

Muon simulation codes MUSIC and MUSUN for underground physics

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Abstract

The paper describes two Monte Carlo codes dedicated to muon simulations: MUSIC (MUon SIMulation Code) and MUSUN (MUon Simulations UNderground). MUSIC is a package for muon transport through matter. It is particularly useful for propagating muons through large thickness of rock or water, for instance from the surface down to underground/underwater laboratory. MUSUN is designed to use the results of muon transport through rock/water to generate muons in or around underground laboratory taking into account their energy spectrum and angular distribution.

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1 Introduction

Muon transport through matter plays an important role in many areas of particle and astroparticle physics. Cosmic-ray muons are detected at large depths underground and underwater (here and hereafter we use the term underwater that includes also under-ice experiments). They are used to study the energy spectrum and composition of primary cosmic rays and calculations of their fluxes, energy and angular distributions are the key element of this research (see, for instance, Refs. [1, 2, 3, 4, 5]).

Experiments with high-energy muon neutrino beams from accelerators require accurate simulations of muon transport from the point of neutrino interaction to the detector. Similarly, neutrino telescopes are detecting (or expecting to detect) muons from atmospheric and astrophysical neutrinos, and three-dimensional propagation of muons from their production point to the detector is crucial for the interpretation of experimental data [6, 7, 8].

Cosmic-ray muons are also a background in experiments looking for rare events at low and high energies deep underground or underwater. Atmospheric down-going muons can be erroneously reconstructed as upward-going muons that mimic neutrino-induced events in a search for astrophysical neutrinos at GeV-TeV energies or in an atmospheric neutrino detection for neutrino oscillation studies. Cosmic-ray muons also produce secondary neutrons (with MeV-GeV energies) by interacting with rock. These neutrons can mimic low-energy (keV-MeV) events in detectors looking for WIMP (Weakly Interacting Massive Particle) dark matter, neutrinoless double-beta decay and neutrinos (solar, geophysical, supernova neutrinos, etc.) (see Ref. [9] for a review and Refs. [10, 11, 12, 13] for example calculations of muon-induced neutron fluxes underground) . High-energy (GeV) neutrons from muons can produce events with a signature similar to proton decay.

There are a few more applications from different areas of science. A morphological reconstruction of mountains and natural caves using atmospheric muons was suggested in Ref. [14]. A search for hidden chambers in pyramids was discussed in Ref. [15]. A ‘muon radiography’ using multiple scattering of cosmic-ray muons was proposed recently [16] to discriminate between low-A and high-A materials in cargo.

All applications mentioned above require accurate calculations of muon spectra and scattering beyond a slab of material. Most of them involve muon transport through large thickness of matter. Hence the CPU time should be reduced to a minimum without compromising the accuracy of calculations.

Several Monte Carlo codes are able to transport muons through matter with high accuracy. The codes can be split in two categories: (i) multipurpose particle transport codes, such as GEANT4 [17] and FLUKA [18], and (ii) codes developed specifically for muon propagation through large thickness of material, such as PROPMU [19], MUSIC [20, 21], MUM [22] and MMC [23].

Significant progress has recently been achieved in the development of the multipurpose transport codes for particle physics applications. The codes have become faster, more robust, flexible and accurate. However, their flexibility requires a good knowledge of physics and programming skills from a user. GEANT4, for instance, is designed as a powerful toolkit but a good knowledge of the code including models and programming language is

needed to use it properly. Significant efforts and time are required to become familiar with such a toolkit. These codes are absolutely necessary when simulating events consisting of many particles that should be produced, transported and detected practically at the same time. Meanwhile, some tasks, for instance muon transport through a homogeneous material, may be accomplished without using multipurpose codes. If a user is interested in transporting muons without following secondary particles produced by them, it is enough to consider accurately only muon interactions and muon energy losses neglecting the fate of secondaries. This is the idea implemented in specially developed muon transport codes.

In this paper we describe the three-dimensional muon propagation code MUSIC. Although the first version of the code was released in 1997 [20] and several modifications were reported since then [21], we believe that the recent developments and improvements made to the code and the variety of applications should be described in a separate paper. The second part of the paper is dedicated to the code MUSUN written to sample muons underground [11] or underwater [24] using the results of muon transport carried out with MUSIC.

2 Muon transport through large thickness of matter: MUSIC

The first version of MUSIC (MUon SIMulation Code), written in FORTRAN, has been released in 1997 [20]. It has been used in the interpretation of data from the LVD experiment in the Gran Sasso Laboratory, namely in the reconstruction of the muon energy spectrum at surface from the measured depth – vertical muon intensity relation [3] and in the evaluation of the fraction of prompt muons in the high-energy muon flux [4]. Several improvements have been done to the code and new features have become available since then. The basic features of the code and improvements are described below.

The code takes into account the energy losses of muons due to four processes: ionisation (using Bethe-Bloch formula) including knock-on electron production, bremsstrahlung (or braking radiation), electron-positron pair production and muon-nucleus inelastic scattering (or photonuclear interactions). The cross-section of bremsstrahlung was taken from Ref. [25] in the first version of MUSIC with an option to use the cross-section from Refs. [26, 27]. The effect of different cross-sections on the MUSIC results was studied in Ref. [20]. The correction to the Born approximation (Coulomb correction) for bremsstrahlung cross-section was not taken into account. It was shown [27] that this correction does not exceed 1% even for heavy nuclei. The pair production cross-section was taken from Ref. [28] in the first version of MUSIC. New parameterisation of the pair production on atomic electrons [29] has been implemented in the second version of the code [21]. Original parameterisation of muon inelastic scattering cross-section [30] has been complemented by a more accurate treatment suggested in Ref. [31]. An option has been added to calculate this cross-section using the deep-inelastic scattering formalism and nucleon structure functions suggested in Ref. [32]. The ALLM parameterisations [33] of structure functions have been implemented. The default options in MUSIC (the recommended cross-sections) are: (i) bremsstrahlung – Ref. [25]; (ii) pair production – Refs. [28, 29]; (iii) inelastic scattering – Ref. [31]. All results presented here have been obtained with this set of

cross-sections.

Muon interaction cross-sections are calculated in MUSIC at the beginning of the first run (or by the code developer) for all elements present in a material and specified by a user, and are averaged using the weights (a fraction of each element by mass) provided by the user.

There are two major versions of MUSIC existing and developed in parallel: (i) ‘standard’, dedicated for muon transport through large thickness of matter; (ii) ‘thin slab’, developed for muon transport through thin slabs of materials.

The standard version of MUSIC considers all interaction processes stochastically if the fraction of energy lost by a muon in the interaction exceeds a pre-defined value of a parameter, v_{cut} (see Ref. [20] for the full code description and tests). The value of v_{cut} can vary from 10^{-5} to 1 and can be set by a user but the recommended value taken as a compromise between the accuracy of the code and its speed, is 10^{-3} as in the first version of the code [20]. The program evaluates the mean free path of a muon between two subsequent interactions (with $v > v_{cut}$) using the sum of integrated cross-sections, where integrals are computed between v_{cut} and 1. Then it samples the real path of the muon to the next interaction using a random number generator from the CERN library (RANLUX). Another random number determines the type of the interaction. The third random number is used to select the fraction of muon energy lost in the interaction v . Then the code calculates the continuous energy losses of the muon between the two interactions, i.e. mean energy losses due to the four aforementioned processes with $v < v_{cut}$. Mean energy loss due to ionisation is computed using Bethe-Bloch formula for $v < v_{cut}$. Knock-on electron production is added to stochastic processes at $v \geq v_{cut}$ including corrections to the ionisation energy loss due to e -diagrams for muon bremsstrahlung on an electron suggested in Ref. [25]. In this process the photon is emitted by the electron and is accompanied by the high-energy recoiling electron.

Muon deflection due to multiple Coulomb scattering in the plane perpendicular to the initial muon direction is calculated between every two interactions with $v \geq v_{cut}$ [20]. The process is treated in the Gaussian approximation [34] that was also used in Ref. [19]. More accurate treatment of muon angular deviation in the framework of Molière theory results in a similar distribution of muon scattering angles beyond the large thickness of rock with a small increase of the mean deflection angle [20]. However, the original Molière theory was developed for angular deviation only and does not provide the lateral displacement that is sometimes more important from the experimental point of view (for instance, for muon bundles underground).

Although multiple Coulomb scattering dominates over stochastic processes in the muon deflection [20], muon deviation due to other interactions is also taken into account in MUSIC. The angular deviation due to muon inelastic scattering is computed using double-differential cross-section [30]. The muon scattering angle due to bremsstrahlung and pair production is calculated following the parameterisations suggested in Ref. [35] (see also Ref. [20] for detailed description).

Note that the muon transport code MUM [22] is one-dimensional and does not take into account muon deflection in the plane perpendicular to the initial direction. The code MMC [23] considers only muon deviation due to multiple scattering and not many details

or results are given in the original paper [23]. PROPMU [19] also treats only multiple scattering. Muon transport codes MUSIC, MUM and MMC were found to agree with each other giving similar muon energy distributions beyond large thickness of rock or water and similar muon survival probabilities [22]. MUSIC and PROPMU are in agreement for muon transport in rock [20] but results obtained with PROPMU in water were found to be different from those obtained with MUSIC and MUM [22].

The MUSIC code allows the transport of muons with energies up to 10^7 GeV. All muons are considered to be ultrarelativistic. If the total muon energy becomes smaller than the muon mass, the muon is considered to be stopped. The code consists of two files and is arranged as consecutive calls to two or three subroutines written in FORTRAN from the ‘main’ user program. The call to the first subroutine is optional: it allows calculation of the muon cross-sections and energy losses and can be done at the beginning of the first run. The files with cross-sections and energy losses can also be supplied by the author. The call to the second subroutine allows reading the muon cross-sections from the computer disk. A single call to the third subroutine transports one muon with given initial parameters (energy, coordinates and direction cosines) to a specified distance in a material with previously calculated cross-sections. The code (the third subroutine) returns the muon parameters at the end of the muon path in the material. If the muon has stopped before reaching the end of the material, the zero value for the muon energy is returned together with approximate coordinates of the point where the muon has stopped.

A special version of MUSIC (‘thin slab’) has been developed for muon transport through thin slabs of materials. Originally it has been written for water as part of the software for the ANTARES experiment [8]. This version has been aimed at providing muon energy, position and direction at the end of every small segment of muon path in water and at passing the muon energy loss at the segment to another part of software that generated Cherenkov photons. Since then this version has also been used to estimate muon deflection in high-A materials (iron, lead and uranium) for possible security applications (searching for hidden high-A materials, like uranium, in cargo) [36]. The ‘standard’ version of MUSIC, in the absence of stochastic interactions on the small segment, always returns the mean value for continuous energy loss at the end of the segment. In the ‘thin slab’ version the cut that separates stochastic and continuous parts of the energy loss is reduced to 1 MeV, meaning that practically all muon interactions are stochastic. The ionisation energy loss is calculated using Landau distribution (call to a function from the CERN library). This version of the code allows the transport of muons with energies up to 10^9 GeV but without taking into account the LPM effect. Both versions of MUSIC (standard and thin slab) give consistent results for thick slabs of matter but the ‘thin slab’ version is more CPU consuming because of the lower value of v_{cut} . The thin slab version gives more accurate results for energy spectra and angular deviation beyond thin slabs of material.

Although the energy loss due to muon pair production by muons is not included in the code because of its small value compared to the electron-positron pair production, there is a possibility to generate muon pairs along the muon path [21].

A few results from muon transport through standard rock ($Z=11$, $A=22$, $\rho = 2.65$ g/cm³) and pure water are shown in Figures 1-3. Muons with initial energies ranging from 10^2 GeV to 10^7 GeV were transported through 15 km w. e. of standard rock and water and their energies at different depths (distances from initial point) were recorded. 10^5

muons were propagated for each value of initial energy. This means that for a survival probability (defined as a probability for a muon with a certain initial energy to traverse a pre-defined distance) of 0.01, the statistical error is about 3%. Survival probabilities as functions of muon energy at surface for different depths are presented in Figure 1 for standard rock (black solid curves) and water (red dashed curves). Numbers to the right from each solid curve show the depths in km w. e. for standard rock. Survival probability curves for water are shifted to the right (for small depths) or to the left (for depths larger than 2 km w. e.) relative to those for standard rock. This behaviour is due to the presence of hydrogen in water. Hydrogen has the ratio of $Z/A \approx 1$ whereas most other materials have $Z/A \leq 0.5$. Ionisation energy loss is proportional to Z/A whereas energy losses due to pair production and bremsstrahlung are approximately proportional to $Z(Z + 1)/A$. At low muon energies (below 1 TeV) and small depths ionisation energy loss dominates over other processes, muon energy losses in water are bigger than in standard rock and muon survival probability for a fixed initial energy is smaller in water than in standard rock. At intermediate depths (between 1 and 3 km w. e.) the muon survival probabilities in water are smaller for low energies and bigger for higher energies compared to standard rock. At large depths (high muon energies) the energy loss due to pair production and bremsstrahlung dominate over ionisation and the muon survival probabilities in water are bigger than those in standard rock for all energies.

Muon energy spectra at vertical at different depths in standard rock (black solid curves) and water (red dashed curves) are presented in Figure 2. Numbers above the curves for standard rock show the depth in km w. e. The spectra have been calculated by convoluting muon energy distributions underground obtained with MUSIC, with muon energy spectra at surface taken in the form that fits the data from the LVD experiment [3, 4] (for full description of the procedure see Section 3 below). Curves for water are shifted above or below those for standard rock due to the presence of hydrogen in water (see discussion above).

MUSIC has been extensively tested against experimental data. It has first been used in the analysis of muon intensities measured by the LVD experiment [3, 4]. Since there are several factors that affect the calculation of muon intensity underground (muon cross-sections, muon energy spectrum at surface, slant depth distribution and rock composition), comparison between measured and calculated muon intensities does not provide an accurate test of the muon transport code. In fact the energy spectrum of muons at surface has been reconstructed from the measured intensities assuming that other factors are known. However, the fact that measured intensities agree with simulations over a large range of zenith angles and slant depths, provides a strong evidence for the validity of the muon transport code.

Depth – vertical muon intensity relation (muon intensity at vertical as a function of depth) is shown in Figure 3 for standard rock and water. Muon intensities have been calculated by integrating muon energy spectra underground over energy. The simulated curves agree well with the measurements both in rock (black triangles – compilation of data points from Ref. [37]) and water (blue open circles – [6], blue filled circles – [7]).

A comprehensive comparison of calculated (using MUSIC) muon intensities underground with measurements has been done in Ref. [38]. Data points were found to be scattered symmetrically around the calculated depth–intensity curve showing the overall consistency

of the muon transport. Large spread of data around simulations may be explained by the complexity of factors involved in data interpretation, such as, rock composition, procedure of data conversion to standard rock etc.

Muon intensities and mean muon energies, calculated with the MUSIC transport code and the LVD parameterisation for the muon spectrum at surface [3, 4] (for full description of the procedure see Section 3 below) are given in Table 1 for standard rock and water.

MUSIC has been used in the analysis of SNO [39] and MACRO [5] data. The code has also been applied for the calculation of expected background induced by cosmic-ray muons in deep underground experiments, such as KamLAND, Super-Kamiokande, etc.

Comparison of energy losses as calculated by MUSIC, GEANT4 and FLUKA is discussed in Ref. [40]. All codes agree well in calculating energy distributions for high-energy muons transported through small and large slabs of materials. Figure 4 shows the energy spectrum of muons with initial energy of 2 TeV transported through 3 km of water using MUSIC, GEANT4 [41] and FLUKA. Distributions look very similar except for small difference at high energies. The survival probability is equal to 0.779 (MUSIC), 0.793 (GEANT4) and 0.756 (FLUKA) with a statistical error of about 0.001. The mean energy of survived muons is 323 GeV (MUSIC), 317 GeV (GEANT4) and 344 GeV (FLUKA). Figures 5 and 6 show distributions of angular deviation and lateral displacement, respectively, for muons with initial energy of 2 TeV transported through 3 km of water. GEANT4 predicts larger number of muons to be scattered to high angles and moved to large distances in the plane perpendicular to the muon direction. The mean scattering angle is equal to 0.22° (MUSIC) and 0.27° (GEANT4), whereas the mean displacement in the plane perpendicular to the initial muon direction is found to be 2.6 metres (MUSIC) and 3.3 metres (GEANT4). Unfortunately it is practically impossible to obtain data on high-energy muon scattering beyond very large thicknesses of matter to test codes, since the lateral separation of muon bundles underground is largely dominated by the scattering angle of the muon parent in the atmosphere at the interaction point where this parent is produced.

Similar transport code has also been developed for tau-leptons: TAUSIC (TAU Simulation Code).

3 Simulations of muons in underground laboratories using MUSUN

MUSUN (MUon Simulations UNderground) is the muon generator useful for sampling muons in underground laboratories according to their energy spectrum and angular distribution. It uses the results of muon transport through matter carried out with MUSIC, convoluted with the muon energy spectrum and angular distribution at surface.

At the first stage muons with various initial energies (from 100 GeV to 10^7 GeV with a step of $\Delta \log E = 0.025$) are propagated through matter and their energy distributions at distances from the initial point ranging from 100 m w. e. to 15000 m w. e. are written on the computer disk for further processing. This is usually done by the code developer following instructions from a user about the rock composition and other possible specific

features such as mountain profile etc. In a standard version of MUSUN the vertical depth should be more than 500 m w. e. There is no strict upper limit for the vertical depth, but the maximum slant depth should not exceed 15 km w. e. At larger depths neutrino-induced muon flux dominates over atmospheric muons and the calculation of the atmospheric muon intensity is not required.

In a simple version of MUSUN, the flat profile is assumed for the surface above the underground site (the curvature of the Earth is taken into account but other possible fluctuations of the slant depth are ignored).

After muon transport the differential muon intensities underground, $I_\mu(E_\mu, X, \cos \theta)$, are calculated using the equation:

$$I_\mu(E_\mu, X, \cos \theta) = \int_0^\infty P(E_\mu, X, E_{\mu 0}) \frac{dI_{\mu 0}(E_{\mu 0}, \cos \theta^*)}{dE_{\mu 0}} dE_{\mu 0} \quad (1)$$

where $\frac{dI_{\mu 0}(E_{\mu 0}, \cos \theta^*)}{dE_{\mu 0}}$ is the muon spectrum at sea level at zenith angle θ^* (zenith angle at surface, θ^* , is calculated from the zenith angle underground, θ , taking into account the curvature of the Earth), and $P(E_\mu, X, E_{\mu 0})$ is the probability for a muon with an initial energy at surface $E_{\mu 0}$ to have an energy E_μ at a depth X .

The energy spectrum at sea level can be taken either according to the parameterisation proposed by Gaisser [1] (modified for large zenith angles [3]) or following the best fit to the ‘depth – vertical muon intensity’ relation measured by the LVD experiment [3]. The first parameterisation [1] has the power index of the primary all-nucleon spectrum 2.70, while the second one [3] uses the index of 2.77 with the normalisation to the absolute flux measured by LVD. For small depths (less than 2–3 km w. e.) that correspond to low muon energies at surface (less than 1 TeV) it is recommended to use the original Gaisser’s parameterisation with an additional factor that takes into account muon decay in the atmosphere [42] if necessary. For larger depths the LVD parametrisation is the preferred option since it agrees with experimental data of the LVD [3] and MACRO experiments [2].

The ratio of prompt muons (from charmed particle decay) to pions is recommended to be set to 10^{-4} , which is well below an upper limit set by the LVD experiment [4]. Note, however, that prompt muon flux does not affect much muon intensities even at large depths.

To calculate integral muon intensity for normalisation, an integration of $I_\mu(E_\mu, X, \cos \theta)$ over dE_μ and $\cos \theta$ is carried out.

MUSUN offers the choice of the muon energy spectrum (as described above), the fraction of prompt muons, the vertical depth of the laboratory, the range of zenith and azimuthal angles, and the range of energies. No additional muon propagation is required for different options. Different types of rocks (rock compositions), however, require separate muon transport.

MUSUN is organised as a set of subroutines written in FORTRAN that are called from the user-defined ‘main’ program. The first call is made to a subroutine that calculates differential and integrated muon intensities for a specific vertical depth (assuming flat surface). The intensity as a function of energy and zenith angle is stored in the computer

memory as a two-dimensional array. Subsequent calls to a 'sampling' subroutine return muon parameters (energy and direction cosines) sampled following energy and zenith angle distribution. Azimuthal angle is sampled randomly as evenly distributed between 0 and 2π since the assumption of the flat surface leads to the spherical symmetry. The muon charge is generated according to the ratio measured for high-energy muons $\mu^+/\mu^- \approx 1.3$.

For practical purposes (for instance, when these muons are used in multipurpose event generators GEANT4 or FLUKA) it is useful to generate muons on the surface of a rectangular parallelepiped or a sphere with predefined dimensions. MUSUN offers a possibility to generate muon positions on the surface of a rectangular parallelepiped with dimensions specified by the user.

Muon parameters are written on the disk and can be passed later on to the multipurpose event generators.

Several underground laboratories (for instance, LNGS at Gran Sasso and LSM at Modane) are located in the transport tunnels under mountains with complex mountain profiles. For these labs special versions of the MUSUN code have been developed that took into account the slant depth distribution as seen from the underground laboratory. Here we present a few graphs with the results of muon production using MUSUN for the Gran Sasso Laboratory.

Figure 7 shows the azimuthal distribution of single muon intensities in the underground Gran Sasso Laboratory for zenith angles up to 60° as measured by LVD [4, 43] (data points with error bars) and generated with MUSUN (dashed curve). Good agreement is seen over the whole range of angles and intensities. Similar conclusion has been achieved in Ref. [44] when comparing azimuthal distributions for the whole range of zenith angles.

Figure 8 shows the energy spectrum of muons at Gran Sasso as generated with MUSUN. This spectrum looks different from Figure 2 because the number of muons is given here per energy bin which is constant on the logarithmic scale but increases with energy on the linear scale, whereas in Figure 2 the spectrum is given per constant energy bin on the linear scale (1 GeV). The mean muon energy at the Gran Sasso Laboratory is calculated as 273 GeV, in good agreement with the measured value of 270 ± 3 (stat.) ± 18 (syst.) GeV [45]. A lego plot of the number of generated muons versus zenith and azimuthal angles is presented in Figure 9.

At present the versions of the MUSUN code exist for the underground sites at Gran Sasso (LNGS), Modane (LSM), Boulby and Soudan. It has been used to study muon-induced neutron background for experiments looking for rare events, such as WIMPs (see, for instance, [11, 12, 46, 13, 47, 48]).

4 Conclusions

The two Monte Carlo codes MUSIC and MUSUN dedicated to muon simulations have been described. MUSIC, a package for muon transport through matter, can be used for propagating muons through large thickness of rock or water, for instance from the surface down to underground/underwater laboratory. It can also be implemented in the event generators for large underwater/under-ice neutrino telescopes or other neutrino detectors.

MUSUN uses the results of muon transport through rock/water to generate muons in or around underground laboratory taking into account their energy spectrum and angular distribution. Various tests showed good agreement of the codes' results with experimental data and other packages.

Since there are several versions of both codes the author finds impractical to submit all of them to the code library. Any specific version can be obtained by request to v.kudryavtsev@sheffield.ac.uk. There is a possibility to adapt the codes to specific needs of a user as was done on several occasions in the past.

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Table 1: Muon intensities and mean energies at various depths underground for standard rock and water calculated using the MUSIC code for muon transport and the LVD parameterisation for the muon spectrum at surface [3, 4]. Flat surface relief was assumed but the curvature of the Earth was taken into account. Column 1 – depth, X , in kilometres of water equivalent, km w. e.; column 2 – vertical muon intensity in standard rock, I_{μ}^{vert} ; column 3 – mean muon energy for the muon flux in standard rock at vertical, E_{μ}^{vert} ; column 4 – global intensity (integrated over solid angle for a spherical detector) in standard rock for flat surface, I_{μ} ; column 5 – mean muon energy for the global muon flux in standard rock, E_{μ} ; column 6 – vertical muon intensity in water (ice); column 7 – mean muon energy for the muon flux in water at vertical; column 8 – global intensity (integrated over solid angle for a spherical detector) in water (ice) for flat surface; column 9 – mean muon energy for the global muon flux in water.

X km w.e.	Standard rock				Water			
	I_{μ}^{vert} $\text{cm}^{-2}\mu\text{s}^{-1}\text{sr}^{-1}$	E_{μ}^{vert} GeV	I_{μ} $\text{cm}^{-2}\text{s}^{-1}$	E_{μ} GeV	I_{μ}^{vert} $\text{cm}^{-2}\mu\text{s}^{-1}\text{sr}^{-1}$	E_{μ}^{vert} GeV	I_{μ} $\text{cm}^{-2}\text{s}^{-1}$	E_{μ} GeV
0.5	1.06×10^{-5}	69	2.07×10^{-5}	93	7.86×10^{-6}	80	1.58×10^{-5}	111
1.0	1.47×10^{-6}	120	2.56×10^{-6}	150	1.14×10^{-6}	144	2.09×10^{-6}	186
2.0	1.38×10^{-7}	197	1.99×10^{-7}	225	1.22×10^{-7}	246	1.90×10^{-7}	291
3.0	2.56×10^{-8}	248	3.09×10^{-8}	271	2.61×10^{-8}	322	3.53×10^{-8}	362
4.0	6.16×10^{-9}	284	6.38×10^{-9}	301	7.40×10^{-9}	379	8.81×10^{-9}	412
5.0	1.70×10^{-9}	308	1.53×10^{-9}	319	2.44×10^{-9}	421	2.58×10^{-9}	447
6.0	5.06×10^{-10}	324	4.02×10^{-10}	332	8.78×10^{-10}	453	8.32×10^{-10}	474
7.0	1.58×10^{-10}	335	1.12×10^{-10}	341	3.35×10^{-10}	477	2.87×10^{-10}	492
8.0	5.07×10^{-11}	344	3.23×10^{-11}	347	1.33×10^{-10}	495	1.03×10^{-10}	506
9.0	1.67×10^{-11}	349	9.61×10^{-12}	351	5.41×10^{-11}	508	3.84×10^{-11}	516
10.0	5.55×10^{-12}	351	2.91×10^{-12}	353	2.24×10^{-11}	519	1.46×10^{-11}	524

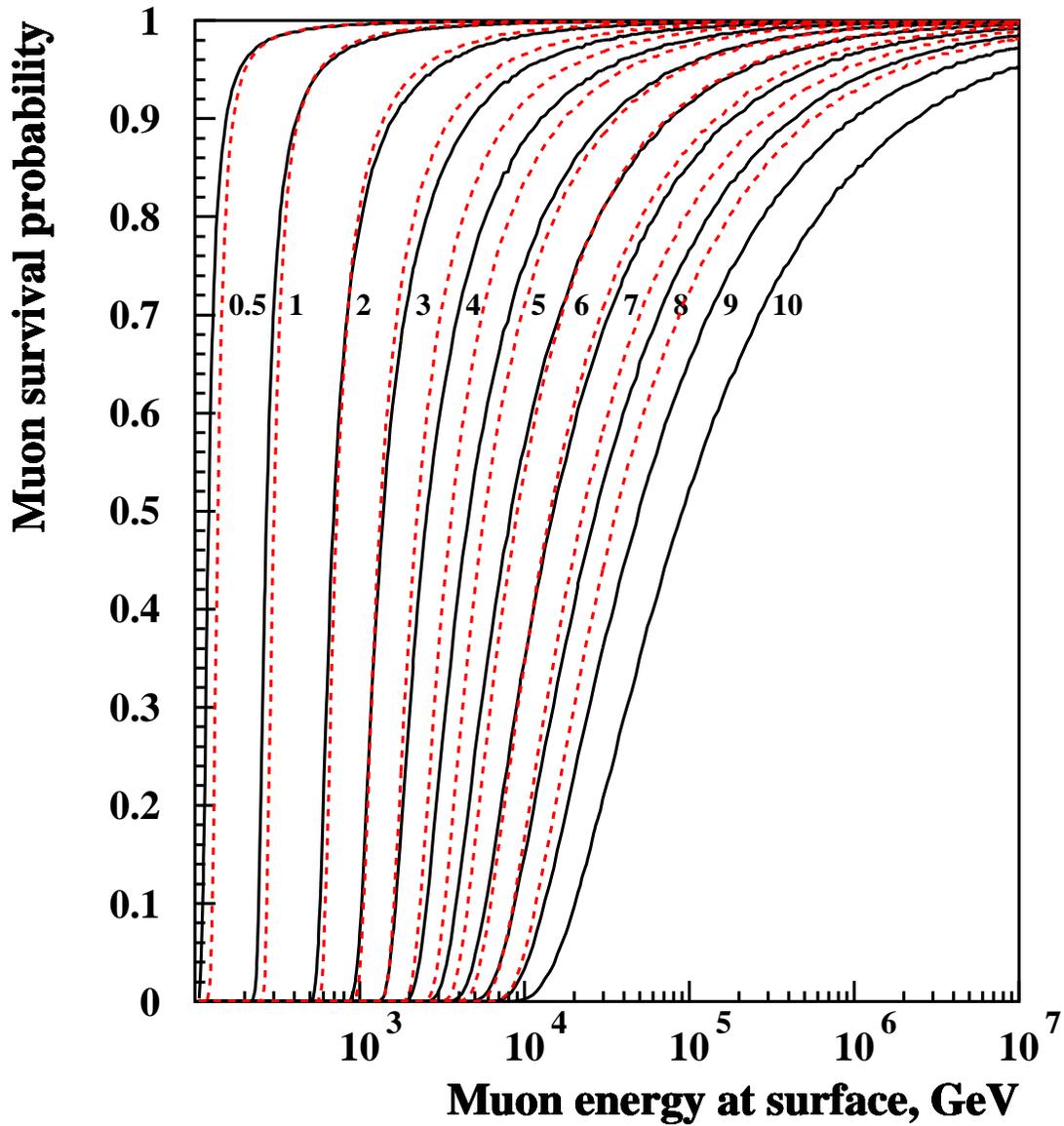


Figure 1: Survival probabilities as functions of muon energy at surface for different depths (from 0.5 to 10 km w. e.) in standard rock (black solid curves) and water (red dashed curves). Numbers to the right from each solid curve show the depths in km w. e. for standard rock. Survival probability curves for water are shifted to the right (for small depths) or to the left (for depths larger than 1 km w. e.) relative to those for standard rock.

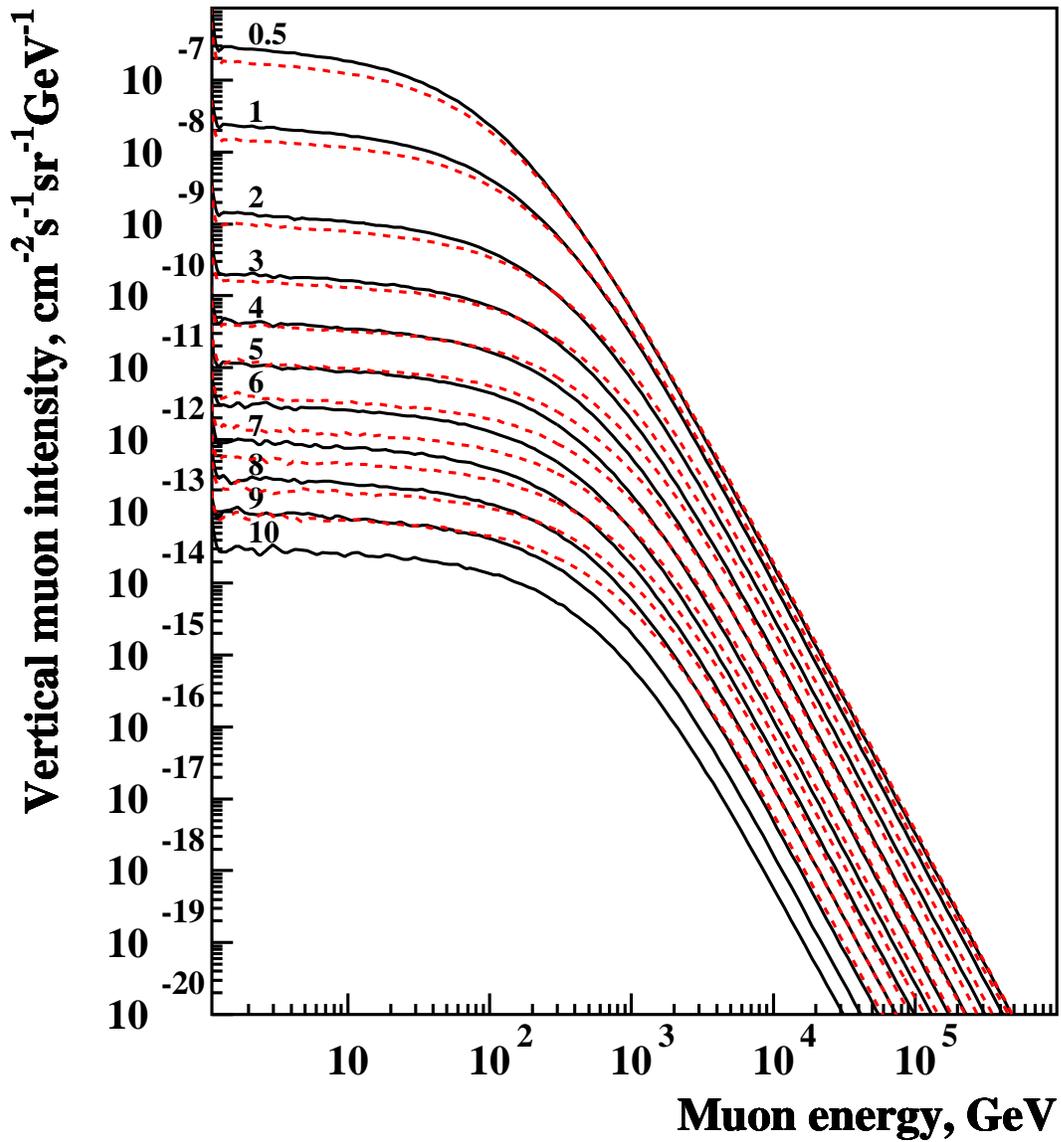


Figure 2: Muon energy spectra at vertical at different depths in standard rock (black solid curves) and water (red dashed curves). Numbers above the curves for standard rock show the depth in km w. e. Curves for water are shifted above or below corresponding curves for standard rock.

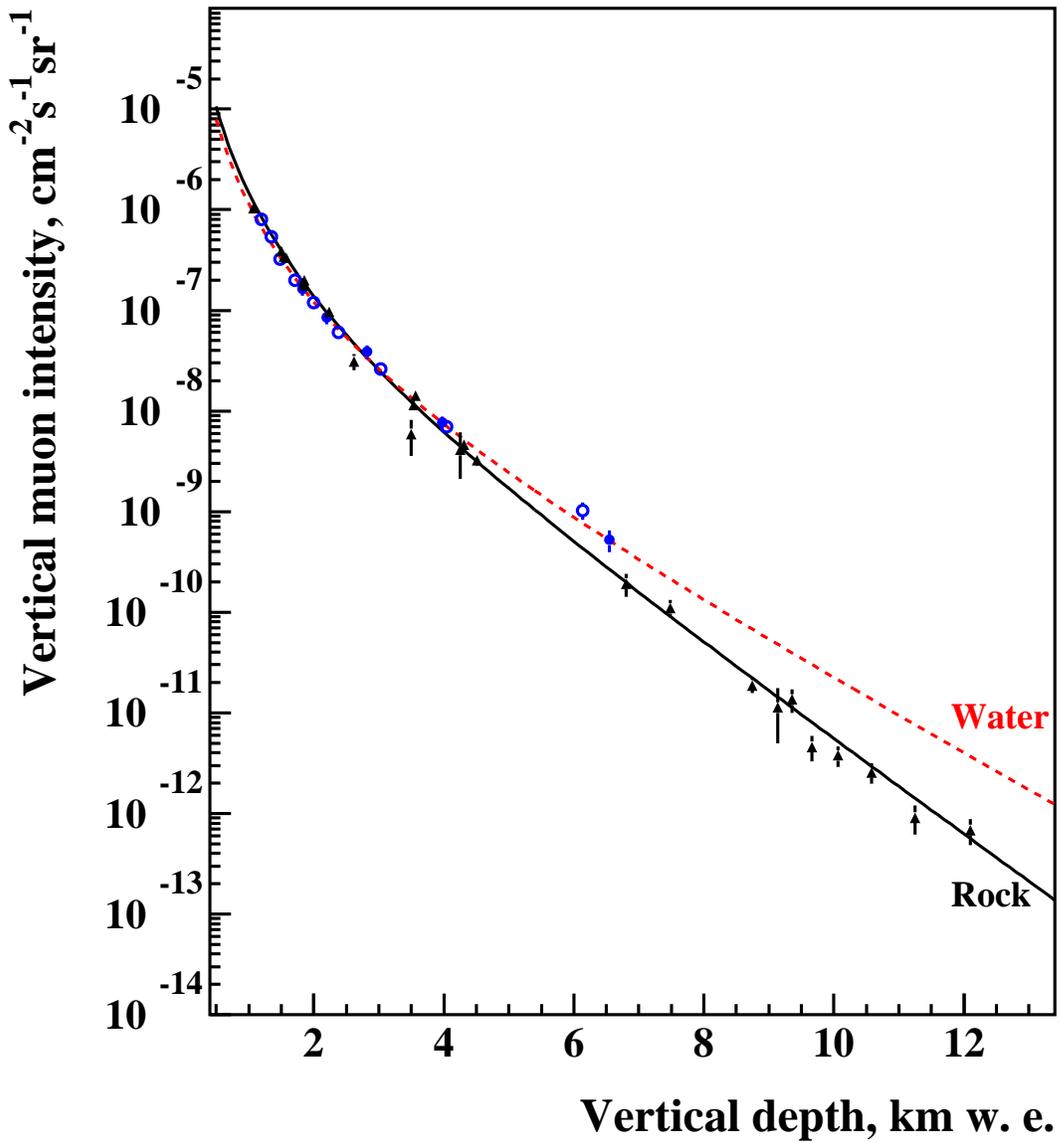


Figure 3: Depth – vertical muon intensity relation (muon intensity at vertical as a function of depth) for standard rock and water. The data points for standard rock (black triangles) are from the compilation of experimental results [37]. The data points for water are from the Baikal [6] (blue open circles) and AMANDA [7] (blue filled circles) experiments.

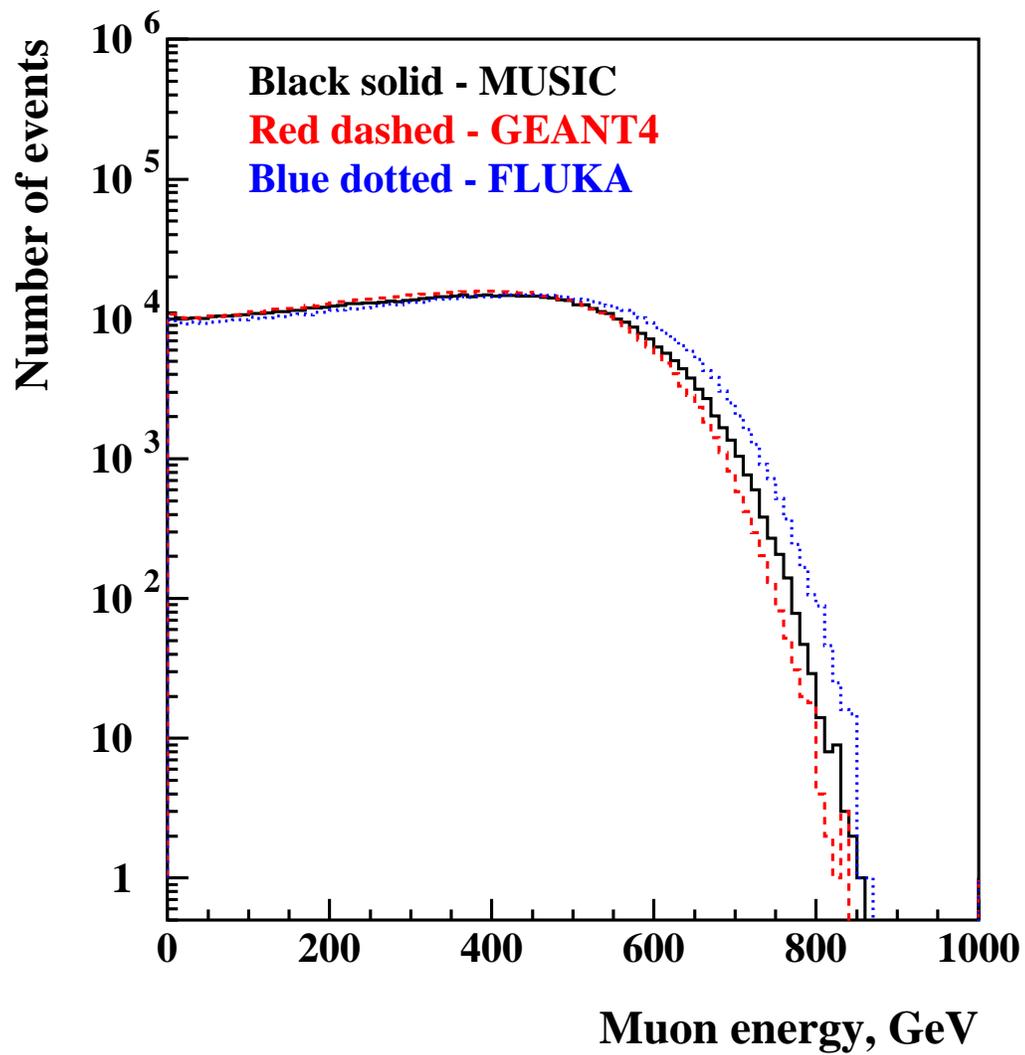


Figure 4: Energy distribution of muons with initial energy of 2 TeV transported through 3 km of water using MUSIC (black solid curve), GEANT4 (red dashed curve) and FLUKA (blue dotted curve).

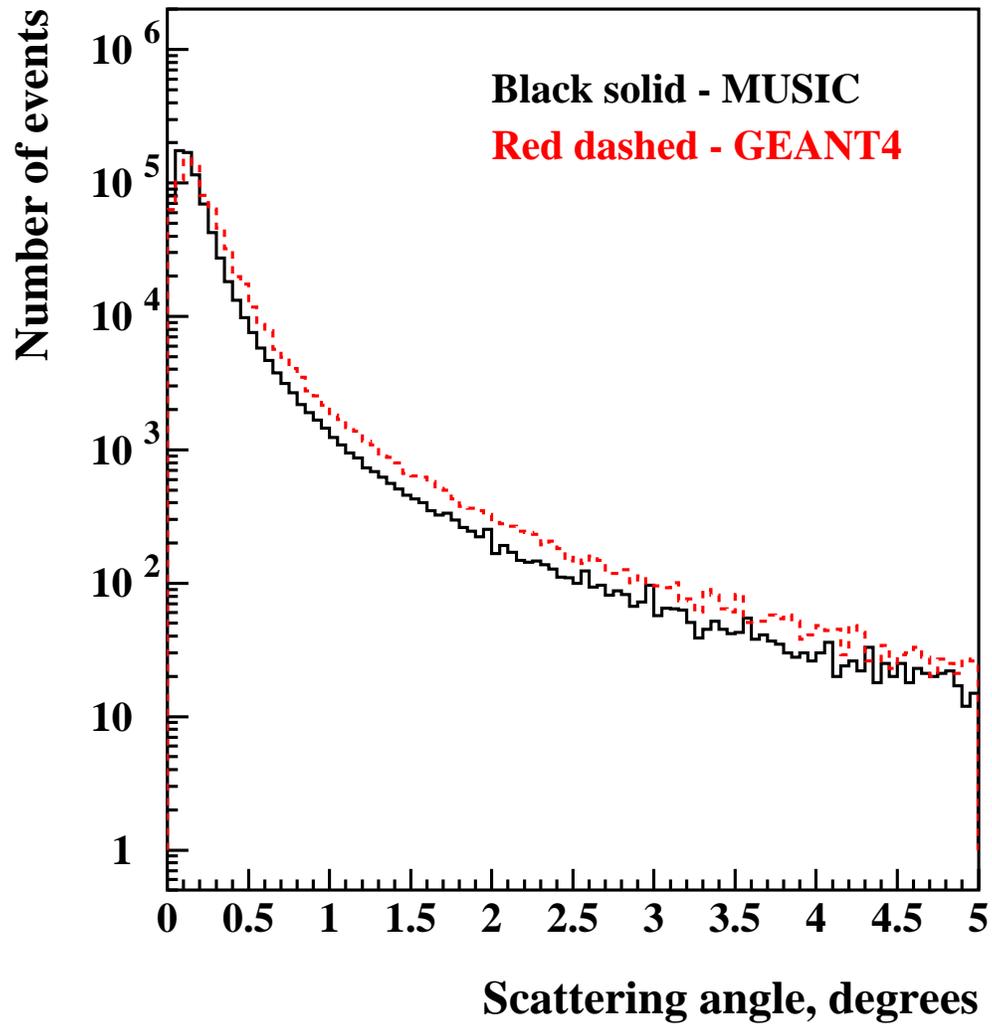


Figure 5: Distribution of angular deviation for muons with initial energy of 2 TeV transported through 3 km of water using MUSIC (black solid curve) and GEANT4 (red dashed curve).

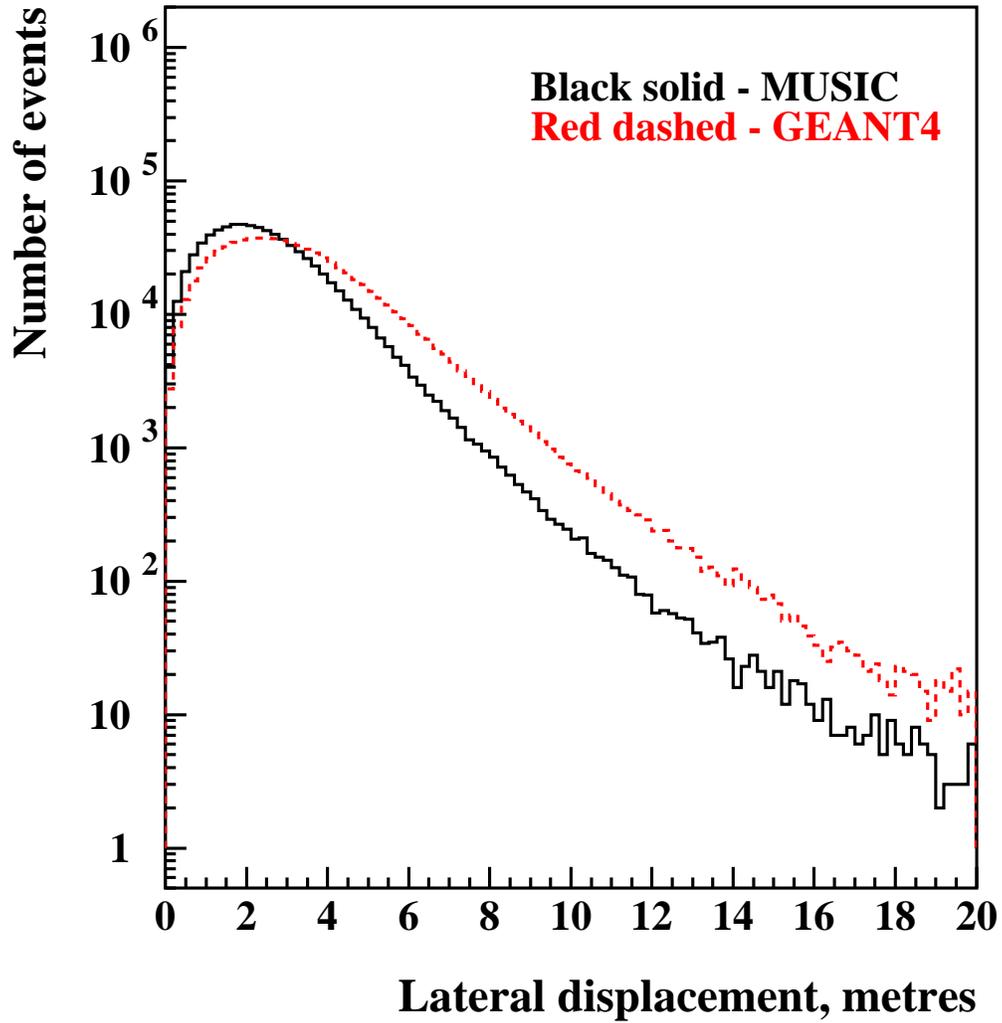


Figure 6: Distribution of lateral displacement for muons with initial energy of 2 TeV transported through 3 km of water using MUSIC (black solid curve) and GEANT4 (red dashed curve).

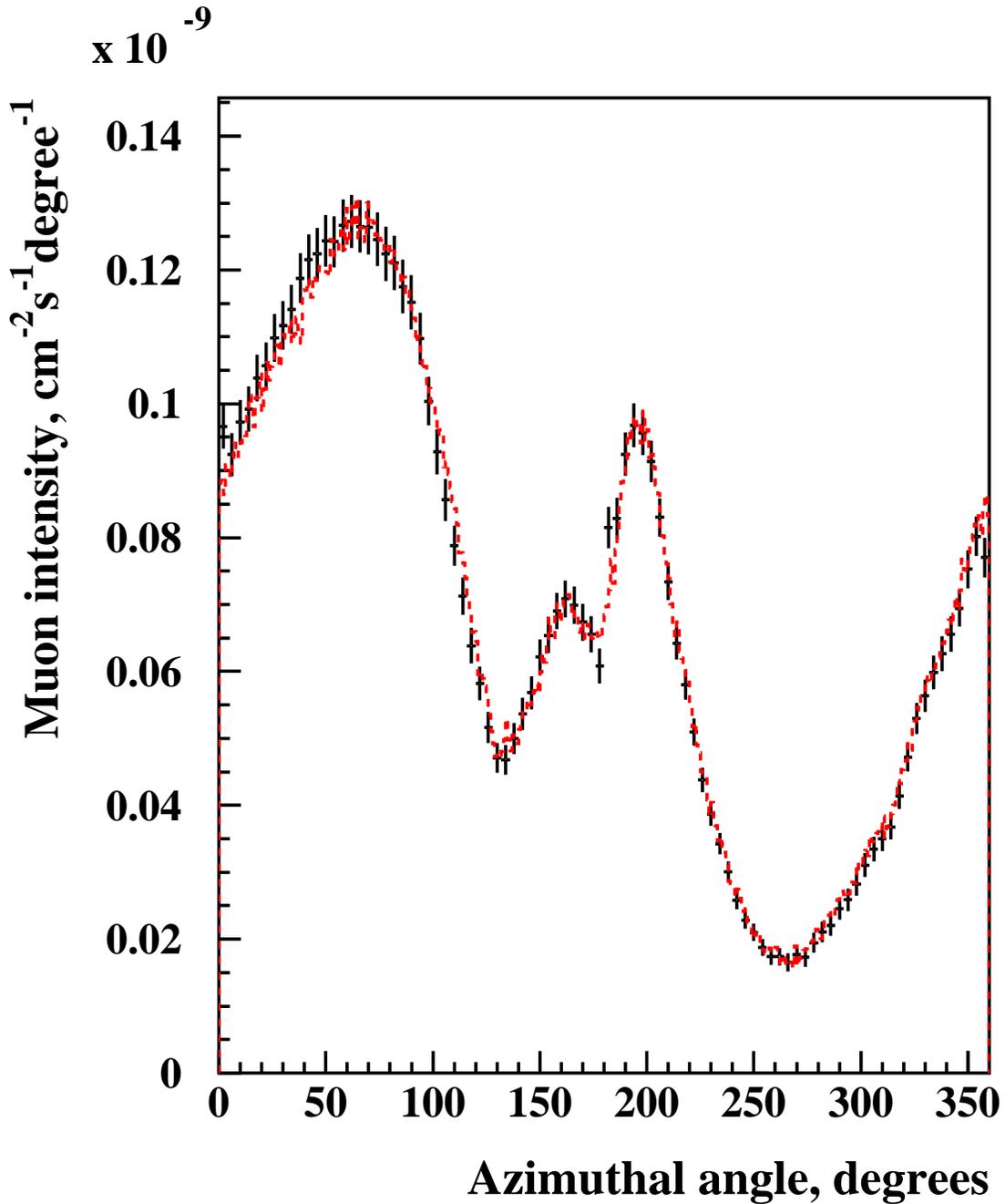


Figure 7: Azimuthal distribution of single muon intensities in the underground Gran Sasso Laboratory for zenith angles up to 60° as measured by LVD [4, 43] (data points with error bars) and generated with MUSUN (dashed curve). LVD acceptance as a function of zenith and azimuthal angles has been taken into account when generating muons. Azimuthal angle is calculated in the LVD reference system [3, 4, 43].

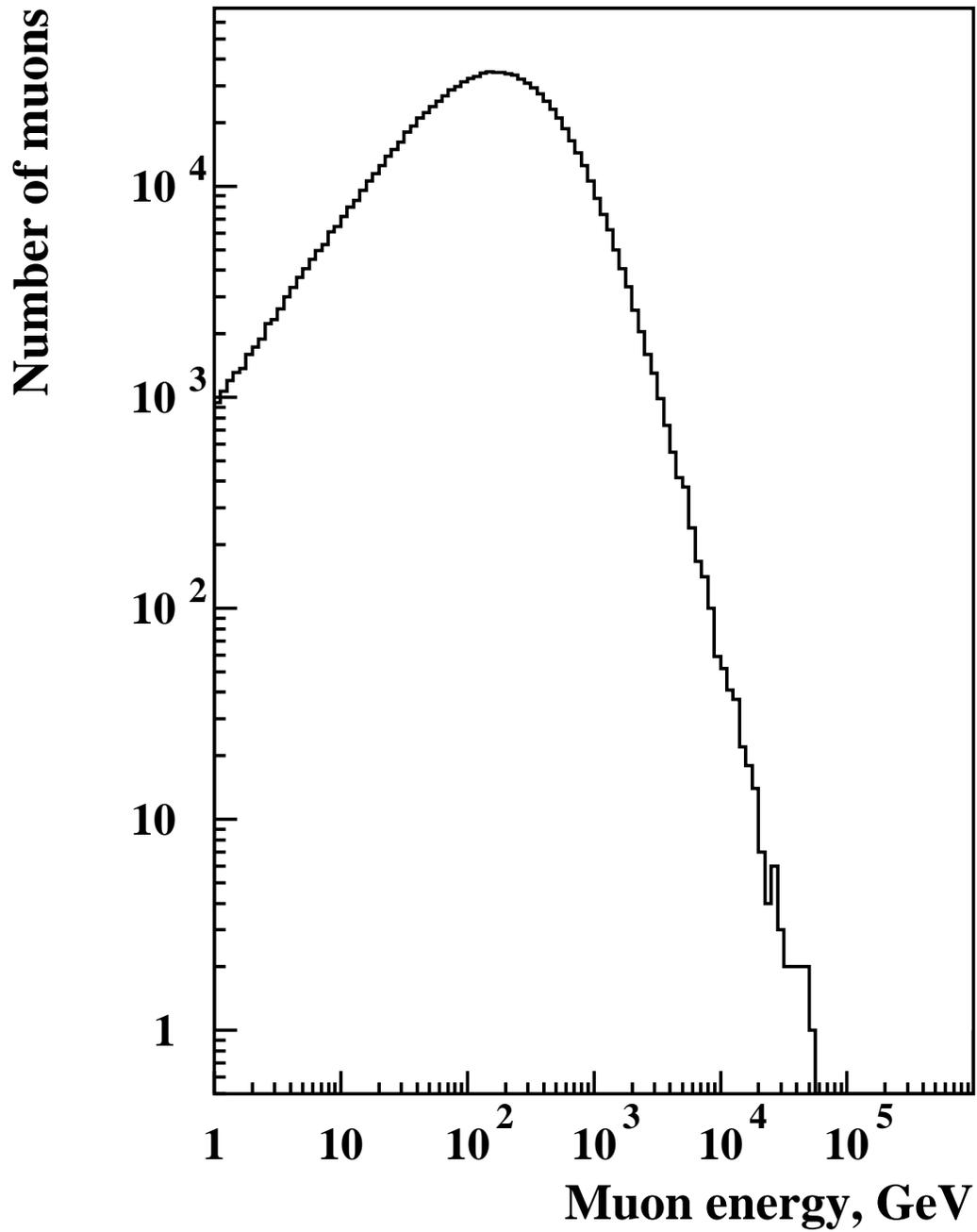


Figure 8: Energy spectrum of muons as generated by MUSUN for the underground Gran Sasso Laboratory.

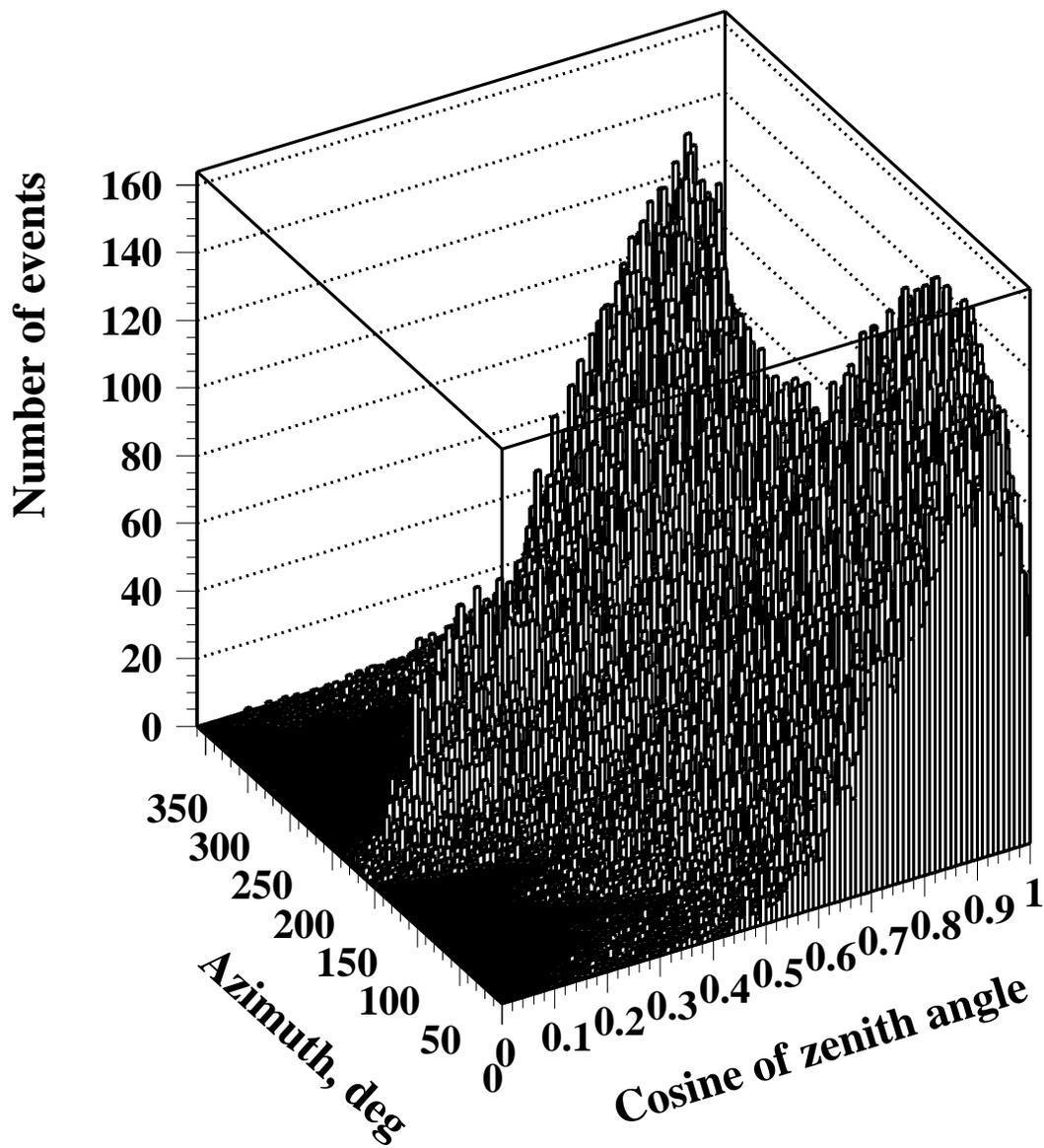


Figure 9: A lego plot of the number of generated muons (using MUSUN) as a function of zenith and azimuthal angles in the Hall A of the underground Gran Sasso Laboratory. Azimuthal angle is calculated in the LVD reference system [3, 4, 43].