume that a packet transmitted by  $i$  is successfully received by  $j$  if and only if  $\text{SINR}(i, j) \geq \gamma_j$ , where  $\gamma_j$  is a threshold characteristic of node  $j$ . Moreover, the wireless channel is subject to fading. Let  $P_{tx}(i)$  be the transmitting power of node  $i$  and  $r_{ij}$  be the distance between  $i$  and  $j$ . Then, the power received by  $j$  when  $i$  transmit is  $P_{rx}(i, j) = A(i, j)g(i, j)$ , where  $A(i, j)$  is a random variable representing channel fading and under Rayleigh fading it is exponentially distributed [30]. The receiver power factor  $g(i, j)$  is given by  $g(i, j) = P_{tx}(i)r_{ij}^{-\alpha}$ , where  $\alpha$  is the path loss exponent with typical values between 2 and 4. The average success probability of a packet over link  $ij$  when the transmitting nodes are in  $T$  is given by [30]

$$P_{i/T}^j = \exp\left(-\frac{\gamma_j \eta_j}{v(i, j)g(i, j)}\right) \prod_{k \in T \setminus \{i, j\}} \left(1 + \gamma_j \frac{v(k, j)g(k, j)}{v(i, j)g(i, j)}\right)^{-1},$$

where  $v(i, j)$  is the parameter of the Rayleigh random variable for fading.

**Remark 1.** In this work, the MPR case is considered only with simulations since the analytical expressions even for the case of one relay are rather complicated [24]. Additionally, with small values of the SINR threshold  $\gamma$  is more likely to have more successful simultaneous transmissions comparing to larger  $\gamma$ . More specifically, if  $\gamma < 1$  it is possible for two or more users to transmit successfully at

the same time, comparing to  $\gamma > 1$  which that probability is almost zero in the considered system setup.

### C. Queue Stability

We adopt the definition of queue stability used in [31].

**Definition 1.** Denote by  $Q_i^t$  the length of queue  $i$  at the beginning of timeslot  $t$ . The queue is said to be stable if

$$\lim_{t \rightarrow \infty} \Pr[Q_i^t < x] = F(x) \text{ and } \lim_{x \rightarrow \infty} F(x) = 1.$$

If  $\lim_{x \rightarrow \infty} \lim_{t \rightarrow \infty} \inf \Pr[Q_i^t < x] = 1$ , the queue is substable. If a queue is stable, then it is also substable. If a queue is not substable, then we say it is unstable.

Loynes' theorem [32] states that if the arrival and service processes of a queue are strictly jointly stationary and the average arrival rate is less than the average service rate, then the queue is stable.

## III. ANALYSIS

In this section we will present the analysis for the collision channel model. We obtain analytical equation for the arrival and the service rate of the two relays and also the stability region of the system. Additionally, we obtain the throughput per user as well as the aggregate throughput of the system.

In order to proceed further we need to calculate the average arrival rates at the queues of the relays.

There is an arrival at the queue of relay  $R_1$  if both relays are silent, only one user transmits, and its transmission is successfully received by  $R_1$  but not by the destination. When both relays receive the packet then the first will store it in its queue with probability  $\frac{1}{2}$  otherwise the second relay will store it.

The probability that both relays are silent depends on the state of the queues at the relays. Both relays are silent when their queues are empty, which happens with probability  $\Pr(Q_{R_1} = 0, Q_{R_2} = 0)$ , when the  $i$  relay has a non-empty queue but the queue at the  $j \neq i$  relay is empty then the probability that both relays are silent is  $(1 - q_{R_i})\Pr(Q_{R_i} > 0, Q_{R_j} = 0)$ . The probability that both relays are silent when their queues are not empty is  $(1 - q_{R_1})(1 - q_{R_2})\Pr(Q_{R_1} > 0, Q_{R_2} > 0)$ . The average arrival rate at the first relay,  $\lambda_{R_1}$ , is



$$\begin{aligned} \lambda_{R_1} = & [\Pr(Q_{R_1} = 0, Q_{R_2} = 0) + (1 - q_{R_1})\Pr(Q_{R_1} > 0, Q_{R_2} = 0) + (1 - q_{R_2})\Pr(Q_{R_1} = 0, Q_{R_2} > 0) + \\ & + (1 - q_{R_1})(1 - q_{R_2})\Pr(Q_{R_1} > 0, Q_{R_2} > 0)] \sum_{i=1}^N q_i p_{iR_1} (1 - p_{id}) \left[ (1 - p_{iR_2}) + \frac{1}{2} p_{iR_2} \right] \prod_{j=1, j \neq i}^N (1 - q_j), \end{aligned} \quad (3)$$

which after simple manipulation becomes

$$\begin{aligned} \lambda_{R_1} = & [1 - q_{R_2}\Pr(Q_{R_2} > 0) - q_{R_1}\Pr(Q_{R_1} > 0) - q_{R_1}q_{R_2}\Pr(Q_{R_1} > 0, Q_{R_2} > 0)] \times \\ & \times \sum_{i=1}^N q_i p_{iR_1} (1 - p_{id}) \left[ (1 - p_{iR_2}) + \frac{1}{2} p_{iR_2} \right] \prod_{j=1, j \neq i}^N (1 - q_j). \end{aligned} \quad (4)$$

Symmetrically, we have that the arrival rate at the second relay,  $\lambda_{R_2}$ , is

$$\begin{aligned} \lambda_{R_2} = & [1 - q_{R_1}\Pr(Q_{R_1} > 0) - q_{R_2}\Pr(Q_{R_2} > 0) - q_{R_1}q_{R_2}\Pr(Q_{R_1} > 0, Q_{R_2} > 0)] \times \\ & \times \sum_{i=1}^N q_i p_{iR_2} (1 - p_{id}) \left[ (1 - p_{iR_1}) + \frac{1}{2} p_{iR_1} \right] \prod_{j=1, j \neq i}^N (1 - q_j). \end{aligned} \quad (5)$$

With the previous expressions for  $\lambda_{R_1}$  and  $\lambda_{R_2}$  we cannot proceed further, since each rate depends on the joint probability density function of the queues. This is a well known non-tractable problem, and in order to bypass this difficulty we will deploy the stochastic dominance technique [28] in order to decouple the queues. The stochastic dominance technique was initially developed to overcome the intractability arising in the analysis of the inseparable multidimensional Markov chain for finite-user buffered slotted ALOHA<sup>2</sup>.

#### A. Computation of Arrival and Service Rate

The stochastic dominance approach implies the construction of two hypothetical dominant systems. In the first system, say  $\mathcal{S}_1$ , the relay  $R_1$  reverts to the transmission of “dummy packets” with the same probability, when its queue is empty. All the other characteristics and assumptions of the original system

<sup>2</sup>The stochastic dominance technique was introduced in [28] however, a brief introduction can be found in [33].

remain exactly the same. Similarly, in the second system  $\mathcal{S}_2$  the relay  $R_2$  reverts to the transmission of “dummy packets” with the same probability, when its queue is empty.

1) Dominant system  $\mathcal{S}_1$  – Relay  $R_1$  transmits “dummy packets”: In the first dominant system, the relay  $R_1$  transmits “dummy packets” when its queue is empty, thus  $\Pr(Q_{R_1} > 0) = 1$ . The average arrival rates  $\lambda_{R_1}$  and  $\lambda_{R_2}$  are

$$\lambda_{R_1} = (1 - q_{R_1}) [1 - q_{R_2} \Pr(Q_{R_2} > 0)] \sum_{i=1}^N q_i p_{iR_1} (1 - p_{id}) \left[ (1 - p_{iR_2}) + \frac{1}{2} p_{iR_2} \right] \prod_{j=1, j \neq i}^N (1 - q_j) \quad (6)$$

$$\lambda_{R_2} = (1 - q_{R_1}) [1 - q_{R_2} \Pr(Q_{R_2} > 0)] \sum_{i=1}^N q_i p_{iR_2} (1 - p_{id}) \left[ (1 - p_{iR_1}) + \frac{1}{2} p_{iR_1} \right] \prod_{j=1, j \neq i}^N (1 - q_j). \quad (7)$$

The service rate of the relay  $R_2$  in the system  $\mathcal{S}_1$  is given by

$$\mu_{R_2} = q_{R_2} p_{R_2d} (1 - q_{R_1}) \prod_{i=1}^N (1 - q_i), \quad (8)$$

and  $\mu_{R_1}$  is given by (1). The probability that the queue at the  $R_2$  is not empty is  $\Pr(Q_{R_2} > 0) = \frac{\lambda_{R_2}}{\mu_{R_2}}$  which can be obtained from Little’s law. However, since the average arrival rate for the second relay,  $\lambda_{R_2}$ , depends on the  $\Pr(Q_{R_2} > 0)$  we cannot directly apply the previous expression. Furthermore,  $\lambda_{R_1}$  and the service rate  $\mu_{R_1}$  depend on the state of the queue of the second relay. Thus, we follow the procedure described in [19]. We model the queue at the  $R_2$  as a Discrete Time Markov Chain (DTMC) with infinite states in order to describe the queue evolution. The DTMC is depicted in Fig. 2. The arrival rate of relay  $R_2$  depends on whether its queue is empty or not. If the queue is empty the arrival rate is denoted by  $\lambda_{R_2,0}$  and by  $\lambda_{R_2,1}$  if it is not. Thus, the average arrival rate  $\lambda_{R_2}$  can be expressed also as

$$\lambda_{R_2} = \Pr(Q_{R_2} = 0) \lambda_{R_2,0} + \Pr(Q_{R_2} > 0) \lambda_{R_2,1}. \quad (9)$$

If the queue at the relay  $R_2$  is empty then, we can easily show that the probability of arrival  $\lambda_{R_2,0}$  is

$$\lambda_{R_2,0} = (1 - q_{R_1}) \sum_{i=1}^N q_i p_{iR_2} (1 - p_{id}) \left[ (1 - p_{iR_1}) + \frac{1}{2} p_{iR_1} \right] \prod_{j=1, j \neq i}^N (1 - q_j). \quad (10)$$

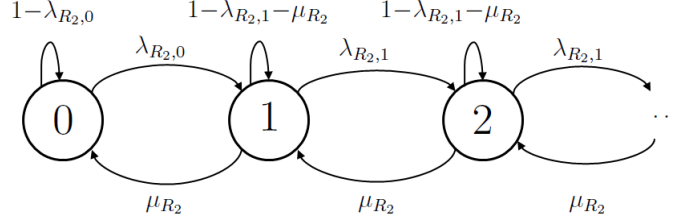


Fig. 2: The Discrete Time Markov Chain model of the queue at the relay  $R_2$  for the first dominant system  $\mathcal{S}_1$ .

If the queue is not empty then the arrival rate is

$$\lambda_{R_2,1} = (1 - q_{R_2})\lambda_{R_2,0}. \quad (11)$$

By following the same methodology as in [19] and applying the method of balance equations, we can compute the stationary distribution of the states  $i$ . The stationary distribution of the Markov Chain exists if and only if  $\lambda_{R_2,1} < \mu_{R_2}$ . Thus, the probability that the queue at the  $R_2$  is empty is given by

$$\Pr(Q_{R_2} = 0) = \frac{\mu_{R_2} - \lambda_{R_2,1}}{\mu_{R_2} - \lambda_{R_2,1} + \lambda_{R_2,0}}. \quad (12)$$

From (10), (11), (12) and (9) we obtain

$$\lambda_{R_2} = \frac{\mu_{R_2}\lambda_{R_2,0}}{\mu_{R_2} - \lambda_{R_2,1} + \lambda_{R_2,0}}. \quad (13)$$

Combining (9), (10), (11) and (13) we obtain the expression of the arrival rate  $\lambda_{R_2}$ , shown in (14), from which we see that  $\lambda_{R_2}$  does not depend on  $q_{R_2}$ , the probability of transmission of  $R_2$ .

$$\lambda_{R_2} = \frac{p_{R_2d} \prod_{i=1}^N (1 - q_i) [\sum_{i=1}^N q_i p_{iR_2} (1 - p_{id}) (1 - q_{R_1}) \prod_{j=1, j \neq i}^N (1 - q_j) [(1 - p_{iR_1}) + \frac{1}{2} p_{iR_1}]]}{p_{R_2d} \prod_{i=1}^N (1 - q_i) + \sum_{i=1}^N q_i p_{iR_2} (1 - p_{id}) \prod_{j=1, j \neq i}^N (1 - q_j) [(1 - p_{iR_1}) + \frac{1}{2} p_{iR_1}]}. \quad (14)$$

2) Dominant system  $\mathcal{S}_2$  – Relay  $R_2$  transmits “dummy packets”: By following exactly the same procedure as in system  $\mathcal{S}_1$ , we obtain the expressions for  $\mu_{R_1}$ ,  $\lambda_{R_1}$ ,  $\mu_{R_2}$  and  $\lambda_{R_2}$ .

### B. Necessary and Sufficient Stability Conditions

The stability region of the system is defined as the set of arrival rate vectors  $(\lambda_{R_1}, \lambda_{R_2})$  for which the queues in the system are stable. In order to derive the stability region we need to characterize the average

arrival rates  $\lambda_{R_1}$  and  $\lambda_{R_2}$  as well as the average service rates  $\mu_{R_1}$  and  $\mu_{R_2}$ . A tool to obtain stability condition for a queue is the Loyne's criterion [32], which states that if the arrival rate is less than the service rate then the queue is stable. The average service rates are given by (1) and (2) thus the service rate of each queue depends on the state of the other, thus we cannot apply the Loyne's criterion directly. We will apply the stochastic dominance technique, which was presented in the previous subsection for decoupling the queues and to obtain necessary and sufficient conditions for the stability. Recall that there are two dominant systems  $S_1$  and  $S_2$ . In  $S_i$  where in the relay  $R_i$  transmits dummy packets when its queue is empty and all the other assumptions remain unaltered. Note that the expressions for the average arrival and service rates change from one dominant system to another since they depend on the probability that a queue is empty.

In the first dominant system  $S_1$ , we have that the queues are stable if  $\lambda_{R_1} < \mu_{R_1}$  and  $\lambda_{R_2} < \mu_{R_2}$ . The expression for the service rate  $\mu_{R_2}$  is given by (8) and the service rate  $\mu_{R_1}$  by (1). Thus, using Little's law we obtain that

$$\Pr(Q_{R_2} > 0) = \frac{\lambda_{R_2}}{q_{R_2} p_{R_2} d (1 - q_{R_1}) \prod_{i=1}^N (1 - q_i)}. \quad (15)$$

After replacing (15) into (1) we obtain that

$$\mu_{R_1} = q_{R_1} p_{R_1} d \left( 1 - q_{R_2} \frac{\lambda_{R_2}}{q_{R_2} p_{R_2} d (1 - q_{R_1}) \prod_{i=1}^N (1 - q_i)} \right) \prod_{i=1}^N (1 - q_i). \quad (16)$$

Now we can apply Loyne's criterion for both queues and obtain the region  $\mathcal{R}_1$  from the first dominant system which is given by

$$\mathcal{R}_1 = \left\{ (\lambda_{R_1}, \lambda_{R_2}) : \lambda_{R_1} < q_{R_1} p_{R_1} d \left( 1 - q_{R_2} \frac{\lambda_{R_2}}{q_{R_2} p_{R_2} d (1 - q_{R_1}) \prod_{i=1}^N (1 - q_i)} \right) \prod_{i=1}^N (1 - q_i), \right. \\ \left. \lambda_{R_2} < q_{R_2} p_{R_2} d (1 - q_{R_1}) \prod_{i=1}^N (1 - q_i) \right\}. \quad (17)$$

From the above condition and after using (6), (7) and (14) we can further obtain the expression for the transmission probability,  $q_{R_1}$  where

$$q_{R_1} > q_{R_1, \min} \Leftrightarrow q_{R_1} > \frac{\sum_{i=1}^N q_i p_{iR_1} (1 - p_{id}) \prod_{j=1, j \neq i}^N (1 - q_j) [(1 - p_{iR_2}) + \frac{1}{2} p_{iR_2}]}{\sum_{i=1}^N q_i p_{iR_1} (1 - p_{id}) \prod_{j=1, j \neq i}^N (1 - q_j) [(1 - p_{iR_2}) + \frac{1}{2} p_{iR_2}] + p_{R_1} d \prod_{i=1}^N (1 - q_i)}. \quad (18)$$

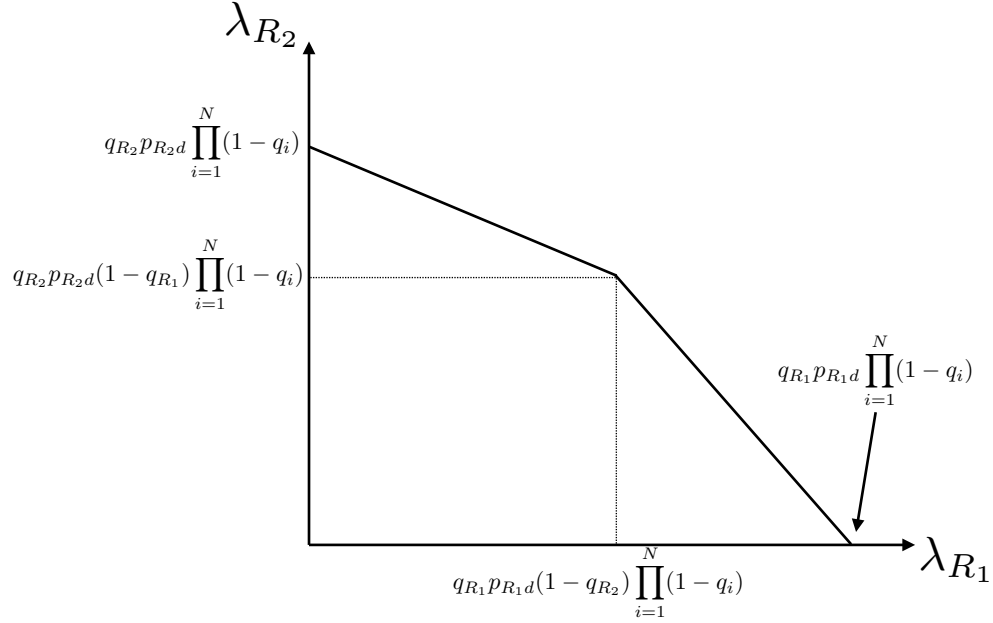


Fig. 3: The stability region,  $\mathcal{R}$ , of the system. The boundaries of the region are given by (17) and (20).

So, the  $q_{R_1, \min}$  for which the queue is stable is given by

$$q_{R_1, \min} = \frac{\sum_{i=1}^N q_i p_{iR_1} (1 - p_{id}) \prod_{j=1, j \neq i}^N (1 - q_j) [(1 - p_{iR_2}) + \frac{1}{2} p_{iR_2}]}{\sum_{i=1}^N q_i p_{iR_1} (1 - p_{id}) \prod_{j=1, j \neq i}^N (1 - q_j) [(1 - p_{iR_2}) + \frac{1}{2} p_{iR_2}] + p_{R_1 d} \prod_{i=1}^N (1 - q_i)}. \quad (19)$$

From the second dominant system,  $\mathcal{S}_2$ , symmetrically we obtain the stability region  $\mathcal{R}_2$ .

$$\mathcal{R}_2 = \left\{ (\lambda_{R_1}, \lambda_{R_2}) : \lambda_{R_2} < q_{R_2} p_{R_2 d} \left( 1 - q_{R_1} \frac{\lambda_{R_1}}{q_{R_1} p_{R_1 d} (1 - q_{R_2}) \prod_{i=1}^N (1 - q_i)} \right) \prod_{i=1}^N (1 - q_i), \right. \\ \left. \lambda_{R_1} < q_{R_1} p_{R_1 d} (1 - q_{R_2}) \prod_{i=1}^N (1 - q_i) \right\}. \quad (20)$$

Following exactly the same procedure as in  $\mathcal{S}_1$  we obtain expressions and bounds for  $q_{R_2}$ , and  $q_{R_2, \min}$  similar to (18) and (19) respectively with  $R_1$  and  $R_2$  interchanged. Similarly to system  $\mathcal{S}_1$ , we observe that  $q_{R_2}$  does not depend on  $q_{R_1}$ . The queue of the relay  $R_2$  is stable if  $q_{R_2}$  satisfies the inequality

$$q_{R_2, \min} < q_{R_2} < 1. \quad (21)$$

Finally the stability region of the system,  $\mathcal{R}$ , is  $\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$  and is shown in Fig. 3.

It is interesting to note that in [28], the stability conditions obtained by the dominant systems are not

merely sufficient, but sufficient and necessary for the stability of the original system. The proof relies on the *indistinguishability* argument which also applies in our case. By considering the properties of the dominant system  $S_1$ , we can see that the queue sizes of the two relays cannot be smaller than those in the original system, provided the queues start with identical initial conditions in both systems. By Loynes' Theorem, the stability condition of a queue is given by  $\lambda < \mu$ . Therefore, given that  $\lambda_{R_2} < \mu_{R_2}$ , if for some  $\lambda_{R_1}$  the queue  $R_1$  in the dominant system  $S_1$  is stable, then the queue is also stable in the original system. Conversely, if for some  $\lambda_{R_1}$  in the dominant system  $S_1$  the queue  $R_1$  is unstable, then it will not transmit any "dummy packets" and as long as the queue does not empty, the dominant and the original systems behave identically and as a consequence, the queue is unstable in the original system as well.

### C. Throughput Per User

In this part, we will give the expression for the average user throughput, which is the average rate of packets departing from each user. There is a departure of a packet from a node if it transmits whereas the two relays and all the other users are silent, and its transmission is either successfully received by the destination or if unsuccessful, it is successfully received by  $R_1$  or  $R_2$ . Thus, the throughput rate  $\mu_i$  seen by the user  $i$  is

$$\begin{aligned} \mu_i = q_i [p_{id} + (1 - p_{id})(p_{iR_1} + p_{iR_2} - p_{iR_1}p_{iR_2})] [\Pr(Q_{R_1} = 0, Q_{R_2} = 0) + (1 - q_{R_1})\Pr(Q_{R_1} > 0, Q_{R_2} = 0) + \\ + (1 - q_{R_2})\Pr(Q_{R_1} = 0, Q_{R_2} > 0) + (1 - q_{R_1})(1 - q_{R_2})\Pr(Q_{R_1} > 0, Q_{R_2} > 0)] \prod_{j=1, j \neq i}^N (1 - q_j), \end{aligned} \quad (22)$$

which, after some simplifications is given by

$$\begin{aligned} \mu_i = q_i [p_{id} + (1 - p_{id})(p_{iR_1} + p_{iR_2} - p_{iR_1}p_{iR_2})] \times \\ \times [1 - q_{R_2}\Pr(Q_{R_2} > 0) - q_{R_1}\Pr(Q_{R_1} > 0) - q_{R_1}q_{R_2}\Pr(Q_{R_1} > 0, Q_{R_2} > 0)] \prod_{j=1, j \neq i}^N (1 - q_j). \end{aligned} \quad (23)$$

We observe that the throughput per user depends on whether both queues are empty or not. So, it is not tractable to find an explicit expression of the throughput per user in closed form. Instead, we will

find an upper and a lower bound and by simulation we will study the tightness of these bounds.

In order to find an upper bound, we will consider the case when the two relays do not interfere with the users. This provides an upper bound because if the relays do not interfere with the users, the interference in the system is less and thus we get higher throughput per user. This is the case when we assume that the relays operate in a different channel than the users. This upper bound is given by

$$\mu_{i,upper} = q_i [p_{id} + (1 - p_{id})(p_{iR_1} + p_{iR_2} - p_{iR_1}p_{iR_2})] \prod_{j=1, j \neq i}^N (1 - q_j). \quad (24)$$

In order to find a lower bound, we will assume that the two relays have always packets in their queues, their queues never empty. This can be the case when the relays are highly utilized. The lower bound is given by

$$\mu_{i,lower} = \mu_{i,upper}(1 - q_{R_1})(1 - q_{R_2}). \quad (25)$$

However, the way we treated the relays so far, that can be reached from any user, can be sub-optimal thus, in the following subsection we consider the case of using one relay per cluster of users.

#### *D. Improving the Throughput Per User by Clustering Users*

In order to improve the throughput per user of the system, we consider the case that we divide the users into two clusters served by relays  $R_1$  and  $R_2$ . We assume that due to the distance between clusters the users of the first cluster do not interfere with the users of the second cluster at their relay. If two users transmit simultaneously we will have a collision at the destination. We also assume that when a relay transmits simultaneously with the users, the users' transmissions do not affect the relay's transmission to the destination node whereas their transmissions to the destination fail. That is because of the shorter distance between the relay and the destination and also the higher transmit power of the relay compared to that of the users'. Furthermore, when both relays transmit simultaneously we have a collision at the destination. We divide the users equally to both clusters and we assume that each cluster has  $N_k$  users with  $k = 1, 2$  where  $N_1 = N_2 = \frac{N}{2}$ .

The throughput per user of the system described depends again on whether both queues are empty or not. Thus, we find an upper and a lower bound and we will show that the results of the simulation

of that system lie between those two bounds. The upper bound of the throughput per user  $i$  of cluster  $k$  is given by:

$$\mu_{i,k,upper} = q_i p_{id} \prod_{j=1, j \neq i}^N (1 - q_j) + q_i (1 - p_{id}) p_{iR_k} \prod_{j=1}^{N_k} (1 - q_j). \quad (26)$$

The lower bound for the throughput is given by

$$\mu_{i,k,lower} = \mu_{i,k,upper} (1 - q_{R_1}) (1 - q_{R_2}). \quad (27)$$

**Remark 2.** *As presented earlier, the throughput per user is given by (23). In order to obtain the inner bounds given by (25) and (27) we did the assumption that the relays have saturated queues. These bounds become tight when the relays' queues approach saturation. The outer bounds for the throughput can be obtained by assuming that the relays' queues are always empty thus, the relays do not cause interference to the users' transmissions. Apparently, the obtained outer bounds are tight when the queues at the relays are underutilized.*

#### IV. NUMERICAL AND SIMULATION RESULTS FOR THE COLLISION CHANNEL MODEL

In this section, we present the numerical results for the per user and aggregate throughput of the system with two stable relays for the collision channel model. We directly verify (in Fig. 5) that the throughput per user for the cases of two relays lies between the upper and lower bounds given in (22)-(25). Then, we compare these two cases with the system without relay and the system with one relay. The results presented below are averages of at least 10,000 runs on each scenario verifying the accuracy of the analysis in the previous sections. We consider that all  $N$  users and both the relays have the same link characteristics and transmission probabilities for both the simple and the scenario with user clustering. All parameters in our testing are given in Table I.

The stability region for the considered scenario for  $N = 2, 4, 8$  users is depicted in Fig. 4. As the number of users increases we see that the boundary of the region shrinks, which is expected since the number of collisions increases in the network.



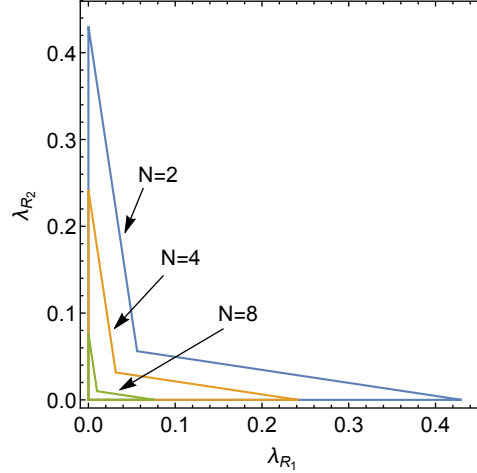


Fig. 4: The stability region of the system described in Table I for  $N = 2, 4, 8$  users.

#### A. Throughput Per User

The plots of Fig. 5 present the throughput per user versus the number of users of the “simple” scenario described in Section III-C and the “clustering” scenario of Section III-D. As expected the simulations lie between the lower and the upper bounds defined in previous sections. In Fig. 5a, when the number of the users in the system increases, we see that the throughput per user tends to the lower bound. This is because the relays’ queues are approaching saturation as the number of users increases. In this case the lower bound becomes tight.

In Fig. 5b, the throughput per user tends to the upper bound, because the relays’ queues tend to be empty most of the time. Thus, we have a better utilization of the system with clustering due to the reduction of concurrent transmissions per relay resulting in less collisions. Furthermore, one can observe in the simulation curves, the result of orthogonalizing user transmissions to the relays via clustering as an effective doubling of the throughput for more than 4 users.

#### B. The Benefit of Using a Second Relay

The plots of Fig. 6 present the per user and aggregate throughput versus the number of users for the cases of no relay, one relay and two relays (with and without clustering), obtained by simulation. We observe that the simple system with two relays does not offer any advantage over the system with one relay. This is expected because the insertion of a second relay with high probability to attempt

TABLE I: Parameters for the collision channel model results.

| <i>Notation</i>                       | <i>Explanation</i>   | <i>Value in “Simple” Scenario</i>          | <i>Value in “Clustering” Scenario</i>                         |
|---------------------------------------|--|--|---|
| $p_{iR_j}, i = 1, \dots, N, j = 1, 2$ | Success probability of transmission from user $i$ to relay $j$                         | $p_{iR_1} = p_{iR_2} = 0.9, i = 1 \dots N$ | $p_{iR_1} = p_{jR_2} = 0.9, i = 1 \dots N_1, j = 1 \dots N_2$ |
| $p_{R_jd}, j = 1, 2$                  | Success probability of transmission from relay $j$ to the destination                  | $p_{R_jd} = 0.9, j = 1, 2$                 |   |
| $p_{id}, i = 1, \dots, N$             | Success probability of transmission from user $i$ to the destination                   | $p_{id} = 0.25$                            |   |
| $q_{R_j}, j = 1, 2$                   | Probability that relay attempts to transmit in a timeslot, (if its queue is not empty) | $q_{R_1} = q_{R_2} = 0.85$                 |   |
| $q_i, i = 1, \dots, N$                | Probability that user $i$ attempts to transmit in a timeslot                           | $q_i = 0.25, i = 1, \dots, N$              |   |

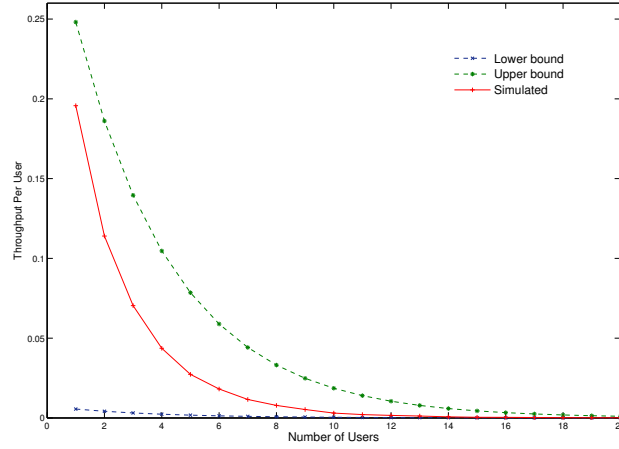
transmission, when its queue is not empty, generates more interference in the system. However, this interference is alleviated in the clustering scenario which thus offers significant advantage over the system with one relay (more than 300% higher aggregate throughput in our specific setup).

### C. Average Per Packet Delay

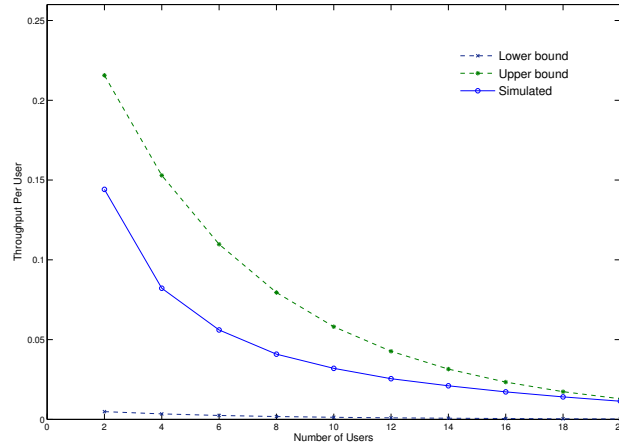
Another important parameter in cooperative systems is the average delay per packet. By delay we refer to the time it takes from the moment a packet has been transmitted until it is delivered to the destination. This parameter is important especially in delay-sensitive networks for real-time services. Thus, here we investigate the average delay per packet of systems with two relays and compare it with the systems with one and no relay.

Fig. 7 presents the average delay per packet (counted in timeslots) versus the number of users for the cases of no relay, one relay and two relays (with and without clustering), obtained by simulation.

The clustered system with two relays provides the lowest average delay compared to the other systems. Furthermore, the clustered systems appears to be more prone to the increase of the number of the users. Above eight users, the no relay system faces lower average delay than the one and the simple two-relay



(a) "Simple" Scenario



(b) "Clustering" Scenario

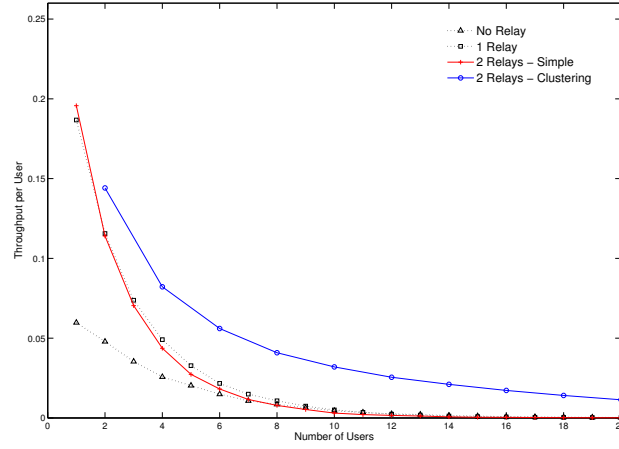
Fig. 5: Throughput per user vs. the number of users.

systems. The reason is that the system without relay does not suffer from additional delays introduced by packets queueing at the relay; this queueing delay increases with the number of users affecting the average per packet delay.

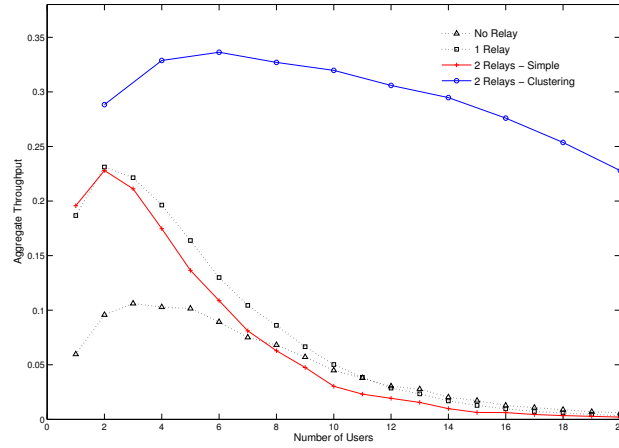
## V. SIMULATION RESULTS FOR THE MPR MODEL

### A. The Performance Benefits of Using a Second Relay

Here we present the aggregate and throughput per user for the cases with no relay, one relay, and two relays in the system for the system with MPR, under different values of an assumed SINR threshold



(a) Throughput per user



(b) Aggregate throughput

Fig. 6: Comparisons of throughput vs. number of users, with the Simple and Clustering Scenarios, against one relay and no relay.

$\gamma$ . We examine two strategies to handle a user's packet successfully received at both relays: either that (a) both relays will store and forward it to the destination (*Simple*) or that (b) the packet is stored by the relay which has the smaller queue size (*Smaller Queue Stores Packet*). If the queue size of the two relays is equal, then the two relays choose randomly and with equal probability which one will store the packet in its queue. Furthermore, as we previously did for the collision channel model we also study the potential impact of dividing the users into two clusters served by relays  $R_1$  and  $R_2$ .

An example topology of a two-relay test network with  $N$  collocated users is depicted in Fig. 8. The

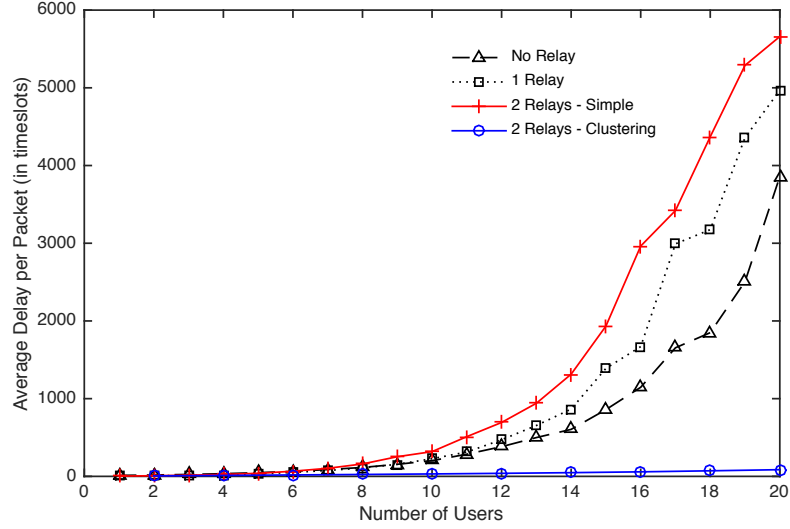


Fig. 7: Comparison of average per packet delay (in timeslots) vs. number of users, with Simple and Clustering scenarios, against one relay and no relay.

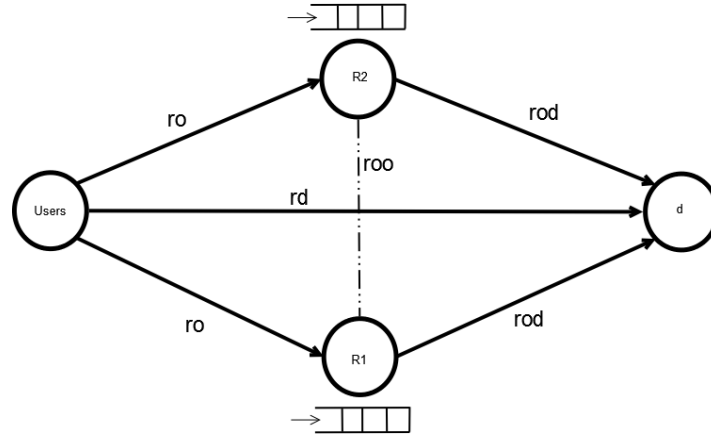


Fig. 8: Two relay nodes with  $N$  users with same link characteristics and transmission probabilities.

parameters used in the simulations for each of the three cases are shown in Table II. To simplify the presentation, we consider that all users have the same transmission probabilities and all links to have the same SINR threshold  $\gamma$ . Note that, with small values of  $\gamma$  it is more likely to have more successful simultaneous transmissions comparing to larger  $\gamma$ . For  $\gamma < 1$  the probability for two or more nodes to transmit successfully at the same time is higher than the same probability when  $\gamma > 1$ , which tends to zero [24].

We note the assumptions made in our simulations. First, that the path loss exponent between users-

TABLE II: Simulation parameters for the MPR model results.

| <i>Notation</i>            | <i>Explanation</i>   | <i>Value</i>                |
|----------------------------|--|-----------------------------|
| $r_d$                      | users-destination distance   | 100m                        |
| $r_R$                      | users-relays distance  | 59m                         |
| $r_{R'}$                   | clustered user-non-serving relay distance  | 88m                         |
| $r_{0d}$                   | relays-destination distance  | 59m                         |
| $r_{00}$                   | inter-relay distance   | 60m                         |
| $\alpha_{id}$              | users-destination path loss exponent   | 4                           |
| $\alpha_{i0}$              | users-relays path loss exponent  | 2                           |
| $\alpha_{0d}$              | relays-destination path loss exponent  | 2                           |
| $P_{tx}(i), i = 1 \dots N$ | Transmit power of user $i$   | 1mW                         |
| $P_{tx}(R_j), j = 1, 2$    | Transmit power of each relay   | 5mW                         |
| $q_i, i = 1 \dots N$       | Probability that user $i$ attempts to transmit in a timeslot                                 | $q_i = 0.25, i = 1 \dots N$ |
| $q_{R_j}, j = 1, 2$        | Probability that relay $j$ attempts to transmit in each timeslot (if its queue is not empty) | $q_{R_1} = q_{R_2} = 0.85$  |

destination as well as between the two relays is 4 while between users-relays and relays-destination is taken to be 2. This so that the relay nodes are more accessible for the users than the destination node. Thus we consider user-relay and user-destination channels that are more reliable than the user-destination one. Otherwise, the presence of the relays would degrade the performance of the network [24]. We also assume that the transmit power of the relays is five times higher than that of the users. For the “Smaller Queue Stores Packet” strategy, we assume that the relays communicate in a separate channel and thus these transmissions do not interfere with those of the system we study. For the clustering scenario, we divide the users equally to both clusters and assume that relay  $R_1$  cannot receive packets from users of cluster 2 and relay  $R_2$  cannot receive packets from users of cluster 1 respectively, this is achieved by first taking the respective path loss exponents to be equal to 4 and the distance between cluster 1 and relay  $R_2$  to be 1.5 times the distance between cluster 1 and relay  $R_1$ , and vice-versa.

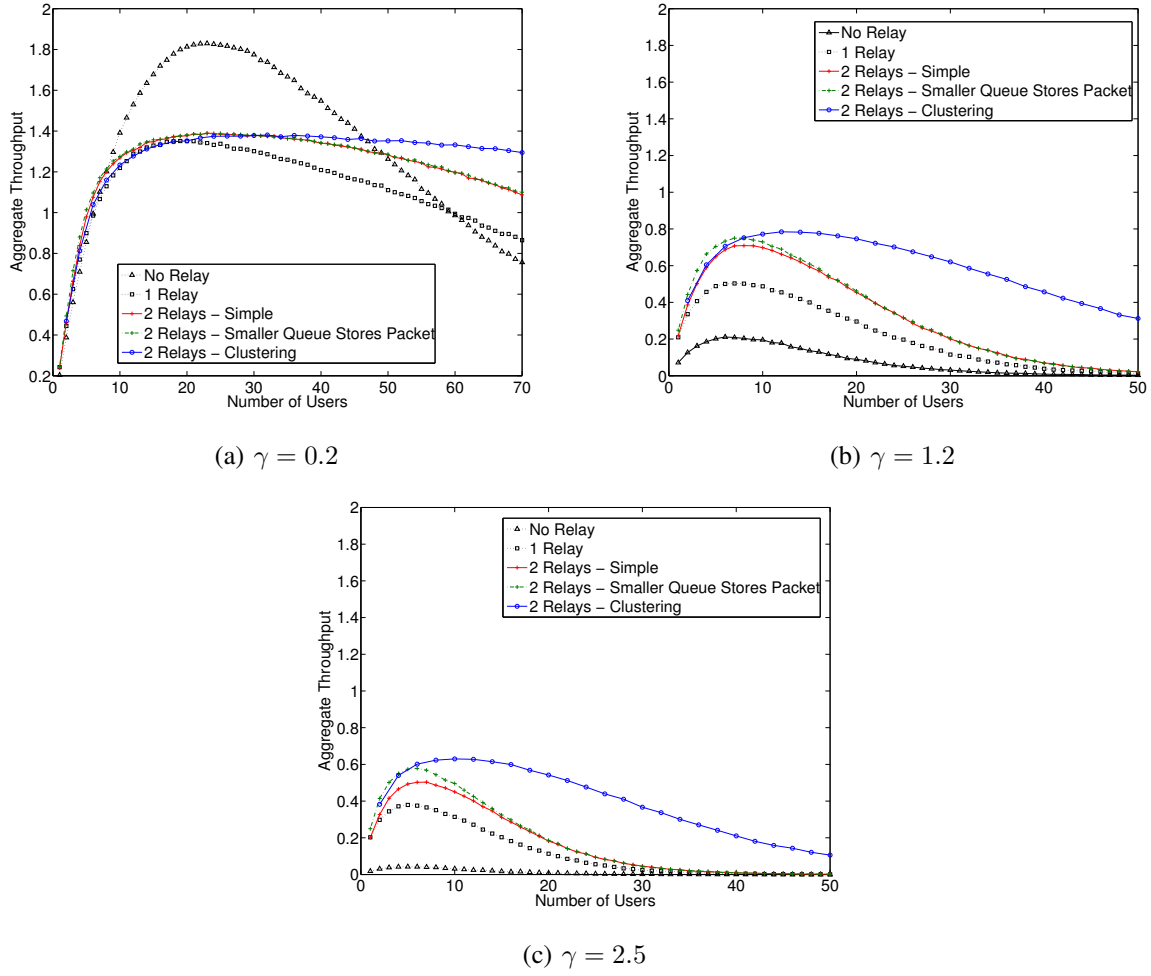


Fig. 9: Aggregate throughput (in packets per slot) vs. the number of users, for different SINR threshold  $\gamma$  values.

For a low SINR threshold value ( $\gamma = 0.2$ ), the aggregate throughput (Fig. 9a) and the throughput per user (Fig. 10a) obtained from the system with two relays are consistently, albeit slightly, higher compared to that of one relay. For the aggregate throughput, this gain increases as more users are inserted in the system. However, it is noteworthy that without relays, the system outperforms those with relay(s) regardless of clustering or forwarding strategy, for 8 users and up to a little over 45 users. For more than that, the performance (always in terms of aggregate and throughput per user) by the system with two relays is higher compared to the system with no relay and increases as more users are inserted in the system. Enabling clustering of users in the two relays starts providing clear benefits over 30 users, by an approximate 15%.

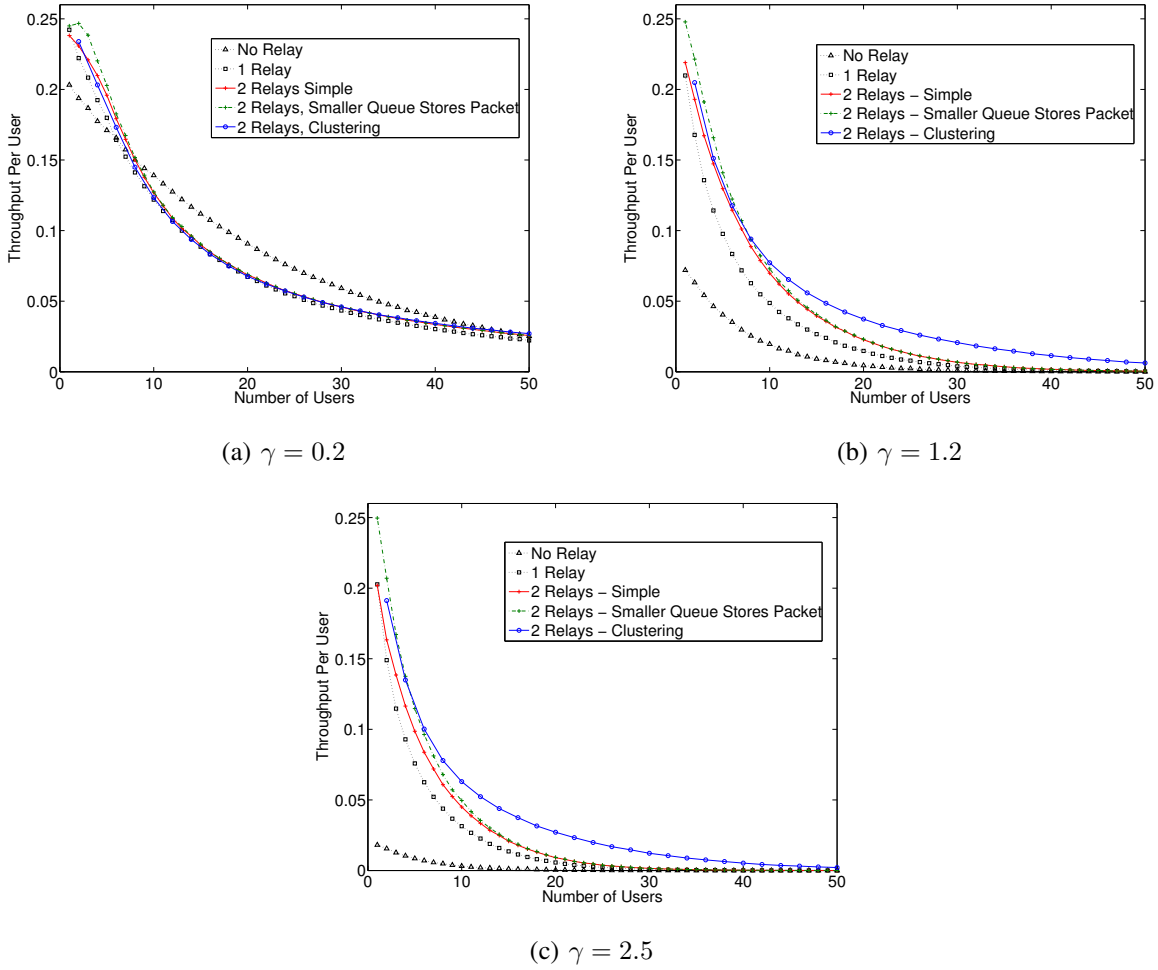


Fig. 10: Throughput per user (in packets per slot) vs. the number of users, for different SINR threshold  $\gamma$  values.

Unlike the limited performance gains observed under  $\gamma = 0.2$  with the higher SINR thresholds (Figs. 9b and 10b for  $\gamma = 1.2$ , and 9c and 10c for  $\gamma = 2.5$ ) we observe that the system with two relays offers significant advantage compared to the networks without or with one relay. This is expected since for higher values of  $\gamma$  the relays, having better channel conditions than the destination, receive a larger percentage of the transmitted packets in their queues to forward to the destination. Regarding the forwarding strategy (be it “Simple” or the “Smaller Queue Stores Packet”), across the threshold value there is a common trend that for a few users (less than 10 in low  $\gamma$ , while less than almost 20 in higher  $\gamma$  values) the latter strategy outperforms the simple one. Furthermore, with two relays and clustering, in higher SINR thresholds significant advantages are observed, for over about 10 users. This



is again expected because in each cluster the users interfere with only half the users of the system in to successfully reach the corresponding relay (the interference caused in each cluster's relay, by the users of the other cluster is almost negligible due to the distance and channel properties).

Note that, the behavior trends across the four relaying schemes remains stable above  $\gamma = 1.2$ , in both aggregate throughput and throughput per user. With this in mind, taking a system design perspective, one can finally point out that given the link characteristics and the transmission probabilities, all relaying schemes reach a maximum aggregate throughput, thus depending on the number a system is expected to serve, the network designer can deploy the most appropriate relaying scheme.

### *B. Average Queue Size*

In cooperative systems with relays, a key parameter to be taken into account is the queue size of the relays. It is important not only to keep the queues of the relays stable but also to keep their sizes as low as possible to limit delays.

The plots of Fig. 11 present the average queue size (in packets) versus the number of users for the systems with one and two relays studied in previous sections for  $\gamma = 0.2$  up to  $\gamma = 2.5$ . For the systems with two relays only the average queue size of the one relay is presented (the average queue size of the second is almost equal because we assume that we have symmetric users in the systems).

The plots in the figure show that the average queue size of the clustered system is higher compared to the other systems for a number of users that reduces with the increase of the threshold  $\gamma$ . This is expected because as each relay serves half the users of the system, the interference between them in the corresponding relay is lower and more simultaneous transmissions to a relay may be successful in a time slot. In that way, the two relays receive more packets resulting in higher queue sizes. It is interesting though to note that the maximum average queue size of the system is below one packet, for the two higher threshold values (about 0.65 packets for  $\gamma = 1.2$  and 0.6 for  $\gamma = 2.5$ ). Moreover, in these cases the average queue sizes of the other three systems tend to become equal with over 25 users.

### *C. Average per Packet Delay*

Fig. 12 presents the average delay per packet (counted in timeslots) versus the number of users, for  $\gamma = 0.2$  up to  $\gamma = 2.5$  for the five relaying cases presented in the previous sections. As expected,

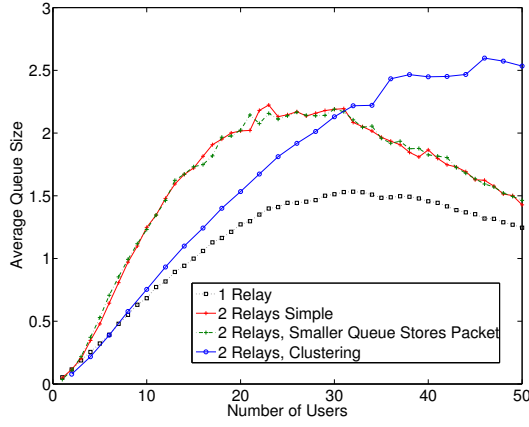
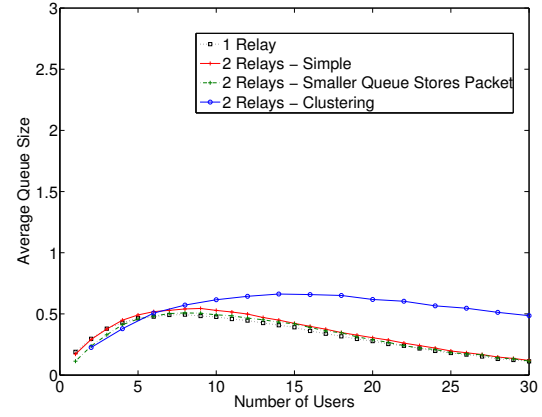
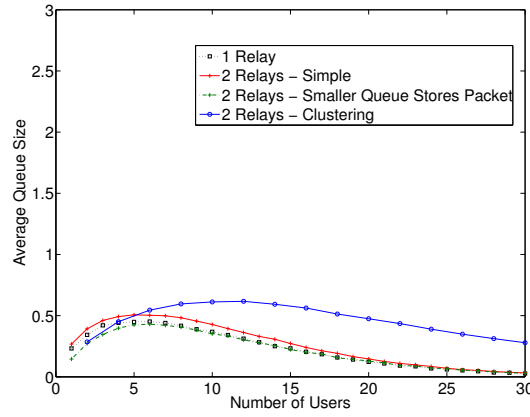
(a)  $\gamma = 0.2$ (b)  $\gamma = 1.2$ (c)  $\gamma = 2.5$ 

Fig. 11: Average relay queue size (in packets) vs. number of users for different SINR  $\gamma$  values in a system with relays.

with more users inserting packets into the system, the average delay per packet increases due to the increased interference. Figs. 12b and 12c show that the systems with two relays provide less average delay, compared to the systems with one and no relay, when the number of users is larger than 10. Specifically, for 30 users and  $\gamma = 1.2$  in Fig. 12b, the clustered system offers the lowest average delay per packet and it is interesting to note that whereas its value increases as the number of users also increases, it does not exceed 50 timeslots, while for the two other cases with two relays its value is about 150 timeslots and for the one relay 240 timeslots and for no relay 900 timeslots. We can make similar observations from Fig. 12c.

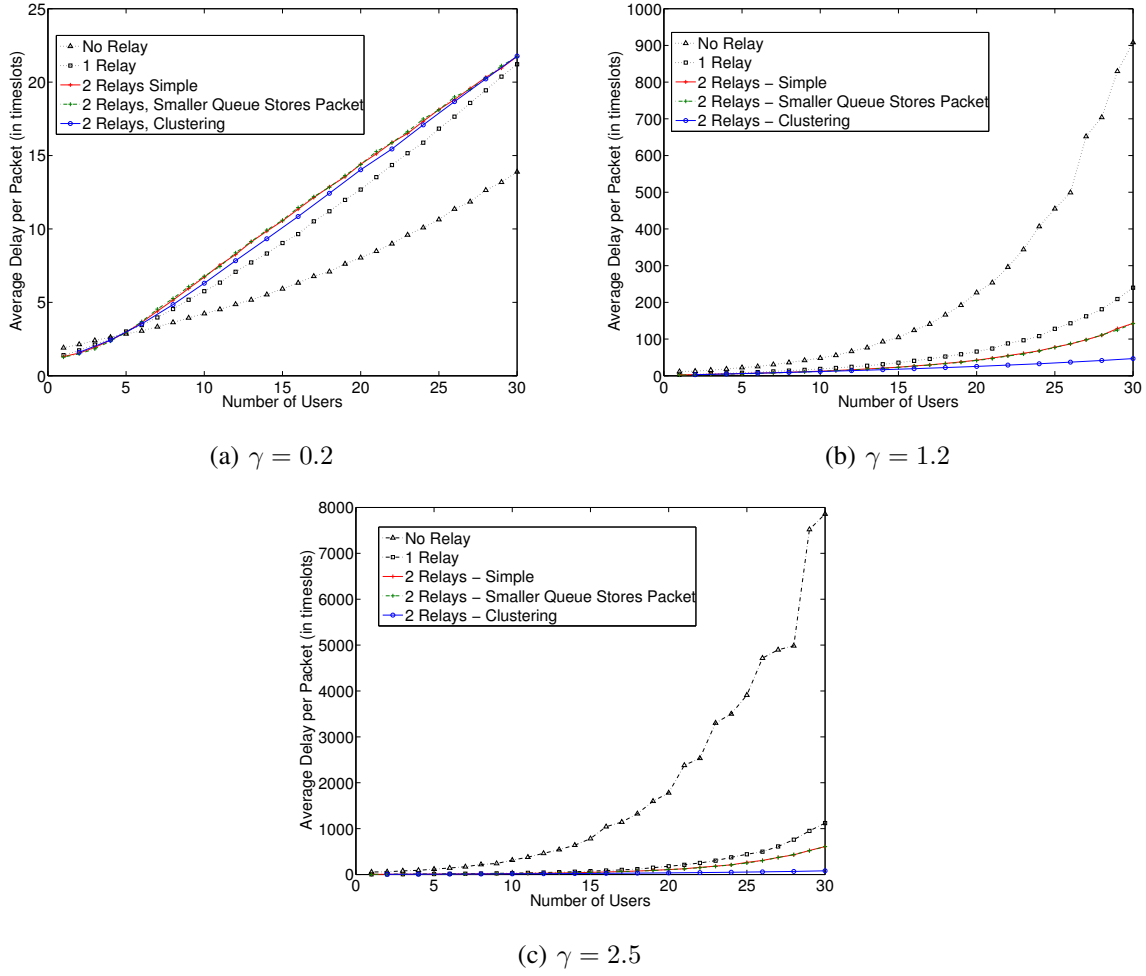


Fig. 12: Average per packet delay (in timeslots) vs. number of users for different SINR  $\gamma$  values in a system with and without relays.

Furthermore, the average delay per packet for all the cases except the clustered one increases excessively, for more than 30 users when  $\gamma = 1.2$  and 25 users when  $\gamma = 2.5$  respectively. Also, due to the fact that the aggregate throughput is fairly low and tends to zero as the number of users tend to 50 (see Figs. 9b and 9c), there are not enough samples in order to make an accurate calculation of the average delay per packet. However, the simulation showed that the average delay per packet obtained from the clustered system for 50 users and  $\gamma = 1.2$  is no more than 160 timeslots and for 50 users and  $\gamma = 2.5$  it is no more than 460 timeslots.

## VI. CONCLUSIONS

In this paper, we examined the potential gains of utilizing two relay nodes to aid the communication of a number of users to a common destination by re-transmitting (when necessary) their packets. Under the classic collision channel model we obtained analytical expressions for the arrival and service rates of the queues of the two relays and also the stability conditions. We further showed that the two relays are free to choose their transmission probabilities independently from each other, provided that these are greater from some minimum values which guarantee the stability of their queues. Employing multi-packet reception made the system intractable, so we conducted a thorough simulation study.

Under both models, we presented a user clustering scenario where the users are divided into two groups, each served by one relay and studied the impact of clustering on the per user and aggregate throughput. Although the insertion of a second relay in a system generally does not offer significantly higher throughput per user in comparison to a system with one relay, the clustered system offers impressive performance gains, in terms of throughput, for large numbers of served users.

Furthermore under the MPR model, we presented two relaying strategies: a simple one, where if both relays receive the same packet they both store it and forward it to the destination, and the Smaller Queue Stores Packet, in which the relay with the smaller queue becomes responsible for forwarding it to the destination. The second strategy offers higher aggregate and throughput per user compared to the first, for limited numbers of users.

These results could be used, for example in cellular and sensor networks, to identify the number of required relays to be deployed and allocate the users among relays. Future extensions of this work can include users with non-saturated queues (i.e. users-sources with external random arrivals) and relays with their own packets and priorities for the users. Other interesting extensions consist of relays which are capable of transmitting and receiving at the same time and the investigation of energy consumption in the total network and in particular at the relay nodes.

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