# Full 1-loop calculation of $\operatorname{BR}\left(B_{s, d}^{0} \rightarrow \ell \bar{\ell}\right)$ in models beyond the MSSM with SARAH and SPheno 

H. Dreiner ${ }^{\text {a }}$, K. Nickel ${ }^{\text {a }}$, W. Porod ${ }^{\text {b }}$, F. Staub ${ }^{\text {a }}$<br>${ }^{a}$ Bethe Center for Theoretical Physics \& Physikalisches Institut der Universität Bonn, Nußallee 12, 53115 Bonn, Germany<br>${ }^{b}$ Institut für Theoretische Physik und Astrophysik, Universität Würzburg, 97074 Würzburg, Germany


#### Abstract

We present the possibility of calculating the quark flavor changing neutral current decays $B_{s}^{0} \rightarrow \ell \bar{\ell}$ and $B_{d}^{0} \rightarrow \ell \bar{\ell}$ for a large variety of supersymmetric models. For this purpose, the complete one-loop calculation has been implemented in a generic form in the Mathematica package SARAH. This information is used by SARAH to generate Fortran source code for SPheno for a numerical evaluation of these processes in a given model. We comment also on the possibility to use this setup for non-supersymmetric models.


## 1. Introduction

With the recent discovery of a bosonic resonance [1, 2] showing all the characteristics of the SM Higgs boson a long search might soon come to a successful end. In contrast there are no hints for a signal of supersymmetric (SUSY) particles or particles predicted by any other extension of the standard model (SM) [3-7]. Therefore, large areas of the parameter space of the simplest SUSY models are excluded. The allowed mass spectra as well as the best fit mass values to the data are pushed to higher and higher values [9]. This has lead to an increasing interest in the study of SUSY models which provide new features. For instance, models with broken $R$-parity [10, 11] or compressed spectra 12 might be able to hide much better at the LHC, while for other models high mass spectra are a much more natural feature than this is the case in the minimal-supersymmetric standard model (MSSM) 13.

However, bounds on the masses and couplings of beyond the SM (BSM) models follow not only from direct searches at colliders. New particles also have an impact on SM processes via virtual quantum corrections, leading in many instances to sizable deviations from the SM expectations. This holds in particular for the anomalous magnetic moment of the muon [14] and processes which are highly suppressed in the SM. The latter are mainly lepton flavor violating (LFV) or decays involving quark flavor changing neutral currents (qFCNC). While the prediction of LFV decays in the SM is many orders of magnitude below the experimental sensitivity [15], qFCNC is experimentally well established. For instance, the observed rate of $b \rightarrow s \gamma$ is in good agreement with the SM expectation and this observable has put for several years strong constraints on qFCNCs beyond the SM [16].

The experiments at the LHC have reached now a sensitivity to test also the SM prediction for $\mathrm{BR}\left(B_{s}^{0} \rightarrow\right.$ $\mu \bar{\mu})$ as well as $\operatorname{BR}\left(B_{d}^{0} \rightarrow \mu \bar{\mu}\right)$ 17]

$$
\begin{align*}
& \operatorname{BR}\left(B_{s}^{0} \rightarrow \mu \bar{\mu}\right)_{\mathrm{SM}}=(3.23 \pm 0.27) \cdot 10^{-9}  \tag{1}\\
& \operatorname{BR}\left(B_{d}^{0} \rightarrow \mu \bar{\mu}\right)_{\mathrm{SM}}=(1.07 \pm 0.10) \cdot 10^{-10} \tag{2}
\end{align*}
$$

[^0]Using the measured finite width difference of the $B$ mesons the time integrated branching ratio which should be compared to experiment is 18

$$
\begin{equation*}
\mathrm{BR}\left(B_{s}^{0} \rightarrow \mu \bar{\mu}\right)_{\text {theo }}=(3.56 \pm 0.18) \cdot 10^{-9} \tag{3}
\end{equation*}
$$

Recently, LHCb reported the first evidence for $B_{s}^{0} \rightarrow \mu \bar{\mu}$. The observed rate 19

$$
\begin{equation*}
\operatorname{BR}\left(B_{s}^{0} \rightarrow \mu \bar{\mu}\right)=\left(3.2_{-1.2}^{+1.5}\right) \times 10^{-9} \tag{4}
\end{equation*}
$$

fits nicely to the SM prediction. For $\operatorname{BR}\left(B_{d}^{0} \rightarrow \mu \bar{\mu}\right)$ the current upper bound: $9.4 \cdot 10^{-10}$ is already of the same order as the SM expectation.

This leads to new constraints for BSM models and each model has to be confronted with these measurements. So far, there exist several public tools which can calculate $\operatorname{BR}\left(B_{s, d}^{0} \rightarrow \ell \bar{\ell}\right)$ as well as other observables in the context of the MSSM or partially also for the next-to-minimal supersymmetric standard model (NMSSM) 20]: superiso [21, SUSY_Flavor [22], NMSSM-Tools 23], MicrOmegas 24] or SPheno 25]. However, for more complicated SUSY models none of the available tools provides the possibility to calculate these decays easily. This gap is now closed by the interplay of the Mathematica package SARAH [26] and the spectrum generator SPheno. SARAH already has many SUSY models incorporated but allows also an easy and efficient implementation of new models. For all of these models SARAH can generate new modules for SPheno for a comprehensive numerical evaluation. This functionality is extended, as described in this paper, by a full 1-loop calculation of $B_{s, d}^{0} \rightarrow \ell \bar{\ell}$.

The rest of the paper is organized as follows: in sec. 22 we recall briefly the analytical calculation for $\operatorname{BR}\left(B_{s, d}^{0} \rightarrow \ell \bar{\ell}\right)$. In sec. 3 we discuss the implementation of this calculation in SARAH and SPheno before we conclude in sec. 4. The appendix contains more information about the calculation and generic results for the amplitudes.

## 2. Calculation of $\operatorname{BR}\left(B_{s, d}^{0} \rightarrow \ell \bar{\ell}\right)$

In the SM this decay was first calculated in ref [27], in the analogous context of kaons. The higher order corrections were first presented in [28]; see also [29]. In the context of supersymmetry this was considered in [30. See also the interesting correlation between $\operatorname{BR}\left(B_{s}^{0} \rightarrow \mu \bar{\mu}\right)$ and $(g-2)_{\mu}$ [31].

We present briefly the main steps of the calculation of $\operatorname{BR}\left(B_{q}^{0} \rightarrow \ell_{k} \bar{\ell}_{l}\right)$ with $q=s, d$. We follow closely the notation of ref. [32]. The effective Hamiltonian can be parametrized by

$$
\begin{equation*}
\mathcal{H}=\frac{1}{16 \pi^{2}} \sum_{X, Y=L, R}\left(C_{S X Y} \mathcal{O}_{S X Y}+C_{V X Y} \mathcal{O}_{V X Y}+C_{T X} \mathcal{O}_{T X}\right) \tag{5}
\end{equation*}
$$

with the Wilson coefficients $C_{S X Y}, C_{V X Y}, C_{T X}$ corresponding to the scalar, vector and tensor operators

$$
\begin{equation*}
\mathcal{O}_{S X Y}=\left(\bar{q}_{j} P_{X} q_{i}\right)\left(\bar{\ell}_{l} P_{Y} \ell_{k}\right), \quad \mathcal{O}_{V X Y}=\left(\bar{q}_{j} \gamma^{\mu} P_{X} q_{i}\right)\left(\bar{\ell}_{l} \gamma_{\mu} P_{Y} \ell_{k}\right), \quad \mathcal{O}_{T X}=\left(\bar{q}_{j} \sigma^{\mu \nu} P_{X} q_{i}\right)\left(\bar{\ell}_{l} \sigma_{\mu \nu} \ell_{k}\right) . \tag{6}
\end{equation*}
$$

$P_{L}$ and $P_{R}$ are the projection operators on left respectively right handed states. The expectation value of the axial vector matrix element is defined as

$$
\begin{equation*}
\langle 0| \bar{b} \gamma^{\mu} \gamma^{5} q\left|B_{q}^{0}(p)\right\rangle \equiv i p^{\mu} f_{B_{q}^{0}} . \tag{7}
\end{equation*}
$$

Here, we introduced the meson decay constants $f_{B_{q}^{0}}$ which can be obtained from lattice QCD simulations 33. The current values for $B_{s}^{0}$ and $B_{d}^{0}$ are given by [34]

$$
\begin{equation*}
f_{B_{s}^{0}}=(227 \pm 8) \mathrm{MeV}, \quad f_{B_{d}^{0}}=(190 \pm 8) \mathrm{MeV} . \tag{8}
\end{equation*}
$$

Since the momentum $p$ of the meson is the only four-vector available, the matrix element in eq. (7) can only depend on $p^{\mu}$. The incoming momenta of the $b$ antiquark and the $s$ (or $d$ ) quark are $p_{1}, p_{2}$ respectively, where
$p=p_{1}+p_{2}$. Contracting eq. (7) with $p_{\mu}$ and using the equations of motion $\bar{b} \not p_{1}=-\bar{b} m_{b}$ and $\not p_{2} q=m_{q} q$ leads to an expression for the pseudoscalar current

$$
\begin{equation*}
\langle 0| \bar{b} \gamma^{5} q\left|B_{q}^{0}(p)\right\rangle=-i \frac{M_{B_{q}^{0}}^{2} f_{B_{q}^{0}}}{m_{b}+m_{q}} . \tag{9}
\end{equation*}
$$

The vector and scalar currents vanish

$$
\begin{equation*}
\langle 0| \bar{b} \gamma^{\mu} q\left|B_{q}^{0}(p)\right\rangle=\langle 0| \bar{b} q\left|B_{q}^{0}(p)\right\rangle=0 \tag{10}
\end{equation*}
$$

From eqs. (7) and (9) we obtain

$$
\begin{equation*}
\langle 0| \bar{b} \gamma^{\mu} P_{L / R} q\left|B_{q}^{0}(p)\right\rangle=\mp \frac{i}{2} p^{\mu} f_{B_{q}^{0}}, \quad\langle 0| \bar{b} P_{L / R} q\left|B_{q}^{0}(p)\right\rangle= \pm \frac{i}{2} \frac{M_{B_{q}^{0}}^{2} f_{B_{q}^{0}}}{m_{b}+m_{q}} \tag{11}
\end{equation*}
$$

In general, the matrix element $\mathcal{M}$ is a function of the form factors $F_{S}, F_{P}, F_{V}, F_{A}$ of the scalar, pseudoscalar, vector and axial-vector current and can be expressed by

$$
\begin{equation*}
(4 \pi)^{2} \mathcal{M}=F_{S} \bar{\ell} \ell+F_{P} \bar{\ell} \gamma^{5} \ell+F_{V} p_{\mu} \bar{\ell} \gamma^{\mu} \ell+F_{A} p_{\mu} \bar{\ell} \gamma^{\mu} \gamma^{5} \ell . \tag{12}
\end{equation*}
$$

Note that there is no way of building an antisymmetric 2-tensor out of just one vector $p^{\mu}$. The matrix element of the tensor operator $\mathcal{O}_{T X}$ must therefore vanish. The form factors can be expressed by linear combinations of the Wilson coefficients of eq. (5)

$$
\begin{align*}
F_{S} & =\frac{i}{4} \frac{M_{B_{q}^{0}}^{2} f_{B_{q}^{0}}}{m_{b}+m_{q}}\left(C_{S L L}+C_{S L R}-C_{S R R}-C_{S R L}\right)  \tag{13}\\
F_{P} & =\frac{i}{4} \frac{M_{B_{q}^{0}}^{2} f_{B_{q}^{0}}}{m_{b}+m_{q}}\left(-C_{S L L}+C_{S L R}-C_{S R R}+C_{S R L}\right)  \tag{14}\\
F_{V} & =-\frac{i}{4} f_{B_{q}^{0}}\left(C_{V L L}+C_{V L R}-C_{V R R}-C_{V R L}\right)  \tag{15}\\
F_{A} & =-\frac{i}{4} f_{B_{q}^{0}}\left(-C_{V L L}+C_{V L R}-C_{V R R}+C_{V R L}\right) \tag{16}
\end{align*}
$$

The main task is to calculate the different Wilson coefficients for a given model. These Wilson coefficients receive at the 1-loop level contributions from various wave, penguin and box diagrams, see Figures B. B. 10 in Appendix B. Furthermore, in some models these decays could also happen already at tree-level 35]. The amplitudes for all possible, generic diagrams which can contribute to the Wilson coefficients have been calculated with FeynArts/FormCalc [36] and the results are listed in Appendix B. This calculation has been performed in the $\overline{\mathrm{DR}}$ scheme and 't Hooft gauge. How these results are used together with SARAH and SPheno to get numerical results will be discussed in the next section.

After the calculation of the form factors, the squared amplitude is

$$
\begin{align*}
(4 \pi)^{4}|\mathcal{M}|^{2} & =2\left|F_{S}\right|^{2}\left(M_{B_{q}^{0}}^{2}-\left(m_{\ell}+m_{k}\right)^{2}\right)+2\left|F_{P}\right|^{2}\left(M_{B_{q}^{0}}^{2}-\left(m_{\ell}-m_{k}\right)^{2}\right)  \tag{17}\\
& +2\left|F_{V}\right|^{2}\left(M_{B_{q}^{0}}^{2}\left(m_{k}-m_{\ell}\right)^{2}-\left(m_{k}^{2}-m_{\ell}^{2}\right)^{2}\right) \\
& +2\left|F_{A}\right|^{2}\left(M_{B_{q}^{0}}^{2}\left(m_{k}+m_{\ell}\right)^{2}-\left(m_{k}^{2}-m_{\ell}^{2}\right)^{2}\right) \\
& +4 \Re\left(F_{s} F_{V}^{*}\right)\left(m_{\ell}-m_{k}\right)\left(M_{B_{q}^{0}}^{2}+\left(m_{k}+m_{\ell}\right)^{2}\right) \\
& +4 \Re\left(F_{P} F_{A}^{*}\right)\left(m_{\ell}+m_{k}\right)\left(M_{B_{q}^{0}}^{2}-\left(m_{k}-m_{\ell}\right)^{2}\right) .
\end{align*}
$$

Here, $m_{\ell}$ and $m_{k}$ are the lepton masses. In the case $k=\ell$, this expression simplifies to

$$
\begin{equation*}
|\mathcal{M}|^{2}=\frac{2}{\left(16 \pi^{2}\right)^{2}}\left(\left(M_{B_{q}^{0}}^{2}-4 m_{\ell}^{2}\right)\left|F_{S}\right|^{2}+M_{B_{q}^{0}}^{2}\left|F_{P}+2 m_{\ell} F_{A}\right|^{2}\right) . \tag{18}
\end{equation*}
$$

| default SM input parameters |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
| $\alpha_{e m}^{-1}\left(M_{Z}\right)=127.93$ | $\alpha_{s}\left(M_{Z}\right)=0.1190$ | $G_{F}=1.16639 \cdot 10^{-5} \mathrm{GeV}^{-2}$ | $\rho=0.135$ | $\eta=0.349$ |  |
| $m_{t}^{\text {pole }}=172.90 \mathrm{GeV}$ | $M_{Z}^{\text {pole }}=91.1876 \mathrm{GeV}$ | $m_{b}\left(m_{b}\right)=4.2 \mathrm{GeV}$ | $\lambda=0.2257$ | $A=0.814$ |  |
| derived parameters |  |  |  |  |  |
| $\mid$ |  |  |  |  |  |
| $m_{t}^{\overline{D R}}=166.4 \mathrm{GeV}$ | $\left\|V_{t b}^{*} V_{t s}\right\|=4.06 * 10^{-2}$ | $\left\|V_{t b}^{*} V_{t d}\right\|=8.12 * 10^{-3}$ | $m_{W}=80.3893$ | $\sin ^{2} \Theta_{W}=0.2228$ |  |

Table 1: SM input values and derived parameters by default used for the numerical evaluation of $B_{s, d}^{0} \rightarrow l \bar{l}$ in SPheno.

Note, the result is independent of the form factor $F_{V}$ in this limit. In the SM the leading 1-loop contributions proceed via the exchange of virtual gauge bosons. They are thus helicity suppressed. Furthermore, since these are flavor changing neutral currents, they are GIM suppressed. The diagrams involving virtual Higgs bosons are suppressed due to small Yukawa couplings. In BSM scenarios these suppressions can be absent.

The branching ratio is then given by

$$
\begin{equation*}
\operatorname{BR}\left(B_{q}^{0} \rightarrow \ell_{k} \bar{\ell}_{l}\right)=\frac{\tau_{B_{q}^{0}}}{16 \pi} \frac{|\mathcal{M}|^{2}}{M_{B_{q}^{0}}} \sqrt{1-\left(\frac{m_{k}+m_{l}}{M_{B_{q}^{0}}}\right)^{2}} \sqrt{1-\left(\frac{m_{k}-m_{l}}{M_{B_{q}^{0}}}\right)^{2}} \tag{19}
\end{equation*}
$$

with $\tau_{B_{q}^{0}}$ as the life time of the mesons.

## 3. Automatized calculation of $B_{s, d}^{0} \rightarrow \ell \bar{\ell}$

### 3.1. Implementation in SARAH and SPheno

SARAH is the first 'spectrum-generator-generator' on the market which means that it can generate Fortran source for SPheno to obtain a full-fledged spectrum generator for models beyond the MSSM. The main features of a SPheno module written by SARAH are a precise mass spectrum calculation based on 2-loop renormalization group equations (RGEs) and a full 1-loop calculation of the mass spectrum. Two-loop results known for the MSSM can be included. Furthermore, also the decays of SUSY and Higgs particles are calculated as well as observables like $\ell_{i} \rightarrow \ell_{j} \gamma, \ell_{i} \rightarrow 3 \ell_{j}, b \rightarrow s \gamma, \delta \rho,(g-2)$, or electric dipole moments. For more information about the interplay between SARAH and SPheno we refer the interested reader to Ref. 37.

Here we extend the list of observables by $\operatorname{BR}\left(B_{s}^{0} \rightarrow \ell \bar{\ell}\right)$ and $\operatorname{BR}\left(B_{d}^{0} \rightarrow \ell \bar{\ell}\right)$. For this purpose, the generic tree-level and 1-loop amplitudes calculated with FeynArts/FormCalc given in Appendix B have been implemented in SARAH. When SARAH generates the output for SPheno it checks for all possible field combinations which can populate the generic diagrams in the given model. This information is then used to generate Fortran code for a numerical evaluation of all of these diagrams. The amplitudes are then combined to the Wilson coefficients which again are used to calculate the form factors eqs. 133 - 16 . The branching ratio is finally calculated by using eq. (19). Note, the known 2 -loop QCD corrections of Refs. [28, 29, 41, are not included in this calculation.

The Wilson coefficients for $B_{s, d}^{0} \rightarrow \ell \bar{\ell}$ are calculated at a scale $Q=160 \mathrm{GeV}$ by all modules generated by SARAH, as this is done by default by SPheno in the MSSM. Hence, as input parameters for the calculation the running SUSY masses and couplings at this scale obtained by a 2-loop RGE evaluation down from the SUSY scale are used. In the standard model gauge sector we use the running value of $\alpha_{e m}$, the on-shell Weinberg angle $\sin \Theta_{W}^{2}=1-\frac{m_{W}^{2}}{m_{Z}^{2}}$ with $m_{W}$ calculated from $\alpha_{e m}\left(M_{Z}\right), G_{F}$ and the $Z$ mass. In addition, the CKM matrix calculated from the Wolfenstein parameters $(\lambda, A, \rho, \eta)$ as well as the running quark masses enter the calculation. To obtain the running SM parameters at $Q=160 \mathrm{GeV}$ we use 2-loop standard model RGEs of Ref. 42]. The default SM values as well as the derived parameters are given in Tab. 1. Note, even if CP violation is not switched on the calculation of the SUSY spectrum, always the phase of the CKM matrix is taken into account in these calculations. This is especially important for $B_{d}^{0}$ decays. All standard model parameters can be adjusted by using the corresponding standard blocks of the SUSY LesHouches Accord 2

| Default hadronic parameters |  |  |  |
| :--- | :--- | :--- | :---: |
| $m_{B_{s}^{0}}=5.36677 \mathrm{GeV}$ | $f_{B_{s}^{0}}=227(8) \mathrm{MeV}$ | $\tau_{B_{s}^{0}}=1.466(31) \mathrm{ps}$ |  |
| $m_{B_{d}^{0}}=5.27958 \mathrm{GeV}$ | $f_{B_{d}^{0}}=190(8) \mathrm{MeV}$ | $\tau_{B_{d}^{0}}=1.519(7) \mathrm{ps}$ |  |

Table 2: Hadronic input values by default used for the numerical evaluation of $B_{s, d}^{0} \rightarrow l \bar{l}$ in SPheno.
(SLHA2) [39. Furthermore, the default input values for the hadronic parameters given in Table 2 are used. These can be changed in the Les Houches input accordingly to the Flavor Les Houches Accord (FLHA) [38] using the following blocks:


While SPheno includes the chiral resummation for the MSSM, this is not taken into account in the routines generated by SARAH because of its large model dependence.

### 3.2. Generating and running the source code

We describe briefly the main steps necessary to generate and run the SPheno code for a given model: after starting Mathematica and loading SARAH it is just necessary to evaluate the demanded model and call the function to generate the SPheno source code. For instance, to get a SPheno module for the B-L-SSM 433 45], use
<< [\$SARAH-Directory]/SARAH.m;
Start["BLSSM"];
MakeSPheno [];
MakeSPheno [] calculates first all necessary information (i.e. vertices, mass matrices, tadpole equations, RGEs, self-energies) and then exports this information to Fortran code and writes all necessary auxiliary functions needed to compile the code together with SPheno. The entire output is saved in the directory

## [\$SARAH-Directory]/Output/BLSSM/EWSB/SPheno/

The content of this directory has to be copied into a new subdirectory of SPheno called BLSSM and afterwards the code can be compiled:

```
cp [$SARAH-Directory]/Output/BLSSM/EWSB/SPheno/* [$SPheno-Directory]/BLSSM/
cd [$SPheno-Directory]
make Model=BLSSM
```

This creates a new binary SPhenoBLSSM in the directory bin of SPheno. To run the spectrum calculation a file called LesHouches.in.BLSSM containing all input parameters in the Les Houches format has to be provided. SARAH writes also a template for such a file which has been copied with the other files to /BLSSM. This example can be evaluated via
./bin/SPhenoBLSSM BLSSM/LesHouches.in.BLSSM
and the output is written to SPheno.spc.BLSSM. This file contains all information like the masses, mass matrices, decay widths and branching ratios, and observables. For the $B_{s, d}^{0} \rightarrow \ell \bar{\ell}$ decays the results are given twice for easier comparison: once for the full calculation and once including only the SM contributions. All results are written to the block SPhenoLowEnergy in the spectrum file using the following numbers:

| 4110 | $\mathrm{BR}^{S M}\left(B_{d}^{0} \rightarrow e^{+} e^{-}\right)$ | 4111 | $\mathrm{BR}^{\text {full }}\left(B_{d}^{0} \rightarrow e^{+} e^{-}\right)$ |
| :---: | :---: | :---: | :--- |
| 4220 | $\mathrm{BR}^{S M}\left(B_{d}^{0} \rightarrow \mu^{+} \mu^{-}\right)$ | 4221 | $\mathrm{BR}^{\text {full }}\left(B_{d}^{0} \rightarrow \mu^{+} \mu^{-}\right)$ |
| 4330 | $\mathrm{BR}^{S M}\left(B_{d}^{0} \rightarrow \tau^{+} \tau^{-}\right)$ | 4331 | $\mathrm{BR}^{\text {full }}\left(B_{d}^{0} \rightarrow \tau^{+} \tau^{-}\right)$ |
| 5110 | $\mathrm{BR}^{S M}\left(B_{s}^{0} \rightarrow e^{+} e^{-}\right)$ | 5111 | $\mathrm{BR}^{\text {full }}\left(B_{s}^{0} \rightarrow e^{+} e^{-}\right)$ |
| 5210 | $\mathrm{BR}^{S M}\left(B_{s}^{0} \rightarrow \mu^{+} e^{-}\right)$ | 5211 | $\mathrm{BR}^{\text {full }}\left(B_{s}^{0} \rightarrow \mu^{+} e^{-}\right)$ |
| 5220 | $\mathrm{BR}^{S M}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)$ | 5221 | $\mathrm{BR}^{\text {full }}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)$ |
| 5330 | $\mathrm{BR}^{S M}\left(B_{s}^{0} \rightarrow \tau^{+} \tau^{-}\right)$ | 5331 | $\mathrm{BR}^{\text {full }}\left(B_{s}^{0} \rightarrow \tau^{+} \tau^{-}\right)$ |

Note, we kept for completeness and as cross-check $\mathrm{BR}^{S M}\left(B_{s}^{0} \rightarrow \mu^{+} e^{-}\right)$which has to vanish. The same steps can be repeated for any other model implemented in SARAH, or the SUSY-Toolbox scripts 37 can be used for an automatic implementation of new models in SPheno as well as in other tools based on the SARAH output.

### 3.3. Checks

We have performed several cross checks of the code generated by SARAH: the first, trivial check has been that we reproduce the known SM results and that those agree with the full calculation in the limit of heavy SUSY spectra. For the input parameters of Tab. 1 we obtain $\operatorname{BR}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)^{S M}=3.28 \cdot 10^{-9}$ and $\operatorname{BR}\left(B_{d}^{0} \rightarrow \mu^{+} \mu^{-}\right)^{S M}=1.08 \cdot 10^{-10}$ which are in good agreement with eqs. (1)-(2). Secondly, as mentioned in the introduction there are several codes which calculate these decays for the MSSM or NMSSM. A detailed comparison of all of these codes is beyond the scope of the presentation here and will be presented elsewhere [40]. However, a few comments are in order: the code generated by SARAH as well as most other codes usually show the same behavior. There are differences in the numerical values calculated by the programs because of different values for the SM inputs. For instance, there is an especially strong dependence on the value of the electroweak mixing angle and, of course, of the hadronic parameters used in the calculation [17. In addition, these processes are implemented with different accuracy in different tools: the treatment of NLO QCD corrections [41, chiral resummation [46, or SUSY box diagrams is not the same. Therefore, we depict in Fig. 1 a comparison between SPheno 3.2.1, Superiso 3.3 and SPheno by SARAH using the results normalized to the SM limit of each program.

It is also possible to perform a check of self-consistency: the leading-order contribution has to be finite which leads to non-trivial relations among the amplitudes for all wave and penguin diagrams are given in Appendix B. 2 and Appendix B. 3 Therefore, we can check these relations numerically by varying the renormalization scale used in all loop integrals. The dependence on this scale should cancel and the branching ratios should stay constant. This is shown in Figure 2, while single contributions can change by several orders the sum of all is numerically very stable.

### 3.4. Non-supersymmetric models

We have focused our discussion so far on SUSY models. However, even if SARAH is optimized for the study of SUSY models it is also able to handle non-SUSY models to some extent. The main drawback at the moment for non-SUSY models is that the RGEs can not be calculated because the results of Refs. 47, 48, which are used by SARAH are not valid in this case. However, all other calculations like the ones for the vertices, mass matrices and self-energies don't use SUSY properties and therefore apply to any model. Hence, it is also possible to generate SPheno code for these models which calculates $B_{s, d}^{0} \rightarrow \ell \bar{\ell}$. The main difference in the calculation comes from the missing possibility to calculate the RGEs: the user has to provide numerical values for all parameters at the considered scale which then enter the calculation. We note that in order to fully support non-supersymmetric models with SARAH the calculation of the corresponding RGEs at 2-loop level will be included in SARAH in the future 49].


Figure 1: The top figure: $R=\operatorname{BR}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right) / \mathrm{BR}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)_{S M}$ for the constrained MSSM and as function of $m_{0}$. The other parameters were set to $M_{1 / 2}=140 \mathrm{GeV}$, $\tan \beta=10, \mu>0$. In the middle $m_{0}$ and $M_{1 / 2}$ were varied simultaneously, while $\tan \beta=30$ was fixed. In the bottom figure we show $\log (R)$ as a function of $\tan \beta$, while $m_{0}=M_{1 / 2}=150 \mathrm{GeV}$ were kept fix. In all figures $A_{0}=0$ and $\mu>0$ was used. The color code is as follows: superiso 3.3 (dotted black), SPheno 3.2.1 (dashed red) and SPheno by SARAH (solid blue).


Figure 2: The figure shows $\left|\sum F_{A}\right|_{\text {penguin }}$ and $\left|\sum F_{A}\right|_{\text {wave }}$ as well as the sum of both $\left|\sum F_{A}\right|$. Penguin and wave contributions have opposite signs that interchange between $Q=10^{2} \mathrm{GeV}$ to $Q=10^{3} \mathrm{GeV}$.

## 4. Conclusion

We have presented a model independent implementation of the flavor violating decays $B_{s, d}^{0} \rightarrow \ell \bar{\ell}$ in SARAH and SPheno. Our approach provides the possibility to generate source code which performs a full 1-loop calculation of these observables for any model which can be implemented in SARAH. Therefore, it takes care of the necessity to confront many BSM models in the future with the increasing constraints coming from the measurements of $B_{s, d}^{0} \rightarrow \ell \bar{\ell}$ at the LHC.

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## Appendix A. Conventions

## Appendix A.1. Passarino-Veltman integrals

We use in the following the conventions of [50] for the Passarino-Veltman integrals. All Wilson coefficients appearing in the following can be expressed by the integrals

$$
\begin{align*}
B_{0}(0, x, y) & =\Delta+1+\ln \left(\frac{Q^{2}}{y}\right)+\frac{x}{x-y} \ln \left(\frac{y}{x}\right)  \tag{A.1}\\
\Delta & =\frac{2}{4-D}-\gamma_{E}+\log 4 \pi  \tag{A.2}\\
B_{1}(x, y, z) & =\frac{1}{2}(z-y) \frac{B_{0}(x, y, z)-B_{0}(0, y, z)}{x}-\frac{1}{2} B_{0}(x, y, z)  \tag{A.3}\\
C_{0}(x, y, z) & =\frac{1}{y-z}\left[\frac{y}{x-y} \log \frac{y}{x}-\frac{z}{x-z} \log \frac{z}{x}\right]  \tag{A.4}\\
C_{00}(x, y, z) & =\frac{1}{4}\left(1-\frac{1}{y-z}\left(\frac{x^{2} \log x-y^{2} \log y}{x-y}-\frac{x^{2} \log x-z^{2} \log z}{x-z}\right)\right) \\
D_{0}(x, y, z, t) & =\frac{C_{0}(x, y, z)-C_{0}(x, y, t)}{z-t}  \tag{A.5}\\
& =-\left[\frac{y \log \frac{y}{x}}{(y-x)(y-z)(y-t)}+\frac{z \log \frac{z}{x}}{(z-x)(z-y)(z-t)}+\frac{z^{2} \log \frac{z}{x}}{(t-x)(t-y)(t-z)}\right] \\
D_{00}(x, y, z, t) & =-\frac{1}{4}\left[\frac{y^{2} \log \frac{y}{x}}{(y-x)(y-z)(y-t)}+\frac{t^{2} \log \frac{t}{x}}{(z-x)(z-y)(z-t)}+\frac{1}{(t-x)(t-y)(t-z)}\right] . \tag{A.6}
\end{align*}
$$

Note, the conventions of ref. [50] (Pierce, Bagger [PB]) are different than those presented in ref. [32] (Dedes, Rosiek, Tanedo [DRT]). The box integrals are related by

$$
\begin{align*}
D_{0} & =D_{0}^{(\mathrm{PB})}=-D_{0}^{(D R T)}  \tag{A.7a}\\
D_{00} & =D_{27}^{(\mathrm{PB})}=-\frac{1}{4} D_{2}^{(D R T)} \tag{A.7b}
\end{align*}
$$

## Appendix A.2. Massless limit of loop integrals

In some amplitudes (i.e. penguin diagrams $(a-b)$, box diagram $(v)$ ) the following combinations of loop integrals appear:

$$
\begin{align*}
& I_{1}=B_{0}\left(s, M_{F 1}^{2}, M_{F 2}^{2}\right)+M_{S}^{2} C_{0}\left(s, 0,0, M_{F 2}^{2}, M_{F 1}^{2}, M_{S}^{2}\right)  \tag{A.8}\\
& I_{2}=C_{0}\left(0,0,0, M_{F 2}^{2}, M_{F 1}^{2}, M_{V 2}^{2}\right)+M_{V 1}^{2} D_{0}\left(M_{F 2}^{2}, M_{F 1}^{2}, M_{V 1}^{2}, M_{V 2}^{2}\right) \tag{A.9}
\end{align*}
$$

The loop functions $B_{0}, C_{0}, D_{0}$ diverge for massless fermions (e.g. neutrinos in the MSSM) but the expressions $I_{1}, I_{2}$ are finite. However, this limit must be taken analytically in order to avoid numerical instabilities. In a generalized form and in the limit of zero external momenta, $I_{i}$ can be expressed by

$$
\begin{align*}
I_{1}(a, b, c) & =B_{0}(0, a, b)+c C_{0}(0,0,0, a, b, c) \equiv B_{0}(0, a, b)+c C_{0}(a, b, c)  \tag{A.10}\\
I_{2}(a, b, c, d) & =C_{0}(0,0,0, a, b, d)+c D_{0}(a, b, c, d) \equiv C_{0}(a, b, d)+c D_{0}(a, b, c, d) \tag{A.11}
\end{align*}
$$

Using eqs. A.1 A. 4 A. 5 we obtain in the limit $a \rightarrow 0$

$$
\begin{align*}
I_{1}(0, b, c) & =B_{0}(0,0, b)+c C_{0}(0, b, c)  \tag{A.12}\\
& =\Delta+1-\log \frac{b}{Q^{2}}+c \frac{1}{b-c} \log \frac{c}{b}  \tag{A.13}\\
& =\Delta+1+\log Q^{2}+\frac{c}{b-c} \log c+\left(-1-\frac{c}{b-c}\right) \log b \tag{A.14}
\end{align*}
$$

The term proportional to $\log b$ vanishes in the limit $b \rightarrow 0$

$$
\begin{equation*}
I_{1}(0,0, c)=\Delta+1-\log \frac{c}{Q^{2}} \tag{A.15}
\end{equation*}
$$

The same strategy works for $I_{2}$ :

$$
\begin{align*}
I_{2}(0, b, c, d) & =C_{0}(0, b, d)+c D_{0}(0, b, c, d)  \tag{A.16}\\
& =\frac{1}{b-d} \log \frac{d}{b}+c \frac{C_{0}(0, b, c)-C_{0}(0, b, d)}{c-d}  \tag{A.17}\\
& =\frac{1}{b-d} \log \frac{d}{b}+\frac{c}{c-d} \frac{1}{b-c} \log \frac{c}{b}-\frac{c}{c-d} \frac{1}{b-d} \log \frac{d}{b}  \tag{A.18}\\
& =\frac{(c-d)(b-c) \log \frac{d}{b}+c(b-d) \log \frac{c}{b}-c(b-c) \log \frac{d}{b}}{(b-d)(c-d)(b-c)} \tag{A.19}
\end{align*}
$$

The denominator of eq. A.19 is finite for $b \rightarrow 0$ and in the numerator, the $\log b$ terms cancel each other:

$$
\begin{equation*}
\left((c-d) c+c d-c^{2}\right) \log b=0 . \tag{A.20}
\end{equation*}
$$

Hence, we end up with

$$
\begin{equation*}
I_{2}(0,0, c, d)=\frac{-c(c-d) \log d-c d \log c+c^{2} \log d}{c d(c-d)}=\frac{\log \frac{d}{c}}{c-d} \tag{A.21}
\end{equation*}
$$

## Appendix A.3. Parametrization of vertices

We are going to express the amplitude in the following in terms of generic vertices. For this purpose, we parametrize a vertex between two fermions and one vector or scalar respectively as

$$
\begin{gather*}
G_{A} \gamma_{\mu} P_{L}+G_{B} \gamma_{\mu} P_{R}  \tag{A.22}\\
G_{A} P_{L}+G_{B} P_{R} \tag{A.23}
\end{gather*}
$$

$P_{L, R}=\frac{1}{2}\left(1 \mp \gamma^{5}\right)$ are the polarization operators. In addition, for the vertex between three vector bosons and the one between one vector boson and two scalars the conventions are as follows

$$
\begin{gather*}
G_{V V V} \cdot\left(g_{\mu \nu}\left(k_{2}-k_{1}\right)_{\rho}+g_{\nu \rho}\left(k_{3}-k_{2}\right)_{\mu}+g_{\rho \mu}\left(k_{1}-k_{3}\right)_{\nu}\right),  \tag{A.24}\\
G_{S S V} \cdot\left(k_{1}-k_{2}\right)_{\mu} \tag{A.25}
\end{gather*}
$$

Here, $k_{i}$ are the (ingoing) momenta of the external particles.

## Appendix B. Generic amplitudes

We present in the following the expressions for the generic amplitudes obtained with FeynArts and FormCalc. All coefficients that are not explicitly listed are zero. Furthermore, the Wilson coefficients are left-right symmetric, i.e.

$$
\begin{equation*}
C_{X R R}=C_{X L L}(L \leftrightarrow R), \quad C_{X R L}=C_{X L R}(L \leftrightarrow R), \tag{B.1}
\end{equation*}
$$

with $X=S, V$ and where $(L \leftrightarrow R)$ means that the coefficients of the left and right polarization part of each vertex have to be interchanged.


Figure B.3: Tree level diagrams with vertex numbering

## Appendix B.1. Tree Level Contributions

Since in models beyond the MSSM it might be possible that $B_{s}^{0} \rightarrow \ell \bar{\ell}$ is already possible at tree-level. This is for instance the case for trilinear $R$-parity violation [35]. The generic diagrams are given in Figure B. 3 . The chiral vertices are parametrized as in eqs. (A.22)-A.23) with $A=1, B=2$ for vertex 1 and $A=3, B=4$ for vertex 2. Using these conventions, the corresponding contributions to the Wilson coefficients read

$$
\begin{array}{rlrl}
C_{S L L}^{(a)} & =16 \pi^{2} \frac{G_{1} G_{3}}{M_{S}^{2}-s}, & & C_{S L R}^{(a)}=16 \pi^{2} \frac{G_{1} G_{4}}{M_{S}^{2}-s} \\
C_{V L L}^{(b)}=16 \pi^{2} \frac{-G_{1} G_{3}}{M_{V}^{2}-s}, & & C_{V L R}^{(b)}=16 \pi^{2} \frac{-G_{1} G_{4}}{M_{V}^{2}-s} \\
C_{S L L}^{(c)}=16 \pi^{2} \frac{-G_{1} G_{3}}{2\left(M_{S}^{2}-t\right)}, & & C_{V L R}^{(c)}=16 \pi^{2} \frac{-G_{2} G_{3}}{2\left(M_{S}^{2}-t\right)} \\
C_{S L R}^{(d)}=16 \pi^{2} \frac{2 G_{2} G_{3}}{M_{V}^{2}-t}, & & C_{V L L}^{(d)}=16 \pi^{2} \frac{-G_{1} G_{3}}{M_{V}^{2}-t} \\
C_{S L L}^{(e)}=16 \pi^{2} \frac{-G_{1} G_{3}}{2\left(M_{S}^{2}-u\right)}, & & C_{V L L}^{(e)}=16 \pi^{2} \frac{G_{2} G_{3}}{2\left(M_{S}^{2}-u\right)} \\
C_{S L R}^{(f)}=16 \pi^{2} \frac{-2 G_{2} G_{4}}{M_{V}^{2}-u}, & & C_{V L R}^{(f)}=16 \pi^{2} \frac{-G_{1} G_{4}}{M_{V}^{2}-u} \tag{B.7}
\end{array}
$$

Here, $s, t$ and $u$ are the usual Mandelstam variables.

## Appendix B.2. Wave Contributions

The generic wave diagrams are given in Figure B.4. The internal quark which attaches to the vector or scalar propagator has generation index $n$. Couplings that depend on $n$ carry it as an additional index. The chiral vertices are parametrized as in eqs. A.22-A.23 with $A=1, B=2$ for vertex $1, A=3, B=4$ for


Figure B.4: Generic wave diagrams. For every diagram there is a crossed version, where the loop attaches to the other external quark.


Figure B.5: Generic wave diagram vertex numbering
vertex $2, A=5, B=6$ for vertex 3 and $A=7, B=8$ for vertex 4 , see also Figure B. 5 for the numbering of the vertices. If a vertex is labelled $3^{\prime}$ for instance, the corresponding couplings are $G_{5}^{\prime}, G_{6}^{\prime}$. Furthermore, we define the following abbreviations:

$$
\begin{align*}
f_{S 1} & =\frac{1}{m_{n}^{2}-m_{i}^{2}}\left(-M_{F}\left(G_{1} G_{3 n} m_{n}+G_{2} G_{4 n} m_{i}\right) B_{0}^{(i)}+\left(G_{2} G_{3 n} m_{n} m_{i}+G_{1} G_{4 n} m_{i}^{2}\right) B_{1}^{(i)}\right),  \tag{B.8}\\
f_{S 2} & =\frac{1}{m_{j}^{2}-m_{n}^{2}}\left(M_{F}\left(G_{2 n} G_{4} m_{j}+G_{1 n} G_{3} m_{n}\right) B_{0}^{(j)}-\left(G_{2 n} G_{3} m_{j}^{2}+G_{1 n} G_{4} m_{j} m_{n}\right) B_{1}^{(j)}\right),  \tag{B.9}\\
\tilde{f}_{S 2} & =\frac{1}{m_{j}^{2}-m_{n}^{2}}\left(M_{F}\left(G_{1 n} G_{3} m_{j}+G_{2 n} G_{4} m_{n}\right) B_{0}^{(j)}-\left(G_{1 n} G_{4} m_{j}^{2}+G_{2 n} G_{3} m_{j} m_{n}\right) B_{1}^{(j)}\right),  \tag{B.10}\\
f_{V 1} & =\frac{1}{m_{n}^{2}-m_{i}^{2}}\left(2 M_{F}\left(G_{1} G_{4 n} m_{n}+G_{2} G_{3 n} m_{i}\right) B_{0}^{(i)}+\left(G_{2} G_{4 n} m_{n} m_{i}+G_{1} G_{3 n} m_{i}^{2}\right) B_{1}^{(i)}\right),  \tag{B.11}\\
f_{V 2} & =\frac{1}{m_{j}^{2}-m_{n}^{2}}\left(2 M_{F}\left(G_{2 n} G_{3} m_{j}+G_{1 n} G_{4} m_{n}\right) B_{0}^{(j)}+\left(G_{2 n} G_{4} m_{j}^{2}+G_{1 n} G_{3} m_{j} m_{n}\right) B_{1}^{(j)}\right),  \tag{B.12}\\
\tilde{f}_{V 2} & =\frac{1}{m_{j}^{2}-m_{n}^{2}}\left(2 M_{F}\left(G_{1 n} G_{4} m_{j}+G_{2 n} G_{3} m_{n}\right) B_{0}^{(j)}+\left(G_{1 n} G_{3} m_{j}^{2}+G_{2 n} G_{4} m_{j} m_{n}\right) B_{1}^{(j)}\right) \tag{B.13}
\end{align*}
$$

The $m_{i}, m_{j}$ are the quark masses and $B_{0,1}^{(i)}=B_{0,1}\left(m_{i}^{2}, M_{F}^{2}, M_{S}^{2}\right)\left(\right.$ or $M_{V}^{2}$ instead of $\left.M_{S}^{2}\right) . m_{n}$ is the mass of the internal quark with generation index $n$. Couplings that involve the internal quark are also labelled with $n$ (e.g. $G_{3 n}$ ). Using these conventions, the contributions to the Wilson coefficients are

$$
\begin{align*}
C_{S L L}^{(a)} & =\frac{G_{7}}{M_{S 0}^{2}-s}\left(G_{5 n} f_{S 1}+G_{5 n}^{\prime} f_{S 2}\right)  \tag{B.14}\\
C_{V L L}^{(c)} & =\frac{G_{7}}{M_{V 0}^{2}-s}\left(-G_{5 n} f_{S 1}-G_{5 n}^{\prime} \tilde{f}_{S 2}\right) \rightarrow \frac{G_{7}}{M_{V 0}^{2}-s} G_{5} G_{1} G_{4} B_{1}\left(0, M_{F}, M_{S}\right) \tag{B.15}
\end{align*}
$$

$$
\left.\begin{array}{rl}
C_{S L L}^{(b)} & =\frac{2 G_{7}}{M_{S 0}^{2}-s}\left(G_{5 n} f_{V 1}-G_{5 n}^{\prime} f_{V 2}\right) \\
C_{V L L}^{(d)} & =\frac{2 G_{7}}{M_{V 0}^{2}-s}\left(-G_{5 n} f_{V 1}+G_{5 n}^{\prime} \tilde{f}_{V 2}\right) \rightarrow \frac{2 G_{7}}{M_{V 0}^{2}-s} G_{5} G_{1} G_{3} B_{1}\left(0, M_{F}, M_{V}\right) \\
C_{S L L}^{(e)} & =\frac{-1}{2\left(M_{S 0}^{2}-t\right)}\left(G_{5 n} G_{7} f_{S 1}+G_{5 n}^{\prime} G_{7}^{\prime} f_{S 2}\right) \\
C_{V L R}^{(e)} & =\frac{-1}{2\left(M_{S 0}^{2}-t\right)}\left(G_{5 n} G_{8} f_{S 1}+G_{6 n}^{\prime} G_{7}^{\prime} \tilde{f}_{S 2}\right) \\
C_{S L R}^{(g)} & =\frac{+2}{M_{V 0}^{2}-t}\left(G_{5 n} G_{8} f_{S 1}+G_{6 n}^{\prime} G_{7}^{\prime} f_{S 2}\right) \\
C_{V L L}^{(g)} & =\frac{-1}{M_{V 0}^{2}-t}\left(G_{5 n} G_{7} f_{S 1}+G_{5 n}^{\prime} G_{7}^{\prime} \tilde{f}_{S 2}\right) \\
C_{S L L}^{(f)} & =\frac{-1}{M_{S 0}^{2}-t}\left(G_{5 n} G_{7} f_{V 1}-G_{5 n}^{\prime} G_{7}^{\prime} f_{V 2}\right) \\
C_{V L R}^{(f)} & =\frac{-1}{M_{S 0}^{2}-t}\left(G_{5 n} G_{8} f_{V 1}-G_{6 n}^{\prime} G_{7}^{\prime} \tilde{f}_{V 2}\right) \\
C_{S L R}^{(h)} & =\frac{+4}{M_{V 0}^{2}-t}\left(G_{5 n} G_{8} f_{V 1}-G_{6 n}^{\prime} G_{7}^{\prime} f_{V 2}\right) \\
C_{V L L}^{(h)} & =\frac{+2}{M_{V 0}^{2}-t}\left(-G_{5 n} G_{7} f_{V 1}+G_{5 n}^{\prime} G_{7}^{\prime} \tilde{f}_{V 2}\right) \\
C_{S L L}^{(i)} & =\frac{-1}{2\left(M_{S 0}^{2}-u\right)}\left(G_{5 n} G_{7} f_{S 1}+G_{5 n}^{\prime} G_{7}^{\prime} f_{S 2}\right) \\
C_{V L L}^{(i)} & =\frac{+1}{2\left(M_{S 0}^{2}-u\right)}\left(G_{5 n} G_{8} f_{S 1}+G_{6 n}^{\prime} G_{7}^{\prime} \tilde{f}_{S 2}\right) \\
C_{S L R}^{(k)} & =\frac{-2}{M_{V 0}^{2}-u}\left(G_{6 n} G_{8} f_{S 1}+G_{6 n}^{\prime} G_{8}^{\prime} f_{S 2}\right) \\
C_{V L R}^{(k)} & =\frac{-1}{M_{V 0}^{2}-u}\left(G_{6 n} G_{7} f_{S 1}+G_{5 n}^{\prime} G_{8}^{\prime} \tilde{f}_{S 2}\right) \\
C_{S L L}^{(j)} & =\frac{-1}{M_{S 0}^{2}-u}\left(G_{5 n} G_{7} \tilde{f}_{V 1}-G_{5 n}^{\prime} G_{7}^{\prime} f_{V 2}\right) \\
C_{V L L}^{(j)} & =\frac{-1}{M_{S 0}^{2}-u}\left(-G_{5 n} G_{8} \tilde{f}_{V 1}+G_{6 n}^{\prime} G_{7}^{\prime} \tilde{f}_{V 2}\right) \\
C_{S L R}^{(l)} & =\frac{-4}{M_{V 0}^{2}-u}\left(G_{6 n} G_{8} \tilde{f}_{V 1}-G_{6 n}^{\prime} G_{8}^{\prime} f_{V 2}\right) \\
C_{V L R}^{(l)} & =\frac{-2}{M_{V 0}^{2}-u}\left(G_{6 n} G_{7} \tilde{f}_{V 1}-G_{5 n}^{\prime} G_{8}^{\prime} \tilde{f}_{V 2}\right)  \tag{B.33}\\
C_{V} \\
C_{V}
\end{array}\right)
$$

## Appendix B.3. Penguin Contributions

Diagrams with scalar propagators have $C_{V X Y}=0$ and those with vector propagators have $C_{S X Y}=0$. The vertex number conventions are given in fig. B. 6 and all possible diagrams are depicted in Figure B. 7 . The chiral vertices are parametrized as in eqs. A.22-A.23 with $A=1, B=2$ for vertex $1, A=3, B=4$ for vertex 2 and $A=7, B=8$ for vertex 4 . Vertex 3 can be a chiral vertex, in this case $A=5, B=6$ is used. Otherwise, we will denote it with a index 5 and give as additional subscript the kind of vertex. The contributions to the Wilson coefficients from these diagrams read

$$
\begin{equation*}
C_{S L L}^{(a)}=\frac{1}{M_{S 0}^{2}-s} G_{1} G_{3} G_{7}\left(G_{6} B_{0}^{(a, b)}+\left(G_{5} M_{F 1} M_{F 2}+G_{6} M_{S}^{2}\right) C_{0}^{(a, b)}\right) \tag{B.34}
\end{equation*}
$$



Figure B.6: Vertex number conventions for a representative penguin diagram

$$
\left.\begin{array}{rl}
C_{S L R}^{(a)} & =\frac{1}{M_{S 0}^{2}-s} G_{1} G_{3} G_{8}\left(G_{6} B_{0}^{(a, b)}+\left(G_{5} M_{F 1} M_{F 2}+G_{6} M_{S}^{2}\right) C_{0}^{(a, b)}\right) \\
C_{V L L}^{(b)} & =\frac{1}{M_{V 0}^{2}-s} G_{1} G_{4} G_{7}\left(G_{6} B_{0}^{(a, b)}+\left(-G_{5} M_{F 1} M_{F 2}+G_{6} M_{S}^{2}\right) C_{0}^{(a, b)}-2 G_{6} C_{00}^{(a, b)}\right) \\
C_{V L R}^{(b)} & =\frac{1}{M_{V 0}^{2}-s} G_{1} G_{4} G_{8}\left(G_{6} B_{0}^{(a, b)}+\left(-G_{5} M_{F 1} M_{F 2}+G_{6} M_{S}^{2}\right) C_{0}^{(a, b)}-2 G_{6} C_{00}^{(a, b)}\right) \\
C_{S L L}^{(c)} & =\frac{1}{M_{S 0}^{2}-s} G_{1} G_{3} G_{5, S S S} G_{7} M_{F} C_{0}^{(c, d)} \\
C_{S L R}^{(c)} & =\frac{1}{M_{S 0}^{2}-s} G_{1} G_{3} G_{5, S S S} G_{8} M_{F} C_{0}^{(c, d)} \\
C_{V L L}^{(d)} & =-\frac{2}{M_{V 0}^{2}-s} G_{1} G_{4} G_{5, S S V} G_{7} C_{00}^{(c, d)} \\
C_{V L R}^{(d)} & =-\frac{2}{M_{V 0}^{2}-s} G_{1} G_{4} G_{5, S S V} G_{8} C_{00}^{(c, d)} \\
C_{S L L}^{(e)} & =-\frac{4}{M_{S 0}^{2}-s} G_{1} G_{4} G_{7}\left(G_{5} B_{0}^{(e, f)}+\left(G_{6} M_{F 1} M_{F 2}+G_{5} M_{V}^{2}\right) C_{0}^{(e, f)}\right) \\
C_{S L R}^{(e)} & =-\frac{4}{M_{S 0}^{2}-s} G_{1} G_{4} G_{8}\left(G_{5} B_{0}^{(e, f)}+\left(G_{6} M_{F 1} M_{F 2}+G_{5} M_{V}^{2}\right) C_{0}^{(e, f)}\right) \\
C_{V L L}^{(f)} & =\frac{2}{M_{V 0}^{2}-s} G_{1} G_{3} G_{7}\left(G_{5} B_{0}^{(e, f)}+\left(-G_{6} M_{F 1} M_{F 2}+G_{5} M_{V}^{2}\right) C_{0}^{(e, f)}-2 G_{5} C_{00}^{(e, f)}\right) \\
C_{V L R}^{(f)} & =\frac{2}{M_{V 0}^{2}-s} G_{1} G_{3} G_{8}\left(G_{5} B_{0}^{(e, f)}+\left(-G_{6} M_{F 1} M_{F 2}+G_{5} M_{V}^{2}\right) C_{0}^{(e, f)}-2 G_{5} C_{00}^{(e, f)}\right) \\
C_{S L L}^{(g)} & =\frac{4}{M_{S 0}^{2}-s} G_{1} G_{4} G_{5, S V V} G_{7} M_{F} C_{0}^{(g, h)} \\
C_{S L R}^{(g)} & =\frac{4}{M_{S 0}^{2}-s} G_{1} G_{4} G_{5, S V V} G_{8} M_{F} C_{0}^{(g, h)} \\
C_{V L L}^{(h)} & =-\frac{2}{M_{V 0}^{2}-s} G_{1} G_{3} G_{5, V V V} G_{7}\left(B_{0}^{(g, h)}+M_{F}^{2} C_{0}^{(g, h)}+2 C_{00}^{(g, h)}\right) \\
C_{V L R}^{(h)} & =-\frac{2}{M_{V 0}^{2}-s} G_{1} G_{3} G_{5, V V V} G_{8}\left(B_{0}^{(g, h)}+M_{F}^{2} C_{0}^{(g, h)}+2 C_{00}^{(g, h)}\right) \\
C_{S L L}^{(i)} & =\frac{1}{M_{S 0}^{2}-s} G_{1} G_{3} G_{5, S S V} G_{7}\left(B_{0}^{(i-l)}+M_{F}^{2} C_{0}^{(i-l)}\right) \\
C_{S L R}^{(i)} & =\frac{1}{M_{S 0}^{2}-s} G_{1} G_{3} G_{5, S S V} G_{8}\left(B_{0}^{(i-l)}+M_{F}^{2} C_{0}^{(i-l)}\right) \\
C_{S L L}^{(k)} & =-\frac{1}{M_{S 0}^{2}-s} G_{1} G_{4} G_{5, S S V} G_{7}\left(B_{0}^{(i-l)}+M_{F}^{2} C_{0}^{(i-l)}\right)  \tag{B.52}\\
(e,
\end{array}\right)
$$



Figure B.7: Generic penguin diagrams

$$
\begin{align*}
C_{S L R}^{(k)} & =-\frac{1}{M_{S 0}^{2}-s} G_{1} G_{4} G_{5, S S V} G_{8}\left(B_{0}^{(i-l)}+M_{F}^{2} C_{0}^{(i-l)}\right)  \tag{B.53}\\
C_{V L L}^{(j)} & =\frac{1}{M_{V 0}^{2}-s} G_{1} G_{4} G_{5, S V V} G_{7} M_{F} C_{0}^{(i-l)}  \tag{B.54}\\
C_{V L R}^{(j)} & =\frac{1}{M_{V 0}^{2}-s} G_{1} G_{4} G_{5, S V V} G_{8} M_{F} C_{0}^{(i-l)}  \tag{B.55}\\
C_{V L L}^{(l)} & =\frac{1}{M_{V 0}^{2}-s} G_{1} G_{3} G_{5, S V V} G_{7} M_{F} C_{0}^{(i-l)}  \tag{B.56}\\
C_{V L R}^{(l)} & =\frac{1}{M_{V 0}^{2}-s} G_{1} G_{3} G_{5, S V V} G_{8} M_{F} C_{0}^{(i-l)} \tag{B.57}
\end{align*}
$$

Here, the arguments of the Passarino-Veltman integrals are as follows, with $s=M_{B_{q}^{0}}^{2}$ :

$$
\begin{array}{lr}
C_{X}^{(a, b)}=C_{X}\left(s, 0,0, M_{F 2}^{2}, M_{F 1}^{2}, M_{S}^{2}\right) & B_{X}^{(a, b)}=B_{X}\left(s, M_{F 1}^{2}, M_{F 2}^{2}\right) \\
C_{X}^{(c, d)}=C_{X}\left(0, s, 0, M_{F}^{2}, M_{S 1}^{2}, M_{S 2}^{2}\right) & \\
C_{X}^{(e, f)}=C_{X}\left(s, 0,0, M_{F 2}^{2}, M_{F 1}^{2}, M_{V}^{2}\right) & B_{X}^{(e, f)}=B_{X}\left(s, M_{F 1}^{2}, M_{F 2}^{2}\right) \\
C_{X}^{(g, h)}=C_{X}\left(0, s, 0, M_{F}^{2}, M_{V 1}^{2}, M_{V 2}^{2}\right) & B_{X}^{(g, h)}=B_{X}\left(s, M_{V 1}^{2}, M_{V 2}^{2}\right) \\
C_{X}^{(i-l)}=C_{X}\left(0, s, 0, M_{F}^{2}, M_{S}^{2}, M_{V}^{2}\right) & B_{X}^{(i-l)}=B_{X}\left(s, M_{S}^{2}, M_{V}^{2}\right) \tag{B.62}
\end{array}
$$



Figure B.8: Vertex number conventions for a set of representative box diagrams

## Appendix B.4. Box Contributions

The vertex number conventions for boxes are shown in figs. B.8, while all possible generic diagrams are given in Figures B. 9 and B.10. All vertices are chiral and they are parametrized as in eqs. A.22-A.23) with $A=1, B=2$ for vertex $1, A=3, B=4$ for vertex $2, A=5, B=6$ for vertex 3 and $A=7, B=8$ for vertex 4. If there are two particles of equal type in a loop (say, two fermions), the one between vertices 1 and 2 ( 2 or 3 ) will be labelled $F 1$ and the other one will be $F 2$. The contributions to the different Wilson coefficients read

$$
\begin{align*}
& C_{S L L}^{(a)}=-G_{1} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} \cdot D_{0}^{(a-c)}  \tag{B.63}\\
& C_{S L R}^{(a)}=-G_{1} G_{3} G_{6} G_{8} M_{F 1} M_{F 2} \cdot D_{0}^{(a-c)}  \tag{B.64}\\
& C_{V L L}^{(a)}=-G_{2} G_{3} G_{6} G_{7} \cdot D_{00}^{(a-c)}  \tag{B.65}\\
& C_{V L R}^{(a)}=-G_{2} G_{3} G_{5} G_{8} \cdot D_{00}^{(a-c)}  \tag{B.66}\\
& C_{S L L}^{(b)}=-G_{1} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} \cdot D_{0}^{(a-c)}  \tag{B.67}\\
& C_{S L R}^{(b)}=-G_{1} G_{3} G_{6} G_{8} M_{F 1} M_{F 2} \cdot D_{0}^{(a-c)}  \tag{B.68}\\
& C_{V L L}^{(b)}=G_{2} G_{3} G_{5} G_{8} \cdot D_{00}^{(a-c)}  \tag{B.69}\\
& C_{V L R}^{(b)}=G_{2} G_{3} G_{6} G_{7} \cdot D_{00}^{(a-c)}  \tag{B.70}\\
& C_{S L L}^{(c)}=\frac{1}{2} G_{1} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(a-c)}  \tag{B.71}\\
& C_{S L R}^{(c)}=-2 G_{1} G_{4} G_{5} G_{8} D_{00}^{(a-c)}  \tag{B.72}\\
& C_{V L L}^{(c)}=-\frac{1}{2} G_{2} G_{4} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(a-c)}  \tag{B.73}\\
& C_{V L R}^{(c)}=-G_{2} G_{3} G_{5} G_{8} D_{00}^{(a-c)}  \tag{B.74}\\
& C_{S L L}^{(d)}=\frac{1}{2} G_{1} G_{3} G_{5} G_{7} M_{1} M_{2} \cdot D_{0}^{(d-f)}  \tag{B.75}\\
& C_{S L R}^{(d)}=2 G_{1} G_{3} G_{6} G_{8} \cdot D_{00}^{(d-f)}  \tag{B.76}\\
& C_{V L L}^{(d)}=-G_{2} G_{3} G_{6} G_{7} \cdot D_{00}^{(d-f)}  \tag{B.77}\\
& C_{V L R}^{(d)}=\frac{1}{2} G_{2} G_{3} G_{5} G_{8} M_{1} M_{2} \cdot D_{0}^{(d-f)}  \tag{B.78}\\
& C_{S L L}^{(e)}=\frac{1}{2} G_{1} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} \cdot D_{0}^{(d-f)}  \tag{B.79}\\
& C_{S L R}^{(e)}=2 G_{1} G_{3} G_{6} G_{8} \cdot D_{00}^{(d-f)}  \tag{B.80}\\
& C_{V L L}^{(e)}=-\frac{1}{2} G_{2} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} \cdot D_{0}^{(d-f)}  \tag{B.81}\\
& C_{V L R}^{(e)}=G_{2} G_{3} G_{6} G_{7} \cdot D_{00}^{(d-f)} \tag{B.82}
\end{align*}
$$



Figure B.9: Generic box diagrams I

$$
\begin{align*}
& C_{S L L}^{(f)}=\frac{1}{2} G_{1} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(d-f)}  \tag{B.83}\\
& C_{S L R}^{(f)}=-2 G_{1} G_{4} G_{5} G_{8} D_{00}^{(d-f)}  \tag{B.84}\\
& C_{V L L}^{(f)}=G_{2} G_{4} G_{5} G_{7} D_{00}^{(d-f)}  \tag{B.85}\\
& C_{V L R}^{(f)}=\frac{1}{2} G_{2} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(d-f)}  \tag{B.86}\\
& C_{S L L}^{(g)}=2 G_{1} G_{3} G_{6} G_{7}\left(C_{0}^{(g-i)}+M_{F 1}^{2} D_{0}^{(g-i)}-2 D_{00}^{(g-i)}\right)  \tag{B.87}\\
& C_{S L R}^{(g)}=2 G_{1} G_{3} G_{5} G_{8}\left(C_{0}^{(g-i)}+M_{F 1}^{2} D_{0}^{(g-i)}-2 D_{00}^{(g-i)}\right)  \tag{B.88}\\
& C_{V L L}^{(g)}=G_{2} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(g-i)}  \tag{B.89}\\
& C_{V L R}^{(g)}=G_{2} G_{3} G_{6} G_{8} M_{F 1} M_{F 2} D_{0}^{(g-i)}  \tag{B.90}\\
& C_{S L L}^{(h)}=-4 G_{1} G_{3} G_{5} G_{7} D_{00}^{(g-i)} \tag{B.91}
\end{align*}
$$



Figure B.10: Generic box diagrams II

$$
\begin{align*}
& C_{S L R}^{(h)}=-4 G_{1} G_{3} G_{6} G_{8} D_{00}^{(g-i)}  \tag{B.92}\\
& C_{V L L}^{(h)}=G_{2} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(g-i)}  \tag{B.93}\\
& C_{V L R}^{(g)}=G_{2} G_{3} G_{6} G_{7} M_{F 1} M_{F 2} D_{0}^{(g-i)}  \tag{B.94}\\
& C_{S L L}^{(i)}=-G_{1} G_{3} G_{5} G_{7}\left(C_{0}^{(g-i)}+M_{S}^{2} D_{0}^{(g-i)}-8 D_{00}^{(g-i)}\right)  \tag{B.95}\\
& C_{S L R}^{(i)}=2 G_{1} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(g-i)}  \tag{B.96}\\
& C_{V L L}^{(i)}=G_{2} G_{3} G_{5} G_{7}\left(C_{0}^{(g-i)}+M_{S}^{2} D_{0}^{(g-i)}-2 D_{00}^{(g-i)}\right)  \tag{B.97}\\
& C_{V L R}^{(i)}=G_{2} G_{4} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(g-i)}  \tag{B.98}\\
& C_{S L L}^{(j)}=2 G_{2} G_{3} G_{5} G_{7}\left(C_{0}^{(j-l)}+M_{F 1}^{2} D_{0}^{(j-l)}-2 D_{00}^{(j-l)}\right)  \tag{B.99}\\
& C_{S L R}^{(j)}=2 G_{2} G_{3} G_{6} G_{8}\left(C_{0}^{(j-l)}+M_{F 1}^{2} D_{0}^{(j-l)}-2 D_{00}^{(j-l)}\right) \tag{B.100}
\end{align*}
$$

$$
\begin{align*}
& C_{V L L}^{(j)}=G_{1} G_{3} G_{6} G_{7} M_{F 1} M_{F 2} D_{0}^{(j-l)}  \tag{B.101}\\
& C_{V L R}^{(j)}=G_{1} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(j-l)}  \tag{B.102}\\
& C_{S L L}^{(k)}=-4 G_{2} G_{3} G_{5} G_{8} D_{00}^{(j-l)}  \tag{B.103}\\
& C_{S L R}^{(k)}=-4 G_{2} G_{3} G_{6} G_{7} D_{00}^{(j-l)}  \tag{B.104}\\
& C_{V L L}^{(k)}=G_{1} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(j-l)}  \tag{B.105}\\
& C_{V L R}^{(k)}=G_{1} G_{3} G_{6} G_{8} M_{F 1} M_{F 2} D_{0}^{(j-l)}  \tag{B.106}\\
& C_{S L L}^{(l)}=-G_{1} G_{3} G_{5} G_{8}\left(C_{0}^{(j-l)}+M_{V}^{2} D_{0}^{(j-l)}-8 D_{00}^{(j-l)}\right)  \tag{B.107}\\
& C_{S L R}^{(l)}=2 G_{1} G_{4} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(j-l)}  \tag{B.108}\\
& C_{V L L}^{(l)}=G_{2} G_{4} G_{5} G_{8}\left(C_{0}^{(j-l)}+M_{V}^{2} D_{0}^{(j-l)}-2 D_{00}^{(j-l)}\right)  \tag{B.109}\\
& C_{V L R}^{(l)}=G_{2} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(j-l)}  \tag{B.110}\\
& C_{S L L}^{(m)}=-G_{1} G_{3} G_{6} G_{7}\left(C_{0}^{(m-o)}+M_{S}^{2} D_{0}^{(m-o)}-\frac{1}{4}\left(13 G_{1} G_{3} G_{6} G_{7}+3 G_{2} G_{4} G_{5} G_{8}\right) D_{00}^{(m-o)}\right)  \tag{B.111}\\
& C_{S L R}^{(m)}=-2 G_{1} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(m-o)}  \tag{B.112}\\
& C_{V L L}^{(m)}=G_{2} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(m-o)}  \tag{B.113}\\
& C_{V L R}^{(m)}=-G_{2} G_{3} G_{6} G_{8}\left(C_{0}^{(m-o)}+M_{S}^{2} D_{0}^{(m-o)}-2 D_{00}^{(m-o)}\right)  \tag{B.114}\\
& C_{S L L}^{(n)}=8 G_{1} G_{3} G_{6} G_{8} D_{00}^{(m-o)}  \tag{B.115}\\
& C_{S L R}^{(n)}=2 G_{1} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(m-o)}  \tag{B.116}\\
& C_{V L L}^{(n)}=-2 G_{2} G_{3} G_{6} G_{7} D_{00}^{(m-o)}  \tag{B.117}\\
& C_{V L R}^{(n)}=G_{2} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(m-o)}  \tag{B.118}\\
& C_{S L L}^{(o)}=-\frac{1}{4}\left(13 G_{1} G_{3} G_{5} G_{7}+3 G_{2} G_{4} G_{6} G_{8}\right) D_{00}^{(m-o)}  \tag{B.119}\\
& C_{S L R}^{(o)}=-2 G_{1} G_{4} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(m-o)}  \tag{B.120}\\
& C_{V L L}^{(o)}=G_{2} G_{4} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(m-o)}  \tag{B.121}\\
& C_{V L R}^{(o)}=2 G_{2} G_{3} G_{5} G_{8} D_{00}^{(m-o)}  \tag{B.122}\\
& C_{S L L}^{(p)}=-G_{2} G_{3} G_{5} G_{7}\left(C_{0}^{(p-r)}+M_{V}^{2} D_{0}^{(p-r)}-\frac{1}{4}\left(13 G_{2} G_{3} G_{5} G_{7}+3 G_{1} G_{4} G_{6} G_{8}\right) D_{00}^{(p-r)}\right)  \tag{B.123}\\
& C_{S L R}^{(p)}=-2 G_{2} G_{3} G_{6} G_{8} M_{F 1} M_{F 2} D_{0}^{(p-r)}  \tag{B.124}\\
& C_{V L L}^{(p)}=G_{1} G_{3} G_{6} G_{7} M_{F 1} M_{F 2} D_{0}^{(p-r)}  \tag{B.125}\\
& C_{V L R}^{(p)}=-G_{1} G_{3} G_{5} G_{8}\left(C_{0}^{(p-r)}+M_{V}^{2} D_{0}^{(p-r)}-2 D_{00}^{(p-r)}\right)  \tag{B.126}\\
& C_{S L L}^{(r)}=8 G_{1} G_{3} G_{5} G_{7} D_{00}^{(p-r)}  \tag{B.127}\\
& C_{S L R}^{(q)}=2 G_{1} G_{3} G_{6} G_{8} M_{F 1} M_{F 2} D_{0}^{(p-r)}  \tag{B.128}\\
& C_{V L L}^{(q)}=-2 G_{2} G_{3} G_{5} G_{8} D_{00}^{(p-r)}  \tag{B.129}\\
& C_{V L R}^{(q)}=G_{2} G_{3} G_{6} G_{7} M_{F 1} M_{F 2} D_{0}^{(p-r)}  \tag{B.130}\\
& C_{S L L}^{(r)}=-\frac{1}{4}\left(13 G_{2} G_{4} G_{5} G_{7}+3 G_{1} G_{3} G_{6} G_{8}\right) D_{00}^{(p-r)}  \tag{B.131}\\
& C_{S L R}^{(r)}=-2 G_{2} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(p-r)}+\frac{3}{4}\left(G_{2} G_{4} G_{5} G_{7}-G_{1} G_{3} G_{6} G_{8}\right) D_{00}^{(p-r)} \tag{B.132}
\end{align*}
$$

$$
\begin{align*}
& C_{V L L}^{(r)}=G_{1} G_{3} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(p-r)}  \tag{B.133}\\
& C_{V L R}^{(r)}=2 G_{1} G_{4} G_{5} G_{8} D_{00}^{(p-r)}  \tag{B.134}\\
& C_{S L L}^{(s)}=-4 G_{2} G_{3} G_{6} G_{7} M_{F 1} M_{F 2} D_{0}^{(s-u)}  \tag{B.135}\\
& C_{S L R}^{(s)}=-4 G_{2} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(s-u)}  \tag{B.136}\\
& C_{V L L}^{(s)}=-4 G_{1} G_{3} G_{5} G_{7}\left(C_{0}^{(s-u)}+M_{F 1}^{2} D_{0}^{(s-u)}-3 D_{00}^{(s-u)}\right)  \tag{B.137}\\
& C_{V L R}^{(s)}=-4 G_{1} G_{3} G_{6} G_{8}\left(C_{0}^{(s-u)}+M_{F 1}^{2} D_{0}^{(s-u)}\right)  \tag{B.138}\\
& C_{S L L}^{(t)}=-4 G_{2} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(s-u)}  \tag{B.139}\\
& C_{S L R}^{(t)}=-4 G_{2} G_{3} G_{6} G_{7} M_{F 1} M_{F 2} D_{0}^{(s-u)}  \tag{B.140}\\
& C_{V L L}^{(t)}=16 G_{1} G_{3} G_{5} G_{7} D_{00}^{(s-u)}  \tag{B.141}\\
& C_{V L R}^{(t)}=4 G_{1} G_{3} G_{6} G_{8} D_{00}^{(s-u)}  \tag{B.142}\\
& C_{S L L}^{(u)}=-4 G_{2} G_{4} G_{5} G_{7} M_{F 1} M_{F 2} D_{0}^{(s-u)}  \tag{B.143}\\
& C_{S L R}^{(u)}=-8 G_{2} G_{3} G_{5} G_{8} D_{00}^{(s-u)}  \tag{B.144}\\
& C_{V L L}^{(u)}=16 G_{1} G_{3} G_{5} G_{7} D_{00}^{(s-u)}  \tag{B.145}\\
& C_{V L R}^{(u)}=2 G_{1} G_{4} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(s-u)}  \tag{B.146}\\
& C_{S L L}^{(v)}=8 G_{2} G_{3} G_{6} G_{7} M_{F 1} M_{F 2} D_{0}^{(v-x)}  \tag{B.147}\\
& C_{S L R}^{(v)}=8 G_{2} G_{3} G_{5} G_{8}\left(C_{0}^{(v-x)}+M_{V 1}^{2} D_{0}^{(v-x)}\right)  \tag{B.148}\\
& C_{V L L}^{(v)}=-4 G_{1} G_{3} G_{5} G_{7}\left(C_{0}^{(v-x)}+M_{V 1}^{2} D_{0}^{(v-x)}-3 D_{00}^{(v-x)}\right)  \tag{B.149}\\
& C_{V L R}^{(v)}=2 G_{1} G_{3} G_{6} G_{8} M_{F 1} M_{F 2} D_{0}^{(v-x)}  \tag{B.150}\\
& C_{S L L}^{(w)}=8 G_{1} G_{3} G_{6} G_{8} M_{F 1} M_{F 2} D_{0}^{(v-x)}  \tag{B.151}\\
& C_{S L R}^{(w)}=32 G_{1} G_{3} G_{5} G_{7} D_{00}^{(v-x)}  \tag{B.152}\\
& C_{V L L}^{(w)}=-2 G_{2} G_{3} G_{6} G_{7} M_{F 1} M_{F 2} D_{0}^{(v-x)}  \tag{B.153}\\
& C_{V L R}^{(v)}=4 G_{2} G_{3} G_{5} G_{8} D_{00}^{(v-x)}  \tag{B.154}\\
& C_{S L L}^{(x)}=-4 G_{1} G_{4} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(v-x)}  \tag{B.155}\\
& C_{S L R}^{(x)}=-8 G_{1} G_{3} G_{5} G_{7}\left(C_{0}^{(v-x)}+M_{V 1}^{2} D_{0}^{(v-x)}-3 D_{00}^{(v-x)}\right)  \tag{B.156}\\
& C_{V L L}^{(x)}=-2 G_{2} G_{3} G_{5} G_{8} M_{F 1} M_{F 2} D_{0}^{(v-x)}  \tag{B.157}\\
& C_{V L R}^{(x)}=-4 G_{2} G_{4} G_{5} G_{7}\left(C_{0}^{(v-x)}+M_{V 1}^{2} D_{0}^{(v-x)}\right)  \tag{B.158}\\
&(v)
\end{align*}
$$

The arguments of the loop functions for the different amplitudes are

$$
\begin{align*}
D_{X}^{(a-c)} & =D_{X}\left(M_{F 1}^{2}, M_{F 2}^{2}, M_{S 1}^{2}, M_{S 2}^{2}\right) & &  \tag{B.159}\\
D_{X}^{(d-f)} & =D_{X}\left(M_{F 1}^{2}, M_{F 2}^{2}, M_{S 1}^{2}, M_{S 2}^{2}\right) & &  \tag{B.160}\\
C_{X}^{(g-i)} & =C_{X}\left(\overrightarrow{0}_{3}, M_{F 2}^{2}, M_{V}^{2}, M_{S}^{2}\right) & & D_{X}^{(g-i)}=D_{X}\left(M_{F 1}^{2}, M_{F 2}^{2}, M_{V}^{2}, M_{S}^{2}\right)  \tag{B.161}\\
C_{X}^{(j-l)} & =C_{X}\left(\overrightarrow{0}_{3}, M_{F 2}^{2}, M_{S}^{2}, M_{V}^{2}\right) & & D_{X}^{(j-l)}=D_{X}\left(M_{F 1}^{2}, M_{F 2}^{2}, M_{S}^{2}, M_{V}^{2}\right)  \tag{B.162}\\
C_{X}^{(m-o)} & =C_{X}\left(\overrightarrow{0}_{3}, M_{F 2}^{2}, M_{F 1}^{2}, M_{V}^{2}\right) & & D_{X}^{(m-o)}=D_{X}\left(M_{F 2}^{2}, M_{F 1}^{2}, M_{S}^{2}, M_{V}^{2}\right)  \tag{B.163}\\
C_{X}^{(p-r)} & =C_{X}\left(\overrightarrow{0}_{3}, M_{F 2}^{2}, M_{F 1}^{2}, M_{S}^{2}\right) & & D_{X}^{(p-r)}=D_{X}\left(M_{F 2}^{2}, M_{F 1}^{2}, M_{V}^{2}, M_{S}^{2}\right)  \tag{B.164}\\
C_{X}^{(s-u)} & =C_{X}\left(\overrightarrow{0}_{3}, M_{F 2}^{2}, M_{V 1}^{2}, M_{V 2}^{2}\right) & & D_{X}^{(s-u)}=D_{X}\left(M_{F 1}^{2}, M_{F 2}^{2}, M_{V 1}^{2}, M_{V 2}^{2}\right) \tag{B.165}
\end{align*}
$$

$$
\begin{equation*}
C_{X}^{(v-x)}=C_{X}\left(\overrightarrow{0}_{3}, M_{F 2}^{2}, M_{F 1}^{2}, M_{V 2}^{2}\right) \quad D_{X}^{(v-x)}=D_{X}\left(M_{F 2}^{2}, M_{F 1}^{2}, M_{V 1}^{2}, M_{V 2}^{2}\right) \tag{B.166}
\end{equation*}
$$

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[^0]:    Email addresses: dreiner@uni-bonn.de (H. Dreiner), nickel@th.physik.uni-bonn.de (K. Nickel), porod@physik.uni-wuerzburg.de (W. Porod), fnstaub@th.physik.uni-bonn.de (F. Staub)

