

# A two-echelon inventory model with lost sales

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## Abstract

This paper considers a single-item, two-echelon, continuous-review inventory model. A number of retailers have their stock replenished from a central warehouse. The warehouse in turn replenishes stock from an external supplier. The demand processes on the retailers are independent Poisson. Demand not met at a retailer is lost. The order quantity from each retailer on the warehouse and from the warehouse on the supplier takes the same fixed value  $Q$ , an exogenous variable determined by packaging and handling constraints. Retailer  $i$  follows a  $(Q, R_i)$  control policy. The warehouse operates an  $(SQ, (S - 1)Q)$  policy, with non-negative integer  $S$ . If the warehouse is in stock then the lead time for retailer  $i$  is the fixed transportation time  $L_i$  from the warehouse to that retailer. Otherwise retailer orders are met, after a delay, on a first-come first-served basis. The lead time on a warehouse order is fixed. Two further assumptions are made: that each retailer may only have one order outstanding at any time and that the transportation time from the warehouse to a retailer is not less than the warehouse lead time. The performance measures of interest are the average total stock in the system and the fraction of demand met in the retailers. Procedures for determining these performance measures and optimising the behaviour of the system are developed.

**Keywords:** Inventory, Multi-echelon, Lost sales, Batch-ordering, Poisson demand, Non-identical retailers, Iterative optimisation.