Keep or Return? Managing ordering and return policies in start-up companies

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Abstract

Start-up companies are considered an important factor in the success of a nation's economy. We are interested in the decisions for long-term survival of these firms when they have considerable cash restrictions. In this paper we analyse several inventory control models to manage inventory purchasing and return policies. The Markov decision models are formulated for both established companies that look at maximising average profit and start-up companies that look at maximising their long-term survival probability. We contrast both objectives, and present properties of the policies and the survival probabilities. We find that start-up companies may need to be riskier if the return price is very low, but there is a period where a start-up firm becomes more cautious than an established company and there is a point, as it accumulates capital, where it starts behaving as an established firm. We compare the various models and give conditions under which their policies are equivalent.

Keywords: Inventory, Markov Processes, Dynamic programming

1 Introduction

Start-up firms are an important component of the growth rate of any nation's economy both in the production and service sector as they increase competition, drive innovation and generate jobs. However, such companies have a high risk of failure in their initial phases where there are serious capital restrictions. There is a need for models that aid decisions and provide insight into policies for long-term survival, where one has to balance levels of inventory against the loss of profit if there is unfulfilled demand. This is particularly the case in retail environments, where there are a large number of franchise start-ups. In these, the entrepreneur has to take the financial risk, including the possibility of running out of capital, while the franchising organisation looks after marketing and quality control issues. Such arrangements occur throughout the retail business from fast foods, through fashion wear to maintenance contracts and professional services. In such retail and franchise operations, there is also the question of what to do with unsold stock. Should it be kept for another period, should all of it be returned (usually at a lower price) to the wholesaler or franchiser, or should some of the stock be returned and some kept. We investigate the characteristics of optimal policies in inventory purchase and stock return for start-ups that are interested in their long-term survival and compare them with those of well-established firms (interested in maximising their average profit). We develop several Markov decision process models to analyse the trade off between keeping unsold items and returning them at lower prices in order to have liquidity to face overhead costs. The overhead costs typically include the recurrent cost of employee wages, equipment lease charges and rent of premises. The average profit case is an extension of the well known newsvendor model (see [7], chapter 10) of

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operations management where it is assumed all unsold stock will be returned each period. Allowing stock to be kept changes the problem from a renewal problem to a Markov decision problem. As well as comparing the three situations — keep all unsold product, return all unsold product and decide how much to return — in the average reward case, we also look at the problem faced by start-up companies of maximising the survival probabilities in these three situations.

This has a connection with the author's previous work on survival strategies (see [1, 5 and 8]). However these papers, together with related work of Buzacott et al [4] and Babich and Sobel [3] have dealt with manufacturing start-ups, where components once purchased have to be kept. Thus, this related problem extends such problems by including return as well as ordering decisions.

The paper is organised as follows. In Section 2 we describe the three types of problems analysed. We present the dynamic programming recursion for the various Markov decision processes both for profit and long-term survival objectives. In Section 3 we analyse the properties under the profit maximising objective for established firms. We find optimal purchasing and return policies and the value of the average expected profit for the models. We also state conditions under which the different types of problem have the same optimal policies. In Section 4 we analyse the properties of the best policies for start-up companies under the probability of survival objective. We prove that if there is a positive average profit then the survival probability in the long run is also positive. The optimal survival policies say the firms should prefer cash to "equivalent" inventory and there are monotonic properties of the survival probability in terms of cash available and time horizon. However the companies may have to gamble and order more than the profit maximising order quantities if they are in a difficult situation but will become more conservative when the situation is improved. When the firm has acquired enough capital, all ordering and return policies will maximise its survival probability, in that it can always recover from any unlucky combination of inventory ordered and demand. Thus thereafter the firm should follow the optimal average reward policy (unless of course a run of unfortunate demands lowers its capital to a level where survival again becomes an issue). In section 5 we look at the special case where there is no holding cost. In this case we are able to show that if we keep all inventory or can decide how much to return then the purchasing policy of start-up franchises should be more cautious than that for established firms. We give our conclusions in Section 6.

2   Models

We consider simplified models where the company purchases only one type of product from a wholesaler. We want to find optimal ordering quantities for this product. Let S denote the selling price per unit, and C the wholesaler purchase price for the product, with H denoting the fixed overhead cost. The overhead costs typically include the recurrent cost of employee wages, equipment lease charges and rent of premises. Let R be the return price of any unsold product which is returned at the end of the period ($R \leq C$). Let h be the cost of holding one unit of inventory for one period. We argue that in the survival probability models this is not really the cost of capital since we explicitly incorporate the capital available in the model, but the holding cost does reflect the real interest rate (difference between inflation and actual interest rate), and the insurance, handling and storage costs involved in holding inventory. We consider the scenarios where the holding cost is applied to:

  i) items in inventory at the beginning of the period;

 ii) items in inventory at the beginning of the period or purchased during the period;

iii) items in inventory at the beginning of the period or purchased during the period that are not sold;

iv) items in inventory at the beginning of the period or purchased during the period that are not sold or returned.

We consider a zero lead-time for replenishment of inventory and assume that the demand for the product is random with independent identical distributions each time period. The stationarity of the demand distribution is reasonable in the case of start-up franchise operations because the general marketing of the product will have created the demand before the franchise begins its operation. Otherwise one is in the situation where the start-up company needs to consider not just its inventory decisions but also its marketing and advertising decisions which can stimulate the demand. A manufacturing version of these problems was considered in [2]. Let $p(d)$ be the probability of there being a demand of d units in a period, and let $P(x \leq d)$ be the probability that the demand is less than or equal to d. Let M be the maximum possible demand that can be satisfied in a period (it can also be interpreted as the maximum number of items that the company can hold in its premises to be sold in a given period); $M = \max\{d|p(d) > 0\}$. We now formulate the different inventory problems as dynamic programming models under the criteria of maximising the survival probability and of maximising the average profit. We will denote by g the average reward and by q the probability of survival. When necessary we will use a sub-index to distinguish between the different models.

2.1 Must return all items at the end of the period

In this model the company is forced to return any unsold items at the end of the period. This model is particularly appropriate when the items are perishable or are quickly out of fashion. If k is the number of items purchased and d is the realised demand in a given period, then $S \min(k, d)$ will be the cash received from sales and $R \min(k - d, 0)$ the cash from returns. With holding cost scenarios 1 and 4, the holding cost clearly has no effect on profit, because the inventory level is zero at the beginning and end of each period. Scenario 2 can be allowed for by modifying the purchase cost $(C \to C + h)$, while scenario 3 can be allowed for by modifying both the purchase cost $(C \to C + h)$ and the selling price $(S \to S + h)$. The following formulations therefore cover all four holding cost scenarios. The maximum average profit can be calculated as

$$g_{PR} = \max_k \left\{ \sum_{d=0}^{M} p(d) \left(S \min(k, d) + R \max(k - d, 0) - Ck - H \right) \right\}. \tag{1}$$

Under the average profit objective this model is equivalent to the well-known newsvendor problem where the shortage cost is zero, see [7] chapter 10. As stated before we believe this is a valid objective for an established firm. If the average reward becomes negative in a given situation then the natural course of action would be to close down its operations. Now consider a start-up company that seeks to maximise its survival probability. Let $q_{SR}(n, x)$ be the maximum probability of surviving n more periods with x units of capital. $q_{SR}(n, x)$ is the optimal function for a finite-horizon dynamic programming problem with countable state space (x can be assumed to have discrete levels) and a finite action space — the amount k to order. Thus, it has an optimal non-stationary policy (see [6], p.90). Hence, the survival probability satisfies the following dynamic programming optimality equation:

$$q_{SR}(n, x) = \max_k \left\{ \sum_{d=0}^{M} p(d) q_{SR} \left(n - 1, x + S \min(k, d) + R \max(k - d, 0) - Ck - H \right) \right\} \tag{2}$$

with boundary conditions $q_{SR}(0, x) = 1$ if $x \geq 0$ and $q_{SR}(n, x) = 0$ for any n and $x < 0$. That is, if the capital is negative at any point in time the probability of survival will be zero. Let $q_{SR}(x)$ be the probability of survival in the long run with capital x, then $q_{SR}(x) = \lim_{n \to \infty} q_{SR}(n, x)$. We denote by k_{PR} and k_{SR} the optimal purchasing policies of the model under profit maximising and survival probability respectively.

2.2 Must keep all items

In this model any unsold items cannot be returned at the end of the period and so are kept in stock. This is an appropriate model when the wholesaler does not receive items back once they are delivered. With holding cost scenario 1, the effect of the holding cost is to reduce the profit by ih where i is the inventory level at the beginning of the period. With scenario 2, profit is reduced by a further kh units, where k is the number of items purchased, but this can be allowed for by modifying the purchase cost $(C \to C + h)$. As returns are not possible in this model, scenarios 3 and 4 are equivalent. After modifying the purchase cost $(C \to C + h)$ and the selling price $(S \to S + h)$, the only effect of the holding cost is to reduce the profit by ih. The following formulations therefore cover all four holding cost scenarios. When $h = 0$, this is in effect the zero-lead time version of the model studied in [1]. If the firm is well established, then we assume its objective is to maximise the average profit per period. In this case, there is no constraint on the amount of capital the firm has since it is assumed it has enough to finance any purchase it wants. Thus, the state of the firm at the start of any period is described completely by the number of components in stock. This is a countable state, finite-action, unichained Markov decision process, and so the standard results for average reward Markov decision processes hold (see [6]). Let g be the average profit and let $v(i)$ be the bias term of starting with i items in stock. Then, the optimality equation for this dynamic programming model satisfies

$$g_{PK} + v_{PK}(i) = \max_k \left\{ \sum_{d=0}^{M} p(d) \left(S \min(i + k, d) - Ck - H - hi + v_{PK}(\max(i + k - d, 0)) \right) \right\}. \quad (3)$$

On the other hand, if the company is a start-up firm with considerable capital restrictions, it will not survive if it uses up all its capital. Hence at any point the state of the firm is described by two variables: i the number of items in stock, and x the capital of the firm. The maximum probability of surviving n more periods with i items in inventory and x capital is

$$q_{SK}(n, i, x) = \max_k \left\{ \sum_{d=0}^{M} p(d) q_{SK} \left(n - 1, \max(i + k - d, 0), x + S \min(i + k, d) - Ck - H - hi \right) \right\} \quad (4)$$

where $q_{SK}(0, i, x) = 1$ if $x \geq 0$ and $q_{SK}(n, i, x) = 0$ for any n and $x < 0$. Let $q_{SK}(i, x)$ be the probability of survival in the long run with inventory i and capital x, then $q_{SK}(i, x) = \lim_{n \to \infty} q_{SK}(n, i, x)$. We denote by k_{PK} and k_{SK} the optimal purchasing policies of the model under profit maximising and survival probability respectively.

2.3 Model with choice of keeping or returning inventory

This model generalises the previous ones and allows a choice between returning or keeping (some or all of the) unsold items. At the end of a period, when there are items left in stock (i.e. if $i + k - d \geq 0$), a decision must be taken to return j items to the wholesaler. After suitable modification to C, S and R, the only effect of each holding cost scenario is to reduce the profit by ih. Scenario 2 requires a modification to the purchase cost $(C \to C + h)$. Scenario 3 requires modification to the purchase cost $(C \to C + h)$ and the selling price $(S \to S + h)$. Scenario 4 requires modification to the purchase cost $(C \to C + h)$, the selling price $(S \to S + h)$ and the return price $(R \to R + h)$. The following formulations therefore cover all four holding cost scenarios. Table 1 summarises the required changes to move between the four scenarios in each of the models. Again for an established firm looking to maximise the average profit per period, denote by g_{PD} the average profit, and by $v_{PD}(i)$ the bias term of starting with components in stock. Then, the optimality equation of the dynamic programming model for this situation is

$$g_{PD} + v_{PD}(i) = \max_k \left\{ \sum_{d=0}^{M} \max_{0 \le j \le i+k-d} p(d) \Big(S \min(i+k, d) \right.$$

$$\left. + Rj - Ck - H - hi + v_{PD}(\max(i+k-d-j, 0)) \Big) \right\}. \quad (5)$$

Similarly, if $q_{SD}(n, i, x)$ is the probability of surviving n more periods with i items in inventory and x capital, then it satisfies the following dynamic programming optimality equation:

$$q_{SD}(n, i, x) = \max_k \left\{ \sum_{d=0}^{M} \max_{0 \le j \le i+k-d} p(d) q_{SD} \Big(n-1, \max(i+k-d-j, 0), \right.$$

$$\left. x + S \min(i+k, d) + Rj - Ck - H - hi \Big) \right\} \quad (6)$$

where $q_{SD}(0, i, x) = 1$ if $x \ge 0$ and $q_{SD}(n, i, x) = 0$ for any n and $x < 0$. Let $q(i, x)$ be the probability of survival in the long run with inventory i and capital x, then $q_{SD}(i, x) = \lim_{n \to \infty} q_{SD}(n, i, x)$. We denote by k_{PD} and k_{SD} the optimal purchasing policies of the model under profit maximising and survival probability respectively.

Model	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Must return	0	$C \to C + h$	$C \to C + h$ $S \to S + h$	0
Must keep	$-ih$	$-ih$ $C \to C + h$	$-ih$ $C \to C + h$ $S \to S + h$	$-ih$ $C \to C + h$ $S \to S + h$
Keep or return	$-ih$	$-ih$ $C \to C + h$	$-ih$ $C \to C + h$ $S \to S + h$	$-ih$ $C \to C + h$ $S \to S + h$ $R \to R + h$

Table 1: Modelling the different holding cost scenarios

3   Properties of the profit maximising objective

In this section we analyse the average profit maximising policies for the different models introduced in the previous section. Let us first state the optimal ordering policy for the model where we must return all unsold items at the end of the period. Throughout we will define $\bar{d}(i) = \sum_{d=0}^{M} p(d) \min\{i, d\}$ which is the expected sales if one has i items available. Moreover we define $P(i) = \sum_{d=0}^{i-1} p(d)$ and $\bar{P}(i) = 1 - P(i)$ to be the cumulative demand distributions. Note that $\bar{d}(i) - \bar{d}(i-1) = \bar{P}(i)$.

Theorem 3.1
The optimal average profit g_{PR} and the ordering policy k_{PR} satisfy

$$g_{PR} = (S - R)\bar{d}(k_{PR}) - (C - R)k_{PR} - H$$

$$k_{PR} = i_{PR}^* = \max\left\{ i | \bar{P}(i) \ge \frac{C - R}{S - R} \right\} = \max\left\{ i | P(i) \le \frac{S - C}{S - R} \right\} \quad (7)$$

Proof:
See [7] Chapter 4 or alternatively note that the k_{PR} defined does maximise R.H.S. of (1).   ◇

In other words, the order quantity should be increased up to the point where the difference between selling price and purchase price is less than or equal to the difference between the selling price and return price times the probability that the last additional unit cannot be sold. As the cumulative distribution function is non-decreasing, it follows that the optimal order increases if the profit or the return cost increases, and decreases if the purchasing cost increases. Note that if d has a discrete distribution with equal probability $(1/b+1)$ of each demand between 0 and b, which in future is denoted by $U(0,b)$ then $k_{PR} = \left\lfloor \dfrac{(b+1)(S-C)}{S-R} \right\rfloor$ where $\lfloor x \rfloor$ means the integer part of x.

Now focus on the model where we must keep all unsold items.

Theorem 3.2

The optimal average reward, g_{PK}, bias terms, $v_{PK}(i)$, and optimal ordering policy, $k_{PK}(i)$ which satisfy the average reward model of (3) are

$$
\begin{aligned}
g_{PK} &= (S - C + h)\bar{d}(i_{PK}^*) - H - h i_{PK}^* \\
v_{PK}(i) &= \begin{cases} (C-h)i \text{ for } i \leq i_{PK}^* \\ S\left(\bar{d}(i) - \bar{d}(i_{PK}^*)\right) - h\left(i - i_{PK}^*\right) + (C-h)\bar{d}(i_{PK}^*) + \displaystyle\sum_{d<i} p(d)v(i-d) \text{ for } i > i_{PK}^* \end{cases} \quad (8) \\
k_{PK}(i) &= \begin{cases} i_{PK}^* - i & i \leq i_{PK}^* \\ 0 & i > i_{PK}^* \end{cases} \text{ where } i_{PK}^* = \max\left\{ i | \bar{P}(i) \geq \frac{h}{S-C+h} \right\} \quad (9)
\end{aligned}
$$

Proof:

The proof uses the policy iteration algorithm for dynamic programming models (see [6]). First we substitute the values of (8) into (3) to confirm that they satisfy that equation if the policy used is that in (9). Then we apply the policy improvement step to check what maximises the R.H.S of (3) if the values of (8) are substituted for g_{PK} and $v_{PK}(i)$. It is straight forward to confirm that the maximisation is given by the policy in (9), which confirms this is the optimal policy.

$\diamond$

Turning to the problem where one can decide how much to return, intuitively there are two courses of action. Either one keeps an existing item and so pays the holding cost h, or one returns it and then buys a new one next period, which costs $C - R$. This is true no matter how many items are in stock and so one would expect if $h > C - R$ one should return all the items each period, while if $h < C - R$ one should not return any items, unless one is far above the ideal inventory. If $h = C - R$, then it does not matter what one does and so all return policies are optimal. This intuition is confirmed in the following theorem.

Theorem 3.3

If g_{PD} is the optimal average profit for the model where there is a choice on how much inventory to keep and how much to return at the end of each period, and if j, $0 \leq j \leq \max\{0, i+k-d\}$ is the amount to return, then

  i) If $C < R + h$, $g_{PD} = g_{PR}$ and $j = \max\{0, i+k-d\}$ so that all items are returned at the end of each period;

  ii) If $C = R + h$, $g_{PD} = g_{PR} = g_{PK}$ and any j is optimal;

  iii) if $C > R + h$, $g_{PD} = g_{PK}$ and, when $i+k-d < i_{PK}^*$, $j = 0$ so no items are returned at the end of each period.

Proof:

i) If $C < R+h$, suppose the optimal policy π^* has average reward g_{PD}. The proof is by contradiction so assume for some i, k and d, the policy π^* chooses to return j^* where $j^* < i+k-d$. Consider the policy $\tilde{\pi}$ which at this point returns j^*+1 and at the next period orders one more than π^*. Thereafter $\tilde{\pi}$ follows policy π^*. It is easy to see that $\tilde{\pi}$ has an immediate extra reward of $R+h-C$ in this state and no change in other states. Thus it is clear that applying policy iteration to π^* will find a better policy and hence π^* cannot be optimal.

iii) The result follows in exactly the same way by assuming the j^* in the optimal policy π^* satisfies $j^* > 0$ and then showing that the policy $\tilde{\pi}$ which returns j^*-1 and then buys one less than π^* at the next period gives an extra total profit of $C-R-h$ more than π^*. Note that when $i+k-d-j < i_{PK}^*$, π^* orders at least one item next period. As mentioned above if i is much larger than i_{PK}^*, it may be optimal to return some items at the end of the period.

ii) If $C = R+h$ then from (7) and (8), $i_{PK}^* = k_{PR}$ and hence $g_{PR} = g_{PK}$. It is trivial to see that the return quantity j does not effect the average reward since it is the same cost whether one keeps an item or sells it off and buys it back again the next period.

$\diamond$

Note that in the decision case the optimal average reward and the optimal order-up-to level are whichever of the must keep or must return average rewards and order-up-to levels are higher.

Corollary 3.1

   i) $g_{PD} = \max\{g_{PR}, g_{PK}\}$

   ii) $k_{PD}(i) = \begin{cases} i_{PD}^* - i & i \le i_{PD}^* \\ 0 & i > i_{PD}^* \end{cases}$ where $i_{PD}^* = \max\{i_{PK}^*, i_{PR}^*\}$

Proof:

i) Since the decision problem includes the must keep and must return cases $g_{PD} \ge \max\{g_{PR}, g_{PK}\}$. Theorem 3.3 then ensures there is equality here.

ii) This follows immediately from Theorem 3.3 and the definitions of i_{PR}^* in (7) and i_{PK}^* in (8).

$\diamond$

The results in this section help us characterise the behaviour of an established company following a profit maximising objective. In the following sections we focus on the survival probability objective which we argue is appropriate for a start-up company. This will give the opportunity to examine the conditions where it is sensible to be more or less conservative in strategy when one is a start-up company than when one is an established one.

4   Properties of the survival probability objective

In this section if we wish to discuss all three models at the same time we use the notation $q_{S\cdot}$ to represent q_{SK}, q_{SR} and q_{SD}. Note that the "must return" model is a special case of the "decision" model. Whatever the choices of R, S, C, H and $p(d)$ in the must return model, if we take the same parameter values in the decision model and let $h \to \infty$, we can guarantee that the optimal return policy will return all the stock at the end of each period, i.e. will become the must return case. Such an argument does not work for the "must keep" case, because if we let $R \to 0$ in the decision case, the holding cost may still be sufficiently large that one will want to return some if not all of the unsold items. However it is clear that any results we prove for q_{SD} also hold for q_{SR} and conversely any counter example for q_{SR} is also a counter example for q_{SD}.

Our first result is to show that $q_{S.}(n, i, x)$ is non-decreasing in n, the number of periods to go. This intuitively reasonable result will then prove the existence of $q_{S.}(i, x)$, the infinite horizon survival probability.

Lemma 4.1

i) $q_{S.}(n, i, x) \geq q_{S.}(n+1, i, x)$ for all $n, x \geq 0$ and, in the SK and SD case, $i \geq 0$.

ii) $q_{S.}(i, x) = \lim_{n \to \infty} q_{S.}(n, i, x)$ exists.

Proof:

i) The proof is by induction on n and we will prove it in the SD case first. Since $q_{SD}(0, i, x) = 1$ when $x \geq 0$ and $q_{SD}(1, i, x) \leq 1$ when $x \geq 0$, the hypothesis holds for $n = 0$. Assuming it holds for n compared with $n - 1$, then from (6):

$$q_{SD}(n+1, i, x) = \max_k \left\{ \sum_{d=0}^M \max_{0 \leq j \leq i+k-d} p(d) q_{SD} \left(n, \max(i+k-d-j, 0), \right. \right.$$

$$\left. x + S \min(i+k, d) + Rj - Ck - H - hi \right) \Big\}$$

$$\leq \max_k \left\{ \sum_{d=0}^M \max_{0 \leq j \leq i+k-d} p(d) q_{SD} \left(n-1, \max(i+k-d-j, 0), \right. \right.$$

$$\left. x + S \min(i+k, d) + Rj - Ck - H - hi \right) \Big\} = q_{SD}(n, i, x)$$

So the relationship goes through and this also establishes the result for q_{SR}. A similar proof holds for q_{SK}.

ii) follows immediately since one has a non-increasing sequence bounded below by zero.

$\diamond$

Note that since the decision case includes all the actions allowed in each of the other two cases, if one has the same cost and demand parameters and equivalent starting positions in all three cases, the decision case must have a higher probability of survival. This observation is formalised in the following lemma.

Lemma 4.2
For all $x \geq 0$ and $i \geq 0$,

$$q_{SD}(i, x) \geq \max \left\{ q_{SK}(i, x), q_{SR}(x + (R - h)i) \right\} \tag{10}$$

Proof:
The optimal policy in the must keep model is a feasible policy in the decision model, so $q_{SD}(i, x) \geq q_{SK}(i, x)$. From state (i, x) in the decision model, consider the decision to order up to k items and to return all unsold items at the end of the period. Compared to ordering k items in state y in the must return model, this results in additional revenue of

$$-hi + C \min(i, k) + S \max(\min(i, d) - k, 0) + R \max(i - \max(k, d), 0)$$

at the end of the period when the demand during the period is d. So each item in inventory at the beginning of the period costs an additional h due to the holding cost, but generates additional revenue of at least R. Hence from state (i, x) in the decision model, the policy of ordering up to

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$k_{SR}(x + (R - h)i)$ items and then following an optimal policy for the must return model results in a survival probability of at least $q_{SR}(x + (R - h)i)$.

$\diamond$

We are now in a position to prove the connection between the objective of maximising the average profit and maximising the survival probability. The next theorem shows that in all three models, if the maximum expected profit is positive then there is a positive chance of survival no matter how little capital or inventory one starts with.

Theorem 4.1

  i) If $g_{PR} > 0$ then $q_{SR}(x) > 0$ for all $x \geq 0$.

 ii) If $g_{PK} > 0$ then $q_{SK}(i, x) > 0$ for all $x \geq \max\{0, (h - C)i\}$ and $i \leq M$.

iii) If $g_{PD} > 0$ then $q_{SD}(i, x) > 0$ for all $x \geq \max\{0, (h - C)i\}$ and $i \leq M$.

Proof:
i) Let k^* be the profit maximising order quantity for the must return model, equation (1), and let $q^k(x)$ be the long term survival probability when ordering k units. We have that

$$q(x) \geq q^{k^*}(x) = \sum_{d=0}^{M} p(d)q^{k^*}(x + S\min(k^*, d) + R\max(k^* - d, 0) - Ck^* - H).$$

This is a difference equation, so the solution is of the form $q(x) = Aa^x$. Substituting in the above gives:

$$Aa^x = \sum_d p(d)Aa^{(x + S\min(k^*, d) + R\max(k^* - d, 0) - Ck^* - H)}.$$

To solve this let

$$f(a) = \sum_d p(d)a^{(S\min(k^*, d) + R\max(k^* - d, 0) - Ck^* - H)} - 1$$

and seek a such that $f(a) = 0$. We have that $f(1) = 0$, $f(0) = \infty$, since for $d = 0$: $(R - C)k^* - H < 0$, so there are negative powers of a, and $f(\infty) = \infty$, since for $d = M$: $(S - C)k^* - H > 0$, so there are positive powers of a. Now the derivative of f at 1 is:

$$f'(1) = \sum_d p(d)(S\min(k^*, d) + R\max(k^* - d, 0) - Ck^* - H) = g_{PR} > 0.$$

Hence, there exists a root of $f(a)$ satisfying $0 < a < 1$, and another one at $a = 1$, so one possible solution is:

$$q^{k^*}(x) = Aa^x + B.$$

As $q^{k^*}(-1) = 0$ and $q^{k^*}(\infty) = 1$, this leads to $q^{k^*}(x) = 1 - a^{x+1} > 0$ for all $x \geq 0$ and hence

$$q_{SR}(x) \geq q^{k^*}(x) > 0.$$

ii) For the must keep model, the profit maximising policy is to order up to i^* given by (9). Let $q^{i^*}(i, x)$ be the long term survival probability under this policy starting with inventory i and capital x, and let $q_1^{i^*}(x)$ be the long term survival probability under this policy starting with capital x when the survival boundary condition is applied after delivery of the order for a period. As in (4), we have:

$$q_1^{i^*}(x) = \sum_{d=0}^{M} p(d)q_1^{i^*}(x + S\min(i^*, d) - H - h\max(i^* - d, 0) - C\min(i^*, d)).$$

This is also a difference equation, so as before its solution is of the form $q_1^{i^*}(x) = Aa^x$, where a must satisfy

$$f(a) = \sum_{d=0}^{M} p(d) a^{(S \min(i^*, d) - H - h \max(i^* - d, 0) - C \min(i^*, d))} - 1 = 0.$$

Again $f(0) = \infty$, $f(\infty) = \infty$, $f(1) = 0$ and

$$f'(1) = \sum_{d=0}^{M} p(d)(S \min(i^*, d) - H - h \max(i^* - d, 0) - C \min(i^*, d)) = g_{PK} > 0.$$

Also $q_1^{i^*}(-1) = 0$ and $q_1^{i^*}(\infty) = 1$, so as in (i) we have $q_1^{i^*}(x) = 1 - a^{x+1} > 0$ since $0 < a < 1$.

Now consider the problem starting in state $(i, x + \max\{0, (h - C)i\})$, where $0 \leq i \leq M$ and $x \geq 0$, with the original timing (end of period) for the survival boundary condition to be checked. Assume that in the initial period one orders $M - i$ and thereafter one orders M for $n - 1$ more periods. If the demand in these n periods is M each period, which happens with probability $p(M)^n$, then in the first period the capital changes by $(C - h)i + (S - C)M - H$ and in every other period it goes up by $(S - C)M - H$. So during this interval, the end of period capital is never negative and after this interval it is at least $x + n[(S - C)M - H]$. If we choose n so that $n[(S - C)M - H] > Ci^*$ then we end up in a better position than if we had started with capital x after delivery, and hence payment, of an order of i^*. Hence

$$q_{SK}(i, x + \max\{0, (h - C)i\}) \geq p(M)^n q_1^{i^*}(x) > 0.$$

iii) If $C \leq R + h$, then from Theorem 3.3 $g_{PD} > 0 \Rightarrow g_{PR} > 0$ which from part (i) means $q_{SR}(x) > 0$ for all x. By (10) this implies $q_{SD}(i, x) > 0$ for all i and $x \geq (h - R)i \geq (h - C)i$. Similarly if $C > R + h$, then from Theorem 3.3 $g_{PD} > 0 \Rightarrow g_{PK} > 0$ and so part (ii) gives that $q_{SK}(i, x) > 0$ for $i \leq M$ and $x \geq \max\{0, (h - C)i\}$. Again the result follows from (10).

$\diamond$

Having shown that non-zero survival probabilities exist, there are some obvious features one would expect of them.

i) Additional resource, capital or inventory, might be expected to always benefit a start-up firm. Theorems 4.2 and 4.3 show this is true of capital, but not necessarily true of inventory.

ii) A start-up firm might be expected to prefer additional capital to additional inventory, as additional capital gives the firm greater flexibility. Theorem 4.4 establishes conditions under which this is true.

iii) In the problem where the start-up firm can decide how much inventory to return, it might be expected either to return everything or nothing. This is the situation if a firm wants to maximise its profit, but example 4.2 shows this is not the case if the firm wants to maximise its chance of survival.

Theorem 4.2

i) $q_{S.}(n, i, x)$ is non-decreasing in x.

ii) $q_{S.}(i, x)$ is non-decreasing in x.

Proof:

i) The proof is again by induction on n and we prove the result for the q_{SD} case first. Trivially since $q_{SD}(0, 1, x)$ equals 1 if $x \geq 0$ and 0 if $x < 0$, the result holds for $n = 0$. Assume the result holds for $q_{SD}(n, i, x)$ and assume $x' > x \geq 0$, then

$$q_{SD}(n+1, i, x)$$
$$= \max_k \left\{ \sum_d \max_j p(d) q_{SD}\left(n, \max(i+k-d-j, 0), x + S \min(i+k, d) + Rj - Ck - H - hi\right) \right\}$$
$$\leq \max_k \left\{ \sum_d \max_j p(d) q_{SD}\left(n, \max(i+k-d-j, 0), x' + S \min(i+k, d) + Rj - Ck - H - hi\right) \right\}$$
$$= q_{SD}(n+1, i, x')$$

and the induction hypothesis holds.

This also proves the result for q_{SR} and a similar argument holds for q_{SK}.

ii) follows by taking the limit as n tends to infinity. $\diamond$

The same result that "more is better" does not necessarily hold for inventory, unless you are able to return unwanted inventory. The next result with its counter example confirms this. We only state the result for the infinite horizon case, but the same proof holds for a finite time horizon.

Theorem 4.3

    i) If $R \geq h$, $q_{SD}(i, x)$ is non-decreasing in i. (Note this condition is always satisfied for holding cost scenario 4.)

    ii) If $R < h$, $q_{SD}(i, x)$ is not necessarily non-decreasing in i.

    iii) $q_{SK}(i, x)$ is not necessarily non-decreasing in i.

Proof:

i) Let π^* be the optimal policy for maximising the long-term survival probability. Suppose when we start in state (i, x), π^* orders k^* and, if at the end of the first period there are $i + k^* - d \geq 0$ in stock, returns $j^*(d)$. Consider the policy π which, when we start in state $(i + 1, x)$, orders k^*, but if at the end of the period there are $i + 1 + k^* - d > 0$ in stock, returns $j^*(d) + 1$, and thereafter follows π^*. The capital available at the end of the first period from π starting in $(i + 1, x)$ is greater than that from π^* starting in state (i, x) by $R - h$ if $d \leq i + k^*$ (one more to pay a holding cost on, but one more item returned) and $S - h$ if $d > i + k^*$ (one more to pay a holding cost on, but one more item sold). However the inventory level in both cases is the same. So by Theorem 4.2 (ii), π starting in $(i + 1, x)$ will give at least as high a chance of survival as the optimal policy starting in (i, x), and the result holds.

ii) If $R < h$, there are many examples where the extra inventory at the start is counter productive. Take the deterministic example with $S = 10$, $C = 0$, $H = 5$, $R = 0$, $h = 5$ and $p(1) = 1$. Under holding cost scenario 1, 2 or 3, $q_{SD}(1, 0) = 1$, since in the first period the holding cost of 5 and the overhead cost of 5 do not exceed the sales revenue of 10, and thereafter a policy of ordering one item each period gives a profit of 5 per period. However $q_{SD}(2, 0) = 0$, since the sales revenue of 10 does not cover the holding cost of 10 and the overhead of 5 in the first period.

iii) Similarly for the "must keep" case there are many counter examples to the monotonicity of the survival probability in i. For example Figure 1 describes the survival probabilities $q_{SK}(i, 0)$ for Problem A below which has $S = 10$, $C = 7$, $H = 5$, $R = 4$, $h = 2$ and $p(d) \sim U[0, 19]$. $\diamond$
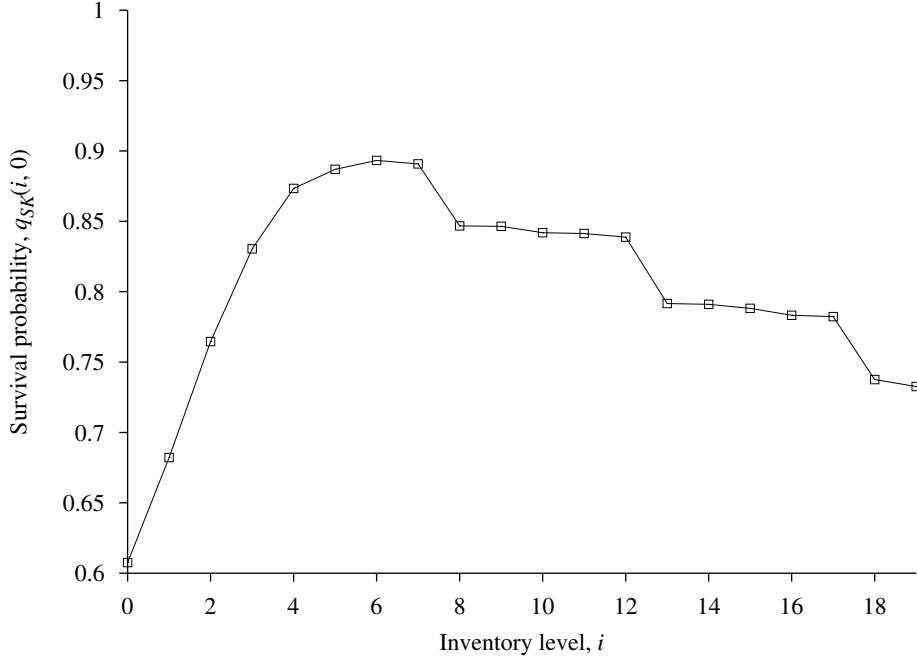
Figure 1: Survival probability as a function of inventory for the must keep model and Problem A

Even though having more inventory is not necessarily an advantage having capital always is. Moreover the next result says you would prefer to have capital free rather than tied up in inventory in both the decision and the must keep situations. Since the must return case has no inventory at the end of a period, such a result does not make sense in that case.

Theorem 4.4 [Start-up firms prefer capital to inventory]
For the survival models in which inventory can be carried over from one period to the next:

$$q_{S\cdot}(i + j, x) \leq q_{S\cdot}(i, x + (C - h)j) \text{ for all } j, x > 0.$$

Proof:
If in state $(i + j, x)$, the optimal policy is to order k^* then, immediately after the arrival of the order, there will be inventory of $i + j + k^*$ and capital of $x - Ck^* - h(i + j)$. Of course income from sales and returns is expected to increase the amount of capital by the end of the period. In state $(i, x + (C - h)j)$, if $k^* + j$ items are ordered then, again when the order arrives, there will be inventory of $i + j + k^*$ and capital of $x + (C - h)j - C(k^* + j) - hi = x - Ck^* - h(i + j)$. So the position is exactly as with the optimal policy from $(i + j, x)$, but this order need not be optimal for the state $(i, x + (C - h)j)$ and so the result holds.

$\diamond$

The following examples provide counter examples to a series of intuitively appealing results that one might imagine hold for the various survival and profit maximising models presented in this paper. The examples are based on the following three problem instances.

Problem A: $S = 10$, $C = 7$, $H = 5$, $R = 4$, $h = 2$ and $p(d) \sim U[0, 19]$.

Problem B: As Problem A except that $h = 0$, so $S = 10$, $C = 7$, $H = 5$, $R = 4$, $h = 0$ and $p(d) \sim U[0, 19]$.

Problem C: $S = 11$, $C = 9$, $H = 2$, $R = 0$, $h = 5$ and $p(d) \sim U[0, 19]$.

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Although the set of problems presented here is limited, we have observed similar behaviour in many other problems in the course of our research.

Example 4.1
There is no simple relationship between $q_{SR}(x)$ and $q_{SK}(0, x)$. In some situations the firm has a greater chance of survival when it is forced to return unsold items at the end of a period, while in others it has a greater chance of survival when it must keep items. The model that gives the greater chance of survival is not even determined entirely by the problem parameters, as it can also be influenced by the amount of capital available to the firm. This is demonstrated in Figure 2 which shows the probability of survival as a function of the capital available for the three survival models and Problem B. When $x \leq 18$, $q_{SR}(x) > q_{SK}(0, x)$, but when $x > 18$, $q_{SK}(0, x) > q_{SR}(x)$. It seems that, in this case, there is a threshold capital above which the firm would be willing to sacrifice the cash obtained from a return for the opportunity to sell the item in the future at a higher price. We have observed this behaviour in many other examples.

When $C > 0$ and $h > 0$ in the decision model, it is clear that the myopic decision which only looks one period ahead would return everything while, if $C < R + h$, Theorems 3.1 and 3.2 imply that profit is maximised by keeping all items. When one has little capital decisions have to be somewhat myopic, while with lots of capital, maximising survival is more akin to maximising profit. Thus it is not surprising that we find this change in survival probabilities, where $q_{SR}(x) > q_{SK}(0, x)$ for small x and $q_{SR}(x) < q_{SK}(0, x)$ for large x, occurs in many cases.

Figure 2 also illustrates the result of equation (10) which says that the survival probability under the decision model is at least as great as that under the other two models.
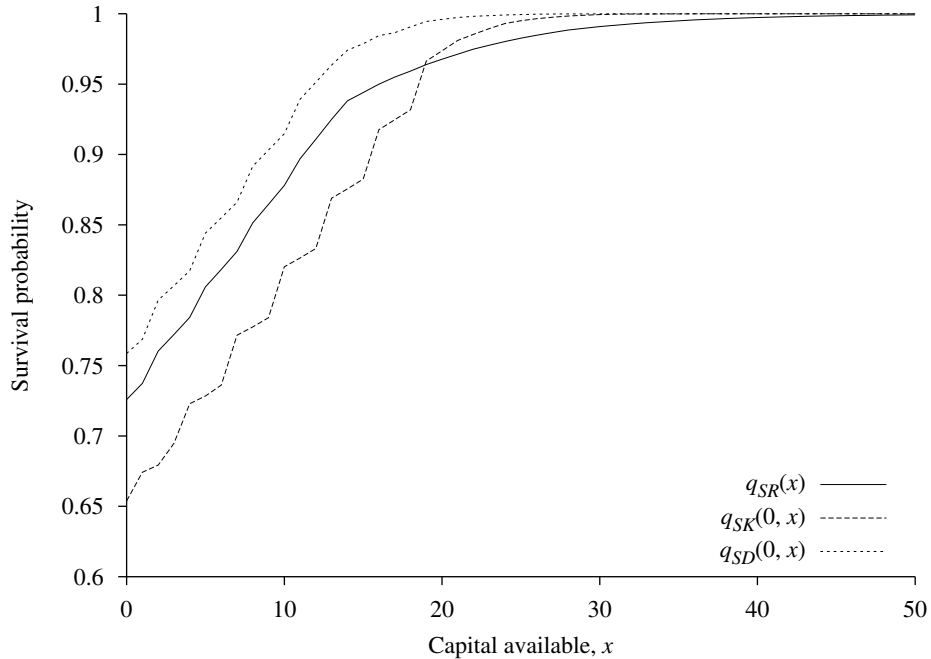

Figure 2: Survival probability as a function of capital available for all three models and Problem B with initial inventory 0

Example 4.2
With the decision model, situations exist under the survival objective where the optimal return policy neither returns all or keeps all unsold items. This is demonstrated in Figure 3 which shows the optimal return quantity as a function of the capital available at the beginning of a period when the inventory level is 5 and no demand occurs. The optimal order quantity is zero regardless of the capital available,
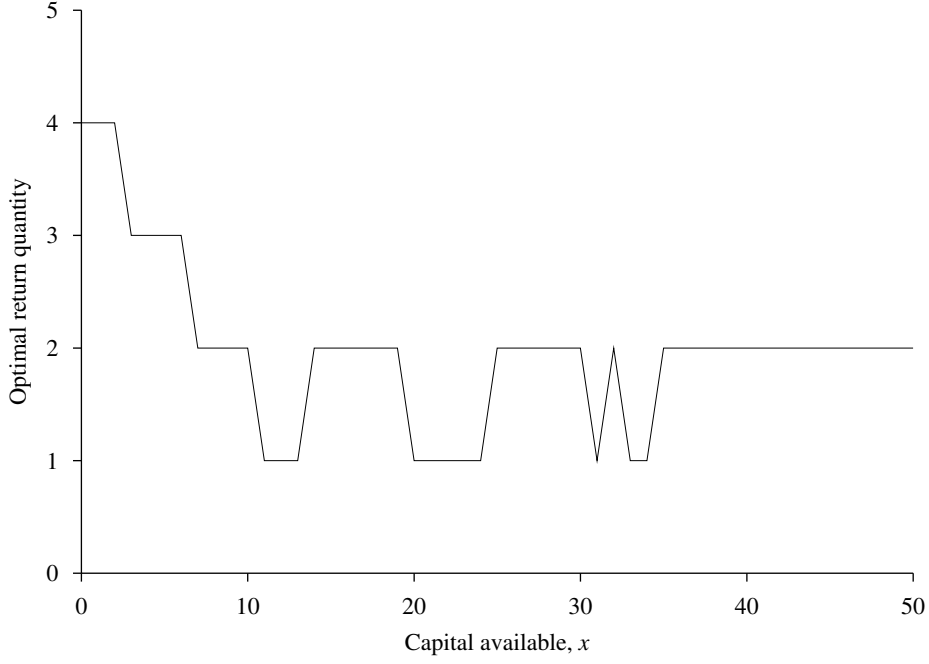
Figure 3: Optimal return quantity under the survival objective as a function of capital available for Problem A with initial inventory 5 and demand 0

so the return quantity must always lie between 0 and 5. When the capital available initially is low it is optimal to return most, but not all of the unsold items. Initially the optimal return quantity gradually falls as the capital available increases. In fact when $x \leq 13$, the optimal policy returns the minimum number of items required to raise the capital at the end of the period to at least 0. This explains the stepwise decrease in the optimal return quantity. When $x > 13$, the optimal policy returns more items than can be explained simply by the need to survive in the current period. We also see that in this case, the optimal return quantity is not monotonic in the amount of capital available.

Note that in Figure 3, it is never optimal to return all or none of the unsold items. As illustrated in Table 2, this is not always the case for the decision model under the survival objective. With initial capital 15 and initial inventory 0, it is optimal to order 3 and return all unsold items, one unsold item or none of the unsold items depending on whether the demand during the period is 0, 1 or 2 respectively.

Demand, d	Optimal return quantity, j
0	3 out of 3
1	1 out of 2
2	0 out of 1

Table 2: Optimal return quantity under the survival objective as a function of demand for Problem A with initial capital 15, initial inventory 0 and optimal order quantity 3

This example shows that the return decision in the survival model is not trivial and there are cases with the same problem parameters where it is optimal to return all, some or none depending on the capital available to the firm. This contrasts with the model under a profit maximising objective for which it is optimal either to keep or return all unsold items depending on the relative values of certain problem parameters (Theorem 3.3).

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Example 4.3

There is no simple relationship between the optimal order quantities for the three models under the survival objective, i.e. k_{SR}, k_{SK} and k_{SD}. This is demonstrated in Figure 4 which shows the optimal order quantity as a function of the capital available for the three survival models and Problem B. The model giving rise to the largest order quantity depends on the capital available to the firm. For example if $x = 20$, k_{SR} is larger than both k_{SK} and k_{SD}, if $x = 1$, k_{SK} is larger than the other two and if $x = 60$, k_{SD} is larger than the other two. As the capital available increases, all three models seem to be approaching an optimal order quantity of 3. This suggests that, in this case, the impact of the revenue from returns on the long-run survival probability decreases as the capital available to the firm increases, and so the distinction between the three model lessens. This is also apparent in the plot of survival probability in Figure 2 which shows that in all three cases the survival probability approaches 1 as the capital available increases. It would be useful to be able to characterise the stable order policy, but all we can say from our numerical experiments is that it appears to be less than or equal to (usually strictly less than) the profit maximising policy, but it is larger than the policy that minimises the probability of making a loss during a period (which for Problem B is to order up to 2 items).

With all three models, the optimal order quantity displays a downward trend as the capital available increases, but in no case is the optimal order quantity monotonic in the capital available. The non-monotonic behaviour often appears to be due to short term survival effects. For example k_{SR} equals 3 when $x = 10$ and 14, but equals 2 when $10 < x < 14$. Under the must return model when $x = 10$, the firm will fail in the next period if it does not sell at least one item regardless of whether it orders
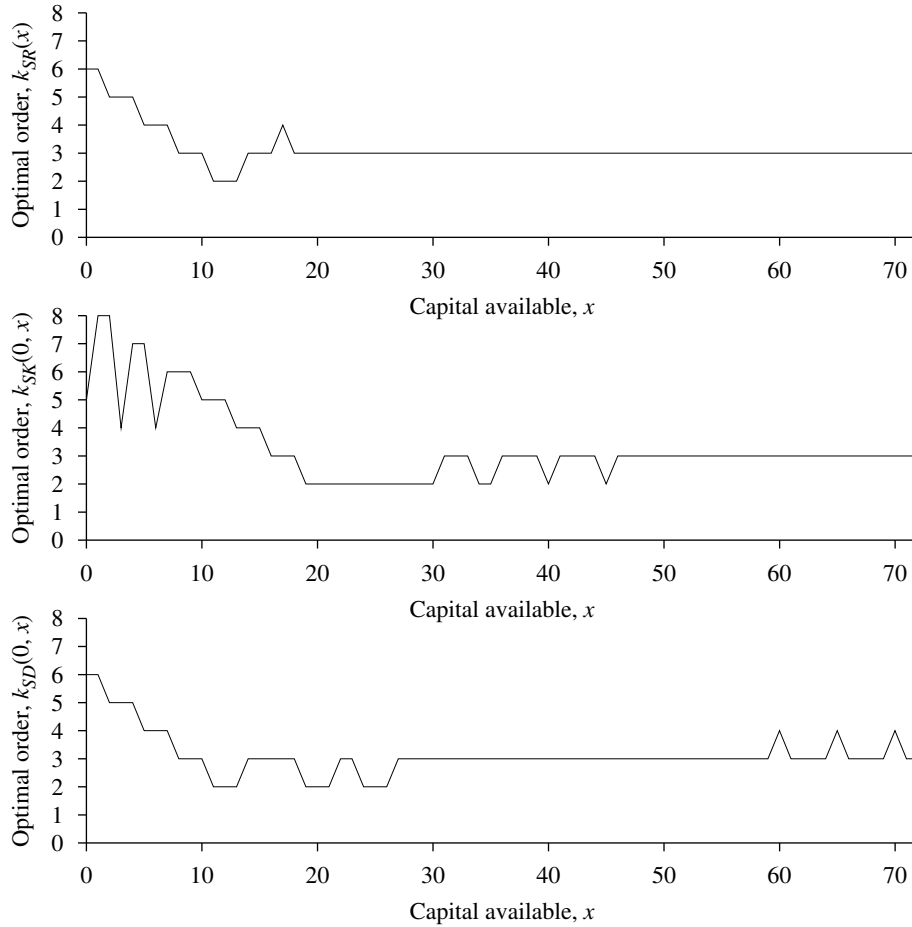

Figure 4: Optimal order under the survival objective as a function of capital available for all three models and Problem B with initial inventory 0

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2 or 3 items. However when $10 < x < 14$, the firm can survive the next period without selling any items if it orders 2 items, but still needs to sell at least one item if it orders 3 items. While if $x = 14$, the firm can survive the next period without selling any items regardless of whether it orders 2 or 3 items. It appears that when $10 < x < 14$, the greater potential for revenue generation when the firm purchases 3 items does not compensate for the greater risk of immediate failure.

Provided the average reward per period remains positive, varying the fixed overhead cost H has no effect on the profit maximising policy. This is not true under the survival objective. Reducing the fixed overhead cost not only improves the chance of survival, it also tends to make the optimal order policy for all three models more conservative when the capital available is low. These observations suggest that as the situation facing the start-up firm becomes harder (e.g. higher overhead costs or, see example 4.4, lower return values or higher holding costs), the firm may have to take riskier decisions when the capital available is highly restricted in order to maximise its chance of survival.

For Problem B, when the initial inventory is 0 and the capital available is within the range shown in Figure 4, the optimal order quantity under the survival objective is unique. However as the capital available increases beyond this range, the models start to have multiple optimal policies and eventually all order quantities give the same probability of survival. This happens with $x \geq 310$ for the must return case, $x \geq 188$ for the must keep case and $x \geq 119$ for the decision model. By this stage the firm has acquired enough capital that its survival is practically ensured regardless of what happens in the next period. As long as the capital available to the firm remains above this level, the firm should focus on profit maximisation.

Example 4.4

There is no simple relationship between k_P and k_S. For Problem C, $\bar{P}(i) = 1 - \dfrac{i}{20}$, $\dfrac{C - R}{S - R} = \dfrac{9}{11}$ and $\dfrac{h}{S - C + h} = \dfrac{5}{7}$. Hence using the results of Section 3, $k_{PR}^* = 3$ and $k_{PK}^*(0) = k_{PD}^*(0) = 5$. Figure 5 compares these optimal order quantities with the optimal order quantities under the survival objective for varying levels of capital. It turns out that for Problem C, the optimal order quantities for the must keep and decision models are the same for all levels of capital.
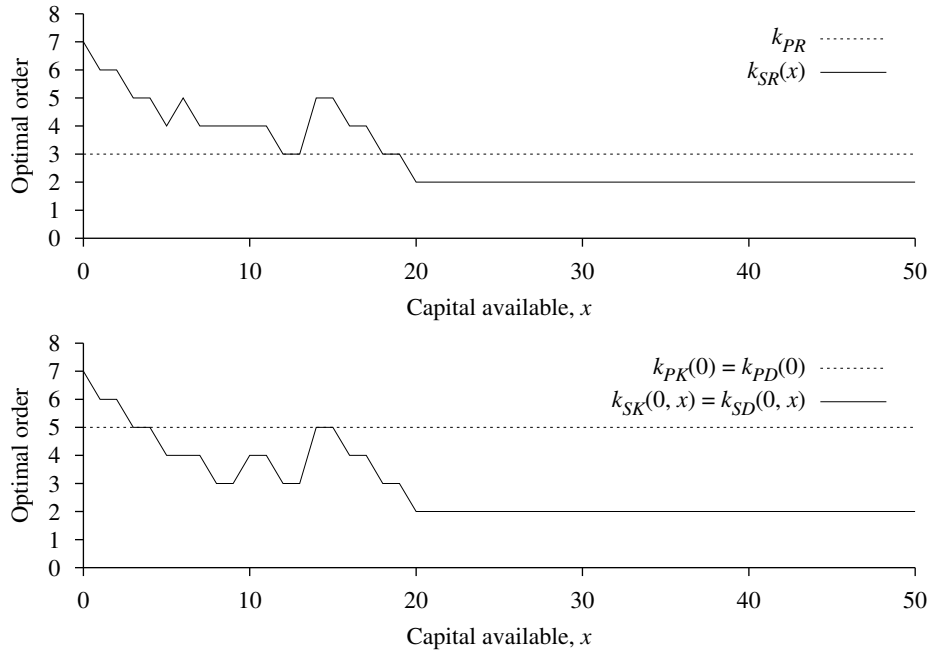

Figure 5: Optimal order under both objectives as a function of capital available for all three models and Problem C with initial inventory 0

Note that when there is enough capital available to the firm ($x \geq 20$), $k_{PR} > k_{SR}(x)$, but when $x \leq 11$ we have that $k_{PR} < k_{SR}(x)$. This example suggests that a start-up firm which has to return any unsold items, should order more when it has little capital than an established firm looking at maximising average profit, making it riskier than an established firm. However, when it has enough capital it is more conservative and orders fewer items than an established firm looking to maximise the average profit. However this behaviour seems to happen for cases where the return price is very low. As the return price increases, we have observed that the optimal order quantity under the survival objective becomes more conservative than the profit maximising policy for any level of capital.

This example also shows that a start-up firm which has to keep any unsold items, may order more when it has little capital than an established firm looking at maximising average profit. When ($x \leq 2$), $k_{PK}(0) < k_{SK}(0, x)$, but when $x \geq 5$ we have that $k_{PK}(0) > k_{SK}(0, x)$. This behaviour seems to happen for cases where the holding cost is relatively high. In fact in Section 5, we show that if $h = 0$, $k_{PK}(i) \geq k_{SK}(i, x)$ for all i and x.

5 Properties of the survival probability objective when there is no holding cost ($h = 0$)

If there is no holding cost then, in the model where we must keep unsold items, one expects the optimal survival probability to be non-decreasing in i. This property is confirmed in the following theorem.

Theorem 5.1
If $h = 0$:

   i) $q_{SK}(n, i, x)$ is non-decreasing in i.

   ii) $q_{SK}(i, x)$ is non-decreasing in i.

Proof:
i) The proof is by induction on n. Since $q(n, i, x) = 0$ when $x < 0$ for all n, $q(0, i, x) = 1$ when $x \geq 0$ and $q(1, i, x) \leq 1$ when $x \geq 0$, the hypothesis holds in the case $n = 0$. Assume the hypothesis holds for n, and use $\max_i\{a_i\} - \max_i\{b_i\} \leq \max_i\{a_i - b_i\}$ to show:

$$q_{SK}(n + 1, i, x) - q_{SK}(n + 1, i + 1, x) \leq$$
$$\max_k \left\{ \sum_{d=0}^{i+k} p(d) \left[q_{SK}(n, i + k - d, x + Sd - Ck - H) - q_{SK}(n, i + 1 + k - d, x + Sd - Ck - H) \right] \right.$$
$$\left. + \sum_{d=i+k+1}^{M} p(d) \left[q_{SK}(n, 0, x + S(i + k) - Ck - H) - q_{SK}(n, 0, x + S(i + 1 + k) - Ck - H) \right] \right\}$$
$$\leq 0 \text{ by inductive hypothesis and Theorem 4.2.}$$

Hence the result holds for $n + 1$.

ii) follows immediately by taking the limit in (i).

$\diamond$

Since the holding cost does not influence the behaviour of the must return model, the example in Figure 5 still applies and so there is no simple relationship between k_{PR} and $k_{SR}(x)$ even when $h = 0$. We now focus on the must keep and decision models and show that, when there is no holding cost, the optimal ordering policies under the profit maximising objective dominate those under the survival objective.

Theorem 5.2 [For must keep and decision models, survival policies are more cautious]

If $h = 0$ the ordering policies in the must keep and decision models are such that:

   i) $k_{SK}(i,x) \leq k_{PK}(i)$ for all x.

   ii) $k_{SD}(i,x) \leq k_{PD}(i)$ for all x.

Proof:

Theorem 3.2 and Corollary 3.1 show that when $h = 0$, $k_{PK}(i) = k_{PD}(i) = \begin{cases} M - i & \text{if } i \leq M \\ 0 & \text{if } i > M \end{cases}$.

Consider state (i,x) and let $k^* = i + k_{PK}(i) = i + k_{PD}(i)$.

i) If the order placed in state (i,x) is $k_{PK}(i) + A$, where $A > 0$, the resulting survival probability is equal to

$$\sum_{d=0}^{M} p(d) q_{SK}(k^* + A - d, x + Sd - C(k_{PK}(i) + A) - H) \leq \sum_{d=0}^{M} p(d) q_{SK}(k^* - d, x + Sd - Ck_{PK}(i) - H)$$

by Theorem 4.4. Note that the right hand side of this inequality equals the survival probability when the order in state (i,x) is $k_{PK}(i)$. Hence $k_{SK}(i,x) \leq k_{PK}(i)$.

ii) By a similar argument, if the order placed in state (i,x) is $k_{PD}(i) + A$, where $A > 0$, the resulting survival probability is equal to

$$\sum_{d=0}^{M} p(d) \max_{0 \leq j(d) \leq k^*+A-d} q_{SD}(k^* + A - d - j(d), x + Sd + Rj(d) - C(k_{PD}(i) + A) - H)$$

$$\leq \sum_{d=0}^{M} p(d) \max_{0 \leq j(d) \leq k^*-d} q_{SD}(k^* - d - j(d), x + Sd + Rj(d) - Ck_{PD}(i) - H)$$

which equals the survival probability when the order in state (i,x) is $k_{PD}(i)$, and the result follows. The inequality holds because, from Theorems 4.2 and 4.4 and the fact that $R \leq C$, $q_{SD}(i, x + jC) \geq \max\{q_{SD}(i + j, x), q_{SD}(i, x + jR)\}$.

$\diamond$

An obvious corollary is:

Corollary 5.1

If $h = 0$, $k_{SK}(i,x) \leq k_{PD}(i)$ for all x.

Proof:

Follows immediately from Theorem 5.2 and Corollary 3.1.

$\diamond$

As the holding cost is zero in Problem B, Examples 4.1 and 4.3 still apply. Hence even when $h = 0$, there is no simple relationship between $q_{SR}(x)$ and $q_{SK}(i,x)$ nor between the optimal order quantities for the three models under the survival objective. It is interesting to note that for the must return model there is a capital level which gives a smaller probability of survival than the must keep model. It seems to indicate that if there is enough capital available to the company, the company is willing to sacrifice the cash obtained from a return to wait and sell the item in the future at a higher price.

6   Conclusions

We have extended and analysed a zero-lead time version of the model presented in [1]. We have shown that in this case the ordering policy is not necessarily more cautious when a company focuses

on survival probability rather than on average profit. This effect appears to occur most often when the capital available to the company is low. Similar behaviour is encountered when the model is generalised to allow a choice of returning part or all of the unsold inventory at the end of a period. In this case it seems that lower return prices also force a start-up company to become riskier in its purchasing strategy. The model characterised by equation (2) represents an extension of the popular newsvendor problem that focuses on long-term survival rather than profit. Again it is interesting to note that, when the available capital and the return price are low, the optimal survival purchasing strategy may be riskier than the optimal profit maximising one. For higher levels of capital, all three models seem to suggest that a firm following a survival objective tends to be more cautious in its ordering policy that a firm aiming to maximise average profit.

We have given optimal ordering policies under the profit maximising objective, and given conditions under which these policies are equivalent for different models, as well as conditions where simple return policies are optimal, Theorem 3.3. We have shown that if it is possible to make a profit in the long run, then there is a positive probability of surviving in the long run.

Although the models presented in this paper are simple ones, they provide a framework to study and give insight about the relationships between inventory control and long-term survival for firms with considerable capital restrictions. One expects that such an analysis will aid the decision-making of start-up firms. In particular, it suggests that using profit maximisation as the sole objective might not be the best approach when there are strong capital constraints. Focusing on the long-term chance of survival might be a suitable alternative objective.

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