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Multi-Dimensional Urban Sprawl in Europe: a Self-Organizing Map Approach

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Multidimensional urban sprawl in Europe: a self-organizing map approach

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Abstract

The paper considers *urban sprawl* in Europe from a multidimensional and exploratory point of view. Several studies on the topic (mainly from urban economics, but also from other fields, such as urban planning or regional science) are reviewed in order to identify the key dimensions of the *urban sprawl* phenomenon in Europe. These comprise the following six elements: connectivity, decentralization, density, scattering, accessibility to open space, and land-use mix. Several indicators are defined and calculated to measure each of these for a representative sample of 209 European cities. The data are next employed in a *self-organizing map* (SOM) algorithm to investigate how the different dimensions interact with each other, and what is their behaviour. The results of the SOM reveal very interesting patterns that could not be unveiled otherwise. The paper also presents neat results of an exploration on the relationship between geographical location and population size, and characterizes the cities considered in terms of their degree and variety of *sprawl*.

JEL C45, R0, R12, R14.

Keywords Urban sprawl, self-organizing maps, Europe

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1 Introduction

At the beginning of the decade, Galster et al. (2001) wrote that: '*urban sprawl is one name for many conditions*'. In its modern usage, the term was first coined in 1937 by Earle Draper (Nechyba and Walsh, 2002) but, ever since then, its use has spread to a point that has made it almost unmanageable. Today, it can be heard in many forums and from very diverse practitioners, who often use it to designate different realities: what *urban sprawl* means to a politician probably differs from what it is for an urban planner, and that is also far from what inspires an environmental activist. Even inside academia, it represents different concepts depending on the discipline: some focus on its social aspects, other researchers see it as the outcome of free market choices, while yet others identify it as an environmental threat. Such a wide range of views and opinions usually leads to ambiguity and confusion. However, one can interpret this situation in at least two positive ways: first, it is an unmistakable proof that *urban sprawl*, whatever we refer to by those two words, is a relevant issue that is present and affects many people's lives; second, this apparent chaotic situation also represents an opportunity to obtain a much richer understanding of what it really is about. The problem does not come from the great number of interpretations per se, but because of the lack of agreement about the meaning and structure of the term.

In this paper we try to help meet the above-mentioned need for clarity and comprehension of the sprawl. We adopt an exploratory and multidimensional approach that explicitly considers the complexity of urban sprawl, and analyse in which way this phenomenon manifests itself in Europe. We construct a working definition that encompasses six dimensions, and originates from conceptualizing the main ideas present in the literature. Such a definition is implemented through the calculation of indices that capture each of the dimensions for a representative sample of 209 European urban regions. Once all of them are computed, we bring together all the information by means of the self-organizing map algorithm, which is a method to analyse multidimensional data that allows for the identification of patterns that would otherwise remain hidden behind the complexity of the data.

There are two main novelties associated with the aim of this paper: one refers to the region of analysis; and the other one has to do with the methodology employed. It is quite remarkable that, even though Europe is one of the most developed regions of the planet and one where sprawl is supposed to be appearing more and more over time, its urban sprawl has received so little attention from urban researchers; in other words, we do not know much about sprawl in this geographical area based on the basis of facts. One probable reason for this is the limited availability of good urban data at the supra-national level, which undoubtedly makes it difficult to perform studies of this kind. It is likely that this is because Europe is made up of different countries, with different structures and institutions for data definition and collection. In this paper we face the challenge of considering Europe as the region of analysis, obtaining data from different sources and merging them into a consistent and coherent database of 209 representative cities in order to overcome the lack of good readily available data sets.

The second contribution of this paper is related to the use of the self-organizing map (SOM) algorithm and is possible because of the exploratory attitude we adopt, geared towards uncovering relevant patterns rather than to confirm already existing hypotheses. As Yan and Thill (2009) state: *"we have relatively few insights into the geographic processes through which different types of SI* [spatial interaction] *systems emerge from different sets of regional arrangements."* In this regard, the present paper is an effort to help close that gap and offer new knowledge about the nature and structure that urban sprawl takes in Europe, with the hope that this will be used by future works to inspire models that clarify the underlying processes at work. Such an approach departs from traditional inference methods which, as Skupin and Agarwal (2007) argue in this context, *"are either failing or have become obstacles in the search for geographic structures, relationships, and meaning"*. One of its advantages is that we can fully exploit and study the complexity that is inherent in this kind of data. The method we have chosen to use in order to articulate this research philosophy is the SOM, an algorithm that is designed to compress highly dimensional data in a way that preserves all the meaningful information but is also tractable for the human brain. This allows us to consider sprawl as a multi-angled phenomenon and to analyse how its different aspects interact with each other. The SOM was developed to study the human brain structure but, over the years, its use has spread across a wide range of fields such as language pattern recognition and engineering. Surprisingly enough, it is a method that has not been used very often in the social sciences. However, we believe it perfectly fits the needs of a study of this kind, since it allows us to fully exploit the characteristics of the data we deal with, instead of simplifying and making them fit into other methodologies.

The remainder of the paper is structured as follows: Section 2 reviews the literature on sprawl and develops a conceptual categorization of the dimensions that comprise *urban sprawl*; Section 3 describes the databases employed and the strategy followed to define the spatial extent of the regions analysed, as well as details of the indices of choice to capture each of the dimensions; in Section 4 we explain the methodology (SOM) used to analyse sprawl in a multidimensional way; Section 5 presents the results and interpretation of the self-organizing map; and Section 6 concludes with some final remarks.

2 *Urban Sprawl*: a Multidimensional and Ambiguous Concept

As pointed out earlier, the term "urban sprawl" is a very loose and ambiguous one. However, this variety of views and understandings may also enrich the analysis if it is performed properly. This section has the purpose of putting order and structure into the ideas proposed by the literature and turn the looseness into richness which can be used to get a better insight about the real world. The first part reviews the existing literature, focusing on that from urban economics but also considering the fields of geography and urban planning. Far from being comprehensive, it aims to summarize the main concepts and suggestions proposed so far. The second part sums up the contributions obtained and structures them to arrive to a multidimensional working definition of urban sprawl that enables for the measurement and quantification of the phenomenon in the subsequent sections.

2.1 Literature review

Urban sprawl is a fairly recent topic whose relevance has become apparent over the second half of the $20^{th.}$ century, and whose importance is still growing today. As such, the research question behind it is an evolving one, and so is the literature concerned. In fact, although there have been plenty of contributions over the last few years, the main issues are still open, and there is a rather interesting debate going on at various levels. Due to the nature of this topic and to the negative connotations that usually come attached and make it an undesirable social outcome, it is interesting to note that most of the work, both empirical and theoretical, has been geared towards producing useful policy reccommendations; the former mainly has considered issues of definition and mainly, while the latter has tried to get to the 'heart' of the question and delve into the underlying mechanisms that give rise to it.

The vast majority of studies on sprawl have considered it from an empirical point of view but, as Clifton et al. (2008) note, this applied work has come from very different disciplines and with as many diverse interests, foci, and ways of proceeding. There is a strand that deals with laying down working definitions of sprawl and how to implement them to quantify it. Galster et al. (2001) offer one of the first structured and accurate definitions of the concept, one that explicitly recognizes its multidimensionality and is based on a set of eight quantifiable aspects, for which they define indices, thus facilitating empirical assessment. The study is complemented by Wolman et al. (2005), who go deeper into the technicalities of how to implement such a definition. A similar approach is taken by Frenkel and Ashkenazi (2008), who define sprawl in terms of three dimensions that can be transformed into two categories and analysed using seven variables, going beyond just establishing a list of the components but also conceptually categorizing them. Carruthers et al. (2009) adopt a novel approach and use proportional hazard models to characterize urban form, and identify sprawl as a variant of it. Another set of papers proposes different methodologies and tools to measure sprawl, such as: accessibility indices (Jungyul Sohn and Songhyun Choi, 2008); remote sensing (Sudhira et al., 2004); spatial autocorrelation statistics (Tsai, 2005); or combinations of all of them (Torrens, 2008). Some works consider not only the measurement of sprawl but also its determinants (Malpezzi, 1999) and others begin with a working definition and focus on the main determinants that cause sprawl (Burchfield et al., 2006). In other cases, the focus is on the effects rather than the causes: Brueckner and Largey (2008) look at how sprawl affects social interactions; Glaeser and Kahn (2004) consider the relation with urban growth; and Cho et al. (2009) deal with the trade-off between the value of shared open space and parcel size (associated here with more sprawl).

Last, since one of the novelties of this work is considering Europe, a note on the usual regions of study is needed. Typically, the US is the main object of focus, although other areas that at first sight might seem secondary have also received attention (Israel, Korea, India). However, it is remarkable that so little work has been devoted to Europe, particularly taking into account its relevance. Uhel (2006) represents a report on the state of the affairs in Europe carried out by the European Environment Agency, and Patacchini and Zenou (2009) is one of the first efforts to face the limitations in the availability of European data and consider sprawl explicitly. Probably the main reason for this is the lack up until very recently of consistent data across the different countries that make up the continent and the quality of the existing ones. However, if instead of several countries, we zoom in to only one nation, more work has been done (see Ritsema van Eck and Koomen, 2008).

Although most of the efforts have been put into measuring and quantifying sprawl, its causes and consequences, some articles have looked at it from a theoretical point of view. In this regard, it has been mainly economists who have utilized already existing theories in the field, such as externalities or market failures, to offer explanations on this topic (Brueckner, 2000, 2001; Harvey and Clark, 1965; Nechyba and Walsh, 2002). Other works have abstracted reality into a model to explicitly tackle sprawl; some of the models (see Brueckner and Fansler, 1982) were developed decades ago, but it has only been in recent years that most attention has been paid to this kind of approach (e.g. Brueckner and Helsley, 2009; Turner, 2007).

2.2 Categorization of the dimensions

The main conclusion we may draw from the previous review is that, as a multidisciplinary and broad issue, urban sprawl is a multidimensional topic that has been looked at from many different points of view and defined in several ways. This of course enriches the understanding but also introduces ambiguity that usually prevents the analysis from moving forward. For the purpose of this study, we need a working definition which is as complete as possible and allows us to consider and measure sprawl in Europe, in order to explore how and where it is most pressing.

With that requirement in mind, we collect the variables that, in our view, are the ones most often used and divide them into two main categories to come up with an accurate and measureable but complete idea. This approach is in line with that of Frenkel and Ashkenazi (2008), in the sense that different variables are conceptualized into categories that offer a better structure to understand the concept, although we employ a different set of categories that encompasses more aspects and includes more diverse variables. Later on, in Section 3.2, we define one index for each of the variables that allows us to operationalize them.

Table 1: Dimensions of urban sprawl

Dimensions	**Category**
Scattering Connectivity Availability of open space	Urban morphology
Density Decentralization Land-use mix	Internal composition

Table 1 shows the proposed conceptual categorization; there are six main dimensions that fall into two main conceptual categories. The first one is *urban morphology*, which includes as variables the scattering of urban development, the connectivity of the area, and the availability of open space. By "morphology", we mean the spatial configuration and the existing linkages between the different components of a city, both between each other and within themselves. If we think of the concept of "urban region" in a broad sense, this encompasses not only areas that, in terms of land-use, are really urban (i.e. residential, industrial, streets, parks, etc.) but also that part of the land in-between which, despite being part of the region, is not devoted to actual urban uses (i.e. forest, agricultural land between urban uses, etc.). If we call the relevant urban pieces of land *patches*, "urban morphology" here refers to how such patches are arranged and connected within the whole extent of the region. Hence the second category refers to the *internal composition* of the parts defined before; in other words, the focus here is not on *what* is the structure but on *how* that structure is *filled*. This involves considering the arrangement of the people who inhabit the region. To capture this, we will use three variables (density, decentralization, and land-use mix) that each represent different aspects of how the population is distributed across the patches.

3 Data and Indices of Sprawl

3.1 Data

In order to empirically assess urban sprawl in Europe, several data sources had to be assembled in order to come up with a coherent, complete and consistent set of urban regions for which we could measure the different dimensions identified in the previous section. Statistical geo-referenced data were combined with raster and vector geospatial data, in a process that uses the database Urban Audit as a starting point, and complements it with spatial information on land-use provided by the Corine and Urban Morphological Zones data sets.

The Urban Audit database is a project coordinated by EUROSTAT that aims to provide a wide range of indicators (around 300) and variables of several socio-economic, environmental, and other aspects for a representative sample of European cities. It covers four periods in which the data were collected: 1989-1993, 1994-1998, 1999-2002, 2003-2006. We have chosen the third of these periods because it has the most complete data availability, and because it represents the best match with the other data sources employed. The spatial dimension of the cities is defined at three different scales: the core city, which encompasses the administrative boundary of the city; the Larger Urban Zone (LUZ from now on), which is an approximation of the functional urban region centred around the core; and the Sub-City District level (SCD), which is a subdivision of the LUZ of 5,000-40,000 inhabitants per unit according to strict criteria (EUROSTAT, 2004). Since urban sprawl is a phenomenon that particularly affects the fringe of the cities and spreads over the whole urban region, not only in the core, the main unit of analysis in this study will be the LUZ. We will use the boundaries of the LUZs provided by the Urban Audit project in order to delineate the maximum extent of each urban region, while we will also employ the core city definitions to calculate some of the dimensions identified in the previous section.

Despite being one of the most solid and comprehensive statistical data sets available to date about European cities, the Urban Audit suffers from many missing values, which in some cases makes the use of variables unfeasible. This is the situation for many of those required to measure the dimensions of sprawl identified in the literature. For that reason, we have opted to employ this source just as a reference point, in order to choose the cities of interest and to delineate the spatial extent of each of them, but we will not rely much on the actual data provided, calculating instead most of the dimensions ourselves in order to have a more complete sample. This is possible because many of the aspects of sprawl are purely spatial and related to land-use issues, which makes it possible to integrate the presented approach with other data sources that provide more accuracy and completeness.

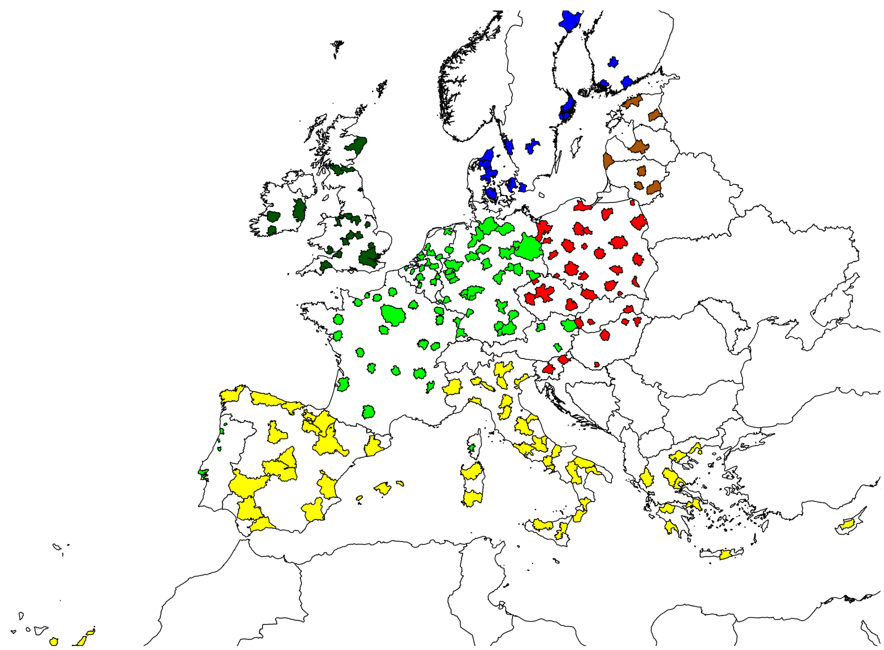
In our case, we make use of two other databases that provide information about the land cover in Europe: the Corine data set and the Urban Morphological Zones. The Corine land-cover project is provided by the European Environment Agency[1] (EEA) and contains data from the year 2000 for the EU-27 and other adjacent countries[2] on land-cover at 100 metres resolution, using the 44 classes of the 3-level Corine nomenclature. The data set Urban Morphological Zones (UMZ hereafter) 2000 is also provided by the EEA and is derived from the Corine. It essentially collects only urban morphological zones, being those

[1]For further details, refer to: `http://www.eea.europa.eu/`

[2]Such countries are: Albania, Bosnia and Herzegovina, Croatia, Iceland, Liechtenstein, Macedonia, the former Yugoslavian Republic of Montenegro, Norway, San Marino, Serbia.

defined as sets of urban areas lieing "less than 200m apart" (Milego, 2007)[3]. Using both sources together provides us with a rich data set on urban land-use in Europe.

Although the Urban Audit data set offers statistics for 258 cities in Europe and the land-use data covers most of Europe, when linking the three datasets, some urban regions had to be dropped in order to obtain a coherent, complete and consistent set. Figure 1 shows the geographical extent of the final set, in which each city has been coloured according to the supra-national region to which it belongs (see Section 5 for more on this). It is composed of 209 LUZs and 215 cores, since some of them are close enough to be part of the same urban area and, although it does not cover all the cities in Europe, the sample is meant to be representative of them[4].



LEGEND: **brown** the Baltic States; **dark green** the British Isles; **red** Central European countries; **yellow** Mediterranean countries; **blue** Northern countries; and **light green** the rest of Western Europe.

Figure 1: Larger Urban Zones (LUZs) employed in the study

As mentioned above, our strategy is to use the LUZ boundaries provided by

[3]For more information about which particular classes are included as urban, please refer to Milego (2007).

[4]More information about the selection process may be found in EUROSTAT (2004).

the Urban Audit project as the spatial extent of each observation; then once each region is delimited, we use the urban patches from the UMZ data to define the actual areas for which we calculate the dimensions of sprawl. The reason we proceed this way is that the LUZ boundaries are approximations of the functional urban region, that is of all the settlements that interact economically with the core, but these areas comprise not only the urban zones but also much in-between land that is not urban; moreover, these boundaries are often administrative, which means that oftentimes some large non-urban areas (e.g. military areas, forests, etc.) are also part of the region. In this way, we calculate indices only for the developed part of each region, which is the aim of the study, and drop out of the analysis those areas that, despite being part of the administrative area, are not classified as urban.

3.2 Different indices to measure different aspects of sprawl

Section 2.2 identifies the six aforementioned concepts (connectivity, decentralization, density, scattering, availability of open space, and land-use mix) as the main components or dimensions of *urban sprawl*. However, as they come from the literature, they are all very broad and, in some cases, fuzzy terms, which may be interpreted in very different ways depending on the context, making it difficult to operationalize them. This section narrows them down into indices that articulate such ideas and enable them to be implemented and quantified with real-world data.

Before starting, a few naming conventions will ease the reading of the formulas. We will employ the subindex r to designate an urban region or, in this particular context, the LUZ; the subindex c refers to the core of the region. Some indices consider measures that, as mentioned before, only apply to the urban patches of land, not to the whole extention of the LUZ; for the cases in which the index is calculated only for the urban patches and then summed (or weighted) across the LUZ, each patch will be designated by the subindex p, P being the total number of patches in the region.

3.2.1 Connectivity

Connectivity refers to the ease of getting from one place to another one within a city. The urban economic literature has related this to the concept of commuting cost, which refers only to trips to work. We will borrow this idea and use the average time of journey to work[5] from the Urban Audit, since it represents the best combination of closeness to the original idea and data availability. However, despite trying to minimize the problem of missing values, the amount of non-reported values for this variable is still 103 (out of 209 regions). In the context of this study, longer than average commutes will be interpreted as more sprawl.

[5] The code of the indicator is `tt1019i`, in the Urban Audit nomenclature.

3.2.2 Decentralization

Decentralization aims at capturing to what extent the population lives outside the core city. With that purpose, we develop the following indicator:

$$decen_r = \frac{Pop_r - Pop_c}{Pop_c}, \tag{1}$$

where *Pop* represents population. It measures the proportion of people who live *decentralized* over that who live in the core, so higher values will imply more sprawl.

3.2.3 Density

Density has been one of the most employed variables to look at the spatial structure of cities. Probably because of that, it has also been one of the phenomena to which more attention has been paid in order to measure sprawl, and several ways have been proposed to capture it, with different levels of complexity and different interests. In our case, we will adopt a conceptually very simple approach but will attempt to implement it in a way that is not distorted by the administrative rules defining the boundaries we use. We will use the following formula:

$$dens_r = \frac{Pop_r}{\sum_{p=1}^{P} A_p}, \tag{2}$$

where A represents the area. This takes the original idea of density (i.e. intensity of a phenomenon in a bounded space) and applies it only to the area of interest, that is the morphologically urban area, avoiding all the non-urban land included in each region. Sprawl has often been related to low of densities, and so will we consider that here as well.

3.2.4 Scattering

Scattering is another variable often considered to define sprawl. Again, several ways to measure it, with different levels of complication, have been proposed. In this case, we will also adopt a simple but clear approach which we believe captures the essence of the concept. We will employ the following expression:

$$scat_r = \frac{P_r}{1000 \times Pop_r}. \tag{3}$$

This measure reflects how fragmented and scattered the urban development is across the whole urban region. Sprawl is then identified as a high number of different patches; we divide by population[6] to correct for the size effect, since it

[6] The reason why it is multiplied by 1000 is to re-scale the variable so as not to obtain very low values for all the cities.

is reasonable to expect larger areas to have more patches, simply because they encompass larger geographical regions.

3.2.5 Availability of open space

Availability of open space in the urban environment is captured here by the percentage of the urban area within the urban region that is classified as urban green space (see Appendix A for more details). The expression is:

$$opsp_r = \frac{\sum_{p=1}^{P} opsp_p}{\sum_{p=1}^{P} A_p}, \tag{4}$$

where *opsp* represents the area of land classified as green space.

3.2.6 Land-use mix

In this context, the goal is to have an index that captures the degree to which different landuses are mixed within one urban region. However, land-use mix may be seen as a particular case of a more global concept which is "diversity". All across science, many disciplines have paid attention to this topic; in particular, ecology has heavily employed the concept to measure, for instance, species diversity in ecosystems. We will, therefore, take advantage of that and borrow one of the ecologist's most popular indices: Simpson's index of diversity (Simpson, 1949) to measure the degree to which different landuses are mixed within an urban region. This index has the nice properties of being simple to calculate, bounded, and possible to interpret in terms of the probability that two observations drawn at random from the sample belong to different classes (Baumgärtner, 2006). The original formula is:

$$S = \sum_{i}^{m} p_i, \tag{5}$$

where $p_i =$ is the proportion of observations in class i in the sample, and m is the number of different classes. This index ranges from $1/m$ (largest diversity) to 1 (maximum concentration). It is usually employed in reverse way (either $1/S$ or $1 - S$) so that larger values of the index imply more diversity. In this case, we will adopt the approach of Torrens (2008), in which the index is defined as:

$$S' = \frac{1 - \sum_{i}^{m} p_i}{1 - \frac{1}{m}}. \tag{6}$$

This way, the index ranges from 0 to 1. As Torrens (2008) puts it: "*The value of the Index (SIEI) nears zero when distribution of area among different activities grows uneven (is dominated by one activity) and reaches zero when the landscape contains only a single patch of activity. The value reaches one when the distribution of area is even, i.e. when proportional abundances for land-use are the same*" (p. 12).

4 Methodology: the Self-Organizing Map Algorithm

Once the measures defined in Section 3.2 are calculated for the database of European cities, our strategy is to bring all the *parts* together to consider the phenomenon of urban sprawl as one, but without losing relevant information about its multidimensionality. The method we use for such a task is the *self-organizing map* (SOM) but, before we delve into the analysis of *urban sprawl* in Europe, let us explain this technique in a little more detail.

The SOM[7] is a special kind of unsupervised computational neural network (Fischer, 2001) that combines both data *projection* (reduction of the number of attributes or dimensions of the data vectors) and *quantization* or clustering (reduction of the number of input vectors) of the input space without loss of useful information and the preservation of topological relationships in the output space. In a less technical way, we can say the SOM is a method that takes multidimensional data and compresses the information contained in these data in order to present it in an understandable way for the human brain. It is usually employed for two main purposes: one, to visualize complex data sets by reducing their dimensionality; and two, to perform cluster analysis to group similar observations into exclusive sets. Because of the first of these purposes, it has been compared with other methods such as principal components or factor analysis; and because of the latter, it has also been associated with techniques like the k-means or hierarchical clustering. It was first developed at the beginning of the 1980s (Kohonen, 1982) with the purpose of explaining the spatial organization of the brain's functions (Kohonen and Honkela, 2007), but the range of fields and applications that the SOM has been used for in recent years has grown exponentially [8]. Although there have been several variants and modifications depending on the kind of data and specific purposes for which it is used, we will consider here the basic algorithm, which performs a "non-linear, ordered, smooth, mapping of high dimensional input data manifolds onto the elements of a regular, low-dimensional array" (Kohonen, 2001, p. 106). Given the characteristics of the data and the purpose of this study, we believe the SOM is worth exploring as a method to analyse urban spatial structure.

[7] This section briefly explains the general idea and functioning of the basic SOM algorithm. For a complete and rigorous treatment of the SOM methods, the reader is referred to Kohonen (2001).

[8] For a bibliography of papers using the SOM algorithm, see Oja et al. (2003).

Before detailing the actual procedure, it is helpful to clarify a couple of concepts that will be used throughout the explanation. The first is that of *input space* (also called *signal*), which refers to the set of input data we employ to feed the algorithm; typically, the observations are multidimensional and are thus expressed by using a vector for each of them. The second concept is that of *output space* (trained network, network or SOM), which defines the low-dimensional universe in which the algorithm represents the input data. It usually (although not necessarily) has two-dimensions, and is composed of a set of elements called *neurons* (or *nodes*) which are interconnected, hence the network. What the algorithm does is to represent the input space onto the output space, keeping all the relevant information and ordering observations in a way such that topological closeness in the output space implies statistical similarity in the input space.

The input space is composed by n-dimensional vectors that we want to visualize/cluster in a low-dimensional (usually two) environment (output space). We can express the input vector t as:

$$x = [\xi_1(t), \xi_2(t), \cdots, \xi_n(t)]^T \in \Re^n, \tag{7}$$

where $\zeta_i(t)$ represents the value for each dimension. There are no specific requirements for the data before they become part of the signal space. However, as in other statistical procedures, depending on the distribution of the dimensions and their scales (if they represent different variables), it may be useful to normalize them so they all range within the same bounds and/or to take a logarithmic transformation to avoid skewness.

The output space is an array of x by y neurons (nodes) topologically connected following a kind of geometrical rule (the most common topologies being squares and hexagons). Each of the nodes is assigned a parametric real vector, which we call *model*, and express as:

$$m_i = [\mu_{in}, \mu_{in}, \cdots, \mu_{in}]^T \in \Re^n, \tag{8}$$

The actual values of m_i may be chosen randomly or assigned with any kind of previous knowledge in order to speed up the computing process. We may also define as $d(x, m_i)$ any distance metric between two vectors x and m_i. The most widely used is the Euclidean distance, although other specifications are also valid.

What we are looking for is a topologically-ordered representation of the signal space into the network. That is done by the SOM in an iterative process called *training*, in which each signal vector is sequentially presented to the output space. The best matching unit (b.m.u.) for x is defined as:

$$c = arg\,\min_i \{d(x, m_i)\}, \tag{9}$$

When c is found, the neuron m_i is activated and an addaptive process starts by which the b.m.u. and its topological neighbours are modified by the following scheme:

$$m_i(t+1) = m_i(t) + h_{ci}(t)[x(t) - m_i(t)], \tag{10}$$

where t and $t+1$ represent, respectively, the initial and the final state after the signal has activated the neuron; $h_{ci}(t)$ is a called *neighbourhood function* and expresses how the b.m.u. and its neighbours are modified when activated by a signal; usually, the linear or Gaussian versions are used. This process is repeated over many cycles before the training is finished. The neighbourhood function depends on several parameters relevant for this stage: the distance between the b.m.u. and the modified neuron (so the further away the neuron is, the smaller the adjustment); a *learning rate* $\alpha(t)$ that defines the magnitude of the adjustment, and gradually decreases as the training cycles advance; and the *neighbourhood radius*, which decides which of the surrounding neurons of the b.m.u. are also modified by the neighbourhood function, and also decreases over the training stage. All together, they make possible the topological preservation of the distances between the input vectors in the output space.

It is important to stress that this aforementioned learning process is at the heart of the method, and comes from the fact that the neighbourhood function does not perfectly adjust the b.m.u. and its neighbours but only partially (this is controlled by the learning rate), and that this process is repeated a fairly large number of times, which leads to the arranging and *self-organization* of the input information onto the output space. It is also of interest to remark that, unlike other clustering or data reduction methods, the SOM does not compare the input vectors directly but only makes comparisons between signal and neuron.

Once the training stage is completed, the network is ready to be used. There are two main (complimentary) approaches which we will consider in order to explore the information provided by a trained SOM: looking at the network created, and comparing it with the original input vectors. The former gives an idea about the general trends and relationships of the data set, while the latter is intended to give information about individual vectors and how they interact with each other.

First, the network itself is a useful tool of analysis. It has been said (Kohonen, 2001, p. 160) that a SOM tries to represent $p(x)$, as if it were a probability surface. A useful way to extract information is by visualizing the values of one of the dimensions (*planes*) on the network. This procedure allows us to see how different values of different dimensions map together and interact. By construction, the SOM preserves the topology, but it tries to fill all the available space, thus distorting actual distances from the elements. In order to visualize such distances and identify clusters, Kraaijveld et al. (1992) propose what they call *U-Matrix*, in which the average distance from each neuron to its neighbours is mapped on a grey scale. Here, a cluster border might be seen as an area where distances are large, although the decision concerning what is an actual cluster, and what is not, is always a subjective one that the researcher must make.

The second option we will explore here is to link the original data on the network using the b.m.u.s for each vector. This technique permits us to see where the input vectors are mapped according to the SOM, and which ones are close, based on the data used.

5 A Multidimensional Approach to European Urban Sprawl

In this section we present the results of the SOM applied to the European cities in order to analyse *urban sprawl*. Once all the indices for the different dimensions were computed for each of the urban regions in the sample, we proceeded to run the algorithm. Table 2 shows the main parameters were chosen. We selected a number of neurons ($20 \times 20 = 400 > 209$) larger than the size of the input data so the cities do not necessarily need to cluster in the same neuron unless they are very similar. The rest of the parameters selected according to several trials that revealed that they were the most suitable.

Table 2: SOM computation specifications

Initialization		
Topology		*hexagonal*
x,y		20,20
Neighbourhood function		*gaussian*
	First part	**Second part**
No. of cycles	10.000	100.000
α	0.04	0.03
Initial radius	10	7

As we said in Section 4, there are mainly two ways to exploit a trained network. The first one consists of analysing how the different dimensions (or *component planes*) are mapped across the network, and where one can find the lowest and highest values, which helps to determine whether there is any pattern between the different dimensions. Figure 2 shows the values of the vectors m_i associated with each of the neurons for each of the six dimensions in a coloured scale of five quantiles from light green (lowest values) to dark blue (highest ones). Each of the six pictures can be thought of as a surface that represents how the values of each dimension are distributed across the network.

The first thing[9] we can observe is that some of them follow a (rough) gradient pattern, in which the lowest values are mapped in one area of the network and, from there, there is more or less a smooth transition to higher values. This

[9]The observations concerning the connectivity dimension should be treated with caution: as we noted before, the time of journey to work was taken from the Urban Audit database, which contains many missing values, so the representation obtained for this is not as accurate as for the other dimensions, for which there are no missing values.

is the case of decentralization, land-use mix, scattering, and, to some extent, accessibility to open space. The case of density is also particular, because high values appear to locate in two corners (the lower-left and the upper-right) which are set apart for a region of low values, while the lowest densities are found in the remaining corners.

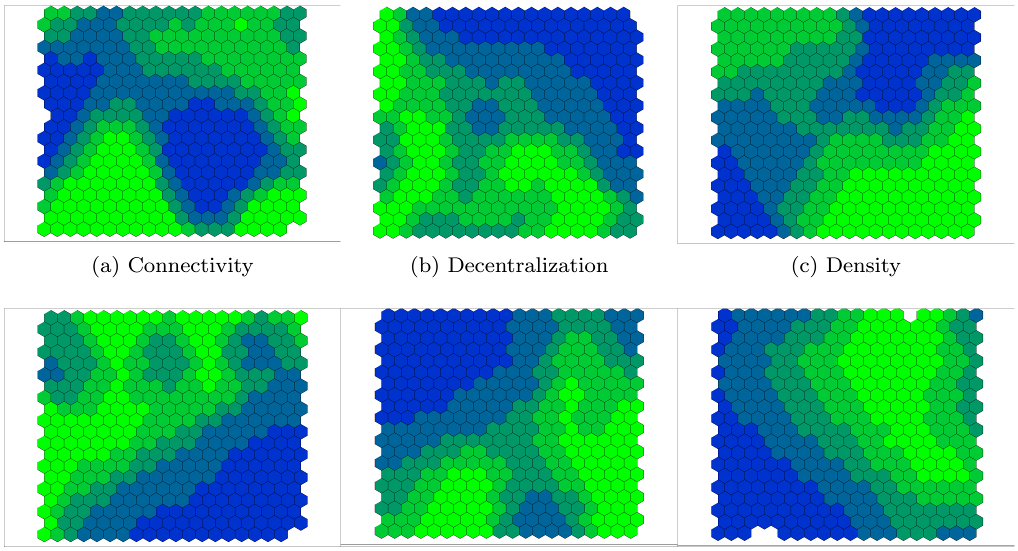
If we bring together the six planes, we can sketch out different regions that approximately correspond to the four corners. There is an area around the upper-right corner in which connectivity (low commuting time), density and decentralization are high (as dark blue is present), and the scattering, the accessibility to open space and land-use mix are somehow medium-low (greater mix of medium blue and green), although the last tendency is less clear. The cities that locate in this area will thus tend to fit the picture of a dense city where people, despite not living in the core, have short commutes, probably the city is compact (not very scattered), but do not have much access to open space. In the same fashion, we can interpret other regions of the map and think of other city profiles by applying a similar approach. Of particular interest is the city profile we come to call the archetypal *sprawling city*: it combines values of high commuting times, medium-high decentralization, low density, high scattering low accessibility to open space and little mix of uses. The *sprawling* profile can be found somewhere in the lower part of the middle-right side of the network, and represents all the dimensions of what the literature has identified as *urban sprawl*, according to the review in Section 2. However, *which European cities* actually locate in the different zones of the SOM we cannot know by looking at Figure 2. For such purpose, we need to tie the original input data to the network.

As stated in Section 4, the way we can link the input 209 European cities to the SOM is by assigning the vector of each one to the statistically closest neuron, that is to its *best matching unit* (b.m.u.). Figure 3 shows the network where each b.m.u. has been marked with a dot coloured according to: (a) geographical location; and (b) population. Having seen how the different dimensions of sprawl interact with each other, we wonder what kind of cities locate in the different regions (profiles) of the SOM.

The first question we would like to address here is whether we can observe regional patterns in Europe, regarding the extent of urban sprawl. Although, over the past few decades, Europe has experienced a process a convergence, it is still the sum of several countries with very diverse backgrounds and histories[10], which implies that, nowadays, there is a wide range of legal structures, urban planning traditions, and social values, to give just a few examples. As a reflection of social processes, city spatial structure is likely to be affected by all these factors, and thus we would expect different degrees of sprawl across different regions.

Figure 3 (a) displays the b.m.u.s of the European cities, where each one has

[10] We could have coloured the b.m.u.s based on nationality but, for ease of pattern-visualization, we have aggregated them into six supra-national regions which still keep some internal homogeneity. Appendix B details the aggregation.



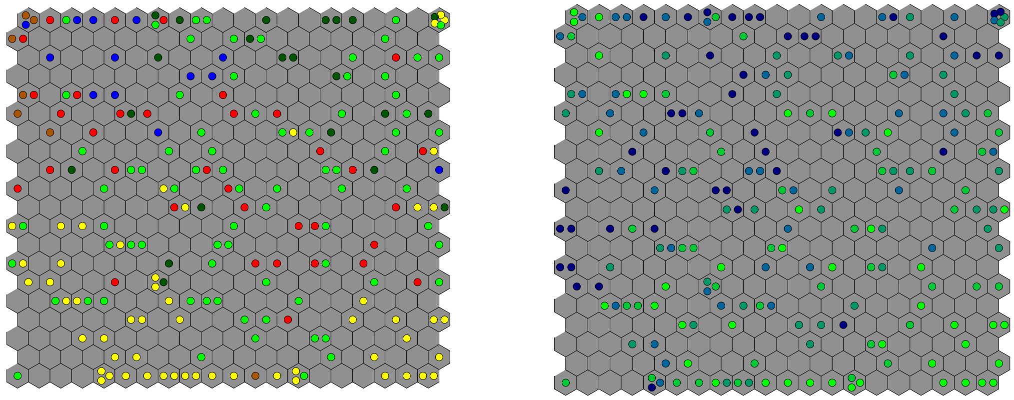
(a) Connectivity (b) Decentralization (c) Density

(d) Scattering (e) Open Space (f) Land-use mix

NOTE: Each figure displays on an intensity scale the value for each of the dimensions in the vector m_i assigned to each hexagonal neuron in the network. Low values are light green and high ones are in dark blue.

Figure 2: Component planes

been coloured in a different way depending on the region to which it belongs. Although some regions, such as those in Western and Central Europe, appear to be spread all over the SOM and we cannot extract clear characteristics from them, the others show very interesting patterns. By linking the locations of the dots to the distribution of the values from the component planes shown in Figure 2, we can delineate different profiles of cities. Both the Baltic States and the Northern countries are mapped around the upper-left area of the network, with the Baltic States along the vertical axis and the Scandinavian countries spread slightly more towards the centre. This depicts a *Baltic city* where people usually live in the core (i.e. low decentralization) but have relatively long commutes, and where density and scattering are medium, and the access to open space and the mix of uses are high. The *Northern city* would be similar, but slightly better connected, denser and more decentralized. Somehow, we could also speak of a *UK city*, which is located around the upper-right quadrant, implying shorter commutes, and a higher density and decentralization, a medium-low scattering and access to open space, and less mix of uses than the other regions. The *Mediterranean city* is located along the bottom part of the SOM and features low average commuting times, density and decentralization, a small amount of



(a) Supra-national regions

(b) Population

NOTE: The dots represent the *best matching unit* (b.m.u.) neuron for each of the input vectors; this is defined as the statistically closest vector m_i in the trained SOM to the values for the dimensions from each city. (a) Legend: **brown** the Baltic States; **dark green** the British Isles; **red** Central European countries; **yellow** Mediterranean countries; **blue** Northern countries; and **light green** the rest of Western Europe. (b) Legend: the colouring represents the population in a gradient of five quantiles from light green (smallest population) to dark blue (largest population).

Figure 3: Best Matching Units (b.m.u.s)

open space, but a high degree of mix of uses, and a mixed pattern of scattering.

Once we know *where* the different kinds of cities are located across space, we then consider the topic of size. There has been an extensive debate in the urban literature about whether the size of the population of a city has effects on a diverse range of topics: for instance, economists have linked this to the theory of agglomerations (see Fujita et al. 2001; Fujita and Thisse 2002) to explain better outcomes for larger cities. Although this theory has not focused on sprawl, we will extend the exploratory approach we have followed so far in order to consider whether there could be any pattern in the relation between urban sprawl and population size. Figure 3 (b) shows the b.m.u.s of the European cities coloured in a gradient of five quantiles from light green (smallest values) to dark blue (highest ones) according to their population. In this case, since we are mapping a continuous variable (population size of the city) instead of a discrete one (region to which the city belongs), it is not as interesting as in the regions' case to comment on each of the bins but to extract (if any) the general tendency of the variable concerned. Although it is not very well differentiated, there are more dark blue dots in the upper-left half of the SOM, and more light green ones in the bottom-right half[11]. This sort of regularity

[11]Several other binnings were tested obtaining similar conclusions.

indicates that population does follow a pattern in which larger cities tend to locate in the upper-left triangle of the trained network and smaller ones in the bottom-right, which implies patterns of population size regarding some of the dimensions of sprawl: for instance, scattering seems to be larger in smaller cities as the component plane for Figure 2 (d) indicates when visually overlaid with Figure 3 (b), and density (Figure 2 (c)) also seems to be higher in larger cities. However, other dimensions (e.g. decentralization, land-use mix) show mixed results, which may be taken as indicative that not every aspect of sprawl is related to size, although some of them seem to have some correlation. This stresses the complexity of the phenomenon and accentuates the necessity of considering its multidimensionality in an explicit way.

After looking at the patterns in geographical location and size, we can return to the archetype of sprawl we mapped in Figure 2 and see which are the cities that better fit the outlined characteristics. This *sprawling city* was located in the lower part of the middle right side of the network; if we look at that part in Figure 3 (a) we see red (Central Europe), light green (Western Europe) and yellow (Mediterranean region) dots; alternatively, if consider Figure 3 (b), we can mostly see green dots, which represent the medium and small cities. According to this, we may profile the most sprawling city in Europe as one of *medium-small population size located in the central, western, or southern part of continental Europe.*

6 Conclusions

The present paper has adopted an exploratory and multidimensional approach to analyse urban sprawl in Europe. A literature review revealed the complexity of the phenomenon and the great variety of points of view from which it has been considered, both outside and inside academia, as well as across many diverse disciplines (economics, geography, urban planning, etc.). The main characteristics concluded from the review were conceptualized into a set of six dimensions of urban sprawl that were divided into two main categories: urban morphology, and internal composition. This provided us with a working definition of sprawl that was both as complete as possible and accurate. In order to be able to apply it to real-world data, we specified an index to capture each of the six dimensions and computed them using a representative sample of European urban regions, in which we distinguished between urban patches and other types of landuses in order to make accurate calculations. Once we had collected all data for each city on each of the dimensions, we brought together all the information through the use of the SOM algorithm, a method that compresses highly dimensional data without loss of relevant information, and makes it possible to visualize them in order to identify patterns and regularities. This approach is novel in two senses: on the one hand, it considers the European region, an issue that has been paid very little attention but, at the same time, it is of great interest, in particular for an issue such as urban sprawl; on the other hand, it uses the

SOM algorithm to compress the data about the different dimensions so that no useful information is lost but it is presented in a way understandable for the human brain.

After the algorithm was run, we obtained a trained network that allowed us to develop the bulk of the analysis. The distribution of the values of the different dimensions across the network confirmed the need for this type of approach, since each of them follow a different pattern, which indicates that just one component does not summarize well the other results. Furthermore, this stage was also useful to identify what we called the archetypal *sprawling city* as one where: commuting times are high; decentralization is medium-high; density, availability to open space and land-use mix are low; and scattering of the urban development is high.

Then we considered the actual input cities, and looked at how they had been mapped across the network by the SOM. We performed two exercises in the search for relevant patterns: the first was intended to link the geographical location of the urban regions to their characteristics in terms of sprawl, while the second aimed to determine whether population size has any kind of association with the variables of sprawl. The main conclusion we could draw from pulling both together was that the *sprawling city* in Europe is one of medium-small size, located in the central, western, or southern part of the mainland.

This work has presented an exploratory analysis of urban sprawl in Europe and, in that regard, it may be seen as an exercise to infer *interesting questions* and suggest new hypotheses. There are several clear paths of research that might start from this point, and we will highlight three of them. One is to introduce time into the analysis; it would be very interesting to delve into the dynamics of sprawl over time, whether it has changed across space or whether the relations between the different dimensions have also evolved. Another interesting research direction would be to put these results into a broader geographical context, and compare them with other relevant regions in the world, such as the US or some of the very rapidly growing Asian economies, for instance. Our last suggestion is to consider a more *explanatory* approach that aims to identify the forces at play that determine urban sprawl, and that confirms (or rejects) some of the hypotheses mentioned here. In any case, in the future we hope to develop more research on the key phenomenon that will shape the structure of our cities in the coming decades.

Appendix

A Land-cover classes for urban green space

Corine code	Class
141	Green urban areas
311	Broad-leaved forest
312	Coniferous forest
313	Mixed forest
322	Moors and heathland
323	Sclerophyllous vegetation
324	Transitional woodland-shrub

B Supra-national classification of countries

Region	Countries
Baltic States	Estonia, Latvia, Lituania
British Isles	UK, Ireland
Central Europe	Czech Republic, Poland, Hungary, Slovenia, Slovakia
Mediterranean countries	Greece, Italy, Spain, Cyprus
Northern countries	Denmark, Sweden, Finland
Rest of Western Europe	Germany, France, Belgium, Netherlands, Austria, Portugal, Luxembourg

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