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Examining deterrence of adult sex crimes: A semi-parametric intervention time series approach

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Abstract

Motivated by recent developments on dimension reduction (DR) techniques for time series data, the association of a general deterrent effect towards South Carolina (SC)'s registration and notification (SORN) policy for preventing sex crimes was examined. Using adult sex crime arrestee data from 1990 to 2005, the the idea of Central Mean Subspace (CMS) is extended to intervention time series analysis (CMS-ITS) to model the sequential intervention effects of 1995 (the year SC's SORN policy was initially implemented) and 1999 (the year the policy was revised to include online notification) on the time series spectrum. The CMS-ITS model estimation was achieved via kernel smoothing techniques, and compared to interrupted auto-regressive integrated time series (ARIMA) models. Simulation studies and application to the real data underscores our model's ability towards achieving parsimony, and to detect intervention effects not earlier determined via traditional ARIMA models. From a public health perspective, findings from this study draw attention to the potential general deterrent effects of SC's SORN policy. These findings are considered in light of the overall body of research on sex crime arrestee registration and notification policies, which remain controversial.

Keywords

Central mean subspace; Nadaraya-Watson kernel smoother; Nonlinear time series; Sex crime arrestee

1. Introduction

Sexual abuse, and sex-related crimes are a major public health burden in the United States (U.S.). Because many of these assaults go unreported and unrecognized, it has been

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rightfully coined as a ‘silentviolent’ modern epidemic. The consequences of sexual abuse are far-reaching and include increased risk of physical and mental health problems for victims (Boney-McCoy and Finklehor, 1996; Letourneau *et. al*, 1996). Massive fiscal costs are incurred to the society flowing directly from these crimes and their consequences (Post *et al.*, 2002). Consequently, prevention of sexual violence has been a public health priority for decades (Mercy *et al.*, 1993).

Yearly rates/counts of any criminal arrests yields auto-correlated time-series data, where the major objective is to infer about the conditional distribution (or moments) of the current observation given the past value(s) for studying the underlying dynamics and providing efficient future forecasts. In an attempt to reduce sexual recidivism rates of targeted sex crime arrestees (i.e., specific deterrence) and rates of first-time sex arrests by potential perpetrators (i.e., general deterrence), the US relies mainly upon state-wide sex crime arrestee registration and notification (SORN) policies, or interventions. In this context, evaluating the effectiveness of these interventions for criminal justice settings (Loftin *et. al.*, 1983; McCleary and Hay, 1980; Vásquez *et. al.*, 2008) are typically handled through an interrupted/intervention time series (Box and Tiao, 1975) approach, which discriminates between the behavior of the time series ‘before’ and ‘after’ the intervention (possibly known, or unknown). A growing body of research suggests that SORN policies fail to reduce sexual recidivism (Letourneau and Levenson, 2010). Moreover, research on South Carolina (SC)’s SORN policy further indicates that registration and notification were each associated with significant and substantial increases in plea bargains and online notification was associated with significant reductions in findings of guilty for adult sex crimes (Letourneau *et al.*, 2010a). Sometimes, these policies resulted in much harsher consequences. For example, SC SORN policy subjects arrestees convicted of virtually any sex crimes to lifetime registration and lifetime notification requirements, which since 1999 have included online notification. There are no mechanisms to reduce the duration of the requirement or to remove oneself from the online registry. Because general deterrent effects depend, in part, on the certainty and noxiousness of the consequence (Wikström, 2008), it is conceivable that harsher and more widely applied SORN policies such as SC’s policy might exert general deterrent effects.

In an evaluation of the general deterrent effect of SC’s SORN policy, Letourneau *et al.* (2010b) conducted univariate Box-Jenkins interrupted auto-regressive integrated moving average (ARIMA) analysis (Box *et al.*, 1994) to understand general deterrence of SC SORN policies on sex crime arrestees, separately for the initial implementation of registration in 1995 and the subsequent implementation of online notification in 1999. They reported a significant intervention effect for 1995, but not for 1999. In comparison to first-time sex-crime arrests from 1990 to 1994 (i.e., prior to initial implementation), results indicated an approximately 11% reduction in first-time sex crime arrests in the post-SORN period from 1995–2005. This decline equated to averting approximately three new sex crime arrests per month. This finding supports the argument that harsher, more widely applied SORN policies can achieve a general deterrent effect. However, it seems counter-intuitive that online notification would fail to enhance a general deterrent effect, given that this mechanism equates with public shaming of arrestees who themselves attribute serious consequences to public notification, including loss of employment, housing, friends and the support of family (Levenson and Cotter, 2005; Wikström, 2008). We argue that Letourneau *et al.* (2010b) original study was limited in that the separate analysis of registration vs. notification implementation might not have captured a sequential (and progressive) effect of these interventions. Moreover, their analyses adhered to traditional Box-Jenkins intervention ARIMA models and did not consider investigating the relevancy (or redundancy) of the information content of the time-series inputs (past values of the series) to arrive at a desirable prediction/forecasting model (Lendasse *et al.*, 2001).

Our current focus in this manuscript is to examine the general deterrence of SC's SORN policy via non-linear intervention time series models, overcoming the limitations mentioned above. Driven by the primary goal of forecasting, development of non-linear time series models (such as our case of intervention time series which might include features like non-linearity, asymmetric cycles, conditional heteroscedasticity, etc) took both parametric (Tiao and Tsay, 1994) and non-parametric regression routes (Tong, 1995; Härdle *et al.*, 1997; Cai *et al.*, 2000 and Fan and Yao, 2003). In the quest of *reducing* the number of time-series inputs variables without sacrificing information and achieve parsimony, Park *et al.* (2009) considered a formal dimension reduction (DR) technique through *central mean subspace* (CMS) as a viable nonparametric alternative for nonlinear time series analysis, motivated by the approach adopted by Cook and Li (2002) for regression. Using notations, for a typical time series x_t where the focus is deriving inference on the conditional distribution $x_t|\mathbf{X}_{t-1}$, where $\mathbf{X}_{t-1} = (x_{t-1}, x_{t-2}, \ldots, x_{t-p})^T$ for some $p \geq 1$, this DR approach focusses on the conditional mean function $E(x_t|\mathbf{X}_{t-1})$ instead of the *time series central subspace* (TSCS) (Park *et al.*, 2010) which studies the dependence of x_t on $\mathbf{X}_{t-1}$. We follow that route in establishing a CMS for non-linear time series (CMS-ITS) analysis of SC sex crime activities which provides an unified framework to accommodate multiple interventions/changepoints, and seek parsimony in using time series inputs for attaining reliable forecasts.

The paper unfolds as follows. Section 2 provides further details on the motivating dataset. Section 3 develops the CMS-ITS methodology, and the estimation strategies. In Section 4, we presents a small numerical study to assess the performance of CMS-ITS for various types on interventions and changes in the mean function, and also apply the methodology to determine the effect of SC SORN on general deterrence. Section 5 discusses the salient features of our methodology with some concluding remarks.

2. Motivation: Data

The data on SC sex crime arrestees were extracted from SC's computerized criminal history records (CCHR) with assistance from the SC Budget and Control Board, Office of Research and Statistics. The CCHR data originated from individual precincts and courts, where initial charges were filed and adjudication decisions rendered, respectively. This information was forwarded to the SC Law Enforcement Division and added to the CCHR database. The current analysis focuses on first (initial) arrests for sex crimes by adult male arrestee defined as 18 years of age or older, that occurred between January 1st, 1990 and December 31st, 2005. To capture general deterrence, any second (and subsequent) sex crime by an arrestee within this time span was dropped, retaining only their first crime arrest. This resulted in

19,060 unique arrestees with a mean age (SD) of 33.96 (11.00) years, with race composition as 53.3% Whites, 44.3% African-Americans, and 2.4% others. Hence, our time-series data spans from 1990 to 2005 where we record average annual charges (or rates) for sex crime arrests per 10,000 adult male population. Population estimates were obtained from SC Community Assessment Network population data tables available at http://scangis.dhec.sc.gov/scan/index.aspx. Figure 1 plots the time-series profile of the rate of sex crime arrests, decomposed into trend, seasonal and error terms. Visual inspection reveals that the crime rates post-1995 appear to be smaller than pre-1995. There is also the effect of the internet-based notification of 1999 that adds to the 1995 effect. The overall goal of this data analysis is to accommodate these intervention effects during model building.

3. Definitions and Estimation Method

3.1. Central mean subspace in intervention time series

Let x_t denote an univariate time series and I_τ, the intervention at a specific time point τ. Suppose $\mathbf{X}_{t-1,l} = (x_{t-1}, \ldots, x_{t-p}, I_{1\tau}, I_{2\tau}, \ldots, I_{l\tau})^T$ for some $p \geq 1$, where l is number of changepoints (fixed, in advance). Following Park *et al.* (2009), we proceed with establishing a central mean subspace for (univariate) intervention time series (CMS-ITS). If this DR is focused on the conditional mean function $\mathrm{E}(x_t|\mathbf{X}_{t-1,l})$ only, we can simply detect intervention effects by extending CMS in time series. In other words, when the conditional mean with fixed number of interventions is our interest, DR depends on estimating a $(p + l) \times q$ matrix $\mathbf{\Phi}_q = (\Phi_1, \ldots, \Phi_q)$, where $q \leq (p + l)$ and l is number of changepoints, such that the q linear combinations $\mathbf{\Phi}_q^T\mathbf{X}_{t-1,l}$ contains all the information about x_t that is available from $\mathrm{E}(x_t|\mathbf{X}_{t-1,l})$. Simply saying, a nonparametric intervention time series type of model can be written as

$$x_t = f(\mathbf{\Phi}_q^T\mathbf{X}_{t-1,l}) + \eta_t \quad (1)$$

where g is an unknown smoothing link function and η_t is an error sequence. We assume that the CMS-ITS, $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}$, exists with $\dim(\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}) = d$. The theoretical framework of CMS-ITS is derived using two definitions and a proposition below.

Definition 1. *Let x_t be a univariate time series and l the number of interventions. Then, a subspace $\mathscr{S}(\mathbf{\Phi_q})$ of $\mathbb{R}^{p+l}$ for which $x_t \perp \mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{\Phi}_q^T\mathbf{X}_{t-1,l}$ holds is called a mean dimension reduction subspace for x_t on $\mathbf{X}_{t-1,l}$, where $\perp$ is the notion of stochastic independence, or orthogonality*.

The conditions in Proposition 1 below are equivalent for Definition 1.

Proposition 1. *(i) $x_t \perp \mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{\Phi}_q^T\mathbf{X}_{t-1,l}$, (ii) $Cov(x_t \mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{\Phi}_q^T\mathbf{X}_{t-1,l})=0$, and (iii) $\mathrm{E}(x_t|\mathbf{X}_{t-1,l})$ is a function of $\mathbf{\Phi}_q^T\mathbf{X}_{t-1,l}$ are all equivalent*.

The second condition indicates that x_t is not correlated to $\mathrm{E}(x_t|\mathbf{X}_{t-1,l})$ given $\mathbf{\Phi}_q^T\mathbf{X}_{t-1,l}$ and the third one implies $\mathrm{E}(x_t|\mathbf{X}_{t-1,l}) = \mathrm{E}(x_t|\mathbf{\Phi}_q^T\mathbf{X}_{t-1,l})$.

Proof: Obviously, (i) implies (ii). If (iii) holds, $\mathrm{E}(x_t|\mathbf{X}_{t-1,l})$ is a constant, given $\mathbf{\Phi}^T\mathbf{X}_{t-1,l}$. That is, it is independent of any other random variable. It is clear that (iii) implies (i). Then, we need to prove that (ii) implies(iii). From (ii), we have

$$\mathrm{E}[x_t\mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{\Phi}^T\mathbf{X}_{t-1,l}]=\mathrm{E}(x_t|\mathbf{\Phi}^T\mathbf{X}_{t-1,l})\mathrm{E}[\mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{\Phi}^T\mathbf{X}_{t-1,l}]. \quad (2)$$

Since

$$\mathrm{E}(x_t|\mathbf{\Phi}^T\mathbf{X}_{t-1,l})=\mathrm{E}[\mathrm{E}(x_t|\mathbf{X}_{t-1,l},\mathbf{\Phi}^T\mathbf{X}_{t-1,l})|\mathbf{\Phi}^T\mathbf{X}_{t-1,l}]=\mathrm{E}[\mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{\Phi}^T\mathbf{X}_{t-1,l}],$$

the right hand side of (2) is $\{\mathrm{E}[\mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{\Phi}^T\mathbf{X}_{t-1,l}]\}^2$. Similarly, the left hand side of (2) is

$$\begin{aligned}
&\mathrm{E}[x_t\mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{\Phi}^T\mathbf{X}_{t-1,l}]\\
&=\mathrm{E}\{\mathrm{E}[x_t\mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{X}_{t-1,l},\mathbf{\Phi}^T\mathbf{X}_{t-1,l}]|\mathbf{\Phi}^T\mathbf{X}_{t-1,l}\}\\
&=\mathrm{E}\{\mathrm{E}[x_t\mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{X}_{t-1,l}]|\mathbf{\Phi}^T\mathbf{X}_{t-1,l}\}\\
&=\mathrm{E}\{[\mathrm{E}(x_t|\mathbf{X}_{t-1,l})]^2|\mathbf{\Phi}^T\mathbf{X}_{t-1,l}\}.
\end{aligned}$$

Therefore, $Var[\mathrm{E}(x_t|\mathbf{X}_{t-1,l})|\mathbf{\Phi}^T\mathbf{X}_{t-1,l}] = 0$, which implies $\mathrm{E}(x_t|\mathbf{X}_{t-1,l})$, given $\mathbf{\Phi}^T\mathbf{X}_{t-1,l}$, is a constant. Therefore, (ii) implies (iii), and hence the proposition.

We propose that any of the three conditions in Proposition 1 could be taken as the definition of a mean DR subspace in intervention time series. In the following, we define the minimal mean dimension reduction subspace in intervention time series.

Definition 2. *Suppose* $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})} = \cap\mathscr{S}_N$, *where* $\cap$ *is over all time series mean dimension reduction subspaces* $\mathscr{S}_N$ *for* $\{x_t\}$. *If* $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}$ *is itself a intervention time series mean dimension reduction subspace, then it is called the CMS in intervention time series*.

The example given in Section 2 of Park *et al.* (2010) might cast some doubt on the existence of CMS-ITS. If $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}$ exists, then $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})} \subseteq \mathscr{S}_{x_t|\mathbf{X}_{t-1,l}}$ because the former is the intersection of a larger collection of subspaces. Therefore, it can reduce the dimension from that of $\mathscr{S}_{x_t|\mathbf{X}_{t-1,l}}$, if we are only interested in $\mathrm{E}(x_t|\mathbf{X}_{t-1})$. Cook (1998) and Yin *et al.* (2008) proved a similar argument for the existence of the CMS in intervention time series. Hence, we assume that the CMS in intervention time series, $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}$, exists with $\dim(\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}) = d$.

3.2. Estimation methods

At the onset, we assume d and p known in order to estimate the basis vectors $\Phi_1, \ldots, \Phi_d$ of CMS-ITS. Consider an objective function $\Upsilon(\mathbf{b}_d)=\mathrm{E}(x_t - f(\mathbf{b}_d^T\mathbf{X}_{t-1,l}))^2$, where $f(\mathbf{b}_d^T\mathbf{X}_{t-1,l})=\mathrm{E}(x_t|\mathbf{b}_d^T\mathbf{X}_{t-1,l})$. The goal is to minimize Υ with respect $\mathbf{b}_d$ such that $\mathbf{b}_d^T\mathbf{b}_d = I_d$. Clearly, $\Upsilon(\mathbf{b}_d) \geq \Upsilon(I_d) = \Upsilon(\mathbf{\Phi}_d)$ for any $\mathbf{b}_d$ and $\mathbf{\Phi}_d = \mathrm{argmin}_{\mathbf{b}_d}\Upsilon(\mathbf{b}_d)$, because of the existence of CMS-ITS and Condition (iii) in Proposition 1. We estimate the unknown mean function f using the well-known Nadaraya-Watson (NW, 1964) estimator with a product

Gaussian kernel. Here, any other nonparametric estimation can be used as long as a loss function can be formed and minimized over the dimensions. Using the NW estimator yields:

$$\hat{f}_{\delta_n}(\mathbf{b}_d^T\mathbf{x})=\frac{\sum_{i=1}^{n}K\left(\frac{\mathbf{b}_d^T\mathbf{x}-\mathbf{b}_d^T\mathbf{x}_{i-1}}{\delta_n}\right)x_i}{\sum_{j=1}^{n}K\left(\frac{\mathbf{b}_d^T\mathbf{x}-\mathbf{b}_d{}^T\mathbf{x}_{j-1}}{\delta_n}\right)}, \quad (3)$$

where K is a kernel function and $\{\delta_n\}$ is a sequence of bandwidths. As a convenient choice, we use a Gaussian kernel function in (3). Supposing $\mathbf{r} = (r_1, \cdots, r_d)^T$ and $\mathbf{W} = (W_1, \cdots, W_d)^T$, the product Gaussian kernel has the form

$$K[(\mathbf{r}-\mathbf{W})/\delta_n]=1/(n\prod_{j=1}^{d}b_{nj})\prod_{j=1}^{d}K[(r_j-W_j)/b_{nj}], \quad (4)$$

with bandwidth $b_{nj} = m_d s_j n^{-1/(d+4)}$ for $j = 1, \cdots, d$ and $m_d = [4/(d + 2)]^{1/(d+4)}$. s_j is the corresponding sample standard deviation of W_j and m_d is from Silverman (1986) or Scott (1992). Next, we minimize a sample version of $\Upsilon(\mathbf{b}_d)$, defined as

$$\hat{\Upsilon}_n(\mathbf{b}_d)=\sum_{t=1}^{n}(x_t-\hat{f}_{\delta_n}(\mathbf{b}_d^T\mathbf{X}_{t-1,l}))^2 \quad (5)$$

with respect to $\mathbf{b}_d$ such that $\mathbf{b}_d{}^T\,\mathbf{b}_d = I_d$. The minimizer $\hat{\Upsilon}_n(\hat{\Phi}_{n,d})$ is defined as the Residual Sum of Squares (RSS), where $\hat{\Phi}_{n,d} = \mathrm{argmin}_{\mathbf{b}_d}\,\hat{\Upsilon}_n(\mathbf{b}_d)$. For details on the computational procedure using `MATLAB`, see Park, Sriram, and Yin (2009). Consistency of the estimators for the CMS-ITS basis vectors follows from similar arguments as in Theorem 1 in Park, Sriram and Yin (2009).

Because the assumption of known dimension d and lag p in CMS-ITS is restrictive, we use the well-known Akaike information criterion (AIC; Burnham & Anderson, 2002) to develop a data-dependent approach for estimating d and p. For a fixed p, we determine $\hat{d}_p$ using the following AIC criteria;

$$\hat{d}_p=\arg\ \min_{1\le d\le p}\{n\log(RSS/n)+2d(p+l)\}. \quad (6)$$

For each p, we compute $\hat{d}_p$, then select the best $\hat{d}$ across all p. Next, using this $\hat{d}$, we find $\hat{p}$ similarly using the following criteria;

$$\hat{p}=\arg\ \min_p\{n\log(RSS/n)+2\hat{d}(p+l)\}. \quad (7)$$

This procedure determines the best estimated d and p. In the following two sections, we investigate how this methodology works in the context of our motivating dataset, and also its performance in a simulation setting.

4. Application

4.1. Simulation study

Before working on the data set, we carry out a small Monte Carlo simulation study to assess finite sample performance of our CMS-ITS proposition. In addition, we illustrate that this method is viable even when mean function is nonlinear, such as the cosine function. Since any real data may not be restricted to the simple linear function, we believe that this will provide a potential research tool for the further studies into this direction. Here, we will use a distance based measure to assess the accuracy of our estimates which calculates the distance between the real CMS and the estimated CMS in the ITS. This was adopted by Ye and Weiss (2003) and is defined as $\rho = |\hat{\mathbf{\Phi}}_{n,d}^T \mathbf{\Phi}_d \mathbf{\Phi}_d^T \hat{\mathbf{\Phi}}_{n,d}|^{0.5}$, where $|\mathbf{C}|$ denotes the determinant of a matrix $\mathbf{C}$. Note that $0 \le \rho \le 1$, and when $\rho = 1$, $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}(\hat{\mathbf{\Phi}}_{n,d}) = \mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}(\mathbf{\Phi}_d)$. Hence, if ρ value is close to one, it implies that the two spaces are closer and the estimates are more accurate. In addition, Xia, *et. al.* (2002) propose a distance measure between $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}(\hat{\mathbf{\Phi}}_{n,q})$ and $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}(\mathbf{\Phi}_d)$ using $m^2 = \left\|(I - \hat{\mathbf{\Phi}}_{n,q}\hat{\mathbf{\Phi}}_{n,q}^T)\mathbf{\Phi}_d\right\|^2$ for $q \ge d$ and $m^2 = \|(I - \mathbf{\Phi}_d \mathbf{\Phi}_d^T)\hat{\mathbf{\Phi}}_{n,q}\|^2$ for $q < d$. Similarly, as m^2 approaches zero, the two spaces are closer and the estimates are more accurate. We simulated three models with varying number (one, two, and three) of fixed (and known) interventions. For sample sizes $n = 100$, 200 and 500, we assessed accuracy of estimates based on 500 Monte Carlo replications. The error term $\{\varepsilon_t\}$ in all our models considered below follows independent standard normal random density. The models are as follows:

Model 1: $x_t = \cos[\frac{\pi}{2}\frac{1}{\sqrt{3}}(x_{t-1} + x_{t-3} - I_{1\tau})] + \varepsilon_t$.

Model 2: $x_t = \cos[\frac{\pi}{2}\frac{1}{\sqrt{4}}(x_{t-1} + x_{t-3} - I_{1\tau} - I_{2\tau})] + \varepsilon_t$.

Model 3: $x_t = \cos[\frac{\pi}{2}\frac{1}{\sqrt{5}}(x_{t-1} + x_{t-3} - I_{1\tau} - I_{2\tau} - I_{3\tau})] + \varepsilon_t$.

each of which has $p = 3$ and $d = 1$. Table 1 reports the average values of ρ and m^2, respectively, for the above models. For all cases, the distance based measures of ρ and m^2 remain close to 1 and 0, respectively, getting further closer with increasing sample size. Even with higher number of interventions, e.g., three changepoints, both measures indicate the estimation procedure to be accurate.

4.2. Data Analysis

Now, we apply our methodology to the motivating time-series dataset on yearly rates of SC sex crime arrests. Similar to the benchmark analysis of the Los Angeles air pollution data in Wei (2006), we build an interrupted ARIMA model including two changepoints corresponding to the 1995 registration and 1999 internet notification. For a genuine forecasting study (as suggested by a reviewer), the sample is split into a training sample (January 1990 – December 2004) for estimation and a hold out sample (January 2005 – December 2005) for forecasting. For the estimation, the period from January 1990 to

December 1994 is assumed to be free of intervention effects and is used to estimate the noise model for N_t. As shown in Figure 2, the sample autocorrelation function (ACF) within this period suggests nonstationary and highly seasonal behavior with period 12. Both ACF and Partial ACF (PACF) of the seasonally differenced series $(1 - B^{12})N_t$, where B is backshift operator, has significant spike at lag 12. Since seasonal moving average component was not significant, it was not included in the model. Finally, we arrive at the following ARIMA$(1, 1, 0)_{12}$ model for the pre-intervention period given by:

$$N_t = \frac{1}{(1-\Phi B^{12})(1-B^{12})} a_t \quad (8)$$

where a_t is a white noise, $(1 - B^{12})$ is first order seasonal differencing and Φ is seasonal autoregressive component at lag 12. After adding two changepoints, we arrive at the final model:

$$x_t = \omega_1 I_{1\tau} + \frac{\omega_2}{(1-B^{12})} I_{2\tau} + \frac{1}{(1-\Phi B^{12})(1-B^{12})} a_t \quad (9)$$

where ω_1 and ω_2 are intervention coefficients; $I_{1\tau}$ and $I_{2\tau}$ are dummy binary variables, where $I_{1\tau}$ is 0 for all months between January 1990 – December 1994, and 1 for all months between January 1995 – December 2004, and $I_{2\tau}$ is 0 for all months between January 1990 – December 1998, and 1 for all months between January 1999 – December 2004. The introduction of dummy variables $I_{1\tau}$ and $I_{2\tau}$ resembles a stepwise (sequential) pattern of incorporating the effects of the two interventions to the time-series. Considering the additive effect of $I_{1\tau}$ and $I_{2\tau}$ on the series, the period from the beginning of the series to December 1994 remains intervention-free, the period from January 1995 – December 1998 is assigned with 'only 1995' effect, while the period from January 1999 – December 2004 receives the effect of 'both 1995 and 1999' interventions. Estimation results of the above model are summarized in Table 2, which indicates there were approximately 7.6% decrease in the rate of sex crime offense charges per 10,000 population post 1995. While ω_1 (the 1995 co-efficient) is statistically significant, ω_2 (the 1999 indicator) is not.

Next, we apply our CMS-ITS method. We proceed with estimating d and p, followed by the estimation of basis vectors of $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,l})}$ in univariate time series with two changepoints, x_t vs $\mathbf{X}_{t-1,2} = (x_{t-1}, \ldots, x_{t-p}, I_{1\tau}, I_{2\tau})^T$. To estimate d, we included the lag x_{t-12} as predictor (to control for the strong seasonality), and then compute the AIC values in (6) for $d = 1, 2, 3, 4$ and $p = 1, \ldots, 13$. For each p, $\hat{d}_p$ is indicated by an asterisk in Table 3. According to AIC criteria in (6), $\hat{d}_p = 1$ for all $1 \le p \le 13$. Since $\hat{d}_p = 1$ uniformly for $1 \le p \le 13$, we decided to use $d = 1$. Under $\hat{d} = 1$, the AIC values in Table 3 indicates that $\hat{p} = 12$ yields the smallest AIC value in (7). From these, we conclude that the estimates of d and p for this data are $\hat{d} = 1$ and $\hat{p} = 12$. This procedure gives the estimate of the 14×1 basis vector Φ_1 of $\mathscr{S}_{\mathrm{E}(x_t|\mathbf{X}_{t-1,2})}$, i.e.

$$\hat{\Phi}_1^T = (0.3788, 0.2861, -0.0017, -0.2259, -0.3386, -0.2305, -0.1288, -0.1340, 0.0240, 0.1926, 0.4076, 0.4509, -0$$

. A time series plot of t vs $d_{1,t} = \hat{\Phi}_1^T \mathbf{X}_{t-1,2}$ and t vs x_t is provided in Figure 3. Evidently, $d_{1,t}$ almost itself nearly captures the periodic fluctuations and dynamics of the entire series, delineating the effect of the two interventions. This happens even prior to the construction of an adequate parsimoniously parameterized model for the data. We now use the estimates above, and the following approach involving plots to build a intervention model.

To test the impact of interventions alone, the CMS-ITS theory requires estimating the direction(s) in the regression of x_t, which includes the (i) coefficients corresponding to the lagged values of x, and (ii) the coefficients corresponding to the interventions. The components corresponding to these two (separate) directions are $d_{2,t}=\hat{\Psi}_1^T\mathbf{X}_{2,t-1}$ (for the lagged x) and $d_{3,t}=\hat{\Theta}_1^T\mathbf{X}_{3,t-1}$ (for the interventions), where $\mathbf{X}_{2,t-1} = (x_{t-1}, \ldots, x_{t-p})^T$, $\mathbf{X}_{3,t-1} = (I_{1\tau}, I_{2\tau})^T$, $\hat{\Psi}_1 = (\hat{\psi}_1, \ldots, \hat{\psi}_{12})$ and $\hat{\Theta}_1 = (\hat{\theta}_1, \hat{\theta}_2)$. Next, we examine the two-dimensional plots of x_t versus $d_{2,t}=\hat{\Psi}_1^T\mathbf{X}_{2,t-1}$ and x_t versus $d_{3,t}=\hat{\Theta}_1^T\mathbf{X}_{3,t-1}$. While the former plot roughly displayed a cubic pattern, the latter one was linear. Finally, since the interactions between $d_{2,t}$ and $d_{3,t}$ was not significant, the regression of x_t on the predictor series $d_{2,t}$ and intervention effect $d_{3,t}$ yields:

$$x_t=0.6571+0.2753d_{3,t}+1.0975d_{2,t}^2-0.8477d_{2,t}^3+\eta_t \quad (10)$$

where η_t is an error sequence and the coefficients in the above model found to be significant with standard error (p-value) estimates being 0.0199 (<2e-16) for the intercept, 0.0449 (6.43e-09) for $d_{3,t}$, 0.1851(1.76e-08) for $d_{2,t}^2$ and 0.2144 (0.0001) for $d_{2,t}^3$. Hence, there is statistical significance for both the changepoints, with an approximate reduction of 8.76% (=0.2753×−0.3180) for 1995 registration and 3.14% (=0.2753×−0.1142) for 1999 internet notification in the rate of sex crime charges per 10,000 population, respectively. The autocorrelation check the error sequence η_t revealed a white noise.

Finally, we compare the forecasting ability of our semi-parametric model in (10) with the interrupted ARIMA model given in (9) using Relative Mean Square Error (RMSE) given by $n^{-1}\sum_{t=1}^{n}\{(x_t-\hat{x}_t)^2/x_t\}$ and the Mean Absolute Percentage Error (MAPE), given as $n^{-1}\sum_{t=1}^{n}\{|x_t-\hat{x}_t|/x_t\}$, using the hold out sample from January to December 2005. The estimates of RMSE and MAPE for our semi-parametric model as in (10) were 0.0133 and 0.1117, while those for the intervention ARIMA model were 0.0156 and 0.1168 respectively. We find evidence of a slightly better predictive performance for our model. Figure 4 compares the predicted (forecasted) values from the fit of the parametric and semi-parametric DR models to the hold out sample, with the true rate of sex crime arrests overlayed.

5. Conclusions

In this paper, we introduced a new semi-parametric DR scheme for ITS modeling when the mean function of the series (rather than the whole conditional distribution) is of interest. Most of the existing DR approaches such as sliced inverse regression (SIR, Duan and Li, 1991) pertain to various contexts of regression, and are not easily applicable to the context of dependent data, such as time-series. Our estimation method uses the Nadaraya-Watson kernel smoother for mean function estimation in the context of CMS-ITS. In our setup, the nonparametric part that uses the NWestimator behaves as an intermediate step to achieve the primary goal of obtaining directions through DR. Hence, the inaccuracy involved in the density estimation can be overlooked as long as the shapes of the estimated densities resemble the true ones, and do not severely affect the estimation of directions. Our Gaussian product kernel behaved adequately in simulation and real data analysis reducing concerns related to choices of kernels and bandwidths typical in the context of nonparametric regression. A data-dependent approach using AIC was used to estimate the dimension d and lag p.

Amidst the challenges posed by the theory of CMS, our proposed methodology is a reasonable contender to the usual intervention ARIMA modeling with respect to model fit and forecasting performance. Our proposed methodology provided a slightly superior forecasting performance, which is the cornerstone of any time-series modeling. Although challenging, future research might consider devising methods of DR in time-series with an unknown number of changepoints, or determining/estimating a changepoint. So far, our methodology focussed on ‘central’ and ‘mean’ subspaces for time-series. However in some situations (such as financial time series), interest might focus on other parts of the data density, say the quantiles, or tails, and one might undertake time series DR starting from a *Quantile* subspace to study the (conditional) quantiles.

The results from this analysis advances the application area to a great extent. Prior time-series analysis of the yearly sex crime rates for SC (Letourneau *et al.*, 2010b) considered separate intervention ARIMA models for 1995 and 1999, and failed to consider the sequential effect of the changepoints on the sex crime arrest rates within an unified framework. Our current ARIMA modeling considers using the two (known) changepoints within the same model. This resulted in a non-significant effect of the internet-based policy of 1999. This is in parity to the Letourneau *et al.* (2010b) analysis. In contrast, our semi-parametricmodel also produced a significant 1999 effect in addition to the previously identified significant 1995 effect. It is worthy to note that the amount of reduction post-1999 intervention was minimal (3.14%) which suggests that the deterrent effect might be driven by the original SORN policy of 1995 while the effect of 1999, though not substantial, still remains. In their original article, Letourneau *et al.* (2010a) concluded that the 1999 internet-based notification should be repealed, considering results that this mechanism was, at best, ineffective (i.e. associated neither with general or specific deterrence) and at worst iatrogenic, given the association these researchers found in separate studies evaluating the effects of notification and registration on plea bargains (which increased significantly following registration and again following notification) and on convictions (which decreased significantly following implementation of SC’s online notification policy; Leteouneau *et. al*, 2010b). If public notification of adult sex crime arrestee indeed fails to deter sexual violence via specific and general deterrence, results in unintended consequences on the prosecution of these cases such that fewer arrestees are convicted of sex crimes, and impairs arrestee’s ability to successfully reintegrate into their communities as law-abiding citizens (as Levenson and Hern, 2007 have forcefully argued), then its repeal is a straightforward policy recommendation. The present study found a slight, albeit significant increase in the deterrence of first-time sex crimes attributable to online notification. The majority of the deterrent effect appears to be due to the original registration policy, which included limited notification. In consideration of the fact that online notification achieves minimal deterrence but is associated with significant harm to the reintegration efforts of ex-arrestees, significant increases in plea bargains from sex to non-sex crimes, and significantly reduced likelihood of guilty convictions for sex crime charges, our original position still seems defensible that SC's online notification should be revised.

We also carefully caution the reader that in the absence of more rigorous research designs (e.g., randomization to conditions), causal attributes cannot be made with certainty. Underlying and hidden factors could have altered other aspects of the justice system resulting in the observed decline in first-time sex crimes. For example, police officers might have altered their charging decisions upon initial arrest or their calculus regarding which crimes to pursue in the first place. Moreover, there is the vexing problem of under-reporting of sex crimes (Smith *et al.*, 2000), although little reason to suspect that underreporting would become more prevalent in the later years in this data set.

The present study is a first of its kind examining broadly the deterrent effects of SC's SORN policy on first time sex crime arrestees using an innovative time-series DR approach. Our findings add to the growing research base evaluating the actual effects of these popular policies, and draws attention to a very wide but mentally disadvantaged section of our society. However, more research related to sex crime policies as implemented in other states and jurisdictions is needed, both to validate the general deterrent effect identified in the present study and to elucidate the parameters under which such deterrent effects are more or less likely to occur. Future research should also evaluate whether any deterrent effect on first-time sex arrests is offset by increases in other types of crime as might be seen in a substitution effect (Ehrlich, 1996). The present study is limited particularly by the absence of additional explanatory variables that might control for confounding factors, e.g., policy changes that applied more broadly to violent crimes (Vujic *et al.*, 2012). It will be important for future evaluations to include such factors and also to include suitable control groups (e.g., another state in the U.S. whose SORN policy was implemented later than, or very differently than, the index state's policy). Such controls might, for example, allow for addressing the counterfactual question of what would have happened to general deterrence in SC without the intervention, facilitating a more reliable estimation of policy effects via multivariate structural time series models (Havey, 1996). Despite these limitations, the present study appears to provide a new and more robust method for assessing policy effects using time series data.

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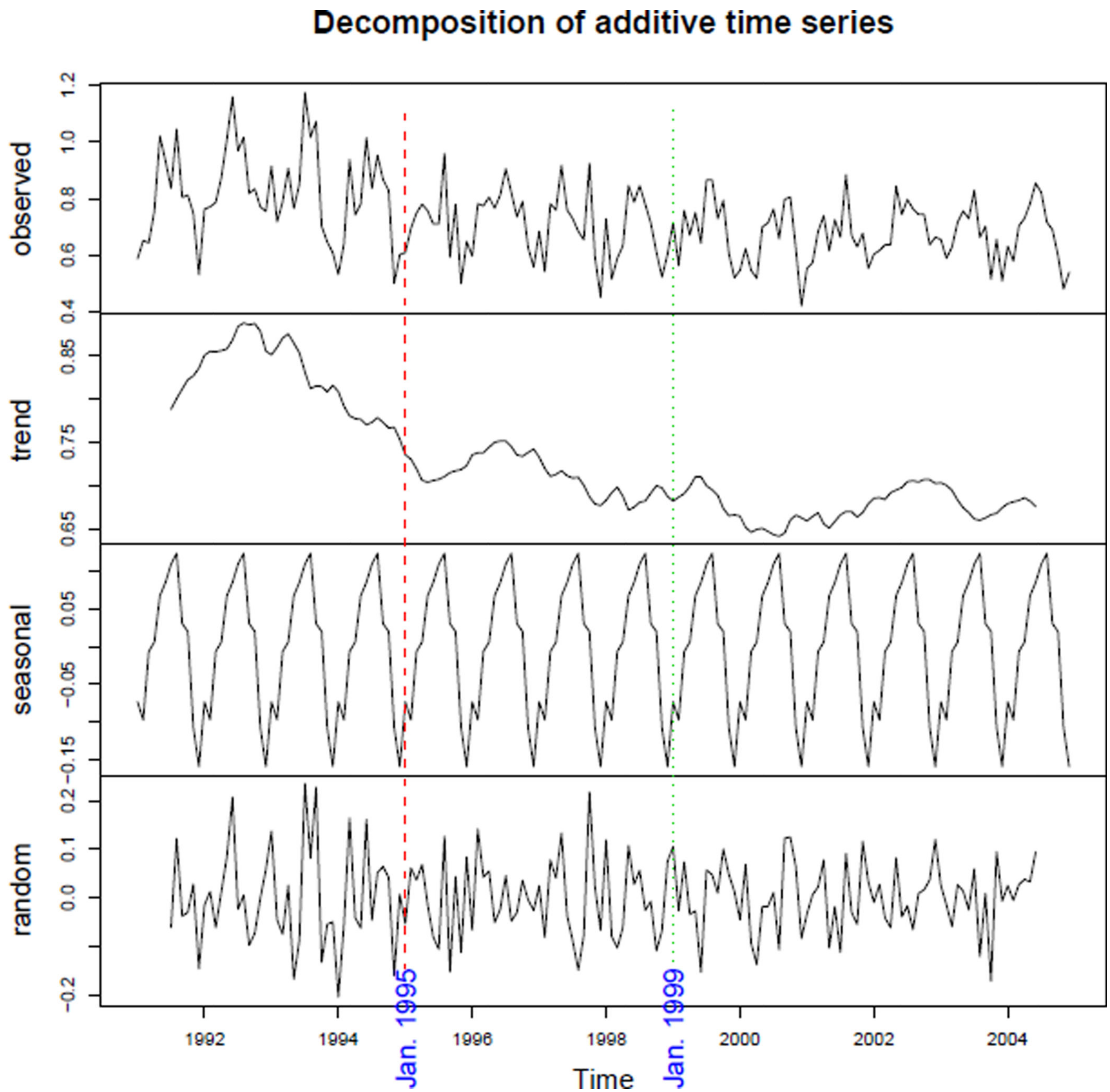

Figure 1.

Time series of the sex crime arrestee data decomposed into trend, seasonal, and error terms. The 1995 and 1999 interventions are indicated via. vertical lines.

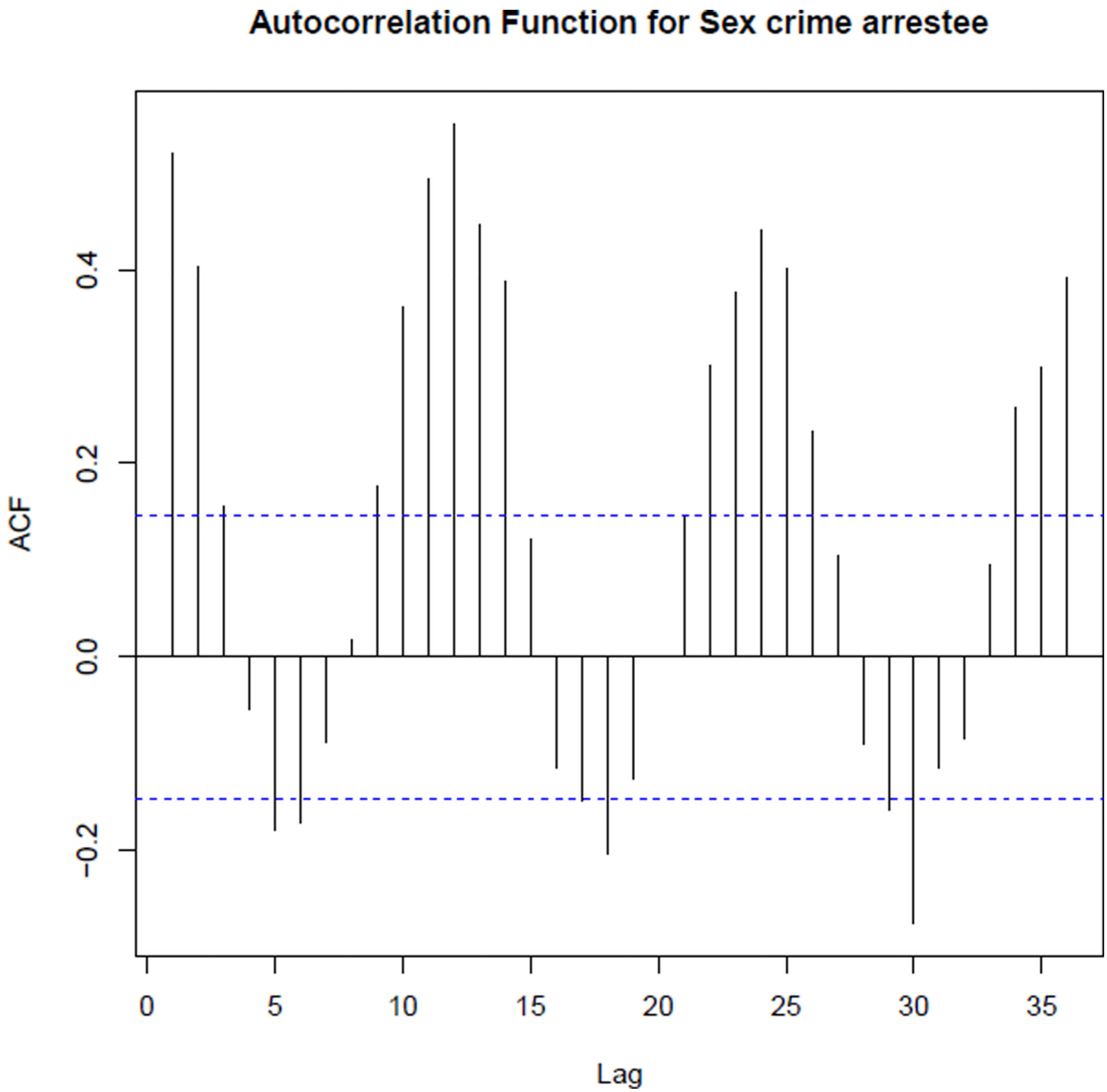

Figure 2.
Autocorrelation Function (ACF) plots from the sex crime arrestee data for the period January 1990 – December 2004.

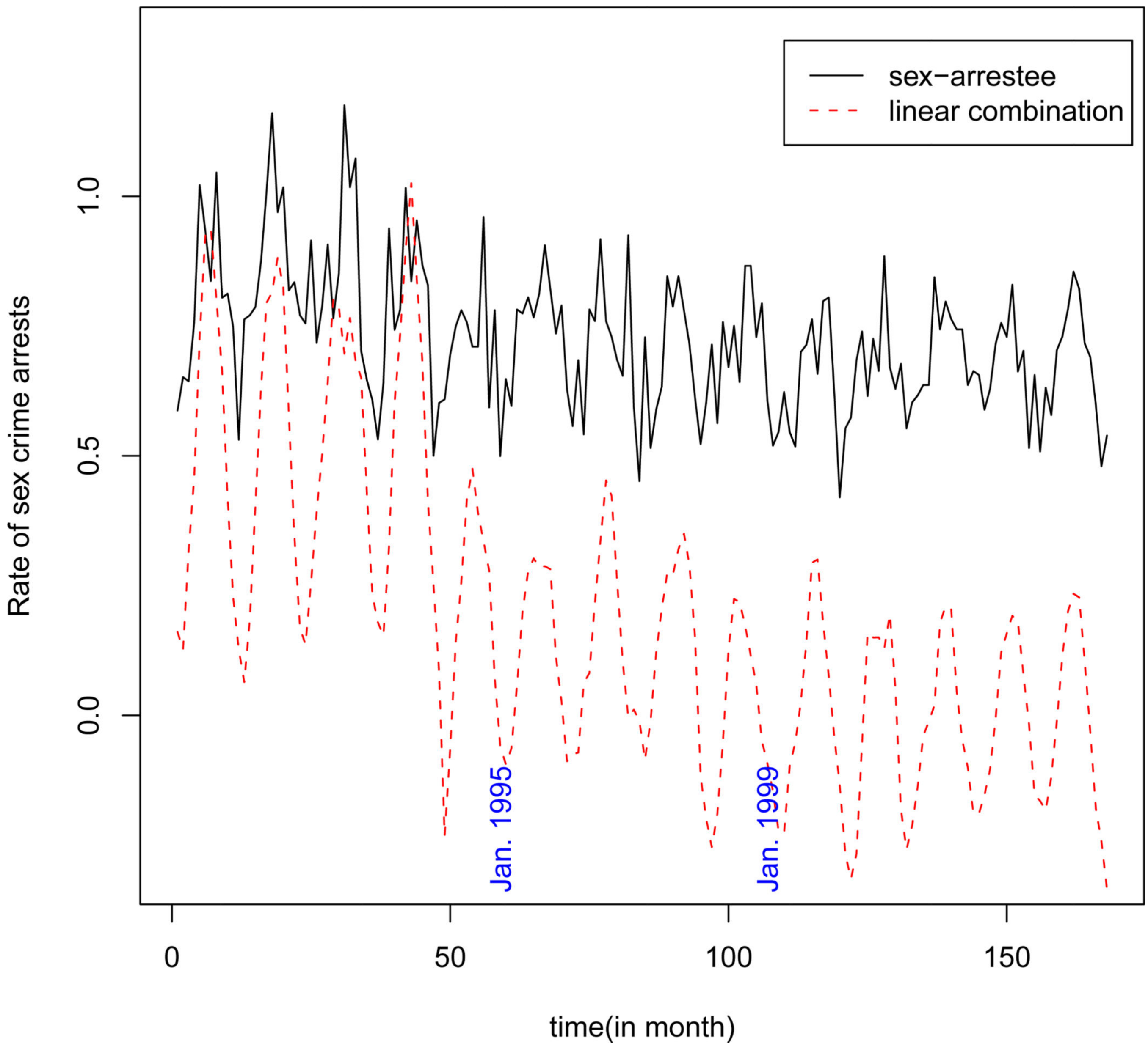

Figure 3.

Time series plot of t vs x_t (solid line) and t vs $d_{1,t}$ (dashed line) for the sex crime arrestee data. The intervention points (Jan, 1995 and Jan, 1999) are indicated.

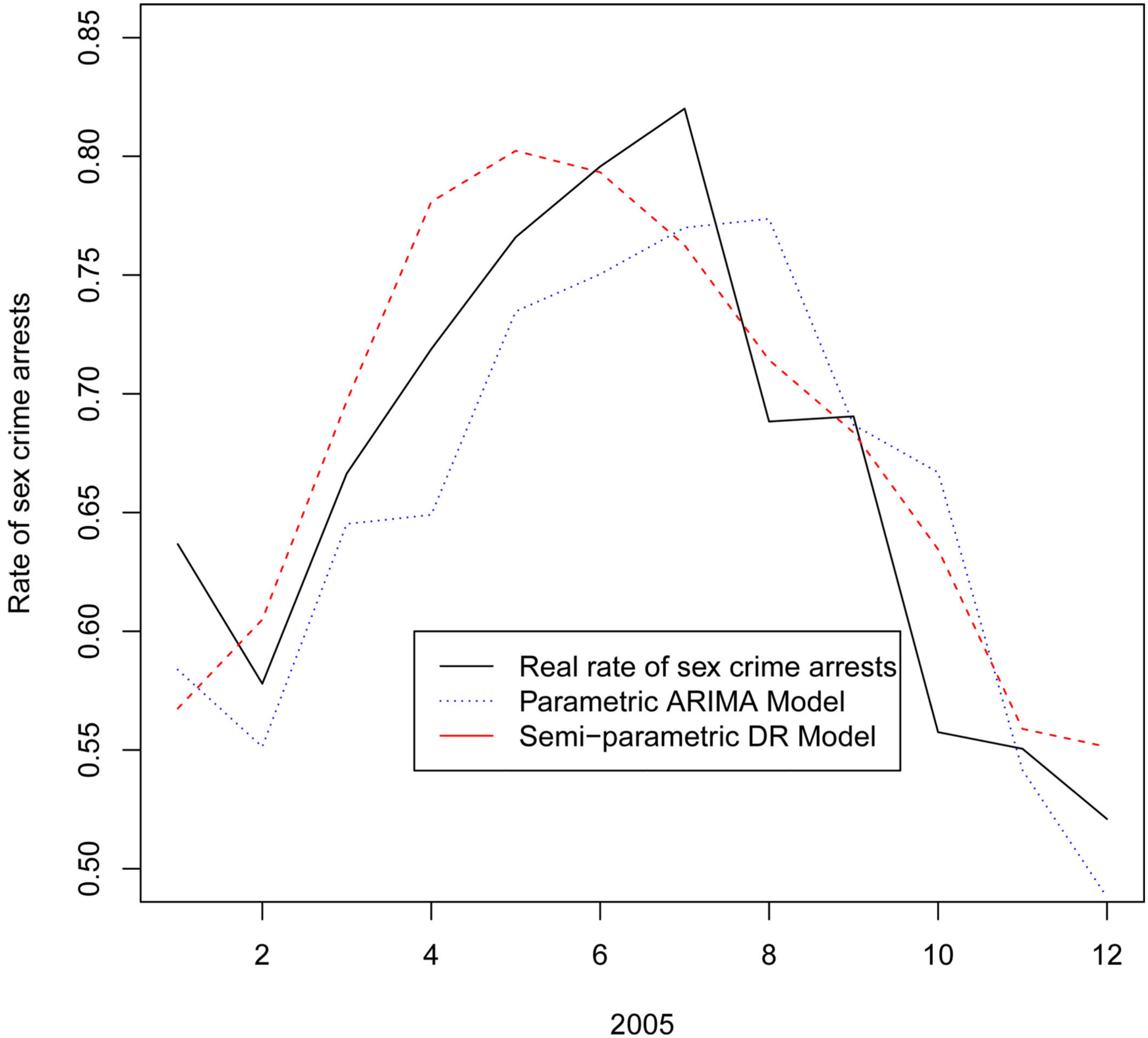

Figure 4.

Comparing forecasts for the period January 2005 – December 2005 from the sex crime arrestee data. The true x_t values are indicated by the solid line, and overlayed with the fitted values from our semi-parametric DR model (10) (dotted line) and the interrupted ARIMA model (9) (dashed line).

Table 1

Simulations results based on 500 Monte Carlo replications. Average values of ρ and m^2 are reported for different number of interventions.

Sample Size	Model	ρ	m^2	Interventions
100	1	0.9596	0.0761	$\tau = 50$
	2	0.9477	0.0962	$\tau = 30, 50$
	3	0.9247	0.1322	$\tau = 30, 50, 70$
200	1	0.9806	0.0381	$\tau = 100$
	2	0.9626	0.0724	$\tau = 50, 150$
	3	0.9585	0.0791	$\tau = 50, 100, 150$
500	1	0.9908	0.0182	$\tau = 250$
	2	0.9876	0.0245	$\tau = 150, 350$
	3	0.9849	0.0299	$\tau = 150, 250, 350$

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Table 2

Parameter estimates from the interrupted ARIMA Model

Parameter	Estimate	St. Error	P-value
ω_1	−0.0758	0.0303	0.0124
ω_2	−0.0035	0.0097	0.7187
Φ	−0.4300	0.0705	<.0001

Table 3

Detecting *d* and *p*:

p	*d*=1	*d*=2	*d*=3	*d*=4
1	−745.21			
2	−747.95*	−747.29		
3	−745.95*	−743.29	−734.55	
4	−752.55*	−750.91	−741.60	−730.96
5	−758.26*	−756.50	−746.76	−734.21
6	−760.25*	−756.76	−746.62	−732.11
7	−759.60*	−752.76	−742.11	−725.62
8	−757.60*	−748.76	−736.11	−719.14
9	−754.25*	−743.33	−730.11	−711.14
10	−753.60*	−742.21	−727.14	−706.22
11	−758.50*	−745.62	−728.93	−706.15
12	−766.62*+	−754.15	−734.44	−711.61
13	−766.11*	−753.42	−731.87	707.14

For fixed $p = 1, \ldots, 13$, the SBC values are presented for $d = 1, 2, 3$ and 4, with the smallest value of SBC (in each row) denoted by a *.

The smallest SBC in the whole table is denoted by +.