

An estimation of cattle movement parameters in the Central States of the US

Phillip Schumm^{1,*}, Caterina Scoglio^{1,*}, H. Morgan Scott²

Abstract

The characterization of cattle demographics and especially movements is an essential component in the modeling of dynamics in cattle systems, yet for cattle systems of the United States (US), this is missing. Through a large-scale maximum entropy optimization formulation, we estimate cattle movement parameters to characterize the movements of cattle across 10 Central States and 1034 counties of the United States. Inputs to the estimation problem are taken from the United States Department of Agriculture National Agricultural Statistics Service database and are pre-processed in a pair of tightly constrained optimization problems to recover non-disclosed elements of data. We compare stochastic subpopulation-based movements generated from the estimated parameters to operation-based movements published by the United States Department of Agriculture. For future Census of Agriculture distributions, we propose a series of questions that enable improvements for our method without compromising the privacy of cattle operations. Our novel method to estimate cattle movements across large US regions characterizes county-level stratified subpopulations of cattle for data-driven livestock modeling. Our estimated movement parameters suggest a significant risk level for US cattle systems.

I. INTRODUCTION

Livestock systems serve significant roles for many regions across the world, yet past outbreaks of disease have demonstrated that they can possess a number of vulnerabilities [1], [2], [3], [4], [5], [6]. The livestock systems of the United States (US), though strictly regulated, may yet be found susceptible to foreign diseases such as Rift Valley Fever [7]. The successful modeling and analysis of livestock epidemics for any region relies heavily on an understanding of the underlying system components. The three most critical elements in a practical epidemic model are the disease progression model, the geo-spatial characterization of the susceptible populations, and the spatial-temporal description of the interactions of individuals within the system [8], [9], [10]. The models of disease progression are several and often independent of the region studied [11], [12], [13], [14], [15]. Data-driven, spatial characterizations of populations are available through regularly conducted censuses (censi) [16], [17]. The third element, the interactions of individuals within the system, represents the set of spatial movements of individuals. When considering system-wide outbreaks of disease, the impact of movement parameters has been shown to be as significant as that of epidemic parameters in metapopulation models [18], [19]. Domestic livestock systems fit well in such metapopulation models as the movements of livestock are controlled and the individuals are restricted to reside within populations rarely defined by their choice. Within the US, livestock movements are controlled by the cattle industries, primarily beef, dairy, breeding, and showmanship.

Within Europe, motivated by outbreaks of Foot and Mouth Disease, a number of governments have designed and implemented animal tracking systems even to the resolution of individuals' daily movements. The databases created by these studies have generated very detailed characterizations of livestock movements for a number of European nations [4], [5], [6], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35]. No similar program has yet to be implemented for the United States, although some have long been in preparation [36]. In the US, a cultural appreciation of personal privacy from the government, strong competition between meat production companies, and a U.S. Federal privacy protection law restrict the ability of the government to collect and release livestock data at a finer spatial resolution than is currently done through the United States Department of Agriculture's (USDA) Census of Agriculture [17]. To address this challenge, a number of survey-based methods have been used to study livestock movements across small regions [37], [38], [39], [40], [41], [42]. However, the national scale of US cattle trade and the potential for livestock diseases to impact the entire country necessitate movement data, models, or estimates to be determined for larger regions. Recently a study has been published of a nation-wide movement estimation based on a 10% sample of veterinary records from State border-crossing cattle shipments [43]. This impressive study, although the first of its magnitude, only captured shipments of cattle that crossed state borders. Although it offers a picture of state-to-state shipment counts, the method used is unable to capture the livestock movements within each state.

¹Kansas State University, Electrical and Computer Engineering Department, Sunflower Networking Group

²Kansas State University, Department of Diagnostic Medicine/Pathobiology

³(*)Corresponding authors: pbschumm, caterina at ksu.edu

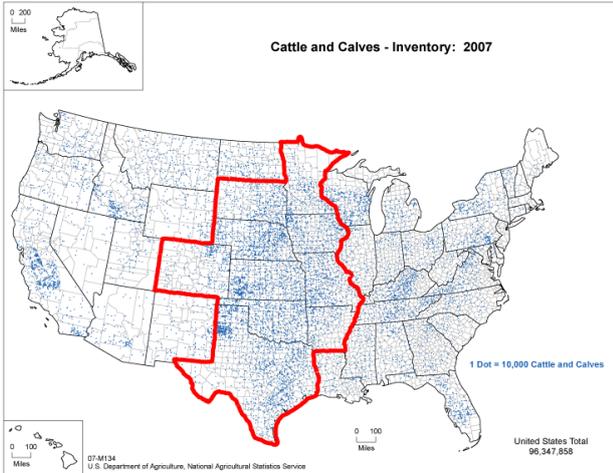


Fig. 1. The 10 Central States of interest are outlined with a red trace over the population distribution of cattle in the United States as provided from the United States Department of Agriculture [44]. Each blue dot represents 10,000 head of cattle.

In this paper, we formulate a large, convex optimization problem to estimate parameters describing the movements of cattle within 10 Central States of the United States. We collect cattle population and aggregated movement data from the United States Department of Agriculture’s database and optimally estimate anonymous data points to construct a database of inputs for an estimation of cattle movement parameters. We design the estimation method to produce a high resolution of cattle demographic and movement parameters and to include the minimal set of assumptions. Our results produce county-to-county movement probabilities among stratified subpopulations as well as birth, slaughter, and expiration rates of cattle for 1034 US counties. In section II, we describe the USDA data structures and challenges present in the database. We estimate non-disclosed data points and discuss the mapping of USDA data to inputs for our estimation formulation. In section III, we formulate the estimation problem and describe the maximum-entropy objective and the flexible set of linear constraints with parameters sculpted to the USDA data set and as few assumptions as possible. We solve the optimal estimation problem and display a subset of the results in section IV. Section V summarizes this paper with a discussion of the results, a series of questions for future agricultural census distributions, and a calculation of critical movement thresholds that demonstrate the potential for a disease outbreak in the cattle of this region.

II. DATA COLLECTION AND STRUCTURE

Every five years, the United States Department of Agriculture (USDA) conducts the United States Census of Agriculture [17]. The National Agricultural Statistics Service (NASS) of USDA then summarizes and publishes a large set of data covering livestock, crops, operator demographics, and much more [44]. As the most comprehensive and clean database of US livestock statistics, the Census of Agriculture as presented in the NASS database is used for our estimation of cattle movement parameters. In particular, we use data from the 2007 Agricultural Census as the 2012 data was not published at the time of this study. The data of interest to this work comes from section 13, titled “Cattle and Calves”, on page 10 of the 2007 Agricultural Census. Section 13 also has a set of related instructions located on page 2 of the instruction sheet appended to the Agricultural Census [17]. From the U.S. Census Bureau and their 2010 Census (of humans) in the United States, we use the centers of human population for each county [16]. We include these geographical points to consider a basic quantification of distance for the cattle movement estimation. Adding this geography to the data from the NASS database, we estimate sets of parameters to characterize cattle movements in the States of Arkansas, Colorado, Iowa, Kansas, Minnesota, Missouri, Nebraska, Oklahoma, South Dakota, and Texas [45], [46]. The US beef production feedlot structure produces more frequent and larger flows of cattle than the typical grazing structure, and notably, these 10 Central States form the core of the US feedlot industry [38], [43]. As outlined in figure 1, these States contain 1034 US counties with more than 51 million head of cattle of the 96.3 million head reported in the 2007 Agricultural Census [44].

A. Data structure of USDA NASS

From section 13 of the 2007 Agricultural Census, we are primarily interested in the responses to questions concerning the populations and movements of cattle. For cattle populations, the Agricultural Census identifies the total number of cattle (Question 3), the number of dairy cows kept for production of milk (Question 2.b), and the number of cattle, including calves, who were in a preslaughter feed program (Question 5) on December 31, 2007. For the

movements of cattle, composed of all sales and shipments, the Agricultural Census captures the total number of cattle “sold or moved” during 2007 (Question 4) and the total number of cattle shipped directly to a slaughter market from a preslaughter feed program during 2007 (Question 6) [17]. Although these data are collected for each individual operation, the statistics of the cattle populations are reported only through aggregated distributions that are delineated by the operation’s county, the type of cattle, and the number cattle of a particular type. The sizes of populations are sorted into seven standard ranges: 1 – 9 cattle, 10 – 19 cattle, 20 – 49 cattle, 50 – 99 cattle, 100 – 199 cattle, 200 – 499 cattle, and 500 or more cattle. The statistics of the cattle movements sort the responses by the total yearly movements (or slaughter) across the same 7 standard ranges, the type of cattle, and the county of the originating operation. It is worth noting that the total dairy cattle population and the total preslaughter population of a given county are subpopulations of the total cattle population for that county. Similarly, the total number of cattle shipped to slaughter from a preslaughter feed program is a fraction of the total cattle movements (sales and shipments) for each county.

According to appendix A of the 2007 Agricultural Census Summary and State Data report, the data presented in the NASS database has undergone some initial processing and systematic error correction [47]. This results in a very consistent database and the potential errors induced by these methods have been quantified in the same appendix. Even with these diligent efforts, there remain two significant challenges in utilizing the data to characterize the movements of cattle. The first concerns the resolution of the timescale of the data. As a summary of the entire year 2007, these data fail to capture any seasonal fluctuations in the cattle populations and movements [48]. This challenge arises from the administration process of the Agricultural Census and we acknowledge its significance; however, we find no comprehensive, data-driven solution to the seasonality challenge and consider only mean-field probabilities in our estimation. The second challenge is posed by the direct sorting of the census responses into the 7 standard ranges rather than preserving any operation-based connections between data points. Therefore, a population of 50 dairy cattle might belong to any operation having a total number of cattle greater than or equal to 50 (4 possible size ranges) without having any connection to the size of its entire operation. Similarly, the sizes of shipments have no direct connections to the size of the originating operations besides a few loose feasibility restrictions.

The data, as it comes from NASS, has been released in such a way that the information of individual farms and cattle operations is not revealed. This is done intentionally by USDA to comply with Title 7 of the U.S. Code [47]. To maintain this anonymity, critically selected elements of the data have not been disclosed. We will estimate these non-disclosed data points through a pair of tightly-constrained convex problems and then include them in the inputs for our main problem, the estimation of cattle movement parameters across the 10 Central States.

B. Data structure for estimation problem

To estimate the non-disclosed entries in the original data, we construct a pair of optimization problems, one for the population data and a second for the movement and slaughter data. The objective of both formulations is a maximum entropy function. For the population distributions and by each State, we maximize the entropy of the distributions of each cattle type given by $Type_A = \{Dairy, Preslaughter, All\ Cattle\}$, where the distributions are normalized by their respective State totals. Likewise, the entropy of the normalized distributions describing the totals of cattle slaughter shipments and cattle movements is maximized across shipment type, $ShipType = \{All\ Shipments, Slaughter\}$, shipment size, $Size_A = \{z1_9, z10_19, z20_49, z50_99, z100_199, z200_499, z500_up\}$, and county. We present the formulations of these problems in appendix A. These problems are solved for each of the 10 States, filling in all non-disclosed data entries. We quantify the dependence of our inputs on these estimations with both the fraction of cattle and the fraction of populations estimated for each State in Appendix B.

We would prefer to represent the system of cattle through three subpopulations rather than the two subpopulations and total population of $Type_A$. The mapping of the first set $Type_A$ to a set of three cattle subpopulation types is not trivial as it requires the expression of a relationship among the 3 cattle types of $Type_A$. For the county totals of these three types, the relationship $Tc_{Dairy,c}^x + Tc_{Preslaughter,c}^x \leq Tc_{AllCattle,c}^x$ holds, where $Tc_{:,c}^x$ represents the respective county total for county c and x indicates that this is a variable to be estimated. However, the relationship is not guaranteed if we consider the stratification of the populations by size as

$$Pop_{Dairy,c,i} + Pop_{Preslaughter,c,i} \leq Pop_{AllCattle,c,i} \quad \forall (c \in County, i \in Size_A) , \quad (1)$$

where $Pop_{:,c,i}$ represents the respective population total for operations with size i and in county c . Rather, Broomfield County in the State of Colorado, as do a number of other counties, reports data in violation of this relationship. Broomfield County is an irregularly cut county on the north side of Denver, Colorado and has 2 dairy farms with total populations in the range $z100_199$. The county, however, reports 2 populations of Dairy cattle in the size range $z50_99$

and no total (*All Cattle*) populations in the size range $z50_99$. Thus the left-hand side of inequality 1 would be non-zero while the right-hand side is identically 0 for $i = z50_99$, $c = Broomfield_Colorado$. This discrepancy arises from other cattle residing at both of these operations that raise the total operation populations into the next size range. We find that through an aggregation of the sizes into 3 ranges, $Size_B = \{z1_19, z20_199, z200_up\}$, it becomes feasible to assume the relationship of inequality 1 for each size j in $Size_B$.

Let us define a new cattle type *Beef*, representing all cattle that are not serving as dairy cattle nor in a preslaughter feed program, as the difference between the total population *All Cattle* and the two subpopulation types, *Dairy* and *Preslaughter*. The *Beef* type represents a diverse set of cattle operations including grazing, backgrounding, and breeding services. The name *Beef* is chosen for simplicity with the assumption that this is the majority role served by cattle in this type. At this point, we describe the cattle subpopulations $Pop_{t,c,j}$ by cattle type t in $Type_B = \{Dairy, Preslaughter, Beef\}$, county c , and size j in $Size_B = \{z1_19, z20_199, z200_up\}$. The set of subpopulations $\{Pop_{t,c,j}^R\}$ results from the solution of the first of the data-patching optimization formulations and represents an aggregation of all cattle type-based subpopulations fitting the subpopulation descriptors, yielding only 9 subpopulations per county. County totals for sales $Tc_{AllMovement,c}^{(s),x}$ and slaughter $Tc_{Slaughter,c}^{(s),x}$, as well as the distributions of yearly totals of sales $Sales_{AllMovement,c,i}^x$ and slaughter $Sales_{Slaughter,c,i}^x$ stratified by county c and size i in $Size_A$ were completed through the solution of the second data-patching optimization problem of appendix A.

III. CATTLE MOVEMENT PARAMETER ESTIMATION

We formulate a non-linear, yet convex, optimization problem with an objective to maximize the entropy of the out-going distributions of each subpopulation [49]. The formulation is nearly linear with the exception of the objective function, or equivalently stated, this formulation contains only linear constraints. The choice of maximum entropy for the problem objective aims to predict the solution having the minimal assumptions (maximum uncertainty) beyond the information contained within the set of constraints [49]. We chose this form of objective to not force any artificial objective in our estimation. Nevertheless, assumptions have been made in the design of both the variables and the constraints. We assume that

- There are no outgoing movements from preslaughter feed programs except for the outgoing movements of cattle for slaughter,
- Cattle classified as dairy cattle do not move into preslaughter feed programs,
- Populations of preslaughter feed cattle having population sizes of 200 head of cattle or more are responsible for all shipments to slaughter that result in yearly totals of 500 or more head shipped from a single premise,
- All sub-populations remain constant on a year-to-year basis, and
- The counties considered form a closed system with no significant movement into or out of the system.

A. Problem formulation

The central portion of this paper revolves around the formulation to estimate the cattle movement parameter $p_{t_1,j_1,t_2,j_2,dist}^x$, which is a probability that represents the movement process from an origin subpopulation of type t_1 and size j_1 to a destination subpopulation of type t_2 and size j_2 with a distance falling in a discrete distance range $dist$ between the origin and destination counties. We mark the decision variables of this formulation with a superscript x to distinguish them from the parameters of the problem. This formulation also estimates the birth bt_{c_1,t_1,j_1}^x , expiration of utility (cull) dt_{c_1,t_1,j_1}^x , and slaughter sl_{c_1,t_1,j_1}^x probabilities for each sub-population of each county. The three types of cattle are now rearranged into the set $Type_B$ as $\{Dairy, Preslaughter, Beef\}$, with size ranges defined by $Size_B = \{z1_19, z20_199, z200_up\}$. The set of discrete distance ranges used in this formulation is called $Distance$ and is defined as $\{d0, d100, d200, d500, d1000, d_{toofar}\}$. The number noted in each distance range between two county centers should be read as the maximum distance in miles of the range with the minimum defined by the previous level, for example, $d500$ indicates a distance between the two county centers falling between 200 and 500 miles. The closest range, $d0$, is assigned for any pair of counties with centers less than 10 miles apart, as well as each county with itself. The formulation has an objective to maximize the entropy of the outgoing distributions of all sub-populations as follows.

Maximize J

where

$$J = \sum_{c_1} \sum_{t_1} \sum_{j_1} \left[-st_{c_1, t_1, j_1}^x \log(st_{c_1, t_1, j_1}^x) - sl_{c_1, t_1, j_1}^x \log(sl_{c_1, t_1, j_1}^x) - dt_{c_1, t_1, j_1}^x \log(dt_{c_1, t_1, j_1}^x) \right. \\ \left. + \sum_{c_2} \sum_{t_2} \sum_{j_2} -p_{t_1, j_1, t_2, j_2, D(c_1, c_2)}^x \log\left(p_{t_1, j_1, t_2, j_2, D(c_1, c_2)}^x + 1.0 - f_{t_1, j_1, t_2, j_2, D(c_1, c_2)}\right) \right]$$

and $c_1 \in \text{County}, t_1 \in \text{Type}_B, j_1 \in \text{Size}_B,$

$$c_2 \in \{\text{County} | D(c_1, c_2) \neq d_{\text{toofar}}\}, t_2 \in \text{Type}_B, j_2 \in \text{Size}_B. \quad (2)$$

The out-going probability distribution for the cattle of each subpopulation is completed through the inclusion of a probability to remain or stay, st_{c_1, t_1, j_1}^x , in the origin subpopulation. The sum of the entropy of these distributions composes our objective function. We have implemented industrial constraints with a set of parameters $\{f_{t_1, j_1, t_2, j_2, dist}\}$ described in the following constraints which forces a subset of the movement probabilities $p_{t_1, j_1, t_2, j_2, dist}^x$ to zero. We account for this by introducing a complimentary $(1.0 - f_{t_1, j_1, t_2, j_2, dist})$ in the logarithm of $p_{t_1, j_1, t_2, j_2, dist}^x$ to avoid the computation of the natural logarithm of zero.

Subject to

Constraints on Statistical rules

$$p_{t_1, j_1, t_2, j_2, dist}^x \leq f_{t_1, j_1, t_2, j_2, dist} \quad \forall (t_1, j_1, t_2, j_2, dist) \quad (3)$$

$$\sum_{c_2 \in \text{County} | D(c_1, c_2) \neq d_{\text{toofar}}} \sum_{t_2 \in \text{Type}_B} \sum_{j_2 \in \text{Size}_B} p_{t_1, j_1, t_2, j_2, D(c_1, c_2)}^x + dt_{c_1, t_1, j_1}^x + sl_{c_1, t_1, j_1}^x + st_{c_1, t_1, j_1}^x = 1.0 \quad \forall (c_1, t_1, j_1) \quad (4)$$

Inequality 3 serves to restrict the probabilities of movement, $p_{t_1, j_1, t_2, j_2, dist}^x$, to be less or equal to 1 as $f_{t_1, j_1, t_2, j_2, dist}$ takes on a value of 1 in the general case. By taking a value of 0, it further prevents the movement of cattle from *Dairy* subpopulations to *Preslaughter* subpopulations ($t_1 = \text{Dairy}$ and $t_2 = \text{Preslaughter} \Rightarrow f_{t_1, j_1, t_2, j_2, dist} = 0$) and the out-going shipments of cattle from *preslaughter* subpopulations ($t_1 = \text{Preslaughter} \Rightarrow f_{t_1, j_1, t_2, j_2, dist} = 0$). Equality constraint 4 ensures that the sum of the out-going probability distributions, the same distributions that are considered in the objective function, is equal to 1.

Constraints on Movement data

$$\sum_{t_1 \in \text{Type}_B} \sum_{j_1 \in \text{Size}_B} \sum_{c_2 \in \text{County} | D(c_1, c_2) \neq d_{\text{toofar}}} \sum_{t_2 \in \text{Type}_B} \sum_{j_2 \in \text{Size}_B} Pop_{t_1, c_1, j_1}^R p_{t_1, j_1, t_2, j_2, D(c_1, c_2)}^x \\ + \sum_{t_1 \in \text{Type}_B} \sum_{j_1 \in \text{Size}_B} Pop_{t_1, c_1, j_1}^R sl_{c_1, t_1, j_1}^x + PN^{mov}(c_1) = \frac{TC_{AllMovements, c_1}^{(s), x}}{RC} \quad \forall (c_1) \quad (5)$$

$$\sum_{j_1 \in \text{Size}_B} Pop_{Preslaughter, c_1, j_1}^R sl_{c_1, Preslaughter, j_1}^x + PN^{slt}(c_1) = \frac{TC_{Slaughter, c_1}^{(s), x}}{RC} \quad \forall (c_1) \quad (6)$$

$$Pop_{Preslaughter, c_1, z200_up}^R sl_{c_1, Preslaughter, z200_up}^x + PN^{slt500}(c_1) \geq \frac{Sales_{Slaughter, c_1, z500_up}^x}{RC} \quad \forall (c_1) \quad (7)$$

$$D_{mov} = \sum_{c_1 \in \text{County}} [|PN^{mov}(c_1)| + |PN^{slt}(c_1)| + PN^{slt500}(c_1)] \quad (8)$$

Equality constraint 5 sums the total sales and shipments originating in each county c_1 and tries to equate the total to the total sales and shipments defined by USDA NASS for county c_1 , allowing a small amount of discrepancy through an error or roughness term $PN^{mov}(c_1)$. This discrepancy is permitted due to data challenges discussed in

section II. The scaling term, $R_C = 52.0$ weeks/year, converts the timescale of the estimation problem from a yearly to weekly basis for the estimated probabilities. Equality constraint 6 equates the total slaughter from preslaughter feed subpopulations in each county c_1 to the respective, scaled data value from USDA NASS, again with a discrepancy term for each county $PN^{slt}(c_1)$. Inequality 7 ensures that the largest yearly slaughter counts (500 or more head) are accredited to the largest (200 or more head) preslaughter subpopulations. This inequality requires a discrepancy term $PN^{slt500}(c_1)$ due to seasonality challenges in the USDA NASS data set. The discrepancy terms are collected in equality 8. Although we represent equality 8 here with absolute value operators, the actual implementation linearizes the terms through a two-variable decomposition of the unrestricted variable that allows us to minimize the resulting value as if it were an absolute value [50]. We retain the absolute value operators for simplicity in the formulation description.

Constraints on Population conservation

$$Leaving(c_1, t_1, j_1,) = Pop_{t_1, c_1, j_1}^R \sum_{c_2 \in County | D(c_1, c_2) \neq d_{toofar}} \sum_{t_2 \in Type_B} \sum_{j_2 \in Size_B} p_{t_1, j_1, t_2, j_2, D(c_1, c_2)}^x \forall (c_1, t_1, j_1) \quad (9)$$

$$Coming(c_1, t_1, j_1,) = \sum_{c_2 \in County | D(c_2, c_1) \neq d_{toofar}} \sum_{t_2 \in Type_B} \sum_{j_2 \in Size_B} Pop_{t_2, c_2, j_2}^R p_{t_2, j_2, t_1, j_1, D(c_2, c_1)}^x \forall (c_1, t_1, j_1) \quad (10)$$

$$Leaving(c_1, t_1, j_1,) - Coming(c_1, t_1, j_1,) + (dt_{c_1, t_1, j_1}^x + sl_{c_1, t_1, j_1}^x - bt_{c_1, t_1, j_1}^x) Pop_{t_1, c_1, j_1}^R + PN^{pop}(c_1, t_1, j_1,) = 0.0 \forall (c_1, t_1, j_1) \quad (11)$$

$$D_{pop} = \sum_{c_1 \in County} \sum_{t_1 \in Type_B} \sum_{j_1 \in Size_B} |PN^{pop}(c_1, t_1, j_1,)| \quad (12)$$

Equality constraints 9 and 10 sum, respectively, the originating and arriving flows of cattle for each subpopulation of each county. Equality constraint 11 then defines the total flux of every subpopulation in the system to be 0 with small exceptions allowed through the discrepancy terms $PN^{pop}(c_1, t_1, j_1,)$. Equality constraint 12 serves to aggregate the discrepancies. Here again in equality 12, we retain the absolute value operator for simplicity in the formulation description [50].

Constraints on Industrial insights and discrepancies

$$D_{mov} + D_{pop} \leq f_{min} P_{AllCattle}^{lot} \quad (13)$$

$$r_{t_1}^{expire-min} \leq dt_{c_1, t_1, j_1}^x \leq r_{t_1}^{expire-max} \quad \forall (c_1, t_1, j_1) \quad (14)$$

$$r_{t_1}^{slaughter-min} \leq sl_{c_1, t_1, j_1}^x \leq r_{t_1}^{slaughter-max} \quad \forall (c_1, t_1, j_1) \quad (15)$$

$$r_{t_1}^{birth-min} \leq bt_{c_1, t_1, j_1}^x \leq r_{t_1}^{birth-max} \quad \forall (c_1, t_1, j_1) \quad (16)$$

In inequality 13, we constrain the total discrepancies (counted in head of cattle) of the movements and net population fluxes to be less than a fraction f_{min} of the total cattle in the system. The value of f_{min} is determined by first solving the linear problem composed of the set of constraints of this formulation with an objective to minimize the system discrepancies. The value of f_{min} is then taken as the ratio of the optimal objective value to the total system population and rounded up to the next highest thousandth. The inequality pairs 14, 15, and 16 provide constraints by cattle type on the feasible probabilities used to describe the respective expiration, slaughter, and birth processes for each subpopulation.

IV. OPTIMIZATION RESULTS

We solved the cattle movement parameter problem of section III using the AIMMS Modeling System of Paragon Decision Technology [51]. The complete formulation is composed of 81142 constraints with 80107 variables and the objective function. For the error limit, a value of $f_{min} = 0.012$ was obtained through the method described following inequality 13, representing a limit of 1.2% of the total number of cattle, 51,252,890. The limits on the demographic probabilities attempt to capture loose bounds on feasible average rates of birth, culling, and slaughter. We assume that dairy cattle are not sent to slaughter houses through a slaughter rate $r_{Dairy}^{slaughter-max} = r_{Dairy}^{slaughter-min} = 0$, but rather through a culling process $r_{Dairy}^{expire-min} = (312 \text{ weeks})^{-1}$, $r_{Dairy}^{expire-max} = (104 \text{ weeks})^{-1}$. We bound the expected birthing rate of dairy cattle as $r_{Dairy}^{birth-min} = (62 \text{ weeks})^{-1}$ and $r_{Dairy}^{birth-max} = (36 \text{ weeks})^{-1}$. The mixed collection of cattle, *Beef*, are allowed a reasonable birth rate as well $r_{Beef}^{birth-max} = (52 \text{ weeks})^{-1}$, $r_{Beef}^{birth-min} = 0$, but the *Preslaughter* individuals are not $r_{Preslaughter}^{birth-min} = r_{Preslaughter}^{birth-max} = 0$. The *Beef* cattle have a maximum average useful lifespan defined by $r_{Beef}^{expire-min} = (520 \text{ weeks})^{-1}$ and they join the other two types in minimal useful life as $r_{Beef}^{expire-max} = r_{Preslaughter}^{expire-max} = (104 \text{ weeks})^{-1}$. The *Preslaughter* population is assumed to not have a minimal expiration rate $r_{Preslaughter}^{expire-min} = 0$, but they have the highest feasible slaughter rate of $r_{Preslaughter}^{slaughter-max} = (2 \text{ weeks})^{-1}$. Lastly the *Beef* populations have a feasible range for their slaughter rates of $r_{Beef}^{slaughter-min} = 0$ to $r_{Beef}^{slaughter-max} = (13 \text{ weeks})^{-1}$. The upper limits on the slaughter rates are quite high, but we explain the need for this later in this section.

A. Cattle movement parameters

As the focus of this study, the cattle movement parameters $p_{t_1, j_1, t_2, j_2, dist}^x$ express the probability that, within a week's duration, an individual in a subpopulation of type t_1 and size j_1 will move or be shipped to a subpopulation of type t_2 and size j_2 at a (county-to-county) distance of $dist$. Table I presents a subset of these probabilities that express the movement of cattle from *Dairy* subpopulations to *Beef* subpopulations for 5 ranges of distance.

TABLE I: Estimated cattle movement parameters $p_{t_1, j_1, t_2, j_2, dist}^x \cdot 10^3$, Dairy to Beef

Source → Destination	$d0$	$d100$	$d200$	$d500$	$d1000$
$D, z1_{19} \rightarrow B, z1_{19}$	0.212302939	0.059831993	0.021823530	0.103473623	0.171239322
$D, z20_{199} \rightarrow B, z1_{19}$	0.0	0.0	0.0	0.0	0.0
$D, z200_{up} \rightarrow B, z1_{19}$	0.043690097	0.007360407	0.000574583	0.0	0.0
$D, z1_{19} \rightarrow B, z20_{199}$	0.184643994	0.054828320	0.021031163	0.108659569	0.275442544
$D, z20_{199} \rightarrow B, z20_{199}$	0.0	0.0	0.0	0.0	0.0
$D, z200_{up} \rightarrow B, z20_{199}$	0.0	0.001677875	0.003638771	0.007653646	0.0
$D, z1_{19} \rightarrow B, z200_{up}$	0.179577791	0.053431956	0.021227743	0.113325182	0.301730295
$D, z20_{199} \rightarrow B, z200_{up}$	0.0	0.0	0.0	0.0	0.002141048
$D, z200_{up} \rightarrow B, z200_{up}$	0.0	0.001111461	0.004969657	0.019264109	0.001843747

A complete table of the cattle movement probabilities is provided in appendix B along with tables that partially present the birth, expiration, and slaughter probabilities of the 9 subpopulations of each county. The entire results are too large for this document as we are studying 1034 counties. The tables in appendix B present results for a sample of 10 counties from each State. Once having obtained the solution, we revisited the movement data released by the USDA NASS. A significant difference exists between the NASS movement data and these movement parameters we've estimated, namely, that the movements of NASS are summarized from individual premises, but our parameters describe movements from and to collections of premises. We simulated 30 years of virtual cattle movements and shipments for slaughter and summarized the movements into the 3 size ranges of $Size_B$. We compare these results for individual counties, considering that shipments originating from aggregated subpopulations ought to usually be larger than shipments from individual operations. This means that the subpopulation-based results should over represent for larger sizes and perhaps under represent for smaller sizes of shipments. For Ellis County in the State of Kansas, figure 2 presents a comparison of our subpopulation based distributions of shipments in blue against the NASS reported yearly totals in red. The three size categories represent the three ranges of $Size_B$, with the smallest range on the left and the largest on the right side. Trego County, also in the State of Kansas, demonstrates one way in which the year-long resolution of the Agricultural Census is insufficient to express the seasonality of the cattle system. At the time of the 2007 census, Trego County reported no large *Preslaughter* populations of cattle. On the year, however, Trego County was responsible for several large shipments of cattle for slaughter. The census happened to catch the finishing yards at a point in time

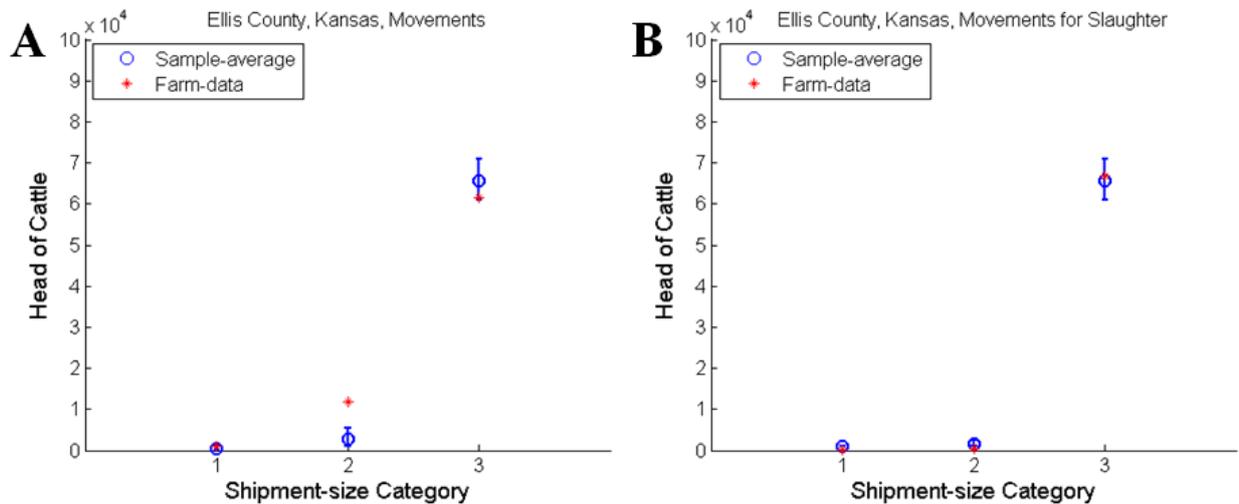


Fig. 2. (A) The average yearly totals of *All Movements* originating from Ellis County, Kansas and generated from the estimated subpopulation cattle movement parameters for the 3 size categories of $Size_B$ are represented with blue circles and their respective 99% confidence intervals are shown with the vertical lines. Shown in red stars are the yearly totals reported by NASS for the same county and aggregated into the ranges of $Size_B$. (B) The yearly totals of cattle shipped for slaughter originating in Ellis County are displayed from both the estimated slaughter rates (in blue) and the NASS database (in red) following the same notations as in A.

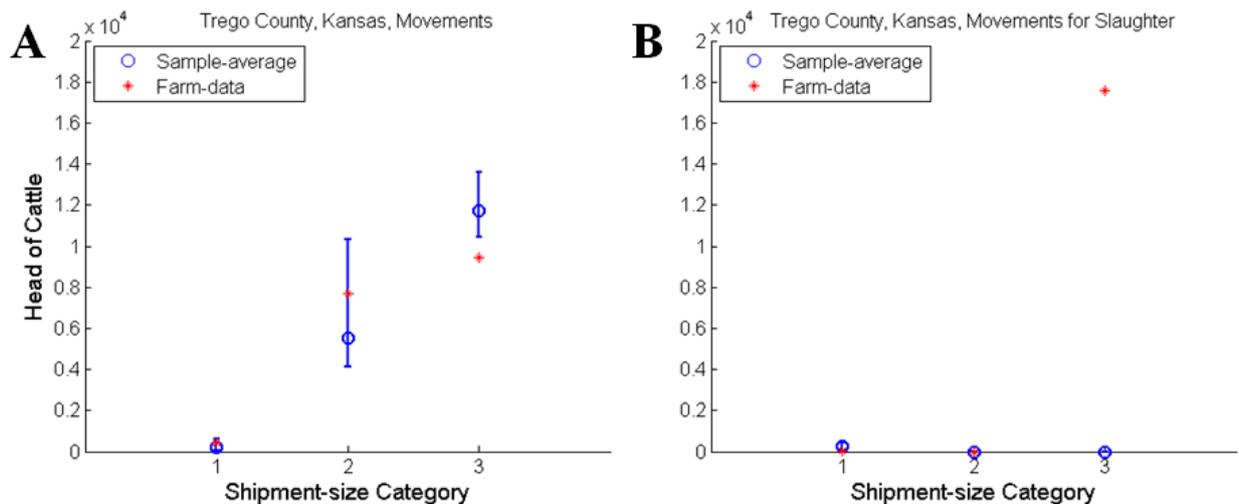


Fig. 3. (A) The average yearly totals of *All Movements* originating from Trego County, Kansas and generated from the estimated subpopulation cattle movement parameters for the 3 size categories of $Size_B$ are represented with blue circles and their respective 99% confidence intervals are shown with the vertical lines. Shown in red stars are the yearly totals reported by NASS for the same county and aggregated into the ranges of $Size_B$. (B) The yearly totals of cattle shipped for slaughter originating in Trego County are displayed from both the estimated slaughter rates (in blue) and the NASS database (in red) following the same notations as in A. A large discrepancy occurs in the third size category between the NASS reported slaughter totals and the generated distribution.

in which they were empty and thus neither the true capacity nor typical population levels of *Preslaughter* cattle are represented in the NASS database. Figure 3.B displays the dramatic mismatch that occurs for the largest slaughter shipment size category.

V. DISCUSSION AND CONCLUSIONS

We have designed and solved a large-scale optimal estimation problem in an attempt to address the privacy challenges and the need for livestock movement data in the United States. Given the resolution limitations of the data available, we do not try to estimate very detailed parameters, but rather adopt a stratified metapopulation approach where we shape the structure of our variables around the structure of the NASS data. Our approach is limited by the timescale resolution of the census report. This leads to several seasonal challenges that include correctly quantifying populations, identifying periods of higher and lower movement rates, and capturing birthing and slaughtering seasons. The demographic bounds used in our formulation depend on advice from industry experts and are flexibly open to further

insights. We simulated and compared subpopulation-based movement distributions with operation-based movements, however this is not a rigorous method of validation. Our problem design includes an assumption on the relationships of cattle types and sizes that proved to not hold true for all counties as demonstrated by Broomfield County in the State of Colorado. Lastly, the parameters we estimate fail to consider individual State laws and Veterinary practices as well as the role of wildlife in the interfacing of subpopulations.

A close examination of the results in table B.1 of appendix B reveals a relatively unrealistic fraction of probabilities that are estimated to take on a value of 0. We believe this to be an artifact that has arisen from the design of the formulation and the nature of the optimization. Without a sufficiently diverse set of constraints, the dimensionality of an optimal solution for the problem will be limited. The objective to maximize entropy would prefer to diversify the results, making as many nonzero as possible, yet it seems too tightly restricted in some way as to allow that to occur in the solution. If we had to select a constraint which would most likely be the cause for these probabilities taking on a value of 0, we would first suspect the tight error limit of inequality 13. All other constraints allow a reasonable amount of flexibility in the set of parameters that would satisfy them. As the objective strives to diversify the distributions, we predict that loosening the error limits would result in fewer movement probabilities taking on a value of 0.

From the challenges that this problem held, a few insights were discovered that might improve the success of a future version of these livestock movement estimation methods without posing any threat to the confidentiality of the data. Having examined the design limitations of this study, we would like to propose 3 new questions to be considered for addition to future versions of the US Census of Agriculture. Those being

- “Of the cattle sold/moved in question 4, excluding those sold for slaughter in question 6, how many went to (a) destinations within the county, (b) destinations in a neighboring county, (c) destinations within the same state, (d) destinations in neighboring (bordering) states, (e) destinations further away?”
- “Of cattle that arrive to this operation during 20XX, how many came from (a) a locations within the county, (b) locations in a neighboring county, (c) locations within the same state, (d) locations in neighboring (bordering) states, (e) locations further away?”
- “How many cattle were born on these premises during the year 20XX?”

Although with many limitations, we have taken a significant first step in tackling the challenges of data in the United States through optimization and computation without compromising anonymity.

Large-scale livestock disease analysis and control is the most immediate use for the parameters estimated by this method. The threat of an livestock epidemic spreading across the region under consideration can be examined using a global epidemic invasion threshold for this system of cattle populations [18], [19]. A recent work has extended this global invasion threshold to directed livestock movements among many populations [52]. This work has derived two global epidemic invasion thresholds, where the first describes the emergence of the disease at a regional scale (p_c) and the second describes the spreading of the disease into the preslaughter locations (p_c^{TS}). We compute a rough approximation of the critical movement rates p_c and p_c^{TS} from the estimated data for the 10 Central States. The computation of the average movement rate $\langle p \rangle$ across the 10 States and the comparison of these values to the critical movement rates reveal the potential for epidemics to break out across these States when considering the current movement rates. We use the same disease parameters for the theoretical comparison: infection rate $\beta = \mu R_0$, recovery rate $\mu = 1(\text{week})^{-1}$, and basic reproductive number $R_0 = 1.2$. From our estimated parameters we determined the approximate average population size $\bar{N}^{approx} = 5483.8$ head and the average death rate $\delta = 0.01516(\text{week})^{-1}$. We simulated 30 years of cattle movements and captured the weekly average outward movement $\langle p \rangle$ for all *Beef* and *Dairy* subpopulations. Concurrently, we measured the resulting network parameters ($\eta_{in}, \eta_{out}, k_i^{in}, k_i^{out}$) for networks representing the weekly movements. We have considered the dynamic network measurements as produced by the realized movements. We have ignored the potential existence of any degree correlations in the resulting networks from the movement-based network construction. Having collected this approximate set of parameters, we computed the critical movement rates p_c and p_c^{TS} and list them in comparison to the system average movement rate $\langle p \rangle$ of the estimated cattle movement parameters in table II.

TABLE II: Average cattle movement parameter and thresholds

	Average	Minimum	Maximum
$\langle p \rangle$	0.131892	0.128170	0.136130
p_c	0.003470	0.003384	0.003565
p_c^{TS}	0.007001	0.006745	0.007282

This first approximation of the incorporation of these two components suggests that the US cattle systems are at a significant level of risk. The average cattle movement rates are roughly 2 magnitudes larger than the thresholds defined by the critical movement rates. This result suggests that an epidemic with parameters similar to the ones implemented would easily invade US cattle populations and reach the cattle's final destinations, possibly compromising the security of the beef supply chain. To impede the progress of this epidemic, a reduction in cattle movement rates of more than 99% would be required. For human influenza-like illnesses, a reduction in mobility rates of one order of magnitude has also been proposed [9]. This amount of reduction is based on an approximated comparison, and further work should be completed to better quantify the risk to US livestock systems.

VI. ACKNOWLEDGMENTS

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VII. APPENDIX A: DATA ESTIMATION

Following the discussion of section II, we formulated a pair of maximum entropy estimation problems that are constrained by straightforward rules defined by the US Department of Agriculture database. The maximum entropy method does not recover a significant amount of diversity with the estimated values, but selects the sets of values that are as homogeneous as possible while obeying all constraints. This method seeks such values because the homogeneity of maximum entropy represents the minimal assumptions possible in the problem design.

A. Population data estimation

With the following formulations, we estimate any undisclosed data elements, namely subpopulations $Pop_{t,c,i}^x$, County totals $Tc_{t,c}^x$, and State totals for each size category $Tz_{t,i}^x$. We defined the sets *County* as the Counties of the considered State, $t \in Type_A = \{Dairy, Preslaughter, All\ Cattle\}$, $i \in Size_A = \{z1_9, z10_19, z20_49, z50_99, z100_199, z200_499, z500_up\}$, and $j \in Size_B = \{z1_19, z20_199, z200_up\}$. From the USDA NASS database, for each State, we collected the numbers of operations $n_{t,c,i}$ of cattle type t having subpopulations with their sizes falling within size range i in county c , the published populations counts $Pop_{t,c,i}^r$ representing all cattle of type t with subpopulations within size range i in county c , the total numbers of cattle $Tc_{t,c}^r$ of type t in county c , the total numbers of cattle $Tz_{t,i}^r$ of type t in size range i in the State, and the total counts of cattle in each type t for the entire State P_t^{tot} . To these parameters, we have added upper limits on the subpopulations $u_{t,i}$ for each cattle type t and size range i , lower limits on the subpopulations $l_{t,i}$ for each cattle type t and size range i , and data indicator parameters, $datPop_{t,c,i}$, $datTc_{t,c}$, and $datTz_{t,i}$, that express the existence of data elements for their respective parameters (The data indicators for non-disclosed data elements are assigned a value of 0 and the remainder are set to 1). The lower and upper limits are set by the limits of the size ranges ($l_{t,z1_9} = 1$, $u_{t,z1_9} = 9$, ...), with the exception of the upper limits on the largest size range, $z500_up$. The largest upper limit $u_{t,z500_up}$ is set to $Tz_{t,z500_up}^r$ if $datTz_{t,z500_up} = 1$, else we set $u_{t,z500_up} = P_t^{tot}$. For the population data of each State, we solve

$$\text{Maximize } \sum_{t \in Type_A} \sum_{c \in County} \sum_{i \in Size_A} \left(-\frac{Pop_{t,c,i}^x}{P_t^{tot}} \log \left[\frac{Pop_{t,c,i}^x}{P_t^{tot}} + 1.0 - datPop_{t,c,i} \text{sign} (Pop_{t,c,i}^r) \right] \right). \quad (17)$$

The objective function maximizes the entropy of the three (by cattle type) population distributions and includes additional terms in the logarithm argument similar to the objective function of the formulation of section III. We include the *sign* function to consider the case when the published data $Pop_{t,c,i}^r = 0$, which would otherwise result in $\log(0)$. Thus the 1.0 is removed from the logarithm argument if and only if $datPop_{t,c,i} = 1$ and the corresponding data value $Pop_{t,c,i}^r$ is strictly positive.

Subject to

Constraints on Population values

$$Pop_{t,c,i}^x \leq datPop_{t,c,i} Pop_{t,c,i}^r + (1.0 - datPop_{t,c,i}) n_{t,c,i} u_{t,i} \quad \forall (t, c, i) \quad (18)$$

$$Pop_{t,c,i}^x \geq datPop_{t,c,i} Pop_{t,c,i}^r + (1.0 - datPop_{t,c,i}) n_{t,c,i} l_{t,i} \forall (t, c, i) \quad (19)$$

The constraints form bounds for the complete set $\{Pop_{t,c,i}^x\}$ even if the data element $Pop_{t,c,i}^r$ is known from the NASS database. We use $datPop_{t,c,i}$ to represent the existence of the data element. Observe that when $datPop_{t,c,i} = 1$ inequalities 18 and 19 converge to act as an equality constraint $Pop_{t,c,i}^x = Pop_{t,c,i}^r$. When $datPop_{t,c,i} = 0$, the upper (lower) bound is defined by the number of subpopulations in $Pop_{t,c,i}^x$ and the upper (lower) limit of the size range. Although these bounds have the potential to be very loose, they help shape the feasible set of values for $Pop_{t,c,i}^x$.

Constraints on County totals

$$Tc_{t,c}^x \leq datTc_{t,c} Tc_{t,c}^r + (1.0 - datTc_{t,c}) uTc_{t,c} \forall (t, c) \quad (20)$$

$$Tc_{t,c}^x \geq datTc_{t,c} Tc_{t,c}^r + (1.0 - datTc_{t,c}) lTc_{t,c} \forall (t, c) \quad (21)$$

$$uTc_{t,c} = \sum_{i \in Size_A} [datPop_{t,c,i} Pop_{t,c,i}^r + (1.0 - datPop_{t,c,i}) n_{t,c,i} u_{t,i}] \forall (t, c) \quad (22)$$

$$lTc_{t,c} = \sum_{i \in Size_A} [datPop_{t,c,i} Pop_{t,c,i}^r + (1.0 - datPop_{t,c,i}) n_{t,c,i} l_{t,i}] \forall (t, c) \quad (23)$$

$$\sum_{i \in Size_A} Pop_{t,c,i}^x = Tc_{t,c}^x \forall (t, c) \quad (24)$$

Inequalities 20 and 21 follow the same data indicator controlled constraint structure as inequalities 18 and 19. The upper and lower limits for the county totals $Tc_{t,c}^x$ are constructed in equality constraints 22 and 23. Given constraints 18, 19, and 24, the bounds computed in 22 and 23 are redundant, however, we included them to help describe the solution space in a more understandable manner.

Constraints on Size totals

$$Tz_{t,i}^x \leq datTz_{t,i} Tz_{t,i}^r + (1.0 - datTz_{t,i}) uTz_{t,i} \forall (t, i) \quad (25)$$

$$Tz_{t,i}^x \geq datTz_{t,i} Tz_{t,i}^r + (1.0 - datTz_{t,i}) lTz_{t,i} \forall (t, i) \quad (26)$$

$$uTz_{t,i} = \sum_{c \in County} [datPop_{t,c,i} Pop_{t,c,i}^r + (1.0 - datPop_{t,c,i}) n_{t,c,i} u_{t,i}] \forall (t, i) \quad (27)$$

$$lTz_{t,i} = \sum_{c \in County} [datPop_{t,c,i} Pop_{t,c,i}^r + (1.0 - datPop_{t,c,i}) n_{t,c,i} l_{t,i}] \forall (t, i) \quad (28)$$

$$\sum_{c \in County} Pop_{t,c,i}^x = Tz_{t,i}^x \forall (t, i) \quad (29)$$

Constraints 25 through 29 repeat the structure of constraints 20 through 24 for the totals $Tz_{t,i}^x$ of each size category.

Constraints on State totals

$$\sum_{c \in County} Tc_{t,c}^x = P_t^{\text{tot}} \forall (t) \quad (30)$$

$$\sum_{i \in Size_A} Tz_{t,i}^x = P_t^{\text{tot}} \forall (t) \quad (31)$$

Equality constraints 30 and 31 state that the totals of the sub-totals must equal the State total P_t^{tot} of the respective population type.

Constraints on Population relations

$$Pop_{t,c,z1_19}^R = Pop_{t,c,z1_9}^x + Pop_{t,c,z10_19}^x \quad \forall (t) \quad (32)$$

$$Pop_{t,c,z20_199}^R = Pop_{t,c,z20_49}^x + Pop_{t,c,z50_99}^x + Pop_{t,c,z100_199}^x \quad \forall (t) \quad (33)$$

$$Pop_{t,c,z200_up}^R = Pop_{t,c,z200_499}^x + Pop_{t,c,z500_up}^x \quad \forall (t) \quad (34)$$

$$Pop_{Dairy,c,j}^R + Pop_{Preslaughter,c,j}^R \leq Pop_{AllCattle,c,j}^R \quad \forall (c \in County, j \in Size_B) \quad (35)$$

Constraints 32-34 aggregate the populations from the size ranges of $Size_A$ to those of $Size_B$. Inequality 35 defines the relationship discussed in section II and enables us to assemble the third cattle type of $Type_B$, *Beef*, as $Pop_{Beef,c,j}^R = Pop_{AllCattle,c,j}^R - Pop_{Dairy,c,j}^R - Pop_{Preslaughter,c,j}^R$ for every county c and $j \in Type_B$.

B. Sales and shipments data estimation

For Sales, Movements, and Slaughter data, we consider another a set of shipment types $ShipType = \{AllShipments, Slaughter\}$. We formulated the parallel problem for the non-disclosed data elements which describe the movements and slaughter of cattle. To estimate shipments $Sales_{q,c,i}^x$, County totals $Tc_{q,c}^{(s),x}$, and State totals for each size category $Tz_{q,i}^{(s),x}$ we solve

$$\text{Maximize} \quad \sum_{q \in ShipType} \sum_{c \in County} \sum_{i \in Size_A} \left(-\frac{Sales_{q,c,i}^x}{S_q^{tot}} \log \left[\frac{Sales_{q,c,i}^x}{S_q^{tot}} + 1.0 - datSales_{q,c,i} \text{sign}(Sales_{q,c,i}^r) \right] \right) \quad (36)$$

Subject to

Constraints on Population values

$$Sales_{q,c,i}^x \leq datSales_{q,c,i} Sales_{q,c,i}^r + (1.0 - datSales_{q,c,i}) n_{q,c,i}^{(s)} u_{q,i}^{(s)} \quad \forall (q \in ShipType, c \in County, i \in Size_A) \quad (37)$$

$$Sales_{q,c,i}^x \geq datSales_{q,c,i} Sales_{q,c,i}^r + (1.0 - datSales_{q,c,i}) n_{q,c,i}^{(s)} l_{q,i}^{(s)} \quad \forall (q, c, i) \quad (38)$$

Constraints on County totals

$$Tc_{q,c}^{(s),x} \leq datTc_{q,c}^{(s)} Tc_{q,c}^{(s),r} + (1.0 - datTc_{q,c}^{(s)}) uTc_{q,c}^{(s)} \quad \forall (q, c) \quad (39)$$

$$Tc_{q,c}^{(s),x} \geq datTc_{q,c}^{(s)} Tc_{q,c}^{(s),r} + (1.0 - datTc_{q,c}^{(s)}) lTc_{q,c}^{(s)} \quad \forall (q, c) \quad (40)$$

$$uTc_{q,c}^{(s)} = \sum_{i \in Size_A} \left[datSales_{q,c,i} Sales_{q,c,i}^r + (1.0 - datSales_{q,c,i}) n_{q,c,i}^{(s)} u_{q,i}^{(s)} \right] \quad \forall (q, c) \quad (41)$$

$$lTc_{q,c}^{(s)} = \sum_{i \in Size_A} \left[datSales_{q,c,i} Sales_{q,c,i}^r + (1.0 - datSales_{q,c,i}) n_{q,c,i}^{(s)} l_{q,i}^{(s)} \right] \quad \forall (q, c) \quad (42)$$

$$\sum_{i \in Size_A} Sales_{q,c,i}^x = Tc_{q,c}^{(s),x} \quad \forall (q, c) \quad (43)$$

Constraints on Size totals

$$Tz_{q,i}^{(s),x} \leq datTz_{q,i}^{(s)} Tz_{q,i}^{(s),r} + (1.0 - datTz_{q,i}^{(s)}) uTz_{q,i}^{(s)} \quad \forall (q, i) \quad (44)$$

$$Tz_{q,i}^{(s),x} \geq datTz_{q,i}^{(s)} Tz_{q,i}^{(s),r} + (1.0 - datTz_{q,i}^{(s)}) lTz_{q,i}^{(s)} \quad \forall (q, i) \quad (45)$$

$$uTz_{q,i}^{(s)} = \sum_{c \in County} \left[datSales_{q,c,i} Sales_{q,c,i}^r + (1.0 - datSales_{q,c,i}) n_{q,c,i}^{(s)} u_{q,i}^{(s)} \right] \quad \forall (q, i) \quad (46)$$

$$lTz_{q,i}^{(s)} = \sum_{c \in \text{County}} \left[\text{datSales}_{q,c,i} \text{Sales}_{q,c,i}^T + (1.0 - \text{datSales}_{q,c,i}) n_{q,c,i}^{(s)} l_{q,i}^{(s)} \right] \forall (q, i) \quad (47)$$

$$\sum_{c \in \text{County}} \text{Sales}_{q,c,i}^x = Tz_{q,i}^{(s),x} \forall (q, i) \quad (48)$$

Constraints on State totals

$$\sum_{c \in \text{County}} Tc_{q,c}^{(s),x} = S_q^{\text{tot}} \forall (q) \quad (49)$$

$$\sum_{i \in \text{Size}_A} Tz_{q,i}^{(s),x} = S_q^{\text{tot}} \forall (q) \quad (50)$$

Constraint on Shipment relations

$$Tc_{Slaughter,c}^{(s),x} \leq Tc_{AllMovement,c}^{(s),x} \forall (c, i) \quad (51)$$

The primary structural difference in this pair of optimization problems is the aggregations and relations of the last constraints of each. The shipment formulation only relates the two types through county totals $Tc_{q,c}^{(s),x}$ and not through the sub-elements as in the population formulation. For the objectives of this paper, there is not a need to add any assumptions to the relationship between the slaughter shipments and the total shipments beyond the assumption-less relationship of inequality 51. The aggregated populations $Pop_{t,c,j}^R$, the county shipment totals $Tc_{q,c}^{(s),x}$, and the largest category of slaughter shipments $Sales_{Slaughter,c,z500_up}^x$ compose the set of inputs for the estimation of cattle movement parameters described in section III.

VIII. APPENDIX B: RESULTS OF OPTIMIZATION

In appendix A, we described the formulations of two optimization problems that estimate all non-disclosed elements of the particular USDA NASS data sets which are used in this chapter. We quantify the amount of estimated data for each State in tables III and IV. Table III presents the numbers of populations estimated for each State by count and percentage as well as the number (head) of cattle assigned across these populations. The populations are counted by summing over all three types of $Type_A = \{Dairy, Preslaughter, All\ Cattle\}$. This method of counting induces double counting since cattle in the first two types also belong to the third type, but offers a systematic quantification of the amount of data estimated. The percentage of cattle assigned through the estimation demonstrates that in situations where many populations are estimated the significance of these populations is less $< 7\%$. The States of Kansas and Texas are exceptions to the trend of small fractions of the total cattle being assigned through estimation. These two States appear to have relatively similar percentages of estimated populations when compared to the other States, yet they assign larger percentages of the cattle totals. These larger numbers of cattle suggest that a higher number of counties in these States possess only a few large cattle operations, where the sparsity and the size of the operations necessitate the non-disclosure of their data elements. Table IV presents a parallel quantification for the estimations of the shipment distributions with counts similarly aggregated over $ShipType = \{All\ Shipments, Slaughter\}$. The results of table IV are comparable to those of table III.

State	Count	Count %	Head	Head %
Arkansas	170	10.79	55,748	3.06
Colorado	221	16.44	178,472	4.46
Iowa	391	18.81	149,802	2.52
Kansas	524	23.76	1,153,577	12.20
Minnesota	387	21.18	132,701	3.83
Missouri	500	31.33	112,753	2.51
Nebraska	399	20.43	552,159	5.89
Oklahoma	330	20.41	381,148	6.55
South Dakota	282	20.35	112,648	2.62
Texas	697	13.07	1,513,115	8.81

TABLE III: Totals and percentages of estimated cattle populations

State	Count	Count %	Head	Head %
Arkansas	139	13.24	75,326	7.56
Colorado	160	17.86	165,694	3.20
Iowa	165	11.90	163,735	2.75
Kansas	327	22.24	1,958,683	13.71
Minnesota	182	14.94	108,631	4.74
Missouri	334	20.93	213,464	8.18
Nebraska	242	15.16	1,144,166	8.98
Oklahoma	200	18.55	407,340	8.74
South Dakota	156	16.88	155,108	4.37
Texas	720	20.25	3,055,251	17.38

TABLE IV: Totals and percentages of estimated cattle shipments

Table V displays the solution for $p_{t_1, j_1, t_2, j_2, dist}^x$ sorted by the origin and destination pairs in the left column and the distances between the county centers in the rows. The cattle types of *Dairy*, *Beef*, and *Preslaughter* are denoted respectively by *D*, *B*, and *P* to be brief.

TABLE V: Estimated cattle movement parameters $p_{t_1, j_1, t_2, j_2, dist}^x \cdot 10^3$

Source → Destination	<i>d</i> 0	<i>d</i> 100	<i>d</i> 200	<i>d</i> 500	<i>d</i> 1000
<i>D</i> , <i>z</i> 1_19 → <i>D</i> , <i>z</i> 1_19	0.317036819	0.182356046	0.045722672	0.137186552	0.195279372
<i>D</i> , <i>z</i> 20_199 → <i>D</i> , <i>z</i> 1_19	2.696121271	0.053763685	0.012881809	0.001997330	0.0
<i>D</i> , <i>z</i> 200_up → <i>D</i> , <i>z</i> 1_19	0.327382526	0.005261100	0.002034078	0.006318463	0.005878289
<i>D</i> , <i>z</i> 1_19 → <i>D</i> , <i>z</i> 20_199	0.033817329	0.000427105	0.000129380	0.001198734	0.009431513
<i>D</i> , <i>z</i> 20_199 → <i>D</i> , <i>z</i> 20_199	0.0	0.0	0.0	0.0	0.0
<i>D</i> , <i>z</i> 200_up → <i>D</i> , <i>z</i> 20_199	0.0	0.0	0.0	0.0	0.0
<i>D</i> , <i>z</i> 1_19 → <i>D</i> , <i>z</i> 200_up	0.123132895	0.050114050	0.017433703	0.097523882	0.135399270
<i>D</i> , <i>z</i> 20_199 → <i>D</i> , <i>z</i> 200_up	0.000725058	0.0	0.0	0.0	0.0
<i>D</i> , <i>z</i> 200_up → <i>D</i> , <i>z</i> 200_up	0.0	0.0	0.0	0.0	0.0
<i>D</i> , <i>z</i> 1_19 → <i>B</i> , <i>z</i> 1_19	0.212302939	0.059831993	0.021823530	0.103473623	0.171239322
<i>D</i> , <i>z</i> 20_199 → <i>B</i> , <i>z</i> 1_19	0.0	0.0	0.0	0.0	0.0
<i>D</i> , <i>z</i> 200_up → <i>B</i> , <i>z</i> 1_19	0.043690097	0.007360407	0.000574583	0.0	0.0
<i>D</i> , <i>z</i> 1_19 → <i>B</i> , <i>z</i> 20_199	0.184643994	0.054828320	0.021031163	0.108659569	0.275442544
<i>D</i> , <i>z</i> 20_199 → <i>B</i> , <i>z</i> 20_199	0.0	0.0	0.0	0.0	0.0
<i>D</i> , <i>z</i> 200_up → <i>B</i> , <i>z</i> 20_199	0.0	0.001677875	0.003638771	0.007653646	0.0
<i>D</i> , <i>z</i> 1_19 → <i>B</i> , <i>z</i> 200_up	0.179577791	0.053431956	0.021227743	0.113325182	0.301730295
<i>D</i> , <i>z</i> 20_199 → <i>B</i> , <i>z</i> 200_up	0.0	0.0	0.0	0.0	0.002141048
<i>D</i> , <i>z</i> 200_up → <i>B</i> , <i>z</i> 200_up	0.0	0.001111461	0.004969657	0.019264109	0.001843747
<i>B</i> , <i>z</i> 1_19 → <i>D</i> , <i>z</i> 1_19	0.110178073	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 20_199 → <i>D</i> , <i>z</i> 1_19	0.325234357	0.004920625	0.0	0.0	0.0
<i>B</i> , <i>z</i> 200_up → <i>D</i> , <i>z</i> 1_19	0.012450382	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 1_19 → <i>D</i> , <i>z</i> 20_199	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 20_199 → <i>D</i> , <i>z</i> 20_199	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 200_up → <i>D</i> , <i>z</i> 20_199	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 1_19 → <i>D</i> , <i>z</i> 200_up	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 20_199 → <i>D</i> , <i>z</i> 200_up	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 200_up → <i>D</i> , <i>z</i> 200_up	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 1_19 → <i>B</i> , <i>z</i> 1_19	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 20_199 → <i>B</i> , <i>z</i> 1_19	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 200_up → <i>B</i> , <i>z</i> 1_19	0.004243014	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 1_19 → <i>B</i> , <i>z</i> 20_199	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 20_199 → <i>B</i> , <i>z</i> 20_199	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 200_up → <i>B</i> , <i>z</i> 20_199	1.010664999	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 1_19 → <i>B</i> , <i>z</i> 200_up	0.0	0.0	0.0	0.0	0.0
<i>B</i> , <i>z</i> 20_199 → <i>B</i> , <i>z</i> 200_up	0.0	0.0	0.0	0.0	0.0

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TABLE V – Continued from previous page

Source → Destination	d0	d100	d200	d500	d1000
<i>B, z200_up</i> → <i>B, z200_up</i>	1.297178111	0.0	0.0	0.0	0.0
<i>B, z1_19</i> → <i>P, z1_19</i>	0.100731885	0.001370626	0.0	0.0	0.004216514
<i>B, z20_199</i> → <i>P, z1_19</i>	0.0	0.0	0.0	0.0	0.0
<i>B, z200_up</i> → <i>P, z1_19</i>	0.0	0.000119848	0.0	0.000241394	0.0
<i>B, z1_19</i> → <i>P, z20_199</i>	0.255802297	0.006266095	0.000343754	0.0	0.000422371
<i>B, z20_199</i> → <i>P, z20_199</i>	0.0	0.0	0.0	0.0	0.0
<i>B, z200_up</i> → <i>P, z20_199</i>	0.144546061	0.0	0.000663091	0.000306429	0.0
<i>B, z1_19</i> → <i>P, z200_up</i>	0.0	0.0	0.0	0.0	0.0
<i>B, z20_199</i> → <i>P, z200_up</i>	0.0	0.0	0.0	0.0	0.0
<i>B, z200_up</i> → <i>P, z200_up</i>	0.633105311	0.0	0.0	0.000817912	0.0

Tables VI, VII, and VIII display the solutions of the slaughter, expiration, and birth probabilities for the first 10 alphabetical counties in each of the 10 States. To receive an electronic copy of the full set of these results, or of the full solutions of the two formulations described in appendix A, please contact the corresponding authors to place your request.

TABLE VI: Estimated cattle slaughter probabilities s_{c_1, t_1, j_1}^J

County_State	Type	z1_19	z20_199	z200_up
Allen_Kansas	P	0.033571193	0.5	0.036699097
	D	0	0	0
	B	0.007507848	0.009829014	0.010425229
Anderson_Kansas	P	0.023255814	0.108863578	0.149660755
	D	0	0	0
	B	0.006436844	0.010131312	0.011449758
Atchison_Kansas	P	0.023255814	0.329251249	0.1943457
	D	0	0	0
	B	0.008294217	0.00953625	0.00626163
Barber_Kansas	P	0.035871536	0.495192349	0.172171946
	D	0	0	0
	B	0.01351212	0.015395286	0.015013007
Barton_Kansas	P	0.043736865	0.495192349	0.185803167
	D	0	0	0
	B	0.005000492	0.012271488	0.01268112
Bourbon_Kansas	P	0.048816568	0.11798054	0.039352856
	D	0	0	0
	B	0.00747556	0.008900366	0.014620185
Brown_Kansas	P	0.025395369	0.5	0.030375102
	D	0	0	0
	B	0.00877883	0.013490409	0.010743065
Butler_Kansas	P	0.041906406	0.082253176	0.040519384
	D	0	0	0
	B	0.014817158	0.016771523	0.02093451
Chase_Kansas	P	0.040341958	0.5	0.06873808
	D	0	0	0
	B	0.013709577	0.020781735	0.017592407
Chautauqua_Kansas	P	0.495192349	0.5	0.031011781
	D	0	0	0
	B	0.00928569	0.011820695	0.013930095
Arkansas_Arkansas	P	0.495192308	0.495192308	0.495192308
	D	0	0	0
	B	0.018786025	0.019806452	0
Ashley_Arkansas	P	0.495192308	0.495192308	0.495192308
	D	0	0	0

Continued on next page

TABLE VI – Continued from previous page

County_State	Type	z1_19	z20_199	z200_up
	B	0.076923077	0.009600608	0.006503108
Baxter_Arkansas	P	0.495192308	0.5	0.495192308
	D	0	0	0
	B	0.013308846	0.016320406	0.010336798
Benton_Arkansas	P	0.495192308	0.5	0.495192308
	D	0	0	0
	B	0.007379618	0.007756007	0.006703147
Boone_Arkansas	P	0.495192308	0.173461238	0.041031942
	D	0	0	0
	B	0.008212355	0.007856942	0.00995242
Bradley_Arkansas	P	0.495192308	0.495192308	0.495192308
	D	0	0	0
	B	0.017529538	0.011132445	0.001517418
Calhoun_Arkansas	P	0.495192308	0.495192308	0.495192308
	D	0	0	0
	B	0.076923077	0.014819211	0.009714514
Carroll_Arkansas	P	0.495192308	0.495192308	0.495192308
	D	0	0	0
	B	0.008607682	0.00825681	0.010122331
Chicot_Arkansas	P	0.495192308	0.495192308	0.495192308
	D	0	0	0
	B	0.015486992	0.029886658	0.000472202
Clark_Arkansas	P	0.495192308	0.023255814	0.495192308
	D	0	0	0
	B	0.009921649	0.00985473	0.007553322
Adams_Colorado	P	0.495192308	0.051798519	0.050894645
	D	0	0	0
	B	0.003854386	0.009244913	0.013484212
Alamosa_Colorado	P	0.495192308	0.400168484	0.495192308
	D	0	0	0
	B	0.003464945	0.03197377	0.012721918
Arapahoe_Colorado	P	0.495192308	0.086493319	0.495192308
	D	0	0	0
	B	0.013202649	0.008962159	0.003452017
Archuleta_Colorado	P	0.495192308	0.495192308	0.495192308
	D	0	0	0
	B	0.021538545	0.076923077	0.017419468
Baca_Colorado	P	0.038064109	0.495192308	0.028683181
	D	0	0	0
	B	0.006636972	0.017203973	0.011915538
Bent_Colorado	P	0.031531953	0.235729296	0.210079799
	D	0	0	0
	B	0.00151656	0.019578763	0.014852309
Boulder_Colorado	P	0.023255814	0.121607622	0.495192307
	D	0	0	0
	B	0	0.009485814	0.009721789
Broomfield_Colorado	P	0.495192307	0.495192307	0.495192307
	D	0	0	0
	B	0.076923077	0.076923077	0.076923077
Chaffee_Colorado	P	0.495192307	0.5	0.495192307
	D	0	0	0
	B	0.006078239	0.014181864	0.015169728
Cheyenne_Colorado	P	0.023255814	0.386135369	0.042243727
	D	0	0	0

Continued on next page

TABLE VI – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	B	0	0.0326771	0
Adair_Iowa	P	0.023255814	0.080292363	0.037199482
	D	0	0	0
	B	0.007889276	0.011576839	0.008302867
Adams_Iowa	P	0.023255814	0.117924252	0.023255814
	D	0	0	0
	B	0.008155424	0.013556774	0.006198316
Allamakee_Iowa	P	0.023255814	0.03171779	0.023255816
	D	0	0	0
	B	0	0.012383564	0.009126368
Appanoose_Iowa	P	0.023255814	0.214178841	0.023255814
	D	0	0	0
	B	0.008260584	0.00923274	0.008508406
Audubon_Iowa	P	0.023255814	0.023255814	0.023255814
	D	0	0	0
	B	0.008213616	0.011263748	0.011165168
Benton_Iowa	P	0.036682697	0.202356581	0.023255814
	D	0	0	0
	B	0.008057512	0.012092271	0.00288699
Black_Hawk_Iowa	P	0.023255814	0.024470922	0.023255814
	D	0	0	0
	B	0.010468598	0.013949151	0.008014421
Boone_Iowa	P	0.023255814	0.076593929	0.048960521
	D	0	0	0
	B	0	0.01067626	0.010369557
Bremer_Iowa	P	0.023255814	0.023255814	0.023255814
	D	0	0	0
	B	0.008723028	0.019542613	0.00133549
Buchanan_Iowa	P	0.023255814	0.099811373	0.023255814
	D	0	0	0
	B	0.010350087	0.017962333	0
Aitkin_Minnesota	P	0.495192307	0.096105026	0.495192307
	D	0	0	0
	B	0.009309037	0.009821998	0.003410523
Anoka_Minnesota	P	0.495192307	0.026761309	0.042671192
	D	0	0	0
	B	0.076923077	0.018587494	0
Becker_Minnesota	P	0.023255814	0.041388208	0.023255814
	D	0	0	0
	B	0.006460702	0.010725619	0
Beltrami_Minnesota	P	0.495192307	0.197447769	0.033485257
	D	0	0	0
	B	0.004637162	0.010487564	0.011040085
Benton_Minnesota	P	0.023255814	0.261671828	0.023255814
	D	0	0	0
	B	0	0.00973622	0
Big_Stone_Minnesota	P	0.023255814	0.142620717	0.023255814
	D	0	0	0
	B	0.020191305	0.027630391	0.011872955
Blue_Earth_Minnesota	P	0.025031289	0.023255814	0.025909508
	D	0	0	0
	B	0	0.01354591	0.00684153
Brown_Minnesota	P	0.024324221	0.23404349	0.023255814
	D	0	0	0

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TABLE VI – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	B	0.003134147	0.011563272	0.013605261
Carlton_Minnesota	P	0.495192308	0.097548014	0.11391064
	D	0	0	0
	B	0.001057179	0.009290557	0
Carver_Minnesota	P	0.023255814	0.023255814	0.023255814
	D	0	0	0
	B	0	0.013540869	0.013255607
Adair_Missouri	P	0.495192307	0.114218014	0.023255814
	D	0	0	0
	B	0.007961366	0.016532786	0.010316696
Andrew_Missouri	P	0.023255814	0.208444035	0.185684308
	D	0	0	0
	B	0.009589729	0.008885841	0.00312721
Atchison_Missouri	P	0.028656358	0.5	0.029618309
	D	0	0	0
	B	0.012808331	0.014000949	0.009477909
Audrain_Missouri	P	0.028213303	0.242757101	0.023255814
	D	0	0	0
	B	0	0.010606543	0
Barry_Missouri	P	0.03491713	0.178992094	0.029262464
	D	0	0	0
	B	0.007380737	0.007998558	0.004833156
Barton_Missouri	P	0.050301409	0.031230044	0.036801372
	D	0	0	0
	B	0.00784318	0.009093817	0.004711364
Bates_Missouri	P	0.050304968	0.281939475	0.023255814
	D	0	0	0
	B	0.002211479	0.008745822	0.010344667
Benton_Missouri	P	0.023255814	0.142049026	0.023255814
	D	0	0	0
	B	0.007923787	0.008850476	0.005560948
Bollinger_Missouri	P	0.033173619	0.058613246	0.033917999
	D	0	0	0
	B	0.008612423	0.007867582	0.007814681
Boone_Missouri	P	0.495192308	0.218880817	0.104016435
	D	0	0	0
	B	0.007971714	0.008471301	0.00815955
Adams_Nebraska	P	0.042548066	0.200602995	0.041206148
	D	0	0	0
	B	0.015253812	0.021832914	0.023326377
Antelope_Nebraska	P	0.033339779	0.074081673	0.056553099
	D	0	0	0
	B	0.006739358	0.016816169	0.015400011
Arthur_Nebraska	P	0.362258126	0.495192308	0.091681228
	D	0	0	0
	B	0	0.076923077	0.039360571
Banner_Nebraska	P	0.026193458	0.5	0.5
	D	0	0	0
	B	0.001572474	0.043622562	0.014663388
Blaine_Nebraska	P	0.023255814	0.495192308	0.495192308
	D	0	0	0
	B	0.006412629	0.06564557	0.024348427
Boone_Nebraska	P	0.032027837	0.081569472	0.048848092
	D	0	0	0

Continued on next page

TABLE VI – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	B	0.006683754	0.022212827	0.011943518
Box_Butte_Nebraska	P	0.039159258	0.5	0.060395373
	D	0	0	0
	B	0.012107592	0.019609292	0.014680446
Boyd_Nebraska	P	0.033045794	0.5	0.057585913
	D	0	0	0
	B	0.004108109	0.026508828	0.013554222
Brown_Nebraska	P	0.043781836	0.5	0.033977332
	D	0	0	0
	B	0.009401976	0.024510091	0.018097803
Buffalo_Nebraska	P	0.038124145	0.220161259	0.023255814
	D	0	0	0
	B	0.007431053	0.01168565	0.01168051
Adair_Oklahoma	P	0.495192309	0.0859375	0.495192309
	D	0	0	0
	B	0.007473255	0.008213802	0.01392568
Alfalfa_Oklahoma	P	0.078730238	0.495192309	0.037011834
	D	0	0	0
	B	0.00413298	0.014720334	0.013769332
Atoka_Oklahoma	P	0.495192309	0.472367853	0.023255814
	D	0	0	0
	B	0.001421418	0.008280264	0.010565404
Beaver_Oklahoma	P	0.028946136	0.139422907	0.039773342
	D	0	0	0
	B	0.016697822	0.019623909	0.020462464
Beckham_Oklahoma	P	0.495192309	0.461884672	0.096797507
	D	0	0	0
	B	0.007522354	0.009596268	0.010796834
Blaine_Oklahoma	P	0.028619376	0.485633937	0.028211712
	D	0	0	0
	B	0.011267806	0.018077846	0.011723377
Bryan_Oklahoma	P	0.025830874	0.072840921	0.023255814
	D	0	0	0
	B	0.002619904	0.007997643	0.010601071
Caddo_Oklahoma	P	0.495192309	0.234356838	0.039039506
	D	0	0	0
	B	0.006250516	0.00883874	0.010990652
Canadian_Oklahoma	P	0.049321851	0.235966396	0.404514456
	D	0	0	0
	B	0.011382511	0.008867903	0.011459877
Carter_Oklahoma	P	0.495192309	0.092623108	0.104025769
	D	0	0	0
	B	0.007534227	0.00789964	0.014235211
Aurora_SouthDakota	P	0.023255814	0.155019831	0.025947217
	D	0	0	0
	B	0.004899654	0.030234257	0.012945114
Beadle_SouthDakota	P	0.023255814	0.285903398	0.023255814
	D	0	0	0
	B	0.006705068	0.022307439	0.015516797
Bennett_SouthDakota	P	0.495192304	0.220017613	0.023255814
	D	0	0	0
	B	0.006863572	0.029148529	0.016392049
Bon_Homme_SouthDakota	P	0.023255814	0.089922811	0.036915325
	D	0	0	0

Continued on next page

TABLE VI – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	B	0.005090991	0.01766321	0.011445517
Brookings_SouthDakota	P	0.023255814	0.311031245	0.023255814
	D	0	0	0
	B	0.008748808	0.017059643	0.01223668
Brown_SouthDakota	P	0.025677008	0.5	0.080357143
	D	0	0	0
	B	0.005615895	0.013638481	0.013165281
Brule_SouthDakota	P	0.023255814	0.139660616	0.039408529
	D	0	0	0
	B	0.004705102	0.027325173	0.020200506
Buffalo_SouthDakota	P	0.074527789	0.495192304	0.023255814
	D	0	0	0
	B	0	0.048829201	0.027976599
Butte_SouthDakota	P	0.495192304	0.5	0.042644015
	D	0	0	0
	B	0.010119128	0.011058908	0.014568834
Campbell_SouthDakota	P	0.032997901	0.191148857	0.023255814
	D	0	0	0
	B	0.005920757	0.076923077	0.018100999
Anderson_Texas	P	0.03631947	0.204080266	0.032203238
	D	0	0	0
	B	0.007782706	0.007196695	0.008817879
Andrews_Texas	P	0.495192302	0.495192302	0.095961538
	D	0	0	0
	B	0.004124906	0.024147624	0.039398783
Angelina_Texas	P	0.495192302	0.064093026	0.02808652
	D	0	0	0
	B	0.008952783	0.007432072	0.007934384
Aransas_Texas	P	0.495192302	0.5	0.495192302
	D	0	0	0
	B	0.004102684	0.012616353	0.02206189
Archer_Texas	P	0.076649947	0.451516835	0.023255814
	D	0	0	0
	B	0.006405842	0.011570082	0.013176313
Armstrong_Texas	P	0.495192302	0.495192302	0.495192302
	D	0	0	0
	B	0.012782341	0.030000496	0.014519382
Atascosa_Texas	P	0.023255814	0.077959736	0.040562325
	D	0	0	0
	B	0.004211175	0.006699459	0
Austin_Texas	P	0.495192302	0.023255814	0.118222788
	D	0	0	0
	B	0.007994126	0.006731931	0.008012339
Bailey_Texas	P	0.054255332	0.5	0.024905863
	D	0	0	0
	B	0.013867849	0.020343573	0.0226384
Bandera_Texas	P	0.495192302	0.495192302	0.495192302
	D	0	0	0
	B	0.01592188	0.007273256	0.004113778

TABLE VII: Estimated cattle culling probabilities dt_{c_1, t_1, j_1}^x

County_State	Type	z1_19	z20_199	z200_up
Allen_Kansas	P	0.009615385	0.009615385	0.009615385
	D	0.008628627	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Anderson_Kansas	P	0.005643688	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Atchison_Kansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Barber_Kansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.003272848	0.009615385	0.009615385
Barton_Kansas	P	0	0.009615385	0.009615385
	D	0.007365086	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Bourbon_Kansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.005748238
Brown_Kansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009252611	0.009615385	0.009615385
Butler_Kansas	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.001923077	0.001923077	0.001923077
Chase_Kansas	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.003239968	0.009615385	0.009615385
Chautauqua_Kansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.007797195	0.009615385	0.009615385
Arkansas_Arkansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Ashley_Arkansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Baxter_Arkansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.004838909	0.001923077	0.009615385
Benton_Arkansas	P	0.009615385	0.009615385	0.009615385
	D	0.006689507	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Boone_Arkansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.008976691	0.009615385	0.009615385
Bradley_Arkansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Calhoun_Arkansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Carroll_Arkansas	P	0.009615385	0.009615385	0.009615385

Continued on next page

TABLE VII – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	D	0.009615385	0.009615385	0.009615385
	B	0.008461196	0.009615385	0.009615385
Chicot_Arkansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Clark_Arkansas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Adams_Colorado	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Alamosa_Colorado	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Arapahoe_Colorado	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Archuleta_Colorado	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.001923077	0.009615385	0.00496589
Baca_Colorado	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Bent_Colorado	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Boulder_Colorado	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Broomfield_Colorado	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Chaffee_Colorado	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Cheyenne_Colorado	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Adair_Iowa	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Adams_Iowa	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Allamakee_Iowa	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Appanoose_Iowa	P	0.009615385	0.009615385	0.004362779
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Audubon_Iowa	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Benton_Iowa	P	0	0.009615385	0.009615385

Continued on next page

TABLE VII – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	D	0.008460672	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Black_Hawk_Iowa	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Boone_Iowa	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Bremer_Iowa	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Buchanan_Iowa	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Aitkin_Minnesota	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.003205128	0.009615385
	B	0.009615385	0.009615385	0.009615385
Anoka_Minnesota	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Becker_Minnesota	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Beltrami_Minnesota	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Benton_Minnesota	P	0.009615385	0.009615385	0.001105323
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Big_Stone_Minnesota	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Blue_Earth_Minnesota	P	0.009615385	0.003663372	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Brown_Minnesota	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Carlton_Minnesota	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Carver_Minnesota	P	0.006758086	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Adair_Missouri	P	0.009615385	0.009615385	0.006414382
	D	0.009615385	0.003205128	0.009615385
	B	0.009372131	0.001923077	0.009615385
Andrew_Missouri	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Atchison_Missouri	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Audrain_Missouri	P	0.009615385	0.009615385	0.009615385

Continued on next page

TABLE VII – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Barry_Missouri	P	0.009615385	0.009615385	0.009615385
	D	0.008572655	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Barton_Missouri	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Bates_Missouri	P	0.009615385	0.009615385	0.009615385
	D	0.009255684	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Benton_Missouri	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.003205128	0.009615385
	B	0.009615385	0.009615385	0.009615385
Bollinger_Missouri	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Boone_Missouri	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Adams_Nebraska	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.001923077	0.001923077	0.001923077
Antelope_Nebraska	P	0	0.009615385	0.009615385
	D	0.007292264	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Arthur_Nebraska	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Banner_Nebraska	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Blaine_Nebraska	P	0.001476284	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.001923077	0.009615385
Boone_Nebraska	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Box_Butte_Nebraska	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.005235557	0.009615385	0.009615385
Boyd_Nebraska	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Brown_Nebraska	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.007617898	0.009615385	0.009615385
Buffalo_Nebraska	P	0	0.009615385	0.009615385
	D	0.00498551	0.009615385	0.009615385
	B	0.009487195	0.009615385	0.009615385
Adair_Oklahoma	P	0.009615385	0.009615385	0.009615385
	D	0.007223185	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.006229949
Alfalfa_Oklahoma	P	0	0.009615385	0.009615385

Continued on next page

TABLE VII – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Atoka_Oklahoma	P	0.009615385	0.009615385	0.003425781
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Beaver_Oklahoma	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.001923087	0.001923077	0.001923077
Beckham_Oklahoma	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009540192	0.009615385	0.009615385
Blaine_Oklahoma	P	0	0.009615385	0.009615385
	D	0.006417325	0.009615385	0.009615385
	B	0.005496637	0.001923077	0.009615385
Bryan_Oklahoma	P	0.009615385	0.009615385	0.002429066
	D	0.006530466	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Caddo_Oklahoma	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Canadian_Oklahoma	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.005373552	0.009615385	0.009615385
Carter_Oklahoma	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.006222852
Aurora_SouthDakota	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Beadle_SouthDakota	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.001923077	0.009615385
Bennett_SouthDakota	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Bon_Homme_SouthDakota	P	0	0.009615385	0
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Brookings_SouthDakota	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.008377778	0.009615385	0.009615385
Brown_SouthDakota	P	0	0.009615385	0.009615385
	D	0.009615385	0.003205128	0.009615385
	B	0.009615385	0.009615385	0.009615385
Brule_SouthDakota	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Buffalo_SouthDakota	P	0.009615385	0.009615385	0.005092425
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Butte_SouthDakota	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.007149193	0.009615385	0.009615385
Campbell_SouthDakota	P	0.007470394	0.009615385	0.009615385

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TABLE VII – Continued from previous page

County_State	Type	z1_19	z20_199	z200_up
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Anderson_Texas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Andrews_Texas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Angelina_Texas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Aransas_Texas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Archer_Texas	P	0.009615385	0.009615385	0.009615385
	D	0.007376245	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Armstrong_Texas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.004471264	0.009615385	0.009615385
Atascosa_Texas	P	0	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Austin_Texas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385
Bailey_Texas	P	0	0.009615385	0.009615385
	D	0.009234928	0.009615385	0.009615385
	B	0.003252872	0.009615385	0.009615385
Bandera_Texas	P	0.009615385	0.009615385	0.009615385
	D	0.009615385	0.009615385	0.009615385
	B	0.009615385	0.009615385	0.009615385

TABLE VIII: Estimated cattle birth probabilities bt_{c_1, t_1, j_1}^x

County_State	Type	z1_19	z20_199	z200_up
Allen_Kansas	P	0	0	0
	D	0.027777778	0.016129032	0.017953942
	B	0.018276606	0.019230769	0.019230769
Anderson_Kansas	P	0	0	0
	D	0.025586227	0.016129032	0.018222653
	B	0.018107286	0.019230769	0.019230769
Atchison_Kansas	P	0	0	0
	D	0.016129032	0.016129032	0.018877302
	B	0.019230769	0.019230769	0.015413338
Barber_Kansas	P	0	0	0
	D	0.02529387	0.016129032	0.016435705
	B	0.019230769	0.019230769	0.019230769
Barton_Kansas	P	0	0	0
	D	0.027777778	0.016129032	0.017263927
	B	0.016907492	0.019230769	0.019230769
Bourbon_Kansas	P	0	0	0
	D	0.016129032	0.016129032	0.018376514

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TABLE VIII – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	B	0.019230769	0.019230769	0.019230769
Brown_Kansas	P	0	0	0
	D	0.016129032	0.016129032	0.018866441
	B	0.019230769	0.019230769	0.019230769
Butler_Kansas	P	0	0	0
	D	0.026170753	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.019230769
Chase_Kansas	P	0	0	0
	D	0.026010979	0.016129032	0.017472421
	B	0.019230769	0.019230769	0.019230769
Chautauqua_Kansas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.019230769
Arkansas_Arkansas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0	0.018969067	0.005159444
Ashley_Arkansas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0	0.019230769	0.014350436
Baxter_Arkansas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.019230769
Benton_Arkansas	P	0	0	0
	D	0.027777778	0.016129032	0.018169074
	B	0.019230769	0.019230769	0.015924444
Boone_Arkansas	P	0	0	0
	D	0.016129032	0.016129032	0.017836563
	B	0.019230769	0.019230769	0.019230769
Bradley_Arkansas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.008637904
Calhoun_Arkansas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0	0.019230769	0.01503217
Carroll_Arkansas	P	0	0	0
	D	0.023605494	0.016129032	0.01820416
	B	0.019230769	0.019230769	0.019230769
Chicot_Arkansas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.006679178
Clark_Arkansas	P	0	0	0
	D	0.016129032	0.016129032	0.016764637
	B	0.019230769	0.019230769	0.016338478
Adams_Colorado	P	0	0	0
	D	0.02469076	0.027777778	0.016129032
	B	0.017015418	0.019230769	0.019230769
Alamosa_Colorado	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.012848176	0.019230769	0.019230769
Arapahoe_Colorado	P	0	0	0
	D	0.016129032	0.016129032	0.01713899
	B	0.016918868	0.019230769	0
Archuleta_Colorado	P	0	0	0
	D	0.016129032	0.016129032	0.016129032

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TABLE VIII – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	B	0.019230769	0.019230769	0.019230769
Baca_Colorado	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.018184486	0.019230769	0.019230769
Bent_Colorado	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.012981611	0.019230769	0.019230769
Boulder_Colorado	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.008281172	0.019230769	0.016561338
Broomfield_Colorado	P	0	0	0
	D	0.016129032	0.016129032	0.017484407
	B	0	0	0
Chaffee_Colorado	P	0	0	0
	D	0.016129032	0.027777778	0.016129032
	B	0.014699315	0.019230769	0.019230769
Cheyenne_Colorado	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.010833001	0.019230769	0.006827021
Adair_Iowa	P	0	0	0
	D	0.016129032	0.016129032	0.018745377
	B	0.019230769	0.019230769	0.017138713
Adams_Iowa	P	0	0	0
	D	0.016129032	0.016129032	0.018897365
	B	0.019230769	0.019230769	0.014620485
Allamakee_Iowa	P	0	0	0
	D	0.027326503	0.016129032	0.01668865
	B	0.011652523	0.019230769	0.01602659
Appanoose_Iowa	P	0	0	0
	D	0.016129032	0.016129032	0.018565048
	B	0.019230769	0.019230769	0.017752919
Audubon_Iowa	P	0	0	0
	D	0.016129032	0.016129032	0.018683005
	B	0.018413723	0.019230769	0.019230769
Benton_Iowa	P	0	0	0
	D	0.027777778	0.016129032	0.017858711
	B	0.019230769	0.019230769	0.01163595
Black_Hawk_Iowa	P	0	0	0
	D	0.023496569	0.016129032	0.017945692
	B	0.019230769	0.019230769	0.016392004
Boone_Iowa	P	0	0	0
	D	0.016129032	0.016129032	0.018607536
	B	0.010042433	0.019230769	0.018242834
Bremer_Iowa	P	0	0	0
	D	0.024740326	0.016129032	0.01782086
	B	0.019230769	0.019230769	0.009736764
Buchanan_Iowa	P	0	0	0
	D	0.016129032	0.027777778	0.017571702
	B	0.019230769	0.019230769	0.009052111
Aitkin_Minnesota	P	0	0	0
	D	0.016129032	0.027777778	0.016129032
	B	0.019230769	0.019230769	0.011425996
Anoka_Minnesota	P	0	0	0
	D	0.016129032	0.016129032	0.016510483

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TABLE VIII – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	B	0	0.019230769	0.006181996
Becker_Minnesota	P	0	0	0
	D	0.023346785	0.027777778	0.01614006
	B	0.016560991	0.019230769	0.008857946
Beltrami_Minnesota	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.015279969	0.019230769	0.019230769
Benton_Minnesota	P	0	0	0
	D	0.024622896	0.027777778	0.016984489
	B	0.011100426	0.019230769	0.007760078
Big_Stone_Minnesota	P	0	0	0
	D	0.019332831	0.016129032	0.016702798
	B	0.019230769	0.019230769	0.019230769
Blue_Earth_Minnesota	P	0	0	0
	D	0.016129032	0.016129032	0.020741612
	B	0.009645197	0.019230769	0.008637162
Brown_Minnesota	P	0	0	0
	D	0.024461055	0.016129032	0.018214839
	B	0.014482383	0.019230769	0.019230769
Carlton_Minnesota	P	0	0	0
	D	0.016251788	0.027777778	0.016129032
	B	0	0.019230769	0.00868384
Carver_Minnesota	P	0	0	0
	D	0.024623555	0.016129032	0.017369622
	B	0.01117836	0.019230769	0.019230769
Adair_Missouri	P	0	0	0
	D	0.016129032	0.027777778	0.016129032
	B	0.019230769	0.019230769	0.019230769
Andrew_Missouri	P	0	0	0
	D	0.016129032	0.016621182	0.019129215
	B	0.019230769	0.019230769	0.012399117
Atchison_Missouri	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.018366686
Audrain_Missouri	P	0	0	0
	D	0.016129032	0.027777778	0.017783347
	B	0.011282715	0.019230769	0.009090127
Barry_Missouri	P	0	0	0
	D	0.027777778	0.027777778	0.018091841
	B	0.019230769	0.019230769	0.01391762
Barton_Missouri	P	0	0	0
	D	0.016129032	0.027777778	0.018188974
	B	0.019230769	0.019230769	0.013882739
Bates_Missouri	P	0	0	0
	D	0.027777778	0.027777778	0.018589607
	B	0.014072685	0.019230769	0.019230769
Benton_Missouri	P	0	0	0
	D	0.016129032	0.027777778	0.018404722
	B	0.019230769	0.019230769	0.014839656
Bollinger_Missouri	P	0	0	0
	D	0.016129032	0.027777778	0.016129032
	B	0.019230769	0.019230769	0.017300568
Boone_Missouri	P	0	0	0
	D	0.021661125	0.016129032	0.016859126

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TABLE VIII – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	B	0.019230769	0.019230769	0.017060361
Adams_Nebraska	P	0	0	0
	D	0.025699882	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.019230769
Antelope_Nebraska	P	0	0	0
	D	0.027777778	0.016129032	0.018085365
	B	0.018668958	0.019230769	0.019230769
Arthur_Nebraska	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.011580267	0	0.019230769
Banner_Nebraska	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.012238099	0.019230769	0.019230769
Blaine_Nebraska	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.018232305	0.019230769	0.019230769
Boone_Nebraska	P	0	0	0
	D	0.016129032	0.016129032	0.018126595
	B	0.018458332	0.019230769	0.019230769
Box_Butte_Nebraska	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.019230769
Boyd_Nebraska	P	0	0	0
	D	0.027062466	0.016129032	0.016848928
	B	0.015796609	0.019230769	0.019230769
Brown_Nebraska	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.019230769
Buffalo_Nebraska	P	0	0	0
	D	0.027777778	0.027777778	0.017695815
	B	0.019230769	0.019230769	0.019230769
Adair_Oklahoma	P	0	0	0
	D	0.027777778	0.027777778	0.018082334
	B	0.019230769	0.019230769	0.019230769
Alfalfa_Oklahoma	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.016268376	0.019230769	0.019230769
Atoka_Oklahoma	P	0	0	0
	D	0.016129032	0.027777778	0.016537072
	B	0.013317166	0.019230769	0.019230769
Beaver_Oklahoma	P	0	0	0
	D	0.016129032	0.027777778	0.016129032
	B	0.019230769	0.019230769	0.019230769
Beckham_Oklahoma	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.019230769
Blaine_Oklahoma	P	0	0	0
	D	0.027777778	0.016129032	0.016938883
	B	0.019230769	0.019230769	0.019230769
Bryan_Oklahoma	P	0	0	0
	D	0.027777778	0.027777778	0.017394324
	B	0.014566378	0.019230769	0.019230769
Caddo_Oklahoma	P	0	0	0
	D	0.016129032	0.016129032	0.016978254

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TABLE VIII – *Continued from previous page*

County_State	Type	z1_19	z20_199	z200_up
	B	0.018372329	0.019230769	0.019230769
Canadian_Oklahoma	P	0	0	0
	D	0.016129032	0.016129032	0.017138516
	B	0.019230769	0.019230769	0.019230769
Carter_Oklahoma	P	0	0	0
	D	0.016129032	0.027777778	0.016129032
	B	0.019230769	0.019230769	0.019230769
Aurora_SouthDakota	P	0	0	0
	D	0.027741076	0.027777778	0.016661265
	B	0.016509084	0.019230769	0.019230769
Beadle_SouthDakota	P	0	0	0
	D	0.02522112	0.016129032	0.017119513
	B	0.018596288	0.019230769	0.019230769
Bennett_SouthDakota	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.018484221	0.019230769	0.019230769
Bon_Homme_SouthDakota	P	0	0	0
	D	0.016129032	0.016129032	0.017964506
	B	0.016561308	0.019230769	0.019230769
Brookings_SouthDakota	P	0	0	0
	D	0.027107932	0.016129032	0.017786228
	B	0.019230769	0.019230769	0.019230769
Brown_SouthDakota	P	0	0	0
	D	0.024652157	0.027777778	0.016129032
	B	0.017324439	0.019230769	0.019230769
Brule_SouthDakota	P	0	0	0
	D	0.024219127	0.016129032	0.016129032
	B	0.016567037	0.019230769	0.019230769
Buffalo_SouthDakota	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.011384863	0.019230769	0.019230769
Butte_SouthDakota	P	0	0	0
	D	0.022164075	0.027777778	0.016129032
	B	0.019230769	0.019230769	0.019230769
Campbell_SouthDakota	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.017423846	0	0.019230769
Anderson_Texas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.018046785
Andrews_Texas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.014572897	0.019230769	0.019230769
Angelina_Texas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.016906416
Aransas_Texas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.012403999	0.019230769	0.019230769
Archer_Texas	P	0	0	0
	D	0.027777778	0.016129032	0.017299292
	B	0.018194069	0.019230769	0.019230769
Armstrong_Texas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032

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TABLE VIII – Continued from previous page

County_State	Type	z1_19	z20_199	z200_up
	B	0.019230769	0.019230769	0.019230769
Atascosa_Texas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.015680453	0.019230769	0.008757709
Austin_Texas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.017517157
Bailey_Texas	P	0	0	0
	D	0.027777778	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.019230769
Bandera_Texas	P	0	0	0
	D	0.016129032	0.016129032	0.016129032
	B	0.019230769	0.019230769	0.01301213

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