Functional tolerancing: virtual material condition on complex junctions

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Abstract: In industry, functional tolerancing of mechanisms is today more and more based on ISO GPS (Geometrical Product Specification) and ASME standards. In this context, the CLIC method (french acronym for “Cotation en Localisation avec Influence des Contacts”) has been developed in our laboratory since 1998. The current standards are incomplete to specify complex shapes, for example to define a datum reference frame on these surfaces. Using specific examples, the present paper outlines six proposals as possible extension of standards of tolerancing to describe the functional need for these links. Two main propositions are developed, the material conditions on complex surfaces and the definition of a new association criterion. Tolerance analysis models are presented; they must be consistent with respect to proposed functional specifications.

Keywords: Functional dimensioning and tolerancing, tolerance analysis, GPS standards, complex links, virtual boundary

1. Scientifical context

The CLIC method [1] enables to elaborate a functional tolerancing based on notion of virtual boundary and a three dimensional tolerance analysis. Junctions between parts are described according to precedence order with primary, secondary and tertiary links. Geometrical functional requirements of the mechanism are imposed by the functional analysis. Intern requirements can be automatically detected by feature recognition (requirement synthesis) or imposed by the designer. Functional tolerancing relative to a given requirement is generated for each requirement (specification synthesis). Next, an equation is established to determine the effect of these tolerances on each functional characteristic (tolerance analysis). The set of equations allows then optimizing the tolerances and nominal dimensions parts in order to decrease the manufacturing cost (tolerance synthesis).

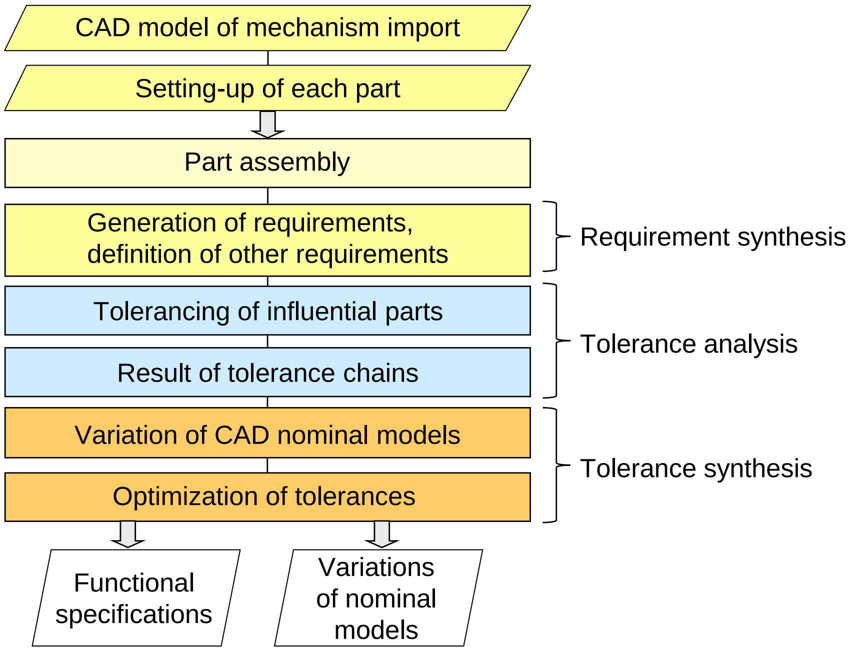


Figure 1: Process flow of CLIC approach

A demonstrator enables to study many mechanisms, taking into account part defects and clearances between parts. However, the studied junctions in these mechanisms are simple and composed of planes or cylinders.
The objective of this work is to extend the CLIC method to complex links composed of prismatic surfaces or free surfaces. It is necessary to propose a functional tolerancing for these junctions in order to respect geometric requirements and to generate transfer equations. For that, the calculation model should be consistent with respect for the definition of the specification.
Several extensions will be proposed to generalize the concept of virtual condition to complex surfaces. Nowadays, these proposed specifications are out of ISO or ASME standards.

The second section reminds the tolerancing method for transfer on simple mechanisms, in order to explain the necessity to add an orientation specification to a position specification and the interest of virtual condition specification. One difficulty is underlined for the mobility of the datum reference frame with least material virtual boundary.
The third section introduces the need of virtual condition on complex surface and proposes a new specification which respects the independence rule.
Finally, the fourth section analyzes a link denoted “hybrid” constituted of both contact feature and fitting feature for the same geometric entity. Therefore, it is necessary to use specific writing with a new association criterion.

2. Basic transfers

2.1 *Presentation of basic mechanism*

In this paper, the approach will be illustrated by an elementary mechanism (fig 2) constituted of two parts noted housing and body. Generally, a junction between two parts is realized by a primary link, a secondary link and eventually a tertiary link. These links can be classified in different type, planar surface, cylindrical surface, prismatic surface, surface of revolution, spherical surface and complex surface. Each link is formed by one or several surfaces.

B. Anselmetti has enumerated for example 30 positioning features (plane, cylinder, cone, sphere, thread, but also, coplanar planes, coaxial cylinders, set of cylinders, groove (symmetrical parallel planes), free surfaces…)[3]. With different precedence order, it is possible to list about 10 000 different junctions. Moreover, the junction can be composed of clearance or interference. Furthermore, forces can orientate the displacements or screws can block the mobilities. Therefore, it is necessary to introduce approaches as generic as possible, for example,by using the concept of set of surfaces and TTRS for Technologically and Topologically Related Surfaces developed by A.Clément [4].
The considered requirement on this mechanism will be a location of an ending surface belonging to the ending part (body) relative to a datum reference frame of the housing. If there is clearance, the requirement must be respected for all positions of the body obtained by the mobility allowed by the clearance. In other words, if there is clearance in the junction, it will be considered as unfavorable to this requirement [1]. This notion of unfavorable or favorable clearance is comparable to the notion of quantifier proposed by J-Y.Dantan [5].

The searched equation must provide the maximum displacement of the hole axis functions of tolerances. For that, the analysis line method [1] determines the displacement of the vertex F_1 and F_2 located at extremities of the hole axis in analysis direction $\mathbf{f_1}$, $\mathbf{f_2}$… This imposes to discretize the directions around the axis.

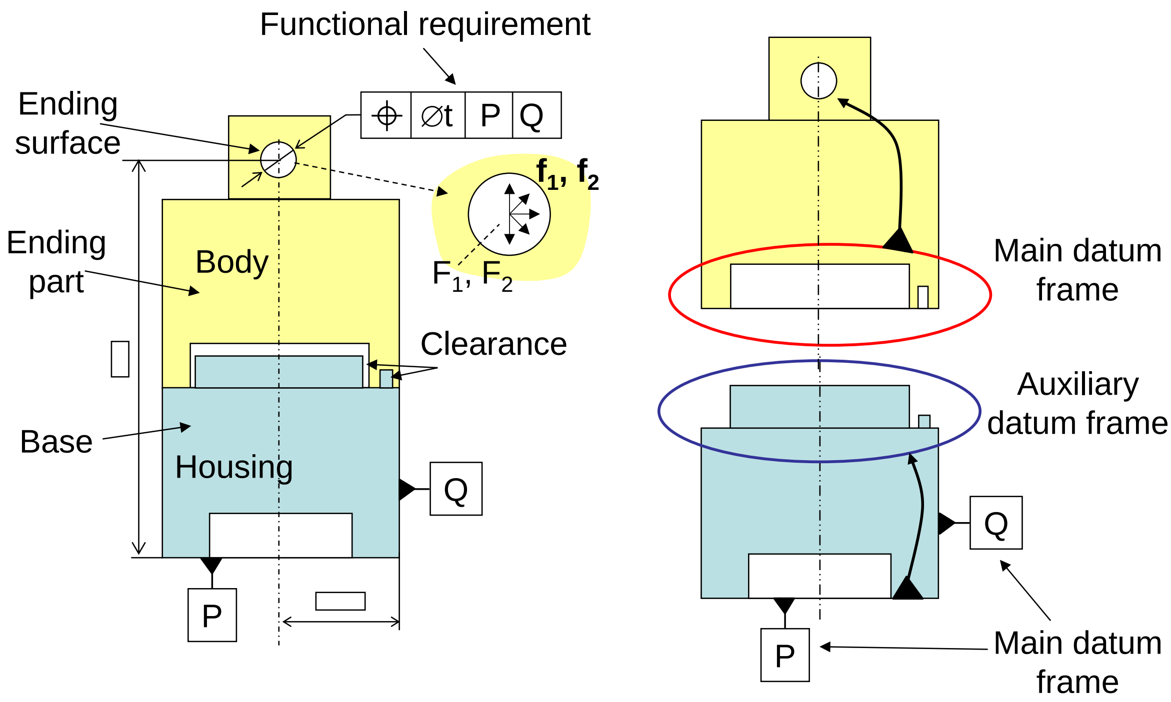


Figure 2: Basic mechanism

The tolerancing of this elementary mechanism must enable to assemble the two parts and assure the required accuracy.
To examine this problem, the classic solution consists in taking into account the fits defined in the standard ISO 286-1:2010 [6] and unidirectional tolerance chain projected in each direction. In this case, the tolerancing is established by a simple plus and minus tolerancing which does not satisfy ISO tolerancing in lots of cases. The approximations can be very significant because angular effects are not taken into account.

In the industries, the most efficient approach is the tolerance analysis by computer aided-tolerancing software 3DCS ® (Dimensional Control Systems), CETOL® (Sigmetrix), VSA ®

and eM-TolMAte ® (Siemens PLM) which are often based on Monte Carlo methods. For that, the designer has to choose geometric specifications applied on parts. The software simulates then a population of components with defects generated by Monte Carlo simulation and assemblies virtually parts. The desired characteristic is measured on final assemblies, which allows estimating the result of the tolerance chain in worst case or in statistic. The quality of these results depends on chosen specification, junction model and different adjustments for random number generator.

Scientific approaches can be classified into four categories.
An easy solution is to model the junction by punctual contacts which form isostatic links (MECAmaster) [7]. The deviation on each vertex represents the clearance effect and location deviation of the bearing surface. So in links with clearance, the designer has to determine contact points between parts function of the studied requirement. The model depends thus on the studied requirement and on chosen analysis direction.

Several authors consider that all surfaces of link have orientation and location defects. Real surfaces are modeled by substituted ideal surfaces (form defect is not taken into account) which have an orientation and location deviation relative to the nominal surface defined in CAD model [8], [9]. For example, the deviation of a plane is expressed function of three parameters, two rotations and one translation. A hexagonal link with six planes imposes consequently 18 parameters. The mobility of the part is modeled by the six degrees of freedom, which enable to calculate the displacement of ending surface vertexes.

Constrains of mating impose constrains between these parameters. The derived relationships show influent deviations relative to the requirement. The designer must then choose specification and tolerance values which permit to control these influent deviations, which allows calculating searched displacements.

Systems of equations can be very complex. M. Giordano [10] and D. Tessandier [11] present results with domains and polytopes, but this can be complex with a great number of parameters.

The third approach consists in simulating local defects of surfaces. J.K Davidson depicts the surface in the form of T-Map® [12]. Samper [13] suggests a modal model which permits to parameterize the form defects. In both cases, defects must be generated in order to determine contact points between the pair of surfaces.

The fourth approach is based on boundary conditions defined in the standard ISO 2692 [14] and ASME 2009 [15]. The major interest is to consider the assembly with perfect form part, at maximum material to check if the assembly is possible or at least material to determine the maximum displacement of the ending surface. The fundamental hypothesis supposes that the displacement will be greater when links are at least material conditions. This approach is very efficient to compute the greatest displacement in worst case but does not allow good statistic evaluation.

The CLIC method refers to this last approach. The tolerancing proceeds in two steps. The tolerancing of junction surfaces enables to create the main datum reference frame on positioning surfaces and an auxiliary datum reference frame on support surfaces. Form specifications assure the quality of the contact. Specifications at maximum material condition guarantee the assembly. In the second step, positioning surfaces are positioned each relative to the others, by locating each surface of the auxiliary datum reference frame with regard to the main datum reference frame, this using the concept of least material condition for fitting features of the junction and finally by locating the ending surface relative to the main datum reference frame of the ending part.

2.2 Transfer with a surfacic link

The figure 3 illustrates a basic mechanism composed of a body and a housing. Plane A of the body is in contact with plane D of the housing. A location requirement imposes to control the maximum height at vertex F. The distance between F and contact face is L. Orientation and location specifications are used to specify the contact surface D of the housing. The real surface has to remain inside these two tolerance zones. The datum plane A of the body bears down the real plane of the housing.

The contact hypothesis considers that the datum plane A of the body remain in the orientation and location tolerance zones of housing. Measurements done by Radouani [16] show that this hypothesis is not perfectly respected and there is an overtaking and a possible interference which depends on the sum of flatness defects or contact surfaces. Generally, the influence of form defects is neglected.

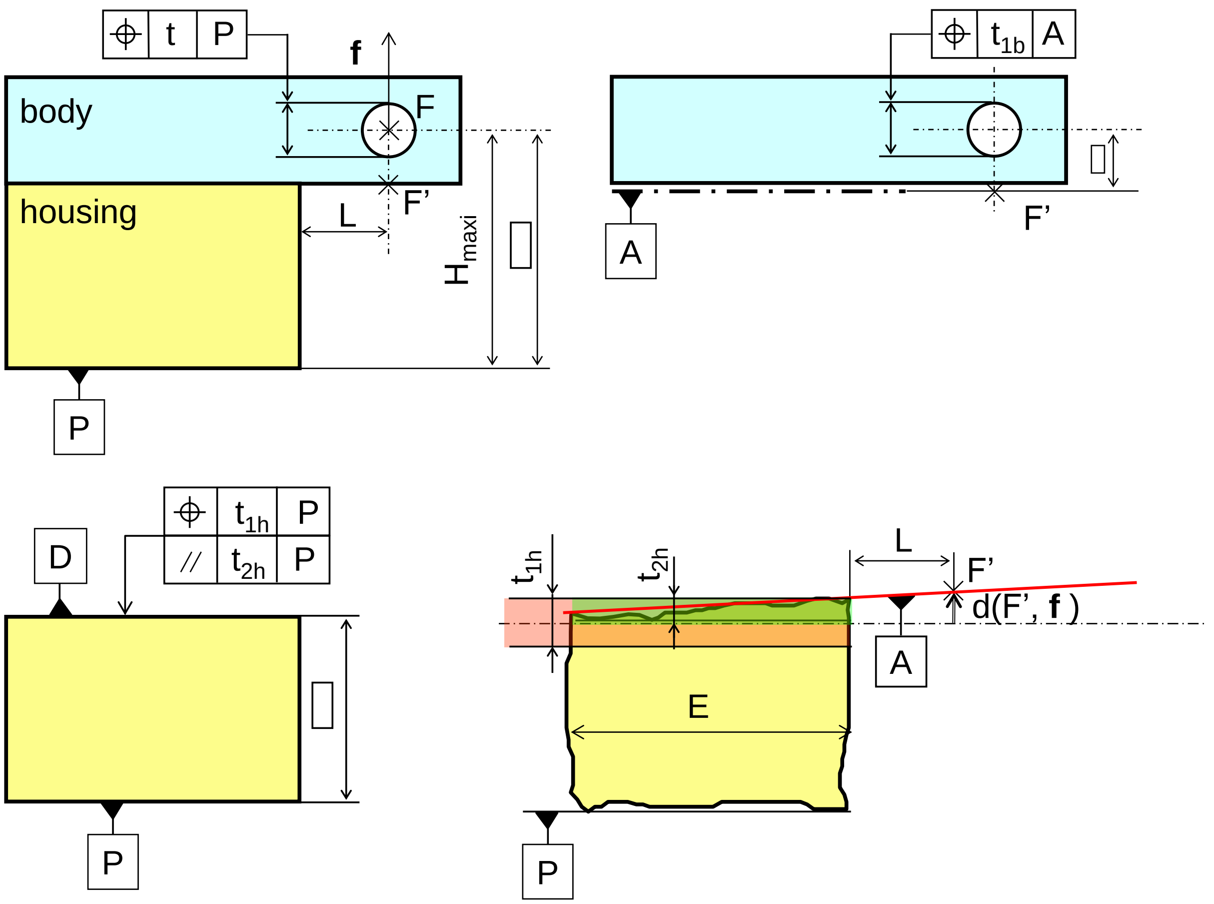


Figure 3: Basic tolerance chain with contact surfaces

The displacement of the vertex F in the direction **f** is equal to the displacement of the point F' which belongs to the datum plane A:

d(F, **f**) = d(F', **f**)= $t_{1h}/2 + t_{2h}.L/E$ (1)

If there was not orientation specification, the inclination of A would be bigger. The displacement of F would be:
$d(F, \mathbf{f}) = d(F', \mathbf{f}) = t_{1h}/2 + t_{1h}.L/E$ (2)
Therefore, the interest of this orientation specification with $t_{2h} < t_{1h}$ is to control the angularity of the datum plane A in the location tolerance zone and to limit thus the displacement of the vertex F.

2.3 Transfer with link with clearance

The figure 4 illustrates a basic mechanism composed of a shaft assembled in the housing with clearance. The studied requirement is a location of the ending cylindrical surface in order to control the position of the point F relative to the datum plane P of the housing.

Both diameter specifications with envelope requirements impose maximum material boundary to ensure the assembly of these two parts.
The new hypothesis in this case is that the displacement of the point F in a direction **f** will be maximum when cylinders A and D will be at least material conditions and when the hole D will be inclined in the tolerance zone. The tolerancing will be at least material condition.

The cylinder A of the shaft at least material condition is a cylinder with a diameter $d_v = d - t_d/2$. The point F is located on the axis of this cylinder.
The hole D of the shaft is a cylinder with a diameter $D_{vo} = D+t_D/2 + t_{2h}$ parallel to P which must be included in the position virtual boundary with a diameter $D_{vL} = D+t_D/2 + t_{1h}$.
The classical hypothesis supposes the least material boundary of the datum A of the body can move inside both least material conditions of the housing.

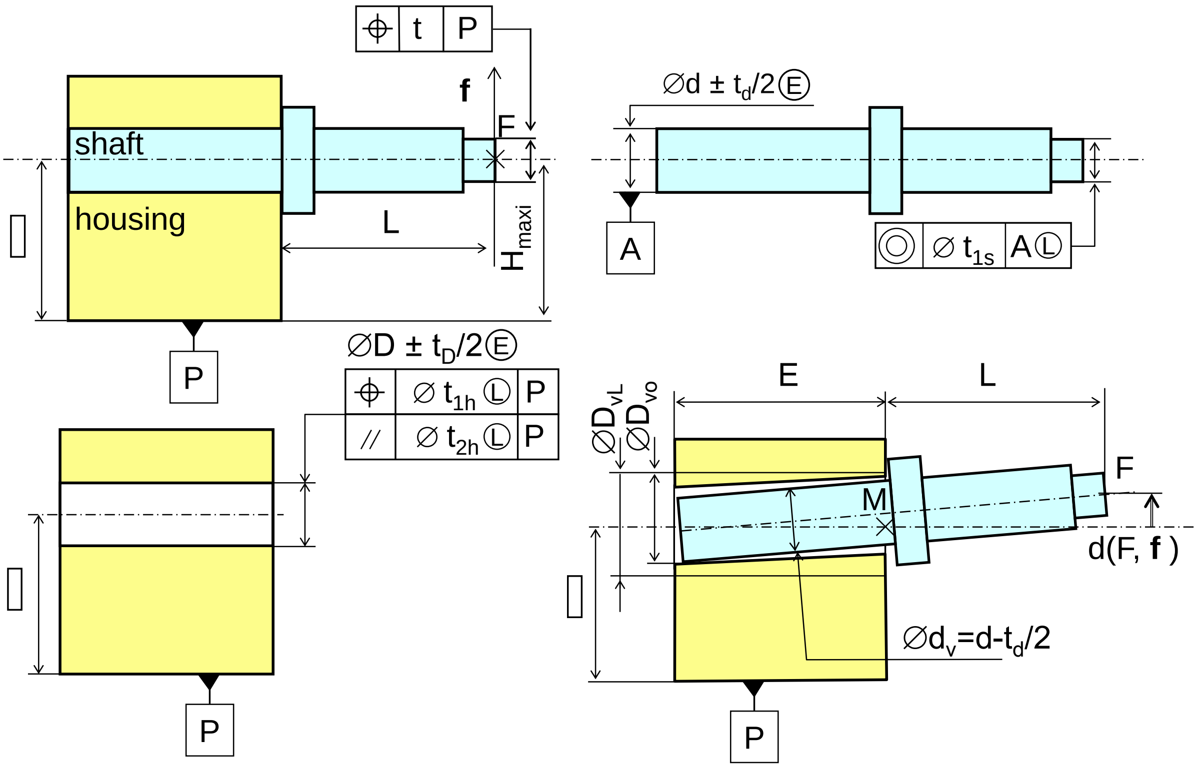


Figure 4: Basic tolerance chain with cylindrical link and clearance

This model lets to simulate the displacement of the cylinder A in the hole D in order to determine the displacement of the point F in the direction **f**.

$d(F, \mathbf{f}) = (D_{vL} - d_v)/2 + (D_{vo} - d_v).L/E$ (3)

Indeed, the cylinder A can lean cause of the diameter differences of virtual boundaries ($D_{vo} - d_v$) and translate of the half of the difference ($D_{vL} - d_v$) at point M.

Without orientation specification, the inclination of A would be bigger. The maximum displacement of F would be:

$d(F, \mathbf{f}) = (D_{vL} - d_v)/2 + (D_{vL} - d_v). L/E$ (4)

Therefore, the interest of this orientation specification with $t_{2h} < t_{1h}$ is to control the angularity of the datum cylinder A in the location tolerance zone and to limit thus the displacement of the point F.

This example shows the importance of the least material boundary concept to calculate the resultant of the 3D tolerance chain. Moreover, if the real cylinder D is smaller than the least material cylinder, the housing will be acceptable, with position and orientation deviations bigger than the tolerance value.

In this last example, P datum reference is fixed, without mobility allowed by material modifier. Next section focuses on the case of a floating datum reference obtained with least material boundary.

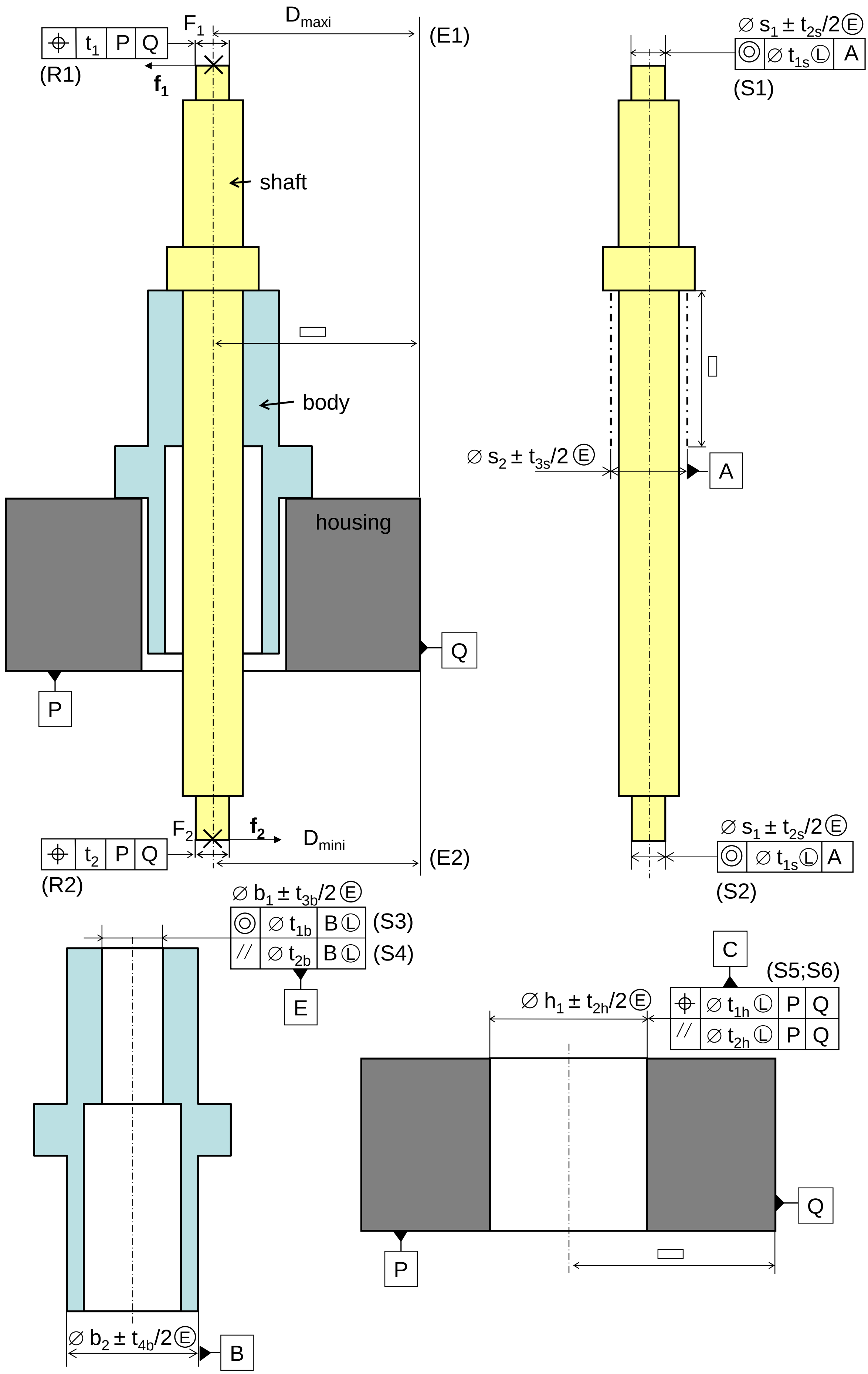
2.4 Transfer with floating datum reference

When the link of the support has clearance, the datum of this link has to be considered at least material boundary. The virtual least material cylinder is thus floating in the real cylinder of the real part. This may be a problem because the orientation virtual boundary and the position virtual boundary are independent. To study this behavior, the figure 5 shows a stacking of three parts composed of two serial cylindrical links.

The primary cylinder A of the shaft is assembled with interference in the hole E of the body.

The primary cylinder B of the body is assembled with clearance in the hole C of the housing.

Both studied requirements R1 and R2 are locations of points F_1 et F_2 with respect to the datum reference frame PQ. For this demonstration, the requirement R1 will be analysed in the direction $\mathbf{f_1}$ and R2 in the direction $\mathbf{f_2}$.



F1
Dmaxi
(E1)
t1 P Q
(R1)
f1
shaft
body
housing
Q
P
F2
f2
Dmini
(E2)
t2 P Q
(R2)
∅ s1 ± t2s/2 Ⓔ
∅ t1s Ⓛ A
(S1)
∅ s2 ± t3s/2 Ⓔ
A
∅ s1 ± t2s/2 Ⓔ
∅ t1s Ⓛ A
(S2)
∅ b1 ± t3b/2 Ⓔ
∅ t1b B Ⓛ (S3)
∅ t2b B Ⓛ (S4)
E
∅ b2 ± t4b/2 Ⓔ
B
C
(S5;S6)
∅ h1 ± t2h/2 Ⓔ
∅ t1h Ⓛ P Q
∅ t2h Ⓛ P Q
Q
P

Figure 5: Mechanism with two primary cylindrical junctions

In figure 5, the tolerancing established with the CLIC method is similar to the figure 4. The envelope requirements guarantee the assembly of parts for the primary cylinders. The study of the two requirements imposes position specification between surfaces. There is not clearance for the link between the shaft and the body. Therefore, there is not material modifier on the datum A for shaft specifications S1 and S2 neither on the toleranced surface of S3 and S4 for the body. On the other hand, the link between the body and the housing has clearance. Consequently, datum for specifications S5 and S6 of the body and toleranced surface of S5 and S6 of the housing are at maximum material condition.

For the position specification S3, the virtual boundary of the datum B is a cylinder with a diameter value b_2 - $t_{4b}/2$ which must be included in the real cylinder B. The axis of the cylinder E must be contained in a cylinder of diameter value t_{1b} coaxial to this virtual boundary.
For the orientation specification S4, the virtual boundary of the datum B is a cylinder with diameter value b_2-$t_{4b}/2$ which must be contained in the real cylinder B. The axis of the cylinder E must be contained in a t_{2b} diameter cylinder parallel to this virtual boundary.

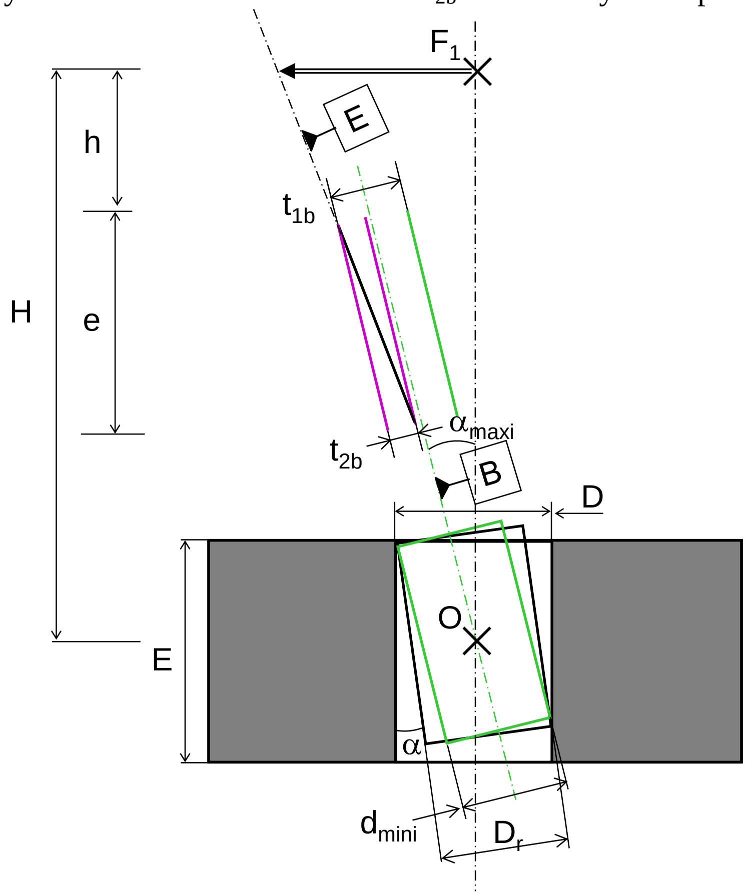


Figure 6: Three-dimensional model for displacement of F_1

The figure 6 shows the configuration which presents the maximum displacement of the point F1 in the direction $\mathbf{f_1}$.
The body at least material condition is composed of the cylinder B with diameter value d_{mini} = b_2-$t_{4b}/2$ inclined in the hole C of the housing of an angle α_{maxi} = (D - d_{mini})/E. The axis of the hole E is simultaneously in the orientation tolerance zone (diameter t_{2b}) which is parallel to the virtual boundary of B and in the position tolerance zone (diameter t_{1b}) which is centered to

the virtual boundary of B. The primary link shaft/body which is with interference, the datum A of the shaft is coincided to the axis of E. The maximum displacement of the point F_1 is:
$d(F_1, \mathbf{f_1}) = (D - d_{mini}).H/E + t_{1b}/2 + t_{2b}.h/e$ (5)

If the diameter D_r of the real cylinder B of the body is bigger than $b_2 - t_{4b}/2$, the inclination of this cylinder is smaller than α_{maxi} in the cylinder C of the housing. On the other hand, the working deviation for the body is bigger. The body remains conform if the real axis of the cylinder E is contained in the two tolerance zones defined relative to the least material boundary with diameter value $b_2 - t_{4b}/2$, which must be contained in the real cylinder B. So, the figure 6 shows well this cylinder B of diameter value D_r containing least material boundary of the cylinder B, with the two tolerance zones.
The maximum displacement of the point F_1 is identical. Then the relationship (5) is confirmed.
Practically, with a D_r diameter cylinder B bigger than the least material condition, the clearance between the housing and the body decreases. That cuts down the angular deviation α. This reduction enables to allow a bigger position deviation of the axis E of the body with respect to cylinder B. This gain on the tolerance is the main advantage of the using of the virtual least material condition on the datum.

Finally, the maximum displacement of the point F_1 in the analysis direction $\mathbf{f_1}$ is the same for a cylinder B at least material and for a cylinder B with a bigger diameter. The calculus hypothesis which considers that the displacement is maximum when the parts are at least material condition, is valid.

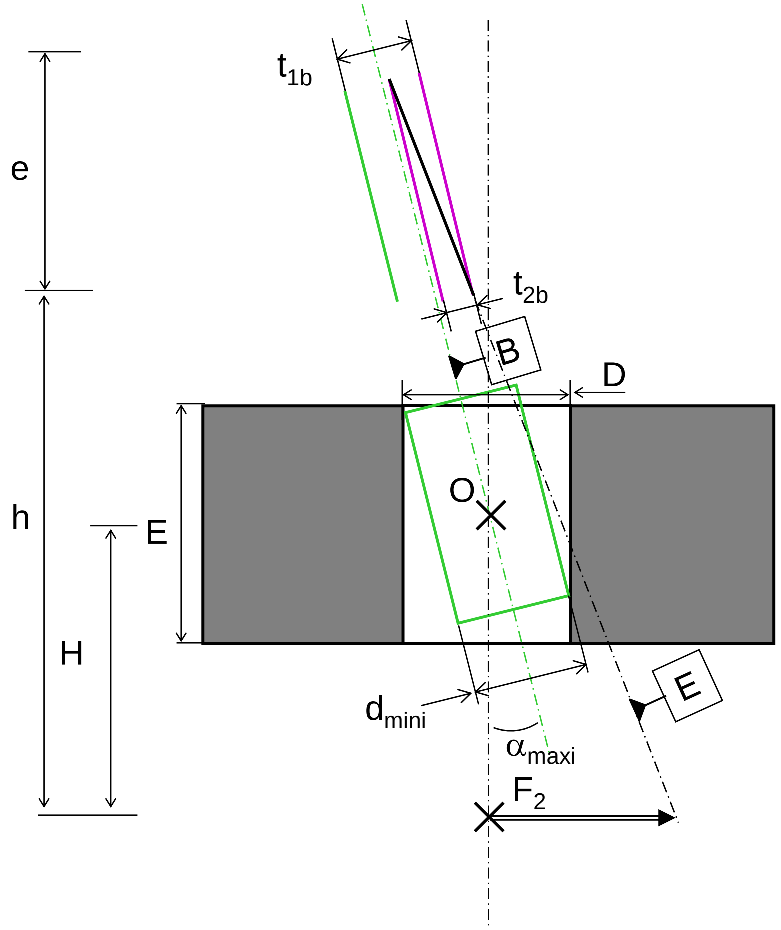


Figure 7: Three-dimensional model for displacement of point F_2 with B at least material condition

The figure 7 shows the configuration of conform parts which simulate the maximum displacement of the point F_2 in the analysis direction $\mathbf{f_2}$ considering the cylinder B at least material condition. The datum B is inclined in the hole C of an angle value $\alpha_{maxi} = (D - d_{mini})/E$. The axis of the hole E must simultaneously be contained in the orientation tolerance zone (diameter t_{2b}) which is parallel to the least material boundary of B and in the position tolerance zone (diameter t_{1b}) which is centered to the least material boundary of B. As the primary link shaft/body is with interference, the datum A of the shaft is coincided with the axis of E. The maximum displacement of the point F_2 is then :

$d(F_2, \mathbf{f_2}) = (D - d_{mini}).H/E + t_{1b}/2 + t_{2b}.h/e$ (6)

The relation is the same than (5).

The figure 8 shows the maximum displacement of the point F_2 with conform parts but with a diameter d_r for the cylinder B lightly superior than minimal diameter d_{mini}. The inclination of his cylinder is $\beta = (D - d_r)/E$.

The conform parts which provide the maximum displacement of the point F_2, are in a very particular configuration permitted by the independence of specifications S3 and S4.

- For the location specification S3 (Fig 5), the axis Δ_1 of the least material boundary of the datum is a cylinder of diameter value $d_{mini} = b_2 – t_{4b}/2$, inclined of $\gamma = (d_r - d_{mini})/E$ in the anti-trigonometric sense, inside real B cylinder (fig 8).
- For the orientation specification S4, the axis Δ_2 of the least material boundary of the datum is a cylinder of diameter value $d_{mini} = b_2 – t_{4b}/2$ inclined of $\gamma = (d_r - d_{mini})/E$ in the trigonometric sense, inside real B cylinder The orientation tolerance zone of E is parallel to Δ_2. This inclination sense maximizes the displacement of the point F_2. The E axis is then leant of $\theta = \alpha + t_{2b}/e$

In this case, the maximum displacement of point F_2 is:

$d^*(F_2, \mathbf{f_2}) = \mathbf{F_2F'_2.f_2} = (\mathbf{F2M}+\mathbf{MN}+\mathbf{NF'2}).\ \mathbf{f_2} = (h – H).(\gamma–\beta) + t_{1b}/2 + \theta.h$

$d^*(F_2, \mathbf{f_2}) = (2d_r – D - d_{mini}).(h - H)/E + t_{1b}/2 + (D - d_{mini}).h/E + t_{2b}.h/e$ (7)

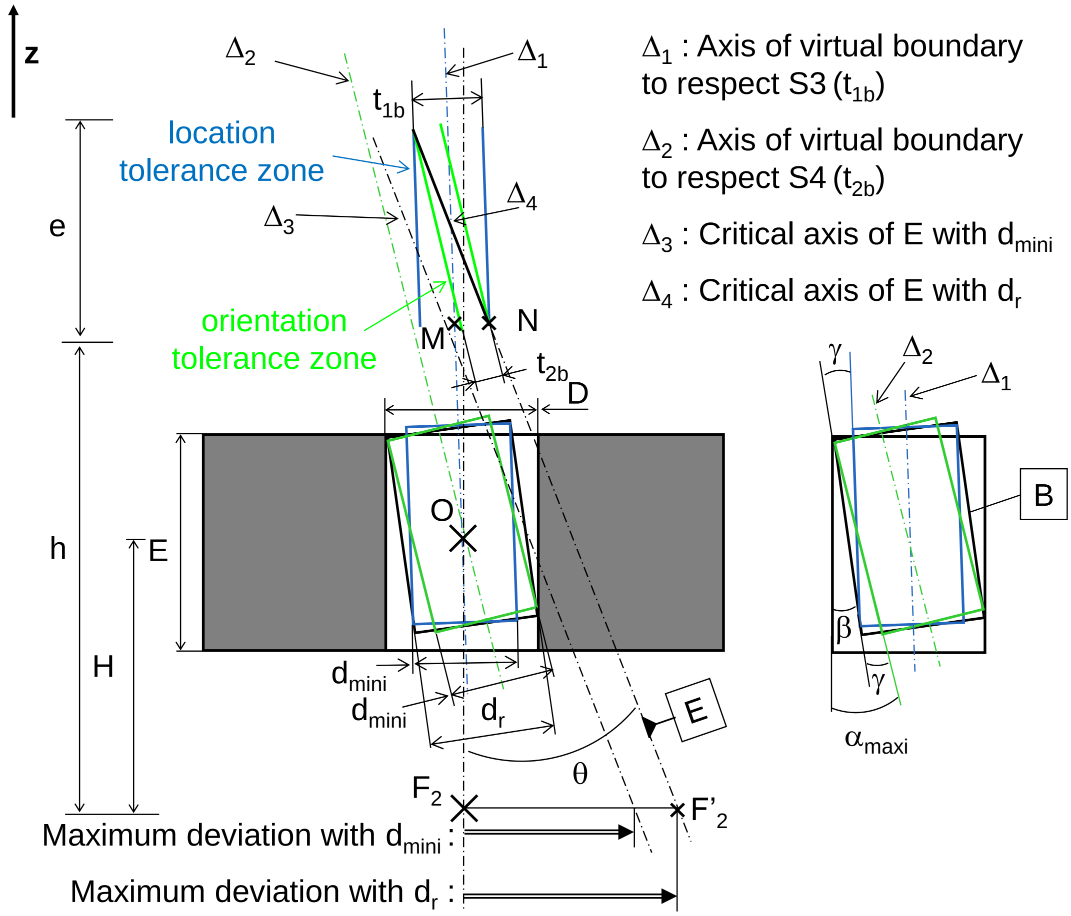


Figure 8: Three-dimensional model for displacement of F2 in general case

The obtained displacement by relation (7) is higher than the one calculated considering B at least material condition given by relation (6). The difference is:

$d^*(F_2, \mathbf{f_2}) - d(F_2, \mathbf{f_2}) = 2(d_r - d_{mini}).(h – H)/E$ (8)

The authorized displacement for F_2 by the specifications is then higher if diameter d_r is large, i.e. when cylinder B is not at least material condition. Then, it contradicts the usual hypothesis. This phenomenon comes from the different positions of least material boundaries for the specification S3 and S4 (figure 5). It appears when floating reference is located between the analysis point (F_2) and the surface of link (E).

2.5 Hypothesis analysis for transfer at least material

Although it is intuitively obvious, the hypothesis considering that the maximum displacement is obtained with parts at least material condition is not verified. This is due to the independence principle with a double specification: the orientation tolerance zone is not parallel to the location tolerance zone. The orientation specification is no more efficient, it does not limit the inclination of the axis in the location tolerance zone.

Four solutions can be discussed to take into account this phenomenon.

1. To keep the proposed tolerancing: The additional term must be taken into account in the calculus of the resultant. This solution would lead to a paradox because the

displacement of the point F_2 is maximum when the datum is at maximum material condition, this is conflicting to the classic notion of mobility due to clearance.

2. Datum without modifier: Specifications would be stricter because they do not permit to benefit from the virtual boundary mobility when parts are not at least material condition.
3. Removal of orientation specification: In this case, point F_2 is:
   $d(F_2, \mathbf{f_2}) = (D - d_{mini}).H/E + t_{1b}/2 + t_{1b}.h/e$ (9)
   This formula is equivalent to (6) with $t_{1b} = t_{2b}$. The location tolerance t_{1b} that limits the inclination has to be lower than the one obtained with both specifications for the same displacement of point F_2. The single specification is then more restrictive.
4. Least material boundary common to specifications S3 and S4. With actual standards, this solution is not directly applicable. A commentary as "Common least material boundary on the reference" has to be added.

The fourth solution seems to be the less restrictive and the most coherent with the tolerancing method which limits the orientation deviation inside the position tolerance zone.

2.6 Proposal of a new tolerancing concept

A new tolerancing concept has to be proposed in order to apply the fourth solution.
<u>Proposal 1:</u> Composed specification
A "composed" specification is the association of several specifications dealing with a single toleranced surface (simple element, group, common zone …) with a single datum reference frame. The toleranced surface has to simultaneously belong to all the tolerance zones defined relatively to the single datum reference frame.

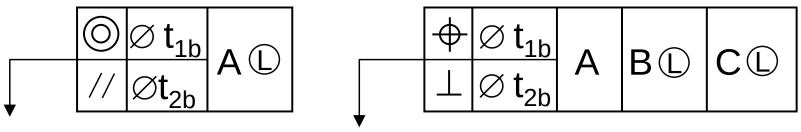


Figure 9: Specification with single datum reference frame.

In case of a datum with a maximum or minimum material modifier, the toleranced surface has to belong simultaneously to all tolerance zones defined relative to a virtual boundary of the unique datum reference frame.
This writing does not contradict to the independence principle, because in fact, the two specifications form a single one composed of two tolerance zones relative to a common datum reference frame.

This concept is very well suitable in order to combine an orientation specification and a position specification, on the same tolerance surface, with the same datum reference frame.

If the functional point is F_1, the worst case is obtained in the case of the figure 6. It is useless to approve the writing of the figure 9, because this could lead to reject conform parts which respect the functional requirement.

The concept proposed figure 9 is near to the notion of composite tolerancing in ASME standard, which associates an orientation specification and a position one. On the other hand, this standard does not mention particular properties about datum reference frame.

This section has shown the interest to add an orientation specification to the position specification in order to control the influence of angular defects in the primary link in case of important overhang. Unfortunately, in current standards, this strategy is not suitable for datum with clearance. Four solutions are so proposed, adding over displacement, only position specification, datum without material modifier or comment to impose a common least material boundary for both specifications.

3. Virtual boundary on complex surfaces

3.1 Introduction

The tolerancing at maximum material condition guarantees the assembly of parts with clearance. The tolerancing at minimum material condition facilitates the calculation of the tolerance chains result.

In the current ISO standards [14], the maximum or minimum material modifiers are associated to the local dimensions and are exclusively useful for fitting features, cylindrical features or symmetrical parallel plane features (a groove for example).

However, lots of mechanisms are composed of more complex junctions with a functional clearance for which the notion of virtual boundary at least or maximum material condition can be absolutely used. This section presents different junctions and some proposals to extend current standard.

3.2 Local dimensions

In the elementary mechanism of the figure 10, the junction is constituted of a primary plane and a secondary cylinder. The requirement R1 is a location of the extremity of the body with respect to the datum reference frame PQ. On the body, this cylinder is in fact composed of three cylindrical sectors.

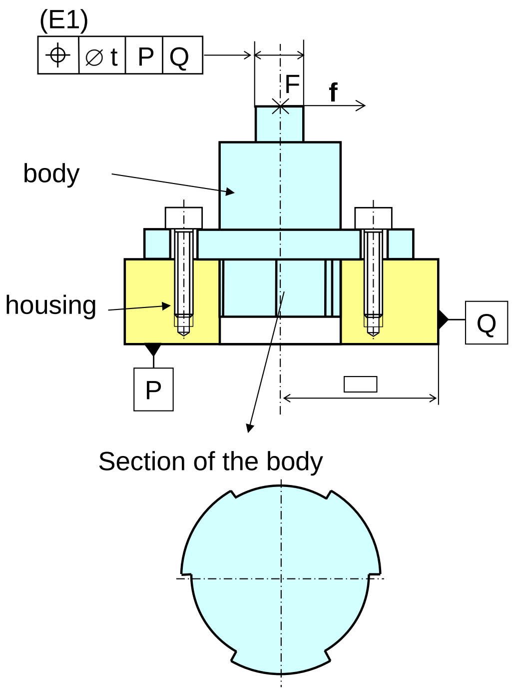


Figure 10: Secondary feature composed of several cylindrical sectors

The figure 11 shows the proposed tolerancing for the body considering B as a cylinder.
- The flatness (S1) of the primary plane A guarantees a good contact with the housing.
- The diameter specification (S2) is a problem because the notion of local dimension does not exist on the body. Indeed, there are not two points face to face on the cylinder in order to measure the local diameter. The envelope requirement can be certified with a 17,98 diameter gauge.
- The specification of perpendicularity at maximum material condition can be perfectly certified with the help of a 18 diameter gauge flattened on the plane A or with a measurement machine.

The specification of perpendicularity S3 guarantees the assembly when the contact plane on primary plane is assured. On the other hand, the envelope requirement ensures a larger clearance to facilitate the assembly of the body in the housing.

For the specifications S4 and S6, the datum B appears alsowith a modifier. According to the standard ISO 2692-2007, the diameter of the least material boundary (S4) is equal to the minimum diameter of B (17.92), and, the diameter of the virtual boundary at maximum material (S6) is equal to the maximum diameter of B (17.98).

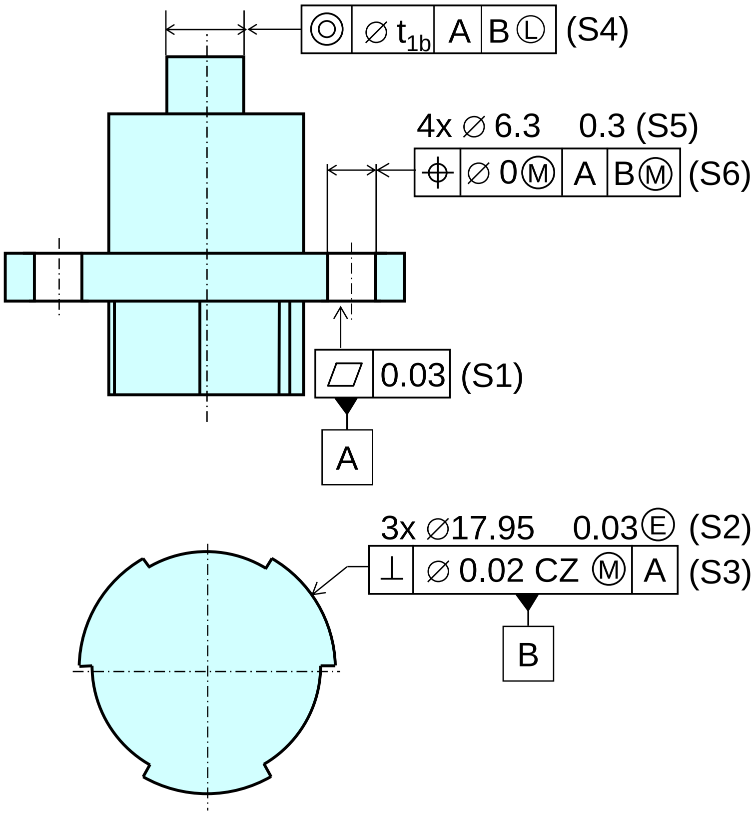


Figure 11: Tolerancing considering B as cylinder

This tolerancing satisfies perfectly the functional need to assure the assembly and the requirement R1. Nevertheless, it is problematical in a sense that the local dimensions are not measurable.
In fact, this example reveals that it is not necessary to measure the local dimensions if the tolerancing is complete, with a tolerancing of the cylinder B at maximum material condition and if B is used as a datum at least material condition. The writing of the diameter is only imposed by the writing way of the virtual boundary diameter in current standard. For the specification S2, it would be possible to put the diameter between brackets, this means that

the value would be just given to determine the size of virtual boundaries but not to certify the local dimensions.

To solve this problem, the standard ASME Y14.5-2009 page 61 proposes to indicate between square brackets directly the size of the virtual boundary on the datum inside the feature control frame. The specification S4 could be written according to the figure 12

(S4) | ◎ | ⌀ t_{1s}Ⓛ | A | BⓁ [⌀17.92] |

Figure 12: Size of virtual boundary on datum frame in ASME standard

Basing on this concept, the rule would be extended to the toleranced surface.

<u>Proposal 2:</u> Size of virtual boundary between square brackets
With a maximum or least material modifier on the tolerance surface or on a datum, the size of the virtual boundary can be given directly between square brackets in the feature control frame. With this writing, the envelope requirement can be expressed by a straightness. The specifications S2, S3, S6 would be thus written according to the figure 13.

(S2) | − | [⌀17.98] CZ Ⓜ |

(S3) | ⊥ | [⌀18] CZⓂ | A |

(S6) | ⌖ | [⌀6]Ⓜ | A | B [⌀18]Ⓜ |

Figure 13 : Size of the virtual boundary in specification

This writing would perfectly respect the independence principle. The direct indication of the size of the virtual boundary on the datum would avoid the ambiguities which has been revealed in the standard ISO 2692 edited in 2007. It would be useless to put a specification of diameter with the notion of local dimension which is a problem. In CAD systems, it does not impose to define parts with medium dimension.

3.3 Maximum and least material on free surfaces

The cylindrical surface B of the body can be considered like a free surface, but the use of material modifiers is not allowed with the current ISO standards. So, the figure 14 is a new proposal which is not described in ISO standards based on the specifications of complex surfaces.

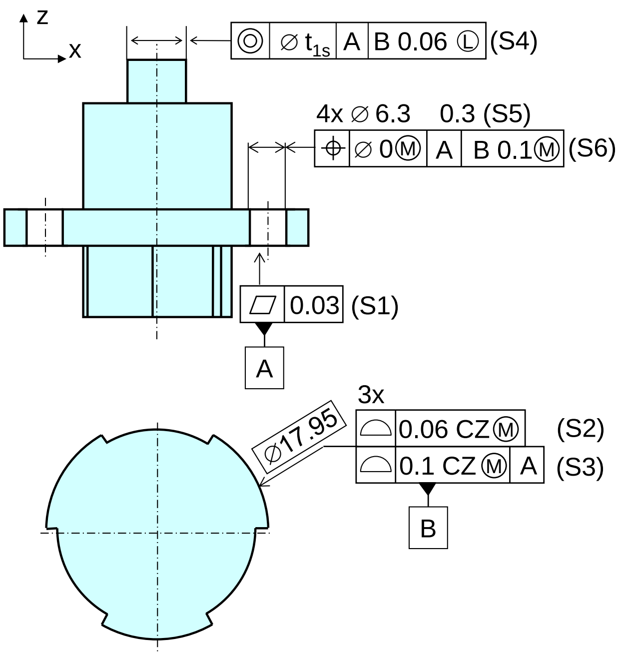


Figure 14: Tolerancing considering B as complex surface

In the figure 14, the nominal surface B is constituted of three cylindrical sectors of nominal diameter value 17.95. Without the maximum material modifier, the specification S2 is completely defined in standards. The tolerance zone is the envelope of a 0.06 diameter sphere which center covers the nominal surface (17.95 diameter cylinder). The tolerance zone is thus limited by two cylinders respectively of diameter value 17.98 and of diameter value 17.92. The real surface must be contained in the 17.98 diameter cylinder which defines the maximum material boundary and corresponds exactly to the envelope requirement of the figure 11. Inversely, 17.92 diameter cylinder must be contained in the material, which corresponds to the virtual boundary at least material for the requirement S4 of the figure 11.

<u>Proposal 3:</u> Virtual boundary at maximum or at least material condition on a surface
For a specification or a datum on a surface defined by the intrinsic nominal characteristics and by basic dimensions, the tolerance zone is defined by the envelope of a sphere of diameter equal to the tolerance and which center covers the nominal surface.
With the maximum material modifier, only the outside boundary must be preserved. This surface is the maximum material boundary. The real surface must respect this virtual boundary.
With the least material modifier, only the inside boundary must be preserved. This surface constitutes the least material boundary. This virtual boundary must be contained in the material.

In the figure 13, the value between square brackets gives the size of the virtual boundary. For the complex surfaces, the notion of local dimension does not exist. Values proposed figure 15 correspond to the classic notion of tolerance and can be distinguished because this value is not between square brackets. It would be possible to define the offset value of the surface relative to the nominal surface. This would enable a negative offset. However, this definition would be new and different from the classic notion of tolerance zone with a coefficient 2 which can create lots of interpretation or measurement problems.
The figure 15 illustrates the proposed explanation for specifications of the figure 14.

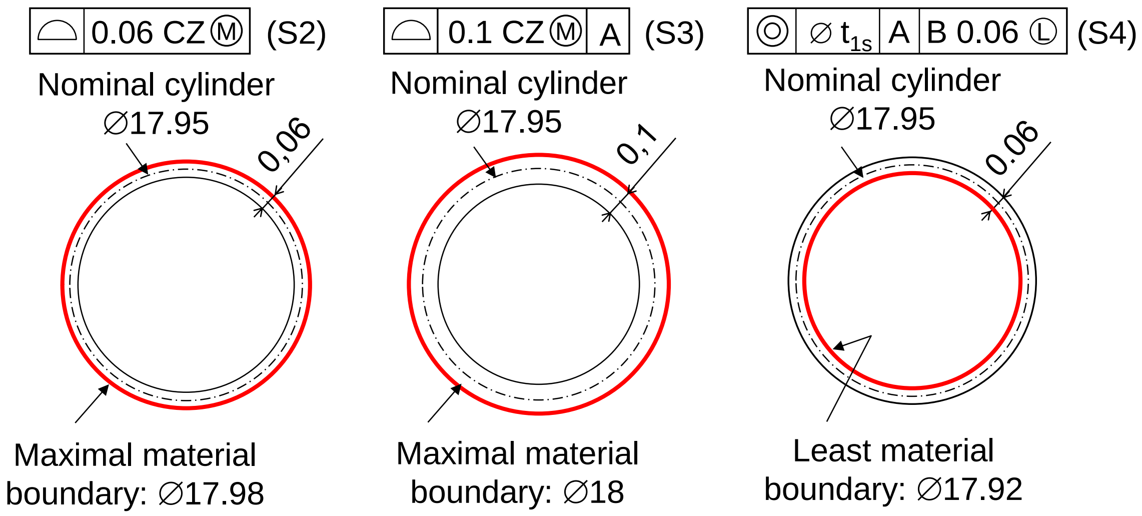


Figure 15: Définition of virtual boundaries

For the S2 specification, the maximum material modifier on the toleranced surface just incites to take only the 17.98 diameter cylinder into account. So the real part must be contained in this boundary which is the maximum material boundary for the form specification of this part.
The S3 specification is a specification location of the surface, but as the datum is a simple plane, degrees of freedom in rotation around z and in translation on axis x and y are free. So, this specification is identical to an orientation specification and corresponds perfectly to the perpendicularity S3 of the figure 11.
For the specification S4, the datum B is also applied on the surface. The least material boundary is defined by the inside boundary generated by a sphere whose center covers the nominal surface. The diameter of the sphere is indicated on the right hand to the datum in the feature control frame. For the specification S4, the virtual boundary is so a 17.92 diameter cylinder which has to be contained in the real surface.

The maximum material boundary ensures the assembly and the least material boundary assures the accuracy of the mechanism. With such coherent tolerancing, there are no more local dimensions to inspect.

3.4 Prismatic junction

In the basic mechanism of the figure 16, the junction is composed of a primary prismatic link with clearance and a secondary plane. The requirement R1 is a location of the conic surface of the body relative to the datum reference frame PQR. This requirement must be respected for all configurations permitted by the mobility dues to the clearance.

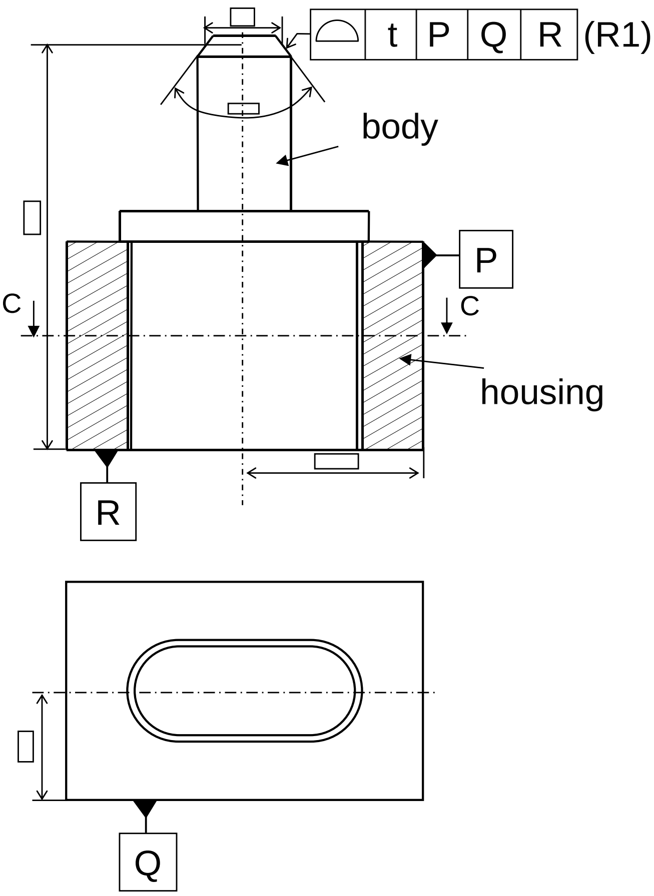


Figure 16: Basic mechanism with prismatic primary datum

The tolerancing proposed by the CLIC method is composed of three specifications, represented in figure 17:

-S1, S2: assembly requirements of the primary surface (at maximum material condition)

-S3, S4: quality of the contact between the two secondary planes knowing that the primary link is assured by the prismatic link with clearance.

- S5, S6, S8: position specifications to respect the studied requirement.

-S7: orientation specification to limit the inclination of the surface D in the position tolerance zone of S6.

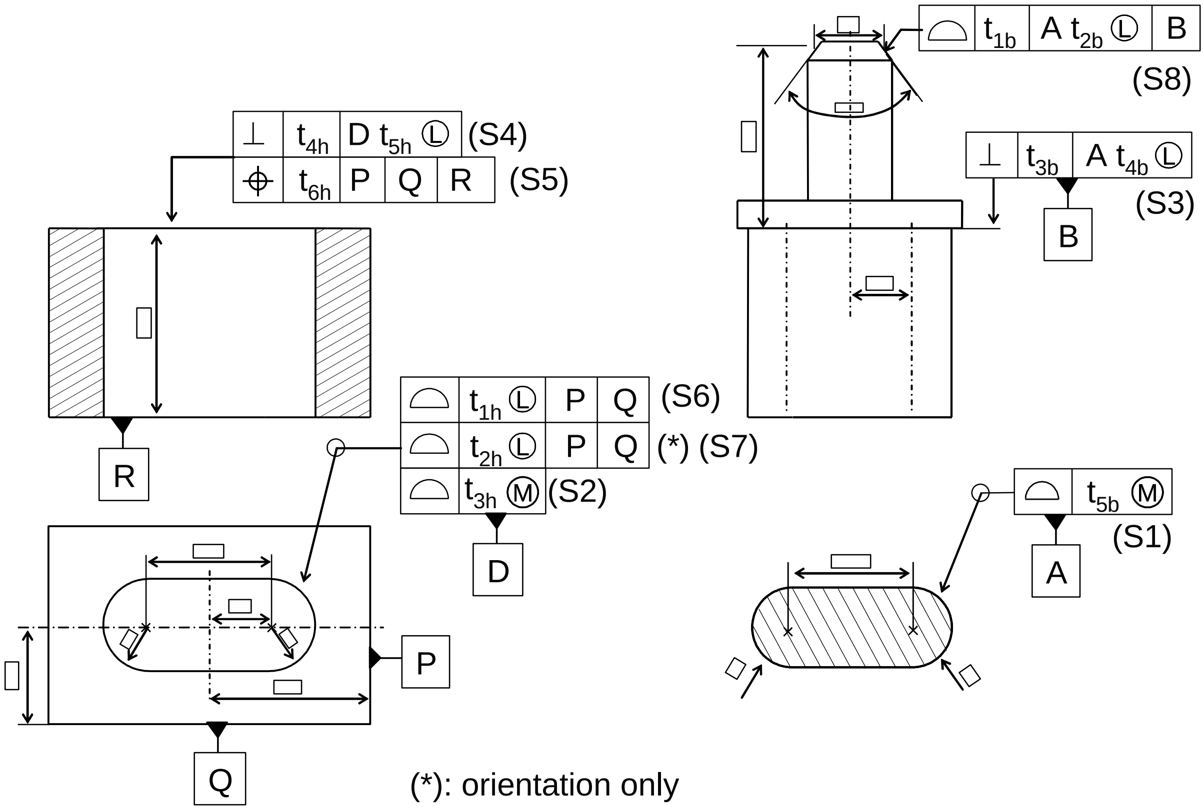


Figure 17: Tolerancing of the housing and of the body

The tolerancing of the housing shows that three specifications of form, orientation and position must be applied on the same surface. In the current state of the standard ISO, the unique useful symbol is the symbol ⌓, which makes impossible to distinguish the orientation specification of the position specification relative to the same datum reference frame. It is necessary to put a comment “orientation only” next to the specification S7.

To solve this difficulty, different authors proposed to use classic symbols of orientation ⊥ // ∠ and position ⌖ ◎ ⌯ considering that the graphic representation indicates that the toleranced surface is a complex surface. This approach seems to be unacceptable, because it would be impossible to distinguish if the specificed element is the surface or the axis of the surface.

For example in the figure 17, for the location S8, the symbol ⌖ woul be acceptable, because the arrow of the specification is not in front of the arrow indicating the angle of the cone or the diameter of the gauging plane. So the toleranced surface is the conic surface. On the other hand, this analysis is no more possible with the tolerancing on the 3D model, for example with the workshop FTA of CATIA® by Dassault Systèmes where the basic dimensions are no more represented. The element pointed by the specification is always the surface of the part. The tolerance element is the axis with the symbol ⌖ or the surface with the symbol ⌓. On the other hand, with material modifiers, the toleranced element is inevitably the surface.

It is important that future standard proposes a specific orientation symbol for complex surfaces. The standard XP E 04-562 [17] has proposed for example the symbol ⌓ori. This paper does not propose voluntarily a new symbol in order to avoid future ambiguities.

In complement of the proposal 3, it is necessary to specify the definition of form, orientation and position specification with modifiers.

Proposal 4: Form, orientation and position at maximum or least material condition
The symbol ⌓ without datum reference frame indicates a form specification. The virtual boundary can move according to the six degrees of freedom in order to certify the real surface.
The symbol ⌓ with a datum reference frame (without comments) indicates a position specification. The virtual boundary can move only according to the degrees of freedom of the datum reference frame in order to certify the real surface.

The symbol ⌓ with a datum reference frame and the comment « orientation only » (or equivalent) indicates an orientation specification. The virtual boundary can move according to the three translations and the degrees of freedom of the datum reference frame in order to certify the real surface.

These definitions allow establishing a rule of tolerancing. If the datum reference frame has degrees of freedom in rotation, to add an orientation specification to a position specification in order to master the angular effects imposes a composed specification in the sense of the proposal 1, in order to guarantee that the two tolerance zones are well parallel.

The figure 18 shows the minimum clearance allocated by form specifications S1 and S2 at maximum material condition represented in figure 17.

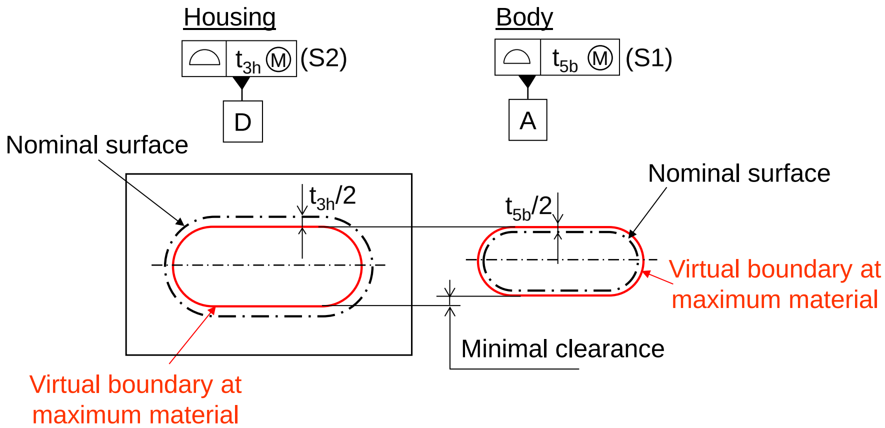


Figure 18: Minimal clearance between maximal material boundaries

For the requirement R1 (fig 16), the accuracy of the primary link will be critical when the parts are at least material condition. On the shaft, this requirement imposes the specification S8 defined in the figure 19. The least material boundary of the primary datum A is an offset of the nominal surface with the value $t_{2b}/2$. This virtual boundary must be contained to the material of the real part.
The secondary datum B is a plane perpendicular to this virtual boundary tangent to the real plane. The difference between the least material boundary of the primary datum and the real surface let a residual mobility in order to introduce the real cone in the tolerance zone. The secondary plane must be tangent but not constrained with a Tchebitchev criterion which minimizes the maximum distance on the secondary plane.

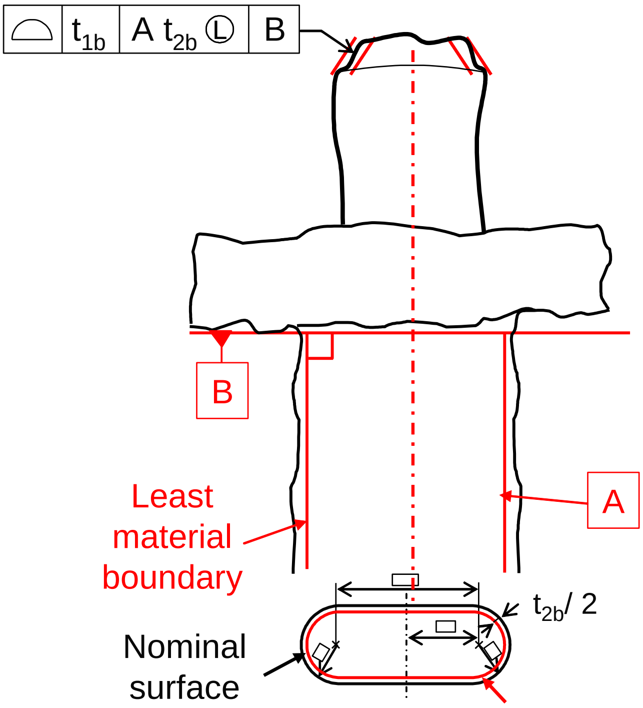


Figure 19: Floating datum reference frame of the shaft

On the housing, this requirement R1 imposes a position specification S6 with tolerance value t_{1h} and an orientation specification of tolerance value t_{2h} with $t_{1h} > t_{2h}$. The least material boundary is located relative to the datum reference frame PQ. The orientation virtual boundary (S7) is smaller and orientated relative to the datum reference frames PQ. The figure 20 shows the virtual housing which gives the maximum displacement for a vertex F in the direction **f**. The hole is defined by the orientation least material boundary moved in the direction **f**, but contained in the location least material boundary. The analysis line being pointed at the top, the surface E is supposed coincided with the superior limit of the location tolerance zone of S5 (fig 17).
The tolerance chain transfer is built with the shaft at least material condition inclined in the orientation least material boundary of the housing to maximize the displacement of the point F. The contact plane B will be tangent to the surface E.

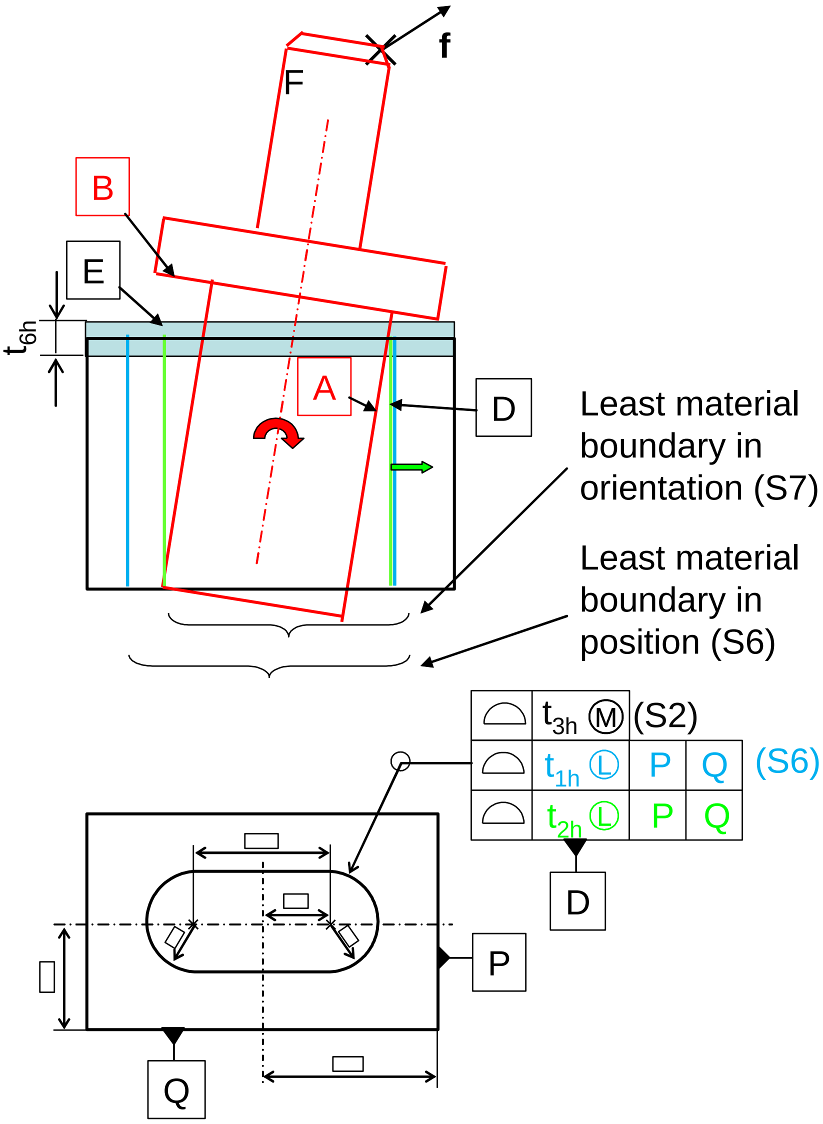


Figure 20: Three dimensional transfer method based on least material model

The three-dimensional model of transfer imposes to study the mobility of a complex virtual boundary in another one and to search contact points between these virtual boundaries. Chavanne uses for that a geometric algorithm or a solver [18].

4. Hybrid junction

4.1 Basic mechanism with an hybrid prismatic junction

The figure 21 presents an basic mechanism with a prismatic link in z direction. In fact, the body is in contact with the housing in the middle surface between J and K and is centered by the two flanks IJ and KL.

The middle surface between J and K has a variable curve but is very flat and does not block the degrees of freedom in translation according to x and in rotation around y. These two last degrees of freedom are blocked by the flanks with a little mobility allowed by the clearance. For the first approach, it would be possible to consider that the middle surface is a prismatic

surface which blocks geometrically all of five degrees of freedom. Then, the flanks have no more role and can not be defined as secondary surface.
Consequently, the entire surface must constitute a primary link qualified of hybrid in the sense of the double behavior (contact and clearance).

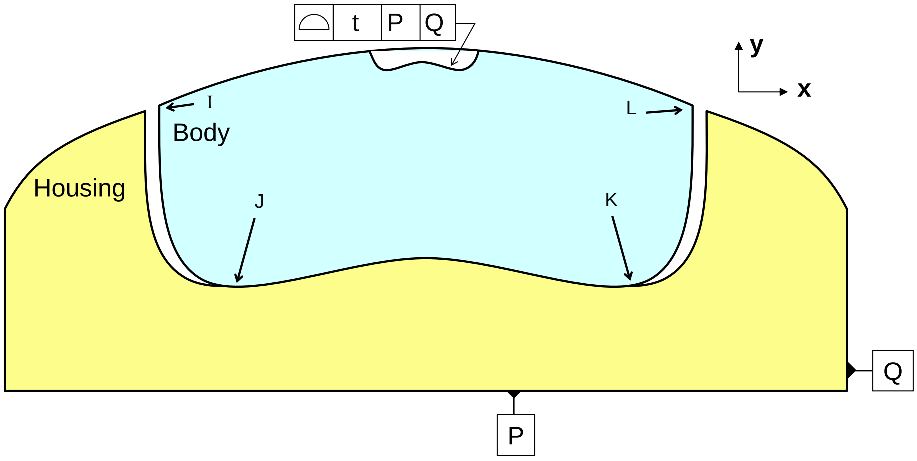


Figure 21: hybrid prismatic link

The figure 21 presents the location requirement of the mark at the top of the body relative to the datum reference frame PQ of the housing. The requirement R1 must be respected for all positions permitted by the clearance.

4.2 Classic tolerancing of parts

The figure 22 presents the classic tolerancing of the two parts considering globally the entire surface.
The form specifications S2 and S4 must ensure the quality of contact and the minimal clearance between parts. The tolerance zones are centered relative to nominal surfaces. To get the required clearance, it is necessary that nominal surfaces of the two parts are different on the flanks. On the other hand, the nominal surfaces are identical for the middle contact surface.

On the body, the ending surface must be located relative to a datum reference frame created on junction surface A.
On the housing, the bearing surface D is located relative to the datum reference frame PQ. It is not necessary to add an orientation specification in order to limit the inclination because there is not overhang between the ending surface and the link surface.

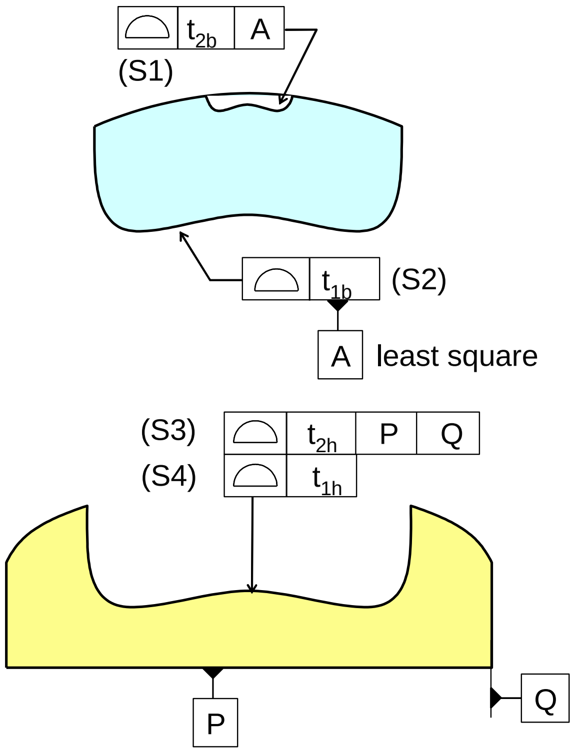


Figure 22: Classical tolerancing of parts

For the specification S1 of the body, standards ISO do not clearly give the definition of the datum on a free surface. The least square criterion is thus indicated on the right hand of the datum feature A. In this case, the datum is an identical surface to the nominal one associated by the least square criterion to the real surface. This criterion minimizes the sum of the square of each deviation and provides a datum perfectly defined, without allowing the mobility corresponding to the clearance between the flanks.

This surface has light angle on the flanks. A datum reference surface identical to nominal surface tangent to exterior material could be away in the real middle surface (fig 23). The Tchebitchev criterion is not suitable for this type of surface.

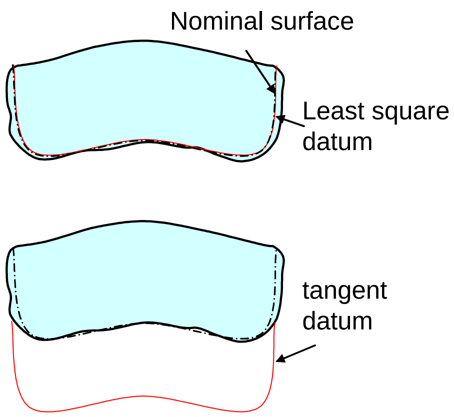


Figure 23: Datum with least square criteria and minmax criterion

4.3 Datum in a link with clearance

The mechanism of figure 24 is similar to the one of figure 21, with a primary plane and a secondary link with clearance. The tolerancing applies the rules of the proposal 3.

Three requirements are taken into account by the proposed tolerancing:

- Quality of the contact on the primary plane
- Minimum clearance to guarantee the assembly of parts
- Location of the mark relative to the datum reference frame PQ for all positions of allowed by the clearance.

The chain transfer must take the maximum clearance into account between the least material boundaries in order to share the tolerances. If the body is bigger than its least material condition, the clearance will be smaller and it will be possible to accept shifted mark. This is allowed by the datum reference frame at least material condition.

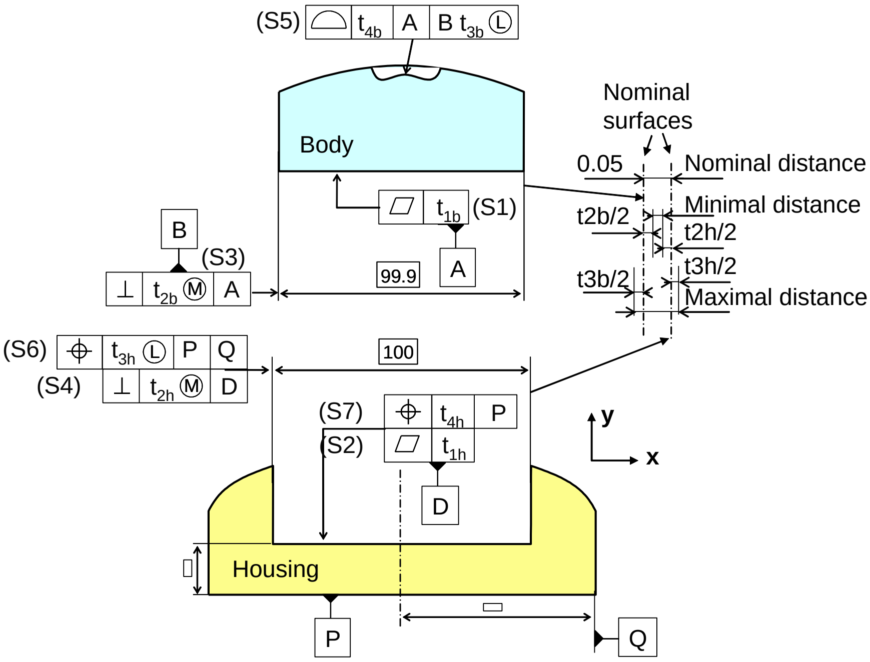


Figure 24: Tolerancing with primary plane and secondary slot

The good contact between the body and the housing is assured by the two form specifications S1 and S2. The gap is the maximal distance between real surfaces when these surfaces are in contact. So, the maximum gap between the two surfaces is the sum of the form tolerance values.

Gap = $t_{1b} + t_{1h}$.

Supposing the body centered in the housing, the nominal distance between the nominal surfaces is defined by basic dimensions:

Nominal distance = (100 – 99.9)/2 = 0.05.

As the part is symmetrical, the clearance is equal to the double of this distance.

The minimum distance between parts is assured by the specifications S3 and S4 which require two maximum material boundaries, offset of $t_{2b}/2$ for the body and offset of $t_{2h}/2$ for the housing.

The minimum distance between the flanks is then:

Minimum distance= nominal distance - $(t_{2b} + t_{2h})/2$

The location requirement of the mark relative to the datum reference frame PQ requires the position specification S5 on the body and the two locations S6 and S7on the housing.

For the specification S5 of the body, the primary plane datum is the tangent plane to the real plane A minimizing the maximal distances. The secondary datum is defined by a virtual boundary composed of two symmetrical parallel planes perpendicular to A and distant of $t_{3b}/2$ to nominal surfaces. This virtual boundary must be contained inside the body surfaces B. The toleranced element is the mark surface. This real surface must be contained in a tolerance zone of wide value t_{4b} theoretically located relative to the primary datum and the least material boundary. The mobility allowed by the space between the virtual boundary and the real surface B enables to move the mark surface in the tolerance zone.
On the housing, the specification S6 defines a least material boundary formed by two planes offseted of $t_{3h}/2$ relative to nominal planes. This virtual boundary must be inside the material.

The maximum distance between the flanks is so:
Maximum distance = Nominal distance+ $(t_{3b} + t_{3h})/2$

4.4 Datum definition in a hybrid link

The figure 26 presents the proposed tolerancing to find the same behavior on the hybrid link. The surface is restricted by letters I, J, K and L to isolate the middle surface and the flanks. Each segment must be separately specified. In standards, the symbol "between" is written under the feature control frame as in case (a) of figure 25.

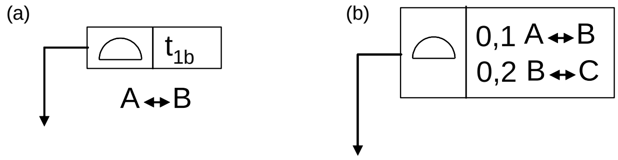


Figure 25: restricted zone inside specification

The specification (b) of the figure 25 introduces a new concept to describe the limit of the restricted zones inside the feature control frame.

Proposal 5: the indication A ↔B located on the right hand of the tolerance value or on the right hand of a datum limits the area of the considered surface. Several indications in the same feature control frame allow modifying the characteristics of the tolerance zone in function of the segment of the considered surface. All tolerance zones thus defined constitute a common zone that must respect the toleranced surface.

This writing is necessary, because it is impossible to write these conditions with independent specifications, particularly for form specifications or floating datum reference frames.

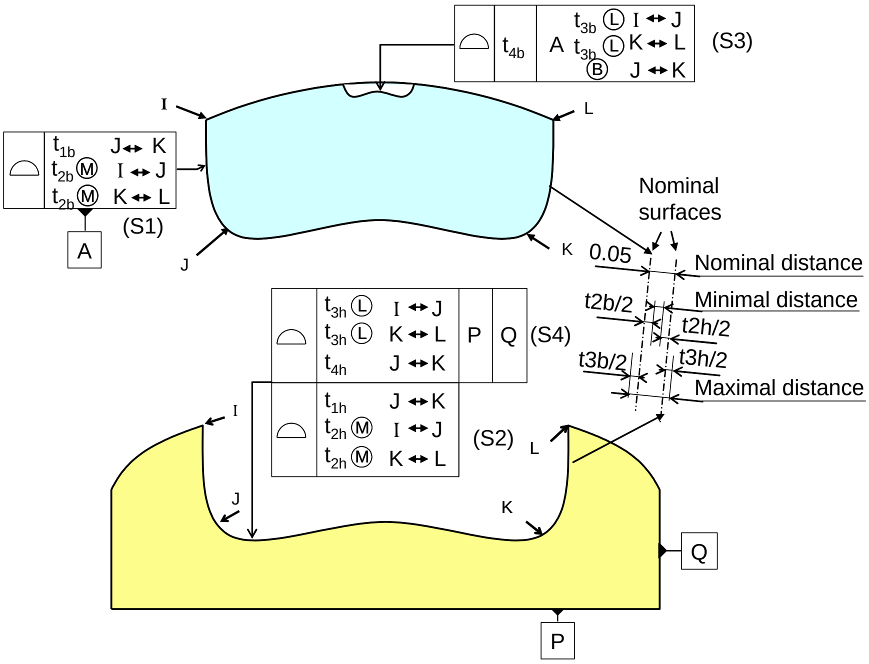


Figure 26 : Tolerancing of hybrid links

The good contact between the body and the housing is assured by the two form specifications S1 and S2 limited to the middle segment between J and K. The maximum gap between the two surfaces is the sum of the form tolerance values
Gap = $t_{1b} + t_{1h}$.

Supposing the body centered in the housing, the nominal distance between the nominal surfaces is defined by nominal surfaces: the distance depends on the considered point.

The part being symmetrical, the clearance is about equal to the double of the minimal distance.

The minimum distance between parts is assured by the specifications S1 and S2 which require two maximum material boundaries, offseted of $t_{2b}/2$ for the body and offseted of $t_{2h}/2$ for the housing.
The minimum distance between the flanks is so:
Minimal distance= nominal distance - $(t_{2b} + t_{2h})/2$

The position requirement of the mark according to the datum reference frame PQ requires the position specifications S3 on the body and S4 on the housing.

For the specification S3 of the body (fig 26), the datum reference is defined on the segment between J and K and with a least material boundary on the flanks. This least material boundary is offset of $t_{3b}/2$ with regard to the nominal surfaces. This one must be contained inside the body surfaces. The toleranced element is the mark surface. This real surface must be contained in a tolerance zone of wide value t_{4b} theoretically located relative to the datum reference frame. The mobility allowed by the space between the virtual boundary and the side real flanks enables to introduce the mark surface in the tolerance zone.
On the housing, the specification S4 defines a maximum material boundary formed by the two surfaces offset of $t_{3h}/2$ relative to nominal surfaces. This virtual boundary must be inside material.

The maximum distance between flanks is so:
Maximum distance = d_{maxi} = Nominal distance + (t3b + t3h)/2

The rules of calculus are similar to those of the tolerancing of the figure 24.

4.5 Definition and measurement of the specification S3

The figure 26 presents the specification of the body. The mark surface is located relative to a datum reference frame composed of a surface zone between J and K and flanks which it is necessary to define a least material boundary on.
We propose a new modifier B like bilimit, because the coupling with the least material condition requires a specific criterion to guarantee the tangency of the datum on the surface part.

<u>Proposal 6:</u> Hybrid datum

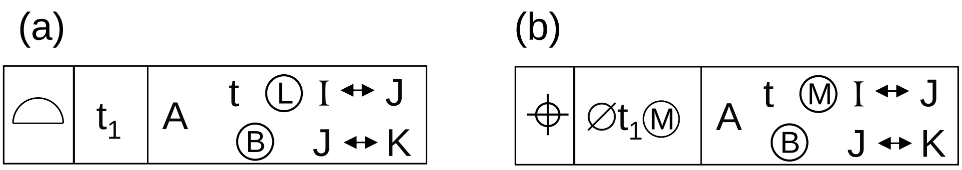


Figure 27: Hybrid datum

In figure 27, case (a), the datum reference is defined by two segments: between J and K, the bilimit modifier (B) imposes the reference to be identical to the nominal surface and tangent to the external real surface minimizing the maximum distance. Between I and J, the least material modifier (L) imposed a reference offset of t/2 with regard to the nominal surface and included in the real part.
Case (b) is similar, the offset is in the opposite direction, and reference must be outside the part.
The association criterion imposes, inside the segment identified with B, to the reference to be tangent to the surface exterior to the material minimizing the maximum distance. The other points are constrained to respect the virtual boundary.

For the body, the floating permitted by the space between the least material boundary and the real part enables the datum reference to slide on the middle surface part without rising in order to move the tolerance element inside the tolerance zone.

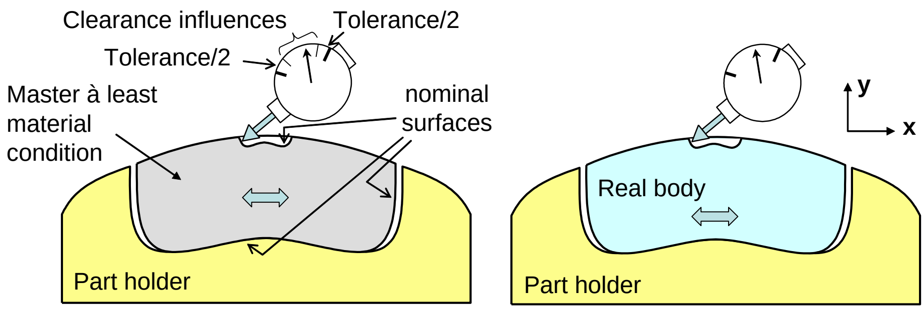


Figure 28: Classical metrology of the body

The figure 28 illustrates the measurement principle of the specification S3 with a simple gauge. The part holder is a perfect part corresponding to the nominal surface of the body in the centered surface part and of a maximum material boundary of the flanks in order to assure

the assembly of the real parts in the part holder. The master is a perfect part corresponding to the nominal surface of the body in the centered surface part and to a least material boundary on the flanks. The mark is also identical to the nominal surface. When the master is translated in the part holder, the gauge moves according to an amplitude which characterizes the influence of the clearance between the master and the part holder at the measurement vertex. The slot identified on the gauge must be enlarged of the tolerance value t_{4b} of the specification S3.
The real part is accepted if the needle of the gauge stays inside the tolerance zone for all positions of the real part in the part holder.
These operations must be realized checking that the master and the part do not peel off the part holder.

The certification of the specification S3 with the help of a three dimensional measurement machine necessitates to measure points B_i in the middle surface part, points L_i on flanks and points C_i in the mark in a measuring reference system (figure 19). The nominal surface is also known in this measuring reference system. For each point, the deviation e_i is determined between the point and the nominal surface. This deviation is negative if the point is inside the material, this deviation is positive if the point is in the exterior material. Then, on the flanks, the deviations relative to the least material boundary are $e_{li} + t_{3b}/2$.
To determine the position deviation of the points C_i, it is necessary to move the datum reference on the points B_i and L_i and to minimize the deviation of points C_i taking advantage of the mobility allowed to the virtual boundary. However, the association criterion imposes also to minimize the maximum distance of points B_i to the datum reference. This double optimization necessitates two successive displacements of the datum. This small displacement of the datum reference is characterized by a torsor with three rotations and two translations [19].

The first step does not take the points Ci into account. The datum reference must be tangent to the surface part respecting the virtual boundary on flanks. The optimization criteria are the following:

Constrains: For points L_i : $e_{li} + t_{3b}/2 \geq 0$ (respect of the virtual boundary)

For points B_i : $e_{bi} \leq 0$ (respect of the tangency)

Target : to minimize the greatest deviations $|e_{bi}|$ of all points B_i

This first step defined the datum reference of the middle of the part. The maximal deviation on points Bi is noted δ.

$$\delta = \max(|e_{bi}|).$$

To minimize the distance of the points C_i to the nominal surface, it is necessary that the datum reference slides on the middle surface of the part. This move cannot be forced by a simple translation. The five degrees of freedom must be free assuring that the datum stay tangent to the surface part and it does peel off the real surface. This is obtained checking that for all B_i, the deviations are included between 0 and $-\delta$.
The second displacement is thus impossible. To allow to the surface sliding, it is necessary to take a additional margin to δ, for example $\varepsilon = 0.002$

The margin ε can generate a slight peeling off of the datum reference relative to the tangent position. A very small value for example equal to the accuracy of the measures is enough to move the surface.

The interest of this double optimization is to allow the mobility of the virtual boundary while leaving the datum reference tangent to the middle surface, without rising of the reference.

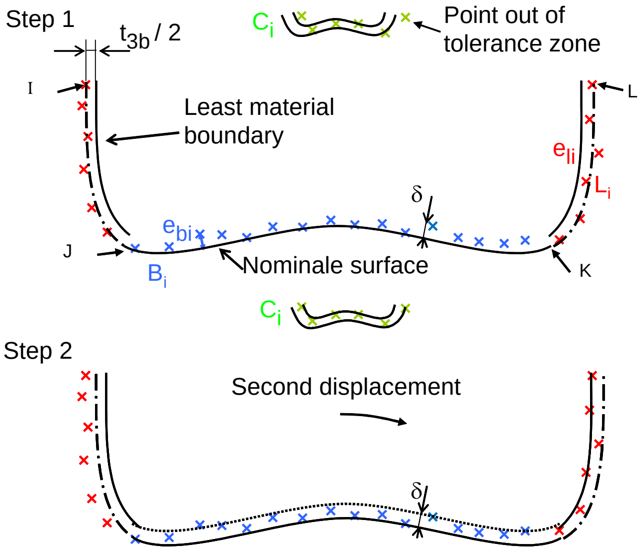


Figure 29: Datum reference building on the body

4.6 Definition of the specification S4

The figure 30 depicts the tolerance zone corresponding to the position specification S4 of the housing. The tolerance zone of the middle surface and the least material boundary are exactly located relative to the datum reference frame PQ.

The real surface of the middle zone will be included inside the tolerance zone. The real flanks will have to respect the least material boundary.

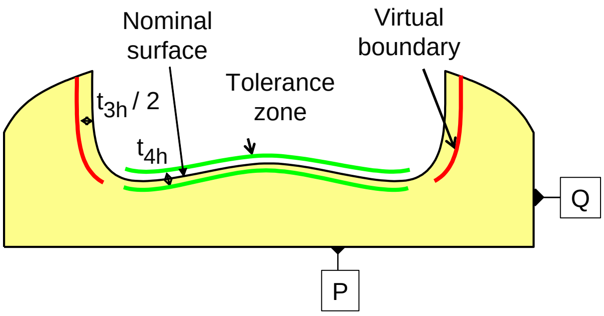


Figure 30: Specification of the housing.

4.7 Transfer model of hybrid surfaces

For the position requirement defined in the figure 21, the transfer principle is based on two hypotheses:

The displacement of the mark due to the junction is maximal when the housing and the body are at least material condition.

The middle segment of the datum frame of the body between J and K stays in the tolerance zone of the housing

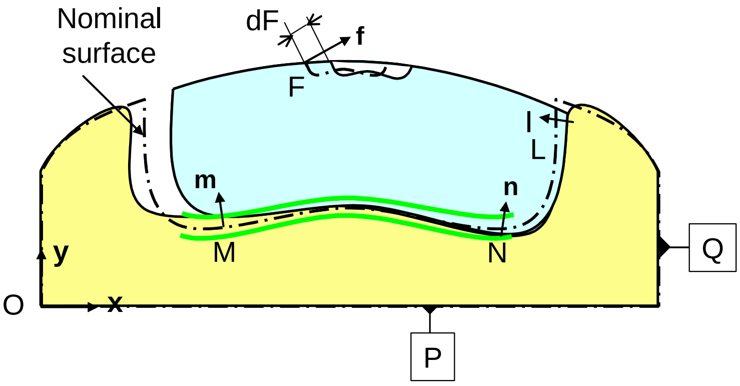


Figure 31: Influence of junction on hybrid surface

The problem is to determinate now the maximum displacement of all vertex F of the mark function of the tolerance values.

The configuration which gives the maximum displacement of the vertex F is represented in the figure 31. In this configuration, the displacement of M is **dM**.**m** = $t_{4h}/2$. The displacement of N is **dN**.**n** = $-t_{4h}/2$.
Supposing flanks are quite perpendicular to the middle surface, the displacement of L is **dL**.**L** = - d_{maxi} (calculated in 4.4).

With regard to the nominal position, the body moves by u in x direction, by v in y direction and rotates of γ around z. Displacement of M, N, L must respect:

dM.**m** = $u.m_x+v.m_y - \gamma.Y_m.m_x + \gamma.X_m.m_y = t_{4h}/2$

dN.**n** = $u.n_x+v.n_y - \gamma.Y_n.n_x + \gamma.X_n.n_y = -t_{4h}/2$

dL.**l** = $u.l_x+v.l_y - \gamma.Y_l.l_x + \gamma.X_l.l_y = -d_{maxi}$

The solution of this system gives the values u, v and γ of the small displacement torsor. This enables to calculate the displacement of the vertex F in the direction **f** function of tolerances of the figure 21.

dF.**f** = $u.f_x+v.f_y - \gamma.Y_f.f_x + \gamma.X_f.f_y$

Therefore, this tolerancing enables to do the tolerance analysis and the tolerance synthesis.

5. CONCLUSION

ISO standards offer suitable solutions to specify mechanisms, for example with the concept of maximum or least material condition. This paper has shown the tolerancing and the transfer method for simple link based on plane and or cylinder. The section 2.4 underlines that the independence principle did not enable to limit the orientation deviation inside a tolerance zone if the datum is at least material condition. This imposes to group the two specifications with the same datum reference frame.
The second section studies the complex link with an extension of the concept of virtual boundary to complex surfaces.
The third section of this paper analyzes links on hybrid prismatic surfaces with a surface contact zone and a zone with clearance. Current standards do not enable to specify these mechanisms assuring the coherence with the 3D transfer models.

This work shows that the concept of maximum or least material condition can be largely extended to complex links.
This paper proposes different writings and concepts which offer the advantage to respect the independence principle and which do not necessitate the use of local dimensions or dynamic diagrams.
These writings are just proposals in a scientific paper and must not be used in an industrial document. It is of course for standards committees to analyze these proposals to formalize these writings in the context of international standards in preparation.

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